WP/15/79

IMF Working Paper

A Simple Multivariate Filter for Estimating Potential Output

by Patrick Blagrave, Roberto Garcia-Saltos, Douglas Laxton, and Fan Zhang

INTERNATIONAL MONETARY FUND

 WP/15/79

IMF Working Paper

Research Department

A Simple Multivariate Filter for Estimating Potential Output

Prepared by Patrick Blagrave, Roberto Garcia-Saltos, Douglas Laxton, and Fan Zhang[1]

Authorized for distribution by Douglas Laxton

April 2015

Abstract

Estimates of potential output are an important ingredient of structured forecasting and policy analysis. Using information on consensus forecasts, this paper extends the multivariate filter developed by Benes and others (2010). Although the estimates in real time are more robust relative to those of naïve statistical filters, there is still significant uncertainty surrounding the estimates. The paper presents estimates for 16 countries and provides an example of how the filtered estimates at the end of the sample period can be improved with additional information.

JEL Classification Numbers: C51, E31, E52

Keywords: Macroeconomic Modeling, Potential Output

Author’s E-Mail Address: PBlagrave@imf.org, RGarciaSaltos@imf.org, DLaxton@imf.org, FZhang@imf.org

[1] The estimates of potential output and the output gap presented in this paper are not official IMF estimates. The views expressed in this paper are those of the authors and do not necessarily represent those of the IMF or IMF policy. The authors would like to thank Troy Matheson, Alasdair Scott, Anke Weber, and Kevin Wiseman for helpful comments. All errors and omissions are our own.

CONTENTS

I. Introduction

Using the Okun (1962) definition of potential output, estimates of the output gap and potential output are presented for a wide range of countries using a modified version of the multivariate filter described in Benes and others (2010). This technique satisfies a few simple criteria, most of which are critical to any discussion of potential output and economic slack. First, the estimates of the output gap are economically plausible, with estimated periods of excess supply and demand coinciding closely with the priors of practitioners. Second, the filter includes some simple economic theory—specifically the structure of the filter relates the output gap to slack in the labor market and changes in inflation. Third, the filter produces more robust real-time estimates of potential and the output gap relative to estimates from an HP filter, though a certain amount of uncertainty in real-time estimates is unavoidable. Fourth, due to the minimal data requirements (GDP, inflation, and unemployment), the filter can be applied to a broad range of countries. Finally, the results can be conditioned in a transparent manner using information from outside of the model at the end of the sample—this is critical given the simplicity of the approach, and the uncertainty surrounding real-time assessment of economic slack. Still, it is important to note that the filter presented in this paper is designed to be 'least bad' among a host of mediocre choices—there is no panacea to the problem of estimating potential output.

The remainder of the paper begins with a brief review of the concept of potential output, contrasting it with concepts of 'sustainable' output recently discussed in the literature, as well as techniques commonly used to estimate potential. Following the discussion in section two, the third section presents the methodology used in this paper; detailed results are presented for two representative countries in section four. Section five presents confidence bands surrounding the estimates of potential output using the multivariate filter, and compares them to those from an HP filter. Section six concludes. Several appendices present the results for each individual country, and also explore the use of additional information at the end of the sample period to help condition estimates from the filter.

II. Potential Output—Brief Overview of Common Estimation Techniques

Potential output is generally thought of as the level of output that can be achieved without giving rise to inflation (Okun, 1962). This definition is particularly prevalent among monetary policy makers, as it allows them to communicate their policy stance in the context of the short-run tradeoff between output and inflation.[2] It is of critical importance that we be concrete in defining the concept of potential output, as this will shape how potential, and the corresponding output-gap estimates, are used by policy makers.

[2] For an example of how this tradeoff is communicated, see Bank of Canada (2009).

Although many practitioners approach potential output with the Okun definition in mind, some recent work has focused on expanding or altering this definition to include consideration of macroeconomic imbalances more broadly (see Alberoa, Estrada, and Santabarbara (2013)), as well as financial imbalances in particular (see Borio, Disyatat, and Juselius (2013)). These measures are perhaps best thought of as gauging the ***path of sustainable future*** output, rather than ***current potential*** output (in the inflation/output tradeoff sense). More specifically, these sorts of imbalances may signal the risk of a future disorderly adjustment wherein output would be substantially lower for a period of time—both the timing of such an adjustment, and whether one would ultimately occur, remain uncertain. For example, in the case of financial-sector imbalances, a strong increase in credit growth often precedes a financial crisis. However, there is no *a priori* reason why rapid credit growth needs to be unsustainable—this sort of credit expansion could equally well be the product of sound economic fundamentals. Given the difficulty of identifying the drivers of a credit expansion in real time, it would not be wise to counsel policy makers to treat all such expansions as bad; rather, these sorts of expansions should be considered carefully, and treated as increasing the (downside) risks around a given baseline. As such, we view approaches which consider financial-sector and broader macroeconomic imbalances as complements to—rather than substitutes for—the Okun concept of potential output.[3]

One of the more prevalent techniques to estimating potential is the use of univariate statistical filters, such as the Hodrick-Prescott (HP) filter, to smooth out fluctuations in output. The appeal of this approach is that it is simple, transparent, and can be applied to any country where GDP data exist. As a result, this technique is widely applied in emerging-market economies, where data scarcity renders many other approaches infeasible. Unfortunately, the approach's relative simplicity brings with it several notable limitations. Chief among these is that the estimates are better thought of as 'trend' (rather than potential) growth, since these filters do not incorporate any economic structure, and thus are not consistent with an economic concept of potential—univariate filters represent a purely statistical approach to approximating potential output. In addition, the estimates which come out of these filters will reflect several statistical features which many users may be unaware of. For example, the estimates of the output gap will be mean zero (over a sufficiently long sample period), and the relative volatility of the cyclical vs. structural component will be determined by the selection of the smoothing parameter (lambda). Finally, univariate filters suffer from a particularly acute 'end-of-sample' problem, with estimates towards the end of a given sample period being subject to significant revisions as more data ultimately become available and the sample is extended.

---

[3] See, for example, Benes, Laxton, and Kumhof (2014a and 2014b), which assesses vulnerabilities associated with excessive credit expansions and asset price bubbles, and the consequences of different macro-prudential policies.

Another common technique to estimating potential output is the production-function approach, in which the inputs of production are considered separately. In its simplest form, this entails specifying a two-factor production function (generally Cobb-Douglas), obtaining data on employment and the capital stock, and then deducing total-factor productivity (TFP) as the residual from the production-function equation. By smoothing the resulting TFP series, and specifying a process for 'potential' employment, one arrives at an estimate for potential output by combining these trends with the estimate of the capital stock.[4] This approach has the benefit of allowing for a more detailed examination of the drivers of potential. However, there are also limitations; in particular, reliable capital-stock data can be hard to obtain, and the estimates of potential arising from this approach are only as good as the filters used to de-trend the TFP and employment components.[5]

Next, a good deal of work has focused on the use of multivariate filters to estimate potential (see Laxton and Tetlow (1992), Kuttner (1994), and more recently Benes and others (2010), among others). This approach adds economic structure to estimates by conditioning them on some basic theoretical relationships (such as a Phillip's curve relating the inflation process to the output gap). One strength of this approach is that estimates of the output gap and potential are consistent with the Okun concept of potential. In addition, in its simplest form this technique is relatively easy to implement requiring only a few variables, and it can be augmented where data availability permits. The shortcomings of the multivariate-filter approach are similar to those facing other methods—there remains an important end-of-sample problem, and the estimates of potential and the output gap are only improved relative to a simple statistical filtration if the structural relationships specified in the filter are valid in the economy in question.

Yet another technique which is gaining popularity in recent years is the use of DSGE models to estimate potential and the output gap (see, for example, Vetlov and others, 2011). This approach is theoretically rigorous, and is thus particularly appealing to academic audiences. Unfortunately, this technique is very difficult to implement, requiring extensive modeling expertise and a great deal of time and effort. In addition, estimates of the output gap and potential output derived from these models tend to be particularly sensitive to the specifications of the DSGE model being used, and they are not always intuitive. This is problematic for policy makers who want to use these estimates to formulate policy.

Finally, there has been some limited work devoted to estimating potential output in emerging markets. As mentioned earlier, data limitations in these countries for the most part preclude

---

[4] For an example of how the production-function approach can be implemented, see D'Auria and others (2010).

[5] As an example, if the employment and TFP series are de-trended using an HP filter, then the resulting estimates of potential output will have almost identical properties to those arising from a direct HP filtration of GDP data.

the use of the production-function approach, or complex multivariate filtration techniques. This has led practitioners to focus on univariate filters as their primary tool of choice (for a brief discussion see DeMasi, 1997).

