Dominant Currency Debt *

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Abstract

We propose a "debt view" to explain the dominant international role of the dollar and provide broad empirical support for it. Within a simple capital structure model in which firms optimally choose the currency composition of their debt, we derive conditions under which all firms issue debt in a single, "dominant" currency. Theoretically, it is the currency that depreciates in global downturns over horizons of typical corporate debt maturity. Both forward-looking and historical covariances suggest that dollar fits this description better than all major currencies, especially for longer horizons. The debt view can jointly explain the fall and the rise of the dollar in international debt markets over the last two decades.

Keywords: Dollar debt, dominant currency, exchange rates, inflation
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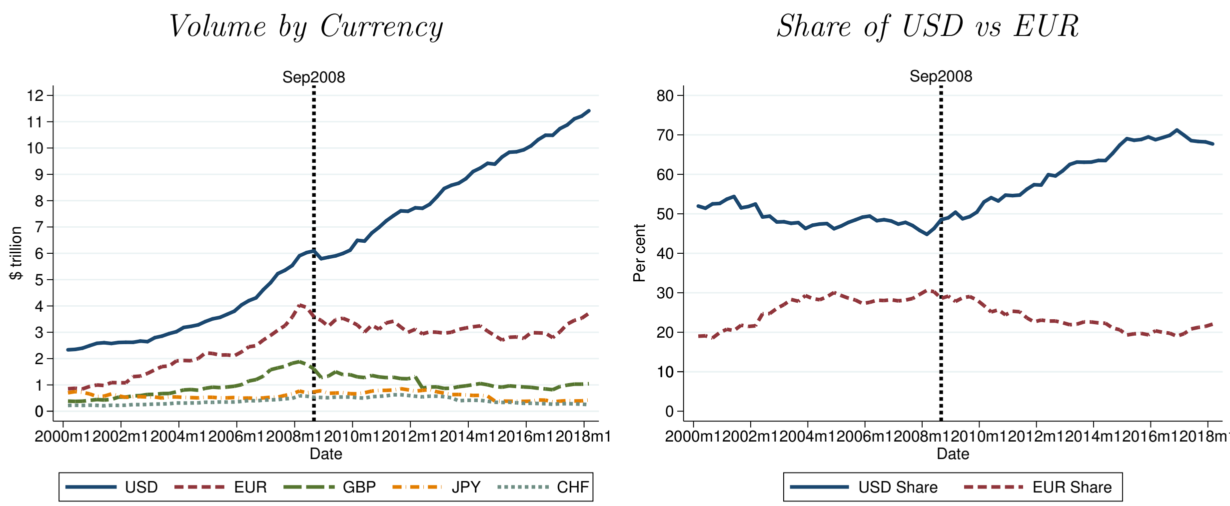
The dollar is the most common currency of choice for debt contracts worldwide. According to the Bank for International Settlements, the dollar-denominated credit to non-banks outside the United States amounts to around \$11.5 trillion. While the dominance of the dollar had declined prior to 2008, the dollar has strengthened its international role since the Global Financial Crisis (Figure 1).[1]

Figure 1: Currency Denomination of International Debt and Cross-Border Borrowings of Non-Banks (Amounts Outstanding)



Source: Bank for International Settlements

In this paper, we study how a single currency can become the most common currency of choice for denominating debt contracts, i.e. *the dominant currency*, why that choice is the dollar, and why the dominance of the dollar may have declined and recovered in the last two decades. To fix ideas, in this paper our focus is not to answer why emerging market firms issue debt in dollars as opposed to local currency. Instead, our main focus is on why large, global firms issue debt in dollars as opposed to other major safe haven currencies, such as the euro or the yen.

[1] Similar patterns were previously documented for debt issuance (see, for example, ECB (2017) and Maggiori, Neiman and Schreger (2018)), and for global cross-border bond holdings (Maggiori, Neiman and Schreger (2018)), as well as for other parts of the global financial system (Maggiori, Neiman and Schreger (2019)).

According to the conventional view, debt issuance in dollars is investor driven. Investors prefer holding safe assets that tend to appreciate in bad times. Therefore, firms choose currency denomination of their debt to cater to investors' demand. There are three major challenges to this view. First, we show empirically in this paper that the dollar is not the "safest" among the major currencies, such as the euro, the Japanese yen or the Swiss franc. Second, nominal interest rates in dollars are higher than those in these other major currencies. Third, the dollar increased its international role after the Bretton Woods, even as it depreciated considerably against other major currencies in 1970s (Gourinchas (2019)).

Motivated by these, we propose the debt view, in which debt insurance in dollars is borrower driven. In our model, firms finance themselves by issuing equity and nominal, defaultable debt to optimize the trade-off between tax benefits of debt and the risk of default.[2] Debt can potentially be issued in any currency and firms issue in dollars if dollar debt maximizes the trade-off. Our main theoretical result is that, independent of the investors' preferences, firms always issue debt in the most "CAPM-risky" currency. It is the currency that, controlling for issuance costs, has the highest covariance with the stock market *over the horizons of debt maturity*. We call it the "dominant" currency. If investors' marginal utility co-moves negatively with the stock market, such debt is unattractive for debt-holders and yet firms still prefer issuing in this currency.

These features of the debt view have two implications and can explain the major challenges to the conventional view. First, dollar debt represents a good hedge for firms against economic downturns, making it easier to repay at times of distress. Second, the currency in which firms prefer issuing debt should have a higher risk premium. As a result, the dollar is the dominant currency for denominating debt, not despite being the riskiest of the major currencies, but precisely because of it. Higher associated risk premium with it leads to higher

[2] We think of our model as being best applicable to large international firms deciding whether to issue debt in one of the major international currencies with comparable market liquidity and issuance costs. Nikolov, Schmid and Steri (2018) show that tradeoff theory efficiently explains capital structure dynamics for large firms. By contrast, other theories need to be developed for smaller firms facing a high degree of informational asymmetry.

nominal dollar interest rates. Finally, it is possible that dollar depreciation after the Bretton Woods incentivized firms to borrow in dollars cementing its dominant international role.

Empirically, we test our prediction that the dollar is the "CAPM-riskiest" among the major international currencies. The first prediction of our model is that the dominant role of the dollar in international debt markets might be attributed to the expectations of market participants of a positive co-movement of the dollar with the stock market *over the horizons of debt maturity of financial and non-financial firms*, which is typically around 5-7 years.[3] We show empirically that this is indeed the case in two ways. First, we compute the historical covariances between the dollar and global stock markets. Second, we use asset-price implied forward looking expectations of market participants regarding the covariance of the euro/dollar exchange rate and the S&P 500, directly computed from so-called quanto contracts (Kremens and Martin, 2019).

Historically, we find that the dollar co-moves positively with the stock market at horizons that typically accord with corporate debt maturity. This pattern does not contradict the well documented tendency of the dollar to appreciate in bad times over shorter horizons (see for example, Gourinchas, Govillot and Rey (2017)). We, indeed, find that the dollar co-moves negatively with the stock market for shorter horizons up to a year, but the sign of the covariance switches for longer horizons which is the more relevant horizon given than typical debt maturities are longer.

To gain a deeper understanding of this pattern, we further decompose the long-horizon covariances into shorter-term contemporaneous and lead-lag relationships between the dollar and the stock market, proxied by S&P 500. While the contemporaneous covariance is negative, we find that S&P 500 robustly predicts the dollar at all horizons (both short and long), and, for longer horizons, this lead-lag relationship is so strong that it overturns the negative contemporaneous covariance.

This pattern of the covariance structure between the dollar and the stock market over dif-

[3]See section 4 and also Cortina, Didier and Schmukler (2018) for typical corporate debt maturities.

ferent horizons has direct implications for the maturity choices of firms for dollar-denominated debt contracts. As the dollar co-movement with the stock market increases over longer horizons, our model predicts that the propensity to issue dollar-denominated debt increases with debt maturity. We use granular bond issuance data to formally test this prediction, and find strong support.

A direct way of computing the forward-looking covariance between the stock market and exchange rates is by using so-called quanto forward contracts (Kremens and Martin, 2019). A euro-quanto forward contract for S&P 500, for example, pays off the level of the S&P 500 index *in euros* when the contract matures. As opposed to a contract that pays off the S&P 500 in dollars, the value of this contract depends on the anticipated covariance between the index and the EUR/USD exchange rate. Hence, the price of this contract reflects the expectations of investors about currency returns and currency risk premia. Kremens and Martin (2019) compute the quanto-implied covariance for contracts with two-year maturity and find that the quanto-implied covariance of the EUR/USD exchange rate with S&P500 exhibited a very strong downward trend in the post-crisis period, and has become negative in the recent years. A negative quanto-implied risk premium (QRP) means that market participants believe that the euro will *appreciate* against the dollar when the S&P 500 falls, corroborating our results based on backward-looking covariances.

Our theoretical characterization of the dominant currency also has direct implications for the time series dynamics of the shares of dollar- and euro-denominated debt. Namely, keeping the distribution of issuance costs constant across firms, our model predicts the share of dollar denominated debt relative to that of euro-denominated debt is negatively related to the QRP. Consistent with the predictions of our model, we find a strong negative relationship between quanto-implied covariances and the share of dollar debt, suggesting that forward looking expectations of currency returns are an important driver of firms' debt currency denomination choice. Moreover, we interpret this fact as strong evidence of a distinctive prediction of our theory, that is, changes to the currency composition of debt can occur in high frequency and are related to forward-looking expectations since our regressions are at a quarterly frequency.

