Chapter 6: TEMPORARILY AND PERMANENTLY MISSING PRICES AND QUALITY CHANGE

INTRODUCTION

The measurement of changes in the level of consumer prices is complicated by the appearance and disappearance of new and old goods and services, as well as changes in the quality of existing ones. If there were no such complications, then a representative sample could be taken of the items households consume in a reference period 0, their prices recorded and compared with the prices of the same matched items in subsequent periods. In this way the prices of like would be compared with like. However, such complications do exist. Items change in quality over time and replacements are of a different quality to the original. New and old models of varieties appear and disappear. For a consumer price index (CPI) to remain representative of what is consumed the sample of items priced each period must remain of the same quality and representative.

Changes in the quality of items should be treated as changes in the volume, as opposed to changes in the price, of the good/service provided. For example, increases over time in the concentration of a detergent (number of washes per 1kg. packet), faster internet service (megabits per second, Mbps), and inclusion of a warranty in the price of a dishwasher all contribute to effective decreases in price; consumers get more for their money. Similarly, quality decreases, for example less legroom in economy flights, when prices remain constant, are effective increases in price.

Statistical agencies go to great lengths to ensure measured price changes are not unduly influenced by changes in the quality of items by measuring the price change of a fixed constant-quality basket of goods and services; they use the matched-models method (MMM). On rebasing, price collectors visit selected outlets with broad details of a product, and record the prices of the most popular, regularly stocked, models sold in each of the outlets and their characteristics including, say, the brand, size, and other price-determining features. The characteristics must be sufficiently detailed to enable a price-collector to uniquely identify the item in subsequently periods and record its matched price.

The measurement of changes in the level of consumer prices by the MMM is complicated by **temporarily unavailable** price quotes, for say one, two, or three months, due to an item being out-of-stock and not yet replenished. A matched price is unavailable in these intervening months. The treatment of prices of temporarily missing items is considered below but typically requires that the missing item's price is imputed for the month(s) in which it is missing using the price changes of similar such goods/services or drawn from a higher level of aggregation. Actual prices are then compared with imputed prices for the measurement of the CPI. Should price quotes become **permanently unavailable** a replacement is required that is preferably comparable in terms of the price-determining specifications of the missing item, or it may be non-comparable. If the replacement is of a comparable quality, its price can be directly compared with the last actual or imputed price for the missing item. If the replacement is non-comparable, say it is of a better quality, the difference in quality has either to be explicitly quantified in terms of its contribution to price, so that an adjustment for its difference in quality can be made to the price and again, the price of like be compared with like. If a reliable explicit quality-adjustment to price is not possible, for data or resource reasons, implicit methods of quality adjustment are available. The appearance and disappearance of new and old goods and services of different quality further complicate the CPI measurement. Explicit and implicit methods of quality adjustment are provided in sections x below and methods for dealing with new and disappearing goods and services illustrated in Chapter 8.

Products that are "strongly" seasonal, that is, missing in particular months when out-of-season but expected to return in the next season, are treated differently to those considered here and are the subject of chapter 9 section x. Strongly seasonal products include some fresh fruits, vegetables and clothing. Also considered in chapter 9 section x is the treatment of "weakly" seasonal products: available throughout the year but prices, sales and quality fluctuate throughout the year. The prices of weakly seasonal products are not missing and are treated differently from strongly seasonal ones, again as discussed in Chapter 9, section x.

The measurement of constant-quality price change is primarily achieved by matching models. When the matching breaks down, that is price quotes are missing, the above techniques are used either as a temporary imputation until the item's temporarily missing prices becomes available, or to introduce a replacement, thus serving to help update the sample. But what of product markets where the matching breaks down on a regular basis because of the high turnover in new models of different qualities to the old ones, say for laptop computers? A failure to match and replace models would lead to a seriously depleted and unrepresentative sample. Yet, a continual process of linking-in new replacement item has been found to lead to a bias in CPI measurement. Section x of this chapter outlines an alternative approach making use of hedonic regressions.

This chapter is primarily concerned with the treatment of temporarily and permanently missing items and their prices. As background to this treatment, the chapter first, outlines why the MMM may fail, the consequences of the failure, and how to ameliorate the effects of such failure on price measurement. Second, a concept of quality is provided to set the chapter against. Third, the role of the price collector becomes increasingly apparent throughout the chapter and while much of this has been the subject of chapter 6, it is revisited in this context of the treatment of missing prices. Fourth, some introductory notes are provided on general measurement issues including the use of additive versus multiplicative quality adjustments, price reference versus current period quality adjustment, short-term (S-T) versus long-term (L-T) comparisons, and geometric aggregation formula.

Having set this background, the chapter then moves to the treatment of temporarily and permanently missing item prices and quality adjustment and the particular needs of price measurement in product markets with a rapid turnover of models of a product, usually in the electronic and high-technology product markets.

BACKGROUND

Why the matched models method may fail

Three potential sources of error arise from the matched models approach: missing items, representativity of sample space, and new products.

Missing items

The first source of error, and the focus of this chapter, is when an item is no longer available in the outlet. It may be temporarily out of stock, discontinued or it may not be available to the same specification—its quality has changed; in either case it is effectively missing in the current period. The item's price may be missing for other reasons. It may be a seasonal item or one whose price does not need to be recorded so frequently, or it may be that the item is a custom-made product or service, supplied each time to the customer's specification.

It is necessary to distinguish between items that are permanently and temporarily missing. Items that are *temporarily* missing are items not available and not priced in the month in question, but that are available and priced in subsequent months. The treatment of items missing because demand and/or supply is seasonal, as is the case with some fruits and vegetables, is the subject of chapter x.

A number of approaches are available for dealing with missing items, including:

- The item may be dropped on the assumption that the aggregate price change of a group of other items reflects the change in the missing item's price—an implicit quality adjustment to price.
- A replacement item may be selected and the replacement item's price may be used for the comparison because the replacement is deemed to be comparable in quality to the missing item.
- The replacement may be deemed to be non-comparable with the missing item, but prices on both the missing and replacement items may be available in an overlap period before the former item was missing, or an imputation of an overlap price made.

The price difference in this overlap period may be used as an estimate of the quality difference to quality-adjust the replacement item's price.

- The replacement price of a non-comparable replacement may be used, with an explicit estimate of the adjustment for the quality difference to extricate the "pure" price and quality change.

In many cases, there is a need to make an explicit quality adjustment to the replacement item's price. A quality adjustment in this instance is an adjustment to the price (price change) of the replacement item (compared with the missing item) to remove that part of the price change that results from quality differences. A quality adjustment can be taken to be a coefficient that multiplies the price of, say, the replacement item to make it commensurate, from the consumer's point of view, with the price of the original.

To take a simple example, suppose that the size (or quantity) an item is sold in is a quality feature. Suppose that the size of the missing item and its replacement differ. For example, if 2 units of the replacement item were equivalent to 3 of the original, the required quality adjustment to be applied to the price of the replacement item is 2/3. Suppose one unit of the replacement actually sells at the same price as one unit of the original, then the price of the replacement, after adjusting for the change in quality, is only 2/3 that of the price of the original. If one unit of the replacement sells for twice the price of the original, then the quality-adjusted price is 4/3 that of the original: the price increase is 33 per cent, not 100 per cent. CPI compilers as a matter of routine convert prices to standard units of measurement and in doing so, undertake explicit quality adjustments.

The approaches listed in above will be discussed later in some detail, along with the assumptions implied by them. By definition, the prices of the unavailable items cannot be determined. The veracity of some of the assumptions about their price changes, had they been available, is therefore difficult to establish. What is stressed here is that the matching of prices of items allows for the measurement of price changes untainted by quality changes. When items are replaced with new ones of a different quality, then a quality-adjusted price is required. If the adjustment is inappropriate, there is an error, and if it is inappropriate in a systematic direction, there is a bias. Careful quality adjustment practices are required to avoid error and bias. Such adjustments are the subject of this chapter.

Sampling concerns

There are four concerns. First, the MMM and the use of replacements is designed to meet the needs of constant-quality price measurement and while the sample of items priced might initially be designed to be representative of price changes of the population of all items, it is effectively following a static sample of items that, over time, can become increasingly unrepresentative. The matching of prices of identical items over time, by its nature, is likely to lead to the monitoring of a sample of items increasingly unrepresentative of the population of transactions. This degradation of the sample is accentuated by the inability of the MMM to incorporate new models/products into the sample except as replacements to obsolete ones.

Substantial developments in, for example, telecommunication hardware and services, embodied in a growing variety of models, are excluded from the sample of models/services covered by the CPI. This omission would not be problematic were the (implicit) quality-adjusted price changes of the excluded items similar to those of the included matched model sample. However, this is unlikely to be the case. The (quality-adjusted) prices of old items being dropped may well be relatively low and the (quality-adjusted) prices of new ones relatively high as part of a sales strategy of dumping old models, at relatively low price to make way for the introduction of new models priced relatively high.

Second, MMM items go missing at the end of their life cycle. Permanently missing items provides an opportunity to improve the representativity of the price index by the inclusion of a more representative replacement. However, because of the additional resources required for quality adjustment to prices, it may be in the interests of the price collectors and desk statisticians, and indeed fall within their guidelines, to avoid making explicit non-comparable replacements and quality adjustments and rely on computer routines using what, as will be seen below, may be inappropriate implicit methods.

A third sampling concern relates to the timing of item substitution: when a replacement item is chosen to substitute for an old one. The prices of items continue to be monitored until they are no longer produced. This means that old items with limited sales are monitored. Such items may exhibit unusual price changes as they near the end of their life cycle, because of the marketing strategies of firms. Firms typically identify gains to be made from different pricing strategies at different times in the life cycle of products, particularly at the introduction and end of their life cycle. The (implicit or otherwise) weight of end-of-cycle items in the index would thus remain relatively high, being based on their sales share when they were sampled. Furthermore, new unmatched items with possibly relatively large sales would be ignored. As a consequence, undue weight would be given to the unusual price changes of matched items at the end of their life cycle.

The final sampling problem with the matching procedure is when the price collector continues to report prices of items until replacements are forced, that is, until the items are no longer available, and has instructions to replace those items with typically consumed or popular items. This improves the coverage and representativity of the sample. But it also makes reliable quality adjustments of prices between the old obsolete and new popular items more difficult. The differences in quality are likely to be beyond those that can be attributed to price differences in some overlap period, as one item is in the last stages of its life cycle and the other in its first. Furthermore, the technical differences between the items are likely to be of an order that makes it more difficult to provide reliable, explicit estimates of the effect of quality differences on prices. Finally, the (quality-adjusted) price changes of very old and very new items are unlikely to meet assumptions of “similar price changes to existing items or classes of items”, as required by the imputation methods. Many of the methods of dealing with quality adjustment for unavailable items may be better served if the switch to a replacement item is made earlier rather than later. Sampling concerns are inextricably linked to quality adjustment methods. This will be taken up in Chapter 8 on item selection and the need for an integrated approach to dealing with both representativity and quality-adjusted prices.

New products

A further potential source of error arises when something new is introduced into the marketplace. When a really new item is introduced, there is an immediate gain in welfare or utility as demand switches from the previous technology and other goods. For example, the introduction of the zip fastener for clothing, instead of buttons, was a completely new good that led to an initial gain in utility or welfare to consumers as they switched from the old to the new technology. This gain from the introduction of zip fasteners, and subsequently of velcro, would not be properly brought into the index by waiting until the index was rebased, or by waiting for at least two successive periods of prices for zip fasteners and linking the new price comparison to the old index. Subsequent prices might be constant or even fall. The initial welfare gain would be calculated from a comparison between the price in the period of introduction and the hypothetical price in the *preceding* period, during which supply would be zero. The practical tools for estimating such a hypothetical price are neither well developed nor practical for CPI compilation, as outlined in more detail in Chapter 21. For a consumer price index built on the concept of a base period and a fixed basket, there is, strictly speaking, no problem. The new good was not in the old basket and should be excluded. Although an index properly measuring an old fixed basket would be appropriate in a definitional sense, it would not be representative of what we buy. Such an index would thus be inappropriate. For a cost of living index concerned with measuring the change in expenditure necessary to maintain a constant level of utility (see Chapter 17), there is no doubt that it would be conceptually appropriate to include the new good and any welfare gain from its introduction, though as outlined in chapter 8, this is highly problematic in practice.

The nature of quality change

This section considers what is meant by quality change. To understand the meaning of quality change requires a conceptual and theoretical platform, so that adjustments to prices for quality differences are made against a well-considered framework.

A starting point is to appreciate that over time the quality of what is produced changes. New automobiles for the large part become more reliable, durable, powerful, economical, have an increasing number of myriad features. In matching the prices of a sample of models selected in a price reference period with the self-same models in subsequent months, the quality mix is kept constant in an attempt to avoid contaminating the price measurement through quality differences. As will be seen later, however, the resulting sample of models is one that gives less emphasis to models subsequently introduced which may have benefited from more recent technological change and have different price changes given the quality of services they provide.

Observed changes in prices arise in theory from a number of sources, including quality changes, changes in tastes and preferences, and changes in the technology of producers, see Chapter 21. More formally, the observed data on prices are the locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possibly varying technologies of production. The separation of the effects of changes in tastes and preferences from quality changes is only possible in highly restrictive circumstances.

Price differences of similar products are often taken to be measures of differences in quality. Yet observed differences in prices are often observed for items of the same quality. This may arise from a number of reasons: (i) some consumers may be unaware of the availability of the self-same items being available at lower prices. There may be "search costs" to their exploring the market to discover lower priced items; (ii) there may be price discrimination because the seller is able to charge different prices to different categories of consumers, such as services such as movie tickets for children and senior citizens; (iii) prices may be sticky with some retailers changing their prices infrequently to avoid the costs of doing so, including adverse customer reaction, or as strategic competitive behavior, such as loss leaders. Different retailers charging prices at different times leads to price variation for the same variety; and (iv) where there are parallel markets, an official one subject to government or official control at which products are rationed and unofficial free market unregulated price one. The unofficial market may be at lower price because it avoids taxes and regulations, or at higher price since the official price is a subsidized one, but has a limited, possibly varying, quantities available for sale (*SNA 2008* paragraphs 15.64-15.75).

The changing mix of the observed characteristics of items is not the only concern. There is also the practical problem of not always being able to observe or quantify characteristics such as the style, reliability, and ease of use and safety of what is produced. The same good provided at a different and more convenient location may command a higher price and be of a higher quality. Furthermore, different times of the day or periods of the year may also give rise to quality differences: For example, electricity or transport provided at peak times must be treated as being of higher quality than the same amount of electricity or transport provided at off-peak times. The fact that peaks exist shows that purchasers or users attach greater utility to the services at these times. Other differences, including the conditions of sale and circumstances or environment in which the goods or services are supplied or delivered, can make an important contribution to differences in quality. A retailer, for example, may attract customers by providing free delivery, credit opportunity or better variety, by being more accessible, by offering shorter order times, smaller tailor-made orders, clearer labeling, better support and advice, more convenient car parking or a wider range of brands, or simply by operating in a more pleasant or fashionable environment. These sorts of benefits are not always specified in the item description, however, that conceptually such quality improvements should be outside the scope of the index. If any such benefits change, including the replacement of an outlet if one closes, a price adjustment for the estimated value of the benefits should be made.

To consider how to adjust prices for quality changes, it is first necessary to ask what is meant by quality. While there may be an intuition as to whether an item consumed in one period is better than its counterpart in the next, a theoretical framework will help in establishing the basis for such comparisons. For example, an item of clothing is sampled and, after a few months, it is missing. One option is to replace it with a similar item. The nearest comparable option may have more cloth in it, or have a lining, be a different color, have different buttons, have better stitching or be considered to be better styled in some fashionable sense. There is a need to put a price estimate on the difference in quality between the old and new items so that the price of like can be compared with like. To propose or criticize a quality adjustment procedure requires some concept of what is ideally required and how the procedure stands up to this. Although such a discussion takes us away from the practicalities of the procedures for a while, its use will become apparent in subsequent sections.

A utility-based approach

In Chapter 17 a cost of living index (COLI) is defined as the ratio of the minimum expenditures in the base and current period required to achieve a given standard of living or "utility". Quality adjustments to prices involve trying to measure the price change for a product that has exhibited some change in its characteristics from an earlier period that provides a different level of utility to the consumer. The equating of the value of a quality change with the change in utility derived by the consumer, while falling naturally under a COLI framework, is not exclusive to it. A cost of a fixed basket of goods index (COGI) can also benefit from regarding quality in this way. While a COGI requires the pricing of a fixed basket of products, some items will become unavailable and the replacement items selected to maintain the sample may not be of the same quality. A GOGI based on a fixed basket concept has the pragmatic need to adjust for quality differences when an item is unavailable, and there is nothing in the definition of a fixed basket index that precludes differences in utility being used as a guideline. If item A is better than its old version, item B, it is because it delivers something more to the consumer who is willing to pay more. That "thing" is called utility.

Note that the definition of a quality change is based on equating some change in characteristics to a different level of utility provided. Consider an example in which a new, improved quality item is substituted for an old one in period *t*, the consumer having to choose between the two. Suppose that after the new quality item appeared, both qualities were offered to a consumer at the same price, say $p^t = 100$. The consumer was then asked to choose between them and naturally preferred the new quality. Say the price of the old quality was then progressively reduced until it reached a point $p^{t*} = 75$, at which the consumer was indifferent as regards the choice between purchasing the old quality at $p^{t*} = 75$ and the new quality at $p^t = 100$. The consumer might then select the old quality at 75 or the new one at 100. Either way, the consumer would obtain the same utility, because of being indifferent as to which to choose. Any further decrease below $p^{t*} = 75$ would cause the consumer to switch back to the old quality.

The difference between p^t and p^{t*} would be a measure of the additional utility that the consumer placed on the new quality as compared with the old quality. It would measure the maximum amount that the consumer was prepared to pay for the new quality over and above the price of the old quality. In economic theory, as will be outlined in Chapter 21, if consumers (or households) are indifferent between two purchases, the utility derived from them is the same. The difference between 75 and 100 must therefore arise from the consumers' valuation of the utility they derive from the two items: their quality difference. The definition is sensible as a conceptual framework. It naturally has problems relating to implementation, but this is not our concern here. Our initial concern is with the provision of an analytical framework on which to ground our thinking and analysis.

The utility-based framework is concerned with the question of how consumers choose between items of different qualities. The answer, in part, is because more utility is derived from an item of higher quality than from an item of lower quality, and thus consumers prefer it. But this does not explain why one item is bought rather than the other. For this it is also necessary to know the relative price of one item with respect to the other, since if the lower-quality item is cheaper, it may still be purchased. The above thought experiment to determine the price below which the old quality would be purchased, $p^{t*} \leq 75$, serves this purpose.

The role of price collectors

Price collectors have a critical role to play in the treatment of missing price observations. They observe and record that a price is missing; whether it is temporarily or permanently missing; if permanently missing, whether a comparable or non-comparable replacement is available, and in the latter case, the price and details of the replacement item. As explained in Chapter 6, on the sampling of monthly prices, the outlets are visited in a process referred to as initiation, to determine the detailed specifications of representative items sold. For example, for the general class of "large white bread, sliced," the more detailed, "large loaf, white, unsliced, brand A, 800gm" may be selected and its details entered along with its price for subsequent monthly repricing. Price collectors have in their possession a checklist of these specifications when visiting outlets in subsequent months in order to (i) help identify the item to be priced; (ii) check the item's specification with those on initiation to ensure the quality has not been changed, it is comparable; and (iii) if the item is non-comparable, use the specifications to identify a replacement item to be priced and record specifications and note differences in quality.

The price collector plays an important role in determining whether the missing price should be treated as temporarily or permanently missing. An item's price can be considered to be temporarily missing if the same item is likely to return to the market within a reasonable time period. On finding there is not an item on sale with the required specifications, the price collector should check with the manager or informed member of staff whether it is temporarily or permanently missing. If temporarily missing, the expected duration: one, two, or more months should be recorded along with the reason for being unavailable and an indication of the likelihood of its return.

Temporarily missing items have their prices imputed; permanently missing ones have a replacement. As the issues and their treatment are different it is therefore important for the price collector to establish whether the unavailability of the item is temporary or permanent. If it is out of stock for, say, three consecutive months, the price collector should be instructed to choose a replacement which matches as closely as possible the item's specification. When the price is regarded as being temporarily missing, it should be imputed using an overall mean imputation, a targeted men imputation, or a class-mean imputation. Some statistical offices use a method referred to as carry-forward (the value of the last observed price). As outlined below, this is not recommended.

Permanent unavailability occurs when the item is withdrawn from the market with no prospect of returning. In some instances, it might be absent the next month and confirmed by the outlet manager/senior staff that it is not going to be replaced. With such information the price collector should immediately look to collecting the price and specifications of a replacement item. In other cases the outlet manager/staff may not be very helpful and it is then that a, say, three-month rule is applied. There may be particular products/circumstances in which the three-month rule can be relaxed, such as a temporary withdrawal of products for health reasons, national emergencies, or the logistics of restocking where there is a sound basis for a belief that the product will return in the near future, albeit in more than three-months.

Decisions on the treatment of missing prices are made by a desk officer informed by the information provided by the price collector and, in some instances, by telephone/visit to the outlet.

Item prices may be missing for products because they are seasonal and out of season, as should be noted by a price collector. Out-of-season products, but expected to return in the next season, are treated differently to those considered here and are the subject of chapter 9 section x. Strongly seasonal products are unavailable when out of season and include some fresh fruits, vegetables, fish, and clothing. Also considered in chapter 9 section x is the treatment of "weakly" seasonal products: available throughout the year but prices, sales and quality fluctuate throughout the year. The prices of weakly seasonal products are not missing and are treated differently from strongly seasonal ones, again as discussed in Chapter 9, section x.