III. Methodology

The multivariate filter approach specified in this paper is relatively simple, requiring data on just three observable variables: real GDP growth, CPI inflation, and the unemployment rate. Annual data is used for these variables for the 16 countries considered.[6] In this section, we present the equations which relate these three observable variables to the latent variables in the model. Parameter values and the variances of shock terms for these equations are estimated using Bayesian estimation techniques.[7]

In the model, the output gap is defined as the deviation of real GDP, in log terms (Y), from its potential level ($\overline{Y}$):

(1) $$y = Y - \overline{Y}$$

The stochastic process for output (real GDP) is comprised of three equations, and subject to three types of shocks:

(2) $$\overline{Y}_t = \overline{Y}_{t-1} + G_t + \varepsilon_t^{\overline{Y}}$$

(3) $$G_t = \theta G^{SS} + (1-\theta)G_{t-1} + \varepsilon_t^G$$

(4) $$y_t = \phi y_{t-1} + \varepsilon_t^y$$

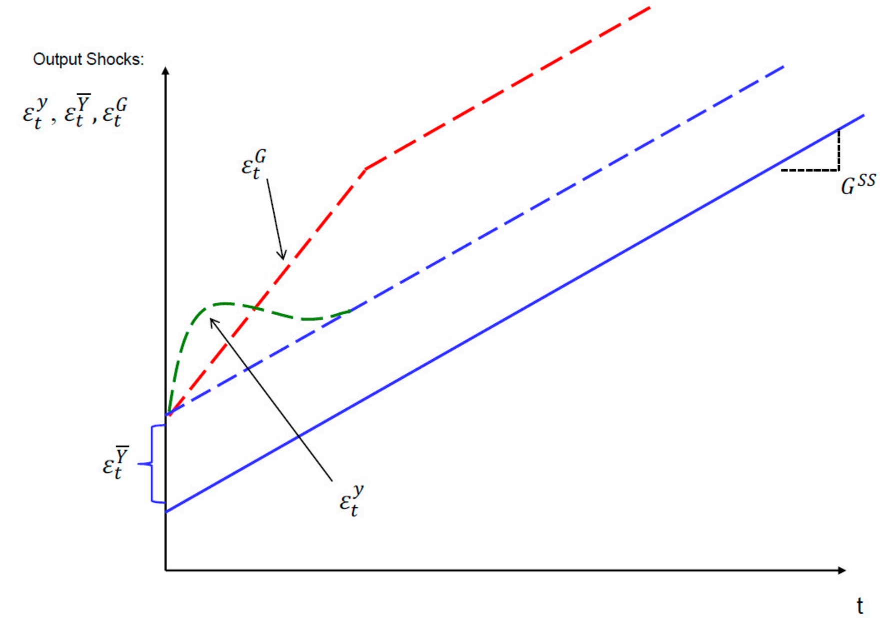
The level of potential output ($\overline{Y}_t$) evolves according to potential growth (G_t) and a level-shock term ($\varepsilon_t^{\overline{Y}}$). Potential growth is also subject to shocks (ε_t^G), with their impact fading gradually according to the parameter θ (with lower values entailing a slower adjustment back to the steady-state growth rate following a shock). Finally, the output-gap is also subject to shocks (ε_t^y), which are effectively demand shocks. The role of each shock term is expressed graphically in Figure 1:

---

[6] More information about sample size and data sources for each country is available in an appendix.

[7] More specifically, we use regularized maximum likelihood techniques (see Ljung, 1999). Also, see Hamilton (1994) for a general discussion of the Kalman filter, which is used to obtain estimates of the unobservable variables as part of the estimation process. Parameter estimates are provided in an appendix.

Figure 1. Shocks to the level and growth rate of potential output, and the output gap.



All else equal, output would be expected to follow its steady-state path, which is shown above by the solid blue line (which has a slope of G^{SS}). However, shocks to: the level of potential ($\varepsilon_t^{\overline{Y}}$); the growth rate of potential (ε_t^G); or the output gap (ε_t^y), can cause output to deviate from this initial steady-state path over time. As shown by the dashed blue line, a shock to the level of potential output in any given period will cause output to be permanently higher (or lower) than its initial steady-state path. Similarly, shocks to the growth rate of potential, illustrated by the dashed red line, can cause the growth rate of output to be higher temporarily, before ultimately slowing back to the steady-state *growth rate* (note that this would still entail a higher *level* of output). And, finally, shocks to the output gap would cause only a temporary deviation of output from potential, as shown by the dashed green line.

In order to help identify the three aforementioned output shock terms, a Phillips Curve equation for inflation is added, which links the evolution of the output gap (an unobservable variable) to observable data on inflation according to the process:[8]

(5) $$\pi_t = \lambda\pi_{t+1} + (1-\lambda)\pi_{t-1} + \beta y_t + \varepsilon_t^\pi$$

[8] Some recent work suggests that the slope of the Phillips curve relationship (β) has flattened over the past several decades (IMF, 2013), whereas other studies suggest that it may have steepened in some countries in recent years (Riggi and Venditti, 2014). Although the methodology in this paper does not allow for time variation in parameter estimates, modest changes in the estimated value of the parameter β, on its own, do not materially change the estimates of potential output and the output gap.

Finally, equations describing the evolution of unemployment are included to provide further identifying information for the estimation of the output gap:

(6) $$\overline{U}_t = (\tau_4 \overline{U}^{ss} + (1 - \tau_4)\overline{U}_{t-1}) + g\overline{U}_t + \varepsilon_t^{\overline{U}}$$

(7) $$g\overline{U}_t = (1 - \tau_3) g\overline{U}_{t-1} + \varepsilon_t^{g\overline{U}}$$

(8) $$u_t = \tau_2 u_{t-1} + \tau_1 y_t + \varepsilon_t^{u}$$

(9) $$u_t = \overline{U}_t - U_t$$

Here, $\overline{U}_t$ is the equilibrium value of the unemployment rate (the NAIRU), which is time varying, and subject to shocks ($\varepsilon_t^{\overline{U}}$) and also variation in the trend ($g\overline{U}_t$), which is itself also subject to shocks ($\varepsilon_t^{g\overline{U}}$)—this specification allows for persistent deviations of the NAIRU from its steady-state value. Most importantly, we specify an Okun's law relationship wherein the gap between actual unemployment (U_t) and its equilibrium process (given by u_t) is a function of the amount of slack in the economy (y_t).

Equations 1-9 comprise the core of the model for potential output. In addition, data on growth and inflation expectations are added, in part to help identify shocks, but mostly to improve the accuracy of estimates at the end of the sample period:

(10) $$\pi_{t+j}^{C} = \pi_{t+j} + \varepsilon_{t+j}^{\pi^C} \quad, j = 0,1$$

(11) $$GROWTH_{t+j}^{C} = GROWTH_{t+j} + \varepsilon_{t+j}^{GROWTH^C} \quad, j = 0,\ldots,5$$

For real GDP growth ($GROWTH$) the model is augmented with forecasts from consensus economics for the five years following the end of the sample period. For inflation, expectations data are added for one year following the end of the sample period. These equations relate the model-consistent forward expectation for growth and inflation (π_{t+j} and $GROWTH_{t+j}$) to observable data on how consensus forecasters expect these variables to evolve over various horizons (one to five years ahead) at any given time ($GROWTH_{t+j}^{C}$). The 'strength' of the relationship between the data on consensus and the model's forward expectation is determined by the standard deviation of the error terms ($\varepsilon_{t+j}^{\pi^C}$ and $\varepsilon_{t+j}^{GROWTH^C}$). In practice, the estimated variance of these terms allows consensus data to influence, but not completely override, the model's expectations, particularly at the end of the sample period. In a way, the incorporation of consensus forecasts can be thought as an heuristic approach to blend forecasts from different sources and methods. The resulting impact of this information on the historical estimates of potential and the output gap is modest, as shown in the following section.

IV. Detailed Results

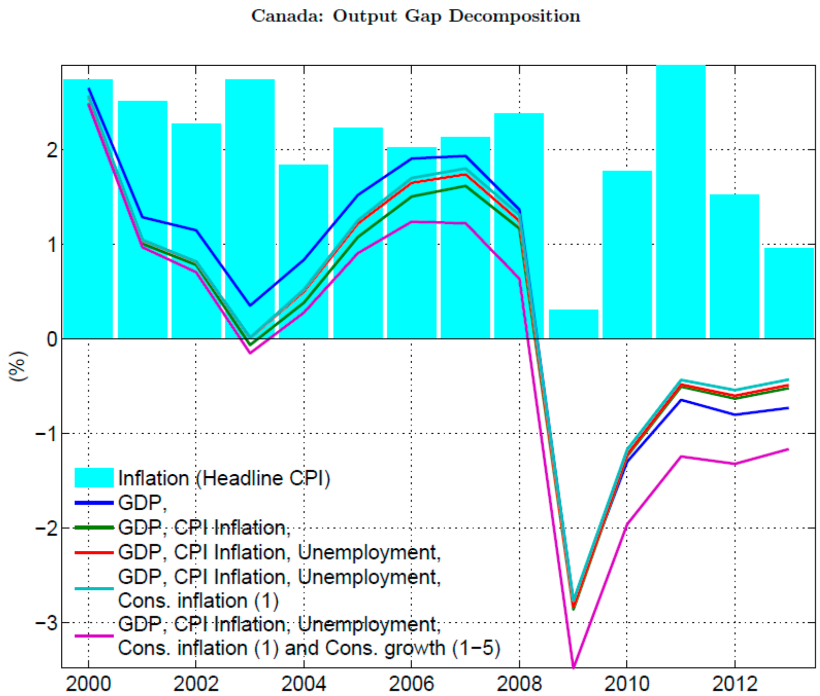
In order to illustrate the approach, detailed results are presented for two different countries, Brazil and Canada. These two countries have been chosen because they are representative of the two different approaches taken in constructing estimates: on the one hand, the prior is that in advanced economies (such as Canada), shocks to output over the cycle will be predominantly associated with fluctuations around the trend,[9] whereas in emerging market economies (such as Brazil), the prior is that there is a more important role for shocks to the trend (potential) in explaining the business cycle, as suggested in the literature discussed earlier (Aguiar and Gopina, 2007). The priors and posterior estimates for both countries are presented in the appendix (Table B1).

In an attempt to shed light on the role of the different components of the model, each marginal step in the construction of the estimates is presented separately for Brazil and Canada. In what follows, we hope to show that the simple model specified in this paper offers several noteworthy advantages (namely, the theoretical coherence of output-gap estimates and inflation, the transparency of the estimates, as well as its end-of-sample revision properties and the robustness of real-time estimates). However, it is far from perfect, and should not be used mechanically to obtain estimates (nor should any model).

Beginning with Canada, the output-gap estimates arising from the simplest filtration of GDP are presented first. In Figure 2, this is depicted by the blue line, which shows the estimates which arise solely as a result of the chosen calibration for the relative incidence of demand/supply shocks over the business cycle (using only equations 1-4 from the preceding section):

[9] Specifically, our prior is that shocks to output in advanced economies will be approximately $^1/_3$ supply/potential, and $^2/_3$ demand/gap. This naïve assumption is a common rule of thumb used at many policy institutions.