The debt view also assigns an important role for monetary policy if relative inflation between two countries is an important driver of exchange rates.[4] Similar to the predictions regarding the QRP, we predict that the share of dollar denominated debt relative to that denominated in euro is positively related to the inflation risk premium of the dollar, and negatively related to that of the euro. In particular, we find that debt currency shares move more tightly with the expectations about risk premia in the Eurozone and that explains debt issuance patterns over the last two decades.[5]

Related literature. The international role of the dollar has received a lot of attention in the recent literature. The dollar is omnipresent in all parts of the global financial system (Gopinath and Stein (2018) and Gourinchas, Rey and Sauzet (2019)). This includes international trade invoicing (see Goldberg and Tille (2008), Gopinath (2015), Casas, Díez, Gopinath and Gourinchas (2017)); global banking (Shin (2012), Ivashina, Scharfstein and Stein (2015), Aldasoro, Ehlers and Eren (2018)); corporate borrowing (Bruno, Kim and Shin (2018), Bruno and Shin (2017), and Giovanni, Kalemli-Ozcan, Ulu and Baskaya (2017)); central bank reserve holdings (Bocola and Lorenzoni (2018) and Ilzetzki, Reinhart and Rogoff (2019)); and global portfolios (Maggiori, Neiman and Schreger (2018)). Our paper adds to the growing literature that studies the international role of the dollar.[6]

[4]See, for example, Imbs, Mumtaz, Ravn and Rey (2005), Chowdhry, Roll and Xia (2005) for evidence in favor of the relative PPP, as well as Chernov and Creal (2019) who argue that PPP is an important driver of long-horizon currency risk premia.

[5]The importance of accommodative monetary policy in helping reduce real debt burdens of firms and the differences across central banks in accomplishing this goal is also acknowledged by the European Central Bank (ECB). See, for example, Praet (2016) and Cœuré (2019).

[6]For example, Matsuyama, Kiyotaki and Matsui (1993), Rey (2001), Caballero and Krishnamurthy (2002), Caballero, Farhi and Gourinchas (2008), Caballero and Krishnamurthy (2009), Devereux and Shi (2013), Chahrour and Valchev (2017), Mukhin (2017), Farhi and Maggiori (2018), He, Krishnamurthy and Milbradt (2019), Bocola and Lorenzoni (2017) and Drenik, Kirpalani and Perez (2018).

Our main contribution is the introduction of the "debt view" in explaining the international role of the dollar. Current explanations can be broadly classified into three categories. First is the "trade view," wherein trade invoicing in dollars is the reason for the dollar's role in the global economy (see, for example, Gopinath and Stein (2018)). Second is the "safe asset view," in which the dollar is dominant because of its safe haven properties (see, for example, He, Krishnamurthy and Milbradt (2019), Farhi and Maggiori (2018), and Jiang, Krishnamurthy and Lustig (2018)) and the global demand for safe assets (Caballero, Farhi and Gourinchas (2008), Caballero, Farhi and Gourinchas (2015), Caballero, Farhi and Gourinchas (2017)). Third is the "vehicle currency view," wherein the dominance of the dollar arises from its international use as a vehicle currency (see for example Goldberg and Tille (2008)).

The debt view of the dollar's dominance assigns an important role to the choice of debt currency denomination of firms, driven by *forward looking expectations about exchange rates and monetary policy*.[7] The debt view focuses on the medium run to account for typical corporate debt maturity, and in that complements other theories which focus on the short run frictions such as price stickiness, or the short-run appreciation of the dollar in bad times as an insurance to investors. In contrast to other theories, we show that a dominant currency equilibrium in the corporate debt market can arise without relying on network effects, price stickiness, pricing complementarities and safety demand.

Three closely related papers to ours are by Gopinath and Stein (2018), Jiang, Krishnamurthy and Lustig (2018), and Liao (2019). Gopinath and Stein (2018) demonstrate how the dollar can emerge as a key international currency starting from its role in trade invoicing and in turn affecting global banking, which in turn affects currency denomination of bank deposits and firm borrowing endogenously. While their main focus is emerging markets and bank-intermediated debt, our results apply mostly for the currency choice of large, global

[7] The main mechanism in our model is similar to the one in Gomes, Jermann and Schmid (2016); however, in an international setting.

firms and applies also to market-based financing, and dollar's dominant role arises due to its risk properties. Jiang, Krishnamurthy and Lustig (2018) find that investors attach a convenience yield for dollar safe assets that can be observed from covered interested parity deviations. An implication of their results is that firms would issue dollar debt to reap the benefits of this convenience yield. Liao (2019) also argues that firms issue dollar debt because of covered interest parity deviations. Our results complement the findings of Gopinath and Stein (2018) and Jiang, Krishnamurthy and Lustig (2018). We find that firms choose the dollar as opposed to other currencies regardless of the preferences of investors and issuance of dollar debt could be determined by its favorable risk properties compared to other major currencies, such as the euro, the Japanese yen or the Swiss franc.

Another emerging strand of literature that is close to our paper in spirit is on the assessment of the safe haven status of US Treasuries. Motivated by the puzzling post-crisis spread between US Treasury yields and risk-free rates implied by the overnight index swaps (OIS), i.e. the negative swap spread, Klinger and Sunderesan (2019) find evidence that during the post-crisis period, US Treasuries have lost their safe haven status compared to German sovereign debt. Augustin, Chernov, Schmid and Song (2019) explain the negative swap spread by the increased riskiness of US Treasuries. Our results are similar in spirit, since we argue that, over longer horizons, the dollar is the riskiest of the major safe haven currencies and that is precisely why it emerges as the dominant currency.

Our paper is also related to the large literature on long-term nominal debt and its real effects, including debt deflation (Fisher (1933)), debt overhang (Myers (1977)) and leverage dynamics (Gomes, Jermann and Schmid (2016)).

1 Theory

Time is discrete, indexed by $t = 0, 1, \cdots$. A large, international firm is infinitely lived and generates cash flows (after tax revenues) of $\Omega_t Z_t$ where Ω_t is the common productivity shock and Z_t is an idiosyncratic shock. We assume that Z_t follows a geometric random walk, $Z_{t+1} = Y_{t+1}Z_t$ where Y_t are i.i.d. and have a density $\ell y^{\ell-1}$, $y \in [0,1]$ and $\ell > 0$. We denote by $\Phi(y) \equiv y^\ell$ the cumulative distribution function of idiosyncratic shocks. All cash flows are priced with a common, exogenously given *dollar* stochastic discount factor $M_{t,t+1}$.

Firms finance themselves by issuing both equity and defaultable nominal bonds in *any* of the N currencies, maturing in one time period.[8] Each bond has a nominal face value of one currency unit, and the firm is required to pay a coupon of c currency units per unit of outstanding debt.[9] We denote by $B_{j,t}$ the stock of outstanding nominal debt at time t denominated in the currency of country j. We also denote by $B_t = (B_{j,t})_{j=1}^N$ the vector of debt stocks in different currencies. As in Gomes, Jermann and Schmid (2016), we assume that coupon payments are shielded from taxes, so that

$$\mathcal{B}_{t+1}(B_t) = ((1-\tau)c+1)\sum_{j=1}^N \mathcal{E}_{j,t+1}B_{j,t}$$

is the total debt servicing cost, net of tax shields. The choice of firm leverage, therefore, depends on the trade-off between tax advantages and the distress costs.[10] Thus, absent default, the nominal distribution to shareholders at time $t+1$ is given by

$$\Omega_{t+1}Z_{t+1} - \mathcal{B}_{t+1}(B_t)\,.$$

[8] We interpret this one single period as the typical maturity of corporate debt of the order of several years. See, for example, Cortina, Didier and Schmukler (2018). In particular, we do not address the known fact that the US dollar tends to appreciate over the short term during crises (see, for example, Maggiori (2017) and Farhi and Maggiori (2018)), making that currency unattractive for short-term borrowing. Below, we provide empirical evidence suggesting that firms are well aware of this risk profile, and they tend to issue long-term dollar-denominated debt.

[9] Apart from the multiple currencies assumption, when modelling the financing side, we closely follow Gomes, Jermann and Schmid (2016). However, our model is static. Empirical findings in Kalemli-Ozcan et al. (2018) suggest that the effects of foreign currency debt on firms' behaviour might be even stronger in a dynamic setting.

[10] For simplicity, as in Gomes, Jermann and Schmid (2016), we assume that tax shields are the only motivation for issuing debt. However, one can also interpret τ as a reduced form of gains from debt issuance, such as the alleviation of adverse selection costs.

If the idiosyncratic shock realization, Y_{t+1}, is below an endogenous default threshold $\Psi_{t+1}(B_t)$, shareholders optimally default on their debt. Upon default, shareholders get zero, debt holders takeover the firm and are able to recover a fraction $\rho < 1$ of debt face value and coupon.[11] Thus, by direct calculation, the nominal price of one unit of debt denominated in currency j is given by

$$\delta^j(B_t) \;=\; E_t\left[M_{t,t+1}\left(1-(1-\rho)\Phi(\Psi_{t+1}(B_t))\right)(1+c)\mathcal{E}_{j,t+1}\right] ,$$

where $\Phi(\Psi_{t+1}(B_t))$ is the default probability conditional on the realization of aggregate variables. We assume that firms face a proportional cost $q(j)$ of issuing in country j currency for $j = 1, \cdots, N$[12] and maximize equity value plus the proceeds from the debt issuance net of issuance costs. Thus, the equity value V_t of a given firm satisfies

$$V_t \;=\; \Omega_t Z_t \;+\; \max_{B_t}\left\{\sum_{j=1}^{N}\delta^j(B_t)B_{j,t}(1-q(j)) + E_t[M_{t,t+1}\max\{V_{t+1} \;-\; \mathcal{B}_{t+1}(B_t),0\}]\right\}.$$

It is then straightforward to show that equity value is homogeneous in Z_t, so that $V_t = Z_t\bar{\Omega}_t$ for some variable $\bar{\Omega}_t$ that is independent of idiosyncratic shocks. Thus, default occurs whenever Y_{t+1} falls below the default threshold

$$\Psi_{t+1}(B_t) \;\equiv\; \frac{\mathcal{B}_{t+1}(B_t)}{\bar{\Omega}_{t+1}} ,$$

[11] We also assume that ρ is sufficiently small relative to ζ, so that debt holders can recover at most what they get from (inefficiently) running the firm net of the (un-modelled) default costs paid to lawyers, etc. We assume that these costs go directly to the representative consumer, and hence, they have no impact on the equilibrium outcomes. There are major differences in these default costs across different countries. See, Favara, Morellec, Schroth and Valta (2017).