As outlined in chapter 6 a coding sheet, such as the simple illustration in Table 6.1, should be completed by the price collector so that the price entry to the database has an appropriate designation as to its need for treatment. Meta data should be collected on the extent and product groups in which, for example, there is a high level of missing prices of different forms. Illustrative item codes are given below and statistical offices should build on the detail required to meet their specific needs.

Illustrative item codes for price collector for missing values	
Collection code	**Description**
T	Temporarily Missing – The item is unavailable but is expected to be available again in the near future.
P	Permanently Missing - The item is no longer available and is unlikely to return.
S	Seasonally Missing – the item (product group) is strongly seasonal and is out of season.
C	Comparable Replacement - A replacement item that is comparable to the old item in all major aspects.
NC	Non-comparable Replacement – A replacement item that is not comparable to the old item.

Some general points

Additive versus multiplicative adjustment

The quality adjustments to prices may be undertaken either by adding a fixed amount or multiplication by a ratio. For example, consider *m*, an old item, and *n* its replacement and, for a price comparison over periods *t, t* + 1, and *t* + 2, the price of *m* is only available in periods *t* and *t* + 1 and *n* only available in periods *t*+1 and *t* + 2. A price relative over periods *t, t* + 1, and *t* + 2 requires an overlap ratio p_n^{t+1} / p_m^{t+1} to be used as a measure of the relative quality difference between the old item and its replacement. This ratio could then be *multiplied* by the price of the old item in period *t*, p_m^t to obtain the quality-adjusted prices p_n^{t*} as follows:

	t	$t+1$	$t+2$
old item *m*	p_m^t	p_m^{t+1}	
replacement item *n*	p_n^{*t}	p_n^{t+1}	p_n^{t+2}

Such multiplicative formulations are generally advised, as the adjustment is invariant to the absolute value of the price. The overlap ratio is, for example, 1.2; the new 20 percent more than the old. Yet there may be some items for which the worth of the constituent parts is not considered to be in proportion to the price. In other words, the constituent parts have their own, intrinsic, absolute, additive worth, which remains constant over time. Retailers selling over web sites may, for example, include the cost of shipping, which in some instances may remain the same in the short to medium-term irrespective of what is happening to price. If the cost of shipping is subsequently excluded from the price, this fall in quality should be valued as a fixed additive sum.

Price reference versus current period adjustment

Two variants of the approaches to quality adjustment are to make the adjustment either to the price in the price reference period or to the price in the current period. For example, in the overlap method, described above, the implicit quality adjustment coefficient was used to adjust p_m^t to p_n^{*t}. An alternative procedure would have been to multiply the ratio p_m^{t+1} / p_n^{t+1} by the price of the replacement item p_n^{t+2} to obtain the quality-adjusted price p_m^{*t+2}, etc. The first approach is more straightforward since, once the base period price has been adjusted, no subsequent adjustments are required. Each new replacement price can be compared with the adjusted base period price. More importantly, the valuation of the quality differential took place in period t+1 can be applied to period t with more confidence than the ongoing periods t+2, t+3, t+4 and so forth.

Long-term versus short-term comparisons

Much of the analysis of quality adjustments in this manual has been undertaken by comparing prices between two adjacent periods, say, month-on-month period t prices with those in a subsequent period t+1. For long-term comparisons the price reference period is taken as, say, period t and the index is compiled by comparing prices in period t first with $t + 1$; then t with $t + 2$; t with $t + 3$, etc. The short-term framework allows long-term comparisons built up as the product of links: t first with $t + 1$; then t+1 with $t + 2$; t +2 with $t + 3$, etc.; built up as a sequence of links joined together by successive multiplication. This chapter focuses on S-T comparisons, for reasons of their inherently better properties and for focus of exposition; chapter 9 provides detail, an illustration, and the relative merits of the use of the L-T and S-T methods.

Aggregation formula for elementary price indices

A ratio of geometric means—the Jevons price index number formula—is used to measure price changes at this un-weighted level. Alternative formula include a ratio of arithmetic means—the Dutot price index number formula—and an arithmetic average of price ratios—the Carli price index number formula. The Jevons price index formula is used here for reasons of its better properties and focus of exposition. As with the previous section, chapter 9 provides detail, an illustration, and the relative merits of the use of the Dutot and Carli price index number formula.

It will be the case that with scanner and other such data, information on prices, expenditure values, and quality characteristics will be available for the vast majority of individual models sold by major outlets. This availability of data on transaction values allows weights to be used at this detailed level of aggregation and thus the use of weighted price index formulas as outlined in chapters 9, ?, and ??.

THE TREATMENT OF TEMPORARILY AND PERMANENTLY MISSING ITEM PRICES

To measure aggregate price changes, a representative sample of items is selected from a sample of outlets, along with a host of details that define each item, their specifications. The items are re-priced each month. The detailed specifications are included on the re-pricing form each month as a prompt to help ensure that the same items are being priced. Detailed checklists of item descriptions should be used, as any lack of clarity in the specifications may lead to errors. Attention should also be devoted to ensuring that the specifications used are not just to identify the item on a subsequent visit, for example its location in the outlet, but contain all pertinent, price-determining elements, otherwise there may be cases in which the quality change would become invisible to the price measurement process.

The MMM succeeds in ensuring the prices of like specifications are compared with like, that is the measurement of price change is not influenced by changes in the quality of the items. However, when a price is missing there is the potential for mis-measurement. The treatment of missing values depends on whether the item is temporarily missing—the product will be available in the near future being, for example, out-of-stock—or permanently missing, the item will not be available. A temporarily missing price say in March should be imputed and compared with its February actual price for a February to March price change, and with its actual price on its return, say after a month out of stock, in April for the March to April price change. A permanently missing price requires a replacement.

When an item is missing in a month a number of approaches may be used though the terminology differs between authors and statistical agencies; they include:

- **imputation:** the price changes of all items in the product group, or targeted similar ones, are assumed to be the same as that for the missing item. Such imputations are to be used for temporarily missing items. Permanently missing items require a comparable or non-comparable replacement;
- **direct comparison:** if an item is permanently missing and a replacement item is directly comparable, that is, it is so similar it can be assumed to have had more or less the same quality characteristics as the missing one, its price replaces the unavailable price. Any difference in price level between the new and old is assumed to arise from a price and not quality change;
- **explicit quality adjustment:** if a replacement item is non-comparable—there are identifiable quality differences—estimates of the effect of the quality differences on prices enable quality-adjusted price comparisons to be made;

- **implicit quality adjustment: overlap:** if a replacement item is non-comparable and no information is available, or resources too limited, to allow reasonable explicit estimates to be made of the effect on price of a quality change, the price difference between the old item and its replacement in the overlap period is then taken to be a measure of the quality differential.

Specific attention will need to be devoted to product areas with relatively high weights, where large proportions of items are turned over. Some of the methods are not straightforward and require a level of expertise. Quality adjustment needs to be implemented by developing a gradual approach on a product-by-product basis. Such concerns should not be used as excuses for failing to attempt to estimate quality-adjusted prices. The practice of statistical agencies in dealing with missing items, even if it is to ignore them, implicitly involves a quality adjustment. Such an implicit approach may not be the most appropriate method, and may even be misleading.

It is apparent that applying quality adjustments to prices is not a simple matter of applying routine methods to prices in specified product areas. A number of alternative approaches are suggested below. Some will be more appropriate than others for specific product areas. An understanding of the consumer market, technological features of the producing industry, and alternative data sources will all be required for the successful implementation of quality adjustments.

Temporarily missing price observations

Overall mean imputation

This method uses the price changes of other similar items as estimates of the price change of the missing item. Consider a Jevons elementary price index, i.e., a geometric mean of price relatives (Chapter 20). The prices of the missing item in the current period, say $t + 1$, is imputed by multiplying its prices in the immediately preceding period t by the geometric mean of the price relatives of the remaining matched items in the product group between these two periods. The comparison is then linked by multiplication to the price changes for previous periods. The method provides the same result as simply dropping the item that is missing from both periods from the calculation. In practice, the series is continued in the database by including the imputed prices; this then forms a complete tableau. The imputations are based on assumptions of similar price movements.

Consider the illustrative example in Table 6.1. A product with broad specifications is sold in six outlets, A to F, with different "tighter" store-specific specifications adopted for each outlet. The price reference period is December 2019 with successive prices collected for each outlet's specification in January, February, and March to July 2020. The price collector finds the item temporarily missing in outlet F's price collection for March 2020, and likely to remain missing for the next month or so, but to return thereafter. Assume the recommended

Jevons elementary index number formula is used, that is, for a period 0 to period *t* price index the index is compiled as geometric mean of price relatives, equivalent to the ratio for geometric means of prices. Chapter 9 provides further examples of imputations and replacements with using the Jevons index, as compared with the Dutot (ratio of arithmetic means) and Carli indices (mean of the ratio of price relatives) and chapter (x) on the relative merits of the three indices. Here the focus is on the recommended Jevons index.

The imputed prices using the Jevons formula are entered in Table 6.1 providing a complete tableau of prices for outlets A to F over the reference price period and subsequent months. The Jevons index number formula is shown in equation (6.1) as direct/long-term (L-T) index comparing, in its second last term, the price of each matched item in the price reference period, hereafter 0, with the current month *t*, and in the last term for the example in Table 6.1, July 2020 with the price reference period = 100.0 of Dec 2019.

$$(6.1)\quad \mathrm{P_J}\left(\mathrm{P^0},\mathrm{P^t}\right)=\prod_{i=1}^{N}\left(\frac{P_i^t}{P_i^0}\right)^{\frac{1}{N}}=\frac{\prod_{i=1}^{N}\left(P_i^t\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(P_i^0\right)^{\frac{1}{N}}}\equiv\frac{\prod_{i=1}^{N}\left(P_i^{Jul20}\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(P_i^{Dec19}\right)^{\frac{1}{N}}}$$

In practice the use of the Jevons formula in this L-T form is not advised. Instead a short-term (S-T) formulation is recommended as the product of month-on-month Jevons indices. The S-T cumulative Jevons index for December 19=100 to July 2020 is:

(6.2)

$$\mathrm{P_J}\left(\mathrm{P}^{Dec19},\mathrm{P}^{Jul20}\right)=\frac{\prod_{i=1}^{N}\left(P_i^{Jan20}\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(P_i^{Dec19}\right)^{\frac{1}{N}}}\times\frac{\prod_{i=1}^{N}\left(P_i^{Feb20}\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(P_i^{Jan20}\right)^{\frac{1}{N}}}\times\frac{\prod_{i=1}^{N}\left(P_i^{Mar20}\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(P_i^{Feb20}\right)^{\frac{1}{N}}}=\ldots.\times\frac{\prod_{i=1}^{N}\left(P_i^{Jul20}\right)^{\frac{1}{N}}}{\prod_{i=1}^{N}\left(P_i^{Jun20}\right)^{\frac{1}{N}}}$$

The (L-T) and (S-T) approaches in equations (6.1) and (6.2) approaches provide the same answer as the numerator of each term on the right hand side of equation (6.2) cancels with the denominator of the next. However, a major advantage of the S-T formulation is that when an individual price is missing, its price can be imputed using the month-on-month price changes of similar or higher-level aggregates rather than the L-T comparison of the current month to the price reference period, which may be many years ago. Indeed, the index for, say July 2020 compared with June 2020 with a missing price in July 2020, can be seen from equation (6.2) to have its pre-existing sample measuring price changes up to the last link, with an imputation only being made for the last term. This contrasts sharply with index results using a L-T approach that will require an imputation for the whole period of December 2019 to July 2020. Again, it stressed that in practice the periods between rebasing can be very long with many missing price observations, severely degrading the sample.

The treatment of missing prices in this Chapter is by means of a S-T index. Calculations for a direct L-T Jevons index, along with counterpart Dutot and Carli indices are illustrated with examples in Chapter 9 along with related issues.

The right-hand-side of equation (6.2) requires a geometric mean of prices to be calculated for each period for a matched sample. Table 6.1 shows the geometric means for the price reference period, December 2020, January and February 2020, but then missing prices for March, April and May with prices returning in June and July. The first task is to impute the missing price for March 2020. This is undertaken using the ratio of geometric mean prices for each of the matched sample of outlets A to E in February and March to provide the short-term (S-T) price relative:[1]

$$(6.3)\quad \mathrm{P_J}\left(\mathrm{P}^{Feb20},\mathrm{P}^{Mar20}\right)=\frac{\prod_{i=A}^{E}\left(P_i^{Mar20}\right)^{\frac{1}{5}}}{\prod_{i=A}^{E}\left(P_i^{Feb20}\right)^{\frac{1}{5}}}=\frac{\left(5.49\times5.25\times5.20\times5.65\times6.90\right)^{\frac{1}{5}}}{\left(5.49\times5.10\times5.20\times5.49\times6.50\right)^{\frac{1}{5}}}=\frac{5.67}{5.54}=1.023765$$

This is the change in the (geometric) mean price for the matched prices A to E is from February to March. This increase of 1.023765 when multiplied by the February price of 5.99 yields an imputed September price of $1.023765\times5.99=6.13$.

The price collector subsequently finds item F's April and May prices to be temporarily missing the respective imputed prices are: $1.00352\times6.13=6.15$ and $1.00365\times6.15=6.17$. The imputed prices are entered in Table 6.1 providing a complete tableau of prices for outlets A to F over the reference price period and subsequent months. Chapter 9 provides an example with the recommended Jevons index as compared with the Dutot (ratio of arithmetic means), and Carli indices (mean of the ratio of price relatives).

The Jevons index can be calculated from this tableau as either long-term comparisons, with each current period compared with a price reference period, or as the product of S-T price relatives. Both yield the same result; the latter is preferred in this context since it facilitated the incorporation of permanently missing price observations.

The short-term month-on-month price relative for February to March 2020 is given as:

$$(6.4)\quad \mathrm{P_J}\left(\mathrm{P}^{Feb20},\mathrm{P}^{Mar20}\right)=\frac{\prod_{i=A}^{F}\left(P_i^{Mar20}\right)^{\frac{1}{5}}}{\prod_{i=A}^{F}\left(P_i^{Feb20}\right)^{\frac{1}{5}}}=\frac{5.74}{5.61}=1.0237$$

with the remaining relatives calculated in Table 6.1. The price index (December 2019 =100.00) to July 2020 is shown below in Table 6.2 and in Table 6.1 as the cumulative product of short-term relatives and for all months.

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[1] Geometric means over large samples are more accurately computed as the equivalent:
$$\exp\left[\tfrac{1}{N}\left(\sum_{i=1}^{N}\ln P_i^t-\sum_{i=1}^{N}\ln P_i^0\right)\right].$$

Table 6.2 Overall mean and targeted mean imputations

	Overall mean imputation	Targeted mean imputation
December 2019	100.00	100.00
January 2020	100.00*1.0137=101.37	100.00*1.0137=101.37
February 2020	101.37*1.0075=102.13	101.37*1.0075=102.13
March 2020	102.13*1.0237=104.55	102.13*1.0237=104.92
April 2020	104.55*1.0035=104.92	104.92*1.0044=105.38
May 2020	104.92*1.0036=105.30	105.38*1.0038=105.78
June 2020	105.30*1.0020=105.52	105.78*1.0041=106.21
July 2020	105.52*1.0081=106.37	106.21*1.0144=107.74

Targeted mean imputation

The overall mean imputation is based on assuming the price change of the temporarily missing item is the same as that of the overall price change at a higher level of aggregation. A targeted form of the method would use price movements of a cell or an aggregate of similar items that is, items expected to have similar S-T price changes. The sample of observations used for the targeting may be specific to a type of outlet and/or region and/or cluster of features, for example, “up-market” television sets. It would generally be a sub-set of items within a higher level of aggregation and decisions as to target the imputation using similar items in the sub-set or use a wider higher level of aggregation will depend in part on the adequacy of the sample size for the sub-set of similar items and the homogeneity of the elementary aggregate at the higher level.

Imputed prices in the illustration in Table 6.1 for the missing item-prices in outlet F for March, April, and May 2020 are based on adjusting the preceding period’s price by the price movements of the remaining matched pairs of prices at other independent traders, D and E, rather than all outlets. Changes in the geometric mean price relative as applied to adjust the preceding period’s price, are:

$$5.99\times\left(5.65\times 6.90\right)^{\frac{1}{2}}/\left(5.49\times 6.50\right)^{\frac{1}{2}}=5.99\times 1.04522=6.26 \text{ for March 2020;}$$

$$6.26\times\left(5.25\times 6.90\right)^{\frac{1}{2}}/\left(5.65\times 6.90\right)^{\frac{1}{2}}=6.26\times 1.00881=6.32 \text{ for April 2020; and}$$

$$6.32\times\left(5.80\times 6.90\right)^{\frac{1}{2}}/\left(5.75\times 6.90\right)^{\frac{1}{2}}=6.32\times 1.00434=6.34 \text{ for May 2020.}$$

The price index is compiled as the cumulative product of the S-T price relatives, as shown in Table 6.1 and 6.2.

The higher levels used at this elementary stage of aggregation would be specific to a country in that it would follow the country’s CPI aggregation structure, as outlined in Chapter 9 (paragraphs 9.5–9.9) and Figure 9.1. The higher level might be a region and type of outlet, for example in Figure 9.1, Brand A of par-boiled long-grain white rice sold in supermarkets in the

Northern region. Should there be an insufficient sample size for Brand A, similar Brands A and B, or all brands, might be used for all types of outlets in the region. Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates where the prices can be expected to move in the same direction. The imputation can be made using all of the remaining prices in the elementary aggregate. This is numerically equivalent to omitting the variety for the immediate period, but it is necessary to make the imputation.

An imputed price should always be directly compared with the actual price on the item's return as this provides a self-correcting measure. For example, if the imputation was badly wrong and showed decreases in prices over the period, when in fact the price of the item either sold elsewhere or, if not sold, being pent-up, was increasing, then a direct comparison between the last imputed and the returning actual price would bring the index back to its longer term trend. The overlap method described below, links-in a replacement item's price change and can be used for permanently missing items. The overlap method does not have this self-correcting feature and should not be used for temporarily missing items.

Carry forward imputation

Carrying forward the last observed price should be avoided wherever possible and is acceptable only for a very limited number of periods. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and product turnover. While simple to apply, carrying forward the last observed price biases the resulting index towards zero change. In addition, when the price of the missing variety is recorded again, there is likely to be a large compensating step-change in the index to return to its proper value. The adverse effect on the index will be increasingly severe if the variety remains un-priced for some length of time. In the illustration in Table 6.1, the missing prices in March 2020 would be imputed as the February 2020 price of 5.99 carried forward, as would be the imputed prices in April, and May 2020. However, on the item's return, in June there would be the step increase in price from May to June of 5.99 to 6.25. In general, to carrying forward is not an acceptable procedure or solution to the problem. Exceptions may be well-established and well-advertised periodic increases of set prices and tariffs and information from a senior store manger.

General considerations

As a general principle, temporarily missing prices require an explicit imputation entered into the data compilation. The overall mean imputation may refer to a higher level of aggregation, a variety, within a region/type of outlet. However, the default should be the higher-level of aggregation and a targeted imputation used if this heading comprises more than one variety some with different price changes. For example, if the missing price observation is for "canned tuna" where the higher weighted level aggregate is "canned fish" which say includes "canned tuna" and "canned salmon," then subject to a sufficient sample size, the imputation should be based on price movements of "canned tuna."

An overall mean imputation not only benefits from an automation of its implementation of imputations, but also serves the integrity of the index. By using an overall imputation statistical offices guard themselves against criticisms of influencing the CPI by their choice of "similar varieties," particularly where there are missing values for heavily weighted elementary aggregates. However, such caution should not be exercised where there are strong a priori or empirical grounds to believe a target imputation to be superior. Statistical offices should have retrospective monthly price data available to them at higher levels of aggregation and be able to examine differences in S-T month-on-month price changes between the missing item's price changes as against price changes of similar items and higher levels of aggregation and choose between an overall and targeted imputation accordingly. In the much-simplified illustration of Table 6.1 price changes of supermarkets are very different from independent traders and an imputation is illustrated for item F using independent traders. Item F might also have been imputed using the price index at a higher level of aggregation or even a single outlet's price. Similar principles of aggregation apply.

The importance of adequate software and the role of price collectors and desk officers, is stressed. Typically, a price collector reports an item price as temporarily missing; this is then passed to a desk officer for confirmation and then, perhaps, further at a higher level. If confirmed, a decision is made as to whether to use a targeted or overall imputation. If overall, an appropriate computer routine is applied entering the imputed figure with a designation as imputed into the tableau. If targeted the desk officer selects the item rows regarded as likely to have similar price changes, and the imputation is applied. In all of this the routine records the decisions made and measures the number of temporarily missing items, by elementary aggregate and their treatment for tabulation for quality assurance.

Permanently missing price observations

Table 6.2 shows imputed temporarily missing item price in March, April, and May 2020 for outlet F using the overall-mean imputation of Table 6.1. However, in June 2020 after 3-months there is neither a return of the item nor expectation by the senior store staff that it will return. The price collector finds a replacement item. The use of replacement items is not only to maintain the original sample sizes at the last rebasing, but also the representativity of the items selected. Senior store staff may have confirmed that the missing item is permanently missing in its first month, and indeed have helped identify a best-selling replacement model(s) expected to have high sales for the foreseeable future; its specifications and how it differ from the old model.

Comparable replacement

A price collector would regard an item as permanently unavailable if verified by a senior member of the outlet's staff or in following a, say three-month rule, the item is no longer available and there is no evidence to the contrary that it will reappear. The price collector should follow the existing item's specification and look to find a comparable item with the same specifications: a washing machine with the same spin-speed, capacity, brand or cluster of equivalent brands. If a comparable replacement exists its detailed specifications should be confirmed by the price collector against the existing specifications. Any changes in the specification deemed to be not sufficiently price-determining should be noted for the desk officer to confirm, say color, trim, and so forth. At its most straightforward level, a comparable replacement may simply be a new model number attributed to what is essentially the existing item.

