# Home Bias in Equities under New Open Economy Macroeconomics

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**Abstract**

This paper presents a potential solution to the home bias puzzle based on a new open economy macroeconomics model. In response to technology shocks, sticky prices generate a negative correlation between labor income and the profits of domestic firms, leading to home bias in equity holdings. In contrast, under flexible prices, labor income and the profits of the domestic firms are positively correlated. Returns on human capital and equities may be positively correlated under sticky prices when the source of shocks is monetary, but this risk is hedged through nominal assets rather than through equities.

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# 1 Introduction

The "home bias" puzzle is one of the major puzzles in international finance. Empirical studies have found that foreign equities comprise a small proportion of investors' portfolios.[1] This finding is puzzling because it appears that investors are forgoing important opportunities for diversification of risk.[2] While there have been many suggested resolutions to the puzzle, none seem able to explain entirely the extent of home bias. We offer a new explanation that may contribute to an understanding of home bias. In a New Keynesian framework in which some nominal prices are sticky, it may be natural for households to bias their portfolios strongly toward home equities as a hedge against shocks to their labor income.

The intuition is straightforward: In a Keynesian framework, in the short run the level of output is demand determined. Productivity shocks have no effect on short-run output. Thus, for example, if home firms experience a positive productivity shock, their demand for labor will decline. Employment and wages will fall, but profits to the firm will increase. An effective hedge against employment and wage risk, then, is ownership of the firm. In a Keynesian model, the short-run returns to labor and firm owners are negatively correlated, in contrast to the usual presumption in neoclassical models.

The fact that productivity shocks create a negative correlation between returns to workers and those to firm owners is the key implication of the Keynesian model. Galí (1999) builds a closed economy model under sticky prices and shows that it can generate a fall in labor hours in response to the positive technology shock, which rarely arises in a flexible price model[3]. His empirical work demonstrates that labor hours decline in response to positive technology shocks in most G7 countries. The related empirical work by Bottazzi, Pesenti and van Wincoop (1996) finds that wages and domestic capital returns are negatively correlated in most OECD countries.[4]

Our explanation can be considered part of one thread of the literature that has attempted

[1] French and Poterba (1991), Tesar and Werner (1995), and Warnock (2002), for example.

[2] Lewis(1999, 2000) surveys the literature on this puzzle and discusses the losses from non-diversification.

[3] For example of flexible price models which generate a negative correlation, see Francis and Ramey (2003) and Dotsey (1999).

[4] The US is one of the exceptions.

to explain home bias as a hedge against non-tradable risks.[5] Our non-tradable risk is fluctuations in labor income. The literature has not yet reached a consensus on how much of home bias can be explained by the need to hedge against non-tradable risk. There is a literature that specifically examines whether home assets can be a hedge for labor risk. But in neoclassical models, because labor income is correlated more with domestic firms' profits than with those of foreign firms', the optimal portfolio will be more foreign-weighted than the classical endowment model predicts, as shown in Baxter and Jermann (1997). Hence, past attempts to explain home bias by using labor income have been largely unsuccessful.[6]

Empirical studies find the unconditional correlation of labor income and profits is not sufficiently negative to explain home bias fully.[7] But in our model, the unconditional correlation need not be negative. In Keynesian models, monetary shocks lead to consumption risk. We show that those shocks can be hedged effectively with bond portfolios (or by taking a forward position in foreign exchange.) Unexpected changes in the relative supplies of money (at home and abroad) create nominal exchange rate changes that, in turn, alter the value of returns on home and foreign bonds. So the real risk created by monetary shocks can be diversified through the nominal asset portfolio. Monetary shocks lead to positively correlated changes in labor payments and profits, but that risk is not hedged with the equity portfolio.

Of course, nominal prices do not remain fixed forever when productivity or monetary shocks occur. Eventually an adjustment is made and neoclassical results obtain in the long run. Indeed, our model has real labor income positively correlated with productivity shocks in the long run. So the ability of our model to explain home bias depends on the persistence of price stickiness, the persistence of productivity shocks, and the weight that households assign to future consumption. We show that home bias is greater when prices adjust more slowly, when productivity shocks are less persistent, and when the future is discounted more

[5]For example, Eldor, Pines and Schwarz (1988), Stockman and Dellas (1989), Tesar (1993), Baxter, Jermann and King (1998), Serrat (2001) and Pesenti and van Wincoop (2002). A related analysis by Obstfeld and Rogoff (2001) argues that transactions costs to trade in international goods can help account for home bias in equities.

[6]See also Jermann (2002). However, Palacios-Huerta (2001) claims that a substantial fraction of home bias can be explained when the differential human capital of stockholders and non-stockholders is taken into account along with human capital frictions.

[7]See Bottazzi et al. (1996) and Pesenti and van Wincoop (2002).

heavily.

We do not claim that our model offers the only explanation for home bias. The literature has taken many different approaches to explaining the phenomenon. In addition to the papers cited above that consider diversification against non-tradable risk, several other avenues have been explored. One group of studies has argued that the gains from international diversification are in fact small, so that small transactions costs of diversification will lead to heavily concentrated portfolios.[8] Others have claimed that acquisition of information about foreign firms is more costly than for information on home firms.[9] Another set of studies shows that home bias can be explained in the context of generalized preferences or prior beliefs.[10] Some claim that home bias is partly due to empirical mismeasurement.[11] All of these factors may help explain home bias.

Our model is in a "new open economy macroeconomics" setting. We study the endogenous portfolio choice in an economy driven by technology shocks and monetary shocks. Using bonds and equities, we can successfully replicate the complete market allocation up to a linear approximation. Equity holdings of households are a function of the discount rate, the elasticity of substitution between home and foreign goods, and the persistence of technology shocks and nominal prices. If technology shocks are i.i.d. or the elasticity of substitution between home and foreign goods is unity, then we have 100 per cent home bias.

In the following sections, we present two kinds of models. The first is static, and gives us the intuition for home bias. The new open economy macro literature has considered two instances of nominal price stickiness – when prices are set in advance in the producers' currency (PCP) and when prices are set in advance in the local currency of consumers (LCP). In the static model, complete home bias is the equilibrium under both types of price settings. The second model is a more realistic dynamic one, in which we focus on persistent technology shocks and differential price stickiness. The dynamic model analyzes

[8]For example, Cole and Obstfeld (1991), Tesar (1995), Butler and Joaquin (2002) and many others. However, van Wincoop(1994, 1999), for example, finds large unexploited gains from international risk sharing.

[9]For example, Kang and Stulz (1997) and Hasan and Simaa (2000).

[10]For example van Wincoop (1994), Aizenman (1999) as examples of the former and Pastor (2000) for the latter.

[11]For example, Rowland and Tesar (2004) find that multinationals may have provide diversification opportunities for some countries.

the conditions under which home bias occurs, and the degree of bias. The last section summarizes our results and suggests some extensions.

## 2 The Simple Static Model

We follow the standard new open economy macroeconomics model setup–that is, a general-equilibrium, two-country model with sticky prices. In this section, we first consider a static model that provides the intuition behind our explanation for home bias. The next section then builds a dynamic model to investigate more realistically what determines whether and how much home bias is generated.

In our model, there are two countries, which we call Home and Foreign. The world population is normalized to unity; half the population lives in Home and half in Foreign. Their preferences are identical. Households provide labor elastically and own firms through equity. Firms use labor as the only input to produce a good monopolistically, and preset their prices in the consumers' currency. Markets are segmented so that only firms can export goods. All goods are tradable and perishable.

We adopt local currency pricing here. First, what we observe in the data, at least for developed countries, is that "consumer prices of tradables" are sticky in the consumers' currencies rather than in the producers' currencies. However, the pricing assumption is not particularly important in determining the equity portfolio. In fact, we would have exactly the same equity portfolio when prices are preset in producers' currencies, even though the number of forward contracts differ.[12]

In our model, we consider two kinds of shocks. One is a monetary shock, which is a "demand" shock, and the other is a technology shock, which is a "supply shock". The distribution of shocks is identical between Home and Foreign.

Finally, we assume that before the realization of shocks, only forward contacts in the foreign exchange and equities are traded.

[12] See Matsumoto (2004).

## 2.1 Households

Households in both countries have identical preferences over the consumption basket, the real money of the domestic country, and leisure. The representative household[13] in the Home country solves

$$\max_{\gamma,\tilde{\delta}} \mathrm{E}_{t-1} \max_{C_t,M_t,L_t} U\left(C_t, \frac{M_t}{P_t}, L_t\right), \quad s.t. \text{ budget constraint,}$$

where $U$ is a contemporaneous utility function, and $\gamma$ and $\tilde{\delta}$ are portfolio choice variables, which will be defined later. $C_t$ denotes the consumption basket for Home, while $M_t$ denotes Home money; $P_t$, the price index; and $L_t$, the labor supply. In the static model, we assume that households choose portfolios and that firms set prices before monetary and technology shocks are realized at time $t$. We use the notation $\mathrm{E}_{t-1}$ to represent expectation formed without knowledge of the shocks. After the realization of shocks, households make choices about consumption, money holdings, and the labor supply, while firms produce goods as demanded.

The utility function has the form

$$U\left(C_t, \frac{M_t}{P_t}, L_t\right) = \frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln\left(\frac{M_t}{P_t}\right) - \frac{\eta}{1+\psi} L_t^{1+\psi}, \tag{2.1}$$

$$\rho > 1,\ \chi > 0,\ \psi > 0, \text{ and } \eta > 0.$$

$C_t$ is a consumption basket of a representative Home household defined as

$$C_t \equiv \left(\frac{1}{2}\right)^{1/(\omega-1)} \left(C_{h,t}^{(\omega-1)/\omega} + C_{f,t}^{(\omega-1)/\omega}\right)^{\omega/(\omega-1)}, \tag{2.2}$$

where $\omega > 0$ is the elasticity of substitution between Home produced goods and Foreign produced goods. $C_{h,t}$ is the consumption basket of Home produced goods and $C_{f,t}$ is that

[13] We will omit the index for households since they are identical.

of Foreign produced goods.

$$C_{h,t} \equiv \left[2^{1/\lambda}\int_0^{1/2} C_{h,t}(i)^{(\lambda-1)/\lambda}di\right]^{\lambda/(\lambda-1)}, \qquad C_{f,t} \equiv \left[2^{1/\lambda}\int_{1/2}^{1}, C_{f,t}(i)^{(\lambda-1)/\lambda}di\right]^{\lambda/(\lambda-1)}, \tag{2.3}$$

where $\lambda$ denotes the elasticity of substitution among varieties, with $\lambda > 1$. Then, we can write the CPI as follows:

$$P_t = \left(\frac{1}{2}\right)^{1/(1-\omega)} \left(P_{h,t}^{1-\omega} + P_{f,t}^{1-\omega}\right)^{1/(1-\omega)}, \tag{2.4}$$

where

$$P_{h,t} = \left[2\int_0^{1/2} P_{h,t}(i)^{1-\lambda}di\right]^{1/(1-\lambda)}, \quad P_{f,t} = \left[2\int_{1/2}^{1} P_{f,t}(i)^{1-\lambda}di\right]^{1/(1-\lambda)}, \tag{2.5}$$

where $P_{h,t}(i)$ is the price of Home goods $i$ sold in Home in terms of the Home currency, and $P_{f,t}(i)$ is the price of Foreign goods $i$ sold in Home in terms of the Home currency.

Home households receive the following: wages ($W_tL_t$, where $W_t$ denotes the wage); dividends; transfers from the government ($Tr_t$) which equal the change in the money supply; and the gains or losses from forward contracts. The ownership of the firms can be shared internationally. Households can choose their equity portfolios before the realization of time $t$ shocks.

Foreign households have an analogous utility function for Foreign quantities and prices, which we will denote by superscript asterisks. Foreign prices are denominated in Foreign currency.

