8. Calculating Consumer Price Indices in Practice

Introduction

8.1 The purpose of this chapter is to provide a general description of the ways in which consumer price indices (CPIs) are calculated in practice. The methods used in different countries are not exactly the same, but they have much in common. There is clearly interest from both compilers and users of CPIs in knowing how most statistical offices actually calculate their CPIs.

8.2 As a result of the greater insights into the properties and behavior of price indices that have been achieved in recent years, it is now recognized that some traditional methods may not necessarily be optimal from a conceptual and theoretical viewpoint. Concerns have also been voiced in a number of countries about possible biases that may be affecting CPIs. These issues and concerns need to be considered in this manual. Of course, the methods used to compile CPIs are inevitably constrained by the resources available, not merely for collecting and processing prices, but also for gathering the expenditure data needed for weighting purposes. In some countries, the methods used may be severely constrained by lack of resources. Nonetheless, there are still methods that should be avoided at all costs because they result in severe bias in the indices.

8.3 The calculation of CPIs usually proceeds in two stages. First, price indices are estimated for the elementary expenditure aggregates, or simply elementary aggregates. Then these elementary price indices are averaged to obtain higher-level indices using the relative values of the elementary expenditure aggregates as weights. This chapter starts by explaining how the elementary aggregates are constructed, and what economic and statistical criteria need to be taken into consideration in defining the aggregates. The index number formulae most commonly used to calculate the elementary indices are then presented, and their properties and behavior illustrated using numerical examples. The pros and cons of the various formulae are considered, together with some alternative formulae that might be used instead. The problems created by disappearing and new varieties are also explained, as well as the different ways of imputing values for missing prices.

8.4 The chapter also discusses the calculation of higher-level indices. The focus is on the ongoing production of a monthly price index in which the elementary price indices are averaged, or aggregated, to obtain higher-level indices. Price-updating of weights, chain linking and reweighting are discussed in a subsequent chapter. Data editing procedures are discussed in the chapter on data collection.

8.5 While the purpose of this chapter is the compilation of CPIs at the various levels of aggregation, statistical offices must keep in mind that the end goal of producing the indices is to disseminate and publish CPIs of high quality. To this end the sampling process for selecting the items that are included in the indices and the price observations that are representative of the product varieties in the consumer markets are critically important in determining the quality of the indices at the elementary and aggregate levels. In this regard, the sampling procedures presented in Chapter 5 are very important to attain the end goal.

The calculation of price indices for elementary aggregates

8.6 CPIs are typically calculated in two steps. In the first step, the elementary price indices for the elementary aggregates are calculated. In the second step, higher-level indices are calculated by averaging the elementary price indices. The elementary aggregates and their price indices are the basic building blocks of the CPI.

Construction of elementary aggregates

8.7 Elementary aggregates are groups of relatively homogeneous goods and services, i.e., similar in characteristics, content, price or price change. They may cover the whole country or separate regions within the country. Likewise, elementary aggregates may be distinguished for different types of outlets. The nature of the elementary aggregates depends on circumstances and the availability of information. Elementary aggregates may therefore be defined differently in different countries. Some key points, however, should be noted:

- Elementary aggregates should consist of groups of goods or services that are as similar as possible, and preferably fairly homogeneous.
- They should also consist of varieties that may be expected to have similar price movements. The objective should be to try to minimize the dispersion of price movements within the aggregate.
- The elementary aggregates should be appropriate to serve as strata for sampling purposes in the light of the sampling regime planned for the data collection.
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8.8 Each elementary aggregate, whether relating to the whole country or an individual region or group of outlets, will typically contain a very large number of individual goods or services, or varieties. In practice, only a small number can be selected for pricing. When selecting the varieties, the following considerations need to be taken into account:

- The varieties selected should be ones for which price movements are believed to be representative of most of the products within the elementary aggregate.
- The number of varieties within each elementary aggregate for which prices are collected should be large enough for the estimated price index to be statistically reliable. The minimum number required will vary between elementary aggregates depending on the nature of the products and their price behavior. However, there should be 8-10 observations for calculating the elementary index as discussed in Chapter 5.
- The object is to try to track the price of the same variety over time for as long as the variety continues to be representative. The varieties selected should therefore be ones that are expected to remain on the market for some time, so that like can be compared with like, and problems associated with replacement of varieties be reduced.

The aggregation structure

8.9 The aggregation structure for a CPI is illustrated in Figure 9.1. Using a classification of consumers' expenditures such as the Classification of Individual Consumption according to Purpose (COICOP), the entire set of consumption goods and services covered by the overall CPI can be divided into groups, such as "food and non-alcoholic beverages". Each group is further divided into classes, such as "food". For CPI purposes, each class can then be further divided into more homogeneous sub-classes, such as "rice". The sub-classes are the equivalent of the basic headings used in the International Comparison Program (ICP), which calculates purchasing power parities (PPPs) between countries. Finally, the sub-class may be further subdivided to obtain the elementary aggregates, by dividing according to region or type of outlet, as in Figure 9.1. In some cases, a particular sub-class cannot be, or does not need to be, further subdivided, in which case the sub-class becomes the elementary aggregate. Within each elementary aggregate, one or more products are selected to represent all the products in the elementary aggregate. For example, the elementary aggregate consisting of bread sold in supermarkets in the northern region covers all types of bread, from which white bread and whole grain bread are selected as representative products. Of course, more representative products might be selected in practice. Finally, for each representative product, a number of specific varieties can be selected for price collection, such as particular brands of white bread. Again, the number of sampled varieties selected may vary depending on the nature of the representative product.

8.10 Methods used to calculate the elementary indices from the individual price observations are discussed below. Working upwards from the elementary price indices, all indices above the elementary aggregate level are higher-level indices that can be calculated from the elementary price indices using the elementary expenditure aggregates as weights. The aggregation structure is consistent, so that the weight at each level above the elementary aggregate is always equal to the sum of its components. The price index at each higher level of aggregation can be calculated on the basis of the weights and price indices for its components, that is, the lower-level or elementary indices. The individual elementary price indices are not necessarily sufficiently reliable to be published separately, but they remain the basic building blocks of all higher-level indices.

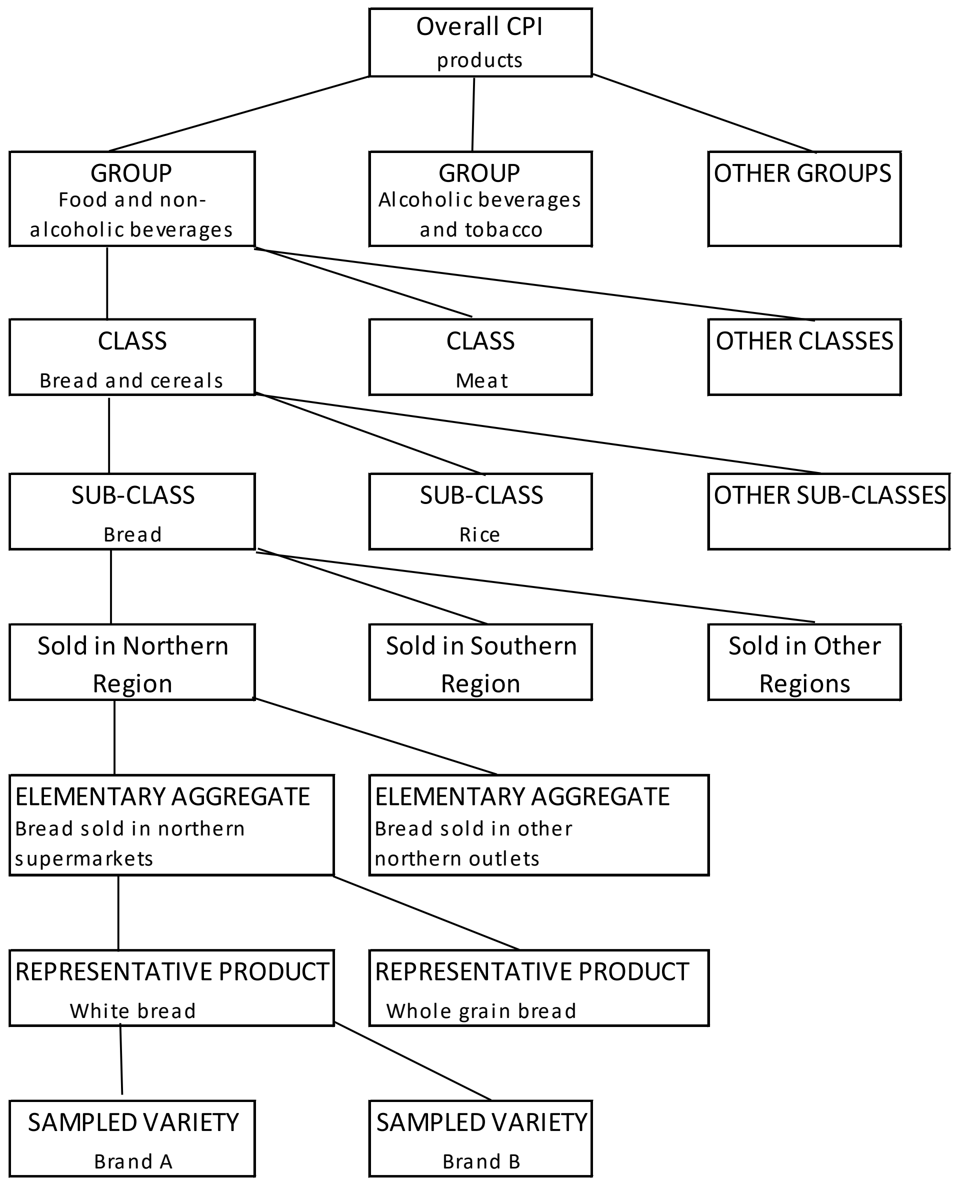
Weights within elementary aggregates

8.11 The ideal index number formula to use for CPI calculations would have weights for each observation at the elementary index level as well as weights for aggregating to higher levels. In a few countries, this approach has been achieved. Most countries that have weights at this level use Laspeyres-type indices which are discussed later in the section on higher level aggregate indices. Also, having weights for both the weight reference period and the current period would be ideal to produce one of the target indices for CPI compilation (Fisher, Törnqvist, or Walsh price indices).

8.12 In most cases, the price indices for elementary aggregates are calculated without the use of explicit expenditure weights. Often, the elementary aggregate is simply the lowest level at which reliable weighting information is available. In this case, the elementary index has to be calculated as an unweighted average of the prices of which it consists. Even in this case, however, it should be noted that when the varieties are selected with probabilities proportional to the size of some relevant variable such as sales, weights are implicitly introduced by the sampling selection procedure.

8.13 For certain elementary aggregates, information about sales of particular varieties, market shares and regional weights may be used as explicit weights within an elementary aggregate. When possible, weights should be used that reflect the relative importance of the sampled varieties, even if the weights are only approximate.

Figure 9.1 Typical aggregation structure of a consumer price index


Overall CPI
products
GROUP
Food and non-alcoholic beverages
GROUP
Alcoholic beverages and tobacco
OTHER GROUPS
CLASS
Bread and cereals
CLASS
Meat
OTHER CLASSES
SUB-CLASS
Bread
SUB-CLASS
Rice
OTHER SUB-CLASSES
Sold in Northern Region
Sold in Southern Region
Sold in Other Regions
ELEMENTARY AGGREGATE
Bread sold in northern supermarkets
ELEMENTARY AGGREGATE
Bread sold in other northern outlets
REPRESENTATIVE PRODUCT
White bread
REPRESENTATIVE PRODUCT
Whole grain bread
SAMPLED VARIETY
Brand A
SAMPLED VARIETY
Brand B

8.14 For example, assume that the number of suppliers of a certain product such as fuel for cars is limited. The market shares of the suppliers may be known from business survey statistics and can be used as weights in the calculation of an elementary aggregate price index for car fuel. Alternatively, prices for water may be collected from a number of local water supply services where the population in each local region is known. The relative size of the population in each region may then be used as a proxy for the relative consumption expenditures to weight the price in each region to obtain the elementary aggregate price index for water. The calculation of weighted elementary indices is discussed in more detail later in the chapter.