Figure 2. Canada: Output Gap Decomposition



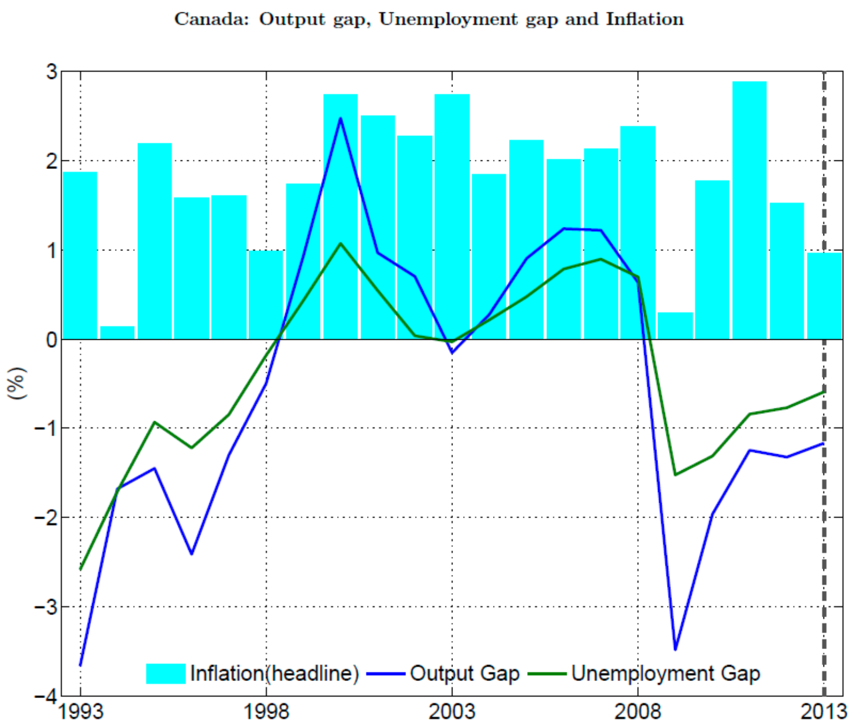
Source: Authors' estimates.

The addition of equation 5 suggests less excess demand in the pre-crisis period, given that inflation was not very far above target (shown by the dark green line). In more recent years, the inclusion of inflation points to less economic slack, which follows from the structure of the filter, where observed increases in inflation are associated with a closing output gap, all else equal. Of course, as practitioners we may not agree with this simple mechanical assessment of the filter, which would motivate the addition of judgment to help condition these estimates at the end of the sample—we will return to this issue in the appendix (section C).

The additions of model structure and data on unemployment (the red line) and inflation expectations (teal line) do little to alter the filter's assessment of the gap in the case of Canada. The latter result is likely due to the fact that expectations seldom deviate from the inflation target, and thus offer little additional information beyond what is obtained from actual inflation data. Adding growth expectations (magenta line) leads to a noteworthy downshift in the output-gap estimates, particularly since the early 2000s. This seems to reflect the fact that consensus forecasts were fairly strong for most of the past decade, with the exception of the crisis period, and have slowed only modestly since the crisis—the filter interprets this as evidence that the observed decline in growth is unlikely to be long lasting, which suggests an important cyclical element to the slowdown (perhaps associated with weak external demand conditions).

The output-gap estimates can also be considered in conjunction with the other measures being used to help identify them. In Figure 3, the estimated output gap (blue line) is shown for Canada alongside the estimates of the unemployment gap (green line) and inflation rate (teal bars). The main takeaway here is the coherence between each of these pieces of information.

Figure 3. Canada: Output gap, Unemployment gap, and Inflation



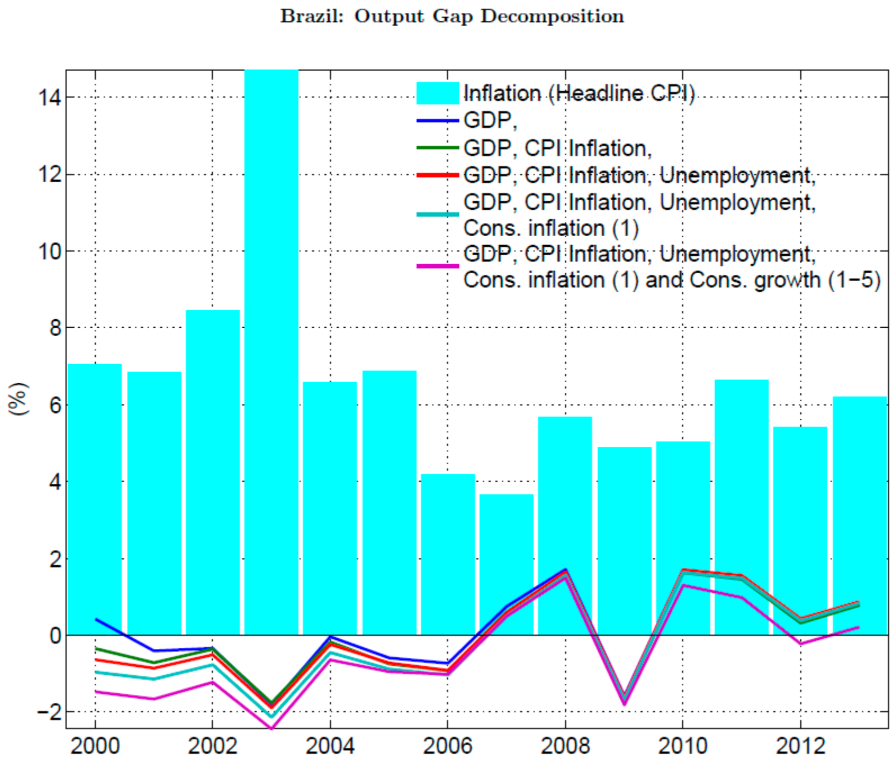
Source: Authors' estimates.

In particular, the early-to-mid nineties in Canada were characterized by considerable slack in the labor and goods markets, which was engineered by the Bank of Canada in order to facilitate the adoption (in 1991) of an inflation targeting regime.[10] Then, much of the pre-crisis 2000s featured somewhat more elevated inflation, partly driven by periods of excess demand as shown by both the output and unemployment gaps. Finally, the global financial crisis plunged the Canadian economy into recession, opening up significant slack in both labor and goods markets, with this slack still not quite eliminated more than 5 years after the crisis.

Turning to Brazil, the same figure is shown as was done for Canada. Again, the blue line in Figure 4 shows the output-gap estimates associated with the chosen relative incidence of supply/demand shocks:

[10] The adoption of this regime involved some initial inflation-reduction targets, where the target was gradually reduced from 4 percent to 2 percent by 1994Q4.

Figure 4. Brazil: Output Gap Decomposition



Source: Authors' estimates.

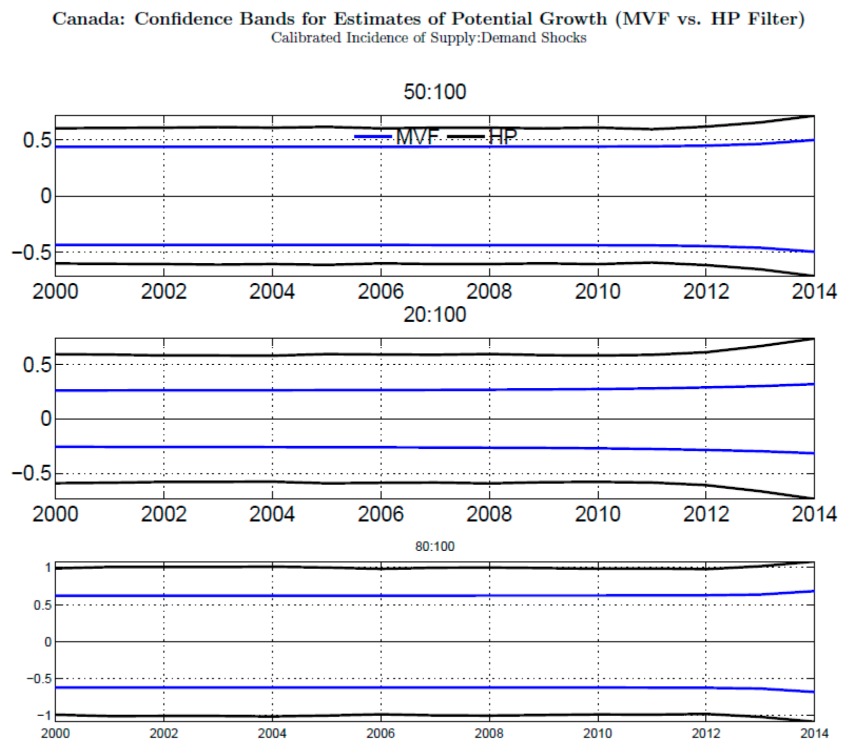
Next, the green line in the graph above shows the role of adding structure on inflation to the filter. Though difficult to see since it is largely obscured by the red line, this additional economic structure results in a slightly more negative output gap pre-crisis, as a result of declining inflationary pressures in 2006-07. The effect of adding information on unemployment is given by the red line, which is nearly identical to the green line suggesting that data on labor-market conditions do not add much identifying information for the output gap in Brazil. From there, information on the expectation for inflation one-year-ahead is added, taken from consensus forecasts (the teal line). Again, looking at the pre-crisis period, this information suggests that there was more slack in the economy, as early as 2005, because forecasters were already anticipating lower inflation in the year ahead, thereby suggesting a negative output gap in that period. Next, the magenta line shows the effect of adding expectations data for growth. The impact at the end of the sample period is significant: with actual growth underperforming expectations in both 2011 and 2012, the filter interprets this weak growth as temporary (related to demand rather than potential), resulting in more economic slack.

Estimates of potential growth and the output gap for 14 additional countries are presented in the appendix (section A). An examination of the results shows that despite considerable heterogeneity amongst the economies under consideration, the estimates of potential growth and the output gap all display similar properties (broadly speaking). In particular, output-gap estimates are related to movements in inflation as a result of the structure underpinning the estimates, which was presented earlier.