The comparable replacement method requires the price collector to make a judgment that the replacement is of a similar quality to the old item and any price changes are untainted by quality changes. In the illustration in Table 6.2 there is a comparable item F1 to replace the item F, both from outlet F. The replacement item is considered by the price collector and confirmed by the desk officer as being directly comparable and thus 6.29 and 6.29 are entered into the data tableau as a continuation of outlet F series for June and July respectively. The price index is calculated using S-T price relatives built into a L-T price index. The price index as at July 2020 (December 2019=100.00) is 106.48, a 6.48 percent increase over this period. The price index remains as a constant-quality index since in June and July 2020 the prices of like quality items continue to be compared with like.

A common practice of manufacturers of electronic goods such as television-sets, household appliances, computers and computer-related hardware and software, automobiles is to have major quality changes in some years but relatively minor ones in other years. A new "comparable" model would have a new model number with a new production run, though nothing much physically has changed. The method of comparable replacement relies on the efficacy of the price collectors, desk officers and, in turn, on the adequacy of the specifications used as a description of the items. Statistical agencies may tend towards designating replacements as comparable since they are wary of sample sizes being reduced by dropping items and also wary of the intensive use of resources to introduce non-comparable replacements or make explicit estimates as outlined below. The use of items of a comparable specification has much to commend it. If the quality of items is improving, however, the preceding item will be inferior to the current one. Continually ignoring small changes in the quality of replacements can lead to an upward bias in the index. The extent of the problem will depend on the proportion of such occurrences, the extent to which comparable items are accepted as being so despite quality differences, and the weight attached to those items. Proposals in Chapter 8 to monitor types of quality adjustment methods by product area provide a basis for a strategy for applying explicit adjustments where they are most needed.

Non-comparable replacements

Non-comparable replacements require implicit or explicit price adjustments for that component of the price change measure between the price of the old model and the price of the new non-comparable replacement that is due to quality differences. The main implicit method is the overlap method. The replacement's price change is linked onto the old items price change using an overlap period that includes both the old and replacement items price. Where an overlap price for the replacement item does not exist it might be imputed. Explicit quality adjustment methods require an estimate of the price differential due to the quality difference. Explicit methods include quantity adjustments, option/feature costs, and "patched" hedonic regression methods. These are considered in turn.

Overlap method

The imputation of an overlap price

Consider for illustration Table 6.3 where in each outlet we define the items to be different "models" of the product with a pre-existing "old" model F and a non-comparable "new" replacement model for the missing item in outlet F in June and July 2020, that is F2, with prices of 5.25 and 5.99 respectively. The prices are lower than would be expected from the prices of F, but this is a non-comparable replacement: it is a replacement item with a major share of the market that is expected to remain on the market for the foreseeable future.

The overlap method requires a price for both "old" model F, and the "new" model F1 in an overlap period: F exists up to and including May, F1 exists in June, July and thereafter. Table 6.3 shows 5.25 to be entered as an estimated price in May 2020 for the new model, to provide an overlap price for the old and replacement models, F and F2. One source of information for this overlap price of 5.25 in May is the price collector.

The price collector may have anticipated falling sales and a switch of consumers to a new model, brand, or variety and recorded the overlap prices for the replacement prior to its adoption, in May rather than June. Price collectors should be trained to anticipate such changes, to corroborate them with senior outlet staff, and relay the information back to the desk officer for possible action, that is, in our example: in May F has dwindling sales and a poorer positioning for display in the outlet and is supplemented by F2. Senior outlet staff has confirmed that F2 is to effectively replace F as a model aimed at that the same segment of the market. The price for F2 and its quality characteristics should be recorded alongside that of F to provide an overlap price for the introduction of F2 in June, or if possible, to enact the replacement in May. As a general principle, the replacement of models is best not undertaken when the old model has limited sales and is at the end of its life cycle.

Alternatively, the price collector may have in June asked senior outlet staff whether the new model was sold in the previous month to obtain an overlap price for May, or if sold in other outlets, whether there is a pricing agreement with the supplier that this outlet would have kept to had it been supplied to them, and what would have been the price. The desk officer should confirm such details by visit or telephone with senior staff of outlet F.

Imputed overlap prices: the overlap price in May of 5.25 might have been imputed. The validity of this imputation is critical to the quality-adjustment methodology. In May 2020 there is a price of 6.18 for the old model and 5.25 for the new model. The method implicitly attributes this difference in the common May overlap period as an indicator of their quality difference.

If the new (old) model was not sold in May (June), an imputation for its May (June) price cane be made to provide an overlap price. The imputation may be an overall-mean or targeted imputation following the principles outlined for temporarily missing item prices as illustrated in Table 6.1. Alternatively, a class-mean imputation might have been used which in principle, is more suited to this context of imputing replacement item prices for permanently missing items, as opposed to temporarily missing ones.

The class-mean imputation method is a specifically designed targeted imputation to use to introduce a replacement when an item's price is permanently missing. The class-mean method of implicit quality adjustment to prices arose from concerns that unusual price were charged at the start of and end of a model's. Thus, the price movement of continuing items appears to be a flawed proxy for the pure price component of the difference between old and replacement items. A class-mean imputation is mainly considered as a means of quality adjustment where there is a relatively high rate of frequent replacements, such as different models of automobiles launched each year. On bringing in a new model the old model might be sold at a lower than normal price to clear stocks and the new at an inflated model to capture segments of the market willing to pay more for the latest model—a strategy referred to as price skimming in marketing.

The class-mean method is similar in procedure to the overall and targeted mean imputation and is a form of targeted imputation. The "target" is measured price changes of replacements for permanently missing products. Only the price changes of "comparable" replacements are used to impute the overlap price, the replacements being limited to those that have exactly the same price□determining characteristics, or those items with replacements that have been declared comparable after review or have already been quality□adjusted through one of the "explicit" methods. For example, when the arrival of a new model of a particular make of motor vehicle forces price collectors to find replacements, some of the replacements will be of comparable quality, others can be made comparable with explicit quality adjustments, but the remaining ones will need imputed prices for an overlap month. Class mean imputation calculates imputed price relatives using only the prices of comparable and, where appropriate, explicitly quality□adjusted varieties or models. In general, it does not use the prices for the varieties or models that were not replaced, because these are likely to be different from those of new models. The prices of old models tend to fall as they become obsolete, while the new models (represented by the replacements) tend to have a higher price before falling.

Class mean-imputations rely on other explicit quality adjustments and comparable replacements. The other explicit quality adjustments may be from available option or feature prices and may be limited in nature, covering only some of the differences in product attributes, available for only a small proportion of unrepresentative model changes, and the availability of comparable replacements limited. Given a substantial churn in the market and difficulties with such imputations and estimates an alternative recommended approach is that of hedonic indices, as outlined in section x below.

It may be the case, however, that sufficiently large samples of comparable substitutes or directly quality-adjusted items are unavailable. Or it may be that the quality adjustments and selection of comparable items are not deemed sufficiently reliable. In that case, a targeted imputation, outlined in section x above, might be considered. The targeted mean is less ambitious in that it seeks only to capture price changes of similar items, irrespective of their point in the life cycle. Yet it is an improvement on the overall mean imputation, as long as sufficiently large sample sizes are used.

The overlap method

Given the overlap price in May 2020, the overlap method is used for a non-comparable replacement, F2 for F in June and July 2020. The price index is measured through and including May 2020 using the prices of the old model: the geometric mean of the prices of the old model in 2020 in May is 5.78 and in April, 5.76, a S-T price relative of 5.78/5.76=1.00359. The price index to May 2020 is the cumulated product of the old index's price relatives, 105.29 (December 2019=100.000).

The index from May onwards no longer uses F, but switches to F2. For this there is need of an overlap: average prices up to and including May using the old F and for May, June, and July onwards, use the new F2. The prices for outlet F in June and July, 5.25 and 5.99 respectively, are based on F2. The overlap in May for the new model F2 is 5.25: Therefore the S-T price relative for May to June in outlet F is 5.25/5.25=1.00000 and for June to July is 5.99/5.25=1.03048. The prices in Table 6.3 for outlet F for June are 1.00000*6.18=6.18 and for July are 1.03048*6.18=7.05. This completes the price tableau. The geometric means are calculated as before, and their ratios form the S-T price relatives, and, in turn, the cumulative product of the price relatives, commencing from December 2019 is the price index, at 108.5 for July 2020 (December 2019=100.00).

The overlap method is only as good as the validity of its underlying assumptions. Consider p_m^{t-1} and p_m^t as the prices of an old item *m* in periods *t*-1 and *t* and p_n^{t+1} is the price of a new replacement item *n* in period *t* + 1, and there is an overlap price, imputed, observed, or informed by the outlet manager, for the new replacement in period *t*, p_n^{t*}. Now item *n* replaces *m,* but is of a different quality. The measured price relative between periods *t*-1 and *t*+1 shown by the right hand side expression in equation (6.5) to be the price change of the old to new, respectively in these two periods, multiplied by (adjusted for) the price overlap for *m* to *n* in period *t* as a measure of the quality differential.

$$(6.5)\quad I^{t-1,t+1} = \frac{p_m^t}{p_m^{t-1}} \times \frac{p_n^{t+1}}{p_n^{t*}} = \frac{p_n^{t+1}}{p_m^{t-1}} \times \frac{p_m^t}{p_n^{t*}}$$

Table 6.4, Introducing a non-comparable replacement via an overlap						
Model	**January**	**February**	**March**	**April**	**May**	**June**
Old (*m*)	25	28				
New (*n*)		**30**	35	38	40	41

For example, with an old model *m* permanently missing in March (*t*+1) and replaced by a new model *n*, with an overlap in February (*t*), the price relative for January (*t*-1) to March (*t*+1) using the overlap method is given by the first term in equation (6.5) and in the second term as an equivalent direct comparison between the new and old with a quality adjustment as the value of the relative prices in the overlap period February (*t*) of the old to the new model, 28/30=0.9333. The overlap method implicitly values the quality difference as the ratio of the two prices in the overlap period.

$$(6.6)\quad I^{Jan,Mar}=\frac{p_m^{Feb}}{p_m^{Jan}}\times\frac{p_n^{Mar}}{p_n^{Feb}}=\frac{p_n^{Mar}}{p_m^{Jan}}\times\frac{p_m^{Feb}}{p_n^{Feb}}=\frac{28}{25}\times\frac{35}{30}=\frac{35}{25}\times\frac{28}{30}=1.30667$$

Moreover, for a longer term price comparison, say February to June, the valuation of the quality difference remains as that of the price ratio at February, the time of the splice.

$$(6.7)\quad I^{Jan,Jun}=\frac{p_m^{Feb}}{p_m^{Jan}}\times\left(\frac{p_n^{Mar}}{p_n^{Feb}}\times\frac{p_n^{Apr}}{p_n^{Mar}}\times\frac{p_n^{May}}{p_n^{Apr}}\times\frac{p_n^{Jun}}{p_n^{May}}\right)=\frac{p_m^{Feb}}{p_m^{Jan}}\times\frac{p_n^{Jun}}{p_n^{Feb}}=\frac{p_n^{Jun}}{p_m^{Jan}}\times\frac{p_m^{Feb}}{p_n^{Feb}}$$

$$=\frac{28}{25}\times\frac{41}{30}=\frac{41}{25}\times\frac{28}{30}=1.530667$$

Of note is that the price of a missing good is, by definition, not usually observed at the same time period as the price of the replaced good since the choice of replacing product is only made once the previous one has disappeared. Indeed, the list of specifications is not always comprehensive since their main aim is to identify the product in the shop rather than comparing the products. However, it may be that the replacement item was on sale in the previous period and senior store staff has a record of its price. Further, the situation with scanner data is different: when we think of replacing a product by another one, it is a very easy task to look back and measure the price of the two products when both were sold (if such a situation has existed) and insofar we have a full set of characteristics, the two products can be compared in terms of extensive listings of characteristics.

The assumption is that the quality difference in any period equates to the price difference at the *time of the splice*. The *timing* of the switch from *m* to *n* is thus crucial. Unfortunately, price collectors usually hang onto an item so that the switch may take place at an unusual period of pricing, near the end of item *m*'s life cycle and the start of item *n*'s life cycle. The analysis is more formally given in Annex 1.

Relative prices may not reflect quality differences. For example, a new replacement model or brand of an improved quality may be stocked and sold at the same price as the old model. The outlet competes in the market in part by changing the quality of what is sold, as opposed to the price. Retailers may also reflect unusual pricing policies aimed at minority segments of the market. For example, the ratio of prices in an overlap period of a generic and a branded pharmaceutical drug may reflect the (perceived or otherwise) needs of two different market segments, rather than quality. The overlap method can be used with a judicious choice of the overlap period. It should if possible be a period before the use of the replacement since in such periods the pricing may reflect a strategy to dump the old model to make way for the new one.

The overlap method has at its roots a basis in the law of one price: that when a price difference is observed it must arise from some difference in physical quality or some such factors for which consumers are willing to pay a premium, such as the timing of the sale, location, convenience or conditions. Economic theory would dictate that such price differences would not persist, given markets made up of rational producers and consumers. However, Chapter 15 of *2008 SNA* (paragraphs 15.70–15.72, pages 303–304) notes three reasons why this might fail:

> "In the first place, purchasers may not be properly informed about existing price differences and may therefore inadvertently buy at higher prices. While they may be expected to search out for the lowest prices, costs are incurred in the process. Given the uncertainty and lack of information, the potential costs incurred by searching for outlets in which there is only a possibility that the same goods and services may be sold at lower prices may be greater than the potential savings, so that a rational purchaser may be prepared to accept the risk that he or she may not be buying at the lowest price…..
>
> Secondly, purchasers may not be free to choose the price at which they purchase because the seller may be in a position to charge different prices to different categories of purchasers for identical goods and services sold under exactly the same circumstances, in other words, to practice price discrimination…..
>
> Thirdly, buyers may be unable to buy as much as they would like at a lower price because there is insufficient supply available at that price. This situation typically occurs when there are two parallel markets. There may be a primary, or official, market in which the quantities sold, and the prices at which they are sold, are subject to government or official control, while there may be a secondary market, either a free market or unofficial market, whose existence may or may not be recognized officially."

The overlap method is commonly used as a default procedure for introducing replacement items when they are permanently missing. Statistical offices that re-base their CPI irregularly, say more than every 5 years, may also experience ta sample degradation with many items becoming permanently missing and, without replacements, the sample becoming increasingly composed of imputed prices. Replacements serve to maintain the sample composition and update the representativity of the items being priced.

Yet replacements in turn, to be linked to the price movements o the CPI based on actual prices, require a methodology for quality adjustment. The widely used overlap method has a major advantage of it not requiring an explicit adjustment. Explicit quality adjustments are considered below and are more resource intensive that the implicit overlap method. Further, the overlap method can be automated with information from the price collector fed through to the desk officer and then to a computational routine. The recommendation is that in accepting the use of the overlap method the desk officer should ensure that relative prices at the time of the overlap reflect quality differences. Where this is not the case an explicit quality-adjustment method should be used.

The overlap method is implicitly employed when samples of items are rotated. That is, the old sample of items is used to compute the category index price change between periods t-1 and t, and the new sample is used between t and $t + 1$. The "splicing" together of these index movements is justified by the assumption that – on a group-to-group rather than item-to-item level – differences in price levels at a common point in time accurately reflect differences in qualities.

The bias in using the overlap method within an elementary aggregate depends on the (i) ratio of missing to total observations and (ii) the difference between the mean of price changes for existing items and the mean of quality-adjusted replacement price changes. The bias decreases as either of these terms decrease. A formal analysis is given in Annex 2.

Linked to show no price change

Table 6.4a, Introducing a non-comparable replacement via an overlap						
Model	**January**	**February**	**March**	**April**	**May**	**June**
Old (*m*)	25	28				
New (*n*)		**35**	35	38	40	41

Returning to the example in Table 6.4, reproduced in Table 6.4a above, the new replacement item is non-comparable—of a different quality—the price difference being quite large. The imputed price in February is the same as its March price, 35. The new item is linked-in to show no price change in the period of replacement: February to March. This linking procedure attributes the price change between the replacement item in the current period and the old item in the preceding period, 28 to 35, to a change in quality, not price. The January to March price change is:

$\frac{28}{25} \times \frac{35}{35} = 1.12$, A 12 percent increase in price.

In equation (6.5) $p_n^{t+1} = p_n^{t*}$ $p_n^{t+1} = p_n^t$ and

$$(6.8)\quad I^{t-1,t+1} = \frac{p_m^t}{p_m^{t-1}} \times \frac{p_n^{t+1}}{p_n^{t*}} = \frac{p_n^{t+1}}{p_m^{t-1}}$$

That is, a price change between January (period t-1) and March (period t-1) is measured as that between January (t-1) and February(t). If (quality-adjusted) prices are rising (falling) the method biases the index downwards (upwards).

The bias is perpetuated through subsequent periods of measurement. For example, if half of the difference in price between the old in February and new in March was due to quality and half due to price change, that is (35-28=) 7/2=3.5. The new price change for April should be:

$$\frac{28}{25}\times\frac{35}{32.5}\times\frac{38}{35}=1.31$$

instead of $\frac{28}{25}\times\frac{35}{35}\times\frac{38}{35}=1.216$

As with the carrying forward, the method is particularly pernicious since it can be readily incorporated into a regular automatic compilation routine and simply not noticed: the price of the replacement is automatically imputed to form the overlap price and the index compiled. Linked to show no price change should not be used.

Explicit methods of quality adjustment

The aforementioned methods do not rely on explicit information on the value of the change in quality. This section discusses the following methods that rely on obtaining an explicit valuation of the quality difference: quantity adjustment; differences in production or option costs; and the hedonic approach.

Quantity adjustment

Quantity adjustment is one of the most straightforward explicit adjustments to undertake. It is applicable when the size of the replacement item differs from that of the available item. In some situations there is a readily available quantity metric that can be used to compare the items. Examples are the number of units in a package (e.g., paper plates or vitamin pills) and the size or weight of a container (e.g., kilogram of flour, litre of cooking oil). Quantity adjustment to prices can be accomplished by scaling the price of the old or new item by the ratio of quantities. The index production system may do this scaling adjustment automatically, by converting all prices in the category to a price per unit of size, weight or number. Scaling is important. For example, if cooking oil is now sold in 5 litre containers instead of 2.5 litre ones, it should not be the case that prices have doubled.

The specification of an item is often to a specific size, for example, 1kilogram packet of flour. If only 2 kilogram packets are sold in a specific outlet, the price collector should choose a representative 2 kilogram packet, but mark the new specification as such and, after confirmation by the desk officer, prices continued to be collected for the 2 kilogram packet but adjusted (halved) to be consistent with the original specification. This is particularly important where price variances are computed or use is made of a Dutot price index number formula—one sensitive to the homogeneity of the items used.[2]

Changes in the size of items sold can be dealt with similarly, however, there are some caveats. In the pharmaceutical context, for example, prices of bottles of pills of different sizes differ. A bottle of 100 pills, each having 50 milligrams of a drug, is not the same as a bottle of 50 pills of 100 milligrams, even though both bottles contain 5,000 milligrams of the same drug. If there is a change, say, to a larger size container, and a *unit* price decrease of 2 per cent accompanies this change, then it should not be regarded as a price fall of 2 per cent if consumers gain less utility from the larger and more inconvenient containers. In practice, it will be difficult to determine what proportion of the price fall is attributable to quality and what proportion to price. A general policy is not to automatically interpret unit price changes arising from packaging size changes as pure price changes, if contrary information is available.

For some products, such as clothing and bedding, increases in size may not reflect increases in utility. For example, shoes may come in a range of adult sizes, but be all sold at the same price. Though larger automobiles can accommodate more people in comfort and may yield utility as a status symbol.

Consider a further example: a branded bag of flour previously available in a 0.5 kilogram bag priced at 1.5 is replaced by a 0.75 kilogram bag priced at 2.25. The main concern here is with rescaling the quantities. The method would use the relative quantities of flour in each bag for the adjustment. The price may have increased by [(2.25/1.5) $\times$100 = 150] 50 per cent but the quality-adjusted prices (i.e. prices adjusted by size) have remained constant [(2.25/1.5)$\times$ (0.5/0.75)$\times$100=100]. The approach can be outlined in a more elaborate manner by recourse to Figure 6.1. The concern here is with the part of the unbroken line between the (price, quantity) coordinates (1.5, 0.5) and (2.25, 0.75), both of which have *unit* prices of 3 (price = 1.5/0.5 and 2.25/0.75). There should be no change in quality-adjusted price. The symbol Δ denotes a change. The slope of the line is β which is Δprice/Δsize = (2.25-1.5)/(0.75-0.50)=3, i.e., the change in price arising from a unit (kilogram) change in size. The quality- (size-) adjusted price in period t-1 of the old m bag is:

$$(6.9)\quad \hat{p}_m^{t-1} = p_m^{t-1} + \beta\Delta\text{size} = 1.5 + 3\,(0.75 - 0.5) = 2.25$$

The quality-adjusted price change shows no change, as before:

$$p_n^t / \hat{p}_m^{t-1} = 2.25 / 2.25 = 1.00$$

The approach is outlined in this form so that it can be seen as a special case of the hedonic approach (discussed below), where price is related to a number of quality characteristics of which size may be only one.