[12] While we do not micro-found these costs, it is not difficult to do so. These costs may originate from underwriting costs, the limited risk bearing capacity of intermediaries (in the case of bank loans), or the actual debt placement costs incurred by the investment banks (such as locating bond investors). These costs can differ drastically depending on the currency in which the debt is issued. For example, according to Velandia and Cabral (2017), "... in the case of Mexico, the average bid-ask spread of the yield to maturity on outstanding USD-denominated international bonds is 7 basis points, compared to 10 basis points for outstanding EUR-denominated bonds. Mexico is also an example with very liquid benchmarks on both currencies."

Everywhere in the sequel, we use $E_t^\$$ and $\mathrm{Cov}_t^\$$ to denote conditional expectation and covariance under the dollar risk neutral measure with the conditional density $E_t[M_{t,t+1}]^{-1}M_{t,t+1}$. Furthermore, for each stochastic process X_t, we consistently use the notation

$$X_{t,t+1} \equiv \frac{X_{t+1}}{X_t}.$$

We need the following assumption to ensure that the leverage choice problem has a non-trivial solution.

Assumption 1 *We have*

$$\begin{aligned}(1-q(j))(1+c) &> (1+c(1-\tau)) \qquad \textit{and} \\ \bar{q}(j,\$) &\equiv \frac{((1-q(j))(1+c)-(1+c(1-\tau)))}{(1-\rho)(1+c)[(1-q(j))+\ell(1-q(\$))]-(1+c(1-\tau))} > 0\end{aligned}$$

for all i, $j=1,\cdots,N$. Let also $\bar{q}(\$) \equiv \bar{q}(\$,\$)$.

The first condition ensures that the cost $q(j)$ of issuing debt is less than the gains, as measured by the value of tax shields, so there is positive debt issuance. The second condition ensures that the recovery rate ρ is sufficiently small: Otherwise, funding becomes so cheap for the firm that the firm may want to issue infinite amounts of debt. The following is true.[13]

[13] As we show in the Appedix (see Proposition C.1), in our model firms never hedge their foreign exchange risk. There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (2018): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." Du and Schreger (2016) also provide evidence that firms do not fully hedge their currency risk exposures. See also Niepmann and Schmidt-Eisenlohr (2017), Bruno and Shin (2017). That being said, Liao (2019) does find evidence that at least 40% of global firms issue currency-hedged foreign debt. While it is known that costly external financing makes hedging optimal (see, for example, Froot, Scharfstein and Stein (1993) and Hugonnier, Malamud and Morellec (2015)), Rampini, Sufi and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less.

Theorem 1.1 *Issuing debt only in dollars is optimal if and only if*

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \;\leq\; \frac{\mathrm{Cov}_t^{\$}\left(\bar{\Omega}_{t+1}^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[\bar{\Omega}_{t+1}^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]} \tag{1}$$

for all $j = 1, \cdots, N$. In this case, optimal dollar debt satisfies

$$b_{\$,t} \;=\; \mathcal{E}_t^{-1} B_{\$,t} \;=\; (1 + c(1-\tau))^{-1} \left(\frac{\bar{q}(\$)}{E_t^{\$}\left[\bar{\Omega}_{t+1}^{-\ell}\right]} \right)^{\ell^{-1}} .$$

Absent heterogeneity in effective issuance costs (that is, when $q(j)$ is independent of j), (1) takes the form of

$$\mathrm{Cov}_t^{\$}\left(\bar{\Omega}_{t+1}^{-\ell}, \mathcal{E}_{j,t,t+1}\right) \;\geq\; 0 \;, \;\; j = 1, \cdots, N\,. \tag{2}$$

Intuitively, at time t, firms, when deciding on the currency composition of their debt, choose to issue in dollars if they anticipate the dollar to depreciate at those times when their time $t+1$ valuation is low; the condition (2) provides a precise formalization of this intuition. Since $\left(\bar{\Omega}_{t+1}\right)^{-\ell}$ attains its largest value when $\bar{\Omega}_{t+1}$ is close to zero, covariance (2) overweighs the distress states: When ℓ is sufficiently high, (2) essentially requires the dollar to depreciate against all its key competitors during times of major economic downturns.

It is also important to note that condition (2) corresponds to the problem a firm faces when choosing between dollar debt and debt denominated in other key currencies, such as e.g., the euro, the yen, the Swiss franc and the pound. For an emerging markets' firm that is choosing between local currency debt and dollar debt, heterogeneity in issuance costs may be as (if not more so) important as the currency risk profile. However, even for the choice between dollar- and euro-denominated debt, ignoring differences in issuance costs puts the dollar at a disadvantage: Existing evidence (see e.g., Velandia and Cabral (2017)) suggests that issuing debt in dollars is significantly cheaper than issuing in euros.

To test the validity of condition 1, we need to find an empirical proxy for $\bar{\Omega}_t$. We suppose for simplicity that the distressed state only lasts for one period, and debt holders run the firm inefficiently, making its output drop. We call this drop "Distress Costs". The following is true.

Proposition 1.2 *Let S_t be the (value-weighted) stock market index (i.e., total market capitalization of all (large and diversified) firms. Then,*

$$S_t \;=\; \bar{\Omega}_t \;-\; Distress\ Costs_t\,.$$

Proposition 1.2 shows that $\bar{\Omega}_t$ is closely related to the stock market index. If distress costs are small relative to the total value of the stock index, then $\bar{\Omega}_t$ can be directly proxied by the corresponding stock market index of "similar" firms. We will therefore use stock market indices in our empirical tests of condition (2).

1.1 Dominant Currency Debt and Inflation Cyclicality

In this section, we derive a link between the characterization in Theorem 1.1 and the inflation risk premium. Denote by $\mathcal{P}_{i,t}$ inflation in country $i,\ i=1,\cdots,N$. We will make the following assumptions about the joint long-term dynamics of inflation and exchange rates at horizons of average corporate debt maturity (about 5-7 years).

Assumption 2
- *Counter-cyclical inflation*

$$\log \mathcal{P}_{i,t,t+1} \;=\; -\,\phi_i \log S_t \;+\; \varepsilon_{i,t+1},$$

where $\varepsilon_{i,t+1} \sim N(0,\sigma_i^2)$ are log real exchange rates and $\phi_i > 0$ is the degree of inflation cyclicality.

- *Relative PPP is an important driver of exchange rates:*

$$\mathcal{E}_{j,t,t+1} \;=\; \mathcal{P}^{-1}_{\$,t,t+1}\mathcal{P}_{j,t,t+1}\, e^{\varepsilon^*_{j,t+1}}$$

where $\varepsilon^*_{j,t+1} \sim N(0, \sigma^2_{i,*})$

- *CAPM-Style stochastic discount factor,*

$$\log M_{t,t+1} \;=\; -\gamma \log S_t \;+\; \varepsilon^M_{t+1},$$

where $\varepsilon^M_{i,t+1} \;\sim N(0, \sigma^2_M)$.

- *all variables* $\varepsilon^M_{i,t+1},\; \varepsilon^*_{j,t+1},\; \varepsilon_{i,t+1},\; S_t$ *are uncorrelated.*

Theorem 1.3 *Suppose that* ϕ_i *are all pairwise different. Then, firms issue all debt the currency of the country with the highest index* ϕ_j*.*

The common shock structure in Assumption 2 allows us to abstract from the choice between local currency and foreign currency debt, and focus on the choice between different global currencies (such as, e.g., the EUR and the Dollar).

In addition to inflation stabilization indices ϕ_i, countries may also differ in the volatility of idiosyncratic shocks. Naturally, firms view this uncertainty as an additional and undesirable form of risk. The following is true:

Proposition 1.4 *Absent heterogeneity in the indices* ϕ_i*, firms always issue in the currency of the country with the lowest idiosyncratic exchange rate volatility* $\sigma^2_i + \sigma^2_{i,*}$*.*

Proposition 1.4 holds true for any shocks to exchange rates that are unrelated to economic fundamentals, for example, monetary policy shocks or temporary demand pressures and liquidity shocks in currency markets. Proposition 1.4 suggests that, in addition to insufficient market liquidity (modelled by high issuance costs), the significant idiosyncratic volatility of emerging market currencies may serve as an additional important mechanism that explains why firms do not want to issue in these currencies, despite the fact that such currencies do tend to significantly depreciate during crises (see also Du, Pflueger and Schreger (2016)). As an illustration, consider a typical emerging market currency, the Argentinian Peso (ARS). During the period of November 1995-September 2018, the standard deviation of the monthly returns on the dollar index was 1.9%, while the standard deviation of monthly returns on the ARS/USD exchange rate was 7.1%. Further, this volatility was almost entirely due to idiosyncratic shocks, as indeed, the R^2 of a regression of the monthly ARS/USD returns on the returns on the dollar index was only 0.0033.

2 Evidence from Backward-Looking Measures

Condition (2) shows that firms prefer to issue in dollars if the dollar exchange rate co-moves positively[14] with their stock market value. Our theory mainly applies to global firms that are exposed mostly to global shocks. We use two stock market indices to test our predictions, namely S&P 500 and the MSCI AC World Index measured in dollars to be consistent with our theoretical conditions. For this sub-section, we use the trade-weighted dollar index against major currencies, including those in Eurozone, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden, as obtained from the FRED database.[15] In the next subsection, we also provide results using the bilateral exchange rates between the dollar and the euro,[16] the yen, the pound and the Swiss franc.

[14]In fact, given that issuing in dollars is cheaper than issuing in any other currency, condition (1) implies that firms would issue all their debt in dollars even if this correlation were negative, but not too negative relative to the cost gain of issuing in dollars.

[15]Our results are robust when we use other indices such as the narrow or the broad dollar index obtained from the BIS.

[16]We use the Deutsche mark prior to the introduction of the euro using the euro/Deutsche mark exchange rate at the time of the inception of the euro.