V. UNCERTAINTY IN ESTIMATING THE OUTPUT GAP AND POTENTIAL

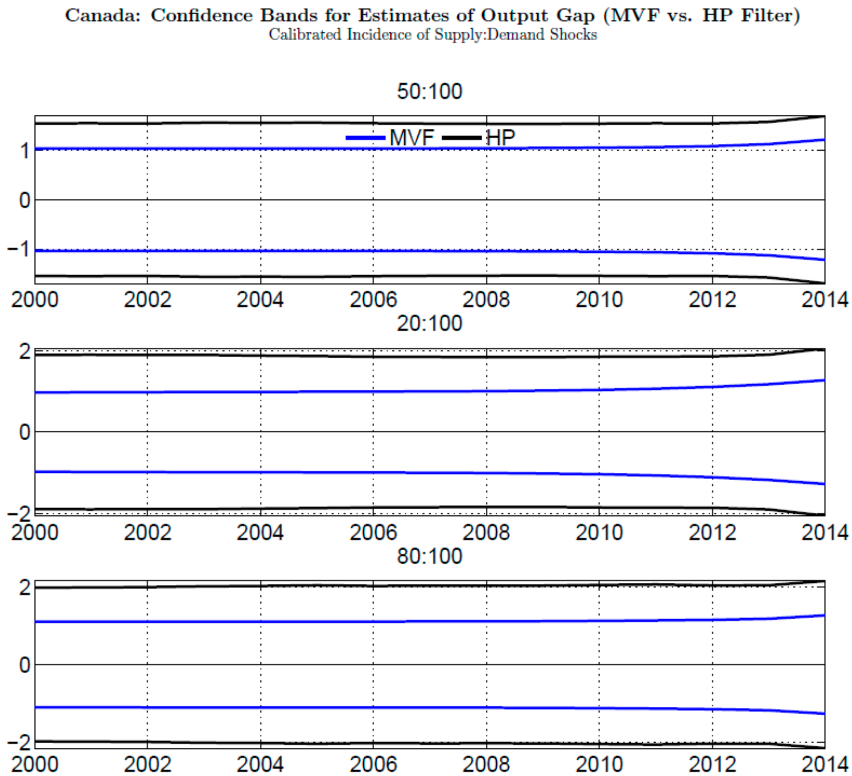
Potential output and the output gap are not variables which can be observed—they can only be estimated, and these estimates are subject to varying degrees of imprecision, depending on the technique used and the amount of information available when the estimates are constructed. To assess the robustness of the MVF estimates of potential, we construct confidence bands for the MVF approach specified in this paper and then compare them to confidence bands for a simple HP filtration of GDP, for Canada: [11]

Figure 5. Canada: Confidence Bands for Estimates of Potential Growth



[11] These confidence bands measure the uncertainty inherent in the model's estimates of the latent variables, and are not intended to capture model or parameter uncertainty, which are broader concepts beyond the scope of this exercise. In these figures, the confidence bands are plotted in deviations from the model's point estimates to allow for easier comparison of MVF and HP. For the construction of the HP-based confidence bands, a Monte Carlo procedure was followed where 5,000 draws of GDP were obtained from simulating historical shocks, which were assumed to follow mean-zero Gaussian processes. To alleviate the burn–in bias in HP estimates, GDP was simulated backwards for a sufficiently long period. We then applied an HP filter with the signal-to-noise ratio (λ) = 6.25 to each sample, and plot +/- 1.96 RMSEs from the assumed true path of the trend growth and cycle components.

Figure 6. Canada: Confidence bands for Estimates of Output Gap (MVF vs. HP Filter) Calibrated Incidence of Supply:Demand Shocks



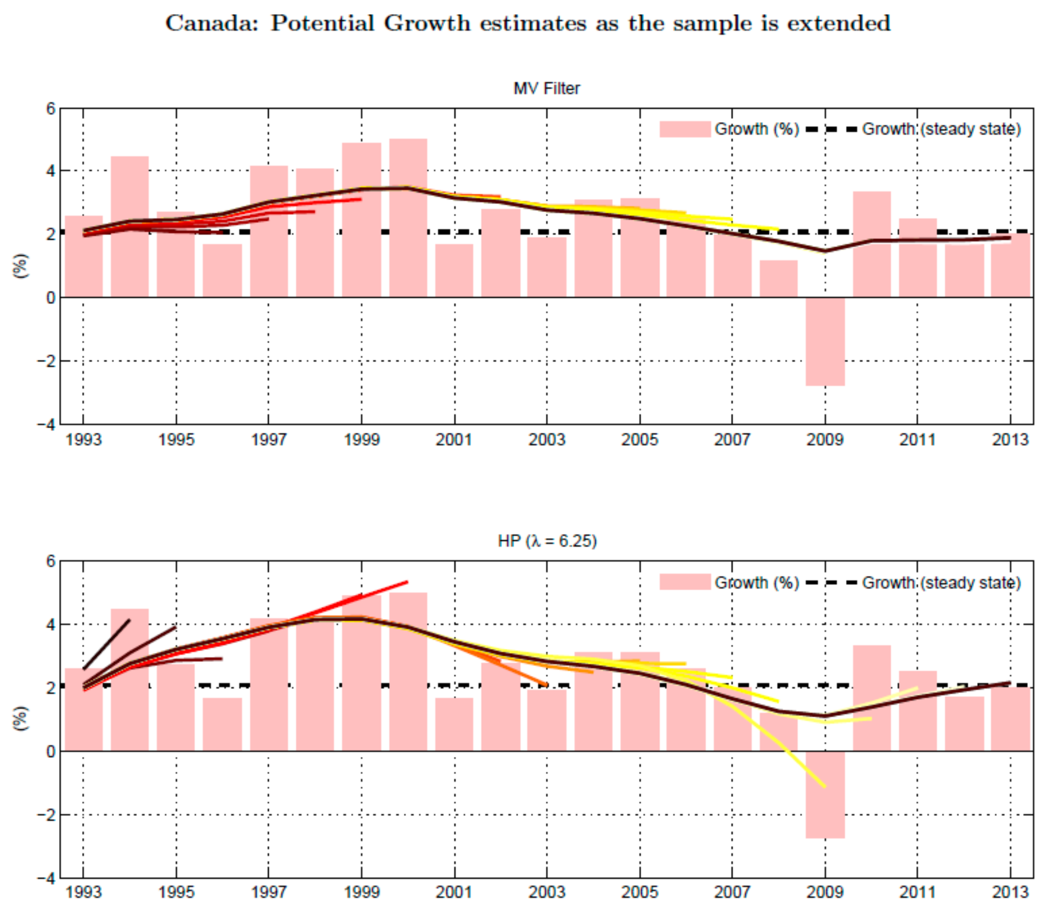
Source: Authors' estimates.

Each panel shows results using three different calibrations for the relative variance of supply and demand shocks (50:100; 20:100; 80:100). The ratio 50:100 is the baseline calibration of the MVF used in this paper (corresponding to 1/3 supply shocks and 2/3 demand shocks), and showing alternative calibrations serves as a robustness check to ensure that the improved fit of the MVF relative to the HP filter is not a function of these relative variances. As shown in the figures, irrespective of the assumed relative incidence of these shocks, the estimates of potential and the output gap coming from the MVF are subject to less uncertainty than are those from an HP filter. This result follows from the fact that more identifying information is used in the MVF than in a simple univariate filter. The end-of-sample problem is also illustrated nicely, with confidence bands widening at the end of the sample period for both techniques. Although the results are shown only for Canada, this same result holds across all countries in our sample.[12]

[12] The *degree* to which the MVF estimates outperform those from the simple HP filter does vary by country, and depends on the strength of the relationship between the output gap and inflation/unemployment in a given economy.

Real-time estimates coming from the MVF are also less prone to revision than are estimates derived from an HP filter. In Figure 7, quasi-real-time[13] estimates of potential output over the past 20 years are plotted, once again for Canada:

Figure 7. Canada: Potential Growth estimates as the sample is extended



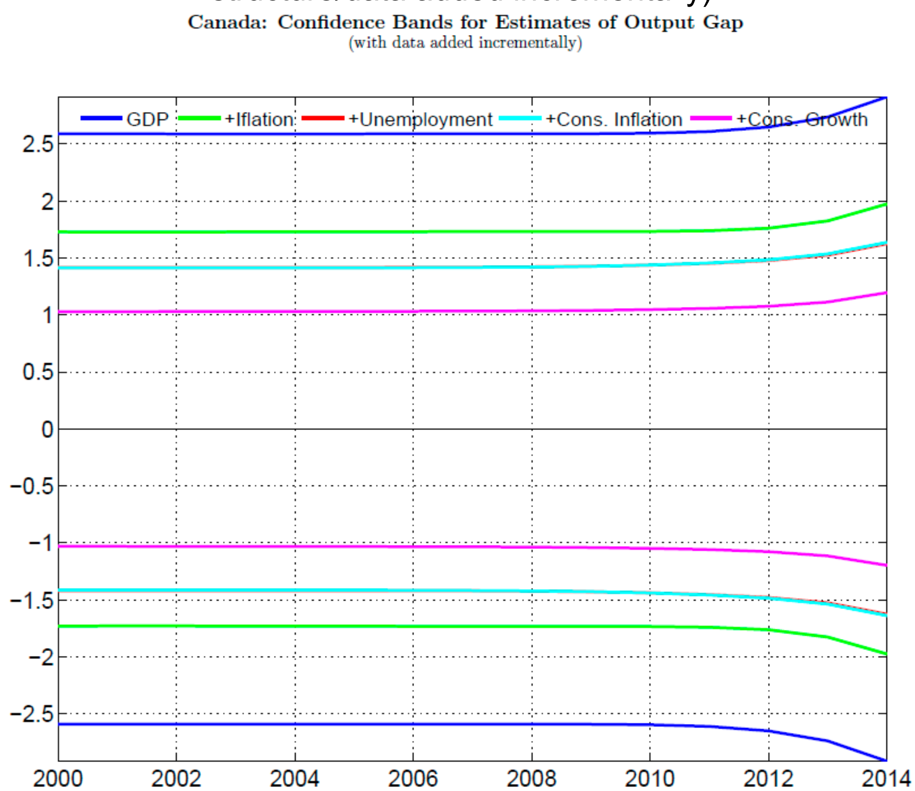
Source: Authors' estimates.