Calculation of elementary price indices

8.15 Various methods and formulae may be used to calculate elementary price indices. This section provides a summary of the methods that have been most commonly used and the pros and cons that statistical offices must evaluate when choosing a formula at the elementary level. Chapter 20 provides a more detailed discussion

8.16 The methods most common in use are illustrated in a numerical example in Tables 9.1 – 9.3. In the example an elementary aggregate consists of seven varieties of an item, and it is assumed that prices are collected for all seven varieties in all months, so that there is a complete set of prices. There are no disappearing varieties, no missing prices and no replacement varieties. This is quite a strong assumption since many of the problems encountered in practice are attributable to breaks in the continuity of the price series for the individual varieties for one reason or another. The treatment of disappearing and replacement varieties is taken up later. It is also assumed that there are no explicit weights available.

8.17 The properties of the three indices (Jevons, Dutot, and Carli) are examined and explained in some detail in Chapter 20 where it is shown that the Jevons is preferred in most circumstances when weights are not available. Here, the purpose is to illustrate how they perform in practice, to compare the results obtained by using the different formulae and to summarize their strengths and weaknesses. These widely used formulae that have been, or still are, in use by statistical offices to calculate elementary price indices are illustrated in Tables 9.1 – 9.3 by using average prices, averages of price relatives and long-term vs. short-term price relative methods. It should be noted, however, that these are not the only possibilities and some alternative formulae are considered later. The first is the Jevons index for i = 1… n varieties. It is defined as the unweighted geometric mean of the price relatives, which is identical to the ratio of the unweighted geometric mean prices, for the two periods, 0 and t, to be compared:

$$P_J^{0:t} = \prod\left(\frac{p_i^t}{p_i^0}\right)^{1/n} = \frac{\prod\left(p_i^t\right)^{1/n}}{\prod\left(p_i^0\right)^{1/n}} \tag{9.1}$$

The second is the Dutot index, defined as the simple, or unweighted arithmetic mean of prices:

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^t}{\frac{1}{n}\sum p_i^0} \tag{9.2}$$

The third is the Carli index, defined as the unweighted ratio of the arithmetic mean of the price relatives, or price ratios:

$$P_C^{0:t} = \frac{1}{n}\sum\left(\frac{p_i^t}{p_i^0}\right) \tag{9.3}$$

8.18 Table 9.1 shows the comparison of the Dutot and Jevons indices using the monthly average prices. The first calculation for the Dutot index uses the average prices in the long-term formula (direct approach) where each month's average (*t*) is compared to the initial base price (*0*), i.e., the base price reference period. The Dutot index is also calculated using the short-term relatives (chained approach) where the *month-to-month* changes in average prices are used to move forward the previous month's index level. The results are the same for both the direct and chained approaches in the Dutot calculations. Similarly, in Table 9.1 the Jevons index uses the geometric average prices in the long-term and short-term formulae to derive the price index levels that are the same for both the long-term and short-term method. The Jevons indices do, however, differ from those calculated using the Dutot formula.

Table 9.1: Jevons and Dutot Price Indexes Using Averages of Prices

	Base	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A	**Prices**							
Variety 1	2.36	2.09	1.93	2.59	2.05	2.85	2.59	2.36
Variety 2	5.02	5.38	5.12	5.52	4.08	4.08	5.52	5.02
Variety 3	5.34	5.07	5.09	5.88	6.29	5.86	5.88	5.34
Variety 4	6.00	5.73	4.27	6.00	4.75	5.27	6.60	6.00
Variety 5	6.12	6.39	5.50	6.12	5.86	6.29	6.74	6.12
Variety 6	2.80	2.72	2.82	3.08	2.85	2.05	3.08	2.80
Variety 7	6.21	5.45	6.95	6.21	5.27	4.75	6.84	6.21
Geometric average price	***4.55***	***4.38***	***4.20***	***4.81***	***4.17***	***4.17***	***5.01***	***4.55***
Long-Term (L-T) Price Relative	*1.000*	*0.963*	*0.923*	*1.056*	*0.917*	*0.917*	*1.100*	*1.000*
Short-Term (S-T) Price Relative	*1.000*	*0.963*	*0.958*	*1.144*	*0.868*	*1.000*	*1.200*	*0.909*
Arithmetic average price	**4.84**	**4.69**	**4.52**	**5.06**	**4.45**	**4.45**	**5.32**	**4.84**
Long-Term (L-T) Price Relative	*1.000*	*0.970*	*0.935*	*1.046*	*0.920*	*0.920*	*1.100*	*1.000*
Short-Term (S-T) Price Relative	*1.000*	*0.970*	*0.964*	*1.118*	*0.880*	*1.000*	*1.196*	*0.909*
Jevons Index (L-T Ratio of Geometric Average Prices)	**100.0**	**96.3**	**92.3**	**105.6**	**91.7**	**91.7**	**110.0**	**100.0**
Dutot Index (L-T Ratio of Average Prices)	**100.0**	**97.0**	**93.5**	**104.6**	**92.0**	**92.0**	**110.0**	**100.0**
Jevons Index (Chained S-T Ratio of Geometric Average Prices)	**100.0**	**96.3**	**95.8**	**114.4**	**86.8**	**100.0**	**120.0**	**90.9**
Dutot Index (Chained S-T Ratio of Average Prices)	**100.0**	**97.0**	**93.5**	**104.6**	**92.0**	**92.0**	**110.0**	**100.0**

8.19 In Table 9.2, the Jevons and Carli indices are calculated using the averages of long-term price relatives from the base period (price reference period). The results for the Carli indices are different from those of both the Jevons and Dutot indices. The Jevons indices are exactly the same whether calculated using average prices or price relatives.

8.20 The properties and behavior of the different indices are summarized in the following paragraphs (see also Chapter 20). First, the differences between the results obtained by using the different formulae tend to increase as the variance of the price relatives, or ratios, increases. The greater the dispersion of the price movements, the more critical the choice of index formula, and method, becomes. If the elementary aggregates are defined in such a way that the price movements within the aggregate are minimized, the results obtained become less sensitive to the choice of formula and method.

8.21 Certain features displayed by the data in Tables 9.1 and 9.2 are systematic and predictable; they follow from the mathematical properties of the indices. For example, it is well known that an arithmetic mean is always greater than, or equal to, the corresponding geometric mean, the equality holding only in the trivial case in which the numbers being averaged are all the same. The direct Carli indices are therefore all greater than the Jevons indices, except in the price reference period, in June when all prices increased by 10 percent above their base prices, and the end period when all prices return to their base period values. In general, the Dutot may be greater or less than the Jevons, but tends to be less than the Carli.

Table 9.2: Jevons and Carli Price Indexes Using Averages of Long-Term Price Relatives

	Base	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A	**Long-Term (L-T) Price Relatives**							
Variety 1	1.000	0.888	0.816	1.100	0.869	1.207	1.100	1.000
Variety 2	1.000	1.072	1.019	1.100	0.813	0.813	1.100	1.000
Variety 3	1.000	0.949	0.953	1.100	1.178	1.097	1.100	1.000
Variety 4	1.000	0.955	0.712	1.000	0.792	0.878	1.100	1.000
Variety 5	1.000	1.044	0.898	1.000	0.957	1.028	1.100	1.000
Variety 6	1.000	0.974	1.008	1.100	1.018	0.733	1.100	1.000
Variety 7	1.000	0.877	1.118	1.000	0.848	0.765	1.100	1.000
Geometric average of L-T price relatives	*1.000*	*0.963*	*0.923*	*1.056*	*0.917*	*0.917*	*1.100*	*1.000*
Jevons index (L-T Geometric Changes)	**100.0**	**96.3**	**92.3**	**105.6**	**91.7**	**91.7**	**110.0**	**100.0**
Arithmetic average of L−T price relatives	*1.000*	*0.966*	*0.932*	*1.057*	*0.925*	*0.931*	*1.100*	*1.000*
Carli Index (L-T Arithmetic Changes)	**100.0**	**96.6**	**93.2**	**105.7**	**92.5**	**93.1**	**110.0**	**100.0**

8.22 The Carli and Jevons indices depend only on the price relatives and are unaffected by the price level. The Dutot index, in contrast, is influenced by the price level. In the Dutot index, price changes are implicitly weighted by the price in the base (price reference) period, so that price changes on more expensive products are assigned a higher weight than similar price changes for cheaper products (this can be seen from equation (9.4)). In Tables 9.1 and 9.3 this is illustrated in the development of the March index where prices for varieties 4, 5, and 7, which have the largest base prices, are the same as in the base month and mitigate the 10 percent price increases of varieties 1, 2, 3, and 6 from the base month. The monthly price Dutot index is 104.6 vs. 1.05.6 in the Jevons, and 1.057 in the Carli. Because of the relative high base prices for varieties 4, 5, and 7, the variety price increase is less in the Dutot index, and the Dutot index level in March is lower than the Jevons.

Table 9.3: Jevons and Carli Price Indexes Using Chained Short-Term Price Relatives

	Base	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A	**Short-Term (S−T) Price Relatives**							
Variety 1	1.000	0.888	0.920	1.347	0.790	1.389	0.911	0.909
Variety 2	1.000	1.072	0.950	1.080	0.739	1.000	1.353	0.909
Variety 3	1.000	0.949	1.004	1.155	1.071	0.931	1.003	0.909
Variety 4	1.000	0.955	0.745	1.405	0.792	1.109	1.253	0.909
Variety 5	1.000	1.044	0.860	1.113	0.957	1.074	1.070	0.909
Variety 6	1.000	0.974	1.035	1.091	0.925	0.720	1.501	0.909
Variety 7	1.000	0.877	1.275	0.894	0.848	0.902	1.438	0.909
Geometric average of S-T price relatives	*1.000*	*0.963*	*0.958*	*1.144*	*0.868*	*1.000*	*1.200*	*0.909*
Jevons index (Chained S-T Geometric Changes)	**100.0**	**96.3**	**92.3**	**105.6**	**91.7**	**91.7**	**110.0**	**100.0**

Arithmetic average of S−T price relatives	*1.000*	*0.966*	*0.970*	*1.155*	*0.875*	*1.018*	*1.219*	*0.909*
Carli Index (Chained S-T Arithmetic Changes)	**100.0**	**96.6**	**93.7**	**108.2**	**94.6**	**96.3**	**117.3**	**106.7**

8.23 Another important property of the indices is that the Jevons and the Dutot indices are transitive, whereas the Carli is not. Transitivity means that the chained monthly indices are identical to the corresponding direct indices. This property is important in practice, because many elementary price indices are in fact calculated as chain indices which link together the month-on-month indices. The intransitivity of the Carli index is illustrated dramatically in Table 9.3 when each of the individual prices in the final month return to the same level as it was in base month, but the chained Carli registers an increase of 6.7 percent over the base month. Similarly, in June, although each individual price is exactly 10 percent higher than base month, the chain Carli registers an increase of 17.3 percent. These results would be regarded as perverse and unacceptable in the case of a direct index, but even in the case of a chain index the results seems so intuitively unreasonable as to undermine the credibility of the chain Carli. The price movements between April and May illustrate the effects of "price bouncing" in which the same seven prices are observed in both periods but they are switched between the different varieties. The monthly Carli index (short-term and long-term) increases from April to May whereas both the Dutot and the Jevons indices are unchanged.