2.1 Why is the dollar the dominant currency? Results with the dollar index

Given that the dollar is the most common currency of denomination in international debt markets, the first prediction of our model is that the returns on the dollar index positively correlate with the returns on the stock market indices at horizons that correspond to the typical corporate debt maturity, that is around 5-7 years (see section 4, Choi, Hackbarth and Zechner (2018), Cortina, Didier and Schmukler (2018)). To test this prediction, we first run the following regressions for the horizons of $h \in \{3, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120\}$ months:[17]

$$Ret\_USD_{t-h,t} = \alpha^h + \beta_h Ret\_StockIndex_{t-h,t} + \epsilon_{t-h,t}\,. \tag{3}$$

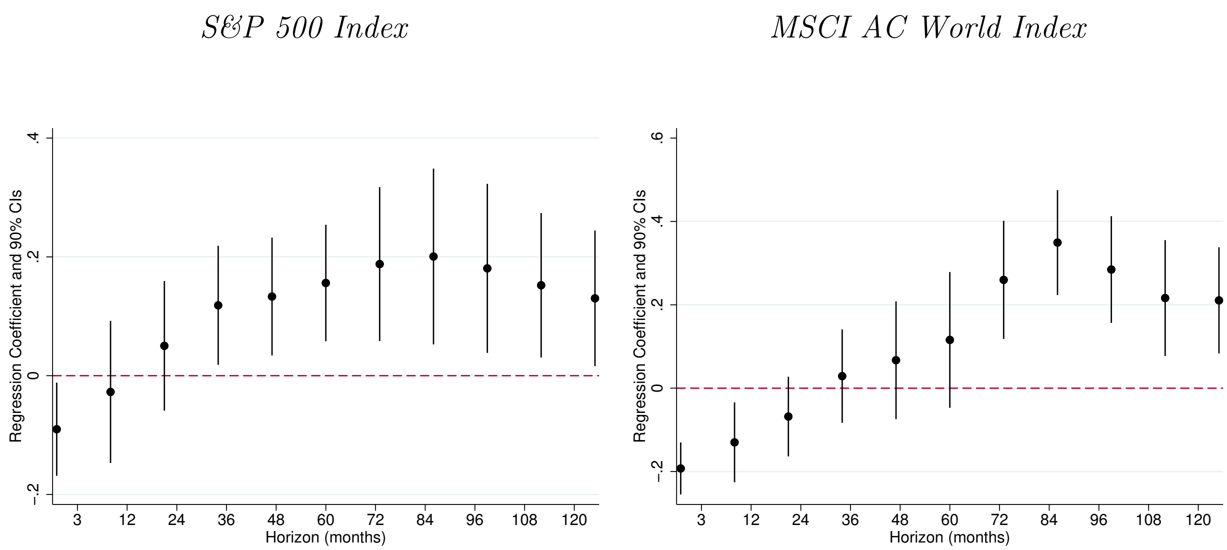
Here, $Ret\_USD_{t-h,t}$ and $Ret\_StockIndex_{t-h,t}$ denote the rolling returns on the dollar index and the two indices we use (in two separate regressions) over h months, respectively. Figure 2 reports the results for the regression coefficient β_h for different horizons and for different stock market indices, together with the 90% confidence intervals for the period between January 1988 and August 2019.[18]

The results show a pattern of negative correlations at short horizons and positive and mostly increasing correlations at longer horizons. The negative correlations for shorter horizons are consistent with the findings in Gourinchas, Govillot and Rey (2017) and Gourinchas (2019), who show that dollar tends to appreciate in bad times. Stavrakeva and Tang (2018) argue that this effect might be driven by the signalling role of the US monetary policy. However, Figure 2 suggests that the sign of the relationship reverts for typical horizons of

[17] We then control for autocorrelation at the respective horizons by using the Newey-West correction with the respective number of lags.

[18] The sample period starts from January 1988 as it is the start date of the MSCI series. In Appendix A.1, we repeat all the exercises for the S&P 500 starting from 1973. All our results are qualitatively similar, though our results from the 1988-2019 sample period are slightly stronger. This suggests that the covariance we are looking at has become stronger since 1988.

Figure 2: Correlation of the USD index with stock market indices



Notes: The graphs report the regression coefficients β_h from the regressions (3). The dots show the corresponding values of the β_h coefficients, while the lines show the 90% confidence intervals for these coefficients. Standard errors are corrected using the Newey-West procedure with the number of lags being equal to the horizon h of returns for each respective regression.

corporate debt maturity.[19] These findings, together with condition (2), suggest that US firms are better off if they borrow in dollars rather than in other major international currencies if their debt maturity exceeds roughly two years, which is the case here.

Why does the sign of the co-movement between the dollar index and the stock market change for longer horizons? To answer this question, we perform a simple covariance decomposition and show that this behaviour is driven by robust lead-lag relationships between the dollar and the stock market. In particular, while the statistical significance of the long horizon covariances in Figure 2 is difficult to establish due to a rather small sample with around 31 years of data, we show that the underlying lead-lag relationships are strong and robust, and hold for a vast majority of both short and long horizons.

[19]Interestingly enough, the same pattern of sign reversal at longer horizons is also observed in the behaviour of UIP deviations. See Valchev (2015) and Engel (2016). Understanding the links between these findings and our results is an interesting direction for future research.

We decompose the covariance between the dollar and the stock market based on the additivity of log-returns: $Ret_{t-h-j,t} = R_{t-h-j,t-h} + R_{t-h,t}$ for any h, $j > 0$. Using this decomposition, we get that

$$\begin{aligned}
&\mathrm{Cov}(Ret_USD_{t-h-j,t},\ Ret_SP500_{t-h-j,t}) \\
&= \underbrace{\mathrm{Cov}(Ret_USD_{t-h-j,t-h},\ Ret_SP500_{t-h-j,t-h}) + \mathrm{Cov}(Ret_USD_{t-h,t},\ Ret_SP500_{t-h,t})}_{co-movement} \\
&+ \underbrace{\mathrm{Cov}(Ret_USD_{t-h-j,t-h},\ Ret_SP500_{t-h-j,t})}_{USD\ leading\ SP500} + \underbrace{\mathrm{Cov}(Ret_SP500_{t-h-j,t-h},\ Ret_USD_{t-h,t})}_{SP500\ leading\ USD}.
\end{aligned} \tag{4}$$

Since the co-movement terms in the covariance decomposition are negative for shorter horizons, while the total covariance is positive for longer horizons (see Figure 2), it has to be that at least one of the lead-lag terms in (4) is positive and sufficiently large to offset the negative co-movement terms. We compute these covariances and their significance by running predictive regressions of S&P 500 on USD and vice-versa. Figure 3 shows that the the terms "SP500 leading USD" are positive and significant for a majority of (h, h) pairs.[20] Thus, the attractiveness of the dollar as a debt issuance currency is driven by the fact that it tends to follow past stock market moves.[21]

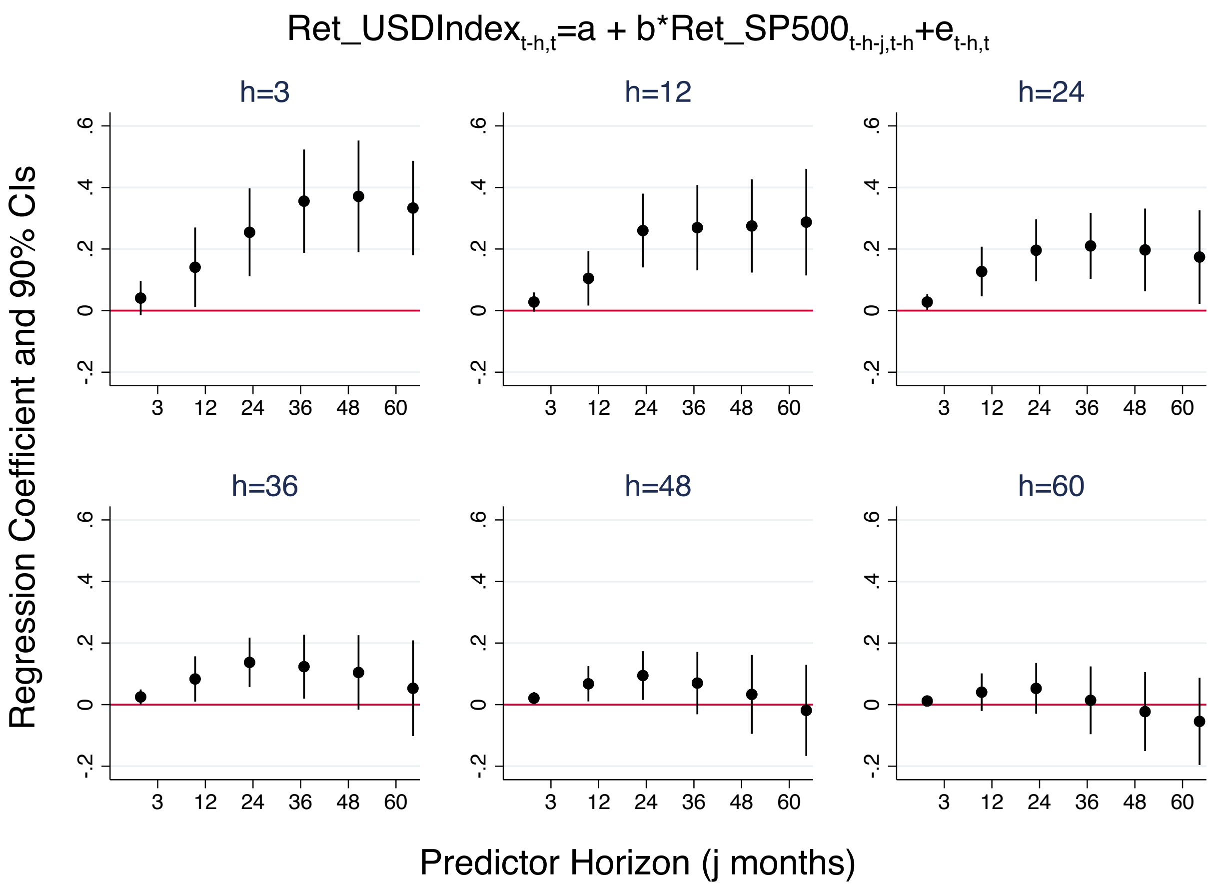
2.2 Why is the dollar the dominant currency? Results with bilateral exchange rates

In this section, we provide the results for the same regressions as in Section 2.1, but using bilateral exchange rates for the dollar against four other major currencies. As Figure 4 shows, the dominant currency condition (2) holds empirically with i=dollar and currency j being

[20] "USD leading SP500" terms are small and insignificant for all values of $h, h,$. We do not report them for brevity, but they are available upon request.