We assume that shocks are symmetric between Home and Foreign. This assumption, together with the assumptions of identical size and identical preferences, guarantees the existence of an equilibrium in which the equity prices of Home and Foreign firms are the same at time $t-1$.[14]

Let $\gamma_h$ denote the weight of Home firms and let $\gamma_f$ denote the weight of Foreign firms

[14]If prices are different, then one country is richer than the other *ex ante*, a situation that contradicts symmetry.

in the equity portfolio of the Home household. Given the symmetry in the model, there is home bias when $\gamma_f < \frac{1}{2}$

Equity dividends received by a Home household are given by

$$\gamma_h \Pi_t + \gamma_f S_t \Pi_t^*$$

where $\Pi_t$ is the profit (dividend) of Home firms and $\Pi_t^*$ is that of Foreign firms in terms of the Foreign currency. [15] $S_t$ is the Home currency price of Foreign currency.

Home and Foreign households are also allowed to trade forward contracts in the foreign exchange. Let $\tilde{\delta}$ denote the number of forward contracts. The forward rate, $F_t$, is know at the time the forward contract is entered into, prior to the realization of shocks. After the shocks are realized, the Home households receives $\tilde{\delta}(S_t - F_t)$ units of Home currency.

Therefore, the budget constraint of a representative Home household is

$$P_t C_t + M_t = \gamma_h \Pi_t + \gamma_f S_t \Pi_t^* + W_t L_t + \tilde{\delta}(S_t - F_t) + Tr_t. \tag{2.6}$$

Given prices and the total consumption basket $C_t$, the optimal consumption allocations are

$$C_{h,t} = \frac{1}{2}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t, \qquad C_{f,t} = \frac{1}{2}\left(\frac{P_{f,t}}{P_t}\right)^{-\omega} C_t, \tag{2.7}$$

$$C_{h,t}(i) = 2\left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\lambda} C_{h,t}, \qquad C_{f,t}(i) = 2\left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\lambda} C_{f,t}. \tag{2.8}$$

[15] Theoretically, profits can be negative in the case of a loss, but we have to assume that the profits of both Home firms and Foreign firms are positive to take logarithms.

The remaining first order conditions are

$$
\begin{aligned}
\frac{M_t}{P_t} &= \chi C_t^{\rho}, && (2.9)\\
W_t &= \frac{\eta}{\chi} M_t L_t^{\psi}, && (2.10)\\
\mathrm{E}_{t-1}\left(S_t \frac{C_t^{-\rho}}{P_t}\right) &= \mathrm{E}_{t-1}\left(\frac{C_t^{-\rho}}{P_t}\right) F_t, && (2.11)\\
\mathrm{E}_{t-1}\left(\Pi_t \frac{C_t^{-\rho}}{P_t}\right) &= \mathrm{E}_{t-1}\left(S_t \Pi_t^{*} \frac{C_t^{-\rho}}{P_t}\right). && (2.12)
\end{aligned}
$$

## 2.2 Firms

Firms engage in monopolistic competition as in Blanchard and Kiyotaki (1987), which is typical of the new open macroeconomics literature. A firm in this economy monopolistically produces a specific good indexed by $i$ using a linear technology:[16]

$$Y_t(i) = A_t L_t(i), \tag{2.13}$$

where $Y_t(i)$ is the production of firm $i$, $A_t$ is the country-specific technology parameter and $L_t(i)$ is the labor input of firm $i$. Labor is assumed to be homogeneous and to be supplied elastically. Home and Foreign markets are segmented, and only the producer can distribute its product. Firms set prices one-period in advance in the consumers' currencies for each country. Firms in each country set prices so as to maximize their expected profits, taking other firms' prices as given, which is equivalent to taking the price level as given since each firm has measure zero on interval $[0, 1]$.

Given the CES utility sub-function, the demand for Home good $i$ from the Home market denoted by $Y_{h,t}(i)$ is

$$Y_{h,t}(i) = \frac{1}{2}\left(\frac{P_{h,t}(i)}{P_h, t}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t, \tag{2.14}$$

[16] Using a Cobb-Douglas technology with other fixed inputs will not change the result if the returns on the other factors belong to the equity holders.

while the demand for Home good $i$ from the Foreign market is

$$Y_{h,t}(i)^* = \frac{1}{2}\left(\frac{P^*_{h,t}(i)}{P^*_h, t}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t. \tag{2.15}$$

Firm $i$'s profit maximization problem is

$$\max_{P_{h,t}(i), P^*_{h,t}(i)} \mathrm{E}_{t-1}\left\{\tilde{D}_t(i)\left[P_{h,t}(i)Y_{h,t}(i) + P^*_{h,t}(i)Y_{h,t}(i)^* - \frac{W_t}{A_t}(Y_{h,t}(i) + Y_{h,t}(i)^*)\right]\right\},$$

where $\tilde{D}_t(i)$ is the stochastic discount factor for the firm $i$. For example, if firms are owned by Home residents, it will be $\dfrac{C_t^{-\rho}}{P_t}$. However, because firms are not always domestically owned, we use a more general notation.

The optimal price of Home goods for the Home market[17] is

$$P_{h,t} = \frac{\lambda}{\lambda - 1}\frac{\mathrm{E}_{t-1}\left(\tilde{D}_t C_t \dfrac{W_t}{A_t}\right)}{\mathrm{E}_{t-1}\,\tilde{D}_t C_t}. \tag{2.16}$$

Similarly, the optimal price of Home goods for the Foreign market is

$$P^*_{h,t} = \frac{\lambda}{\lambda - 1}\frac{\mathrm{E}_{t-1}\left(\tilde{D}_t C^*_t \dfrac{W_t}{A_t}\right)}{\mathrm{E}_{t-1}\left(\tilde{D}_t C^*_t S_t\right)}. \tag{2.17}$$

Because firms are all alike, they will set the identical prices for each market.

The market clearing condition can be obtained by equating the output with the sum of the demands for Home goods:

$$A_t L_t = \frac{1}{2}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t + \frac{1}{2}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t. \tag{2.18}$$

Given these prices we can calculate profits. Using the optimal consumption allocations,

[17] We will omit index $i$ since Home firms are identical.

we can write the profits for the firms in each country in terms of the Home currency as:

$$\Pi_t = \frac{1}{2}P_{h,t}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega}C_t + \frac{1}{2}S_t P^*_{h,t}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega}C^*_t - W_t L_t, \tag{2.19}$$

$$S_t\Pi^*_t = \frac{1}{2}S_t P^*_{f,t}\left(\frac{P^*_{f,t}}{P^*_t}\right)^{-\omega}C^*_t + \frac{1}{2}P_{f,t}\left(\frac{P_{f,t}}{P_t}\right)^{-\omega}C_t - S_t W^*_t L^*_t. \tag{2.20}$$

Firms will pay out all of their profits as dividends.

We assume that $A_t$ and $A^*_t$ are drawn from an identical distribution with $\operatorname{var}\ln A_t = \operatorname{var}\ln A^*_t = \sigma_a^2$, $\operatorname{cov}(\ln A_t, \ln A^*_t) = \sigma_{a,a^*}$. We also assume that $M_t$ and $M^*_t$ are drawn from an identical distribution with $\operatorname{var}_{t-1}\ln M_t = \operatorname{var}_{t-1}\ln M^*_t = \sigma_m^2$, $\operatorname{cov}(\ln M_t, \ln M^*_t) = \sigma_{m,m^*}$, and we assume that the money shocks are independent of the technology shocks.

The labor market is competitive, and the wage moves freely to equate demand and supply of labor after the shocks. The output of each good is determined by demand. Firms adjust output after the shocks to satisfy demand, holding prices constant. The money market is assumed to equilibrate, so money demand equals money supply. We normalize the number of equities in each country to unity so that $\gamma_h + \gamma^*_h = 1$. Equilibrium in the equity market, given our assumptions of initial symmetry, requires

$$\gamma_h + \gamma_f = 1. \tag{2.21}$$

## 2.3 Solution of the Static Model

An equilibrium in the static model satisfies equations (2.4)-(2.21), and their foreign counterparts. These 39 equations (one is redundant by Walras' Law) solve for $C_t$, $C_{h,t}$, $C_{f,t}$, $C_{h,t}(i)$, $C_{f,t}(i)$, $L_t$, $W_t$, $P_t$, $P_{h,t}$, $P_{f,t}$, $P_{h,t}(i)$, $P^*_{h,t}(i)$, $Y_t(i)$, $Y_{h,t}(i)$, $Y^*_{h,t}(i)$, $\Pi_t$, $\gamma_h$, $\gamma_f$, and their foreign counterparts, and $\tilde{\delta}$, $F_t$, and $S_t$.[18]

We will not in fact solve for this equilibrium, but will instead solve the equilibrium for a set of equations that approximate these 39. We will take first-order approximations of the equations that are determined after the realization of shocks (equations (2.4)-(2.10),

[18] We have also implicitly assumed that there is a money market equilibrium condition that holds, but we have not introduced separate notation for money demand and money supply and that there is a forward market clearing condition which can be guaranteed here by setting $\tilde{\delta}^* = -\tilde{\delta}$.

(2.13)-(2.15), and (2.18)-(2.20)) and second-order approximations of the equations that are set prior to the realization of shocks (the portfolio choice equations (2.11)-(2.12), and the price-setting equations, (2.16)-(2.17).) We must use a second-order approximation of the equations determined ex ante because second moments are important in determining portfolio choice and price levels.[19]

Our focus is on the equilibrium portfolio choice of equity shares and forward foreign exchange position. We proceed in this section to construct the equilibrium solutions for these variables in an intuitive manner, to exposit the economic factors that lead to home bias. We will first derive the portfolio demands for households, taking prices as given. With these in hand, we will use equilibrium conditions in goods, labor, and asset markets to derive the equilibrium portfolio positions.

We rely on ex ante symmetry in the derivations below. We use the notation for any variable $X_t$, $x_t \equiv \ln(X_t) - \bar{x}$, where $\bar{x} \equiv E(\ln(X_t))$. We use "var" to denote variance, and "cov" covariance.[20] In the linearized equations below, we suppress the intercept terms for convenience. Approximating the household first-order condition (2.11), where we have used symmetry to give us $f_t = 0$, we get:

$$-\rho \operatorname{cov}(c_t, s_t) + \frac{1}{2} \operatorname{var}(s_t) = 0. \tag{2.22}$$

We can use similar steps, and recognize that symmetry implies that $\bar{\pi} = \bar{\pi}^*$, $\operatorname{var}(\pi_t) = \operatorname{var}(\pi_t^*)$, and $\operatorname{cov}(s_t, \pi_t) = -\operatorname{cov}(s_t, \pi_t^*)$, to derive from equation (2.12):

$$\rho \operatorname{cov}(c_t, \pi_t - (s_t + \pi_t^*)) - \frac{1}{2} \operatorname{cov}(s_t, \pi_t - (s_t + \pi_t^*)) = 0. \tag{2.23}$$

We approximate the budget constraint (2.6), using condition (2.21) to arrive at:

$$c_t = (1-\gamma)(1-\zeta)\pi_t + \gamma(1-\zeta)(s_t + \pi_t^*) + \zeta(w_t + l_t) + \delta s_t, \tag{2.24}$$

where $\zeta \equiv \dfrac{e^{\bar{w}+\bar{l}}}{e^{\bar{\pi}} + e^{\bar{w}+\bar{l}}}$, $\delta \equiv \dfrac{\tilde{\delta}}{e^{\bar{\pi}} + e^{\bar{w}+\bar{l}}}$, and $\gamma \equiv \gamma_f$. We use equation (2.24) to substitute out

[19]While price levels do not play significant roles in our analysis of portfolio choice, they will be important for welfare analysis.