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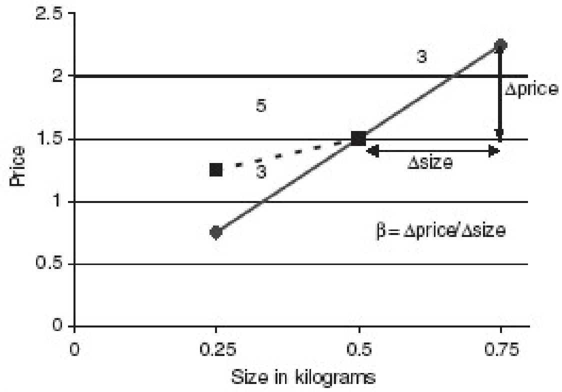
[2] A Dutot elementary price index number formula is a ratio of arithmetic means of matched prices. More weight is given to price changes with higher prices in the reference period. Thus if the price of a container in the price reference period doubles, while others remain the same, so too will the implicit weight given to its price change, see chapter 9.

Figure 6.1 Quality adjustments for different sized items



The method can be seen to be successful on intuitive grounds as long as the unit price of different-sized bags remains constant. Say the 0.5-kilogram bag was missing and a replacement 0.25-kilogram packet used priced at 0.75, as shown by the continuation to coordinate (0.75, 0.25) of the unbroken line in Figure 6.1 the quality-adjusted prices would again not change. Assume, however, that the unit (kilogram) prices were 5, 3 and 3 for the 0.25, 0.5 and 0.65 kilogram bags, respectively, as shown in Table 6.5 and in Figure 6.1 (including the broken line). Then the measure of quality-adjusted price change would depend on whether the 0.5-kilogram bag was replaced by the 0.25 kilogram one (a 67 per cent increase) or the 0.75 kilogram one (no change). This is not satisfactory because the choice of replacement size is arbitrary. The rationale behind the quality adjustment process is to ask: does the difference in unit price in each case reflect different levels of utility? If so, adjustments should be made to the unit prices to bring them into line. If not, adjustments should be made to the unit prices for that proportion attributable to differences in utility gained from, say, more convenient packaging or the availability of smaller lots. It may be obvious from the nature of the product that an item packaged in a very small size with a disproportionately high unit price carries an unusually high profit margin, and that an appropriate replacement for a large-sized item would not be this very small one.

Table 6.5 Example of size, price and unit price of bags of flour

Size (kilograms)	First price	First unit price	Second price	Second unit price
0.25	0.75	3	1.25	5
0.5	1.50	3	1.50	3
0.75	2.25	3	2.25	3

Differences in feature/option costs

Consider an example of the *price* of an option being used to adjust for quality. Let the prices for an item in periods *t*-1 and *t* be 10,000 and 10,500, respectively, but assume the price in period t is for the item with a new feature, as standard, that previously in period t-1 had to be purchased as an "option" for an additional 300. Then between periods t-1 and t that include the feature in both periods the price change would be 10,500/10,300=1.01942 or 1.942 per cent.

Option costs are thus useful in situations in which the old and new items differ by quantifiable characteristics that can be valued in monetary terms by reference to market prices. The valuation of a quantifiable product feature may be readily available from the comparison of different product prices. This is especially, and conveniently, so for some goods and services sold on the Internet which can be identified by their brands and price-determining characteristics.

Consider the addition of a feature to a product – say an automatic icemaker in the door of a refrigerator. Refrigerators for a brand may be sold as standard or with a door- installed automatic icemaker. The price collector may always have collected prices on the standard model, but this may no longer be in production, being replaced by a model with an installed automatic icemaker. The cost of the option is thus known from before and a continuing series developed by simply adjusting the old price in the price reference period to include the option price. Even this process may have its problems. First, the cost of producing something as standard may be lower than when it was an option, say all new refrigerators now have the door-installed automatic icemaker. This saving may be passed on, at least in part, to the consumer. The option cost method would thus understate a price increase. Further, by including something as standard the consumer's valuation of the option may fall since buyers cannot refuse it. Some consumers may attribute little value to the option. The overall effect would be that the estimate of the option cost, priced for those who choose it, is likely to be higher than the implicit average price consumers would pay for it as standard. Estimates of the effect on price of this discrepancy should in principle be made, though in practice are quite difficult.

Quality differences are not necessarily positive; an airline may charge for a second piece of baggage when previously it did not. Again, there will be an option price available for the additional piece of baggage so that the price of like –two pieces of baggage – is compared with like.

Option cost adjustments can be seen to be similar to quantity adjustments, except that instead of size being the additional quality feature of the replacement, the added quality can be any other individual option/feature. The comparison is: $p_n^t / \hat{p}_m^{t-1}$ where $\hat{p}_m^{t-1} = p_m^{t-1} + \beta\Delta z$ for an individual z characteristic where $\Delta z = \left(z_n^t - z_m^{t-1}\right)$. The characteristics may be the size of the random access memory (RAM) of a personal computer (PC) when a specific model of PC is replaced by a model that is identical except for the amount of RAM it possesses. For example, the web pages of sellers of laptops allow buyers to customize their purchase, an extra 4-gigabyte (GB) of RAM, from 8 to 12 GB, for a specific brand and model of a laptop may cost an additional \$70. Say the standard laptop used for CPI measurement has 8GB of memory, costs 899.99 and is not available in the next period. The new standard model in period t has 12 GB but costs the same 899.99, p_n^t. We want to compare the (constant-quality) price of the new model with the old model in t -1, but the latter should have its price adjusted to include an extra 4 GB of RAM. The price of an additional GB of memory for this brand/model in period t-1 is 70/4= 17.5, and its quality-adjusted price in period t-1 are $\hat{p}_m^{t-1}$ = 899.99 + 17.5 (12 - 8) = 969.99. The period t (unchanged) price of 899.99 is now compared with its comparable period t -1 price to yield a constant-quality price change of 899.99/969.99=0.9278, which is a price fall of 7.22 percent, while the package price is constant.

Again, this phrasing of the calculation is more complex than required: the adjustment is to simply add 70 to the old price, 70+899.9=969.99. However, it serves to demonstrate some limitations of these approaches as special cases of the hedonic method, as outlined in the next section.

This calculation conveniently makes the quality adjustment to the old model's price in period t -1 so that new model's price in future months can be directly compared with the *quality-adjusted* old price for the life of the new specification. However, the required information on the value of an extra 4 GB may only be available in period t and not be applicable to a period t -1 adjustment. Statistical offices should ideally keep a record of, say, web customizations of specified items along with comparable/non-comparable replacements especially for products with a high degree of technical change and churn/turnover of models and maintain good relations with senior outlet staff.

If the relationship between price and RAM is linear, the above formulation is appropriate. Many web pages give the price of additional RAM as being independent of other features of PCs, and a linear adjustment is appropriate. Bear in mind that a linear formulation values the worth of an additional fixed additional amount of RAM to be the same, irrespective of the amount of RAM the machine possesses or amount of other features.

The relationship between price and the product features may be non-linear. Denote the price-determining characteristics as z, and assume there are k of them. The change in z is intended to reflect the service flow, but the non-linearity in the price–z relationship may reflect consumer's decreasing marginal utility to the scale of the provision. The price a customer is willing to pay per GB falls as increasing amounts of GB are purchased. For some features there will be economies of scale: supplying much more of a feature makes the price fall, possibly substantially; while for others it may become technically difficult, and more expensive, to compress higher amounts of a feature into the available space. The data should reveal some of this relationship and caution against applying linear relationships outside of the range in which they are warranted. Further, it should give some insight into the required adjustments for such non-linear relationships, though this may be better estimated using a regression formulation and non-linear specification, as considered in the next section.

The similarity between the quantity adjustment and the option cost approaches is apparent since both relate price to some dimension of quality: the size or the option. The option cost approach can be extended to more than one quality dimension. Both approaches rely on the acquisition of estimates of the change in price resulting from a unit change in the option or size: the β slope estimates. In the case of the quantity adjustment, this was taken from an item identical to the one being replaced, aside from the fact that it was of a different size. The β slope estimate in this case was perfectly identified from the two pieces of information. It is as if the nature of the experiment controlled for changes in the other quality factors by comparing prices of what is essentially the same thing except for the quantity (size) change.

The same reasoning applies to option costs. There may be, for example, two items, identical but for the possession of a single feature. Their difference in price allows the value of the feature to be determined. Yet sometimes the value of a feature or option has to be extracted from a much larger data set. This may be because the quality dimension takes a relatively large range of possible numerical values without an immediately obvious consistent valuation. Consider the simple example of only one feature varying for a product, the speed of processing of a PC. It is not a straightforward matter to determine the value of an additional unit of speed. To complicate matters, there may be several quality dimensions to the items and not all combinations of these may exist as items in the market in any one period. Furthermore, the combinations existing in the second period being compared may be quite different to those in the first. Considering these aspects leads to a more general framework, known as the hedonic approach.

Hedonic approach: patching

The hedonic approach is an extension of the two preceding approaches in that, first, the change in price arising from a unit change in quality—the quantity or option/feature—is now estimated from a data set comprising prices and quality characteristic values of a larger number of items. Second, the quality characteristic set is extended to cover, in principle, all major characteristics that might determine price, rather than just the quantity or option/feature adjustment.

The hedonic approach is particularly useful when the market does not reveal the price of the quality characteristics required for the adjustment. Markets reveal prices of items, not quality characteristics, so it is useful to consider items as tied bundles of characteristics. A sufficiently large data set of items with their characteristics and sufficient variability in the mix of characteristics between the items allows the hedonic regression to provide estimates of the implicit prices of the characteristics. For example, the price of (clothes) washing machines will be listed, though a new (replacement) model for a brand may have a (cotton) capacity load size not previously available, say 12 kilograms (kgs.), instead of the preceding model's 10 kgs. To make an explicit quality adjustment we require the price of the additional 2 kgs. The regression approach using a dataset of many models' prices and characteristics can estimate the price of additional kgs. of capacity from data for models of washing machines on their price, capacity, year (age of model), color, running cost, and so forth.

Under the matched models method each price collector needed to select a representative item, record its price and specifications, and re-pricing the self-same item in subsequent periods. The extension required in the hedonic approach is that the prices and price-determining characteristics should be collected for a large sample of, if not all, models. The method is particularly suitable when there are no immediately apparent comparable replacements and the non-comparable ones vary in their characteristics over more than one variable. A new model of car, household appliance, computer or related hardware and software, telecommunication equipment and much more, can differ from the old model in many respects, yet there is only a single price for each new and old model. This approach is particularly necessary when there is a frequent churn, that is, turnover of items in the market. New models with quite different values for their characteristics are frequently replacing old ones

The requirement that data are collected on the prices and specifications of a large sample if not all models is not as demanding as it might appear. Extensive data on prices and characteristics of models of consumer goods and services are readily available on web sites—many comparing prices and salient characteristics—that can be copied with relative ease, and indeed automated using web-scraping.[3] Such detailed information is also available as scanner data, see chapter x.

Figure 6.2 is a scatter diagram relating the price (£ sterling) to the (cotton) capacity (kgs.) of models of washing machines sold in the UK. Data are from the *Which?* Magazine. [4] It is apparent that washing machines with larger capacities command higher prices – a positive relationship. It is also apparent from Figure 6.2 that there are several models of washing machine with the same capacity but quite different prices, resulting from the fact that other things differ. For example, 12 kg. capacity machines' prices range from £754 to £1,349.

To estimate the value given to additional units of capacity an estimate of the slope of the line that best fits the data is required. The equation of a straight line is: $Price = \hat{\beta}_0 + \hat{\beta}_1 z_1$

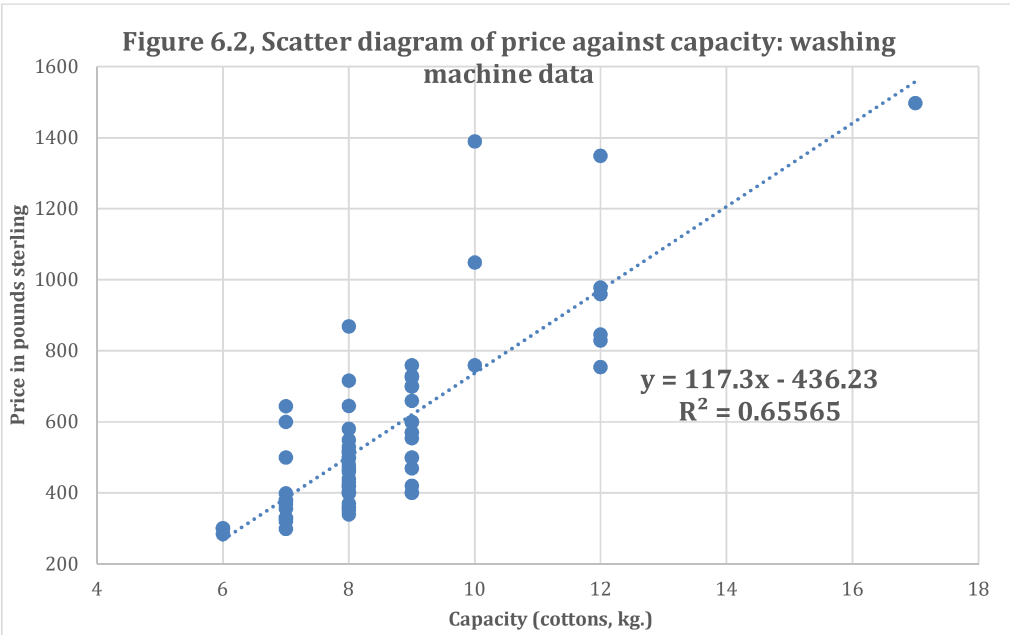
The slope $\hat{\beta}_1$ is a measure of the change in *Price* that arises from a one-unit change in the characteristic, z_1, *Capacity*. The ^ (hat) above $\hat{\beta}_1$ denotes that it is estimated from the data. The estimated slope is from the equation of a line that best fits the data: that best represents

[3] The billion prices project makes extensive use of web-scraped data: "The Billion Prices Project: Using Online Data for Measurement and Research" *Journal of Economic Perspectives*, 31(1) (Spring 2016). http://www.thebillionpricesproject.com.

[4] *Which?* is a brand name used by the *Consumers' Association*, a registered charity based in the United Kingdom. It exists to promote informed consumer choice in the purchase of goods and services by testing products, highlighting inferior products or services, raising awareness of consumer rights and offering independent advice. Data for this illustration was taken from the December 2017 website: https://www.which.co.uk/reviews/washing-machines. The example here is for illustrative purposes only.

(continued)

the underlying pattern of the relationship. In Figure 6.2 the equation of the line that best fits the data was derived using ordinary least squares (OLS) regression. The intercept and slope of the line that best fits the data are estimated as ones that *minimizes the sum of the squared differences* between the individual prices and their counterpart prices predicted by the line: the least squares criterion. Facilities for regression are available on standard statistical and econometric software, as well as spreadsheets.[5]



The estimated (linear) equation in this instance is:

$Price = -436.229 + 117.298\ Capacity.$ $\bar{R}^2 = 0.65$

The coefficient on *Capacity* is the estimated slope of the line: the change in price (£117.30) resulting from a 1 kg. change in *Capacity*. This can be used to estimate quality-adjusted price changes for washing machines of different capacities. The value of $\bar{R}^2$ is 0.65; this indicates that 65 per cent of price variation is explained by variation in *Capacity*. A *t*-statistic to test the null hypothesis of the coefficient being zero was found to be 11.789: recourse to standard tables on *t*-statistics found the null hypothesis was rejected with a *p-value* of 2.00E-16: the decimal point is moved to the left 16 digits (zeros). The fact that the estimated coefficient differs from zero cannot be attributed to sampling errors at this level of significance. There is a miniscule probability that the test has wrongly rejected the null hypothesis.

Hedonic regressions should generally be conducted using a semi-logarithmic formulation. The dependent variable is the (natural) logarithm of the price, but the variables on the right-hand side of the equation are kept in their normal units, hence the semi-logarithmic formulation. A double-logarithmic formulation would also take logarithms of the right-hand side price-determining characteristic variables. However, if any of these variables are dummy variables which take the value of zero in some instances, the double-logarithmic formulation

[5] The illustrative empirical work in this section was undertaken using R, though would be equally applicable with any such standard statistical software including EViews, SAS, and Stata.

would break down because logarithms of zero cannot be taken. The focus is thus on the semi-logarithmic form.

The estimated (semi-logarithmic) regression equation in this instance is:

$\log(Price) = 4.77611 + 0.17374\ Capacity.$ $\bar{R}^2 = 0.61$

The coefficient of 0.17374 has a useful direct interpretation: when multiplied by 100 it is the percentage change in price arising from a 1 unit (kg.) change in capacity. There is a estimated 17.374 per cent change in price for each additional kg. of capacity.

The range of prices for a given capacity was noted to be substantial which suggests that other quality characteristics may be involved. Table 6.6 provides the results of a regression equation that relates price to a number of quality characteristics as listed in the first column.[6] While the results are given for both linear and semi-logarithmic regression specifications, the focus here is on the latter functional form.

A multivariate semi-logarithmic hedonic regression model is given by:

(6.10) $$Price = \beta_0 \beta_1^{z_1} \beta_2^{z_2} \beta_3^{z_3} \ldots\beta_n^{z_n} \varepsilon$$

$$\ln \text{Price} = \ln\beta_0 + z_1 \ln\beta_1 + z_2 \ln\beta_2 + z_3 \ln\beta_3 + \ldots.. z_n \ln\beta_n + \ln\varepsilon$$

where ε is an error term assumed to have the usual properties to satisfy OLS assumptions. Note that for this semi-logarithmic form, logarithms are taken of only the left-hand-side variable, i.e., *Price*. Each of the z characteristics enters the regression without having logarithms taken. This has the advantage of allowing dummy variables for the possession or otherwise of a feature to be included on the right hand side. Such dummy variables take the value of one if the item possesses the feature and zero otherwise. The taking of logarithms of the first equation in (6.10) allows it to be transformed in the second equation to a linear form. This allows the use of a conventional ordinary least squares (OLS) estimator to yield estimates of the logarithms of the coefficients. These are given as the coefficients for the Semi-logarithmic model in Table 6.6. This estimated coefficients in Table 6.6 are based on a multivariate model: for *Capacity*, for example, the estimated coefficient of 0.108452 is of the effect of a unit change in capacity on price, *having controlled for the effect of other variables in the equation*. The scatter diagram in Figure 6.2 clearly showed the inadequacy of relying on a single price-determining variable and this approach has as one of its justifications the challenge of addressing this remiss. The preceding estimated coefficient of 0.17374 was based on only one variable, and is different from this improved result.

[6] These include Age (Months since model was launched); Capacity (for cottons, kg.); Warranty (if 5 years, benchmark: 2 years); Steel (Stainless steel outer); (Annual) Energy cost, £; LG (manufactured, benchmarked on Samsung); Steam (wash/refresh); Hygiene /Allergy program; LED display (benchmarked on LCD); and price for 74 models as advertised on November 2017 in *Which?* for three up-market brands: Bosch, LG, and Samsung. A much larger general regression model with more variables was first estimated but reduced to this smaller specific model using standard econometric principles and practice. The White (studentized) Breusch-Pagan test for homoscedastic residuals was not rejected at conventional significance levels with a *p-value* of 0.2197.

(continued)

When dummy variables are used, the coefficients, when multiplied by 100, are estimates of the percentage change in price, given by $(e^{\beta_1} - 1)100$. For example, from Table 6.6, LG models have a $(e^{-0.219743} - 1)100 = 19.73$ percent lower price than their benchmarked Samsung counterpart, having controlled for other differences in their price-determining characteristics as specified in the regression equation.[7]

Table 6.6, Illustrative hedonic regression estimates for washing machines

	Linear			**Log-linear**		
	Coefficient	**Std. Error**	***p-value***	**Coefficient**	**Std. Error**	***p-value***
(Intercept)	-206.939	112.3	0.06986+	5.21691	0.165379	< 2e-16***
Age	-1.579	2.1	0.44971	-0.005997	0.003059	0.054218+
Capacity	81.024	13.2	5.32e-08***	0.108452	0.019404	4.86e-07***
Warranty	-138.651	48.7	0.00592**	-0.264562	0.071761	0.000466***
Ststeel	144.036	74.0	0.05608+	0.265767	0.109074	0.017575*
Energycost	10.103	3.4	0.00430**	0.018969	0.00503	0.000353***
LG	-115.816	43.9	0.01044*	-0.219743	0.064687	0.001167**
Steam	191.196	92.3	0.04233*	0.257177	0.135987	0.063056+
Hyg_AllergyP	63.627	40.2	0.11842	0.152722	0.05923	0.012198*
LEDdisplay	49.409	52.9	0.35391	0.166143	0.077946	0.036833*
$\overline{R}^2$	0.701			0.721		
F-statistic	20.25			22.22		
p-value	1.06E-15			2.20E-16		

***,**,* and + denote statistically significant at 0.1, 1, 5, and 10 percent levels, respectively.

The value $\overline{R}^2$ =0.721 is the proportion of variation in (the logarithm of) price explained by the estimated equation. More formally, it is 1 minus the ratio of the variance of the residuals, $\Sigma_{i=1}^{N}\left(p_i^t - \hat{p}_i^t\right)^2 / N$, of the equation to the variance of prices, $\Sigma_{i=1}^{N}\left(p_i^t - \overline{p}_i^t\right)^2 / N$. The bar on the term R^2 denotes that an appropriate adjustment for degrees of freedom is made to this expression, which is necessary when comparing equations with different numbers of explanatory variables. A high value of $\overline{R}^2$ can be misleading for the purpose of quality adjustment. First, such values indicate that the explanatory variables account for much of the price variation. This may be over a relatively large number of varieties of goods in the period concerned. This, of course, is not the same as implying a high degree of prediction for an adjustment to a replacement item of a single brand in a subsequent time period. Predicted values depend for their accuracy not just on the fit of the equation, but also on how far the characteristics of the item whose price is to be predicted are from the means of the sample. The more unusual the item, the higher the prediction probability interval. Second, the value $\overline{R}^2$ indicates the proportion of variation in prices explained by the estimated equation. It may be that 0.90 is explained while 0.10 is not explained. If the dispersion in prices is very large,

[7] There is some bias in these coefficients; and in the (semi-) logarithmic equation, half the variance of each coefficient should be added to the coefficient before using it. For the LG coefficient the standard error from Table 6.6 is 0.064687, its variance is 0.064687^2=0.00418: the adjustment is to add 0.00418/2 to -0.219743, giving -0.21765; a lower price of $-(e^{-0.21765} - 1)100 = 19.56$ percent.

this still leaves a large absolute margin of prices unexplained. Nonetheless, a high $\overline{R}^2$ is a necessary condition for the use of hedonic adjustments.