Having established that the MVF estimates of potential output growth are subject to less uncertainty than are those coming from the HP filter, we proceed to investigate how the uncertainty surrounding the MVF estimates of potential output growth change when the model is expanded piece-by-piece, adding each identifying equation one-by-one. Similar to the results shown in the previous exercise, the estimates of potential output growth become

[13] These estimates are constructed by sequentially estimating potential output in each year, using only the data available as of that date. For example, the quasi-real-time estimates of potential in 2007 (for both HP and MVF) would have used data from the beginning of the sample through 2007 only. The estimates are 'quasi' real-time in the sense that actual vintage data are not used for this exercise (but rather only currently-available data, which have been revised over time).

more robust as more information is added to the model in the form of additional structure. The results for Canada are in shown in Figure 8.[14] Relative to the simplest possible formulation of the MVF (with GDP only) shown by the blue line (and using only equations 1-4 from section III), the addition of model structure which relates estimates of the output gap to inflation (equation 5) improves the performance of potential-growth estimates materially (shown by the green line). From there, adding model structure (equations 6-9) and data on unemployment further improves the robustness of estimates (red line). Adding structure (equation 10) and observable data on consensus expectations of inflation yields only a very marginal further improvement (teal line, which is largely obscured by the red line), whereas adding structure (equation 11) and consensus expectations of growth significantly improves the performance (pink line).

Figure 8. Canada: Confidence Bands for Estimates of Output Gap (with structure/data added incrementally)



Source: Authors' estimates.

[14] The lines of the chart are presented in deviations from the Kalman smoother's mean estimate.

VI. CONCLUSION

The methodology presented in this paper draws on previous work applying multivariate filters to the estimation of potential output. By embedding the structural relationship between inflation, unemployment and the output gap, this class of models produces estimates of potential output and economic slack which are intuitive and consistent with basic economic theory. The innovations in this paper are twofold: first, a simplified version of the filter used in Benes and others (2010) is used to estimate potential in a broader range of countries where data limitations preclude richer analysis. This approach is particularly useful in EMs, as it represents an improvement over simple univariate filtration techniques and yet is not onerous in its data requirements. And, second, data on growth expectations have been added in order to help address (though not completely alleviate) the end-of-sample problem. As shown in the preceding section, estimates of potential obtained using this model are more robust than are those resulting from HP-filtering techniques. Even still, the end-of-sample problem remains an issue, particularly around turning points in the business cycle, which motivates the use of additional information taken from outside the model by practitioners when using the results to guide policy.

Future work will focus on extending the methodology to other countries, and experimenting with alternate measures of inflation, such as wage inflation. In addition, the results will be investigated further to gauge whether there are important commonalities in the evolution of potential output in the pre- and post-crisis periods across countries—this will be done by decomposing the existing results using a production-function approach.

VII. References

Aguiar, M., and G. Gopinath, 2007, "Emerging Market Business Cycles: The Cycle Is the Trend", *Journal of Political Economy*, University of Chicago Press, vol. 115, pp. 69–102.

Alberoa, E., A. Estrada, and D. Santabarbara, 2013, "Growth Beyond Imbalances. Sustainable Growth Rates and Output Gap Reassessment," Banco de Espana Documentos de Trabajo no. 1313.

Bank of Canada, 2009, *Monetary Policy Report,* July 2009. http://www.bankofcanada.ca/wp-content/uploads/2010/04/mpr230709.pdf

Bank of Canada, 2014, *Monetary Policy Report,* October 2014. http://www.bankofcanada.ca/wp-content/uploads/2014/07/mpr-2014-10-22.pdf

Barnett, R., S. Kozicki, and C. Petrinec, 2009, "Parsing Shocks: Real-Time Revisions to Gap and Growth Projections for Canada," Federal Reserve Bank of St. Louis Review, pp. 247–265.

Benes, J., K. Clinton, R. Garcia-Saltos, M. Johnson, D. Laxton, P. Manchev, and T. Matheson, 2010, "Estimating Potential Output with a Multivariate Filter," IMF Working Paper WP/10/285.

Benes, J., M. Kumhof, and D. Laxton. 2014, "Financial Crises in DSGE Models: Selected Applications of MAPMOD." Working Paper 14/56, International Monetary Fund, Washington.

Benes, J., M. Kumhof, and D. Laxton. 2014, "Financial Crises in DSGE Models: A Prototype Model." Working Paper 14/57, International Monetary Fund, Washington.

Benes, J., and P. N'Diaye, 2004, "A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to The Czech Republic," IMF Working Paper WP/04/45.

Borio, C., P. Disyatat, and M. Juselius, 2013, "Rethinking Potential Output: Embedding Information about the Financial Cycle," BIS Working paper no. 404.

Butler, L., 1996, "The Bank of Canada's New Quarterly Projection Model, Part 4. A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter," Technical Report no. 77 (Ottawa: Bank of Canada).

Cotis, JP, J. Elmeskov, and A. Mourougane, 2004, "Estimates of Potential Output: Benefits and Pitfalls from a Policy Perspective." Published in *Euro Area Business Cycle: Stylized Facts and Measurement Issues*, pp. 35–60.

D'Auria, F, C. Denis, K. Havik, K. McMorrow, C. Planas, R. Raciborski, W. Roger, A. Rossi, 2010, "The production function methodology for calculating potential growth rates and output gaps" European Commission Economic Papers No 420, July.

De Masi, P. R., 1997, "IMF Estimates of Potential Output: Theory and Practice," IMF Working Paper WP/97/177.

Giorno, C, P. Richardson, D. Roseveare, and P. van den Noord, 1995, "Estimating Potential Output, Output Gaps and Structural Budget Balances," OECD Working Paper no. 152

Hamilton, J, 1994, *Time Series Analysis*, Princeton University Press, New Jersey.

Hodrick, R.J., and E.C. Prescott, 1997, "Post-War U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, Vol. 29(1), pp. 1–16.

International Monetary Fund (IMF), 2013, "The Dog That Didn't Bark: Has Inflation Been Muzzled or Was It Just Sleeping?" World Economic Outlook, April, Washington.

Kuttner, K. N., 1994, "Estimating Potential Output as a Latent Variable" *Journal of Business and Economic Statistics*, Vol. 12, pp. 361–68.

Laxton, D., and R. Tetlow, 1992, "A Simple Multivariate Filter for the Measurement of Potential Output," Technical Report no. 59 (Ottawa: Bank of Canada).

Ljung, L., 1999, "System Identification: Theory for the User," Prentice-Hall: New Jersey.

Mishkin, F.S., 2007, "Inflation Dynamics," Annual Macro Conference, Federal Reserve Bank of San Francisco, March. http://www.federalreserve.gov/newsevents/speech/Mishkin20070323a.htm

Okun, A.M., 1962, "Potential GNP: Its Measurement and Significance," in Proceedings of the Business and Economic Statistics Section, pp. 98-104 (Washington: American Statistical Association).

Prachowny, M., 1993, "Okun's Law: Theoretical Foundations and Revised Estimates," *The Review of Economics and Statistics,* Vol. 75, No. 2, pp. 331–336.

Riggi, M, and F. Venditti, 2014, "Surprise! Euro area inflation has fallen," Banca D'Italia Occasional Paper no. 237.

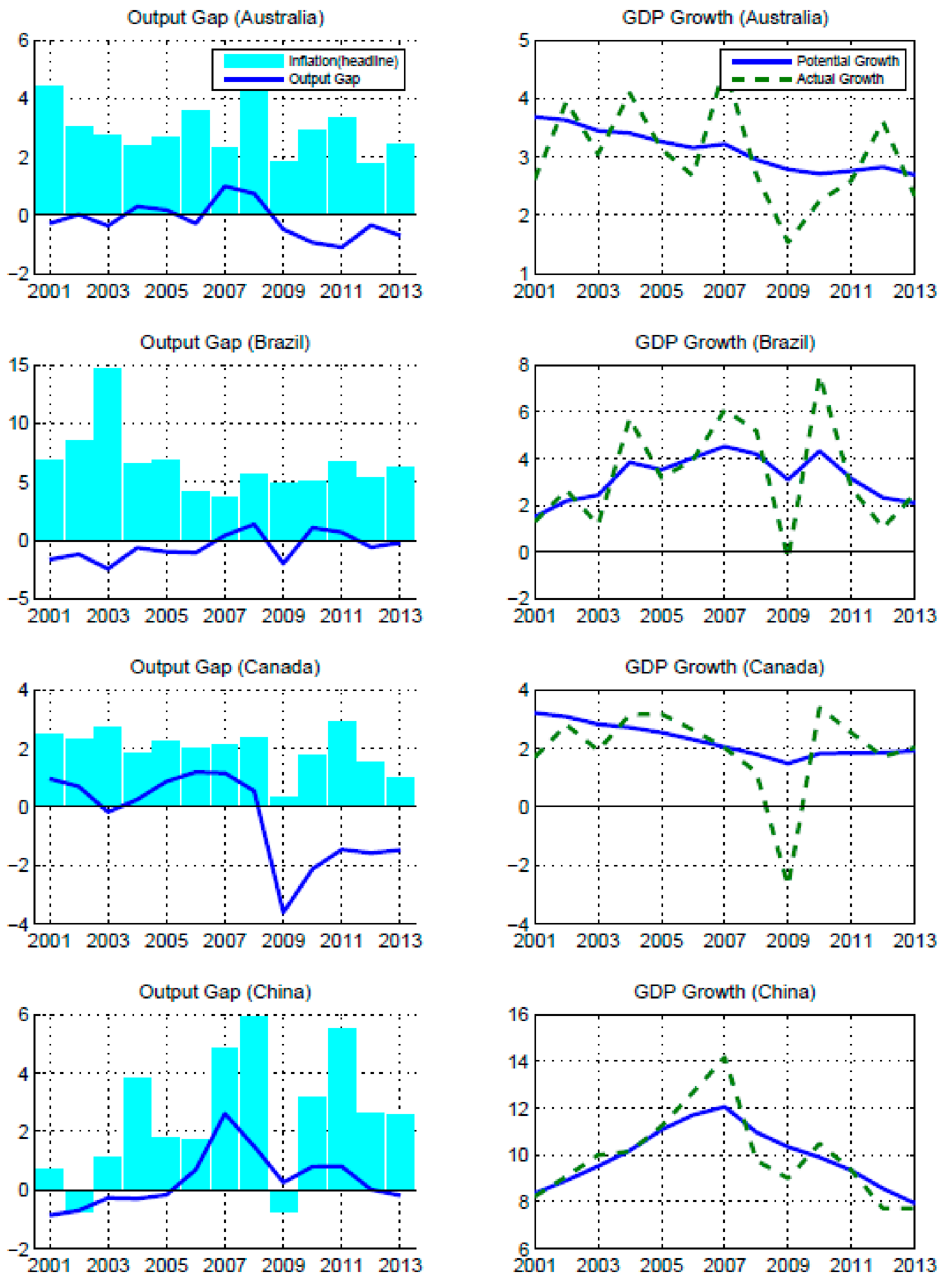
Vetlov, I., T. Hledik, M. Jonsson, H. Kucsera, and M. Pisani, 2011. "Potential Output in DSGE Models," ECB Working Paper Series No. 1351.