8.24 One general property of geometric means should be noted when using the Jevons index. If anyone observation out of a set of observations is zero, their geometric mean is zero, whatever the values of the other observations. The Jevons index is sensitive to extreme falls in prices and it may be necessary to impose upper and lower bounds on the individual price ratios of say 10 and 0.1, respectively, when using the Jevons. This range should be determined after assessing the typical size of price movements and may vary across different product groups. Of course, extreme observations often result from errors of one kind or another, so extreme price movements should be carefully checked anyway.

8.25 The message emerging from this brief illustration of the behavior of just three possible formulae is that different index numbers and methods can deliver very different results. Knowledge of these interrelationships infers that the chained Carli formula is not preferred. However, this information in itself is not sufficient to determine which formula should be used, even though it makes it possible to make a more informed and reasoned choice.[1] It is necessary to appeal to other criteria in order to settle the choice of formula. There are two main approaches that may be used, the axiomatic and the economic approaches, which are presented below. First, however, it is useful to consider the sampling properties of the elementary indices.

Sampling properties of elementary price indices

8.26 The interpretation of the elementary price indices is related to the way in which the sample of goods and services is drawn. Hence, if goods and services in the sample are selected with probabilities proportional to the population expenditure shares in the price reference period,

- the sample (unweighted) Carli index provides an unbiased estimate of the population Laspeyres price index, and
- the sample (unweighted) Jevons index provides an unbiased estimate of the population Geometric Laspeyres price index (see equation (9.6))

[1] Another alternative index discussed in Chapter 20 is the Carruthers-Sellwood-Ward-Dalen index which is the geometric average of a Carli and harmonic mean index. Such an index is an unweighted proxy for a Fisher index. However, the results are the same as the Jevons index which is an unweighted proxy for a Törnqvist index.

8.27 If goods and services are sampled with probabilities proportional to population quantity shares in the price reference period, the sample (unweighted) Dutot index would provide an estimate of the population Laspeyres price index. However, if the basket for the Laspeyres index contains different kinds of products whose quantities are not additive, the quantity shares, and hence the probabilities, are undefined.

Axiomatic approach to elementary price indices

8.28 As explained in Chapters 16 and 20, one way in which to decide upon an appropriate index formula is to require it to satisfy certain specified axioms or tests. The tests throw light on the properties possessed by different kinds of indices, some of which may not be intuitively obvious. Four basic tests will be cited here to illustrate the axiomatic approach:

- *Proportionality test* – if all prices are λ times the prices in the price reference period, the index should equal λ. The data for June, when every price is 10 percent higher than in the base reference preiod, show that all three direct indices satisfy this test. A special case of this test is the *identity test*, which requires that if the price of every variety is the same as in the reference period, the index should be equal to unity, as in the last month in the example.
- *Changes in the units of measurement test (commensurability test)* – the price index should not change if the quantity units in which the products are measured are changed, for example, if the prices are expressed per liter rather than per pint. The Dutot index fails this test, as explained below, but the Carli and Jevons indices satisfy the test.
- *Time reversal test* – if all the data for the two periods are interchanged, the resulting price index should equal the reciprocal of the original price index. The Carli index fails this test, but the Dutot and the Jevons indices both satisfy the test. The failure of the chained Carli to satisfy the test is not immediately obvious from the example, but can easily be verified by interchanging the prices in base period and June, for example, in which case the backwards chained Carli from June back to the base period is 97.0 whereas the reciprocal of the forwards chained Carli is 1/117.3 or 85.2.
- *Transitivity test* – the chain index between two periods should equal the direct index between the same two periods. It can be seen from the example that the Jevons and the Dutot indices both satisfy this test, whereas the chained Carli index does not. For example, although the prices in January have returned to the same levels as the base period, the chain Carli registers 106.7. This illustrates the fact that the chained Carli may have a significant built-in upward bias.

8.29 Many other axioms or tests can be devised, but the above are sufficient to illustrate the approach and also to throw light on some important features of the elementary indices under consideration here and provide evidence of the preference for the Jevons index.

8.30 The sets of products covered by elementary aggregates are meant to be as homogeneous as possible. If they are not fairly homogeneous, the failure of the Dutot index to satisfy the units of measurement or commensurability test can be a serious disadvantage. Although defined as the ratio of the unweighted arithmetic average prices, the Dutot index may also be interpreted as a weighted arithmetic average of the price relatives in which each relative is weighted by its price in the base period. This can be seen by rewriting formula (9.2) above as

$$P_D^{0:t} = \frac{\frac{1}{n}\sum p_i^0 \left(p_i^t / p_i^0\right)}{\frac{1}{n}\sum p_i^0} \qquad (9.4)$$

However, if the products are not homogeneous, the relative prices of the different varieties may depend quite arbitrarily on the quantity units in which they are measured.

8.31 Consider, for example, salt and pepper, which are found within the same sub-class of COICOP. Suppose the unit of measurement for pepper is changed from grams to ounces, while leaving the units in which salt is measured (say kilos) unchanged. As an ounce of pepper is equal to 28.35 grams, the "price" of pepper increases by over 28 times, which effectively increases the weight given to pepper in the Dutot index by over 28 times. The price of pepper relative to salt is inherently arbitrary, depending entirely on the choice of units in which to measure the two goods. In general, when there are different kinds of products within the elementary aggregate, the Dutot index is not acceptable.

8.32 The Dutot index is acceptable only when the set of varieties covered is homogeneous, or at least nearly homogeneous. For example, it may be acceptable for a set of apple prices even though the apples may be of different varieties, but not for the prices of a number of different kinds of fruits, such as apples, pineapples and bananas, some of which may be much more expensive per variety or per kilo than others. Even when the varieties are fairly homogeneous and measured in the same units, the Dutot's implicit weights may still not be satisfactory. More weight is given to the price changes for the more expensive varieties, but in practice they may well account for only small shares of the total expenditure within the aggregate. Consumers are unlikely to buy varieties at high prices if the same varieties are available at lower prices.

8.33 It may be concluded that from an axiomatic viewpoint, both the Carli and the Dutot indices, although they have been, and still are, used by some statistical offices, have serious disadvantages. The chained Carli index fails the time reversal and transitivity tests. In principle, it should not matter whether we choose to measure price changes forwards or backwards in time. We would expect the same answer, but this is not the case for the chained Carli indices that may be subject to a significant upward bias. The Dutot index is meaningful for a set of homogeneous varieties but becomes increasingly arbitrary as the set of products becomes more diverse. On the other hand, the Jevons index satisfies all the tests listed above and also emerges as the preferred index when the set of tests is enlarged, as shown in Chapter 20. From an axiomatic point of view, the Jevons index is clearly the index with the best properties.

Economic approach to elementary price indices

8.34 In the economic approach, the objective is to estimate an economic index – that is, a *cost of living index* for the elementary aggregate (see Chapter 20). The items for which prices are collected are treated as if they constituted a basket of goods and services purchased by consumers, from which the consumers derive utility. A cost of living index measures the minimum amount by which consumers would have to change their expenditures in order to keep their utility level unchanged, allowing consumers to make substitutions between the varieties in response to changes in the relative prices of varieties.

8.35 The economic approach is based on a number of assumptions about consumer behavior, market conditions and the representativity of the sample. These assumptions do not always hold in reality. At the detailed level of elementary aggregates special conditions will often prevail and change over time and the information available about establishments, products and market conditions may be incomplete. Thus, although the economic approach may be useful in providing a possible economic interpretation of the index, conclusions should be made with caution. In general, in the decision of how to calculate the elementary indices one should be careful not to put too much weight on a strict economic interpretation of the index formula at the expense of the statistical considerations.

8.36 In the absence of information about quantities or expenditures within an elementary aggregate, an economic index can only be estimated when certain special conditions are assumed to prevail. There are two special cases of some interest. The first case is when consumers continue to consume the same *relative* quantities whatever the relative prices. Consumers prefer not to make any substitutions in response to changes in relative prices. The cross-elasticities of demand are zero. The underlying preferences are described in the economics literature as "Leontief". In this first case, the Carli index calculated for a random sample would provide an estimate of the cost of living index provided that the varieties are selected with probabilities proportional to the population expenditure shares. If the varieties were selected with probabilities proportional to the population quantity shares (assuming the quantities are additive), the sample Dutot would provide an estimate of the underlying cost of living index.

8.37 The second case occurs when consumers are assumed to vary the quantities they consume in inverse proportion to the changes in relative prices. The cross-elasticities of demand between the different varieties are all unity, the expenditure shares being the same in both periods. The underlying preferences are described as "Cobb-Douglas". With these preferences, the Jevons index calculated for a random sample would provide an unbiased estimate of the cost of living index, provided that the varieties are selected with probabilities proportional to the population expenditure shares.

8.38 On the basis of the economic approach, the choice between the sample Jevons and the sample Carli rests on which is likely to approximate the more closely to the underlying cost of living index: in other words, on whether the (unknown) cross-elasticities are likely to be closer to unity or zero, on average. In practice, the cross-elasticities could take on any value ranging up to plus infinity for an elementary aggregate consisting of a set of strictly homogeneous varieties, i.e., perfect substitutes. It should be noted that in the limit when the products really are homogeneous, there is no index number problem, and the price "index" is given by the ratio of the unit values in the two periods, as explained later. It may be conjectured that the average cross-elasticity is likely to be closer to unity than zero for most elementary aggregates, especially since these should be constructed in such a way as to group together similar varieties that are close substitutes for each other. Thus, in general, the Jevons index is likely to provide a closer approximation to the cost of living index than the Carli. In this case, the Carli index must be viewed as having an upward bias.

8.39 In the economic approach, the Jevons index is strictly speaking not a fixed basket index, since the quantities are assumed to vary over time in response to changes in relative prices. As a result of the inverse relation of movements in prices and quantities the expenditure shares are constant over time. Carli and Dutot, on the other hand, keep the quantities fixed while the expenditure shares vary in response to change in relative prices.

8.40 The Jevons index does not imply that expenditure shares remain constant. Obviously, the Jevons can be calculated whatever changes do, or do not occur in the expenditure shares in practice. What the economic approach shows is that if the expenditure shares remain constant (or roughly constant), then the Jevons index can be expected to provide a good estimate of the underlying cost of living index. Similarly, if the relative quantities remain constant, then the Carli index can be expected to provide a good estimate, but the Carli does not actually imply that quantities remain fixed.

8.41 It may be concluded that, on the basis of the economic approach as well as the axiomatic approach, the Jevons emerges as the preferred index, although there may be cases in which little or no substitution takes place within the elementary aggregate and the direct Carli might be used. The chained Carli should be avoided altogether. The Dutot index may be used provided the elementary aggregate consists of homogenous products. In general, the index compiler should use the Jevons index for the elementary aggregates.

Chain versus direct indices for elementary aggregates

8.42 In a direct elementary index, the prices of the current period are compared directly with those of the price reference period. In a chain index, prices in each period are compared with those in the previous period, the resulting short-term indices being chained together to obtain the long-term index, as illustrated in Tables 9.1-9.3.

8.43 Provided that prices are recorded for the same set of varieties in every period, as in Table 9.1, any index formula defined as the ratio of the average prices will be transitive: that is, the same result is obtained whether the index is calculated as a direct index or as a chain index. In a chain index, successive numerators and denominators will cancel out, leaving only the average price in the last period divided by the average price in the reference period, which is the same as the direct index. Both the Dutot and the Jevons indices are therefore transitive. As already noted, however, a chain Carli index is not transitive and should not be used because of its upward bias. However, the direct Carli is transitive also.

8.44 Although the chain and direct versions of the Jevons and Dutot indices are identical when there are no breaks in the series for the individual varieties, they offer different ways of dealing with new and disappearing varieties, missing prices and quality adjustments. In practice, products continually have to be dropped from the index and new ones included, in which case the direct and the chain indices may differ if the imputations for missing prices are made differently.