[21] See Eren, Malamud and Schrimpf (2019) for a detailed analysis of these phenomena.

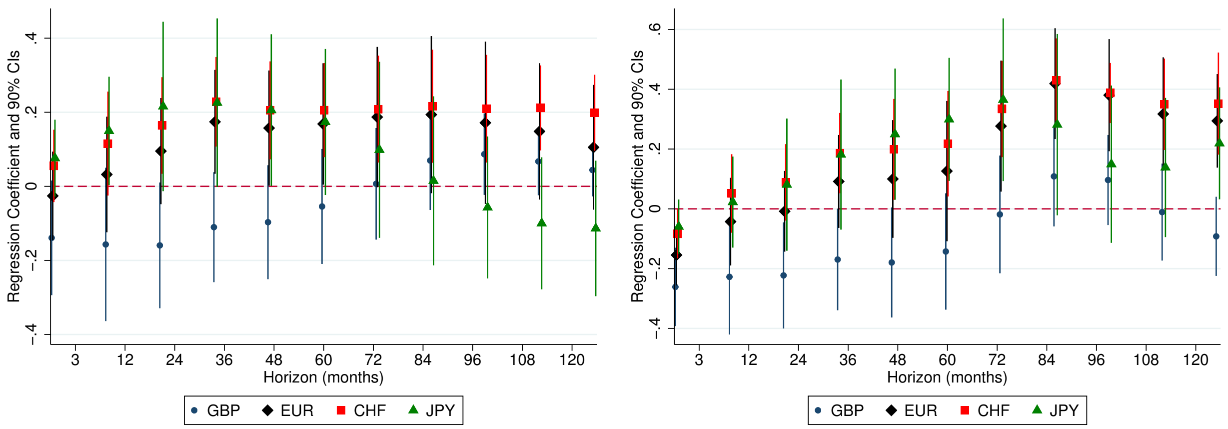
Figure 3: Does the S&P 500 predict the dollar?



the euro (EUR), the yen (JPY), or the Swiss franc (CHF). The only exception is British pound (GBP), for which our empirical proxy estimates in Figure 4 for the covariance in (2) have a negative sign. However, these covariance estimates are statistically insignificantly different from zero at the horizons of average corporate debt maturity. Thus, even absent differences in issuance costs, firms would strictly prefer issuing debt denominated in dollars, even if they could issue in EUR, JPY, or CHF. And even a slight difference in issuance costs favouring dollar to GBP would also make dollar immediately dominate over GBP.

Figure 4: Correlation of the bilateral exchange rate of the dollar against major currencies with stock market indices

S&P 500 Index *MSCI AC World Index*



Notes: The graph on the left-hand side herein reports the regression coefficients β_h from the regressions (3). The graph on the right-hand side reports the regression coefficients from the regressions (**??**). The dots show the corresponding values of the β_h coefficients, while the lines show the 90% confidence intervals for these coefficients. Standard errors are corrected using the Newey-West procedure with the number of lags being equal to the horizon h of returns for each respective regression.

2.3 Yen vs. pound

As we show in Section 2.2, the risk properties of the dollar alone can explain why the dollar dominates the euro, the yen and the Swiss franc in the sense of Theorem 1.1. One notable case is the pound: By Figure 4, the pound has favorable risk properties for debt issuers compared to most of the other major currencies. In reality, there are many reasons why the pound may not be the most obvious competitor to the dollar, such as differences in the size of the economies, lower issuance costs for the dollar etc. However, it is reasonable to compare the dynamics of corporate debt issuance in GBP to that in JPY, since Japan and the Great Britain have similar size in the world economy. In this case, Figure 4 shows that (2) holds empirically with i=GBP and j=JPY, and hence corporates should strictly prefer issuing in GBP to issuing in JPY. Figure 9 in the Appendix confirms this prediction of our model. Indeed, surprisingly, despite the slightly larger share of Japan in the world economy and lower nominal interest rates and inflation in Japan, the share of pound-denominated debt is higher than the share of yen-denominated debt, lending support to the "debt view."

3 Evidence from Forward-Looking Measures

The goal of this section is to understand the pre- and post-crisis trends in the shares of euro- and dollar-denominated debt through the lens of our model. An ideal test of our predictions would be to test the following condition:

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \;\leq\; \frac{\mathrm{Cov}_t^{\$}\left(\left(\bar{\Omega}_{t+1}\right)^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[(\bar{\Omega}_{t+1})^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]},$$

which links debt currency choice to the covariance between the stock market returns and exchange rates (see section 2), for long-horizons using forward-looking measures. According to our model, the currency that *market participants anticipate to co-move* more with the stock market would be chosen by firms as the currency to denominate their debt. If the distribution of issuance costs stays roughly constant across firms, our model implies a tight link between the time variation in this anticipated co-movement and currency denomination of debt issuance. In particular, all else constant, our model would attribute the recent rise in the share of dollar-denominated debt to heightened expectations of market participants of the dollar becoming more highly correlated with the stock market than the euro (that is, the dollar becoming a hedge for borrowers rather than investors). In this section, we provide evidence for the link between debt issuance patterns and such forward-looking market expectations and find support to the debt view, suggesting that firms issue more dollar debt when the dollar becomes more risky from the investors' point of view.

3.1 Quanto-implied risk premia and inflation risk premia

A direct way of computing forward-looking covariance between the stock market and exchange rates is by using so-called quanto forward contracts (Kremens and Martin (2019)), which brings us very close to the ideal test of our theory. A euro-quanto forward contract for S&P 500 with maturity T, for example, pays off the level of the S&P 500 index *in euros*. This means that at initiation the exchange rate is fixed. As opposed to a contract that pays off the S&P 500 in dollars, the value of this contract depends on the anticipated covariance between the index and the EUR/USD exchange rate. Hence, the price of this contract reflects the expectations of investors about currency returns. For example, if a quanto contract on the S&P 500 denominated in euros is more valuable than the S&P 500 denominated in dollars, it means that investors expect the euro to depreciate when the index (in dollars) is low, and vice versa. Formally we have:

$$\mathrm{Cov}_t^{\$}\left(S_{t+1}, \frac{EUR}{USD}\right) = \frac{R_{f,t}^{\$}}{R_{f,t}^{i} P_t}(Q_t - F_t) \tag{5}$$

where Q_t and F_t are quanto and vanilla forward prices, respectively. Using the approximation

$$\mathrm{Cov}_t^{\$}\left(S_{t+1}^{-\ell}, \frac{EUR}{USD}\right) \approx -\ell\,\mathrm{Cov}_t^{\$}\left(S_{t+1}, \frac{EUR}{USD}\right),$$

we get the following empirical prediction:

Prediction 1 *The quanto-implied covariance* (5) *is* **negatively** *related to the shares of dollar- and euro-denominated debt issuance.*

Liquid quanto contracts only exist for maturities of two years and lower. Kremens and Martin (2019) compute the quanto-implied covariance for contracts with two-year maturity and find that the quanto-implied covariance of the EUR/USD exchange rate with S&P500 exhibited a very strong downward trend in the post-crisis period, and has become negative in the recent years. This is exactly the same pattern that we find using backward-looking covariances in the previous section.

While quanto-implied covariance is the most relevant measure for our purposes, data obtained from Kremens and Martin (2019) only cover a period between December 2009 and October 2015. Since our goal is to explain the fall and the rise of the dollar in debt markets over the last two decades, we also resort to a longer time series containing similar information about forward-looking covariances.

We use our model to generate similar predictions that we can test with other available data measuring forward-looking risk premia. To this end, we appeal to Theorem 1.3 that provides a direct link between debt currency denomination and anticipated inflation cyclicality under the assumption that relative PPP is an important determinant of exchange rates at horizons of average corporate debt maturity.[22] The explicit link between relative inflation dynamics and exchange rates is the key element behind Theorem 1.3. The latter states that firms should be issuing dollar debt only if they expect the United States to have the most counter-cyclical inflation over the horizon of their debt maturity. These expectations about inflation cyclicality can be backed out from the inflation risk premium, given by the difference between inflation expectations under the risk-neutral and the physical measures:

$$IRP_{i,t} \;=\; \log\left(\frac{E_t^i[\mathcal{P}_{i,t,t+1}]}{E_t[\mathcal{P}_{i,t,t+1}]}\right) \;=\; \log\left(\frac{e^{r_t}\mathrm{Cov}_t(M_{i,t,t+1},\mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t,t+1}]}\right).$$

The covariance term, $\mathrm{Cov}_t(M_{i,t,t+1},\mathcal{P}_{i,t,t+1})$, reflects the basic intuition, namely, that the inflation risk premium is determined by market expectations regarding inflation cyclicality. It is determined by the co-movement of the household stochastic discount factor and inflation.

The following is true.

[22]While the perfect link between exchange rates and inflation relies on a strong form of PPP, Theorem 1.3 would still hold true even with large PPP deviations, as long as the relative inflation component of the exchange rates contributed significantly to the covariance (2) over the horizons of debt maturity of a typical firm. See Chernov and Creal (2019) for evidence that PPP is an important driver of long horizon currency risk premia.

Proposition 3.1 *Under the hypotheses of Theorem 1.3 , the inflation risk premium, $IRP_{i,t}$, has the largest value for the dominant currency country.*

In our model, IRP can be viewed as a barometer of market expectations of inflation counter-cyclicality, as captured by $\phi_{€,t}$. In this case, under the hypotheses of Theorem 1.3, assuming the the distribution of issuance costs stays constant across firms, Proposition 3.1 implies the following empirical prediction.[23]

Prediction 2 *The dollar debt share is positively related to the dollar IRP and is negatively related to the euro IRP.*

The left panel of Figure 5 shows the quanto-implied risk premium for the EUR/USD exchange rate taken directly from Kremens and Martin (2019). The right hand panel of Figure 5 shows the inflation risk premia for the euro and the dollar for two years and for five years, taken directly from Hördahl and Tristani (2014).[24]

3.2 The fall and the rise of the dollar

In Figure 6, we show the volume and the currency composition of gross issuance patterns of international debt as well as the shares of the dollar and the euro obtained from the BIS International Debt Securities statistics (data includes all sectors except the government). The figure shows that dollar share in debt issuance patterns follow a similar pattern as what is shown in Figure 1 with amounts outstanding. Maggiori, Neiman and Schreger (2018) show a similar pattern for holdings of corporate bonds.