Zmitrowicz, K, and M. Khan, 2014, "Beyond the Unemployment Rate: Assessing Canadian and US Labour Markets Since the Great Recession." *Bank of Canada Review*, Spring 2014

VIII. Appendix

A. Estimates of potential output and the output gap: Graphs[15]

Figure A1



[15] Output gap is % deviation from potential, and inflation and growth rates are in % change.

Figure A2

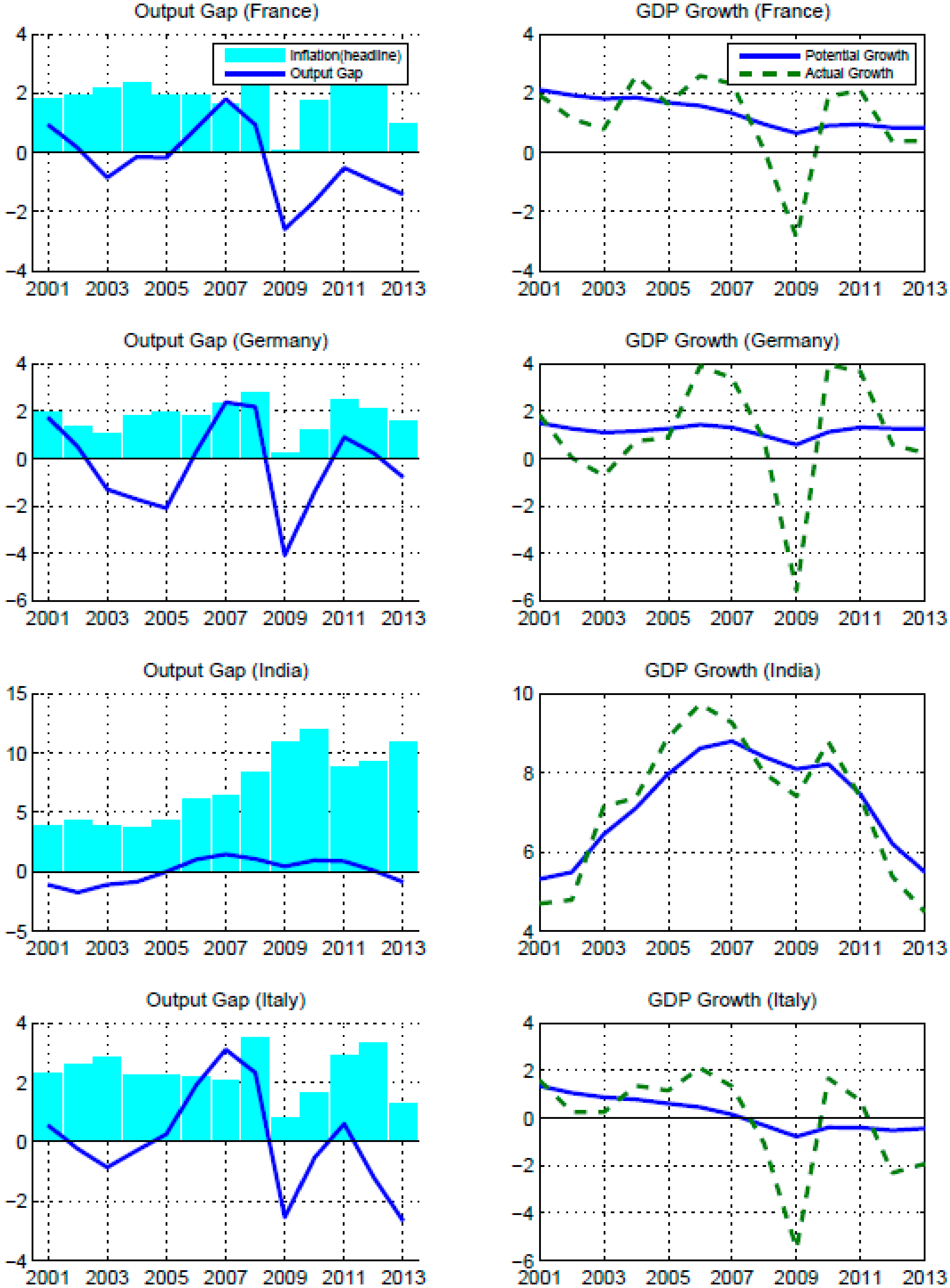

Output Gap (France)
Inflation(headline)
Output Gap
GDP Growth (France)
Potential Growth
Actual Growth
Output Gap (Germany)
GDP Growth (Germany)
Output Gap (India)
GDP Growth (India)
Output Gap (Italy)
GDP Growth (Italy)
2001 2003 2005 2007 2009 2011 2013

Figure A3

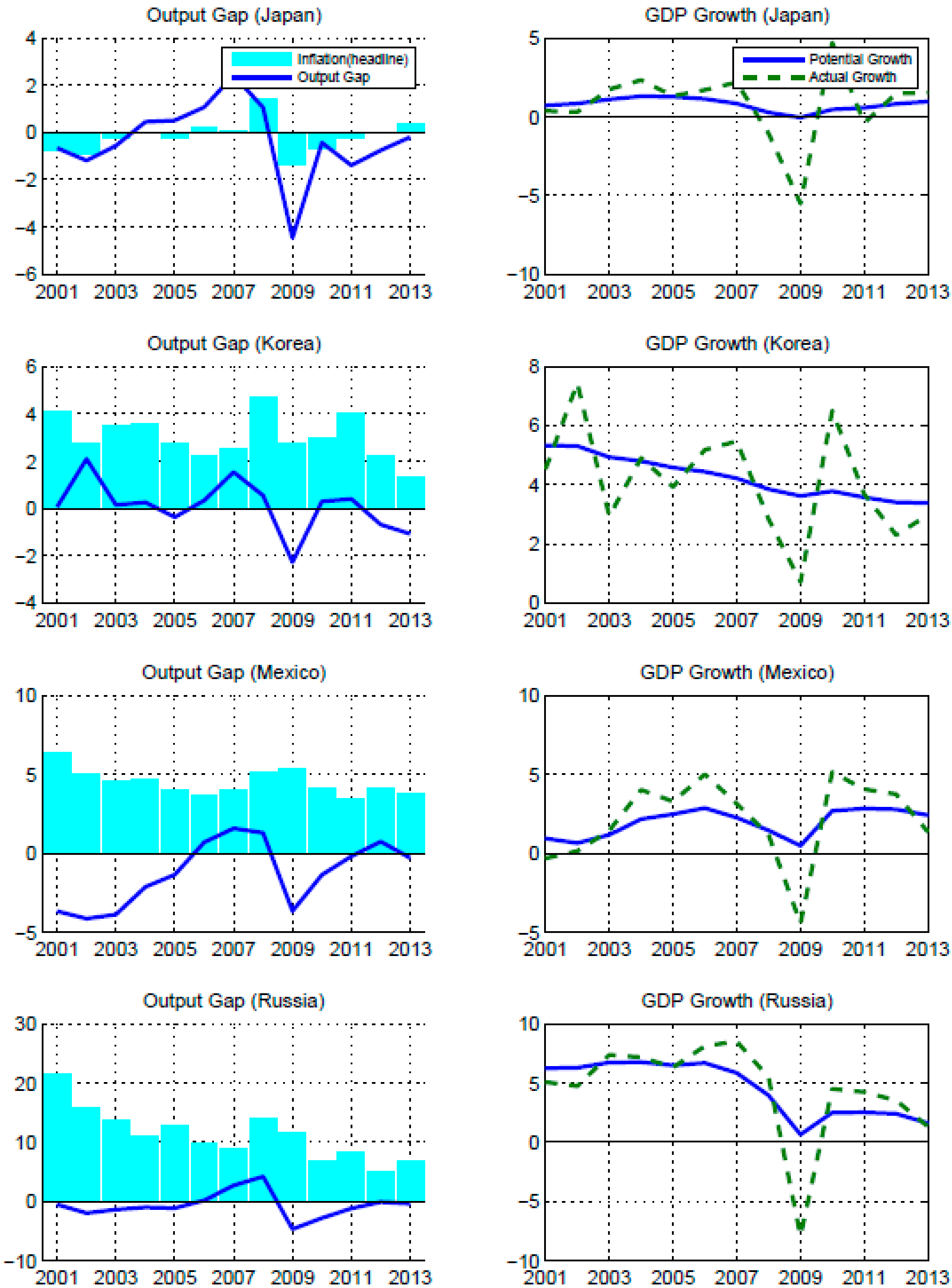
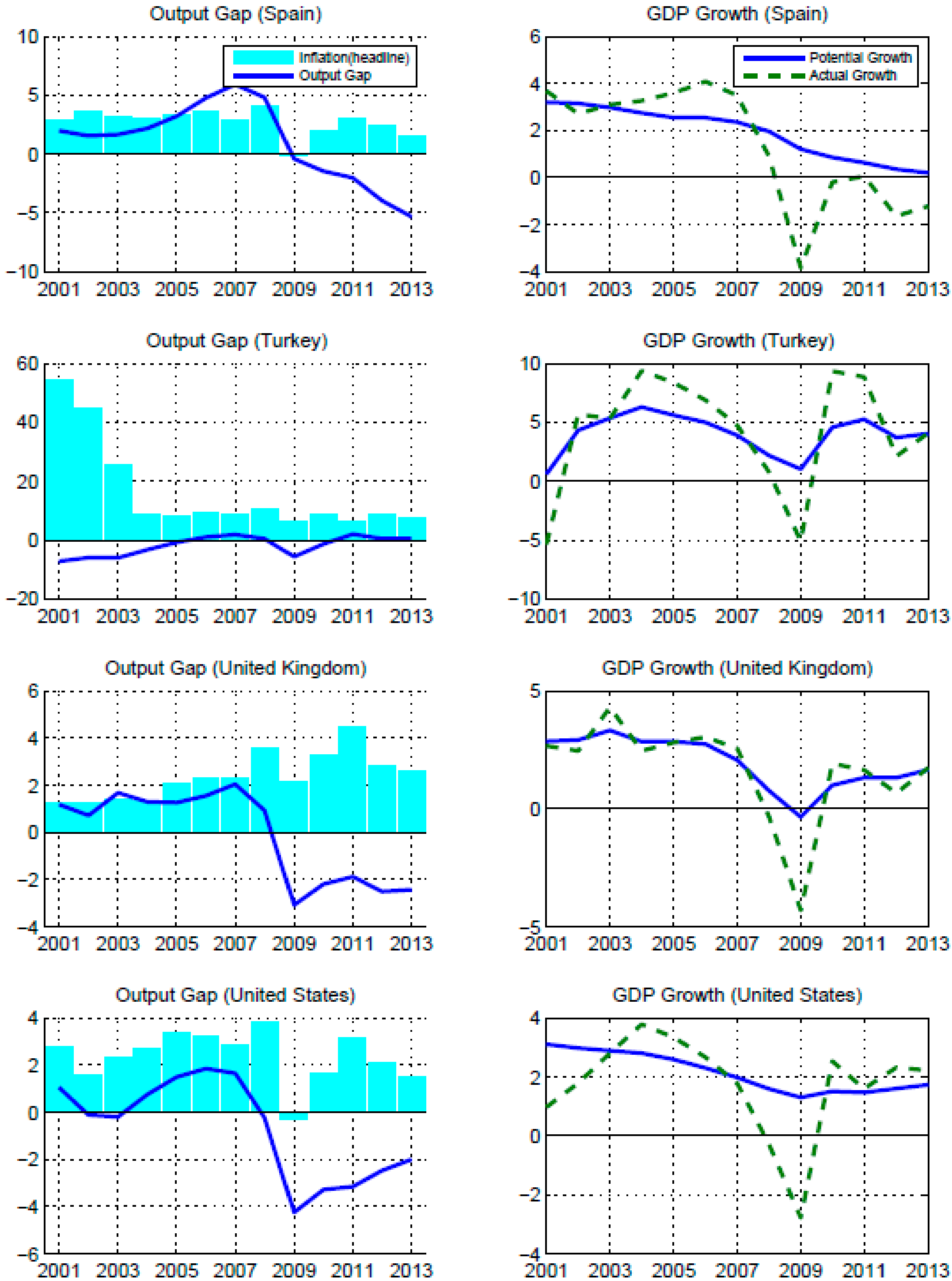