8.45 When a replacement variety has to be included in a direct index, it will often be necessary to estimate the price of the new variety in the price reference (base) period, which may be some time in the past. The same happens if, as a result of an update of the sample, new varieties have to be linked into the index. Assuming that no information exists on the price of the replacement variety in the price reference period, it will be necessary to estimate it using price ratios calculated for the varieties that remain in the elementary aggregate, a subset of these varieties or some other indicator. However, the direct approach should only be used for a limited period of time. Otherwise, most of the reference prices would end up being imputed, which would be an undesirable outcome. This effectively rules out the use of the Carli index over a long period of time, as the Carli can only be used in its direct form anyway, being unacceptable when chained. This implies that, in practice, the direct Carli may be used only if the overall index is chain linked annually, or biannually.

8.46 In a chain index, if a variety becomes permanently missing, a replacement variety can be linked into the index as part of the ongoing index calculation by including the variety in the monthly index as soon as prices for two successive months are obtained. Similarly, if the sample is updated and new products have to be linked into the index, this will require successive old and new prices for the present and the preceding months. For a chain index, the substitute variety for a missing observation would also have to have prices for the current and previous period. However, if the previous price is not available, it will have an impact on the index for two months, since the substitute observation cannot be used until the subsequent month. It also is possible to impute the price of the missing item in the first missing month so that the next period price can be compared to the imputed price.

8.47 A missing price does not have such a problem in the case for a direct index. In a direct index a single, non-estimated missing observation will only have an impact on the index in the current period. For example, for a comparison between periods 0 and 3, a missing price of the substitute in period 2 means that the chain index excludes the variety for the last link of the index in periods 2 and 3, while the direct index includes it in period 3 since a direct index will be based on varieties whose prices are available in periods 0 and 3 (unless an imputation is made). In general, however, the use of a chain index can make the estimation of missing prices and the introduction of replacements easier from a computational point of view, whereas it may be inferred that a direct index will limit the usefulness of overlap methods for dealing with missing observations.

8.48 The direct and the chain approaches also produce different by-products that may be used for monitoring price data. For each elementary aggregate, a chain index approach gives the latest monthly price change, which can be useful for both data editing and imputation of missing prices. By the same token, however, a direct index derives average price levels for each elementary aggregate in each period, and this information may be a useful by-product. Nevertheless, because the availability of cheap computing power and of spreadsheets allows such by-products to be calculated whether a direct or a chained approach is applied, the choice of formula should not be dictated by considerations regarding by-products.

Consistency in aggregation

8.49 Consistency in aggregation means that if an index is calculated stepwise by aggregating lower-level indices to obtain indices at progressively higher levels of aggregation, the same overall result should be obtained as if the calculation had been made in one step. For presentational purposes this is an advantage. If the elementary aggregates are calculated using one formula and the elementary aggregates are averaged to obtain the higher-level indices using another formula, the resulting CPI is not consistent in aggregation. It may be argued, however, that consistency in aggregation is not necessarily an important or even appropriate criterion, or that it is unachievable when the amount of information available on quantities and expenditures is not the same at the different levels of aggregation. In addition, there may be different degrees of substitution within elementary aggregates as compared to the degree of substitution between products in different elementary aggregates.

8.50 The Carli index would be consistent in aggregation with the Laspeyres index if the varieties were to be selected with probabilities proportional to expenditures in the reference period. This is typically not the case. The Dutot and the Jevons indices are not consistent in aggregation with a higher-level Laspeyres. As explained below, however, the CPIs actually calculated by statistical offices are usually not true Laspeyres indices anyway, even though they may be based on fixed baskets of goods and services. If the higher-level index were to be defined as a geometric Laspeyres, consistency in aggregation could be achieved by using the Jevons index for the elementary indices at the lower level, provided that the individual varieties are sampled with probabilities proportional to expenditures. Although unfamiliar, a geometric Laspeyres has desirable properties from an economic point of view and is considered again later.

Missing price observations

8.51 The price of a variety may fail to be collected in some period either because the variety is missing temporarily or because it has permanently disappeared. The two classes of missing prices require different treatment as noted previously in Chapter 7. Temporary unavailability may occur for seasonal varieties (particularly for fruit, vegetables and clothing), because of supply shortages or possibly because of some collection difficulty (say, an outlet was closed or a price collector was ill). The treatment of seasonal varieties raises a number of particular problems. These are dealt with in Chapter 22 and will not be discussed here.

Treatment of temporarily missing prices

8.52 In the case of temporarily missing observations for non-seasonal varieties, one of four actions may be taken:

- Omit the variety for which the price is missing so that a matched sample is maintained (like is compared with like) even though the sample is depleted
- Carry forward the last observed price
- Impute the missing price by the average price change for the prices that are available in the elementary aggregate
- Impute the missing price by the price change for a particular comparable variety from another similar outlet

8.53 Omitting an observation from the calculation of an elementary index is equivalent to assuming that the price would have moved in the same way as the average of the prices of the varieties that remain included in the index. Omitting an observation changes the implicit weights attached to the other prices in the elementary aggregate.

8.54 Carrying forward the last observed price should be avoided wherever possible and is acceptable only for a very limited number of periods. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and product turnover. While simple to apply, carrying forward the last observed price biases the resulting index towards zero change. In addition, when the price of the missing variety is recorded again, there is likely to be a compensating step-change in the index to return to its proper value. The adverse effect on the index will be increasingly severe if the variety remains unpriced for some length of time. In general, to carry forward is not an acceptable procedure or solution to the problem.

8.55 Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates where the prices can be expected to move in the same direction. The imputation can be made using all of the remaining prices in the elementary aggregate. As already noted, this is numerically equivalent to omitting the variety for the immediate period, but it is useful to make the imputation so that if the price becomes available again in a later period the sample size is not reduced in that period. In some cases, depending on the homogeneity of the elementary aggregate, it may be preferable to use only a subset of varieties from the elementary aggregate to estimate the missing price. In some instances, this may even be a single comparable variety from a similar type of outlet whose price change can be expected to be similar to the missing one. (See Chapter 7 on imputation methods.)

8.56 Tables 9.4a and 9.4b illustrate the calculation of the price index for the elementary aggregate where the price for variety 6 is missing in March. The long-term (direct) indices are therefore calculated on the basis of the six varieties with reported prices. The short-term (chained) indices are calculated on the basis of all seven prices from January to February and from April to July. From February to March and from March to April the monthly indices are calculated on the basis of six varieties only.

Table 9.4a: Jevons and Dutot Elementary Price Indexes Using Averages with Missing Prices

	Base	Match	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A	**Prices**								
Variety 1	2.36	2.36	2.09	1.93	2.59	2.05	2.85	2.59	2.36
Variety 2	5.02	5.02	5.38	5.12	5.52	4.08	4.08	5.52	5.02
Variety 3	5.34	5.34	5.07	5.09	5.88	6.29	5.86	5.88	5.34
Variety 4	6.00	6.00	5.73	4.27	6.00	4.75	5.27	6.60	6.00
Variety 5	6.12	6.12	6.39	5.50	6.12	5.86	6.29	6.74	6.12
Variety 6	2.80		2.72	2.82		2.85	2.05	3.08	2.80
Variety 7	6.21	6.21	5.45	6.95	6.21	5.27	4.75	6.84	6.21
Geometric average price (7 obs)	***4.55***		***4.38***	***4.20***		***4.17***	***4.17***	***5.01***	***4.55***
Geometric average price (6 matched obs)		***4.94***		***4.49***	***5.18***	***4.45***			
Long-Term (L-T) Price Relative	*1.000*		*0.963*	*0.923*	*1.049*	*0.917*	*0.917*	*1.100*	*1.000*
Jevons Index (direct)	**100.0**		**96.3**	**92.3**	**104.9**	**91.7**	**91.7**	**110.0**	**100.0**
Geometric average S−T price relatives	*1.000*		*0.963*	*0.958*	***1.153***	***0.859***	*1.000*	*1.200*	*0.909*
Jevons Index (chained averages)	**100.0**		**96.3**	**92.3**	**106.4**	**91.4**	**91.4**	**109.7**	**99.7**
Arithmetic average price (7 obs)	**4.84**		**4.69**	**4.52**		**4.45**	**4.45**	**5.32**	**4.84**
Arithmetic average price (6 matched obs)		**5.18**		**4.81**	**5.39**	**4.72**			
Long-Term (L-T) Price Relative	*1.000*		*0.970*	*0.935*	*1.041*	*0.920*	*0.920*	*1.100*	*1.000*
Dutot Index (direct)	**100.0**		**97.0**	**93.5**	**104.1**	**92.0**	**92.0**	**110.0**	**100.0**
Short-Term (S−T) Price Relatives	*1.000*		*0.970*	*0.964*	*1.121*	*0.875*	*1.000*	*1.196*	*0.909*
Dutot Index (chained averages)	**100.0**		**97.0**	**96.4**	**108.1**	**94.6**	**94.6**	**113.1**	**102.8**

Table 9.4b: Jevons and Carli Elementary Price Indexes Using Relatives with Missing Prices

	Base	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A	**L-T Price relatives**							
Variety 1	1.000	0.888	0.816	1.100	0.869	1.207	1.100	1.000
Variety 2	1.000	1.072	1.019	1.100	0.813	0.813	1.100	1.000
Variety 3	1.000	0.949	0.953	1.100	1.178	1.097	1.100	1.000
Variety 4	1.000	0.955	0.712	1.000	0.792	0.878	1.100	1.000
Variety 5	1.000	1.044	0.898	1.000	0.957	1.028	1.100	1.000
Variety 6	1.000	0.974	1.008		1.018	0.733	1.100	1.000
Variety 7	1.000	0.877	1.118	1.000	0.848	0.765	1.100	1.000
Geometric average pr rel (7 obs)	***1.000***	***0.963***	***0.923***		***0.917***	***0.917***	***1.100***	***1.000***
Geometric average pr rel (6 obs)				***1.049***				
Jevons Index (avg L-T pr rel)	**100.0**	**96.3**	**92.3**	**104.9**	**91.7**	**91.7**	**110.0**	**100.0**
Arithmetic average pr rel (7 obs)	*1.000*	*0.966*	*0.932*		*0.925*	*0.931*	*1.100*	*1.000*
Arithmetic average pr rel (6 obs)				*1.050*				
Carli Index (avg L-T pr rel)	**100.0**	**96.6**	**93.2**	**105.0**	**92.5**	**93.1**	**110.0**	**100.0**

Item A	**S-T Price relatives**							
Variety 1	*1.000*	*0.888*	*0.920*	*1.347*	*0.790*	*1.389*	*0.911*	*0.909*
Variety 2	*1.000*	*1.072*	*0.950*	*1.080*	*0.739*	*1.000*	*1.353*	*0.909*

Variety 3	*1.000*	*0.949*	*1.004*	*1.155*	*1.071*	*0.931*	*1.003*	*0.909*
Variety 4	*1.000*	*0.955*	*0.745*	*1.405*	*0.792*	*1.109*	*1.253*	*0.909*
Variety 5	*1.000*	*1.044*	*0.860*	*1.113*	*0.957*	*1.074*	*1.070*	*0.909*
Variety 6	*1.000*	*0.974*	*1.035*			*0.720*	*1.501*	*0.909*
Variety 7	*1.000*	*0.877*	*1.275*	*0.894*	*0.848*	*0.902*	*1.438*	*0.909*
Geometric average pr rel (7 obs)	***1.000***	***0.963***	***0.958***			***1.000***	***1.200***	***0.909***
Geometric average pr rel (6 matched obs)				*1.153*	*0.859*			
Jevons Index (chained S-T pr rel)	**100.0**	**96.3**	**92.3**	**106.4**	**91.4**	**91.4**	**109.7**	**99.7**
Arithmetic average pr rel (7 obs)	***1.000***	***0.966***	***0.970***			***1.018***	***1.219***	***0.909***
Arithmetic average pr rel (6 matched obs)				***1.166***	***0.866***			
Carli Index (chained S-T pr rel)	**100.0**	**96.6**	**93.7**	**109.2**	**94.6**	**96.2**	**117.3**	**106.6**

8.57 The average prices (both arithmetic and geometric) are calculated using the six available prices for the base period, February, March, and April in Table 9.4a. The direct Jevons and Dutot indices use the average of the six prices in March and the base period to derive the March index (104.9 and 104.1, respectively). This calculation uses a matched sample for the prices available in each period (March and the base period) to derive the averages. In April, all seven prices are again available so the direct indices are derived by comparing the averages of the seven prices to their average in the base period.