[20]We drop the $t-1$ subscript on expectations for the rest of this section.

for c in equations (2.22) and (2.23). Then we solve out for $\gamma$ and $\delta$:

$$\begin{aligned}\gamma =& \frac{\operatorname{cov}(\pi_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))} \\ &+ \frac{\zeta}{1-\zeta}\frac{\operatorname{cov}(w_t + l_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))} + \frac{1}{1-\zeta}(\delta - \frac{1}{2\rho})\frac{\operatorname{cov}(s_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))}\end{aligned}$$

$$\begin{aligned}\delta =& \frac{-(1-\zeta)\operatorname{cov}(\pi_t, s_t)}{\operatorname{var}(s_t)} - \frac{\zeta \operatorname{cov}(w_t + l_t, s_t)}{\operatorname{var}(s_t)} + \gamma\frac{(1-\zeta)\operatorname{cov}(\pi_t - s_t - \pi_t^*, s_t)}{\operatorname{var}(s_t)} + \frac{1}{2\rho} \\ =& -(1-\zeta)\beta_{\pi,\,s} - \zeta\beta_{w+l,\,s} + \gamma(1-\zeta)\beta_{\pi-s-\pi^*,\,s} + \frac{1}{2\rho}\end{aligned} \tag{2.25}$$

where we have used the notation $\beta_{x,\,s} \equiv \dfrac{\operatorname{cov}(x_t,\ s_t)}{\operatorname{var}(s_t)}$.

We can then rewrite the last term in the expression for $\gamma$ as:

$$\begin{aligned}&\frac{1}{1-\zeta}\left(\delta - \frac{1}{2\rho}\right)\frac{\operatorname{cov}(s_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))} \\ =& \left(-\beta_{\pi,\,s} + \gamma\beta_{\pi-s-\pi^*,\,s} - \frac{\zeta}{1-\zeta}\beta_{w+l,\,s}\right)\frac{\operatorname{cov}(s_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*))}.\end{aligned}$$

We can then use this to solve out for $\gamma$:

$$\gamma = \frac{\operatorname{cov}(\pi_t - \beta_{\pi,\,s}s_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*) - \beta_{\pi-s-\pi^*,\,s}s_t)} + \frac{\zeta}{1-\zeta}\frac{\operatorname{cov}(w_t + l_t - \beta_{w+l,\,s}s_t, \pi_t - (s_t + \pi_t^*))}{\operatorname{var}(\pi_t - (s_t + \pi_t^*) - \beta_{\pi-s-\pi^*,\,s}s_t)}.$$

Using the properties of orthogonal projections, this can be simplified to:

$$\gamma = \frac{\operatorname{cov}(\pi_t - \beta_{\pi,\,s}s_t, \pi_t - \pi_t^*)}{\operatorname{var}(\pi_t - \pi_t^* - \beta_{\pi-\pi^*,\,s}s_t)} + \frac{\zeta}{1-\zeta}\frac{\operatorname{cov}(w_t + l_t - \beta_{w+l,s}s_t, \pi_t - \pi_t^*)}{\operatorname{var}(\pi_t - \pi_t^* - \beta_{\pi-\pi^*,\,s}s_t)}. \tag{2.26}$$

Consider expression (2.26). From the point of view of the household, the equity position is determined by the covariances and variances of shocks to profits and labor income that are orthogonal to exchange rates. Any variance in the portfolio that is attributable to exchange rate changes is hedged through the forward position, so the equity position is determined only by those risks that are uncorrelated with exchange rate risk.

If the component of labor income that is orthogonal to exchange rates were uncorrelated with relative profits of home and foreign firms, the second term in equation (2.26) would drop out. Then the share $\gamma$ of equities held in foreign firms would increase as home profits (orthogonal to the exchange rate) have a higher covariance with relative home and foreign profits. Under our symmetry assumption, this term will equal 1/2, so the portfolio would be balanced between home and foreign equities if only the first term mattered. It is the second term of equation (2.26) that will determine home bias.

That term tells us that the share of foreign equities will be larger the greater the covariance between wage income and home profits relative to foreign profits. If this covariance is positive, there will be anti-home bias ($\gamma > \frac{1}{2}$ ), as in Baxter and Jermann (1997). In that case, returns to home equities (compared to returns on foreign equities) are positively correlated with labor income, so the variance of total income (returns to equities and human capital) is reduced by holding a relatively large share of foreign equities. There is home bias when that covariance is negative. In that case, home equities serve as a hedge against labor income shocks.

So far, to arrive at equation (2.26), we have only used the households' first-order conditions and budget constraints, along with the symmetry assumption and the assumption that nominal prices are fixed. Now we can bring in one more equation from the rest of the economy, the linearization of the profit equation for home firms. We have from (2.19):

$$(1-\zeta)\pi_t + \zeta(w_t + l_t) = c_t^W + \frac{1}{2}s_t, \tag{2.27}$$

where $c_t^W = \frac{1}{2}(c_t + c_t^*)$. In deriving this, we use symmetry to get $\bar{c} = \bar{c}^*$. Taking covariances on both sides of equation (2.27), we get

$$\text{cov}\left(\pi_t + \frac{\zeta}{1-\zeta}(w_t + l_t), \pi_t - \pi_t^*\right) = \frac{1}{2(1-\zeta)}\text{cov}(s_t, \pi_t - \pi_t^*), \tag{2.28}$$

where we have used symmetry to infer that $\text{cov}(c_t^W, \pi_t - \pi_t^*) = 0$. Also,

$$\text{cov}\left(\pi_t + \frac{\zeta}{1-\zeta}(w_t + l_t), s_t\right) = \frac{1}{2(1-\zeta)}\text{var}(s_t), \tag{2.29}$$

using symmetry to infer that $\text{cov}(c_t^W, s_t) = 0$. Dividing (2.29) through by $\text{var}(s_t)$, we can write

$$\beta_{\pi,s} + \frac{\zeta}{1-\zeta}\beta_{w+l,s} = \frac{1}{2(1-\zeta)}. \tag{2.30}$$

Substitute (2.28) and (2.30) onto the right side of (2.26), and we derive $\gamma = 0$.

To get the equilibrium value of $\delta$, substitute $\gamma = 0$ into equation (2.25), and use equation (2.29):

$$\delta = \frac{-(1-\zeta)\operatorname{cov}(\pi, s)}{\operatorname{var}(s)} - \frac{\zeta \operatorname{cov}(w+l, s)}{\operatorname{var}(s)} + \frac{1}{2\rho} = -\frac{1}{2} + \frac{1}{2\rho}. \tag{2.31}$$

We find complete home bias in equity holdings, $\gamma = 0$. Equation (2.26) indicates that the share of equities held in the foreign firm is determined by the covariance of the component of home firm revenues $((1-\zeta)\pi_t + \zeta(w_t + l_t))$ that is orthogonal to the exchange rate with the relative profits of home to foreign firms. If that covariance is zero, then no foreign equities are held. In that case, returns to home equities are a perfect hedge for labor income.

In fact, the residual from projecting $(1-\zeta)\pi_t + \zeta(w_t + l_t)$ on $s_t$ is orthogonal to $\pi_t - \pi_t^*$. That is because equation (2.27) tells us that the revenue of the home firm, $(1-\zeta)\pi_t + \zeta(w_t + l_t)$, is determined by world consumption and the exchange rate: $c_t^W + \frac{1}{2}s_t$. Output is demand determined. Demand depends on the overall level of consumption at home and abroad. Additionally, the home-currency revenue of the home firm increases when the currency depreciates, because the depreciation increases the home-currency value of foreign sales. The projection residual is simply world consumption, $c_t^W$, and that is uncorrelated with relative profits by symmetry.

Note that if we substitute the solutions for $\gamma$ and $\delta$ back into the budget constraint (2.24), we obtain

$$c_t = (1-\zeta)\pi_t + \zeta(w_t + l_t) + \left(\frac{1}{2\rho} - \frac{1}{2}\right) s_t = c_t^W + \frac{1}{2\rho}s_t. \tag{2.32}$$

Using the definition of world consumption, this expression can be written as:

$$\rho c_t = s_t + \rho c_t^*. \tag{2.33}$$

This condition indicates that asset markets are complete. As is well known, when asset markets are complete (and assuming symmetry), the marginal utility of a unit of home (or foreign) currency is equalized between home and foreign residents:

$$\frac{C_t^{-\rho}}{P_t} = \frac{C_t^{*-\rho}}{S_t P_t^*}.$$

Equation (2.33) is the log-linearized version of this condition, using the fact that prices are preset in consumers' currencies. The trading of home and foreign equities and forward contracts for foreign exchange are enough to deliver the complete markets allocation.

We have derived the complete home bias result using only the nominal price stickiness assumption, the definition of home profits, the budget constraint of home households, and the two first-order conditions (2.11 and 2.12) that pertain to asset choice. (The derivations in this subsection all arise from equations (2.22), (2.23), (2.24), and (2.27), which are the approximated versions of the two first-order conditions for asset choice, the household budget constraint, and the definition of firm profits. In performing the approximations, we have used the fact that prices are preset.)

We have not relied on other features of the model, so our home bias result is robust to alternative assumptions. For example, the result does not depend on money demand arising from real balances in the utility function. Other specifications that maintain equations (2.11) and (2.12) will deliver the same result. As long as symmetry is maintained, the result does not depend on the assumptions about monetary policy (that money supplies are determined exogenously with shocks that are independent of equity shocks.) The result also does not depend on our specification of the labor market as competitive with flexible wages. For example, a sticky-wage model in which employment was demand-determined would not alter the conditions that we used in the derivation of the home-bias result.

Further insights can be obtained from making use of some of the other equations of the model. Specifically, linearizing the first-order condition for holdings of money balances (and again using the fact that nominal prices are preset), we have:

$$m_t = \rho c_t. \tag{2.34}$$

Using this equation along with its foreign counterpart, and equation (2.33), we derive:

$$s_t = m_t - m_t^*. \tag{2.35}$$

Exchange rates are determined by relative money supplies.

The fact that equity demand depends only on the covariances after projecting on the exchange rate means that the equity portfolio is used only to hedge productivity shocks. Productivity shocks do not influence the amount of product the firm sells, which is demand determined in a sticky-price model. Nor do productivity shocks affect the exchange rate, which influences firm revenue as well. So firm revenue depends only on monetary shocks. A positive productivity shock, for example, allows the firm to produce the quantity demanded with less labor. Both wages and employment fall in equilibrium. Profits increase by the exact amount of the drop in labor income. But the effect of those shocks on household income is fully hedged when home households hold 100 percent of home firms.

Monetary shocks have real consequences in this model. Indeed, equation (2.34) shows that in equilibrium, consumption is determined only by money supplies. As we have noted, productivity shocks only affect the distribution of revenues between labor income and profits, but in equilibrium the effects of that redistribution is nullified by the complete home bias in equity holdings. The real effects of monetary shocks are hedged through the forward position in foreign exchange.

Suppose, for example, that there is a negative home money shock. In equilibrium, income of home households falls because both labor and profit income fall. But the drop in the home money supply also causes a home currency appreciation ($s_t$ declines.) The equilibrium value of $\delta$ is negative, given our assumption of $\rho > 1$. In this case, a decline in $s_t$ leads to a positive pay-off from the forward position. That is, when $\delta$ is negative, the home resident is short in foreign currency and long in home currency. So an appreciation yields a positive payoff, which hedges the effects of monetary shocks on labor and profit income.

It is interesting that our model implies that investors take a long position in home currency. Another well-established empirical fact is that home residents hold a disproportionate share of home-currency denominated bonds in their portfolios. In the static setting, of course there are no net bond holdings. Subject to the constraint of zero net bond holdings, home bias in nominal assets implies being long in home assets and short in foreign assets. This is the configuration implied by a negative value of $\delta$.