On the interpretation of estimated hedonic coefficients

Some mention should first be made of the interpretation of the coefficients from hedonic regressions. The matter is discussed in detail in Chapter 21. There used to be an erroneous perception that the coefficients from hedonic methods represented estimates of user value as opposed to resource cost. The former is the relevant concept in constructing a consumer price index, while for producer price index compilation it is the latter. Yet hedonic coefficients may reflect both user value and resource cost – both supply and demand influences. There is what is referred to in econometrics as an identification problem; the observed data do not permit the estimation of the underlying demand and supply parameters. What is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production.

It is thus necessary to take a pragmatic stance. In many cases the implicit quality adjustment to prices arising from the use of the overlap method, as described above, may be inappropriate because the implicit assumptions are unlikely to be valid. In such instances, the practical needs of economic statistics require explicit quality adjustments. However, use of the hedonic approach would only be warranted, due to the cost of implementing the method, when the weight, churn and extent of the quality adjustment is substantial. Not to do anything on the grounds that the measures are not conceptually appropriate would be to ignore quality change and provide wrong results. or some high-technology products their profile is such that a hedonic approach is warranted simply to maintain the credibility of the CPI.

The proper use of hedonic regression requires an examination of the coefficients of the estimated equations to see if they make sense. It might be argued that the very multitude of distributions of tastes and technologies, along with the interplay of supply and demand, that determine the estimated coefficients (Chapter 21) make it unlikely that "reasonable" estimates will arise from such regressions. A firm may, for example, cut a profit margin relating to a characteristic for reasons related to long-run strategic plans; this may yield a coefficient on a desirable characteristic that may even be negative. This does not negate the usefulness of examining hedonic coefficients as part of a strategy for evaluating estimated hedonic equations. First, there has been extensive empirical work in this field and the results for individual coefficients are, for the most part, quite reasonable. Over time, individual coefficients can show quite sensible patterns. Unreasonable coefficients on estimated equations are the exception and should be treated with some caution. Second, one can have more faith in an estimated equation whose coefficients make sense and which predicts well, than one which may also predict well but whose coefficients do not make sense. Third, if a coefficient for a characteristic does not make sense, it may be due to multicollinearity, a data problem, and should be examined, say, using variance inflation factors, to see if this is the case (see Appendix 21.1 to Chapter 21).

On the implementation of a hedonic quality adjustment

The implementation of hedonic methods to estimate quality adjustments for matched non-comparable replacements can take two forms. The first is what we refer to as "patching": undertaking a quality adjustment to the price of the old model to make it comparable with the new model. For many items it can be seen as a one-off process for individual varieties within the lifetime of rebasing a sample. The second is the more wholesale process for rapidly changing high-technology products whose changes in quality are substantial within relatively short period periods; this is considered in section xx below.

Patching is the term used here for introducing non-comparable replacements, that is replacements of a different quality, via hedonic regression estimates. Consider items *l, m* and *n* where item *l* is available in all periods, the "old" item *m* is only available in periods *t*, *t*+1, and *t*+2 and the replacement item *n* only in period *t*+3 and subsequently. The items are defined by their *z* quality characteristics; for item *m*, for example, these are z_m^t and the price of item *m* in period *t* is p_m^t. There is no problem with comparing the prices of matched item *l* with characteristics z_l, for they have the same quality characteristics. But there is a problem with item *m*. Its replacement *n* is non-comparable so p_m^{t+2} cannot be directly compared with p_n^{t+3}. What is required is a price in period *t*+2 for item *n;* but item *n* does not have a recorded price in period *t*+2, indeed it may not have been sold then. A hedonic *imputation* approach would predict the price of item *n* in period *t*+2 using a hedonic regression estimated in period *t*+2 and the characteristics of the new item *n,* taken from period *t*+3, i.e. the predicted price of item *n* in period *t*+2, $\hat{p}_n^{t+2}$—the hat over the price, "$\hat{p}$", denotes a predicted value from the regression. The predicted prices are for the characteristics of the replacement item *n*. It is an estimate of what the characteristics of the new replacement item would have been priced at had it been sold in period *t*+2.

Table 6.7a, hedonic regression imputation of new item's price

Item/period	t	t+1	t+2	t+3	t+4
l	p_l^t	p_l^{t+1}	p_l^{t+2}	p_l^{t+3}	p_l^{t+4}
m	p_m^t	p_m^{t+1}	p_m^{t+2}		
n			$\hat{p}_n^{t+2}$	p_n^{t+3}	p_n^{t+4}

For short-term comparisons an overlap method is used with a price relative for *t*+2 compared with *t*+1 given by p_m^{t+2} / p_m^{t+1} and for *t*+3 compared with *t*+2 given by $p_n^{t+3} / \hat{p}_n^{t+2}$ and subsequently, without the need for an imputation, by p_n^{t+4} / p_n^{t+3}. The implicit assumption in the overlap method is that the difference in prices between *m* and *n* is an indicator of the difference in their quality: $p_m^{t+2} / \hat{p}_n^{t+2}$ —the numerator is the (actual) price in period *t*+2 of the old item's characteristics while the denominator is the (predicted) price in period *t*+2 of the new item's characteristics.

The simple example outlined above using data on washing machines sold in the United Kingdom is used here to illustrate the methodology: a linear hedonic regression of price on a single characteristic—their capacity in kg. for a cotton load. The linear ordinary least squares (OLS) estimated regression equation of price on capacity for three up-market models of washing machines is:

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-436.2293196	86.30954526	-5.054241896	3.09227E-06
Capacity	117.2976003	9.949378608	11.78943981	1.43937E-18

Where *p* is price and *c* is capacity, the predicted price, $\hat{p}$ from the estimated regression equation is: $\hat{p} = 117.3c - 436.23$

The coefficient on (cotton load) capacity *c*, 117.3, is the estimated slope of the line: the increase in price (£117.3) resulting from a 1 kg. increase in capacity. Assume the regression equation was estimated using period *t*+2 data, the old model *m* had a capacity of 10kg., but the new model *n* in period *t*+3 to have a capacity of 12 kg., model *n*'s price in period *t*+2 could be predicted as $\hat{p} = 117.3 \times 12 - 436.23 = 971.37$. The ratio of actual price of model *m* in period *t*+2, say (£750) to predicted price in period 2 is the quality adjustment shown for the overlap method in equation (6.5), though for period *t*+2 in this example, $\frac{p_m^{t+2}}{p_n^{t+2*}}$, that is: $\frac{750}{971.37} = 0.7721$. The models are not comparable. The new model in period *t*+2 is more expensive even when its superior quality, its capacity, has been taken into account.

Given we have an estimate of the how much an extra unit of capacity is worth, an alternative approach would be to simply add 2 *times* 117.3 to the period *t*+2 price of *m*, rather than use predicted prices. Such use of individual coefficients is not recommended. In practice a hedonic regression will include several explanatory price-determining variables. These may be linearly related and thus not strictly independent; larger (higher capacity) washing machines may also have higher spin-speeds, be more likely to have a steam feature, and so forth. The estimated coefficient of each such muliticollinear variable would be imprecise, though the predicted price of a regression equation that includes them would be unbiased, a matter also referred to in paragraph 6.?.

With the option cost/ feature example, the quality adjustment might be for a single characteristics and an explicit valuation of the price of further units of this characteristic, such as a GB of storage, available from another source. Hedonic regressions are used where the market does not reveal the shadow implicit prices of individual characteristics; these shadow prices have to be estimated from price data for many varieties with differing bundled sets characteristics.

The method makes use of short-term month-on-month comparisons: predicting the price of item n in period t+2, had it been on sale then, is only for this one-off period as the new item replaces the old, with a quality adjustment. Item n's characteristics are held constant for month-on-month comparisons from t+2 onwards, and item m's characteristics are held constant for month-on-month comparisons from period t up to, and including, period t+2.

Alternatively, item m might have its price predicted from a hedonic regression run on period t+3 data, $\hat{p}_m^{t+3}$. As with the preceding methodology, a predicted price is only required for the overlap period, after which the replacement item forms the continuing index. It is not obvious which of the two approaches, predicting prices for m or n, is preferred. Resources permitting, a geometric mean of the two would be defensible, as would a clear rule from the outset as to the method applied based on some retrospective research on the outcome of using either method for particular product groups.

Table 6.7b, hedonic regression imputation of old item's price					
Item/period	t	t+1	t+2	t+3	t+4
l	p_l^t	p_l^{t+1}	p_l^{t+2}	p_l^{t+3}	p_l^{t+4}
m	p_m^t	p_m^{t+1}	p_m^{t+2}	$\hat{p}_m^{t+3}$	
n				p_n^{t+3}	p_n^{t+4}

A refinement to these approaches is to uses predicted values for, say items m and n, in the overlap period, for example, $\hat{p}_m^{t+3} / \hat{p}_m^{t+2}$. Consider a misspecification problem in the hedonic equation. For example, there may be an interaction effect between a brand dummy and a characteristic. Possessions of a characteristic for a particular brand may be priced higher than all other brands, say a 5 percent premium. The use of $\hat{p}_m^{t+3} / p_m^{t+2}$ would be misleading since the actual price in the denominator would incorporate the premium, while the one predicted from the hedonic regression would not. It is stressed that, in adopting this approach, a recorded actual price is being replaced by an imputation. This is not desirable, but neither is the omitted variable (interaction term) bias. The dual imputation approach is preferred whenever there are concerns about the suitability of the regression equation's specification to fully model prices, as would generally be the case.

A further approach would be to not use a replacement item. Item m's characteristics would be held constant in the comparison from period t+2 onwards. This would require a hedonic regression being run for each subsequent period, $\hat{p}_m^{t+3}$, $\hat{p}_m^{t+4}$. It would also lead to a continuing degradation of the sample as an obsolete old item m has its characteristics repeatedly priced into the future, rather than being replaced by a new item. The method is not recommended.

In the above examples short-term (S-T) overlaps are used. They are much preferred to long-term (L-T) ones. A L-T equivalent of Table 6.7a is shown in Table 6.7c. A predicted price for any replacement item *n* in its month of introduction is estimated for the reference period *t* using a hedonic regression using that period's data. The regression is estimated using period *t* prices and characteristics, but the predicted prices are for the characteristics of the replacement item *n* in *t*+3and subsequently. It is an estimate of what the characteristics of the new replacement item would have been priced at had it been sold in period *t*. The L-T method has the significant advantage of only requiring a hedonic regression to be estimated in the single reference period. For periods *t*+3 and *t*+4 the price relatives are $p_n^{t+3} / \hat{p}_n^t$ and $p_n^{t+4} / \hat{p}_n^t$ respectively. However, as time passes, such comparisons become less meaningful, for example, comparing the actual price this current month of a model of a laptop with one predicted say 18 months ago using the hedonic, market valuations of each characteristic estimated then to be applied to the characteristic set of a laptop sold now. Indeed, the need for a double imputation becomes more important as time passes by, yet a double imputation requires monthly estimation of hedonic regressions that forestalls the very advantage of this approach. If hedonic regressions are to be used on this L-T basis it is important that the regressions are re-estimated regularly at a rate that will depend on the rate of the technological innovations, and changes in consumer preference specific to that product. For example, it may be that consumer's valuations of characteristics of washing machines, including spin-speed, front-loaders, capacity, number and types of wash programs and so forth, are fairly constant over time, even if the technology is itself changing rapidly. Frequent, say monthly, updating of estimated hedonic regression equations is not required. Prior empirical studies on the stability over time in hedonic characteristics would be valuable in this respect. As a general principle, S-T hedonic imputations are preferred to L-T ones.

Table 6.7c, hedonic regression imputation of new item's price

Item/period	*t*	*t*+1	*t*+2	*t*+3	*t*+4
l	p_l^t	p_l^{t+1}	p_l^{t+2}	p_l^{t+3}	p_l^{t+4}
m	p_m^t	p_m^{t+1}	p_m^{t+2}		
n	$\hat{p}_n^t$			p_n^{t+3}	p_n^{t+4}

Limitations of the hedonic approach

The limitations of the hedonic approach should be borne in mind (see also Chapter 21). First, the approach requires statistical expertise for the estimation of the hedonic regression equations. The availability of user-friendly statistical/econometric software with regression facilities makes this less problematic. Statistical and econometric software carry a range of diagnostic tests to help judge if the final formulation of the model is satisfactory. These include $\overline{R}^2$ as a measure of the overall explanatory power of the equation, *F*-test and *t*-test statistics to enable tests to be conducted as to whether the differences between the estimated coefficients of the explanatory (price-determining) variables are jointly and individually different from zero at specified levels of statistical significance. These statistics make use of the errors from the estimated regression equation. The regression equation can be used to predict prices for each item by inserting the values of the characteristics of the items against the estimated coefficients of the explanatory variables. The differences between the actual prices and these predicted results are the residual errors. Statistical/econometric software calculated predicted values and residuals as a matter of routine. A hedonic regression equation estimated using ordinary least squares (OLS) requires assumptions as to the nature of the distribution of these residual errors. These include: (i) the error term has a constant variance. When this assumption is violated, the errors are heteroscedastic; standard tests of statistical significance can be biased and unreliable; (ii) that explanatory variable(s) are not correlated with the error term, they are endogenous. This is particularly important when explanatory price-determining characteristics are omitted from the hedonic regression. If an omitted variable is correlated with an included one, the estimated coefficient on the included one is biased; (iii) when price-determining explanatory independent variables are not truly independent, but correlated with each other—multicollinearity—the coefficient estimates and their tests become sensitive to change in the model and/or data. While the estimated coefficients are imprecise, the predicted prices in a hedonic regression would be unbiased. A full account of all OLS assumptions, consequences, means of detection of violation, and treatment, that may involve use of alternative (to OLS) estimators, can be found in any introductory econometrics/statistical text. Modern software provides the appropriate tests for, and means of surmounting, breaches of these assumptions and thus, validation of the hedonic model used. It is recommended that a background paper by the statistical office responsible for the CPI be published on the hedonic regression model used and it's supporting diagnostic statistics to demonstrate the validity of the model and satisfy the need for transparency.

Second, the estimated coefficients should be updated regularly. Say the predicted price is for the new model in a reference period, as in Table 6.7c. There is, at first sight, no need to update the estimated coefficients each month. Yet the valuation of characteristics in the price reference period may be quite out of line with their valuation in the new period. A fall in the price of substantial increases in storage and processing speed, among other attributes, of computers makes the valuation of additional MBs or GBs of a new model, introduced a few years after the hedonic regression was estimated, a less meaningful exercise. Continuing to use the coefficients from some far-off period to make adjustments to prices in the current period is akin to using out-of-date reference period weights. The comparison may be well defined, but have little meaning. There is a need to update the hedonic regression estimates if they are considered to be out of date, say because of changing tastes or technology, and splice the new estimated comparisons onto the old. The regular updating of hedonic estimates when using imputations or adjustments is thus recommended, especially when there is evidence of instability in the parameter estimates of the hedonic regression over time.

Third, the sample of prices and characteristics used for the hedonic adjustments should be suitable for the purpose. If they are taken from a particular outlet or outlet type, trade source or web page and then used to adjust non-comparable prices for items sold in quite different outlets, then there must at least be an intuition that the marginal utilities for characteristics are similar between the outlets. A similar principle applies for the brands of items used in the sample for the hedonic regression. It should be borne in mind that high $\overline{R}^2$ statistics do not alone ensure reliable results. Such high values arise from regressions in periods prior to their application and indicate the proportion of variation in prices across many items and brands. They are not in themselves a measure of the prediction error for a particular item, sold in a specific outlet, of a given brand in a subsequent period, though they can be an important part of this.

Fourth, there is the issue of functional form and the choice of variables to include in the model. Simple functional forms generally work well. These include linear, semi-logarithmic (logarithm of the left-hand side) and double-logarithmic (logarithms of both sides) forms. Semi-logarithmic models are often employed since many of the price-determining explanatory variables are binary, 1 or 0, depending on whether or not a model has a particular feature. Such issues are discussed in Chapter 21. The specification of a model should include all price-determining characteristics. Typically, a study would start with a large number of explanatory variables and a general econometric model of the relationship, while the final model would be more specific, having dropped a number of variables. The dropping of variables would depend on the result of experimenting with different formulations, and seeing their effects on diagnostic test statistics, including the overall fit of the model and the accordance of signs and magnitudes of coefficients with prior expectations.

Finally, regarding resources, several requirements for the successful design and use of hedonic quality adjustment in the consumer price include:

- Intellectual competencies and sufficient time to develop and re-estimate the model, and to employ it when products are replaced;
- Access to detailed, reliable information on product characteristics;
- A suitable organization of the infrastructure for collecting, checking and processing information.

Hedonic methods may also improve quality adjustment in the consumer price index by indicating which product attributes do *not* appear to have material impacts on price. That is, if a replacement item differs from the old item only in characteristics that have been rejected as price-determining variables in a hedonic study, this would support a decision to treat the items as comparable. Care has to be exercised in such analysis because a feature of multicollinearity in regression estimates is an imprecision of the estimated parameter estimates. This may give rise to statistical tests that do not reject null hypotheses that are false. However, econometric/statistical software provides the tools to explore the nature and extent of multicollinearity; these include variance inflation factors (VIF). The results from VIFs provide valuable information on the nature and extent to which different explanatory variables (characteristics) are inter-related and this in turn can help in the selection of replacement items. The results from hedonic regressions thus have a role to play in identifying price-determining characteristics and may be useful in the design of quality checklists in price collection.

Choice between quality adjustment methods

Choice of method for quality adjustments to prices is not straightforward. The analyst must consider the technology and market for each commodity and devise appropriate methods. This is not to say the methods selected for one product area will be independent of those selected for other areas. Expertise built up using one method may encourage its use elsewhere, and intensive use of resources for one commodity may lead to less resource-intensive methods for others. The methods adopted for individual product areas may vary between countries as access to data, relationships with the outlet managers, resources, expertise and features of the production, and market for the product vary. Guidelines on choice of method arise directly from the features of the methods outlined above. A good understanding of the methods, and their implicit and explicit assumptions, is essential to the choice of an appropriate method.

Figure 6.3 provides a guide to the decision-making process. Assume that the matched models method is being used. If the item is matched for re-pricing in a subsequent period, there is no change in the specifications and no quality adjustment is required. This is the simplest of procedures. However, a caveat applies. If the item belongs to a product area where model replacement is rapid, and replacements non-comparable, the matched sample may become unrepresentative of the universe of transactions. Continued long-term matching would deplete the sample. This a matter for the frequent re-basing and maintenance of the sample, chapter ?.

Consider an item found to be **temporarily missing**. This would require a price imputation and if subsequently determined to be permanently missing—either from information from a senior outlet staff member or use of a three-month rule—a replacement found. **Overall or targeted price imputations** for temporarily missing prices maybe used, though the carry forward method is not recommended unless for controlled or regulated prices.

For **permanently missing** item prices, the selection of a comparable item is preferred, to the same specification of the old item, and the use of its price as a **comparable replacement**. Strictly, this would require that none of the price difference is attributable to quality and confidence that all price-determining factors are included in the specification. In practice, items may be taken to be deemed comparable if there are limited price-determining differences, as might be the case with styling, color, even some more substantial technical changes including performance and reliability that may not be immediately apparent to the consumer. A decision as to the comparability or otherwise of a replacement must be made by a desk officer with appropriate information on product differences supplied by the price collector. A comparable replacement item should also be representative and account for a reasonable proportion of sales. Caution is required when replacing near obsolete items with unusual pricing at the end of their life cycles with similar ones that account for relatively low sales, or with ones that have quite substantial sales but are at different points in their cycle. Strategies for ameliorating such effects are discussed below and in Chapter 8, including early substitutions before pricing strategies become dissimilar. With comparable replacements the price of the old item is directly compared with the price in the next period of the comparable replacement.

Figure 6.3 illustrates the case where **non-comparable replacements** are only available but the quality differences between the replacement and missing item can be explicitly quantified. **Explicit estimates** of quality differences are generally considered to be more reliable, although they are also more resource intensive, at least initially. Once an appropriate methodology has been developed, they can often be easily replicated. General guidelines are more difficult here as the choice depends on the host of factors discussed above, which are likely to make the estimates more reliable in each situation. Central to all of this is the quality of the data upon which the estimates are based. Estimates based on objective data are preferred. Good **production cost** estimates in industries with stable technologies and identifiable constant retail mark-ups and where differences between the old and replacement items are well specified and exhaustive are, by definition, reliable. Estimates of the retail mark-up are, however, prone to error and the **option cost** approach is generally preferable. This requires that the old and new items differ by easily identifiable characteristics that are or have been separately priced as options.

The replacement item may differ from the old one by its **possession of a feature**. Often it is the **price collector** who is best placed to provide an estimate of the price difference in quality of a non-comparable replacement. Say a specified brand of a bottle of tomato ketchup used for pricing is missing in the current period. However, a non-comparable replacement of the same brand is available, though the bottle has been restyled to now stand on its head, and label reversed. The price collector might note that other brands have both sizes on sales with a say 25cent price margin for the new one. The price collector in selecting a non-comparable replacement might also provide the basis for the desk officer to make an explicit quality adjustment. A desk officer might also make **use of the Internet** to identify the percentage markup for a quality characteristic, for example, for additional memory for a computer, blue-tooth technology in their automobile, and so forth.