8.58 For the chained Jevons and Dutot indices that use the short-term price relatives, the average prices for the six varieties available in March are compared to the average prices of the six available varieties in February. The resulting price relatives are multiplied by the February indices to derive the March indices (106.4 for the Jevons and 108.1 for the Dutot). The same holds true for April's compilation—the average of the six prices that were available in both March and April are used to derive the April indices (91.4 for the Jevons and 94.6 for the Dutot).

8.59 For both the Jevons and the Dutot indices, the direct and chain indices now differ from March onward. The first link in the chain index (January to February) is the same as the direct index, so the two indices are identical numerically. The direct index for March completely ignores the price increase of variety 6 between January and February, while this is taken into account in the chain index. As a result, the direct index is lower than the chain index for March. On the other hand, in April, when all prices are again available, the direct index captures the price development for the full sample, whereas the chain index only tracks the long-term development in the 6-price sample.

8.60 Table 9.4b shows the compilation of the Jevons and Carli indices using the long-term (L-T) and short-term (S-T) average of price relative (pr rel) methods. The L-T Carli index shows similar effects in March and April as those for the Jevons index in missing the long-term price change for variety 6. The S-T Carli, however, shows a significant upward bias as it increased to 106.6 when all the prices return to their base period levels in July.

8.61 As Tables 9.4a and 94.b demonstrate, the Jevons, Dutot, and Carli direct indices return to 100.0 in the final period when all prices return to the their base period levels. The chained versions do not, with the Dutot and Carli showing an upward drift by the end month and the Jevons with a slight downward drift.

8.62 The problem with the chain index will be resolved if the missing price is imputed using the average short-term change of the other observations in the elementary aggregate. In Table 9.5a the missing price for variety 6 in March is imputed by the geometric average price changes of the remaining varieties from February to March. While the imputation might be calculated using long-term relatives, i.e., comparing the prices of the present period with the base period prices, the imputation of missing prices should be made on the basis of the price change from the preceding to the present period, as shown in the table. Imputation on the basis of the average price change from the base period to the present period should not be used as it ignores the information about the price change of the missing variety that has already been included in the index. The treatment of imputations is discussed in more detail in Chapter 7.

Table 9.5a: Jevons and Dutot Elementary Price Indexes Using Averages with Imputed Prices

	Base	Match	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A					**Prices**				
Variety 1	2.36	2.36	2.09	1.93	2.59	2.05	2.85	2.59	2.36
Variety 2	5.02	5.02	5.38	5.12	5.52	4.08	4.08	5.52	5.02
Variety 3	5.34	5.34	5.07	5.09	5.88	6.29	5.86	5.88	5.34
Variety 4	6.00	6.00	5.73	4.27	6.00	4.75	5.27	6.60	6.00
Variety 5	6.12	6.12	6.39	5.50	6.12	5.86	6.29	6.74	6.12
Variety 6	2.80		2.72	2.82	3.25	2.85	2.05	3.08	2.80
Variety 7	6.21	6.21	5.45	6.95	6.21	5.27	4.75	6.84	6.21
Geometric average price (7 obs)	***4.55***		***4.38***	***4.20***	***4.84***	***4.17***	***4.17***	***5.01***	***4.55***
Geometric average price (6 obs)		***4.94***		***4.49***	***5.18***				
Long-Term (L-T) Price Relative	*1.000*		*0.963*	*0.923*	*1.064*	*0.917*	*0.917*	*1.100*	*1.000*
Jevons Index (direct)	**100.0**		**96.3**	**92.3**	**106.4**	**91.7**	**91.7**	**110.0**	**100.0**
Geometric avg. S−T price relatives	*1.000*		*0.963*	*0.958*	*1.153*	*0.861*	*1.000*	*1.200*	*0.909*
Jevons Index (chained averages)	**100.0**		**96.3**	**92.3**	**106.4**	**91.7**	**91.7**	**110.0**	**100.0**
Arithmetic average price (7 obs)	**4.84**		**4.69**	**4.52**	**5.08**	**4.45**	**4.45**	**5.32**	**4.84**
Arithmetic average price (6 obs)		**5.18**		**4.81**	**5.39**				
Long-Term (L-T) Price Relative	*1.000*		*0.970*	*0.935*	*1.051*	*0.920*	*0.920*	*1.100*	*1.000*
Dutot Index (direct)	**100.0**		**97.0**	**93.5**	**105.1**	**92.0**	**92.0**	**110.0**	**100.0**
Short-Term (S−T) Price Relatives	*1.000*		*0.970*	*0.964*	*1.124*	*0.875*	*1.000*	*1.196*	*0.909*
Dutot Index (chained averages)	**100.0**		**97.0**	**93.5**	**105.1**	**92.0**	**92.0**	**110.0**	**100.0**

Table 9.5b: Jevons and Dutot Elementary Price Indexes Using Relatives with Imputed Prices

	Base	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.
Item A				**L-T Price relatives**				
Variety 1	1.000	0.888	0.816	1.100	0.869	1.207	1.100	1.000
Variety 2	1.000	1.072	1.019	1.100	0.813	0.813	1.100	1.000
Variety 3	1.000	0.949	0.953	1.100	1.178	1.097	1.100	1.000
Variety 4	1.000	0.955	0.712	1.000	0.792	0.878	1.100	1.000
Variety 5	1.000	1.044	0.898	1.000	0.957	1.028	1.100	1.000
Variety 6	1.000	0.974	1.008	1.162	1.018	0.733	1.100	1.000
Variety 7	1.000	0.877	1.118	1.000	0.848	0.765	1.100	1.000
Geometric average pr rel (7 obs)	***1.000***	***0.963***	***0.923***	***1.064***	***0.917***	***0.917***	***1.100***	***1.000***
Jevons Index (avg L-T pr rel)	**100.0**	**96.3**	**92.3**	**106.4**	**91.7**	**91.7**	**110.0**	**100.0**
Arithmetic average pr rel (7 obs)	***1.000***	***0.966***	***0.932***	***1.066***	***0.925***	***0.931***	***1.100***	***1.000***
Carli Index (avg L-T pr rel)	**100.0**	**96.6**	**93.2**	**106.6**	**92.5**	**93.1**	**110.0**	**100.0**

Item A				**S-T Price relatives**				
Variety 1	*1.000*	*0.888*	*0.920*	*1.347*	*0.790*	*1.389*	*0.911*	*0.909*
Variety 2	*1.000*	*1.072*	*0.950*	*1.080*	*0.739*	*1.000*	*1.353*	*0.909*
Variety 3	*1.000*	*0.949*	*1.004*	*1.155*	*1.071*	*0.931*	*1.003*	*0.909*
Variety 4	*1.000*	*0.955*	*0.745*	*1.405*	*0.792*	*1.109*	*1.253*	*0.909*
Variety 5	*1.000*	*1.044*	*0.860*	*1.113*	*0.957*	*1.074*	*1.070*	*0.909*
Variety 6	*1.000*	*0.974*	*1.035*	*1.153*	*0.876*	*0.720*	*1.501*	*0.909*
Variety 7	*1.000*	*0.877*	*1.275*	*0.894*	*0.848*	*0.902*	*1.438*	*0.909*
Geometric average pr rel (7 obs)	***1.000***	***0.963***	***0.958***	***1.153***	***0.861***	***1.000***	***1.200***	***0.909***

Jevons Index (chained S-T pr rel)	**100.0**	**96.3**	**92.3**	**106.4**	**91.7**	**91.7**	**110.0**	**100.0**
Arithmetic average pr rel (7 obs)	***1.000***	***0.966***	***0.970***	***1.164***	***0.868***	***1.018***	***1.219***	***0.909***
Carli Index (chained S-T pr rel)	**100.0**	**96.6**	**93.7**	**109.0**	**94.6**	**96.2**	**117.3**	**106.6**

8.63 The calculations in Tables 9.5a and 9.5b show that when the missing price for variety 6 is imputed using the short-term price change of the other varieties, The trend of the Jevons, Dutot, and Carli indexes reflect the changes for all the observations using the direct and L-T relative methods. For the Jevons and Dutot indexes, the chained method gives the same results as the direct method. However, the chained Carli is significantly upward biased demonstrating that this method should not be used for index compilation.

Treatment of permanently disappeared varieties

8.64 Varieties may disappear permanently for a number of reasons. The variety may disappear from the market because new varieties have been introduced or the outlets from which the price has been collected have stopped selling the product. Where products disappear permanently, a replacement product has to be sampled and included in the index. The replacement product should ideally be one that accounts for a significant proportion of sales, is likely to continue to be sold for some time, and is likely to be representative of the sampled price changes of the market that the old product covered.

8.65 The timing of the introduction of replacement varieties is important. Many new products are initially sold at high prices which then gradually drop over time, especially as the volume of sales increases. Alternatively, some products may be introduced at artificially low prices to stimulate demand. In such cases, delaying the introduction of a new or replacement variety until a large volume of sales is achieved may miss some systematic price changes that ought to be captured by CPIs. It is desirable to avoid making replacements when sales of the varieties they replace are significantly discounted in order to clear out inventory. In such cases, disappearing variety's price should be returned to its last non-discounted price as the new variety is introduced.

8.66 Table 9.6 shows an example where variety A disappears after March and variety D is included as a replacement from April onward. Varieties A and D are not available on the market at the same time and their price series do not overlap.

Table 9.6 Disappearing varieties and their replacements with no overlapping prices

	January	February	March	April	May
Item B			*Prices*		
Variety A	6.00	7.00	5.00		
Variety B	3.00	2.00	4.00	5.00	6.00
Variety C	7.00	8.00	9.00	10.00	9.00
Variety D				9.00	8.00
Geometric average	**5.01**	**4.82**	**5.65**	**7.66**	**7.56**
Arithmetic average	**5.33**	**5.67**	**6.00**	**8.00**	**7.67**
(a) No imputations for missing prices					
Jevons index – the ratio of geometric mean prices = geometric mean of price relatives					
Direct index	100.00	96.15	112.62	152.86	150.79
Month-to-month change	1.0000	0.9615	1.1713	1.3572	0.9865
Chained m/m index	100.00	96.15	112.62	152.86	150.79
Dutot index – the ratio of arithmetic mean prices					
Direct index	100.00	106.25	112.50	150.00	143.75
Month-to-month change	1.0000	1.0625	1.0588	1.3333	0.9583

Chained m/m index	100.00	106.25	112.50	150.00	143.75

(b) Imputation for missing prices

Jevons index – the ratio of geometric mean prices = geometric mean of price relatives

*Impute the price of variety A in April using the S-T relative of average prices: 5.00 x[(5x10)/(4x9)]$^{0.5}$ = **5.89***
The April average price is derived as (5.89 x 5 x10)$^{1/3}$=6.65
The April index is derived using the January geometric average price (6.65/5.01) = 1.2740 x 100 = 127.40
A new imputed average price is calculated for January by taking the April average price of varieties B, C and D (5x10x9)$^{1/3}$ = 7.66 and deflating the value using the April L-T price change (7.66/1.3273) = 5.77
The May index is then calculated as (7.56/5.77)x100 = 130.94

	January	February	March	April	May
Item B			*Prices*		
Variety A	6.00	7.00	5.00	5.89	
Variety B	3.00	2.00	4.00	5.00	6.00
Variety C	**7.00**	**8.00**	**9.00**	**10.00**	**9.00**
Variety D	6.78			9.00	8.00
Geometric averages	5.01	4.82	5.65	6.65	
Adjusted average	***5.77***			7.66	7.56
Direct index	100.00	96.15	112.62	132.73	130.94