Notice that the forward position does not completely eliminate the effects of monetary shocks on income. From equation (2.27) we have that $(1-\zeta)\pi_t + \zeta(w_t + l_t)$ falls by $\frac{1}{2\rho} + \frac{1}{2}$ times the decrease in $m_t$ (because $c_t^W$ falls by $\frac{1}{2\rho}$ and $\frac{1}{2}s_t$ by $\frac{1}{2}$.) Including returns from the forward position solved from equation (2.31), $\delta = \frac{1}{2\rho} - \frac{1}{2}$, we find that income still falls by $\frac{1}{\rho}$ times the drop in $m_t$. Why? In this model, the Home and Foreign consumption markets are completely segmented. A change in the exchange rate causes a change in the relative prices paid by Home and Foreign households for identical goods, because nominal prices are set in advance in consumers' currencies and do not respond to shocks. So home prices rise relative to foreign prices (expressed in a common currency) when $s_t$ falls. But households cannot trade goods to arbitrage the difference in goods prices. As is well known, when consumer products are not tradable, the efficient configuration of consumption (achievable by complete markets) has consumption levels lower in the Home country (relative to the Foreign country) in those states of the world in which its goods prices are higher than those in the Foreign country. That is why the complete markets equilibrium condition (2.33) does not achieve perfect consumption correlation. So with a negative Home monetary shock, ceteris paribus, Home income falls and Home consumption declines.

# 3 Dynamic Model

In this section, we build an infinite-horizon model, which allows us to examine the effects of persistent technology shocks and different degrees of price stickiness. Most of the assumptions are the same as in the static model.

The price-setting rule is modified as follows. A fraction $\tau$ of firms in each country set prices in advance, and the rest of the firms can adjust their prices in each period after the realization of shocks. This approach allows us to study the portfolio allocation with or without sticky prices, and we can learn how different degrees of price stickiness affect the portfolio. There are different types of firms in each country but we assume the equities of all firms in each country bundled together.

An important question under the dynamic model is, how will persistent shocks affect the optimal portfolio? In a flexible price setting, the optimal portfolio is more foreign skewed than it is in the classic endowment economy case, as shown in Baxter and Jermann (1997). This effect decreases the degree of home bias in our model as well. In the dynamic model, when the elasticity of substitution between Home and Foreign goods is more than unity ($\omega > 1$), the optimal Home portfolio should be less home biased than it is in the static model because households must take into account the future after prices have been adjusted.

## 3.1 Household Problem

Home households maximize their expected utility:

$$\max \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, L_t\right),$$

subject to the following budget constraint:

$$\begin{aligned} & P_t C_t + M_t + Q_t \gamma_{h,\,t+1} + S_t Q_t^* \gamma_{f,\,t+1} \\ = & \gamma_{h,t}(Q_t + \Pi_t) + \gamma_{f,t} S_t (Q_t^* + \Pi_t^*) + (S_t - F_t)\tilde{\delta}_t + W_t L_t + M_{t-1} + Tr_t, \end{aligned} \tag{3.1}$$

where $Q_t$ ($Q_t^*$) denotes the price of Home (Foreign) equities. The utility function and consumption baskets are the same as in the static model. Households enter time $t$ with money $M_{t-1}$, equities ($\gamma_{h,t}, \gamma_{f,t}$), and forward contracts $\tilde{\delta}_t$. After the realization of shocks, households choose the consumption level, real money balances, and labor supply. The dividends from firms are paid at time $t$ depending on $\gamma_{h,t}$ and $\gamma_{f,t}$, and households get the payoff from the forward contract. They receive the transfer from the government as well. Finally, household will choose forward contracts $\tilde{\delta}_{t+1}$ and equity holdings $\gamma_{h,\,t+1}$, $\gamma_{f,\,t+1}$, which will determine the dividends households receive in time $t+1$.

The first order conditions for the households are

$$
\begin{aligned}
\frac{\chi}{M_t} &= \frac{C_t^{-\rho}}{P_t} - \mathrm{E}_t\,\beta\frac{C_{t+1}^{-\rho}}{P_{t+1}}, && (3.2)\\
\eta L_t^{\psi} &= \frac{C_t^{-\rho}}{P_t}W_t, && (3.3)\\
\mathrm{E}_{t-1}\left(\frac{C_t^{-\rho}}{P_t}S_t\right) &= \mathrm{E}_{t-1}\left(\frac{C_t^{-\rho}}{P_t}\right)F_t, && (3.4)\\
\frac{C_{t-1}^{-\rho}}{P_{t-1}}Q_{t-1} &= \mathrm{E}_{t-1}\left(\beta\frac{C_t^{-\rho}}{P_t}(Q_t+\Pi_t)\right), && (3.5)\\
\frac{C_{t-1}^{-\rho}}{P_{t-1}}S_{t-1}Q_{t-1}^{*} &= \mathrm{E}_{t-1}\left(\beta\frac{C_t^{-\rho}}{P_t}S_t(Q_t^{*}+\Pi_t^{*})\right). && (3.6)
\end{aligned}
$$

First, let $D_{t,\,t+s} \equiv \frac{C_{t+s}^{-\rho}}{P_{t+s}}/\frac{C_t^{-\rho}}{P_t}$. The no-bubble solution for equity prices implies that

$$
Q_t = \sum_{s=1}^{\infty}\mathrm{E}_t\,\beta^s D_{t,\,t+s}\Pi_{t+s}, \qquad S_tQ_t^{*} = \sum_{s=1}^{\infty}\mathrm{E}_t\,\beta^s D_{t,\,t+s}S_{t+s}\Pi_{t+s}^{*}. \tag{3.7}
$$

These are simply discounted sums of expected future dividends.

Let

$$
\begin{aligned}
V_t \equiv& \gamma_{h,t+1}Q_t + \gamma_{f,t+1}S_tQ_t^{*}, && (3.8)\\
H_t \equiv& \sum_{s=1}^{\infty}\beta^s E_t D_{t,\,t+s}W_{t+s}L_{t+s}, && (3.9)\\
R_t \equiv& \frac{\beta(Q_t+\Pi_t)}{Q_{t-1}}, && (3.10)\\
R_t^{H} \equiv& \frac{\beta(H_t+W_tL_t)}{H_{t-1}}, && (3.11)\\
\gamma_{t+1} \equiv& \frac{\gamma_{f,t+1}S_tQ_t^{*}}{V_t} = \left(1-\frac{\gamma_{h+1,t}Q_t}{V_t}\right). && (3.12)
\end{aligned}
$$

These are, respectively, financial wealth, human capital, the rate of return on financial wealth and human capital and a share of foreign equity in equity portfolio.

We can rewrite the budget constraint (3.1) for time $t$:

$$P_tC_t + V_t + H_t = V_{t-1}(1-\gamma_t)\beta^{-1}R_t + V_{t-1}\gamma_t\beta^{-1}\frac{S_t}{S_{t-1}}R_t^* + H_{t-1}\beta^{-1}R_t^H + \tilde{\delta}_t(S_t - F_t). \quad (3.13)$$

Assuming $\mathrm{E}_t(M_{t+s}^{-1}) = M_t^{-1}$, we get

$$\frac{C_t^{-\rho}}{P_t} = \chi M_t^{-1} + \mathrm{E}_t\,\beta\frac{C_{t+1}^{-\rho}}{P_{t+1}} = \frac{\chi}{1-\beta}M_t^{-1}; \quad (3.14)$$

therefore, $D_{t,\,t+s} = \dfrac{M_t}{M_{t+s}}$. The first order conditions for equity holdings can be summarized as

$$\mathrm{E}_{t-1}\left(\frac{M_{t-1}}{M_t}R_t\right) = \mathrm{E}_{t-1}\left(\frac{M_{t-1}}{M_t}\frac{S_t}{S_{t-1}}R_t^*\right) = 1. \quad (3.15)$$

## 3.2 Firms

Firms use the same linear technology as in the previous section. We have two types of firms in each country. A fraction $\tau$ of firms set the price in advance, and the rest set the price after the realization of shocks. The profit maximization problem of the Home firm with price flexibility is

$$\max P_{h,t}(i)Y_{h,t}(i) + S_tP_{h,t}^*(i)Y_{h,t}^*(i) - \left(\frac{W_t}{A_t}\right)[Y_{h,t}(i) + Y_{h,t}^*(i)].$$

Because $Y_{h,t}(i)$ is not a function of $P_{h,t}^*(i)$, and $Y_{h,t}(i)^*$ is not a function of $P_{h,t}(i)$, the problem is easy to solve:

$$P_{h,t}(i) = \frac{\lambda}{\lambda-1}\frac{W_t}{A_t} \equiv P_{flex,h,t}, \quad P_{h,t}(i)^* = \frac{\lambda}{\lambda-1}\frac{W_t}{A_tS_t} \equiv P_{flex,h,t}^*, \quad (3.16)$$

where $P_{flex,h,t}$ is the optimal price for the Home market of the Home goods produced by the firms that can adjust prices after they observe shocks. $P_{flex,h,t}^*$ is the optimal price for the Foreign market.

The other optimal prices are

$$P_{preset,h,t} \equiv \frac{\lambda}{\lambda-1}\frac{\mathrm{E}_{t-1}\left[\tilde{D}_t\frac{W_t}{A_t}\left(\frac{1}{P_{h,t}}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega}C_t\right]}{\mathrm{E}_{t-1}\left[\tilde{D}_t\left(\frac{1}{P_{h,t}}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega}C_t\right]}, \tag{3.17}$$

$$P^*_{preset,h,t} \equiv \frac{\lambda}{\lambda-1}\frac{\mathrm{E}_{t-1}\left[\tilde{D}_t\frac{W_t}{A_t}\left(\frac{1}{P^*_{h,t}}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega}C^*_t\right]}{\mathrm{E}_{t-1}\left[\tilde{D}_tS_t\left(\frac{1}{P^*_{h,t}}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega}C^*_t\right]}, \tag{3.18}$$

where $\tilde{D}$ is the stochastic discounted factor, and $P_{preset,h,t}$ is the optimal price for the Home market at time $t$ of the goods produced by the firms that set prices in advance. Now we can rewrite the price indexes as follows:

$$P_{h,t} = \left[(1-\tau)P^{1-\lambda}_{flex,h,t} + \tau P^{1-\lambda}_{preset,h,t}\right]^{\frac{1}{1-\lambda}}, \tag{3.19}$$

$$P_{f,t} = \left[(1-\tau)P^{1-\lambda}_{flex,f,t} + \tau P^{1-\lambda}_{preset,f,t}\right]^{\frac{1}{1-\lambda}}. \tag{3.20}$$

Since we have CES sub-utility functions, the market clearing condition can be obtained by equating the output with the sum of the demands for Home goods:

$$A_tL_t = \frac{1}{2}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega}C_t + \frac{1}{2}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega}C^*_t. \tag{3.21}$$

While flexible-price firms will have higher profit than preset-price firms in general, CES sub-utility makes the aggregate profit of each countries the same as before:

$$\Pi_t = \frac{1}{2}P_{h,t}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega}C_t + \frac{1}{2}S_tP^*_{h,t}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega}C^*_t - W_tL_t, \tag{3.22}$$

$$S_t\Pi^*_t = \frac{1}{2}S_tP^*_{f,t}\left(\frac{P^*_{f,t}}{P^*_t}\right)^{-\omega}C^*_t + \frac{1}{2}P_{f,t}\left(\frac{P_{f,t}}{P_t}\right)^{-\omega}C_t - S_tW^*_tL^*_t. \tag{3.23}$$

We assume that

$$m_{t+1} = m_t + \nu_t^m, \qquad m_{t+1}^* = m_t^* + \nu_t^{m^*}, \tag{3.24}$$

$$a_{t+1}^W = \varrho_W a_t^W + \nu_t^W, \qquad a_{t+1}^R = \varrho_R a_t^R + \nu_t^R, \tag{3.25}$$

where $\varrho_W \in [0,1]$, $\varrho_R \in [0,1)$ are degrees of persistence in world and relative technology levels and where $\nu$s are zero-mean i.i.d. shocks. We assume $\text{var}(\nu^m) = \text{var}(\nu^{m*}) = \sigma_m^*$ and $\text{cov}(\nu^m, \nu^{m*}) = \sigma_{m,m^*}$. Also $\text{var}(\nu^W) = \sigma_W^2$ and $\text{var}(\nu^R) = \sigma_R^2$ and $\text{cov}(\nu^W, \nu^R) = 0$. We assume initial symmetry between Home and Foreign, that is $a_0^R = 0$, $m_0^R = 0$. Here we allow the world technology to be a unit root process following the recent literature of business cycle. However, we assume the relative technology to be a mean reverting process. This assumption is realistic given technology diffusion between two countries.