Output Gap (Japan)
Inflation(headline)
Output Gap
GDP Growth (Japan)
Potential Growth
Actual Growth
Output Gap (Korea)
GDP Growth (Korea)
Output Gap (Mexico)
GDP Growth (Mexico)
Output Gap (Russia)
GDP Growth (Russia)
2001 2003 2005 2007 2009 2011 2013

Figure A4



Source: Authors' estimates.

B. Estimation priors, posteriors, and data sources

Table B1. Estimated Parameters

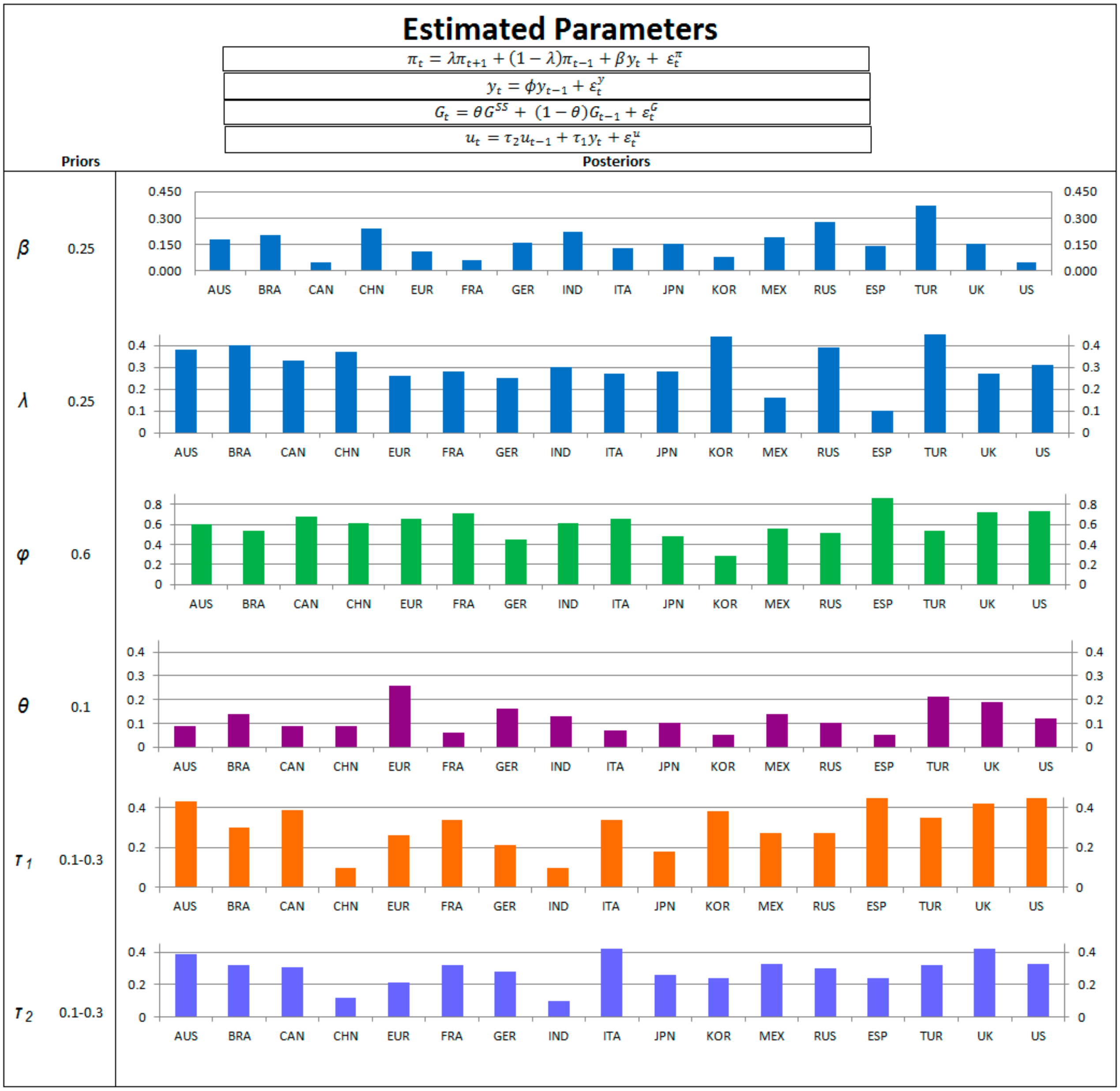


Table B2. Calibrated Parameters

Calibrated Parameters

$$G_t^{\overline{U}} = (1-\tau_3)G_t^{\overline{U}} + \varepsilon_t^{G^{\overline{U}}}$$

$$\overline{U}_t = \left(\tau_4\,\overline{U}^{ss} + (1-\tau_4)\overline{U}_{t-1}\right) + G_t^{\overline{U}} + \varepsilon_t^{\overline{U}}$$

	Advanced Economies	***Emerging Market Economies***
	AU, CA, EU, FR, DE, IT, US, KO, JP, SP, UK	BR, CH, MX, RU, TU
	Prior/Posterior	Prior/Posterior
RES_Y	0.8-1	1
RES_LGDP_BAR	0.1-0.4	0.5
RES_G	0.2-0.4	0.5
RES_UNR_GAP	0.3-0.5	0.8
RES_UNR_BAR	0.1-0.3	0.3
RES_G_UNR_BAR	0.1-0.2	0.3
tau 3	0.1	0.1
tau 4	0.1	0.1

Table B3. Data Sources

Indicator	**Source**
Inflation expectations	Consensus Economics
Gross Domestic Product growth expectations (constant prices)	Consensus Economics
Gross Domestic Product (constant prices)	IMF, World Economic Outlook Database
CPI Inflation	IMF, World Economic Outlook Database
Unemployment Rate	IMF, World Economic Outlook Database

C. Augmenting the raw estimates with additional information

Why additional information is used

As discussed in this paper and elsewhere, real-time estimates of the output gap are frequently revised as new information becomes available. Although the technique used in this paper offers some improvement over standard univariate estimation techniques, it does not resolve the problem. The imprecision at the end of the sample is particularly problematic since policy makers rely on real-time estimates of the economy's cyclical position to set policy. Fortunately, in many cases additional information exists which can be used to condition the filter's estimates at the end of the sample, though even when such information is incorporated considerable uncertainty remains. Discussion of the assessment of the cyclical position at the end of the sample, and in particular the uncertainty surrounding this estimate, is a critical part of the communication strategy of policy institutions who use the potential-output framework to relay their policy decisions to economic agents. Below, we draw upon the Bank of Canada's analysis of the output gap in the immediate aftermath of the financial crisis to provide an example of how this is done in practice (Bank of Canada, 2009):

Excess supply in the Canadian economy increased substantially as the recession deepened. The Bank's conventional measure of the output gap reached -4.3 per cent in the second quarter of 2009. In assessing excess capacity, the Bank considers the conventional measure in conjunction with several other indicators, particularly since this measure tends to have a higher margin of error around turning points…

…After reviewing all the indicators of capacity pressures and taking into account the weakness in potential output associated with the ongoing restructuring of the Canadian economy, the Bank judges that the economy was operating about 3.5 per cent below its production capacity in the second quarter of 2009…

…The substantial excess supply in the economy is expected to result in lower core inflation over the next few quarters…

…[An] upside risk to inflation is the possibility that potential output will be lower than the Bank's revised estimate, if the extensive restructuring in certain sectors is more protracted and the investment response more delayed than currently envisaged.