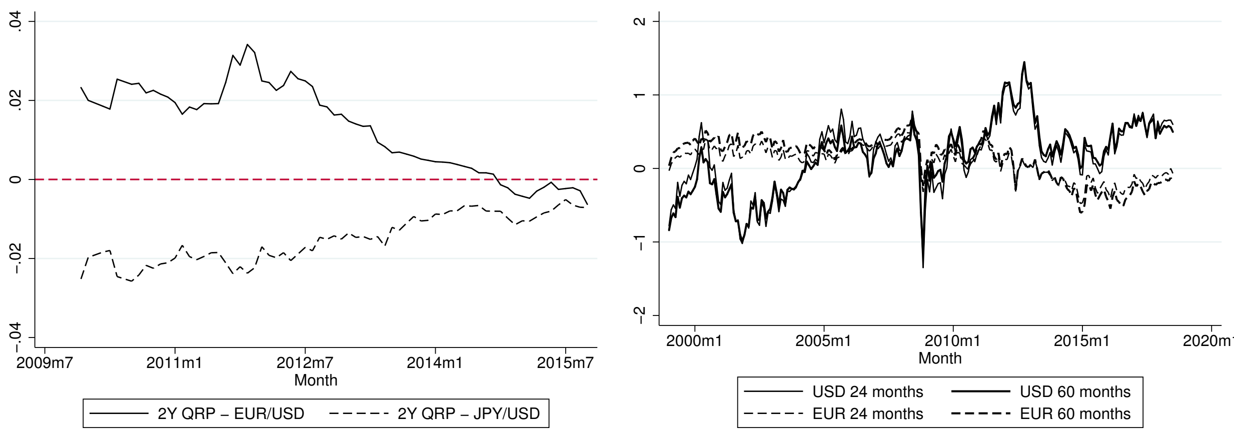
Broad trends shown in Figure 1 for amounts outstanding, in Figure 6, and in Figure 5 for international debt issuance, in Maggiori, Neiman and Schreger (2018) for holdings together

[23] While our model is silent about the origins of these expectations, one might speculate that the observed pattern in inflation risk premia between the euro and the dollar, shown in Figure 5, may be due to declining expectations of inflation stabilization and an increasing expected risk of deflation in Eurozone following the Global Financial Crisis in 2008 and the European Debt Crisis in 2011.

[24] The same pattern is present for longer maturities. Moreover, the difference between the dollar and the euro is more pronounced for longer maturities.

Figure 5: Two-year quanto-implied risk premium and two- and five- year inflation risk premia in the US and the Eurozone

2-Year QRP *2- and 5-Year IRP*

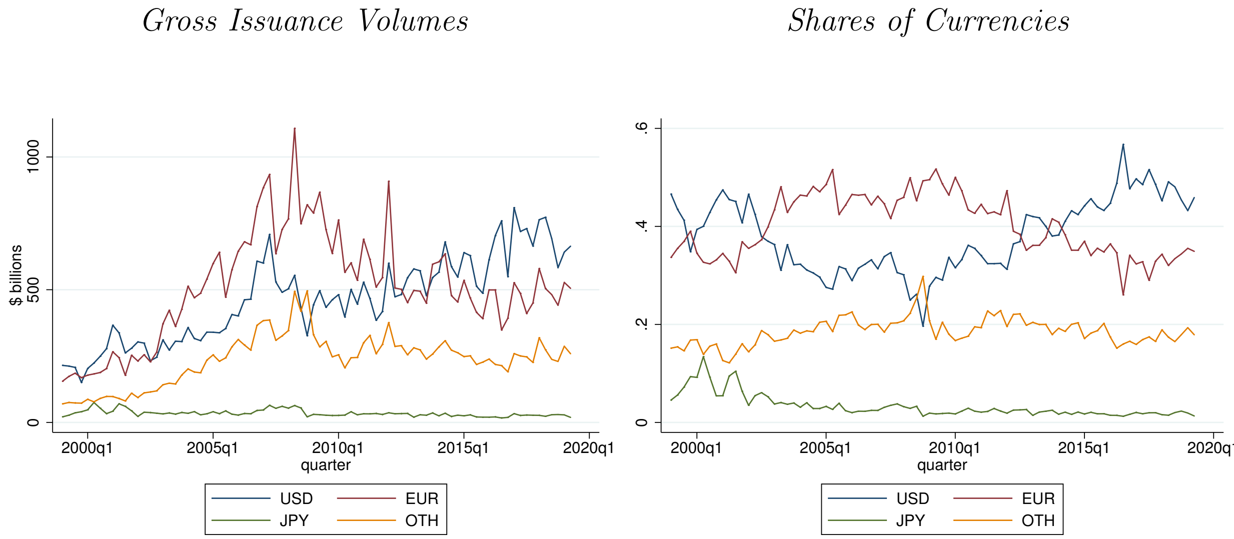


Source: QRP data are from Kremens and Martin (2019) and IRP data are from Hördahl and Tristani (2014).

with trends shown in Figure 5 lend support to the debt view (Predictions 1 and 2). In particular, in the pre-crisis period, IRP for the euro was higher than the IRP for the dollar, and after the crisis this reversed. In line with our predictions, share of dollar debt was in decline before the crisis and increased after the crisis. Moreover, during the period for which we have data, the QRP of the euro against the dollar declined strongly.

Both the QRP and the IRP dynamics suggest that debt holders caring about inflation and foreign exchange risk should dislike holding dollar denominated debt and prefer hold euro-denominated debt in the post-crisis period. Indeed, at horizons beyond two years, euro debt is more safe with respect to its inflation risk and foreign exchange risk in the post-crisis period. However, if firms share the same expectations about inflation cyclicality and foreign exchange risk as investors, the debt view implies that they will prefer issuing debt in dollars because of the attractive risk properties of the dollar due to anticipations of dollar depreciation in market downturns, lowering the probability of default.

Figure 6: Gross Issuance of International Debt Securities



Source: BIS International Debt Securities

3.3 QRP, IRP, and debt currency at higher frequency

Broader trends in debt currency choice, and currency and inflation risk premia are clearly in line with our predictions about the average levels of debt issuance. Next, we test Predictions 1 and 2 about the response of dollar debt issuance to movements in risk premia. One distinctive feature of our theory is that changes to debt issuance currency are driven by expectations and could change quickly. We regress various measures of dollar's share in debt markets to changes in risk premia at a quarterly frequency. First, we test Prediction 1: As the euro becomes less of a hedge for firms, i.e. QRP declines, do they issue more dollar debt? Second, we test Prediction 2: as EUR IRP becomes lower than USD IRP, do firms issue more dollar debt?

We report the results in Table 1. Column (1) shows that one standard deviation decrease in $QRP^{2Y}_{€/\$,t}$, which is around 0.01, is associated with around 3 percentage points higher dollar share in debt issuance in a given quarter. Note that average of the total issuance

Table 1: QRP, IRP, debt currency choice

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Full	Full	Full	Full	Full	Full
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP^{2Y}_{€/\$,t}$	-3.481***	-1.597**	-3.503***			
	(0.332)	(0.643)	(0.311)			
$IRP^{2Y}_{\$,t}$				-0.0168	-0.0197	0.0229
				(0.0190)	(0.0237)	(0.0231)
$IRP^{2Y}_{€,t}$				-0.181***	-0.174***	-0.182***
				(0.0320)	(0.0428)	(0.0285)
Trend		X			X	
Control			X			X
Period	09q4-15q3	09q4-15q3	09q4-15q3	99q1-18q3	99q1-18q3	99q1-18q3
Observations	24	24	24	79	79	79
R^2	0.705	0.781	0.763	0.286	0.287	0.412

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes all sectors except the government. Latest observed values of $QRP^{2Y}_{€/\$,t}$, $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ in a given quarter are used. $QRP^{2Y}_{€/\$,t}$ data come from Kremens and Martin (2019), and $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

is $1,284 billion, hence 3 percentage points amount to around $38 billion in a quarter. In column (2), we rerun the regression with a linear time trend, and in column (3), we control instead for total issuance. The results are qualitatively similar. In columns (4), (5), and (6), we rerun the same type of regressions for dollar share in debt issuance and inflation risk premia in the United States and the Eurozone. The results suggest that while debt issuance patterns do not move much with inflation risk premia in the United States, they mostly react to the inflation risk premia movements in the Eurozone. These results suggest that the decline of the euro as a preferred currency for debt issuance might be due to rising deflation risk in the Eurozone after the European sovereign debt crisis. The magnitude is also sizable as the results suggest that a one standard deviation decrease in the Eurozone IRP (which is 0.22), corresponds to around 3-4 percentage point higher dollar share in debt issuance.[25]

Overall, while we believe that getting the patterns of pre- and post-crisis period right in terms of the relationship between the dollar share and the risk premia is our main empirical finding, regression results with quarterly data are broadly consistent with the predictions of the debt view. Moreover, finding these effects in a relatively high frequency also lends support to another prediction of our theory that changes to debt issuance currency respond to expectations and could change relatively quickly.

4 Debt currency and maturity choice

Our results in section 2 have direct implications on the link between debt maturity and the incentives to issue dollar-denominated debt. Namely, as the dollar's co-movement with the stock market increases over longer horizons, we expect that firms would not be indifferent between issuing short-term dollar debt and rolling it over, and issuing long-term debt. In particular, we predict that firms would prefer issuing their longer maturity debt in dollars.

We use data at the bond issuance level in order to formally test the hypothesis that the propensity to issue dollar-denominated debt increases with debt maturity. We restrict our attention to non-financial corporations that issued bonds between 1999 (the introduction of the euro) and 2019.[26,27]

[25] In the Appendix, we report the results for non-bank and bank debt issuance separately. The results are similar for both sectors.

[26] The dataset includes perpetual bonds as well. In order to include them in the analysis, we winsorize the maturity of the bonds at 5%, both at the lower and upper tail of their maturity distribution. Winsorizing the maturity at 10%, 2.5% or winsorizing only the right tail of the distribution do not change the results materially.