## 3.3 Solution of the Dynamic Model

To solve the model, we use approximations similar to those in the static model. We take the first order approximation of fundamental variables that are determined after the shocks and take the second order approximation for asset holdings and pre-set price levels.[21] We denote $x_t$ as the deviation of $\ln(X_t)$ from its conditional mean, and we will also denote the world variables as $x_t^W = \frac{1}{2}x_t + \frac{1}{2}x_t^*$ , and the relative variables as $x_t^R = x_t - x_t^*$.

The Appendix presents the solution to the model. There, the equilibrium is defined and solutions for all the endogenous variables are given. It shows that the equilibrium conditions are satisfied for those solutions. The derivation of the solution is extremely algebra intensive. Here we discuss the salient features of the solution.

An important feature of the solution is that we are able to replicate the complete market allocation up to a linear approximation. We have two kinds of assets (equities and forward currency contracts) that span the space generated by $a_t^R$ and $m_t^R$. In that case, we have

$$\rho(c_t - c_t^*) = s_t + p_t^* - p_t, \tag{3.26}$$

[21]The second moments of the price levels do not play significant roles here.

This equation is the familiar condition that arises in complete markets in which consumer price levels are not equal (see, for example, Chari, Kehoe and McGrattan (2002).) Pushing the time subscripts one period forward and taking expectations at time $t$, we get

$$\mathrm{E}_t(c_{t+1}) = \mathrm{E}_t(c^*_{t+1}), \tag{3.27}$$

This equation follows because prices are sticky for at most one period, so purchasing power parity holds in expectation.

Equation (3.27) demonstrates a key sort of stationarity that emerges from our dynamic solution. Even though consumption levels might differ between Home and Foreign households at any time, looking forward they are always expected to be equal. That follows because, as we show,

$$\zeta v_t^R + (1-\zeta) h_t^R - s_t = 0. \tag{3.28}$$

This equation means that relative total wealth, which is the sum of financial wealth and human capital, is equalized between Home and Foreign households. To be clear, $V_t$ is defined as the value of equities that the home household acquires at time $t$ and carries into period $t+1$, and $H_t$ is the expected value at time $t$ of returns to work from $t+1$ onward. So equation (3.28) says that the wealth levels of Home and Foreign households at the end of period $t$ are equal.

This equality of wealth occurs even though in equilibrium Home and Foreign households hold different equity portfolios. Since the conditionally expected return on equities depends on the realization of shocks, $v_t^R \neq 0$ in general. That is, the conditionally expected discounted payoffs on the Home and Foreign equity portfolios differ. In addition, $h_t^R \neq 0$. The value of human capital for Home and Foreign households also depends on the realization of shocks, and so they are not in general equal.

Why then is relative total wealth equal? Suppose there is a positive relative technology shock, $a_t^R > 0$, but no change in world productivity so that Home productivity rises and Foreign productivity falls. Hold monetary shocks equal to zero. In this case, we can show that neither Home nor Foreign consumption levels will be changed by the $a_t^R$ shock in equilibrium, which is convenient for this example.

Period $t$ wage income of Home workers falls when prices are sufficiently sticky, and period $t$ wage income of Foreign workers rises, as in the static model. The period $t$ profits of Home firms rise and period $t$ profits of Foreign firms fall. The current income of Home relative to Foreign might rise or fall. On the one hand, Home's relative labor income falls, but the profits Home households reap may be greater than that of Foreign households when there is home bias in equity holdings. Nonetheless, under the parameter configuration that delivers home bias, the overall income of Home falls relative to Foreign - the relative loss in wage income must outweigh any relative gain in profit income.

But, in this situation in which home bias arises, the relative decline in current income for Home is precisely offset by the gains Home gets in the value of its human wealth and the gain in the value of the equities that it carries into period t. The positive realization of $a_t^R$ pushes up $Q_t$ relative to $Q_t^*$ and $H_t$ relative to $H_t^*$. Home's total wealth - the sum of the income it receives in period $t$ from labor and profits, plus the value (after the realization of $a_t^R$ ) of the equity position it carries into period t, plus the value of its human wealth - is unchanged relative to Foreign. Since consumption levels are not affected by $a_t^R$ shocks, the relative wealth of Home and Foreign at the end of period $t$ is unchanged.

As a result of this stationarity, we show that $\delta_t$ and $\gamma_t$ are constant over time:

$$\delta \equiv \delta_t = \frac{1}{2}\left(\frac{1}{\rho} - 1\right)\tau. \tag{3.29}$$

$$\gamma \equiv \gamma_t = \gamma_t^* = \frac{1}{2}\frac{(\omega - 1)\left[\dfrac{(1-\tau)}{1+\omega(1-\tau)\psi} + \dfrac{1}{\omega\psi + 1}\dfrac{\beta\varrho_R}{1-\beta\varrho_R}\right]}{\dfrac{\tau\zeta}{1+\omega(1-\tau)\psi} + (1-\zeta)(\omega-1)\left[\dfrac{(1-\tau)}{1+\omega(1-\tau)\psi} + \dfrac{1}{\omega\psi+1}\dfrac{\beta\varrho_R}{1-\beta\varrho_R}\right]}. \tag{3.30}$$

Home bias can be optimal in our model given certain parameter values. For example, when $\omega = 1$, the terms of trade adjustment insures against the effects on relative wealth from productivity shocks. The share of Home or Foreign goods in consumption expenditure does not change because of the Cobb-Douglas sub-utility function. Hence, households care only about the distribution between labor and firms, as is the case in the static model. Therefore, we get 100 per cent home bias: $\gamma = 0$. However, if we set up the model with flexible prices by letting $\tau = 0$, then the optimal equity portfolio is $\gamma = \frac{1}{2}\frac{1}{1-\zeta} > \frac{1}{2}$. This outcome is similar to the theoretical result obtained by Baxter and Jermann (1997)– "the international diversification puzzle worse than you think." If $\omega = 1$ and all prices are flexible, then $\gamma$ is indeterminate. This is similar to the model by Obstfeld and Rogoff (2002), in which asset trade is not needed because of the Cobb-Douglas specification for the consumption index of Home and Foreign goods.

In order to have home bias, or $\gamma < 1/2$, we generally need[22]

$$\frac{1-\omega(1-\tau)}{1+\omega(1-\tau)\psi} - \frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R} > 0. \tag{3.31}$$

Notice that the condition (3.31) does not depend on $\rho$ or $\zeta$, while $\zeta$ determines the level of home bias. To examine the role of different degrees of price stickiness, we find it useful to consider some extreme cases. It is apparent that if $\beta$ is close to zero and $\tau = 1$, then strong home bias should be optimal. In this model, one period corresponds to the time needed for firms to adjust their prices. This result is not surprising because if prices are very sticky, then this model behaves as if it were a static model.

On the other hand, if $\beta$ is close to one and $\tau = 1$, then $\omega$, the elasticity of substitution between Home and Foreign goods, plays an important role. What role does this parameter ($\omega$) play? The technology shock will have a significant impact once prices adjust if Home and Foreign goods are substitutes for one another. When Home receives a negative technology shock, the demand for Home goods shifts to Foreign goods after prices are adjusted. This fall in demand for Home goods implies that Home firms will cut their labor inputs. In order to hedge against this employment risk, a Home household wants to have Foreign equities because Foreign firms will generate more profit than will Home firms suffering from the negative technology shock. Thus, sticky prices lead to home bias, as we have seen in static model, while flexible prices lead to foreign bias. If the effect from price stickiness is bigger, then home bias will be optimal. Under flexible prices, a positive technology shock enables firms to produce goods more cheaply and to sell them more cheaply so that nominal sales

[22]We omit the case in which the denominator is non-positive: this case can happen only if the price is very flexible and $\omega \leq 1$.

will increase if $\omega > 1$. Although the demand for labor will decrease from the direct effect of the technology shock, the demand for goods will increase and thus indirectly increase the demand for labor.

Persistence in the relative technology shock, $\varrho_R$ , affects the optimal portfolio in precisely the same way as the discount factor, $\beta$. Indeed, it is only the product of the two, $\beta\varrho_R$ , that enters expression (3.30). When productivity shocks are more persistent, there is less home bias. In the limit, as $\beta\varrho_R \to 1$ , the portfolio approaches the flexible price value, $\gamma = \frac{1}{2}\frac{1}{1-\zeta}$. On the other hand, as $\beta\varrho_R \to 0$ , the portfolio approaches $\gamma = \frac{1}{2}\frac{(\omega-1)(1-\tau)}{\tau\zeta + (1-\zeta)(\omega-1)(1-\tau)}$. This latter value is precisely the level $\gamma$ would take in the static model if a fraction $\tau$ of prices were preset.

## 3.4 Empirical Support for the Model

We consider three types of empirical support for the model. First, we calibrate the amount of home bias implied by the equilibrium equity share given in equation (3.30). Second, we review the macroeconomic empirical evidence on the negative correlation of returns to human capital and equity returns. Third, we discuss the implications of the model for home bias from the partial equilibrium standpoint of the investor, and point to how our model relates to the relevant empirical evidence.

We can calibrate the amount of home bias implied by the model. The share of the Home household's equity portfolio held in foreign shares, $\gamma$ , depends on the price stickiness parameter, $\tau$ ; labor's share, $\zeta$ ; the elasticity of substitution between home and foreign aggregates, $\omega$ ; the discount factor, $\beta$ ; the persistence of relative productivity shocks, $\varrho_R$ ; and, the elasticity of labor supply, $\psi$.

We set $\tau = 1$ and then calibrate the length of a period by using estimates of the speed of price adjustment. With $\tau = 1$ , the half-life of price adjustment is one-half of a period. In our model, the speed of price adjustment determines the rate of convergence toward purchasing power parity. Rogoff (1996) has noted that studies of purchasing power parity imply a half-life of the real exchange rate of 3-5 years. We will pick a much faster speed of adjustment of 1 year, which is far below the lower end of the range cited by Rogoff. This implies that one period is equal to two years.

Following Backus, Kehoe and Kydland (1992), we set $\zeta = 2/3$. The estimates of Backus et al. (1992) give us on quarterly data that the autocorrelation of relative productivity shocks is 0.855, so we set $\varrho_R = (0.855)^8 \approx 0.286$. Likewise, the quarterly discount factor in Backus et al. is 0.99, so we take $\beta = (0.99)^8 \approx 0.923$. We follow Backus, Kehoe and Kydland (1994) and Chari et al. (2002) and set $\omega = 1.5$. We follow Obstfeld and Rogoff (2002), and set $\psi = 1$.

With this baseline set of parameters, we find $\gamma \approx 0.052$. That is, the model is capable of explaining a substantial amount of home bias. The model is perfectly symmetric between home and foreign countries, so an unbiased portfolio would be $\gamma = 0.5$. We have been fairly conservative in picking the degree of price stickiness. A greater degree of price stickiness would imply even more home bias.

In our model, negative conditional correlation between labor hours and productivity conditioning on productivity shock is the key driving force for home bias. However, because households can hedge demand shock through forward contracts, the unconditional correlation can be positive. It is important to distinguish between conditional and unconditional correlation in our model.