The month-to-month changes are calculated from the geometric average of price changes of varieties A, B, C from January through April. The monthly change in May is calculated on the geometric average of price changes for varieties B, C, D in April and May

Month-to-month change	1.0000	0.9615	1.1713	1.1785	0.9865
Chained m/m index	100.00	96.15	112.62	132.73	130.94

Dutot index – the ratio of arithmetic mean prices

*Impute the price of variety A in April using the S-T relative of average prices: 5.00 x(5+10)/(4+9) = **5.38***
The April average price is derived as (5.38+5+10)/3=6.79
The April index is derived using the January average price (6.79/5.33) = 1.2740 x 100 = 127.40
*A new imputed average price is calculated for January by taking the April arithmetic average price of varieties B, C and D (5+10+9)/3 = 8 and deflating the value using the April L-T price change (8/1.2740) = **6.28***
The May index is then calculated as (7.67/6.28)x100 = 122.10

	January	February	March	April	May
Item B			*Prices*		
Variety A	6.00	7.00	5.00	***5.38***	
Variety B	3.00	2.00	4.00	5.00	6.00
Variety C	7.00	8.00	9.00	10.00	9.00
Variety D				9.00	8.00
Arithmetic averages	**5.33**	**5.67**	**6.00**	**6.79**	
Adjusted average	***6.28***			8.00	**7.67**
Direct index	100.00	106.25	112.50	127.40	122.10

The month-to-month changes are calculated from the average price for varieties A, B, C from January through April. The monthly change in May is calculated on the average price for varieties B, C, D in April and May

Month-to-month change	1.0000	1.0625	1.0588	1.1325	0.9583
Chained m/m index	100.00	106.25	112.50	127.40	122.10

Carli index – the arithmetic mean of price relatives

Average S-T price relative is (5/4+10/9)x0.5 = 1.1806
*Impute the price of variety A in April as 5.00 x1.1806 = **5.90**, so that L-T relative is (5.90/6) = 0.9838*
Average L-T relative for elementary index is (0.9838+1.6667+1.4286)/3=1.359 7x100 =135.97, the April index
*Impute the price for variety D in January as 9/1.3597 = 6.62 to derive the May index(7.67/6.62 *100) = 149.81*

	January	February	March	April	May
Item B			*Prices*		

Variety A	6.00	7.00	5.00	***5.90***	
Variety B	3.00	2.00	4.00	5.00	6.00
Variety C	7.00	8.00	9.00	10.00	9.00
Variety D				9.00	8.00
			Price ratios		
Variety A	1	1.1667	0.8333	***0.9838***	
Variety B	1	0.6667	1.3333	1.6667	2.0000
Variety C	1	1.1429	1.2857	1.4286	1.2857
Variety D					1.2086
Direct index	100.00	99.21	115.08	135.97	149.81

8.67 To include the new variety in the index from April onward, an imputed price needs to be calculated for March. The imputation will differ based on the formula used. For the Jevons index the geometric average of short-term relatives is used; for the Dutot index the short-term relative of average prices is used; while for the Carli index the arithmetic average of short-term relatives is used. If a direct index is being calculated from average prices, the imputed price must be included in calculating the average prices. In the Jevons and Dutot examples in Table 9.6, the average base price used in the direct calculation must be adjusted for the relative difference between the level of the old sample's average price and level of the new average price using the new sample of varieties. The adjusted base price in these examples is derived by deflation of the new average price level by the long-term trend of the elementary index. From another perspective, the adjusted base price is estimated by applying the ratio of the new sample's average price to the old sample's average price to the old base price. This implicitly assumes that the difference in the average prices reflects the difference in quality.

8.68 If a chain index is calculated, the imputation method ensures that the inclusion of the new variety does not, in itself, affect the index and an adjustment of the base price is not necessary. In the case of a chain index, imputing the missing price by the average change of the available prices gives the same result as if the variety is simply omitted from the index calculation. However, by storing the imputed price as an observation, it can be used with a reported price for index calculation in the subsequent month as previously demonstrated in Table 9.5a. Thus, the chain index is compiled by simply chaining the month-to-month price movement between periods t-1 and t, based on the matched set of prices in those two periods, onto the value of the chain index for period t-1. In the example, no further imputation is required after April, and the subsequent movement of the index is unaffected by the imputed price change between March and April.

8.69 The situation is somewhat simpler when there is an overlap month in which prices are collected for both the disappearing and the replacement variety. In this case, it is possible to link the price series for the new variety to the price series for the old variety that it replaces. Linking with overlapping prices involves making an implicit adjustment for the difference in quality between the two varieties, as it assumes that the relative prices of the new and old varieties reflect their relative qualities. For perfect or nearly perfect markets this may be a valid assumption, but for certain markets and products it may not be so reasonable. The question of when to use overlapping prices is dealt with in detail in Chapter 7. The overlap method is illustrated in Table 9.7.

Table 9.7 Disappearing and replacement varieties with overlapping prices

	January	February	March	April	May
Item B			*Prices*		
Variety A	6.00	7.00	5.00		
Variety B	3.00	2.00	4.00	5.00	6.00
Variety C	7.00	8.00	9.00	10.00	9.00
Variety D			10.00	9.00	8.00
Geometric average price					
A,B,C; (*B,C,D*)	5.01	4.82	5.65 (*7.11*)	(*7.66*)	(*7.56*)
Arithmetic average price					
A,B,C; (*B,C,D*)	5.33	5.67	6.00 (*7.67*)	(*8.00*)	(*7.67*)

Jevons index – the ratio of geometric mean prices = geometric mean of price ratios

Chain the monthly indices based on matched prices

Month-to-month change	1.0000	0.9615	1.1713	*1.0772*	*0.9865*
Chained m/m index	100.00	96.15	112.62	*121.32*	*119.68*

For the direct index, a new imputed average price is calculated for January by taking the average price of varieties B, C and D in March $(4x9x10)^{1/3}$ *= 7.11 and deflating by the March L-T index (1.1262) to derive the adjusted base price (6.32). This calculation maintains the level of the March index. The adjusted base price is used to compile the April and May indices.*

Direct index	100.00	96.15	112.62	121.32	119.68

Dutot index – the ratio of arithmetic mean prices

Chain the monthly indices based on matched prices

Month-to-month change	1.0000	1.0625	1.0588	*1.0435*	*0.9583*
Chained m/m index	100.00	106.25	112.50	*117.39*	*112.50*

For the direct index, a new imputed average price is calculated for January by taking average price of varieties B, C, and D in March((4+9+10)/3 = 7.67 and deflating by the March L-T relative (1.1250) to derive the adjusted base price (6.81).This calculation maintains the level of the March index. This adjusted base price is used to compile the April and May indices

Direct index	100.00	106.25	112.50	117.39	112.50

Carli index – the arithmetic mean of price relatives

Variety A	1.0000	1.1667	0.8333		
Variety B	1.0000	0.6667	1.3333	1.6667	2.0000
Variety C	1.0000	1.1429	1.2857	1.4286	1.2857
Variety D	1.0000			1.0357	0.9206
L-T relative		0.9921	1.1508	1.3770	1.4021

Average L-T relative for elementary index in March is (0.8333+1.333+1.2857)/3=1.1508 x100 =115.08, the March index

Impute the price of variety D in January as 10.00 /1.1508 = ***8.69****, keeping the L-T relative as 1.1508 so that the introduction of variety D does not affect the March index level. The new L-T relatives for variety D in April and May are 1.3770 (9.00 /8.69) and 0.9206 (8.00/8.69).*

The average L-T relative for varieties B, C, and D are used to calculate the April and May indices.

Direct index	100.00	99.21	115.08	137.70	140.21

8.70 In the example in Table 9.7, overlapping prices are obtained for varieties A and D in March. There is now an overlapping sample for March—one using varieties A, B, C, and the other using varieties B, C, and D. A monthly chain Jevons index of geometric mean prices will be based on the prices of varieties A, B and C until March, and from April onwards on the prices of varieties B, C and D. The replacement variety is not included until prices for two successive periods are obtained. Thus, the monthly chain index has the advantage that it is not necessary to carry out any explicit imputation of a reference (base) price for the new variety. The same approach applies to the Dutot chain index.

8.71 If a direct index is defined as the ratio of the arithmetic (geometric) mean prices, the price in the base period needs to be adjusted by deflation of the new average in March by the long-term index so that the March index level is maintained and the new sample does not affect the long-term price change through March. If a new base period price of variety D for January was imputed, different results would be obtained because the price changes are implicitly weighted by the relative base period prices in the Dutot index, which is not the case for the Carli or the Jevons indices. The April and May index change in the

Dutot index is lower than the Jevons because the declines in price of varieties C and D have larger implicit weights in the Dutot (39 and 43 percent) versus the Jevons (33 and 33 percent).[2]

8.72 If the index is calculated as a direct Carli, the January base period price for variety D must be imputed by dividing the price of variety D in March (10.00) by the long-term index change for March (1.1508). This deflation of the variety D price maintains the index level in March. The long-term relative for replacement variety D in April and May is calculated by dividing the prices by the estimated base price (8.69) of variety D in January.

Calculation of elementary price indices using weights

8.73 The Jevons, Dutot, and Carli indices are all calculated without the use of explicit weights. However, as already mentioned, in certain cases weighting information may be available that could be exploited in the calculation of the elementary price indices. Weights within elementary aggregates may be updated independently and possibly more often than the elementary aggregate weights themselves.

8.74 A special situation occurs in the case of tariff prices. A tariff is a list of prices for the purchase of a particular kind of good or service under different terms and conditions. One example is electricity, where one price is charged during daytime while a lower price is charged at night. Similarly, a telephone company may charge a lower price for a call at the weekend than in the rest of the week. Another example may be bus tickets sold at one price to ordinary passengers and at lower prices to children or old age pensioners. In such cases, it is appropriate to assign weights to the different tariffs or prices in order to calculate the price index for the elementary aggregate.

8.75 The increasing use of electronic points of sale in many countries, in which both prices and quantities are scanned as the purchases are made, means that valuable new sources of information may become increasingly available to statistical offices. This could lead to significant changes in the ways in which price data are collected and processed for CPI purposes. The treatment of scanner data is examined in Chapters 6 and Annex 1.

8.76 If the reference period expenditures for all the individual items within an elementary aggregate, or estimates thereof, were to be available, the elementary price index could itself be calculated as a Laspeyres price index, or as a geometric Laspeyres as discussed in the section on calculation of higher level indices.

Other formulae for elementary price indices

8.77 Another type of average is the harmonic mean. In the present context, there are two possible versions: either the harmonic mean of price relatives or the ratio of harmonic mean prices. The harmonic mean of price relatives is defined as:

$$P_{HR}^{0:t} = \frac{1}{\frac{1}{n}\sum \frac{p_i^0}{p_i^t}} \qquad (9.5)$$

The ratio of harmonic mean prices is defined as:

[2] The new sample starts in March as the price reference. The Dutot implicit weights are 17.4 (4/23), 39.1 (9/23), and 43.5 (10/23) percent, respectively, for varieties B, C, and D.

$$P_{RH}^{0:t} = \frac{\sum \frac{n}{p_i^0}}{\sum \frac{n}{p_i^t}} \qquad (9.6)$$

Formula (9.6), like the Dutot index, fails the commensurability test and would only be an acceptable possibility when the varieties are all fairly homogeneous. Neither formula appears to be used much in practice, perhaps because the harmonic mean is not a familiar concept and would not be easy to explain to users. Nevertheless, at an aggregate level, the widely used Paasche index is a weighted harmonic average.