[27] We exclude data on the government sector and focus only on private sector bond issuance.

We use data from Dealogic where observations are at the ISIN level of bond issuance. In order to keep the timing of our analysis similar to the previous sections, we restrict the sample to bonds issued between January 1999 and July 2019. Our dataset includes a total of 688,579 bonds, issued by 60,097 firms that are headquartered in 120 different countries.

The dataset includes information on the identity of the firm, the country where it is headquartered, the industry as well as information on the bonds, such as the currency denomination, date of issuance, maturity date, issued amount denominated in the local currency of the firm's headquarters, and whether the bond is investment-grade or is not. In the full sample, the mean of the winsorized maturity is 3,376 days, with a standard deviation of 3,458 days; the minimum value is 365 days and the maximum value is 11,474 days.[28]

The dollar co-movement with the stock market increases over longer horizons as documented in section 2. An implication of this result through the lens of our model is that the propensity to issue dollar-denominated debt should increase with debt maturity. Following our results in section 2, we test the hypothesis that longer debt maturity is associated with a higher propensity to issue dollar-denominated debt using micro-level data on bond issuance.

To measure the propensity to issue dollar-denominated debt, we use $\mathbb{1}(USD)$, which is a dummy variable that takes the value 1 if the currency denomination of the bond is the dollar. Then, the independent variables of interest in our regressions become: $Maturity_w$, which is the winsorized and standardized value of maturity at the 5% level. According to our hypothesis, we expect a positive coefficient for this variable.

Other control variables are the size of the issuance and a dummy variable that is equal to 1 if the bond is investment-grade. Moreover, depending on the specification, we include $Industry * Month$, $Country * Month$ and $Firm * Month$ fixed effects. We cluster the standard errors at the $Country * Year$ level.[29]

[28]We present the summary statistics regarding debt maturity by different currencies in **??** in a box plot in the Appendix.

[29] In the benchmark specification, we use the country where the headquarters of the parent company of

We run different linear regressions, varying the fixed effects used and making different cuts of the sample in order to test the predictions of our theory. Table 2 presents the results.

Table 2: Debt maturity and currency choice

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	NB	B	NB	B	NB†	B†
	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$
$Maturity_w$	0.0156***	0.0302***	0.0167	0.0776***	0.0359**	0.0742***
	(0.00343)	(0.00802)	(0.0139)	(0.0128)	(0.0157)	(0.0157)
Controls	X	X	X	X	X	X
Industry*Month FE	X	X				
Country*Month FE	X	X				
Firm*Month FE			X	X	X	X
Observations	441,460	241,192	45,662	66,455	31,259	48,802
R-squared	0.688	0.666	0.451	0.433	0.476	0.472
Mean of Dep. Var	0.557	0.534	0.378	0.348	0.433	0.351

Notes: Standard errors clustered by $Country * Year$ in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. $\mathbb{1}(USD)$ is a dummy variable that takes the value 1 if the currency of the issued bond is the dollar. $Maturity_w$ is the standardized value of maturity winsorized at 5% and 95% levels. Controls include the size of the issuance and a dummy variable for the status of investment-grade status of the bond. NB refers to the sample of non-bank financials and non-financial corporations. B refers to the sample of banks. Columns (3), (4), (5), and (6) only include firms that issue in at least two currencies in a given month. † means that the sample is further restricted only to those firms that are from the United States, the Eurozone, Japan, Switzerland or the Great Britain.

The first two columns control for bond characteristics as well as $Industry * Month$ and $Country*Month$ fixed effects for non-banks (in the first column) and for banks (in the second column). The coefficient on $Maturity_w$ suggests that a one standard deviation increase in

the issuer is located. As a robustness check, we use the residence of the issuer instead. Our results then are virtually unchanged.

maturity increases the likelihood of the currency denomination of the bond to be dollars by 1.5 percentage points for non-banks and 3 percentage points for banks.

Next, as part of our identification strategy, we rely on firms that issue multiple bonds in at least two different currencies in a given month. This choice allows us to tightly identify that the same firm that has access to multiple markets chooses to issue the longer maturity bond in dollars as opposed to issuing in other currencies. In columns (4) and (5), we run a similar regression for non-banks and banks, respectively, with $Firm * Month$ fixed effects. While the result for non-banks is of a similar magnitude for non-banks, it is statistically insignificant. On the other hand, the result goes through for banks.

Finally, in columns (5) and (6), we further restrict the sample to firms that are from the United States, the Eurozone, Japan, Switzerland or the Great Britain and repeat the exercise in (3) and (4). This aims to address a potential concern that our results in (3) and (4) are driven by the fact that firms in emerging markets could only access dollar bond issuance markets. Focusing only on the five countries with liquid and deep capital markets alleviates this concern as these firms could potentially issue in their home currency, or any other major currency. The results from these regressions are in line with our hypotheses both for banks and non-banks.

5 Conclusion

Motivated by two facts, namely the dominant international role of the dollar in debt markets, and the fall and the subsequent rise of the dollar in these markets over the last two decades, we address two questions. First, of all the major international currencies, why is the dollar the dominant currency? Second, what explains the fall and the rise of the dollar?

We propose a "debt view" to explain the dominant international role of the dollar and provide broad empirical support for it. We develop a simple capital structure model in which firms optimally choose the currency composition of their debt. Independent of the lenders' stochastic discount factor, borrowers behave as if they have a “CAPM discount factor,” whereby the debt currency choice of borrowers depend on how each currency co-moves with the stock value of the firm. We compare major international currencies which have markets with similar depth and liquidity. In this sense, borrowers prefer debt issuance in the riskiest of the international currencies. Both forward-looking and historical covariances suggest that dollar fits this description better than all major currencies, especially for longer horizons. Moreover, the debt view can jointly explain the fall and the rise of the dollar in debt markets.

Our results have some policy implications. First, it is commonly believed that an exchange rate depreciation could help an economy in downturns mainly through its effect on terms of trade. Our results imply that exchange rate depreciations could also help an economy through reducing the probability of default of indebted firms. Second, our results imply that if a country wishes to gain a dominant currency status for debt issuance, it is important that the currency of that country is not the “safest haven” currency, and riskier than its counterparts. Moreover, an important role for the central bank arises, which is not to have realized inflation undershoot inflation expectations in downturns, and which would generate appreciation pressures for the currency.

What do our results imply for the future of the dollar? Many explanations of the dominant role of the dollar in the international monetary system feature arguments like inertia, size, network externalities, and market liquidity. All these arguments suggest that changes in the dominance status of a currency occur very slowly. By contrast, our results suggest that the dollar can lose its dominance if the expectations about the risk properties of dollar and other currencies change. As this relies on the beliefs of market participants, changes might occur abruptly. Our evidence from quarterly regressions suggests that this is a relevant channel.

This paper fits into a broader research agenda that aims to study the use of various currencies in different parts of the economy through the lens of their risk properties. Our model can be extended in multiple directions. First, addressing the interactions between the role of the dollar in trade, banking and finance may shed important light on how debt issued in a dominant currency could affect other parts of the economy through the lens of the debt view. Second, modelling the demand for safe assets would help in understanding the role of the dollar for financial intermediation and household balance sheets as well as for firms issuing debt jointly. We leave these questions for future research.

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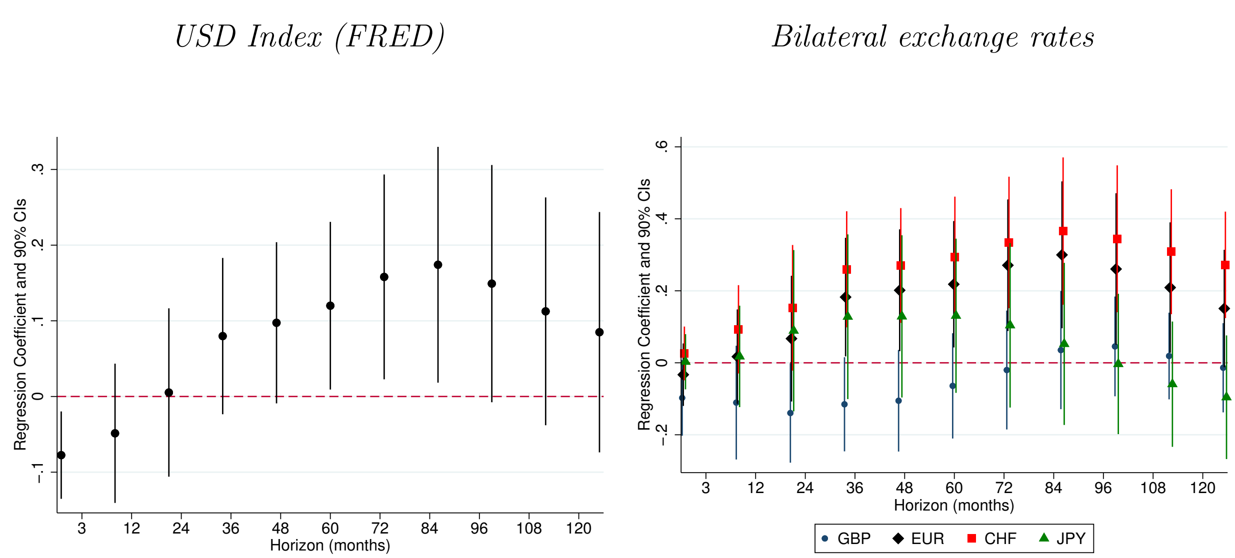
Internet Appendix