Galí (1999) has addressed precisely this issue. He has noted that real business cycle models tend to imply a positive correlation between hours and productivity. He shows in a simple closed-economy New Keynesian macroeconomic model that there is a negative correlation between hours and output per worker when there is a productivity shock. The reasoning is much the same as that in our model.

Galí goes on to derive empirical support for this implication of sticky-price models. He estimates a structural bivariate VAR on total labor hours and labor productivity using U.S. data.[23] The model was estimated on quarterly data from 1948:I to 1994:IV. There are two types of shocks in the model, which Galí classifies as technology shocks and non-technology shocks. The non-technology shocks can be associated with aggregate demand shocks. Under his identification scheme, only technology shocks can permanently increase

[23]He also uses employment instead of labor hours, and finds the same result holds for all G7 countries except Japan.

labor productivity.

Galí finds that the conditional correlation between labor hours and productivity is negative for technology shocks, while the unconditional correlation is positive. Rotemberg (2003) finds similar results. If prices were flexible, in traditional real business cycle models, the correlation conditional on technology shocks would be positive - as it is in our model in the long run.

Galí's findings have not gone unchallenged.[24] Christiano, Eichenbaum and Vigfusson (2003) substitute labor hours per capita for Galí's total labor hours and reverse Galí's finding on the conditional correlation. However, Francis and Ramey (2003) use the same measure, but quadratically detrended, and find the negative correlation between hours per capita and productivity conditional on technology shocks. Galí, López-Salido and Vallés (2003) find a similar result, using first-differences in hours per capita. Francis and Ramey (2004) create a new measure of hours per capita and confirm that a positive technology shock will reduce labor hours in the short run. While there is no consensus yet on the sign of the conditional correlation, there is some significant empirical support for the contention that it is negative.

We can also consider home bias from the perspective of the partial equilibrium of the household that takes the returns to human capital and equities as given. That is, instead of calibrating the full general equilibrium model, we can consider the implications of our set-up for the portfolio of the individual investor. Indeed, Bottazzi et al. (1996) have found evidence in favor of home bias generated by a negative covariance between human capital and the relative returns to equities. The second term on the right-hand-side of our equation (2.26) from the static model is very similar to the expression estimated by Bottazzi et al.. Their measure of home bias (the deviation from the symmetric portfolio) can be written as:

$$\frac{\zeta}{1-\zeta}\frac{\mathrm{cov}(r_t^H, r_t^e - r_t^{e*})}{\mathrm{var}(r_t^e - r_t^{e*})}.$$

In the notation here, $r_t^H$ is the return on human capital, $r_t^e$ is the return on home equities, and $r_t^{e*}$ is the return on foreign equities. In the Bottazzi et al. model, assets have

[24]See Galí and Rabanal (2004) for details.

real payoffs - as if they are indexed to inflation. All of the returns in the above expression are real returns in their model and estimation. Our static model implies an identical measure of home bias, except that in our model assets pay off in nominal terms. All of the variables above should be interpreted as the residual from projecting the nominal returns (expressed in the home currency) onto the exchange rate change. Recall that in our model, risk arising from nominal exchange rate fluctuations is hedged by forward contracts in foreign exchange.

We summarize in Table 2 the findings of Bottazzi et al. That studies uses two different measures of the returns to equities - one measured from economic fundamentals and the other using returns from equity markets. We report results for the fundamentals measure in the first panel, and the financial measure in the second panel. $Corr(r^H, r)$ refers to the correlation of returns to human capital with returns to domestic physical capital in each of the countries listed, and $Corr(r^H, r^*)$ refers to the correlation of the returns to human capital with returns to physical capital in the rest of the world for each country. *Home bias* is the deviation from the standard portfolio implied by their calculation. (Roughly speaking, if *home bias* = 0.5 with country size =0.5, there would be complete home bias in equity holdings. A value greater than its country size implies a portfolio that is short in the foreign equity.) Their findings indicate an average home bias of 0.35 based on these calculations. Thus a substantial amount of home bias can potentially be explained by the negative correlation of wages with returns to capital.

# 4 Conclusion

As we have demonstrated, our model can generate home bias in equity holdings under reasonable assumptions. The key assumption we need is price stickiness. In our model, output is demand determined when prices are sticky. An increase in Home productivity will reduce the demand for labor, but Home firms become prosperous thanks to lower labor costs. These opposing effects on labor income and the profit of domestic firms induce home bias. In a dynamic model, persistent technology shocks will reduce this effect because once prices are adjusted, both firms and households can benefit from the positive technology shock. Nonetheless, home bias in equity holdings can still exist in the presence of persistent technology shocks for reasonable ranges of the parameters. This theoretical result is supported by the empirical findings of Bottazzi et al. (1996), which indicate that, as shown in Table 2, most OECD countries should have home bias ($0 < \gamma < 1$) based on a partial-equilibrium model that uses a continuous-time VAR with wages, home profits, and foreign profits. The source of this negative correlation between returns on human capital and returns on domestic equities in our model is the negative correlation between labor and technology shocks which is also supported by empirical findings in Galí (1999) and Francis and Ramey (2003), among others.

In both the static and dynamic models, the allocation replicates complete markets up to a linear approximation. This is in a sense a shortcoming of our model since the complete markets allocation leaves other puzzles unsolved. That is, models incorporating complete asset markets do not explain the high volatility of the observed exchange rate or the consumption-real exchange rate anomaly as described in Chari et al. (2002).

Although we believe that our model provides an important theoretical foundation for home bias, we also believe that there are other factors, such as information costs, that may explain home bias. The economic forces that lead to home bias in our model do not require the exclusion of other considerations that have been raised in the literature. Our model may provide fertile ground for further investigation into the behavior of international equity markets, since it delivers an algebraic solution to a fully dynamic optimizing general equilibrium model with sticky nominal prices.

# Appendix

## A Solution of the Dynamic Model

An equilibrium satisfies the first order conditions, budget constraint and market clearing conditions. First we define an equilibrium formally. Then, we will list the linearized first order conditions and redefine equilibrium in reduced linearized form. Finally, we show that a complete market allocation satisfies those equilibrium conditions.

*Definition A*

An *equilibrium* is a set of sequences[25] $\{C_t,\ L_t,\ W_t,\ \tilde{\delta}_t,\ \gamma_t,\ C_{h,t},\ C_{f,t},\ C_{h,t}(i),\ C_{f,t}(i),$ $P_{flex,h,t},\ P_{flex,f,t},\ P_{preset,h,t},\ P_{preset,h,t},\ P_t,\ P_{h,t},\ P_{f,t},\ Q_t,\ V_t,\ H_t,\ R_t,\ R_t^H,\ \Pi_t, \gamma_{h,t},\ \gamma_{f,t}\}_{t=1}^{\infty}$ and their Foreign counterparts and $\{S_t,\ F_t\}$, which solves the system of 50 equations[26] consisting of (2.4), (2.7), (2.8), (3.3), (3.4), (3.7)-(3.22), and their foreign counterparts plus 3 asset markets clearing conditions,[27] given stochastic sequences $\{A_t, A_t^*, M_t, M_t^*\}$ and initial conditions $A_0 = A_0^*$, $M_0 = M_0^*$, $\gamma_0 = 0$, and $\gamma_0^* = 0$.

### A.1 Approximated System

In this section, we approximate the core first order conditions.

We denote $x_t$ as the deviation from the unconditional mean and $\hat{x}_t$ as the deviation from the conditional mean–that is, $\hat{x}_t \equiv x_t - \mathrm{E}_{t-1}\, x_t$ and $\hat{\mathrm{E}}_t x_{t+s} = \mathrm{E}_t \ln X_{t+s} - \mathrm{E}_{t-1} \ln X_{t+s}$. We will also denote the world variables as $x_t^W \equiv \frac{1}{2}x_t + \frac{1}{2}x_t^*$ and the relative variables as $x_t^R \equiv x_t - x_t^*$.

#### A.1.1 The first order conditions for households

The first order condition for consumption (3.14) can be linearized,

$$c_t = \frac{1}{\rho}(m_t - p_t). \tag{A.1}$$

[25] There are $24 \times 2 + 2$ variables.

[26] The number of equation should be 51, but one is redundant by Walras' Law.

[27] $\gamma_{h,t} + \gamma_{h,t}^* = 1$, $\gamma_{f,t} + \gamma_{f,t}^* = 1$,

Using above, equation (3.3) can be expressed as

$$w_t = \psi l_t + m_t. \tag{A.2}$$

The first order conditions for asset allocations[28], equations (3.4) and (3.15), can be approximated using second order approximations:

$$\mathrm{cov}_{t-1}(m_t^R, s_t) = \mathrm{var}_{t-1}(s_t), \tag{A.3}$$

$$\mathrm{cov}_{t-1}(-m_t, r_t) + \frac{1}{2}\,\mathrm{var}_{t-1}(r_t) = \mathrm{cov}_{t-1}(-m_t, s_t + r_t^*) + \frac{1}{2}\,\mathrm{var}_{t-1}(s_t + r_t^*). \tag{A.4}$$

### A.1.2 The first order conditions for firms

Firms set their prices optimally. The firs order conditions can be linearized as

$$\begin{aligned} p_{flex,h,t} &= (w_t - a_t), && \text{(A.5)}\\ p_{flex,f,t} &= (w_t^* - a_t^* + s_t) && \text{(A.6)}\\ p_{preset,h,t} &= \mathrm{E}_{t-1}(w_t - a_t) && \text{(A.7)}\\ p_{preset,f,t} &= \mathrm{E}_{t-1}(w_t^* - a_t^* + s_t). && \text{(A.8)} \end{aligned}$$

Thus, the prices of each category of goods (3.19 and 3.20) can be expressed as following:

$$\begin{aligned} p_{h,t} &= \tau\,\mathrm{E}_{t-1}(w_t - a_t) + (1-\tau)(w_t - a_t), && \text{(A.9)}\\ p_{f,t} &= \tau\,\mathrm{E}_{t-1}(w_t^* - a_t^* + s_t) + (1-\tau)(w_t^* - a_t^* + s_t). && \text{(A.10)} \end{aligned}$$

Combining these two, we get the expression for price index:

$$\begin{aligned} p_t =& \frac{1}{2}[\tau\,\mathrm{E}_{t-1}(w_t - a_t) + (1-\tau)(w_t - a_t)] \\ &+ \frac{1}{2}[\tau\,\mathrm{E}_{t-1}(w_t^* - a_t^* + s_t) + (1-\tau)(w_t^* - a_t^* + s_t)]. \end{aligned} \tag{A.11}$$

In order to determine the labor demand, we use the goods market clearing condition.

[28]Here we get $f_t = E_{t-1}s_t$ as part of the first order approximation.

Equation (3.21) can be linearized as

$$
\begin{aligned}
l_t &= \frac{1}{2}\{-\omega(p_{h,t} - p_t) + c_t\} + \frac{1}{2}\{-\omega(p^*_{h,t} - p^*_t) + c^*_t\} - a_t \\
&= -(1-\tau)\omega\frac{1}{2}(w^R_t - a^R_t - s_t) - \tau\omega\frac{1}{2}\,\mathrm{E}_{t-1}(w^R_t - a^R_t - s_t) + c^W_t - a_t
\end{aligned}
\tag{A.12}
$$

### A.1.3 The budget constraint

We log-linearize the budget constraint (3.13), to get

$$
\begin{aligned}
&p_t + c_t + \frac{\beta}{1-\beta}(1-\zeta)v_t + \frac{\beta}{1-\beta}\zeta h_t \\
=&\frac{1}{1-\beta}(1-\zeta)\left\{v_{t-1} + r_t - \gamma_{t-1}(r^R_t - \hat{s}_t)\right\} + \frac{1}{1-\beta}\zeta(h_{t-1} + r^H_t) + \delta_t\hat{s}_t.
\end{aligned}
\tag{A.13}
$$

Here, we use $f_t = E_{t-1}s_t$.