8.78 The ranking of the three common types of mean is always arithmetic ≥ geometric ≥ harmonic. It is shown in Chapter 20 that, in practice, the Carli index (the arithmetic mean of the price ratios) is likely to exceed the Jevons index (the geometric mean) by roughly the same amount that the Jevons exceeds the harmonic mean. The harmonic mean of the price relatives has the same kinds of axiomatic properties as the Carli index, but with opposite tendencies and biases. It fails the transitivity, time reversal and price bouncing tests.

8.79 As referenced earlier, the Carruthers-Sellwood-Ward-Dalen (CSWD) index might be calculated as an unweighted approximation to a Fisher index. The Carli index (equation 9.2) is upward biased and the harmonic mean index of relatives (equation 9.5) is downward biased by about the same amount. In taking the geometric average of these indices, the biases will be offset.

$$P_{CSWD}^{0:t} = \sqrt{P_C^{0:t} \times P_{HR}^{0:t}} \qquad (9.7)$$

The CSWD and Jevons index calculations produce almost identical results. Thus, for compilation purposes, it is easier to simply calculate the Jevons index.

8.80 In recent years, attention has focused on formulae that can take account of the substitution that may take place within an elementary aggregate. As already explained, the Carli and the Jevons indices may be expected to approximate a cost of living index if the cross-elasticities of substitution are close to 0 and 1, respectively, on average. A more flexible formula that allows for different elasticities of substitution is the unweighted Lloyd-Moulton (LM) index:

$$P_{LM}^{0:t} = \left[\sum \frac{1}{n} \left(\frac{P_i^t}{P_i^0} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \qquad (9.8)$$

where σ is the elasticity of substitution. The Carli and the Jevons indices can be viewed as special cases of the LM in which σ = 0 and σ = 1. The advantage of the LM formula is that σ is unrestricted. Provided a satisfactory estimate can be made of σ, the resulting elementary price index is likely to be approximate to the underlying cost of living index. The LM index reduces "substitution bias" when the objective is to estimate the cost of living index. The difficulty is the need to estimate elasticities of substitution, a task that will require substantial development and maintenance work. The formula is described in more detail in Chapter 17.

Unit value indices

8.81 The unit value index is simple in form. The unit value in each period is calculated by dividing total expenditure on some product by the related total quantity. It is clear that the quantities must be strictly additive in an economic sense, which implies that they should relate to a single homogeneous product. The unit value index is then defined as the ratio of unit values in the current period to that in the reference period. It is not a price index as normally understood, as it is essentially a measure of the change in the average price of a *single* product when that product is sold at different prices to different consumers, perhaps at different times within the same period. Unit values, and unit value indices, should not be calculated for sets of heterogeneous products. Unit value methods are discussed in more detail in Chapter 6 and Annex 1 on Unit Values.

Formulae applicable to scanner data

8.82 Scanner data obtained from electronic points of sale are becoming an increasingly important source of data for CPI compilation. Their main advantage is that the number of price observations can be enormously increased and that both price and quantity information is available in real time. There are, however, many practical considerations to be taken into account, which are discussed in other chapters of this manual. Scanner data application and formulae are discussed in more detail in Chapter 6 and Annex 1.

8.83 Access to detailed and comprehensive quantity and expenditure information within an elementary aggregate means that there are no constraints on the type of index number that may be employed. Not only Laspeyres and Paasche but superlative indices such as Fisher and Törnqvist may be envisaged. As noted at the beginning of this chapter, it is preferable to introduce weighting information as it becomes available rather than continuing to rely on simple unweighted indices such as Carli and Jevons. Advances in technology, both in the retail outlets themselves and in the computing power available to statistical offices, suggest that traditional elementary price indices may eventually be replaced by superlative indices, at least for some elementary aggregates in some countries. The methodology must be kept under review in the light of the resources available.

The calculation of higher-level indices

8.84 As shown in Figure 9.1, the elementary indices are the starting point (building blocks) for calculating the CPI. These indices are then aggregated to successively higher levels, e.g., city, region, class, group, etc., to derive the national all items index. These higher level indices are derived by aggregations using weights that are generally derived from an HES. The aggregation formulae can take several forms such as arithmetic (linear) and geometric (exponential) depending on the target index. Laspeyres–type indices tend to use arithmetic aggregations while the superlative indices such as the Törnqvist index use geometric aggregations.

8.85 A statistical office must decide on the target index at which to aim. Statistical offices have to consider what kind of index they would choose to calculate in the ideal hypothetical situation in which they had complete information about prices and quantities in both time periods compared. If the CPI is meant to be a *cost of living index*, then a superlative index such as a Fisher, Walsh, or Törnqvist would have to serve as the theoretical target, as a superlative index may be expected to approximate the underlying cost of living index.

8.86 Many countries do not aim to calculate a cost of living index and prefer the concept of a fixed *basket index*, sometimes also referred to as a *pure price index* or an *inflation index*. A basket index is one that measures the change in the total value of a given basket of goods and services between two time periods. This general category of index is described here as a *Lowe index* (see Chapter 15) in which the weight reference period precedes the price reference period of the index. It should be noted that, in general, there is no necessity for the basket to be the actual basket in one or other of the two periods compared. If the target index is to be a basket index, the preferred basket might be one that attaches equal importance to the baskets in both periods; for example, the Walsh index. Thus, the same index may emerge as the preferred target in both the basket and the cost of living approaches.

8.87 In Chapters 15-17 the superlative indices Walsh, Fisher and Törnqvist show up as being "best" in all the approaches to index number theory. These three indices, and the Marshall-Edgeworth price index, while not superlative, give very similar results so that for any practical reason it will not make any difference which one is chosen as the preferred target index. In practice, a statistical office may prefer to designate a basket index that uses the actual basket in the earlier of the two periods as its target index on grounds of simplicity and practicality. In other words, the Laspeyres index may be the preferred target index. Similarly, if the quantities in both periods are available, the Walsh index, which is also a fixed basket index, might be the target.

8.88 The theoretical target index is a matter of choice. In practice, it is likely to be either a Laspeyres or some superlative index. Even when the target index is the Laspeyres, there may be a considerable gap between what is actually calculated and what the statistical office considers to be its target. Chapters 15-17 present the alternatives from a theoretical point a view. It is also shown that some combination of an arithmetic index such a Young index and a geometric index such as geometric Lowe may approximate the superlative Fisher and Törnqvist indices. Such an approach may be the ideal solution since both of these indices can be produced in real time. What many statistical offices tend to do in practice is use the Laspeyres index as their target.

Consumer price indices as weighted averages of elementary indices

8.89 A higher-level index is an index for some expenditure aggregate above the level of an elementary aggregate, including the overall CPI itself. The inputs into the calculation of the higher-level indices are:

- The elementary aggregate price indices
- The expenditure shares of the elementary aggregates

8.90 The higher-level indices are calculated simply as weighted averages of the elementary price indices. The weights typically remain fixed for a sequence of at least 12 months. Some countries revise their weights at the beginning of each year in order to try to approximate as closely as possible to current consumption patterns, but many countries continue to use the same weights for several years. The weights may be changed only every five years or so. The use of fixed weights has the considerable practical advantage that the index can make repeated use of the same weights. This saves both time and money. Revising the weights can be both time-consuming and costly, especially if it requires new household expenditure surveys to be carried out.

Examples of Laspeyres Price Indices

8.91 The most often referenced formula for calculation of higher level aggregate indices is the Laspeyres indices. The Laspeyres price index is defined as:

$$P_L^{0:t} = \frac{\sum p_i^t \cdot q_i^0}{\sum p_i^0 \cdot q_i^0} = \sum w_i^0 \cdot \left(\frac{p_i^t}{p_i^0}\right) , \quad w_i^0 = \frac{p_i^0 \cdot q_i^0}{\sum p_i^0 \cdot q_i^0} \qquad (9.9)$$

w_i^0 indicates the expenditure shares for the individual varieties in the reference period. As the quantities are often unknown, the index usually will have to be calculated by weighting together the individual price relatives by their expenditure share in the price reference period, w_i^0. The available weighting data may refer to an earlier period than the price reference period, but may still provide a good estimate. A more general version of (equation 9.9) would be that of a Lowe or a Young index, where the weights are not necessarily those of the price reference period. These two indices are discussed in more details later in this chapter. Note that if all shares are equal, equation 9.9 reduces to the Carli index. If the shares are proportional to the prices in the reference period, it reduces to the Dutot index.

8.92 The geometric version of the Laspeyres index is defined as:

$$\mathrm{P}_{GL}^{0:t} = \prod \left(\frac{p_i^t}{p_i^0}\right)^{w_0^i} = \frac{\prod \left(p_i^t\right)^{w_i^0}}{\prod \left(p_i^0\right)^{w_i^0}} , \quad \sum w_i^0 = 1 \qquad (9.10)$$

where the weights, w_i^0, are again the expenditure shares in the reference period. When the weights are all equal, equation (9.10) reduces to the Jevons index. If the expenditure shares do not change much between the weight reference period and the current period, then the geometric Laspeyres index approximates a Törnqvist index. A more general version of (equation 9.10) would be that of a Geometric Young index, where the weights are not necessarily those of the price reference period.

8.93 Table 9.8 provides an example of calculations of aggregate Laspeyres indices. The group consists of three items for which prices are collected monthly. The expenditure shares are estimated to be 0.80, 0.17 and 0.03.

Table 9.8 Calculation of a weighted elementary index

	Weight	December	January	February	Pct. Change Dec. – Feb.
Item *A*	0.80	7	7	9	28.6
Item *B*	0.17	20	20	10	-50.0
Item *C*	0.03	28	28	12	-57.1
Weighted arithmetic mean of price relatives (Laspeyres)					**Index**
((9/7) x 0.8 + (10/20) x0.17 + (12/28) x0.03) x 100 =					112.64
Weighted geometric mean of price relatives (geometric Laspeyres)					
$((9/7)^{0.8}$ x $(10/20)^{0.17}$ x $(12/28)^{0.03})$ x 100 =					105.95

8.94 One option is to calculate the index as the weighted arithmetic mean of the price relatives, which gives an index of 112.64. The individual price changes are weighted according to their explicit weights, irrespective of the price levels. This corresponds to the calculation of a Laspeyres price index, where the price relatives and the weights refer to the same reference month. The index may also be calculated as the weighted geometric mean of the price relatives, the geometric Laspeyres index, which gives an index of 105.95.

Index Reference Periods

8.95 It is useful to recall that three kinds of reference periods may be distinguished:

- *Weight reference period*. The period covered by the expenditure statistics used to calculate the weights. Usually, the weight reference period is a year.
- *Price reference period*. The period whose prices are used as denominators in the index calculation (also referred to as the base price).
- *Index reference period*. The period for which the index is set to 100.

8.96 The three periods are generally different. For example, a CPI might have 2016 as the weight reference year, December 2018 as the price reference month and the year 2015 as the index reference period. The weights typically refer to a whole year, or even two or three years, whereas the periods for which prices are compared are typically months or quarters. The weights are usually estimated on the basis of an expenditure survey that was conducted some time before the price reference period. For these reasons, the weight reference period and the price reference period are invariably separate periods in practice.

8.97 The index reference period is often a year; but it could be a month or some other period. An index series may also be re-referenced to another period by simply dividing the series by the value of the index in that period, without changing the rate of change of the index. The expression “base period” can mean any of the three reference periods and is ambiguous. The expression “base period” should only be used when it is absolutely clear in context exactly which period is referred to.