The use of **hedonic regressions for patching** price changes due to quality differences is most appropriate where data on price and characteristics are available for a range of models and where the characteristics are found to predict and explain price variability well in terms of *a priori* reasoning and econometric terms. Their use is appropriate where the cost of an option or change in characteristics cannot be separately identified and has to be gleaned from the prices of items sold with different specifications in the market. The estimated regression coefficients are the estimate of the contribution to price of a unit change in a characteristic, having controlled for the effects of variations in the quantities of other characteristics. The estimates are particularly suited to valuing changes in the quality of an item when only a given set of characteristics changes and the valuation is required for changes in these characteristics only. The results from hedonic regressions may be used to target the salient characteristics for item selection. The synergy between the selection of prices according to characteristics defined as price determining by the hedonic regression, and their subsequent use for quality adjustment, should reap rewards. The method should be applied where there are high ratios of non-comparable replacements, though not a frequent churn as outlined below, and where the differences between the old and new items can be well defined by its characteristics.

If explicit estimates of quality are unavailable, and no replacement items are deemed appropriate, the **implicit estimates** might be used. One such method is that the use of **imputations** as applied to temporarily missing items is continued. Such use is not recommended as a default procedure in such instances. It may be used to extend the period of search for a replacement, though the absence of the old item and the unavailability of a replacement should indicate to the desk officer that the weight for that item might be better attributed to a quite different item. Such changes naturally take place on re-basing an index, chapter ?, though chapter 8 outlines alternative procedures.

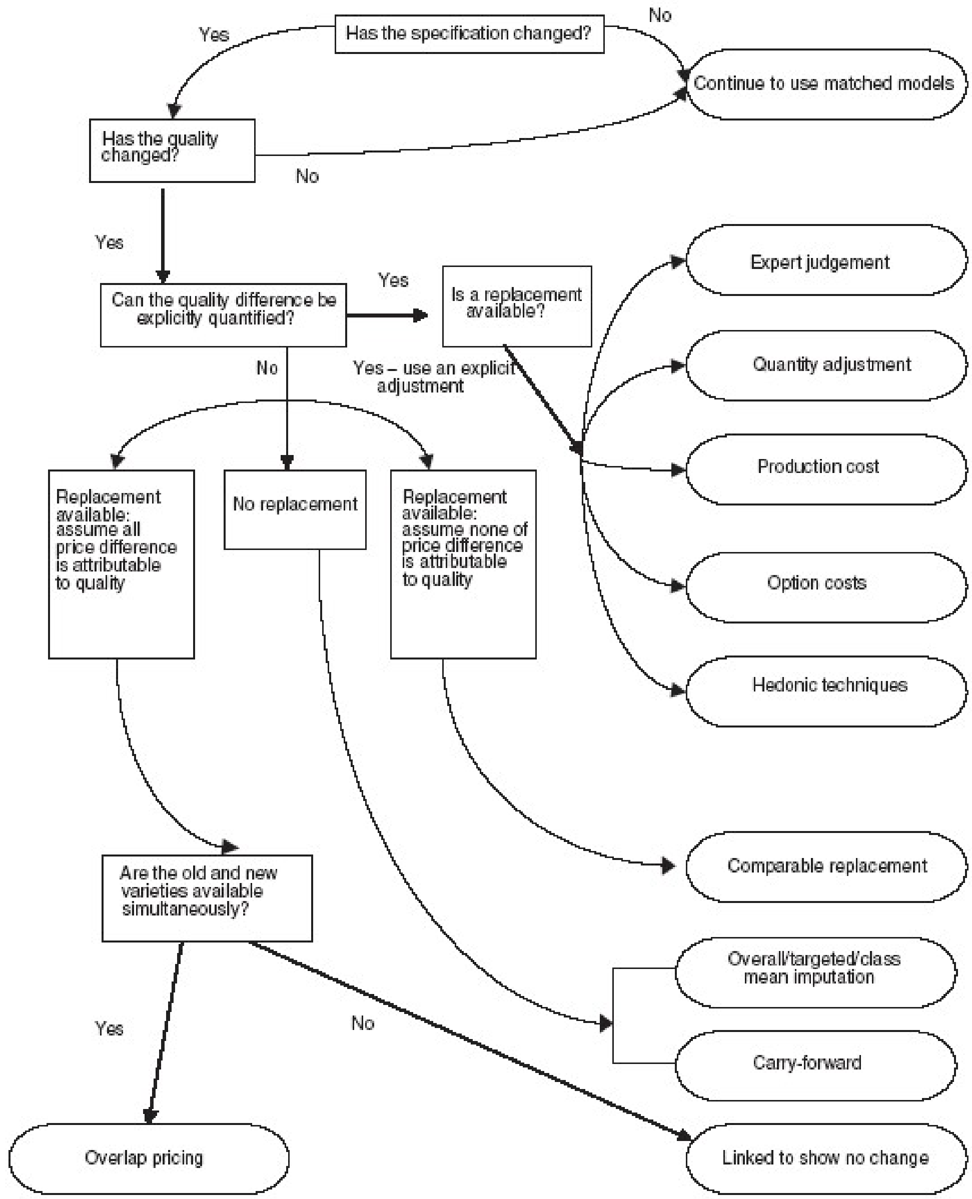
The use of *imputations* has much to commend it resource-wise. It is relatively easy to employ. It requires no judgment (unless targeted) and is therefore objective. Targeted mean imputation is preferred to overall mean imputation as long as the sample size upon which the target is based is adequate. Yet the bias from using imputations for permanently missing item prices is directly related to the proportion of missing items and the difference between quality-adjusted prices of available matched items and the quality-adjusted prices of unavailable ones (see Table 6.2 on page 110). The nature and extent of the bias depends on whether short-term or long-term imputations are being used (the former being preferred) and on market conditions (see paragraphs 6.159 to 6.173). Imputation, in practical terms, produces the same result as deletion of the item for an elementary aggregate. The inclusion of imputed prices may give the illusion of larger sample sizes. Imputation should by no means be the overall catch-all strategy, and statistical agencies are strongly advised against its use as a default device which may lead to serious sample degradation.

If the old and replacement items are available simultaneously, and if the quality difference cannot be quantified, an implicit approach can be used whereby the price difference between the old and replacement items in a period in which they both exist is assumed to be attributable to quality. This **overlap method**, in replacing the old item by a new one, takes the ratio of prices in a period to be a measure of their quality difference. It is implicitly used when new samples of items are taken. The assumption of relative prices equating to quality differences at the time of the splice is unlikely to hold if the old and replacement items are at different stages in their life cycles and different pricing strategies are used at these stages. For example, there may be deep discounting of the old item to clear inventories, and price skimming of market segments that will purchase new models at relatively high prices. As with comparable replacements, early substitutions are advised so that the overlap is at a time when items are at similar stages in their life cycles. It may well be the case that overlap prices are unavailable. In such cases a range of imputation approached are available to estimate an overlap price.

For the reasons discussed, the use of the **linked to show no change** method for permanently and the **carry-forward** method for temporarily missing item prices are not generally advised for making quality adjustment and imputations, unless the implicit assumptions are exceptionally deemed to be valid.

This flow chart is misleading. Will redo.

Figure 6.3 Flowchart for making decisions on quality change



Source: Chart developed from a version by Fenella Maitland-Smith and Rachel Bevan, OECD; see also a version in Triplett (2002).

While Figure 6.3 is appropriate for the treatment of temporarily and permanently missing prices in the routine compilation of a CPI, there is a context in which a quite different strategy is required. The context is where there is a rapid turnover or "churn" in the models of items sold. For example, a television sets are sold by several manufacturers each having a range of models with different features. Over time many new phases of technological development have occurred including the cathode ray tube (CRTs), color TVs, wireless remotes, plasma, liquid-crystal-display televisions (LCD), digital, high definition (HD), larger screens, smart functions, 3D, light-emitting diodes (LEDs), Ultra HD resolution, OLED (Organic Light Emitting Diode), and roll-up OLED. Throughout the life cycle of this Manual there will no doubt be many further phases. New features and restyling extend the life cycle of each model in each phase. As with automobiles, computers, computer-related hardware and software, telecommunication, household appliances and much more, the market is characterized by different manufacturers producing several products of different qualities aimed at different segments of the market each of which has a different quality, say screen size, and, over time, a rapid turnover in its quality characteristics. The methods outlined above if applied to these markets may lead to a biased CPI. The next section considers CPI measurement for these product markets.

HIGH-TECHNOLOGY AND OTHER SECTORS WITH A RAPID TURNOVER OF MODELS

The measurement of price changes of items unaffected by quality changes is primarily achieved by matching models, the above techniques being applicable when the matching breaks down. But what of industries where the matching breaks down on a regular basis because of the high turnover in new models of different qualities to the old ones? The matching of prices of identical models over time, by its nature, is likely to lead to a seriously depleted sample. There is both a dynamic universe of all items consumed and a static universe of the items selected for re-pricing. If, for example, the sample is initiated in December, by the subsequent May, for a L-T price comparison, the static universe will be matching prices of those items available in the static universe in both December and May, but will omit the unmatched new items introduced in January, February, March, April and May, and the unmatched old ones available in December but unavailable in May. For December to May cumulative month-on-month S-T comparisons, similar considerations apply. Although there will be improved imputations for temporarily missing item prices and an improved more timely introduction of replacements, the replacements only borrow from the dynamic universe of new models on a one-on-one basis. The example here is for a December to Jnauary matched price comparison. For many countries matching may effectively continue for many years until the CPI is rebased leaving an extremely degraded sample on rebasing. Two empirical questions show whether there will be any significant bias. First, is sample depletion substantial? Substantial depletion of the sample is a necessary condition for such bias. Second, are the unmatched new and unmatched old items likely to have quality-adjusted prices that substantially differ from those of the matched items in the current and the base periods?

The matching of prices of identical models over time may lead to the monitoring of a sample of models that is increasingly unrepresentative of the population of transactions. Some of the old models that existed when the sample was drawn are not available in the current period; and new models that enter the sample are not available in the base period. It may be that the models that are going out have relatively low prices, while the entrants have relatively high ones. By ignoring these prices, a bias is being introduced. Using old low-priced items and ignoring new high-priced ones has the effect of biasing the index downwards. In some industries, the new item may be introduced at a relatively low price though the old one may continue at a relatively high price, serving a minority segment of the market. In this case, the bias would take the opposite direction. The nature of the bias will depend on the pricing strategies of firms for new and old items. Some strategies for the introduction of new models, and implications for CPI measurement, are considered in Annex 3.

This sampling bias exists for most products. Our concern here, however, is with product markets where the statistical agencies are finding the frequency of new item introductions and old item obsolescence sufficiently high that they may have little confidence in their results. Three procedures will be considered: an extensive use of the matched model (overlap) technique, class-mean imputation, and the use of hedonic price indices (as opposed to the partial, hedonic patching discussed above).

Matching and the overlap method for markets with rapid turnover of models

Table 6.8, Illustration of rapid model turnover

Model	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan
1	25	25	25	25	25	25	25	25	25	25	20		
1R											29.2	30	30
2	35	35	35	35	35	32							
2R						37	37	37	37	37	37	40	40
3	30	30	30	30	30	30	30	30	30	30	27		
3R											31.7	33	33
4	29	29	29	29	29	28							
4R						30	30	30	30	30	30	30	30
5	29	29	29	29	29	29	29	29	29	29	29	29	29

This approach is simply a more extensive use of the overlap approach outlined in section x for permanently missing prices. Its adoption here is for permanently missing items that occur frequently as is usual for changes in models of electronic goods and automobiles. Matching prices of a few representative items becomes less feasible in this context.

Consider Model 1: in November there is no overlap price for the new model 1R, so its price is imputed "backwards" by using the ratio of geometric means of the December to November prices but only including those for which matched models exist, that is models 2R, 4R, and 5. These are all constant-quality price comparisons; of like with like. For model 1, the imputed price of its replacement in November is based on the ratio of geometric means for models 2R, 4R, and 5:

(6.11) $$\frac{\left(37\times 30\times 29\right)^{\frac{1}{3}}}{\left(40\times 30\times 29\right)^{\frac{1}{3}}}=\left(\frac{37}{40}\times\frac{30}{30}\times\frac{29}{29}\right)^{\frac{1}{3}}=0.974$$

and its imputed price $0.974\times 30=29.2$.

The imputed price for the replacement model 2R in June is based on the price changes of matched models 1, 3, and 5 for June and July, that is: $\frac{\left(25\times 30\times 29\right)^{\frac{1}{3}}}{\left(25\times 30\times 29\right)^{\frac{1}{3}}}=1.00$

and its imputed price: $1.00\times 30=30$.The imputed prices for 3R in November, and 4R in June are 37 and 31.6 respectively.

The overall price relatives for each model, and its linked-in replacement, can now be computed as the product of S-T month-on-month price changes, that is model 1 and its replacement 1R, for January to January, using the overlap month of November:

(6.12a)
$$\left[\frac{p_1^{Feb}}{p_1^{Jan}}\times\frac{p_1^{Mar}}{p_1^{Feb}}\times\frac{p_1^{Apr}}{p_1^{Mar}}\times\frac{p_1^{May}}{p_1^{Apr}}\times\frac{p_1^{Jun}}{p_1^{May}}\times\frac{p_1^{Jul}}{p_1^{Jun}}\times\frac{p_1^{Aug}}{p_1^{Jul}}\times\frac{p_1^{Sep}}{p_1^{Aug}}\times\frac{p_1^{Oct}}{p_1^{Sep}}\times\frac{p_1^{Nov}}{p_1^{Oct}}\times\frac{p_{1R}^{Dec}}{p_{1Rimp}^{Nov}}\times\frac{p_{1R}^{Jan}}{p_{1R}^{Dec}}\right]\times 100$$

$$=\left[\frac{p_1^{Nov}}{p_1^{Jan}}\times\frac{p_{1R}^{Dec}}{p_{1Rimp}^{Nov}}\times\frac{p_{1R}^{Jan}}{p_{1R}^{Dec}}\right]\times 100$$

(6.12b)
$$=\left[\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{25}{25}\times\frac{20}{25}\times\frac{30}{29.2}\times\frac{30}{30}\times\frac{30}{30}\right]\times 100$$
$$=\left[\frac{20}{25}\times\frac{30}{29.2}\times\frac{30}{30}\times\frac{30}{30}\right]\times 100=82.19$$

The price relative for model 1 from January in the current year from January in the preceding year shows a $\left(1-\frac{82.19}{100}\right)\times 100=17.81$ percent price *decrease*. It is clear from Table 6.8 that the price for model 1 have been constant up to October, there was a price fall in November, but this was to clear the market for the replacement. There should be a counteracting November-December price increase for the old model 1 to the new replacement 1R that reflects that part of the price change not due to quality differences. But the imputation is based on the constant price movements of models 4R and 5 and a coincidental price increase in model 2R; it is based on an assumption that S-T price movements of matched pairs will proxy the price change of model 1. However, in this context, the constant price changes of matched models are an inappropriate proxy badly biasing the measured price change downwards. At fault are first, the use of the unrepresentative price for model 1 at the end of its life cycle in November, and second, the inappropriate imputation for the replacement item.

The January-January price *decreases* for models 2, 3, and 4, using replacements, and 5 are respectively: 1.2, 6.3, and 3.4 percent and no change for model 5. With a substantial churn of models, possibly more frequently than annual, the bias from using overlaps can be substantial.

In its favor, the method is simply an extension of the linking-in of new products and can be readily applied by a statistical office, especially one with limited resources. Yet in basing the imputations on price changes of matched items not subject to the price changes that occur on the replacement of a model, it biases the CPI.

The overlap method, outlined above in section x, may be subject to bias if applied to where there is substantial churn in the product market and an active policy by the supplier of introducing upgraded replacement models. The nature and extent of the bias depends on the pricing strategy. Table 6.8 illustrated a policy of lower pricing at the end of the life cycle and a higher price at the start. Importantly, the example had no price change for other matched models, from which the imputation was drawn, and thus a biased imputation at this critical overlap period. The bias was substantial and downwards. Alternative pricing strategies are given in Annex 3 along with their implication for bias from using the overlap matching. That the method can introduce substantial bias under quite reasonable conditions has been demonstrated in empirical studies. That the nature and extent of the bias depends on business pricing strategies that may change over time and are unpredictable, is worrying for CPI compilation in this important product area. The method is not recommended for product markets with a high rate of model churn.

Use of a class-mean imputation

It was shown in section x above that an imputation based on price movements of other matched models not at the end of their life cycle, could introduce bias. An alternative, though more resource intensive method, is to base the imputations not on price changes of matched items but use, where possible, explicit quality adjustments for linked-in non-comparable replacement. For example, Internet web pages of prices of similar products may show the difference in characteristics and prices of the old and replacement models. At its simplest, the replacement may simply have a higher value of some performance characteristic or feature the price of which is available as a, say, option. If a sufficient number of explicit quality adjustments can be made, imputations might be better made on the basis of only those models that have had explicit quality adjustments to their price.

However, the very nature of the high frequency of replacements makes the procedure resource intensive and, in some instances, not viable due to the absence of explicit information on prices of features. However, should sufficient models have an explicit adjustment, an average of their price change could be used to impute the price change of other models being replaced. This is the basis of using class-mean imputations. The method requires care that the linking-in of replacement models not take place at the end of the model's life cycle, when pricing might be abnormally low for an item relatively few are purchasing. This not only has a detrimental affect on the quality adjustment methodology, but also on the representativity of the models upon which the prices change measurement is based.

A class-mean imputation was outlined in section x. It is similar in procedure to the overall and targeted mean imputation; it is a form of targeted imputation. The "target" is measured price changes of replacements for permanently missing products. Only the price changes of "comparable" replacements are used to impute the overlap price, the replacements being limited to those that have exactly the same price-determining characteristics, or those items with replacements that have been declared comparable after review or have already been quality-adjusted through one of the "explicit" methods. For example, when the arrival of a new model of a particular make of automobile forces price collectors to find replacements, some of the replacements will be of comparable quality, others can be made comparable with explicit quality adjustments, but the remaining ones will need imputed prices for an overlap month. Class mean imputations are use imputed price relatives only from the prices of comparable and, where appropriate, explicitly quality-adjusted models. In general, it does not use the prices for the models that were not replaced, because these are likely to be different from those of new models. The prices of old models tend to fall as they become obsolete, while the new models (represented by the replacements) tend to have a higher price before falling.

Class mean-imputations rely on other explicit quality adjustments and comparable replacements. The other explicit quality adjustments may be from available option or feature prices and may be limited in nature, covering only some of the differences in product attributes, available for only a small proportion of unrepresentative model changes, and the availability of comparable replacements limited. Given a substantial churn in the market and difficulties with such imputations and estimates an alternative recommended approach is that of hedonic indices.

Hedonic price indices

It is important to distinguish between the use of hedonic regressions for patching, to make adjustments to individual item prices for quality differences when a non-comparable substitute is used, as outlined in paragraphs ? to ?, and their use in their own right as *hedonic price indices,* which are measures of quality-adjusted price changes. Hedonic price indices are suitable when the pace and scale of replacements of items are substantial because, first, an extensive use of these overlap quality adjustments may lead to bias and, second, the sampling will be from a static matched/replacement universe likely to be biased. With new models being continually introduced and old ones disappearing, the coverage of a matched sample may deteriorate and bias may be introduced as the price changes of new/old models differ from those of the matched ones. What is required is a sample to be drawn in each month and price indices constructed; but instead of controlling for quality differences by matching, they will be controlled for, or "partialled out", in the hedonic regression. Note that all the indices described below use a fresh sample of the data available in each period. If there is a new item in a period, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old items drop out, they are still included in the data for the indices in the periods in which they exist. Paragraphs 6.110 to 6.115 stress the need for caution in the use of hedonic regressions for quality adjustments; some of the theoretical and econometric aspects are considered in Chapter 21.

In Chapter 17, theoretical price indices are defined and practical index number formulae are considered as bounds or estimates of these indices. Theoretical index numbers are also defined in Chapter 21 to include goods made up of tied characteristics, so something can be said about how such theoretical indices relate to different forms of hedonic indices. A number of forms are considered in Chapter 21; they are summarized below.

Consider a price comparison between two adjacent time periods, say periods t and t+1. The models sampled do not have to be matched. They may simply be all recorded models on sale in the two periods, albethey comprising a different mix of qualities. The hedonic formulation regresses the price of model i, p_i, on the k=2,….K characteristics of the items z_{ki}. A single regression is estimated on the data in the two time periods compared, the equation also including a dummy variable D^{t+1} being 1 in period t+1, zero otherwise:

The time dummy variable approach

A single hedonic regression equation is estimated with observations across models over adjacent time periods, including the reference period 0 and a subsequent periods t. (The logarithm of) prices of individual models are regressed on their characteristics and a dummy variables for time, taking the values of $D_i^1 = 1$ if the model is sold in period 1 and 0 otherwise. A log-linear specification is given by:

$$(6.13)\ldots \ln \hat{p}_i^{0,t} = \hat{\beta}_0 + \sum_{k=1}^{K} z_{k,i}^{0,t} \ln \hat{\beta}_k + \sum_{t=1}^{T} \hat{\delta}^t D_i^t$$

The $\hat{\delta}^t$ are estimates of the *proportionate* change in price arising from a change between the excluded reference period *t*=0 and successive periods *t*=1,T having controlled for changes in the quality characteristics via the term $\sum_{k=1}^{K} z_{k,i}^{0,t} \ln \hat{\beta}_k$.