## A.2 Definition of Approximated Equilibrium

*Definition B*

An approximated equilibrium is a set of sequences $\{c_t,\ l_t,\ w_t,\ \delta_t,\ \gamma_t,\ p_t\}$ and their Foreign counterparts, and $\{s_t\}$ solve the system of equation (A.1)-(A.4), (A.11)-(A.13), and their Foreign counterparts, given sequences $\{m_t, m^*_t, a_t, a^*_t\}$ and initial conditions $a^R_0 = 0$, $m^R_0 = 0$, and $\gamma_0 = \gamma^*_0 = 0$. An approximated equilibrium is a reduced form of *Definition A*. Most omitted part can be easily verified and should not be confusing.

## A.3 Equilibrium Allocation

We conjecture that the following allocation is an equilibrium.

$$\begin{aligned} l_t^R =& \frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} a_t^R + \frac{\omega\tau}{1+\omega(1-\tau)\psi} \frac{\psi+1}{1+\omega\psi} \, \mathrm{E}_{t-1} \, a_t^R \\ =& \frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} a_t^R + \frac{\omega\tau}{1+\omega(1-\tau)\psi} \frac{\psi+1}{1+\omega\psi} \varrho_R a_{t-1}^R \end{aligned} \tag{A.14}$$

$$\begin{aligned} l_t^W =& \frac{1}{\rho+(1-\tau)\psi} \left\{ (1-\tau-\rho) a_t^W + \tau m_t^W + \tau \, \mathrm{E}_{t-1} \left[ \frac{\rho(\psi+1)}{\rho+\psi} a_t^W - m_t^W \right] \right\} \\ =& \frac{1}{\rho+(1-\tau)\psi} \left\{ (1-\tau-\rho) a_t^W + \tau m_t^W + \tau \left[ \frac{\rho(\psi+1)}{\rho+\psi} \varrho_W a_{t-1}^W - m_{t-1}^W \right] \right\} \end{aligned} \tag{A.15}$$

$$w_t^R = \psi \left\{ \frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} a_t^R + \frac{\omega\tau}{1+\omega(1-\tau)\psi} \frac{\psi+1}{1+\omega\psi} \varrho_R a_{t-1}^R \right\} + m_t^R \tag{A.16}$$

$$w_t^W = \frac{\psi}{\rho+(1-\tau)\psi} \left\{ (1-\tau-\rho) a_t^W + \tau \left[ \frac{\rho(\psi+1)}{\rho+\psi} \varrho_W a_{t-1}^W - m_{t-1}^W \right] \right\} + \frac{\rho+\psi}{\rho+(1-\tau)\psi} m_t^W \tag{A.17}$$

$$p_t^R = \tau m_{t-1}^R + (1-\tau) m_t^R \tag{A.18}$$

$$p_t^W = -\frac{\rho\tau}{\rho+(1-\tau)\psi} \left[ \frac{\rho(\psi+1)}{\rho+\psi} \varrho_W a_{t-1}^W - m_{t-1}^W \right] - (1-\tau) \frac{\rho+\psi}{\rho+(1-\tau)\psi} \left[ \frac{\rho(\psi+1)}{\rho+\psi} a_t^W - m_t^W \right] \tag{A.19}$$

$$c_t^R = \frac{1}{\rho} \tau (m_t^R - m_{t-1}^R) \tag{A.20}$$

$$c_t^W = \frac{\tau}{\rho+(1-\tau)\psi} \left[ \frac{\rho(\psi+1)}{\rho+\psi} \varrho_W a_{t-1}^W + (m_t^W - m_{t-1}^W) \right] + (1-\tau) \frac{\psi+1}{\rho+(1-\tau)\psi} a_t^W \tag{A.21}$$

$$s_t = m_t^R. \tag{A.22}$$

$$\delta \equiv \delta_t = \frac{1}{2}\left(\frac{1}{\rho} - 1\right)\tau. \tag{A.23}$$

$$\gamma \equiv \gamma_t = \gamma_t^* = \frac{1}{2}\frac{(\omega-1)\left[\frac{(1-\tau)}{1+\omega(1-\tau)\psi} + \frac{1}{\omega\psi+1}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]}{\frac{\tau\zeta}{1+\omega(1-\tau)\psi} + (1-\zeta)(\omega-1)\left[\frac{(1-\tau)}{1+\omega(1-\tau)\psi} + \frac{1}{\omega\psi+1}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]}. \tag{A.24}$$

Notice that this allocation replicates the complete markets allocation.

$$\rho(c_t - c_t^*) = s_t + p_t^* - p_t, \tag{A.25}$$

## A.4 Proof

We will show this allocation satisfies the equilibrium conditions.

### A.4.1 Fundamental Variables

We now prove that the first order conditions for fundamental variables and labor market clearing conditions are in fact satisfied.

It is immediate to confirm that equations (A.14) and (A.16) satisfies relative version of equation (A.2) and that equations (A.15) and (A.17) satisfies world version of equation (A.2).

Using equations (A.18) and (A.20), we can see the relative version of equation (A.1), $c_t^R = \frac{1}{\rho}(m_t^R - p_t^R)$, is satisfied. Using (A.21) and (A.19) we can also verify that the world version of (A.1) is satisfied.

We can also verify that (A.14), (A.16) and (A.22) satisfies relative version of labor market clearing condition (A.12):

$$l_t^R = -(1-\tau)\omega(w_t^R - a_t^R - s_t) - \tau\omega\,\mathrm{E}_{t-1}(w_t^R - a_t^R - s_t) - a_t^R \tag{A.26}$$

It is tedious but straightforward to verify that (A.15), and (A.21) satisfies the world version of labor market clearing condition (A.12):

$$l_t^W = c_t^W - a_t^W. \tag{A.27}$$

Using equations (A.17) and (A.19), and using (A.18) and (A.22), we can show

$$\begin{aligned} p_t^W =& \tau \,\mathrm{E}_{t-1}(w_t^W - a_t^W) + (1-\tau)(w_t^W - a_t^W), && \text{(A.28)} \\ p_t^R =& \tau \,\mathrm{E}_{t-1}\, s_t + (1-\tau) s_t. && \text{(A.29)} \end{aligned}$$

are satisfied.

So far, we have proved equations (A.1), (A.2), (A.11), (A.12) are satisfied.

### A.4.2 Returns on assets

In order to show that these allocation in fact satisfies the first order conditions for asset holdings, we want to calculate the rate of return on assets – human capital and equities.

Human capital can be expressed as,

$$h_t = \sum_{s=1}^{\infty} \mathrm{E}_t\, \beta^s (w_{t+s} + l_{t+s}) \tag{A.30}$$

Since $w_{t+s} + l_{t+s} = (\psi+1)\left(l_{t+s}^W + \frac{1}{2} l_{t+s}^R\right) + m_{t+s}^W + \frac{1}{2} m_{t+s}^R$, the return on the human capital is

$$\begin{aligned} r_{H,t} =& (1-\beta)\sum_{s=0}^{\infty} \hat{\mathrm{E}}_t \beta^s \left[(\psi+1)\left(l_{t+s}^W + \frac{1}{2} l_{t+s}^R\right) + m_t^W + \frac{1}{2} m_t^R\right] \\ =& (1-\beta)(\psi+1)\Bigg\{\frac{1}{\rho+(1-\tau)\psi}\left[(1-\tau-\rho)\hat{a}_t^W + \tau \hat{m}_t^W\right] + \frac{1-\rho}{\rho+\psi}\frac{\beta\varrho_W}{1-\beta\varrho_w}\hat{a}_t^W \qquad \text{(A.31)} \\ &+ \frac{1}{2}\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]\hat{a}_t^R\Bigg\} + \left(\hat{m}_t^W + \frac{1}{2}\hat{m}_t^R\right). \end{aligned}$$

Subtracting the foreign counterpart, we get the relative return on human capital:

$$r_t^{HR} = (1-\beta)(\psi+1)\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]\hat{a}_t^R + \hat{m}_t^R. \tag{A.32}$$

Now we will express return on the Home equity in terms of exogenous variables. Log-linearizing equation (3.10), we get

$$r_t = (1-\beta)\sum_{s=0}^{\infty}(\beta^s \hat{\mathrm{E}}_t \pi_{t+s}). \tag{A.33}$$

Following similar step as in the return on human capital, we get the return on equity:

$$\begin{aligned} r_t =&(1-\beta)(\psi+1)\left\{\left[\frac{(1-\tau)(1-\rho)}{\rho+(1-\tau)\psi}+\frac{1}{1-\zeta}\frac{\tau\rho\zeta}{\rho+(1-\tau)\psi}+\frac{1-\rho}{\rho+\psi}\frac{\beta\varrho_W}{1-\beta\varrho_W}\right]\hat{a}_t^W\right.\\ &\left.+\frac{1}{2}\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi}+\frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi}+\frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]\hat{a}_t^R\right\}\\ &+\left\{1+\frac{1-\beta}{1-\zeta}\frac{1-\rho-\zeta(\psi+1)}{\rho+(1-\tau)\psi}\tau\right\}\hat{m}_t^W+\frac{1}{2}\hat{m}_t^R. \end{aligned} \tag{A.34}$$

Subtracting the foreign counterpart, we get

$$r_t^R =(1-\beta)(\psi+1)\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi}+\frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi}+\frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]\hat{a}_t^R+\hat{m}_t^R \tag{A.35}$$

### A.4.3 Asset Allocation

Since we replicate complete markets, these allocations should satisfy the first order conditions for the asset allocation as expressed in equations (3.4) and (3.15). We will prove that linearized version of them (A.3) and (A.4) are satisfied. Recall these second order approximations are

$$\mathrm{cov}_{t-1}(m_t^R, s_t) = \mathrm{var}_{t-1}(s_t), \tag{A.36}$$

$$\mathrm{cov}_{t-1}(-m_t, r_t) + \frac{1}{2}\mathrm{var}_{t-1}(r_t) = \mathrm{cov}_{t-1}(-m_t, s_t + r_t^*) + \frac{1}{2}\mathrm{var}_{t-1}(s_t + r_t^*). \tag{A.37}$$

Given $s_t = m_t^R$, we can easily see the first equation is satisfied.

Using $r_t = r_t^W + \frac{1}{2}r_t^R$, the second equation can be rewritten as,

$$\mathrm{cov}_{t-1}(m_t, r_t^R - m_t^R) + \frac{1}{2}\mathrm{var}_{t-1}\left(m_t^R + r_t^W - \frac{1}{2}r_t^R\right) - \frac{1}{2}\mathrm{var}_{t-1}\left(r_t^W + \frac{1}{2}r_t^R\right) = 0 \tag{A.38}$$

It is easy to see that the first term is zero.

$$\mathrm{cov}_{t-1}(m_t, r_t^R - m_t^R) = \mathrm{cov}_{t-1}\left(m_t, \Upsilon a_t^R\right) = 0 \tag{A.39}$$

where,

$$\Upsilon = (1-\beta)(\psi+1)\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi} + \frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi} + \frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right].$$

The intuition is the same as static model. Because forward contracts provide hedge against monetary shocks, the relative return on equity after adjusted monetary shocks does not correlated with Home monetary shocks.

The second and third terms

$$\mathrm{var}_{t-1}\left(m_t^R + r_t^W - \frac{1}{2}r_t^R\right) - \mathrm{var}_{t-1}\left(r_t^W + \frac{1}{2}r_t^R\right) = \mathrm{var}_{t-1}(m_t^R) - 2\,\mathrm{cov}_{t-1}\left(m_t^R, \frac{1}{2}r_t^R\right) \tag{A.40}$$

Because $\mathrm{cov}_{t-1}\left(m_t^R, \frac{1}{2}r_t^R\right) = \frac{1}{2}\,\mathrm{var}_{t-1}(m_t^R)$, we confirm that this allocations in fact satisfies the first order conditions for asset allocations.