Typical Calculation Methods for Higher-level Indices

8.98 The most common method for calculating a CPI does not involve individual prices or quantities. Instead, a higher-level index is calculated by averaging the elementary price indices by their pre-determined weights. The formula 9.9 can be written as follows:

$$P^{0:t} = \sum w_j^b P_j^{0:t} \ , \quad \sum w_j^b = 1 \tag{9.11}$$

where $P^{0:t}$ denotes the overall CPI, or any higher-level index, from period *0* to *t*, w_j^b is the share weight attached to each of the elementary price indices where the shares sum to 1. $P_j^{0:t}$ is the corresponding elementary price index. The elementary indices are identified by the subscript *j*, whereas the higher-level index carries no subscript at this point. As already noted, a higher-level index is any index, including the overall CPI, above the elementary aggregate level. (Later an *I* will be used as a subscript to indicate the item index at the national level and an *N* to indicate the all items index at the national level.) The weights are derived from expenditures in period *b*, which in practice precedes period 0, the price reference period. If the weights are updated for price change from *b* to 0, which keeps the quantity shares fixed, the index is called a *Lowe index*. If the period *b* weights are used directly in the index as expenditure shares in period 0, the index is known a *Young index*. Both are named for the nineteenth-century index number pioneers who advocated these indices. This is similar to the attribution given to the more noted *Laspeyres index* where the $b = 0$ and the *Paasche index* where period *t* weights are used in a harmonic mean formula. Whether the Lowe or Young index should be used depends on how much price change occurs between the weight and price reference period. This is discussed in detail in Chapter XX.

8.99 Provided the elementary aggregate indices are calculated using a transitive formula such as Jevons or Dutot, but not chained Carli, and provided that there are no new or disappearing varieties from period *0* to *t*, equation (9.11) is equivalent to:

$$P^{0:t} = \sum w_j^b P_j^{0:t-1} P_j^{t-1:t} \ , \quad \sum w_j^b = 1 \tag{9.12}$$

The difference is that equation (9.11) is based on the direct elementary indices from *0* to *t*, while (9.12) uses the chained elementary indices. $P_j^{t-1:t}$ is the short-term price relative for the elementary aggregate between *t*-1 and *t*. The advantage of the latter is that it allows the sampled products within the elementary price index from *t*-1 to *t* to differ from the sampled products in the periods from *0* to *t*-1. Hence, it allows replacement items and new items to be linked into the index from period *t*-1 without the need to estimate a price for period *0*. For example, if one of the sampled varieties in periods *0* and *t*-1 is no longer available in period *t*, and the price of a replacement product is available for *t*-1 at *t*, the new replacement product can be included in the index using the overlap method.

8.100 Equations (9.11) and (9.12) are additive and apply at each level of aggregation. That is, a higher-level index is the same whether calculated on the basis of the elementary price indices or on the basis of the intermediate higher-level indices. The additivity also facilitates the presentation of the index.

8.101 An alternative method for aggregating elementary indices would be geometric aggregation. Geometric aggregation is similar to arithmetic aggregation, but involves weighting each elementary index by the power of its share weight as shown in equation 9.10. Another form of aggregation using shown in equation 9.13 is to convert the elementary indices to natural logarithms and use linear weighting of the logarithms. In this case, the result of the aggregation must be converted from natural logarithm to a real number (the antilog or exponential function).

$$P_G^{0:t} = \exp\left[\sum w_j^b \ln\left(P_j^{0:t}\right)\right] \tag{9.13}$$

8.102 If the weights reference period *b*, the index is a geometric Young index; if they reference period 0, the index is a geometric Laspeyres index, and if they reference the average of periods 0 and *t*, it is a Törnqvist index. Recent empirical research discussed in Chapter 15 has indicated that a geometric average of, for example, a Young index and a geometric Lowe index may closely approximate the Fisher index. The reason for this close fit is that the possible upward bias in the arithmetic Young is offset by a possible downward bias in the geometric Lowe index.

Calculation of Geographic and National Indices

8.103 CPIs are often calculated for individual areas within a country and then aggregated to provide a national index based on the price movements in the individual areas. The aggregation approach is the same where elementary aggregates are combined using weights for each item index in the area to derive the all items CPI for the area. The elementary item indexes are then aggregated using their area weights to derive the national item index. The formula for aggregation items in areas to derive a national item index is:

$$P_I^{0:t} = \sum_j w_{j,a}^b \left(P_{j,a}^{0:t}\right) / \sum_j w_{j,a}^b \qquad 9.14$$

where $P_I^{0:t}$ is national index for item *I* from period 0 to *t*
$P_{j,a}^{0:t}$ is the area index for item *j* in area *a* from the period 0 to *t*
$w_{j,a}^b$ is the weight for item *j* in area *a* from the weight reference period *b*

The national all items index can be compiled by the aggregation of items across areas using their area weights:

$$P_N^{0:t} = \sum_j \sum_a w_{j,a}^b \left(P_{i,a}^{0:t}\right) / \sum_a w_{j,a}^b \qquad 9.15$$

The same result is obtained if the national item indexes are aggregated using the national item weights:

$$P_N^{0:t} = \sum_j w_j^b (P_I^{0:t}) / \sum_j w_j^b \qquad 9.16$$

Where: $P_N^{0:t}$ is the national all items index from the period 0 to *t*
w_j^b is the national weight for item *j* in the weight reference period

Numerical examples

8.104 Table 9.9 illustrates the calculation of higher-level indices using arithmetic aggregation where the weight and the price reference periods are identical, i.e. *b* = 0. The index consists of five elementary aggregate indices and two intermediate higher-level indices, G and H. The overall index and the higher-level indices are all calculated using (9.11). Thus, for example, the overall index for April can be calculated from the two intermediate higher-level indices for April as:

$$P^{Jan:Apr} = (0.6 \cdot 103.92) + (0.4 \cdot 101.79) = 103.06$$

or directly from the five elementary indices as:

$$P^{Jan:Apr} = (0.2 \cdot 108.75) + (0.25 \cdot 100) + (0.15 \cdot 104) + (0.1 \cdot 107.14) + (0.3 \cdot 100) = 103.06$$

Table 9.9 Aggregation of elementary price indices (arithmetic)

	Weight	January	February	March	April	May	June
Month-to-month elementary price indices							
Item A		100.00	102.50	104.88	101.16	101.15	100.00
Item B		100.00	100.00	91.67	109.09	101.67	108.20
Item C		100.00	104.00	96.15	104.00	101.92	103.78
Item D		100.00	92.86	107.69	107.14	100.00	102.67
Item E		100.00	101.67	100.00	98.36	103.33	106.45
Direct or chained monthly elementary price indices with January = 100							
Item A	0.20	100.00	102.50	107.50	108.75	110.00	110.00
Item B	0.25	100.00	100.00	91.67	100.00	101.67	110.00

Item C	0.15	100.00	104.00	100.00	104.00	106.00	110.00
Item D	0.10	100.00	92.86	100.00	107.14	107.14	110.00
Item E	0.30	100.00	101.67	101.67	100.00	103.33	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00
Higher-level indices							
G=A+B+C	0.6	100.00	101.83	99.03	103.91	105.52	110.00
H=D+E	0.4	100.00	99.47	101.25	101.79	104.28	110.00
Total		100.00	100.89	99.92	103.06	105.03	110.00

8.105 Table 9.10 illustrates the calculation of higher-level indices using geometric aggregation where the weight and the price reference periods are identical, i.e. $b = 0$. The index consists of the same five elementary aggregate indices and two intermediate higher-level indices, G and H. The overall index and the higher-level indices are all calculated using (9.13). Thus, for example, the overall index for April can be calculated from the two intermediate higher-level indices for April as:

$$P^{Jan:Apr} = \exp[(0.6 \cdot \ln(103.85)) + (0.4 \cdot \ln(101.74))] = 103.00$$

or directly from the five elementary indices as:

$$\begin{aligned} P^{Jan:Apr} &= \exp[(0.2 \cdot \ln(108.75)) + \big(0.25 \cdot \ln(100)\big) + (0.15 \cdot \ln(104)) + (0.1 \cdot \ln(107.14)) + (0.3 \\ &\qquad \cdot \ln(100))] \\ &= 103.00 \end{aligned}$$

Table 9.10 Aggregation of elementary price indices (geometric)

	Weight	January	February	March	April	May	June
Month-to-month elementary price indices							
Item A		100.00	102.50	104.88	101.16	101.15	100.00
Item B		100.00	100.00	91.67	109.09	101.67	108.20
Item C		100.00	104.00	96.15	104.00	101.92	103.78
Item D		100.00	92.86	107.69	107.14	100.00	102.67
Item E		100.00	101.67	100.00	98.36	103.33	106.45
Direct or chained monthly elementary price indices with January = 100							
Item A	0.20	100.00	102.50	107.50	108.75	110.00	110.00
Item B	0.25	100.00	100.00	91.67	100.00	101.67	110.00
Item C	0.15	100.00	104.00	100.00	104.00	106.00	110.00
Item D	0.10	100.00	92.86	100.00	107.14	107.14	110.00
Item E	0.30	100.00	101.67	101.67	100.00	103.33	110.00
Total		100.00	100.84	99.77	103.00	104.99	110.00
Higher-level indices							
G=A+B+C	0.6	100.00	101.82	98.79	103.85	105.47	110.00
H=D+E	0.4	100.00	99.39	101.25	101.74	104.27	110.00
Total		100.00	100.84	99.77	103.00	104.99	110.00

8.106 Table 9.11 illustrates the calculation of higher-level indices using arithmetic aggregation where the weight and the price reference periods are identical, i.e. $b = 0$. The index consists of five elementary aggregate indices in two geographic areas. The area all items indices are calculated using equation 9.11 in which the weights are the items' share within the area. The national-level item indices are all calculated using equation 9.14. The national all items index can be calculated using either equation 9.15 or 9.16. Thus, for example, the area B index for April is calculated from the five item-level indices for April as:

$$P_a^{Jan:Apr} = [(0.08 \cdot 108.21) + (0.10 \cdot 99.50) + (0.06 \cdot 103.48) + (0.04 \cdot 106.61) + (0.12 \cdot 99.50)]/\,0.6 = 102.55$$

The national item index for item A is calculated from the two area indices for item A:

$$P_I^{Jan:Apr} = [(0.12 \cdot 103.91) + (0.08 \cdot 101.70)]/0.20 \; = 108.53$$

The national all items index is calculate using equation 9.15 as:

$$P^{Jan:Apr} = [(0.2 \cdot 108.53) + (0.25 \cdot 99.8) + (0.15 \cdot 103.79) + (0.1 \cdot 106.93) + (0.3 \cdot 99.8)]/1.0 = 102.86$$

Table 9.11 Aggregation of elementary price indices (arithmetic) across locations

	Weight	January	February	March	April	May	June
Area 1							
Item A	0.12	100.00	102.50	107.50	108.75	110.00	110.00
Item B	0.15	100.00	100.00	91.67	100.00	101.67	110.00
Item C	0.09	100.00	104.00	100.00	104.00	106.00	110.00
Item D	0.06	100.00	92.86	100.00	107.14	107.14	110.00
Item E	0.18	100.00	101.67	101.67	100.00	103.33	110.00
All items	**0.60**	*100.00*	*100.89*	*99.92*	*103.06*	*105.03*	*110.00*
Area 2							
Item A	0.08	100.00	103.53	108.04	108.21	110.82	110.55
Item B	0.10	100.00	101.00	92.13	99.50	102.43	102.17
Item C	0.06	100.00	105.04	100.50	103.48	106.79	106.53
Item D	0.04	100.00	93.79	100.50	106.61	107.94	107.67
Item E	0.12	100.00	102.69	102.18	99.50	104.11	103.85
All items	**0.40**	*100.00*	*101.90*	*100.42*	*102.55*	*105.82*	*105.55*
National							
Item A	0.20	100.00	102.91	107.72	108.53	110.33	110.22
Item B	0.25	100.00	100.40	91.85	99.80	101.97	106.87
Item C	0.15	100.00	104.42	100.20	103.79	106.32	108.61
Item D	0.10	100.00	93.23	100.20	106.93	107.46	109.07
Item E	0.30	100.00	102.08	101.87	99.80	103.64	107.54
All items	***1.00***	***100.00***	***101.29***	***100.12***	***102.86***	***105.34***	***108.22***