In principle, the index $100 \times \exp(\hat{\delta}^t)$ requires an adjustment for it to be a consistent (and almost unbiased) approximation of the proportionate impact of the time dummy.[8] In practice, it usually has little effect.

The method implicitly restricts the coefficients on the quality characteristics to be constant over time: for example, for an adjacent period January and February regression, for *k*= 1,…,*K* characteristics and where period 0 and *t* are January and February respectively, $b_k = b_k^{Jan} = b_k^{Feb}$. Should be greek beta and delta. The (relative) valuation of a characteristic, for example for a washing machine with an additional 100 rpm spin speed, is the same in January as in February. The index, $100 \times \exp\left(d_t\right)$, is an estimate of the RPPI for February (January=100).

The characteristics/repricing approach

A hedonic regression is run to determine the price-determining characteristics of models in a say reference period 0. The average model in period 0 can then be defined as a tied bundle of the averages of each price-determining characteristic, for example for washing machines: Spin-speed: 1,375 rpm; Capacity (cotton load): 8.5 kg.; Annual energy cost: £36.5 (pounds sterling); Steam facility: 4 percent; LG brand: 15 percent; Warranty period: 5.4 years; Run-time (cotton): 18.8 mins; and so forth. These are the $\bar{z}_k$ averages for each of the *k* price-determining characteristics.

These average values of each characteristic are held constant in each period but valued in turn using period 0 and period *t* hedonic regressions. One form of the (average) *characteristics* approach is as a measure of the price change of a set of average period 0 characteristics

[8] An estimate of the proportionate impact of the period *t* time dummy for this log-linear form is the consistent (and almost unbiased) approximation:□ $\left[\exp\left(\hat{\delta}_t\right) \big/ \exp(V(\hat{\delta}_t / 2))\right] - 1$ where $\hat{\delta}_t$ *is* the OLS estimator of δ_t and $V(\hat{\delta}_{tt})$ its estimated variance. The approximation is accurate, even for quite small samples, though in practice it has been found to have a relatively small effect.

valued first, at period *t* hedonic valuations, and second, at period 0 hedonic valuations? A ratio of the results is a constant (period 0 characteristics)
quality price index. The numerator provides an answer to a counterfactual question: what would be the estimated transaction price of a model with period 0 average characteristics, were it on the market in period t?

A constant-quality *hedonic geometric mean characteristics* (HGMC) price index from a log-linear hedonic regression equation is a ratio of geometric means with average characteristics held constant in the *reference* period 0, $\bar{z}_k^0$:

$$(6.14)\ldots.\ P^{0\rightarrow t}_{HGMB:\bar{z}^0} = \frac{\prod_{k=0}^{K}\left(\bar{z}_k^0\right)^{\hat{\beta}_k^t}}{\prod_{k=0}^{K}\left(\bar{z}_k^0\right)^{\hat{\beta}_k^0}} = \frac{\exp\left(\sum_{k=0}^{K}\bar{z}_k^0 \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^{K}\bar{z}_k^0 \ln \hat{\beta}_k^0\right)} \text{ where } \bar{z}_k^0 = 1/N^0 \sum_{i\in N^0} z_{i,k}^0$$

Equation (6.14) holds the (quality) characteristics constant in period 0, though a similar index could be equally justified by valuing in each period a constant period *t* average quality set:

$$(6.15)\ (4)\ldots.P^{0\rightarrow t}_{HGMC:\bar{z}^t} == \frac{\prod_{k=0}^{K}\left(\bar{z}_k^t\right)^{\hat{\beta}_k^t}}{\prod_{k=0}^{K}\left(\bar{z}_k^t\right)^{\hat{\beta}_k^0}} = \frac{\exp\left(\sum_{k=0}^{K}\bar{z}_k^t \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^{K}\bar{z}_k^t \ln \hat{\beta}_k^0\right)} \qquad \text{where } \bar{z}_k^t = 1/N^t \sum_{i\in N^t}^{N^t} z_{i,k}^t$$

Neither a period 0 constant-characteristics index nor a period *t* constant-characteristic quantity basket can be considered to be superior, both acting as bounds for their theoretical counterparts. Some average or compromise solution is required. An index making symmetric use of period 0 and period *t* characteristics values is intuitive:

$$(6.16)\ldots.\ P^{0\rightarrow t}_{HGMC:\sqrt{\bar{z}^0\bar{z}^t}} = \frac{\prod_{k=0}^{K}\left(\bar{z}_k^\tau\right)^{\hat{\beta}_k^t}}{\prod_{k=0}^{K}\left(\bar{z}_k^\tau\right)^{\hat{\beta}_k^0}} = \frac{\exp\left(\sum_{k=0}^{K}\bar{z}_k^\tau \ln \hat{\beta}_k^t\right)}{\exp\left(\sum_{k=0}^{K}\bar{z}_k^\tau \ln \hat{\beta}_k^0\right)} \quad \text{where } \bar{z}_k^\tau = \left(\bar{z}_k^0 + \bar{z}_k^t\right)/2.$$

Note that equations (6.16) , (6.17), and (6.18 all use predicted prices in both the denominator and numerator. This follows the advice to use dual imputations outlined in section x.

However, it also entails running hedonic regressions in each month. Yet a fortuitous result is that a feature of the OLS estimator is that the mean of actual prices is equal to the mean of predicted prices: $\prod_{i\in N^0}\left(\hat{p}^0_{i|z_i^0}\right)^{\frac{1}{N^0}} = \prod_{i\in N^0}\left(p_i^0\right)^{\frac{1}{N^0}}$ and $\prod_{i\in N^t}\left(\hat{p}^t_{i|z_i^t}\right)^{\frac{1}{N^t}} = \prod_{i\in N^t}\left(p_i^t\right)^{\frac{1}{N^t}}$. Thus while the numerator of equations (6.14) and denominators of equation (6.15) must be counterfactual, the denominator of equations (6.14) and numerator of (6.15) can use actual prices. This leaves us with the important results that equation (6.15) does not require a hedonic regression to be estimated in every current period *t*, only in the price reference period 0. This is an important result since, it aids the practical work of compilers who do not have to estimate a hedonic regression equation in each period, but maybe once every one or two years, depending on the amount of churn in the market and shifting technologies and preferences. The hedonic indices from one regression can be chained to its preceding hedonic indexes, and so forth, using successive multiplication.

The hedonic imputation approach

In contrast to the characteristics approach, the *imputation* approach works at the level of individual items/models, rather than the average values of their characteristics. The rational for the imputation approach lies in the matched model method. Consider a set of models transacted in period 0. We want to compare their period 0 prices with the prices of the same matched models in period *t*. In this way there is no contamination of the measure of price change by changes in the quality-mix of models transacted. However, not all of the period 0 models were sold in period *t*—there is no corresponding period *t* price. The solution—in the numerator of equation (6.17)—is to predict the period *t* price of each *i* period 0 model, $\hat{p}^t_{i|z^0_i}$.

We use a period *t* regression to predict prices of models sold in period 0 to answer the counterfactual question: what would a model with period 0 characteristics have sold at in period *t*?

A constant-quality *hedonic geometric mean imputation* (HGMI) price index from a log-linear hedonic regression equation is a ratio of geometric means with characteristics held constant in the *reference* period 0, $\overline{z}^0_k$:

(6.17)

$$P^{0\to t}_{HGMI:z^0_i} = \frac{\prod_{i\in N^0}\left(\hat{p}^t_{i|z^0_i}\right)^{\frac{1}{N^0}}}{\prod_{i\in N^0}\left(\hat{p}^0_{i|z^0_i}\right)^{\frac{1}{N^0}}} = \frac{\exp\left(1/N^0 \sum_{i\in N^0}\ln \hat{p}^t_{i|z^0_i}\right)}{\exp\left(1/N^0 \sum_{i\in N^0}\ln \hat{p}^0_{i|z^0_i}\right)}$$

Alternatively, the value in the numerator of equation (6.17) is the geometric mean of the period *t* price of period *t* price-determining characteristics, $z^t_{i,k}$. This is compared, in the denominator, with the geometric mean of the period 0 predicted price of the self-same period *t* price-determining characteristics, $z^t_{i,k}$. For each model, the quantities of characteristics are held constant in period *t*, $z^t_{i,k}$; only the characteristic prices change.

$$(6.18)\ldots\ldots\quad P^{0\to t}_{HGMI:z^t_i} = \frac{\prod_{i\in N^t}\left(\hat{p}^t_{i|z^t_i}\right)^{\frac{1}{N^t}}}{\prod_{i\in N^t}\left(\hat{p}^0_{i|z^t_i}\right)^{\frac{1}{N^t}}} = \frac{\exp\left(1/N^t \sum_{i\in N^t}\ln \hat{p}^t_{i|z^t_i}\right)}{\exp\left(1/N^t \sum_{i\in N^t}\ln \hat{p}^0_{i|z^t_i}\right)}$$

As with the characteristics approach, a compromise solution as to whether period 0 or period t constant characteristics should be used is to use an average of the two. However, as with the characteristics approach, equation (6.18) has the advantage of only requiring a single hedonic regression to be estimated in the price reference period 0. Should this be used, the regression should be re-estimated every year or so, the frequency being determined by the turnover of products.

The three approaches have different, yet valid, intuitions. Yet as long as the functional form of the aggregator is aligned to the hedonic regression in the manner shown in Table 6.9 below, the imputation and characteristics approaches yield the same result. This consolidation not only markedly narrows down the choice between approaches, but also validates the measure as one resulting from quite different intuitions.

Table 6.9, Equivalences of hedonic approaches		
Hedonic regression: functional form	**Characteristics approach: form of average of characteristics**	**Imputation approach: Form of average of predicted prices**
Linear	Arithmetic mean	Arithmetic mean
Log-linear	Arithmetic mean	Geometric mean
Log-log	Geometric mean	Geometric mean

For a log-linear functional form of a hedonic regression, the requirements are that (i) for the characteristics approach, $\overline{z}_k^0$ and $\overline{z}_k^t$ are arithmetic means of characteristic's values, the right-hand-side (RHS) of the hedonic regression, and (ii) for the imputation approach, the ratio of average predicted prices is a ratio of geometric means, the left-hand-side (LHS).

The important feature of these hedonic indices is that they require no matching of individual models in the periods compared. Matching is required so that the price of a model in period 0 can be compared with that in period t, without a concern that the price change is affected by changes in quality. Such matching restricts the sample and, importantly in this context of a high level of churn in models and where prices change when models change, can lead to bias. This was illustrated using the example in Table 6.6. The price comparison of matched models effectively removes from the sample price changes in the important period of a price comparison as and when models change. The imputation for November to December for model 1 in Table 6.6 is based on matched prices only. Hedonic indices adjust for quality change not by any painstaking matching and, for that matter, identification of replacements, but by applying a hedonic regression to value constant-quality characteristics.

Hedonic indices use data on matched and unmatched observations and, again importantly, can naturally be applied to large monthly data sets, such as scanner and web-scraped data, as opposed to a small sample of what may have been in some long-past reference period, a representative item.

A more detailed and discursive account of the use of hedonic methods for high-technology goods subject to market churn is given in Triplett (2006) with further methodological developments in Silver (2016), albeit in the context of house price indexes. Importantly, are overviews and worked illustrations in the former and detailed derivations and further developments, including explicit weighting systems at this elementary level, in the latter.

An advantage of the imputation approach over the dummy variable approach is that explicit weighting systems can be more readily, accurately, and intuitively applied at this elementary level. For example, equation (6.) may be defined for models *i* over a set of models of television sets sold in period *t*. The formula gives equally weight to each model sold. A major improvement would be to apply to each model's quality-adjusted price change the weight of that price change, that is, the individual model's share of transaction expenditure values, say from scanner data. Silver (2016) outlines the methodology for the imputation approach, again in the context of house price indices, to include quasi-superlative and superlative formulations. The weighted imputation approach also has a correspondence to a weighted characteristics approach, and the more intuitive application of weights, if formulated as in Table 6.7.

A final issue to note is that hedonic indices are estimated for large data sets of models, say web-scraped or scanner data (see chapter x) for which there is no matching or items. It is at initiation that a price collector selects a representative item and matches its characteristics in subsequent period in order to track the price of this self-same item. In doing so the sample of prices collected is highly restricted to what may be a single price. With hedonic indices it is the varying values of the characteristics of the models that enable a constant quality price change. There may be datasets in which accurately matched sampled prices form part of the sampled data. In such a case there would be no need for predicted prices to be used for constant quality price change. The overall measure for this data set would contain: (i) actual price changes for the matched sample; (ii) hedonic price changes for the period 0 models not sold in period *t* (as, for the hedonic imputation approach, in equation (6.17)); and (ii) hedonic price changes for the period *t* models not sold in period 0 (as, for the hedonic imputation approach, in equation (6.18)). Each of these terms would be weighted by their relative expenditure shares, if available. It is from the aforementioned measure of all three components that the difference between the matched models method and hedonic indices becomes apparent.

The difference between hedonic indices and matched indices

An advantage of hedonic indices over matched comparisons was the inclusion by the former of un-matched data. Consider a data set of prices and characteristics over two successive time periods, say periods 0 and *t*. Assume there are *m* matched models in both periods 0 and *t*, *o* old models in period 0, but disappearing thereafter, and *n* new models appearing in period *t*, and subsequently.

	Period 0	**Period *t***
Matched models (*m*)	*m*	*m*
Old model (*o*)	*o*	
New model (*n*)		*n*

A constant-quality, period 0 to *t*, price index, from a hedonic imputation approach, is made up of three terms:

- **The change in the geometric mean price of the *m* matched models**, with no need for quality adjustment, their being matched;

- **The change in the constant-quality geometric mean price of the old models with actual prices in period 0 and counterfactual ones in period *t***. The counterfactual constant-quality price in period *t* has to be estimated since there is only a price in period 0. A prediction is required of what each old model's price in period 0 would have been had it been sold in period *t*. A period *t* hedonic regression is estimated and a predicted price estimated for each model by inserting its period 0 characteristic z_k^0 values into the right hand side of the estimated regression equation. A geometric mean is compiled of these predicted values, $\prod_{i \in o} \left(\hat{p}^t_{i|z_i^0} \right)^{\frac{1}{No}}$, and compared with the period 0 geometric mean, $\prod_{i \in o} \left(p_i^0 \right)^{\frac{1}{No}}$, as in equation (6.17).
- **The change in the constant-quality geometric mean price of the new model in period *t***. The counterfactual constant-quality price in period 0 has to be estimated since there is only a price in period *t*. A prediction is required of what each new model's price in period *t* would have been had it been sold in period 0. A period 0 hedonic regression is estimated and a predicted price estimated for each model by inserting its period *t* characteristics z_k^t values into the right hand side of the estimated regression. A geometric mean is compiled of these predicted values, $\prod_{i \in n} \left(\hat{p}^0_{i|z_i^t} \right)^{\frac{1}{Nn}}$, and compared with the period *t* geometric mean, $\prod_{i \in n} \left(p_i^t \right)^{\frac{1}{Nn}}$, as in equation (6.18).

The overall index can be phrased as a weighted average of these three elements with the matched comparison having a weight of $2N_m/(2N_m+N_o+N_n)$, the old of $N_o/(2N_m+N_o+N_n)$, and the new, $N_n/(2N_m+N_o+N_n)$, though preferably the weights should be expenditure shares rather than the numbers of each model.

The matched model method effectively ignores the last two elements of the bullet points. This procedure would result in no bias if the imputed quality-adjusted price change of new and old items were the same as that for matched models. However, as illustrated in Table 6.6, there may be substantial differences for goods and services where there is a great deal of churn in the models bought and sold. The matched model method might be appropriate if the number of new and number of old models—or their expenditure weights—is small relative to matched models. This would be the case for the hedonic patching of permanently missing model prices outlined in section x, but not for this context where there is a high and frequent turnover in models.

Even if the MMM is used with replacements, something of the dynmic universe of models is brought into the measure, but only insofar as there is a one-on-one item replacement. Further, hedonic indices employ a consistent basis for the explicit quality adjustment for non-comparable replacements.

The deficiency of the matched model method against a hedonic index has been shown above in terms the hedonic imputation approach, though the self-same considerations apply to a time dummy variable approach. Consider an adjacent period time dummy variable hedonic index of the form of equation (6.?), with the index change captured by the coefficient on the dummy variable for time. A sample of models of washing machines for periods t and t+1 would have in the regression the (log of) price on the left hand side and price-determining characteristics on the right hand side (RHS). Also on the RHS would be a dummy variable denoting whether the observation is drawn from period t or t+1. The hedonic regression includes matched, new and old models and the quality adjustment is achieved through the term $\sum_{k=2}^{K} \beta_k z_{ki}$ in equation (6.13). A matched model measure of price change would again only measure the price change for the more limited sample of matched models, though would not require a quality adjustment. The hedonic dummy variable approach, in its inclusion of unmatched old and new observations will likely differ from a geometric mean of matched prices changes, the extent of any difference depending, in this unweighted formulation, on the proportions of old and new items leaving and entering the sample and on the price changes of old and new items relative to those of matched ones. If the market for products is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes. Different market behavior will lead to different forms of bias, see Annex 3.

The use of the geometric mean

Throughout this chapter use has been made of the unweighted geometric mean for aggregating prices and price changes of items—a Jevons price index number formula. Alternative unweighted index number formula include the Dutot price index, a ratio of arithmetic means, and the Carli price index, an arithmetic average of price ratios. The Carli index has a well-established bias, notably in its chained form, which can lead t substantial drift in the results. The Dutot index fails an important axiomatic test—the commensurability or units of measurement test—and its use is restricted to homogeneous items. Illustrations of the calculations of all three formulae and their features have been delayed to Chapters 8 and ?? to provide a more focused account in this chapter of the treatment of missing values.

Treatment of missing item prices and quality adjustment within an elementary aggregate: two-stage (short-term) comparisons

This chapter has also used a S-T framework of comparing month-on-month prices rather than a L-T framework of comparing the current period's price with a fixed price reference period. The use of matching is particularly problematic for long-term (L-T) price comparisons. For L-T price comparisons a selection of representative models in period 0, say the year (or a month within) 2020, has their prices compared with those in January 2021; in February, for the 2020-February price relative; March, for the 2020-March price relative; continuing for what may be in some countries, several years. The sample is increasingly depleted over time as 2020 items become obsolete.

Some mention has been made of the advantages of the S-T approach. Illustration of the S-T as against the L-T approaches has also been delayed until chapter 8, to maintain a focus in this chapter on the methods for the treatment of missing prices, but also to avoid repetition. This chapter 6 was concerned with temporarily and permanently missing item prices. Temporarily missing item prices return to the sample, as do seasonal ones, whose treatment is outlined in chapter ? There is no issue here with maintaining the sample. However, permanently missing item prices need a replacement item otherwise, over time, the sample becomes increasingly depleted and degraded. Yet such one-on-one replacement is unlikely to sufficient to maintain the representativity of the sample, something based on an initiation of item selection at the last rebasing (chapter 9), or sample rotation, that may for some countries be many years ago. Since this initiation many more new items/products may have been introduced and old ones become obsolete. Maintaining the representativity of the sample is addressed in chapter 7 and a related issue of using electronic data sources in chapter 10. The treatment of permanently missing item prices was set within the context of an elementary aggregate, where weights were neither available nor used. A two-stage Lowe price index number was used, though not explicit. We provide its formulas below and in more detail in chapter 7.

The recommended procedure in practical higher-level index number compilation is instead of basing the aggregation on long-run price changes, compiled in a single stage, a two-stage or "modified" formulation is used.

For example, consider a Lowe index in which, for each elementary aggregate the say 2018 weights are price-updated to price reference period of 2019=100.0 as outlined in chapter 9, given here by w_i^{2019*}. The January 2020 price index for each elementary aggregate i has, for example, January's 2020 to 2019 mean monthly price (=100.00) is multiplied by its weight, as shown by the first term below. The CPI is the sum of the weights times the associated price changes. In February, we start the two-stage procedure: first for each elementary aggregate i its weight is multiplied by the preceding month's price change (for 2012 to January 2013) taken from the previous

$$\sum_i w_i^{2019*} \frac{p_i^{Jan2020}}{p_i^{2019}}$$

$$\sum_i \left[w_i^{2019*} \frac{p_i^{Jan2019}}{p_i^{2019}} \right] \frac{p_i^{Feb2019}}{p_i^{Jan2019}} = \sum_i \left[w_i^{2019*} \frac{p_i^{Feb2020}}{p_i^{2019}} \right]$$

$$\sum_i \left[w_i^{2019*} \frac{p_i^{Jan2020}}{p_i^{2019}} \times \frac{p_i^{Feb2020}}{p_i^{Jan2020}} \right] \left(\frac{p_i^{Mar2013}}{p_i^{Feb2013}} \right) = \sum_i \left[w_i^{2008:10*} \frac{p_i^{Mar2013}}{p_i^{2012}} \right]$$

$$\sum_i \left[w_i^{2018*} \frac{p_i^{Jan2013}}{p_i^{2012}} \times \frac{p_i^{Feb2013}}{p_i^{Jan2013}} \frac{p_i^{Mar2013}}{p_i^{Feb2013}} \right] \left(\frac{p_i^{Apr2013}}{p_i^{Mar2013}} \right) = \sum_i \left[w_i^{2008:10*} \frac{p_i^{Apr2013}}{p_i^{2012}} \right]$$

…………

$$\sum_i \left[w_i^{2018*} \frac{p_i^{t-1}}{p_i^{2012}} \right] \frac{p_i^{t}}{p_i^{t-1}}$$

period, to form an "uprated weight." Second, the price change between the current month and its preceding month, January to February, is calculated and multiplied by the previous period's up-rated weight, to form a new uprated weight. For March, the procedure is repeated: the "new" up-rated weight is taken from the previous period and multiplied by the February:March price change. In each month, all we need is the preceding period's up-rated weights, readily available from last month's calculation, and the current to preceding month's price change. As can also be seen, the mean prices for each *i* in successive numerators cancel with those in the denominators to show this is equivalent to a direct long-run price index. This seeming equivalence masks two major advantage of the formula.