### A.4.4 Budget Constraint

In order for us to show that complete market allocation is an equilibrium allocation, we have to show that budget constraint is also satisfied for any realization of exogenous variables.

We log-linearize the budget constraint (3.13), we get

$$\begin{aligned} &p_t + c_t + \frac{\beta}{1-\beta}(1-\zeta)v_t + \frac{\beta}{1-\beta}\zeta h_t \\ =&\frac{1}{1-\beta}(1-\zeta)\left\{v_{t-1} + r_t - \gamma_{t-1}(r_t^R - \hat{s}_t)\right\} + \frac{1}{1-\beta}\zeta(h_{t-1} + r_t^H) + \delta_t\hat{s}_t. \end{aligned} \tag{A.41}$$

First, world budget constraint expressed in Home currency is following.

$$\begin{aligned} &p_t^w + c_t^w + \frac{\beta}{1-\beta}\{(1-\zeta)v_t^W + \zeta h_t^W\} \\ &= \frac{1}{1-\beta}(1-\zeta)\left(r_t^W + v_{t-1}^W\right) + \zeta\frac{1}{1-\beta}\left[{r_t^H}^W + h_{t-1}^W\right]. \end{aligned} \tag{A.42}$$

where we have used $\gamma_t = \gamma_t^*$.

The world budget constraint holds with any realization of $a_t^W$ and $m_t^W$ since equation (A.42) simply indicates that total world wealth carried over into the next period is equal to the value of previous wealth, plus returns, less world consumption. More explicitly, because

$$v_t^W + h_t^W = \frac{1-\beta}{\beta} E_t \sum_{s=1}^{\infty} \beta^s (\pi_{t+s}^W + w_{t+s}^W + l_{t+s}^W) = \frac{1-\beta}{\beta} E_t \beta^s (p_{t+s}^W + c_{t+s}^W), \tag{A.43}$$

both sides of the equation are the sum of future consumption.

Finally, we examine relative budget constraint.

$$\begin{aligned} & p_t^R + c_t^R - \hat{s}_t + \frac{\beta}{1-\beta}[(1-\zeta)v_t^R + \zeta h_t^R] \\ & = \frac{1}{1-\beta}(1-\zeta)\left[r_t^R - \hat{s}_t + v_{t-1}^R - (\gamma_t + \gamma_t^*)(r_t^R - \hat{s}_t)\right] + \zeta \frac{1}{1-\beta}\left[r_t^{HR} - \hat{s}_t + h_{t-1}^R\right] + 2\delta_t \hat{s}_t. \end{aligned} \tag{A.44}$$

Here we use mathematical induction to show first that $(1-\zeta)v_t^R + \zeta h_t^R = 0$, for all $t$.

1. $(1-\zeta)v_t^R + \zeta h_t^R = 0$, for $t = 0$ by assumption.
2. Assume $(1-\zeta)v_{t-1}^R + \zeta h_{t-1}^R = 0$, for $t-1$.
3. Prove $(1-\zeta)v_t^R + \zeta h_t^R = 0$, for $t$

Recall $s_t = m_t^R$. Using these and $\gamma = \gamma^*$, now we can rewrite budget constraint.

$$\begin{aligned} & (\frac{1}{\rho} - 1)\tau \hat{m}_t^R + \frac{\beta}{1-\beta}[(1-\zeta)v_t^R + \zeta h_t^R] \\ & = \frac{1}{1-\beta}(1-\zeta)\left[r_t^R - \hat{m}_t^R - 2\gamma_t(r_t^R - \hat{m}_t^R)\right] + \zeta \frac{1}{1-\beta}\left[r_t^{HR} - \hat{m}_t^R\right] + 2\delta_t \hat{m}_t^R. \end{aligned} \tag{A.45}$$

Using relative returns (A.32) and (A.35), we get

$$\left[\left(\frac{1}{\rho}-1\right)\tau-2\delta_t\right]\hat{m}_t^R+\frac{\beta}{1-\beta}[(1-\zeta)v_t^R+\zeta h_t^R]$$
$$=\left\{(1-2\gamma_t)(1-\zeta)(\psi+1)\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi}+\frac{1}{1-\zeta}\frac{\tau}{1+\omega(1-\tau)\psi}+\frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]\right.$$
$$\left.+\zeta(\psi+1)\left[\frac{\omega(1-\tau)-1}{1+\omega(1-\tau)\psi}+\frac{\omega-1}{1+\omega\psi}\frac{\beta\varrho_R}{1-\beta\varrho_R}\right]\right\}\hat{a}_t^R. \tag{A.46}$$

By substituting,

$$\delta = \frac{1}{2}\left(\frac{1}{\rho}-1\right)\tau$$
$$\gamma_t = \frac{1}{2}\frac{(\omega-1)\left[\dfrac{1-\tau}{1+\omega(1-\tau)\psi}+\dfrac{1}{1+\omega\psi}\dfrac{\beta\varrho_R}{1-\beta\varrho_R}\right]}{\dfrac{\tau\zeta}{1+\omega(1-\tau)\psi}+(1-\zeta)(\omega-1)\left[\dfrac{1-\tau}{1+\omega(1-\tau)\psi}+\dfrac{1}{1+\omega\psi}\dfrac{\beta\varrho_R}{1-\beta\varrho_R}\right]}.$$

into above, we can easily see that given this asset allocation, Home and Foreign relative total wealth, sum of human capital and financial wealth,

$$(1-\zeta)v_t^R+\zeta h_t^R=0.$$

Since the budget constraint holds in both relative and world forms, we show that our conjectured allocation satisfies all the first order conditions, market clearing conditions and budget constraints up to linear approximation. Q.E.D.

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Table 1: List of Notation

| parameter | description |
|---|---|
| $\beta$ | discount factor |
| $\rho$ | risk averse and inverse of the infratemporal substitution parameter |
| $\chi$ | real balance parameter |
| $\psi$ | inverse of the labor supply elasticity |
| $\omega$ | elasticity of substitution between Home goods and Foreign goods |
| $\lambda$ | degree of monopolistic power (also related to the labor share) |
| $\varrho_R$, $\varrho_W$ | persistence of the technology shock |
| $\zeta$ | the labor share in the national income; $\zeta \approx \frac{\lambda-1}{\lambda}$ |
| **any variable** | **description** |
| $X_t$ | Home variables |
| $X_t^*$ | Foreign variables corresponding to Home variable $X_t$ |
| $x_t$ | log deviation from the symmetric steady state |
| $x_t^R$ | relative variables; $x_t^R = x_t - x_t^*$ |
| $x_t^W$ | world variables; $x_t^W = nx_t + (1-n)x_t^*$ |
| $\hat{x}_t$ | unexpected log deviation ; $\hat{x}_t \equiv x_t - \mathrm{E}_{t-1}\, x_t$ |
| **variable** | **description** |
| $\gamma_h$ | Home equity share in the equity portfolio |
| $\gamma_f$ | Foreign equity share in the equity portfolio |
| $\gamma$ | degree of risk sharing; $\gamma = \frac{\gamma_f}{1-n} = \frac{1-\gamma_h}{1-n}$ |
| $\tilde{\delta}$ | number of forward contract |
| $\delta$ | normalized number of forward contract |
| $\tau$ | ratio of firms setting price in advance in the dynamic model |
| $\Pi_t$ | nominal profit of Home firms = dividend |
| $A_t$ | productivity |
| $B_t$ | bond holdings at the end of time $t$ |
| $C_t$ | consumption basket; $C_t = \left(n^{1/\omega} C_{h,t}^{(\omega-1)/\omega} + (1-n)^{1/\omega} C_{f,t}^{(\omega-1)/\omega}\right)^{\omega/(\omega-1)}$ |
| $C_{h,t}$ | consumption of Home goods; $C_{h,t} = \left[(\frac{1}{n})^{1/\lambda} \int_0^n C_{h,t}(i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}$ |
| $C_{f,t}$ | consumption of Foreign goods; $C_{f,t} = \left[(\frac{1}{n-1})^{1/\lambda} \int_n^1 C_{f,t}(i)^{(\lambda-1)/\lambda} di\right]^{\lambda/(\lambda-1)}$ |
| $C_{h,t}(i)$ | consumption of Home good $i$ |
| $C_{f,t}(i)$ | consumption of Foreign good $i$ |
| $F_t$ | forward rate delivered at time $t$ and set before the realization of a shock at time $t$ |
| $i_t$ | nominal interest rate (on domestic currency) from time $t$ to $t+1$ |
| $L_t$ | supply and demand of labor |
| $M_t$ | money balance |
| $P_t$ | price index for the consumption basket; $P_t = \left[nP_{h,t}^{1-\omega} + (1-n)P_{f,t}^{1-\omega}\right]^{1/(1-\omega)}$ |
| $P_{h,t}$ | price subindex for Home goods; $P_{h,t} = \left[\frac{1}{n} \int_0^n P_{h,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}$ |
| $P_{f,t}$ | price subindex for Foreign goods; $P_{f,t} = \left[\frac{1}{1-n} \int_n^1 P_{f,t}(i)^{1-\lambda} di\right]^{1/(1-\lambda)}$ |
| $P_{h,t}(i)$ | price of Home good $i$ |
| $P_{f,t}(i)$ | price of Foreign good $i$ |
| $Q_t$ | price of the Home stock in the dynamic model |
| $R_t$ | returns on Home equities |
| $S_t$ | nominal exchange rate |
| $T_t$, $Tr_t$ | transfer from the government |
| $W_t$ | nominal wage rate |

Table 2: Results From Bottazi et al.(1996)

| Country[a] | Fundamentals[b] | | | Financial[b] | | |
|---|---|---|---|---|---|---|
| | $Corr(r^H, r)$ | $Corr(r^H, r^*)$ | Home Bias[c] | $Corr(r^H, r)$ | $Corr(r^H, r^*)$ | *Home Bias* |
| Belgium | -0.57 | 0.17 | 1.01 | -0.63 | -0.14 | 0.85 |
| Canada | -0.50 | -0.33 | 0.84 | -0.84 | -0.64 | 1.82 |
| France | -0.23 | -0.06 | 0.15 | -0.25 | -0.06 | 0.02 |
| Germany | 0.53 | -0.13 | -1.22 | -0.01 | 0.26 | 0.25 |
| Italy | -0.76 | 0.09 | 0.48 | -0.27 | -0.48 | -0.40 |
| Japan | -0.97 | 0.35 | 0.55 | -0.52 | -0.64 | 0.05 |
| Netherlands | -0.90 | -0.34 | 1.14 | -0.14 | -0.02 | 0.43 |
| Switzerland | -0.79 | 0.14 | 0.49 | -0.38 | 0.16 | 0.18 |
| UK | -0.39 | 0.13 | 0.38 | -0.47 | -0.30 | 0.09 |
| US | 0.96 | -0.31 | -0.17 | -0.40 | -0.05 | 0.19 |
| Average[a] | -0.34 | 0.04 | 0.31 | -0.39 | -0.19 | 0.35 |

Source. Bottazzi et al. (1996), table 3 and table 5.

[a] We omit some countries that were included in the fundamentals approach; for data on all included countries, see Bottazzi et al. (1996). The averages for the fundamental approach are calculated using countries not listed here.

[b] Roughly speaking, the fundamentals approach uses aggregate data to calculate the return on capital, while the financial approach uses the stock market index.

[c] "Home Bias" defined here is the difference between the percentage of stock market wealth invested domestically in the Bottazzi et al. model and the hypothetical percentage of domestic equity in a well-diversified international portfolio in a standard asset-only model. $r^H$ measures the return on human capital. $r$ and $r^*$ measure, respectively, the return on capital of home and the return on capital of the rest of the world.