The takeaway here is that whatever the method for assessing the degree of slack in the economy, estimates are subject to heightened uncertainty at the end of the sample, thereby motivating the consideration of additional information from outside of a simple model. This judgment can then be made part of the broader communication strategy of the policy institution.

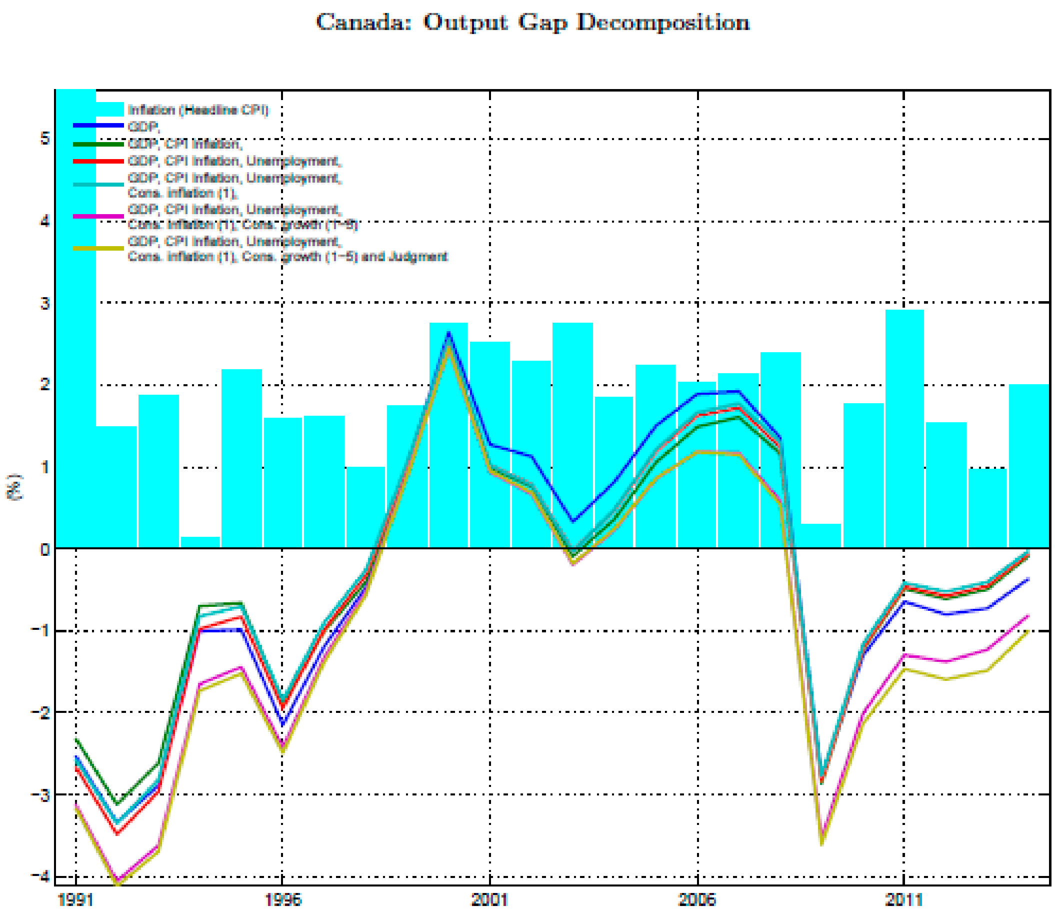
How additional information is applied to the raw filtered output

One important advantage of the technique presented in this paper is that it is flexible, and the estimates at the end of the sample can easily be conditioned using information from off-model analysis. This judgment can be informed by many different sources, ranging from hard data on the components of inflation or the components of GDP to indicator data on such things as capacity utilization or the labor market. To better illustrate the process via which judgment is imposed, and how it can be applied to the filter, we consider our estimates for Canada.

At the time of writing (Fall 2014), there is considerable (though not complete) information about the state of the Canadian economy in 2014. This means that in order to construct estimates for the current year, existing monthly and quarterly data can be utilized and then augmented with assumptions about how the rest of the year will play out – this is done using standard near-term forecasting techniques. Once a near-term forecast is in place, estimates can be computed for 2014, which are shown by the magenta line in the figure presented in section IV.

For the output gap, the assessment of information outside of the simple filter is subject to a large degree of uncertainty, but when done properly it will improve the plausibility and robustness of output-gap estimates at the end of the sample period. In the case of Canada, analysis done outside of the multivariate filter suggests that recent increases in headline inflation are likely understating the degree of slack in the economy, since they are being driven in part by recent Canadian-dollar depreciation, and partly by other temporary sector-specific considerations such sharp increases in prices of communications services and meat products (Bank of Canada, 2014). In addition, more detailed analysis of labor markets suggests that recent improvements in the unemployment rate may be overstating the true degree of improvement in economic conditions, as employed labor may be underutilized (Zmitrowicz and Kahn, 2014). Together, these factors point to slightly more economic slack than what is given by the pure model results – to be concrete, our judgmental assessment is that the output gap in 2014 in Canada is -1.0, slightly below the estimate of the filter (-0.8). Once we have settled on the judgmental adjustment to the filter's estimates of the amount of slack in the economy at the end of the sample, the filter can be used to examine the new results for the entire sample period (given by the yellow line in the figure shown below).

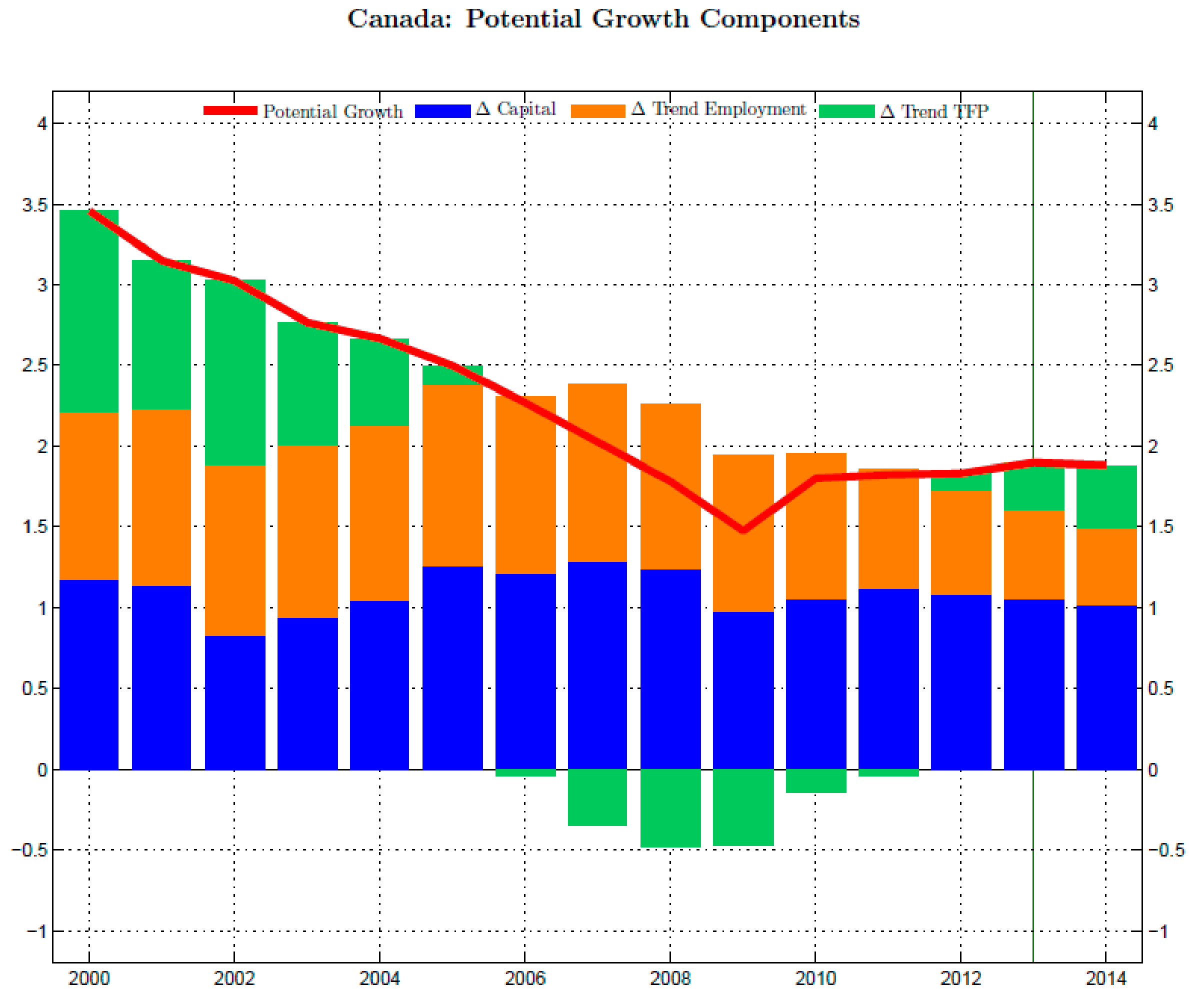
Figure C1. Conditioning Estimates with Additional Information



Source: Authors' estimates.

Then, as a cross-check of the results from the multivariate filter, data on capital and labor inputs can be used to decompose the filter's estimates of potential growth into its component parts using a simple Cobb Douglas production function approach, given by: $\overline{Y} = \overline{A}\, K_t^{\alpha} \overline{L}_t^{(1-\alpha)}$ (alpha = 1/3). The decomposed estimates of potential output growth provide information about the plausibility of the estimated path from the multivariate filter, in view of information on the production-function components. In the case of Canada, the estimated path for potential growth in recent years appears plausible – with the contribution from trend labor inputs (given by the orange bar) expected to fade slightly at the end of the sample, and capital deepening expected to hold steady, any acceleration in potential growth will need to be driven by trend total factor productivity (green bar). The fact that the projected contribution from this factor in 2014 is relatively small, and well in line with recent historical estimates, should provide some confidence that the estimates of potential that come out of the filter are plausible.

Figure C2. Canada: Potential Growth Components



Source: Authors' estimates.