First, each S-T price relative for an elementary aggregate i, say in April 2020, $\frac{p_i^{Apr\,2020}}{p_i^{Mar\,2020}}$, are (geometric) mean prices in April 2020 compared with geometric mean prices in March of matched items. Table 6.6 might illustrate an individual elementary aggregate with geometric mean prices compiled from several outlets. If an item is temporarily no longer available an imputation can be based on S-T month-on-month price relative, rather than a L-T price that might assume similar price movements over several years.

Similarly, for permanently missing prices where imputations are used to form an overlap comparison for the missing item price and its replacement, assumptions based on similar S-T month-on-month price movements are more reasonable than the much less plausible ones based on L-T price movements. This mechanism facilitates the inclusion of new specifications when old specifications become obsolete and enables the index to better represent the dynamic changes taking place in consumer choice. A direct comparison between the price of a new replacement item specification in April 2020 with its old specification in 2019 is likely to be fraught with difficulties given the quality differences between the two item specifications over a long period.

The two-stage Lowe will differ—be improved—from its fixed base (L-T) counterpart in equation (6.) since the monthly imputations on the left hand side will differ to those on the right.

Second, the use of $\frac{p_i^t}{p_i^{t-1}}$ in the compilation facilitates data verification since outliers in short-run changes are more readily identifiable than those in long-run ones. In practice, a computer routine for an index need only maintain as active files the previous month's up-rated weights and the previous and current month's prices and price change.

This chapter's work has for the large part been concerned with S-T price relatives compiled within an elementary aggregate, *i*. The larger picture of weighted aggregation across elementary aggregates is for this context of missing prices and sample representativity, considered in chapters 7, with illustrative calculations of the aggregation formulas in chapter 8, and the introduction of new weights in chapter 9.

Recommendations:

There are a good number of prescriptive recommendations to be drawn from this chapter, such as

- Adopt a modified (two-stage) calculation procedure for the Lowe index to facilitate the introduction of replacement item specifications/items.

Annex 1 Overall mean (or targeted) imputation

Consider $i = 1 \ldots m$ items where, as before, p_m^t is the price of item *m* in period *t*, p_n^{t+1} is the price of a replacement item *n* in period *t* + 1. Now *n* replaces *m,* but is of a different quality. There are (*m*-1) matched prices and a single replacement price such that *m*=(*m*-1)+1 Let *A*(*z*) be the quality adjustment to p_n^{t+1} which equates its quality services or utility to p_m^{t+1} such that the quality-adjusted price $p_m^{*t+1} = A(z)\, p_n^{t+1}$. For the imputation method to work, the average price changes of the *i*=1….*m* items, including the quality-adjusted price p_m^{*t+1}, given on the left-hand side of equation (A6.1.1), must equal the average price change from just using the overall mean of the rest of the *i*=1….*m*-1 items, on the right-hand side of equation (A6.1.1). The discrepancy or bias from the method is the balancing term *Q*. It is the implicit adjustment that allows the method to work. The arithmetic formulation is given here, though a similar geometric one can be readily formulated. The equation for one unavailable item is given by:

$$\text{(A6.1.1)}\quad \frac{1}{m}\left[\frac{p_m^{*t+1}}{p_m^t} + \sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}\right] = \left[\frac{1}{(m-1)}\sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}\right] + Q$$

$$Q = \frac{1}{m}\frac{p_m^{*t+1}}{p_m^t} - \frac{1}{m(m-1)}\sum_{i=1}^{m-1}\frac{p_i^{t+1}}{p_i^t}$$

and for *x* unavailable items by:

$$\text{(A6.1.2)}\quad Q = \frac{1}{m}\sum_{i=m-x+1}^{m}\frac{p_m^{*t+1}}{p_m^t} - \frac{x}{m(m-x)}\sum_{i=1}^{m-x}\frac{p_i^{t+1}}{p_i^t}$$

The relationships are readily visualized if r_1 is defined as the arithmetic mean of price changes of items that continue to be recorded and r_2 of quality-adjusted unavailable items. For the arithmetic case, where

$$\text{(A6.1.3)}\qquad r_1 = \left[\sum_{i=1}^{m-x} p_i^{t+1}/p_i^t\right] \div (m-x) \text{ and } r_2 = \left[\sum_{i=m-x+1}^{m} p_i^{*t+1}/p_i^t\right] \div x$$

then the bias of arithmetic mean of ratios from substituting equations (A6.1.3)in (A6.1.2) is:

$$\text{(A6.1.4)}\quad Q = \frac{x}{m}(r_2 - r_1)$$

which equals zero when $r_1 = r_2$. The bias depends on the ratio of unavailable values and the difference between the mean of price changes for existing items and the mean of quality-adjusted replacement price changes. The bias decreases as either (x/m) or the difference between r_1 and r_2 decreases. Furthermore, the method is reliant on a comparison between price changes for existing items and quality-adjusted price changes for the replacement or unavailable comparison. This is more likely to be justified than a comparison without the quality adjustment to prices. For example, suppose there were *m* = 3 items, each with a price of 100 in period *t*. Let the *t* + 1 prices be 120 for two items, but assume the third, i.e., *x* = 1, is unavailable and is replaced by an item with a price of 140, of which 20 is attributable to quality differences. Then the arithmetic bias as given in equations (A6.1.3) and (A6.1.4), where $x = 1$ and *m* = 3, is

$$\text{(A6.1.5)}\quad \frac{1}{3}\left[(-20+140)/100 \;-\; \left(\frac{120}{100}+\frac{120}{100}\right)/2 = 0\right]$$

Had the bias depended on the unadjusted price of 140 compared with 100, the imputation would be prone to serious error. In this calculation, the direction of the bias is given by $(r_2 - r_1)$ and does not depend on whether quality is improving or deteriorating, in other words whether $A(z)<1$ or $A(z)>1$. If $A(z)<1$, a quality improvement, it is still possible that $r_2 < r_1$ and for the bias to be negative.

The analysis here is framed in terms of a short-term price change framework. That is, the short-term price changes between the prices in a period and those in the preceding period are used for the imputation. This is different from the long-term imputation where a base period price is compared with prices in subsequent months, and where the implicit assumptions are more restrictive.

Table A6.1.1 provides an illustration in which the (mean) price change of items that continue to exist, r_1, is allowed to vary for values between 1.00 and 1.5 – corresponding to a variation between no price change and a 50 per cent increase. The (mean) price change of the quality-adjusted new items compared with the items they are replacing is assumed not to change, i.e., r_2 = 1.00. The bias is given for ratios of missing values of 0.01, 0.05, 0.1, 0.25 and 0.5, both for arithmetic means and geometric means. For example, if 50 per cent of price quotes are missing and the missing quality-adjusted prices do not change, but the prices of existing items increase by 5 per cent (r_1=1.05), then the bias for the geometric mean is represented by the proportional factor 0.9759; i.e., instead of 1.05, the index would be 0.9759 x 1.05 = 1.0247. For an arithmetic mean, the bias is –0.025; instead of 1.05 it should be 1.025.

Equation (A6.1.4) shows that the ratio x/m and the difference between r_1 and r_2 determine the bias. Table A6.1.1 shows that the bias can be quite substantial when x/m is relatively large. For example, for x/m = 0.25, an inflation rate of 5 per cent for existing items translates to an index change of 3.73 per cent and 3.75 per cent for the geometric and arithmetic formulations, respectively, when r_2 = 1.00, i.e., when quality-adjusted prices of unavailable items are constant. Instead of being 1.0373 or 1.0375, ignoring the unavailable items would give a result of 1.05. Even with 10 per cent missing (x/m = 0.1), an inflation rate of 5 per cent for existing items translates to 4.45 per cent and 4.5 per cent for the geometric and arithmetic formulations, respectively, when r_2 = 1.00. Considering a fairly low ratio of x/m, say 0.05, then even when r_2 = 1.00 and r_1 = 1.20, Table A6.1.1 shows that the corrected rates of inflation should be 18.9 per cent and 19 per cent for the geometric and arithmetic formulations, respectively. In competitive markets, r_1 and r_2 are unlikely to differ by substantial amounts since r_2 is a price comparison between the new item and the old item after adjusting for quality differences. If r_1 and r_2 are the same, then there would be no bias from the method even if x/m = 0.9. There may, however, be more sampling error. It should be borne in mind that it is not appropriate to compare bias between the arithmetic and geometric means, at least in the form they take in Table A6.1.1 The latter would have a lower mean, rendering comparisons of bias meaningless.

Table A6.1.1 Example of the bias from implicit quality adjustment when the (mean) price change of quality-adjusted new items compared with the items they are replacing is assumed not to change (r_2=1.00)

	Geometric mean					Arithmetic mean				
	Ratio of missing items, *x/m*					Ratio of missing items, *x/m*				
	0.01	0.05	0.1	0.25	0.5	0.01	0.05	0.1	0.25	0.5
r_1										
1	1	1	1	1	1	0	0	0	0	0
1.01	0.999901	0.999503	0.999005	0.997516	0.995037	-0.0001	-0.0005	-0.001	-0.0025	-0.005

1.02	0.999802	0.99901	0.998022	0.995062	0.990148	-0.0002	-0.001	-0.002	-0.005	-0.01
1.03	0.999704	0.998523	0.997048	0.992638	0.985329	-0.0003	-0.0015	-0.003	-0.0075	-0.015
1.04	0.999608	0.998041	0.996086	0.990243	0.980581	-0.0004	-0.002	-0.004	-0.01	-0.02
1.05	0.999512	0.997563	0.995133	0.987877	0.9759	-0.0005	-0.0025	-0.005	-0.0125	-0.025
1.1	0.999047	0.995246	0.990514	0.976454	0.953463	-0.001	-0.005	-0.01	-0.025	-0.05
1.15	0.998603	0.993036	0.986121	0.965663	0.932505	-0.0015	-0.0075	-0.015	-0.0375	-0.075
1.2	0.998178	0.990925	0.981933	0.955443	0.912871	-0.002	-0.01	-0.02	-0.05	-0.1
1.3	0.99738	0.986967	0.974105	0.936514	0.877058	-0.003	-0.015	-0.03	-0.075	-0.15
1.5	0.995954	0.979931	0.960265	0.903602	0.816497	-0.005	-0.025	-0.05	-0.125	-0.25

r_1 =(mean) price change for items that continue to exist.

An awareness of the market conditions relating to the commodities concerned is instructive in understanding likely differences between r_1 and r_2. The concern here is when prices vary over the life cycle of the items. Thus, for example, at the introduction of a new model, the price change may be quite different from price changes of other existing items. Thus assumptions of similar price changes, even with quality adjustment, might be inappropriate. For example, if new computers enter the market at prices equal to, or lower than, prices of previous models, but with greater speed and capability, an assumption that $r_1 = r_2$ could not be justified. Or if new clothing enters the market at relatively high quality-adjusted prices, while old, end-of-season or out-of-style clothes are being discounted. Again there will be bias, as r_1 differs from r_2.

Some of these differences arise because markets are composed of different segments of consumers. Indeed, the very training of consumer marketers involves consideration of developing different market segments and ascribing to each appropriate pricing, product quality, promotion and place (method of distribution) – the 4Ps of the marketing mix as taught in introductory marketing. In addition, consumer marketers are taught to plan the marketing mix for the life cycle of items. Such planning allows for different inputs of each of these marketing mix variables at different points in the life cycle. This includes "price skimming" during the period of introduction, when higher prices are charged to skim off the surplus from segments of consumers willing to pay more. The economic theory of price discrimination would also predict such behaviour. Thus the quality-adjusted price change of an old item compared with a new replacement item may be higher than price changes of other items in the product group. After the introduction of the new item its prices may fall relative to others in the group. There may be no law of one price change for differentiated items within a market..

There is thus little in economic or marketing theory to support any expectation of similar (quality-adjusted) price changes for new and replacement items, as compared to other items in the product group. Some knowledge of the realities of the particular market under study would be helpful when considering the suitability of this approach. Two aspects need to be considered in any decision to use the imputation approach. The first is the proportion of replacements; Table A6.1.1 provides guidance here. The second is the expected difference between r_1 and r_2. It is clear from the above discussion that there are markets in which they are unlikely to be similar. This is not to say the method should not be used. It is a simple and expedient approach. What arguably should not happen is that it is used by default, without any prior evaluation of expected price changes and the timing of the switch. Furthermore, its use should be targeted, by selecting items expected to have similar price changes. The selection of such items, however, should take account of the need to include a sufficiently large sample so that the estimate is not subject to undue sampling error.

The manner in which these calculations are undertaken is also worth considering. In its simplest form, the pro forma setting for the calculations, say on a spreadsheet, would usually have each item description and its prices recorded on a monthly basis. The imputed prices of the missing items are inserted into the spreadsheet, and are highlighted to show that they are imputed. The need to highlight such prices is, first, because they should not be used in subsequent imputations as if they were actual prices. Second, the inclusion of imputed values may give a false impression of a larger sample size than actually exists. Care should be taken in any audit of the number of prices used in the compilation of the index to code such observations as “imputed”.

The method described above is an illustration of a short-term imputation. As is discussed in chapter 8, there is a strong case for using short-term imputations as against long-term ones.

Annex 2 Quality adjustment using a replacement and price overlap

Consider $i = 1 \ldots m$ items where p_m^t is the price of item *m* in period *t*, p_n^{t+1} is the price of a replacement item *n* in period *t* + 1; *n* replaces *m,* but is of a different quality. Let there be overlap prices for *m* and *n* in period *t* and let $A(z^{t+1})$ be the quality adjustment to p_n^{t+1} which equates its quality to p_m^{t+1} such that the quality-adjusted price $p_m^{*t+1} = A(z^{t+1})p_n^{t+1}$. The index for the item in question over the period *t*-1 to *t* + 1 is: Now the quality adjustment to prices in period *t* + 1 is defined $p_m^{*t+1} = A(z^{t+1})p_n^{t+1}$ which is the adjustment to p_n in period *t* + 1 which equates it to p_m in period *t* + 1 (had it existed then).

A desired measure of price changes between periods *t*-1 and *t* + 1 is thus:

$$\text{(A6.2.1)}\ \left(p_m^{*t+1} / p_m^{t-1}\right)$$

The overlap formulation equals this when:

$$\text{(A6.2.2)}\ \frac{p^{*t+1}}{p_m^{t-1}} = A(z^{t+1})\frac{p_n^{t+1}}{p_m^{t-1}} = \frac{p_n^{t+1}}{p_n^t} \times \frac{p_m^t}{p_m^{t-1}}$$

$A(z^{t+1}) = \dfrac{p_m^t}{p_n^t}$ and similarly for future periods of the series

$A(z^{t+i}) = \dfrac{p_m^t}{p_n^t}$ for $\dfrac{p_m^{*t+i}}{p_m^{t-1}}$ for $i = 2, \ldots T$

But what if the assumption does not hold? What if the relative prices in period *t*, $R^t = p_m^t / p_n^t$, do not equal $A(z^t)$ in some future period, say $A(z^{t+i}) = \alpha_i R^t$? If $\alpha_i = \alpha$, the comparisons of prices between future successive periods, say between *t* + 3 and *t* + 4, are unaffected, as would be expected, since item *n* is effectively being compared with itself,

$$\text{(A6.2.3)}\ \frac{p_m^{*t+4}}{p_m^{t-1}} / \frac{p_m^{*t+3}}{p_m^{t-1}} = \frac{\alpha R^t}{\alpha R^t}\frac{p_n^{t+4}}{p_n^{t+3}} = \frac{p_n^{t+4}}{p_n^{t+3}}$$

However, if differences in the relative prices of the old and replacement items vary over time, then:

$$\text{(A6.2.4)}\ \frac{p_m^{*t+4}}{p_m^{t-1}} / \frac{p_m^{*t+3}}{p_m^{t-1}} = \frac{\alpha_4}{\alpha_3}\frac{p_n^{t+4}}{p_n^{t+3}}$$

Note that the quality difference here is not related to the technical specifications or resource costs, but to the relative prices consumers pay.

Annex 3: The nature and extent of the index number bias if only matched items are used?

Section x (The difference between hedonic indices and matched indices) showed that sample degradation and differences in the (quality- adjusted) prices of unmatched new, unmatched old and matched models can lead to bias in matched models price indexes. The nature and extent of such bias depends on the frequency with which manufacturers turn over their models and the pricing strategy retailers employ over the life cycle of the models. If, say, the quality-adjusted prices of new unmatched models in period t=2 are higher than their matched counterparts in period 2, and if the quality-adjusted prices of old unmatched models in period 1 are lower than their matched counterparts in period 1, then there will be a *larger* fall in the matched models index between periods 1 and 2 compared with a hedonic index that uses all of the data. Similarly, if the quality-adjusted prices of unmatched new models, are *below* matched ones in period 2, and the quality-adjusted prices of old models *above* matched ones in period 1, there will be a *smaller* fall in the matched models index compared with a hedonic index that uses all of the data. The nature and extent of the matched models index bias thus depends on the pricing strategy adopted for new and old models. Indeed if hedonic-adjusted prices are consistently above or consistently below average prices for unmatched new *and* old models some of the bias will cancel.

The case for old unmatched models having below average quality-adjusted prices is based on an inventory-clearing argument. For an old model near or at the end of its life cycle retailers want to clear out the remaining inventory from both their warehouses and store shelves so they have room to stock and display the replacement model. They do not wish the old model to cannabalize some of the sales of the new model which may well have a higher price (profit) margin. The extent of any such cannibalization will depend on the cross-price elasticities between the new and old models. This inventory clearing argument is noted in quadrant IV of Figure A6.3.1.

New and old models may coexist for some time and the case for existing models having their *postentry* prices *increased* following the introduction of a new model is of interest. In principle, our focus is on unmatched old items no longer available for the matched price comparison, and hence they have no postentry prices. However, the logic behind such postentry pricing applies to a pricing strategy for a multi-product firm that anticipates the introduction of a new model. A *multi-product* monopolist can increase the prices of existing models because some of the demand for existing models that would usually be lost to competitors, due to the price increase, will now not go to the competitor's products (an assumption upon which the existing prices were set) but will go instead to the firm's new model. The new model will cannalibalize some of the existing model's sales that would otherwise be lost due to the price increase in the existing model (Figure A6.3.1, quadrant II). However, it has been argued that any such effect may be outweighed by the need to cut the prices of the existing models to prevent the existing model's sales cannibalizing sales of the new, more profitable, model (Figure A6.3.1, quadrant IV). Old, branded, pharmaceutical drugs can increase after the expiration of a patent and introduction of new generic models. This is because of price discrimination with some market segments remaining with particularly strong preferences for the old models willing to pay higher prices (Figure A6.3.1, quadrant IV). A study of computer processors and disk drives, found that with the introduction of products embodying new the prices for older products decline rapidly to permit an older technology to compete with a newer one for a limited time, but the old technology is eventually driven out..

New models may have *above* average quality-adjusted prices in their period of introduction because firms 'price-skim' market segments willing to pay a premium for the new model over and above that due to its improved quality (Figure A6.3.1, quadrant I). Indeed marketing texts advocate price-skimming as one of two 'new product' pricing strategies. The alternative strategy is 'market-penetration' pricing for which a low initial price is set for a new model to attract a large number of buyers quickly to win market share and take advantage of falling costs due to scale economies. Such pricing may initially be possible because the new model is based on new, lower cost components that can provide a feature set that is comparable to existing models, but at a lower price point. In either event quality-adjusted prices of new models may have *below* average prices (Figure A6.3.1, quadrant III).

Figure A6.3.1 summarizes these positions. Consider the right-hand side of equation(15). The first summation is for matched models. The second summation is for new models $m \in S(2\neg 1)$ which has a positive sign and potential upwards bias to matched models index. The third summation is for old models $m \in S(1\neg 2)$ which have a potential negative, downward bias. The combination of above average prices for new models in I and below average prices for old models in III leads to an overall net upwards bias. Similarly pricing in quadrants II and III lead to match models indexes which are biased downwards. However, pricing in quadrants I and II lead to an indeterminate bias, with countervailing positive bias from the new above average priced models and negative bias from old average priced models. The bias from pricing in quadrants III and IV is also indeterminate, positive bias from the new above average priced old models and negative bias from the old below average priced models.

It is possible to say something about the likely pricing strategies of different products. Consider the case of digital cameras compared to film-based cameras. Given the current differential in product costs for the two technologies, and where the two categories are in their respective life-cycles, we can speculate that relatively greater effort is likely to be placed on R&D in new models which reduce unit costs for digital cameras compared to R&D in film-based camera models that reduce unit costs. Products in a mature stage of their category life cycle, where R&D development is relatively small and product enhancing, as opposed to cost reducing, may be more likely to have above average quality-adjusted prices for new models.

The nature and the extent of bias from using matched models is dictated by the pricing and production strategies of the retailers and manufacturers.

Figure A6.1.3: Matched models price index bias and pricing strategies

	New unmatched models	**Old** unmatched models	Pricing	Matched models price index bias
Quality-adjusted prices **above** matched models prices	Market skimming [*quadrant I*]	Multiproduct monopoly pricing strategy; price discrimination to segments with sticky downwards pricing; old technologies reduce prices. [*quadrant* II]	I and IV II and III	Upwards downwards
Quality-adjusted prices **below** matched models prices	Market penetration pricing; low unit costs, new producing technology [*quadrant III*]	Inventory clearing. [*quadrant* IV]	I and II III and IV	Countervailing Countervailing