A Additional results and further evidence

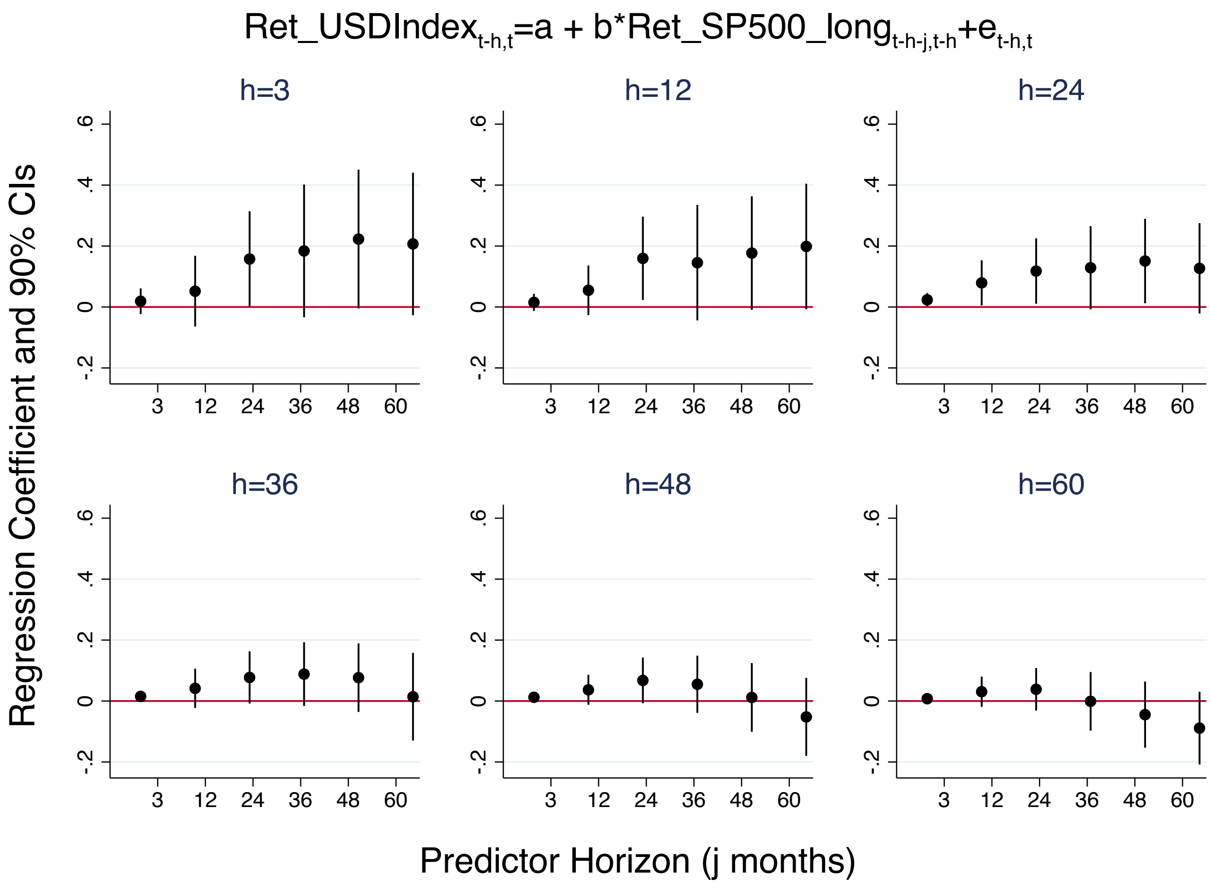
A.1 Correlation between the S&P 500 Index and the dollar since 1973 and predictive regressions

Figure 7: Correlation of the USD index and bilateral exchange rates with the S&P 500 since 1973



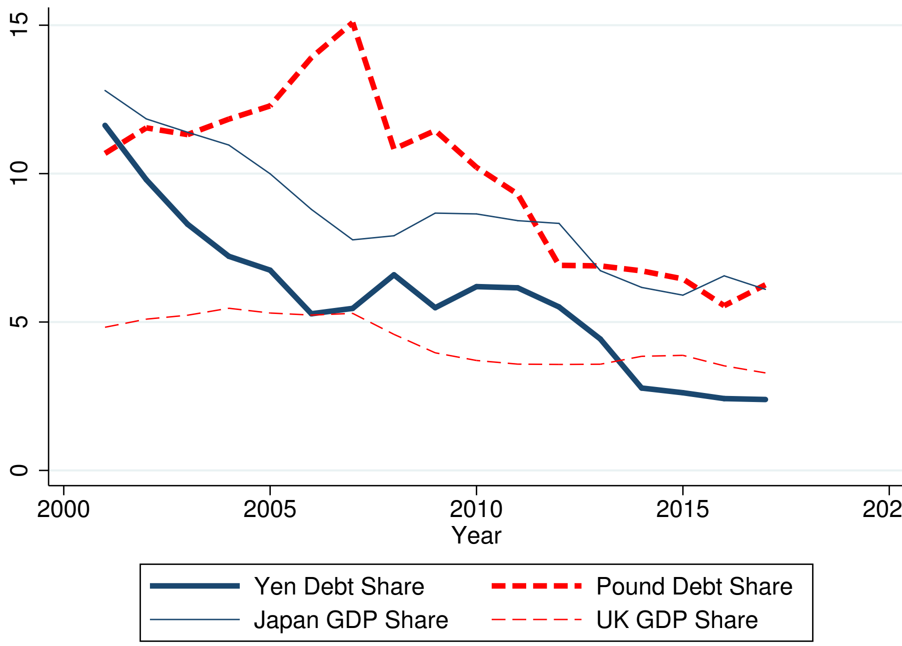
Notes: The graphs report the regression coefficients β_h from the regressions (3) between January 1973 and August 2019. The dots show the corresponding values of the β_h coefficients, while the lines show the 90% confidence intervals for these coefficients. Standard errors are corrected using the Newey-West procedure with the number of lags being equal to the horizon h of returns for each respective regression.

Figure 8: Does the S&P 500 predict the dollar since 1973?



A.2 Yen vs. Pound

Figure 9: The yen versus the pound

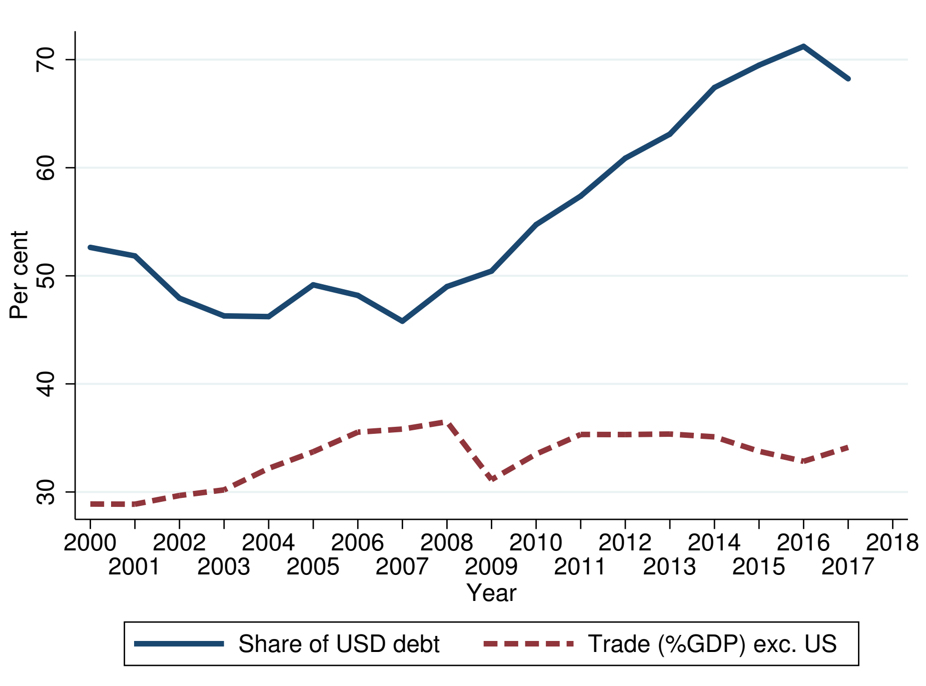


Source: BIS, IMF WEO, authors' calculations

A.3 Dollar debt and international trade

Our model also makes predictions about the relationship between international trade and dollar-denominated debt. As we show in the Appendix (see the proof for Proposition **??**), in the dominant currency debt equilibrium of Theorem 1.3, an increase in the coefficient ϕ of monetary policy effectiveness of the dominant currency country's central bank is always associated with more issuance of debt denominated in the dominant currency; and (ii) a drop in the conditional expectations for the amount of international trade. This result is intuitive: An aggressive monetary policy provides incentives for firms to choose higher leverage, which ex post leads to more debt overhang and a drop in international demand. Thus, in the extended version of the model that is discussed in the previous section, shocks to $\phi_{\$,t}$ should move trade and the amount of debt denominated in the dominant currency in opposite directions. Figure 10 shows the joint dynamics of dollar-denominated debt and international trade over the last two decades. Consistent with our theory, the pre- and post-crisis trends in Figure 1 move-to-one with opposite trends in international trade.[30]

Figure 10: Trade and Debt



Source: BIS, World Bank, FRED, authors' calculations
Notes: $TotalTrade(\%GDP)_t^{exUS}$ is the total trade to world GDP, excluding the US.

[30]Indeed, regressions of yearly data for the share of dollar debt on differences in dollar and Euro IRP and total trade to world GDP, excluding the US, yields a positive and significant coefficient for IRP differences and a negative coefficient for trade in line with our predictions. We omit this for brevity, but the results are available upon request.

A.4 Local currency and dollar debt mix

In this section, we take as given the dominance of dollar among the major global currencies, and we investigate whether debt view can be used to explain the mixture of dollar- and local-currency denominated debt. We developed and test the predictions of our model using a cross-section of the emerging market economies for which data on corporate debt in different currencies are available.[31] To this end, we prove the following extension of Theorem 1.1 for the case wherein firms issue a mixture of local currency (LC) and dollar-denominated debt (see Theorem D.1 in the Appendix for the proof. Note that, while Proposition A.1 is a partial equilibrium result, it still holds true in general equilibrium when debt overhang costs are sufficiently small).

Figure 11 shows the mean of the debt ratio, $\frac{L\bar{C}U}{USD}_i$, for each country in our sample. The left-hand panel shows several outliers: China and the EU countries in the sample (Czechia, Hungary, and Poland), while the right-hand panel shows the rest of the countries. We exclude outliers from our regressions and focus only on the sample of countries listed in the right-hand panel.

Proposition A.1 *Suppose that (1) $q = q(\$)$ (that is, issuing in LC costs the same as issuing in dollars); (2) the variance of all shocks is sufficiently small; and (3) issuing debt in both LC and dollars is optimal; (4) ℓ is close to 1. Then,*

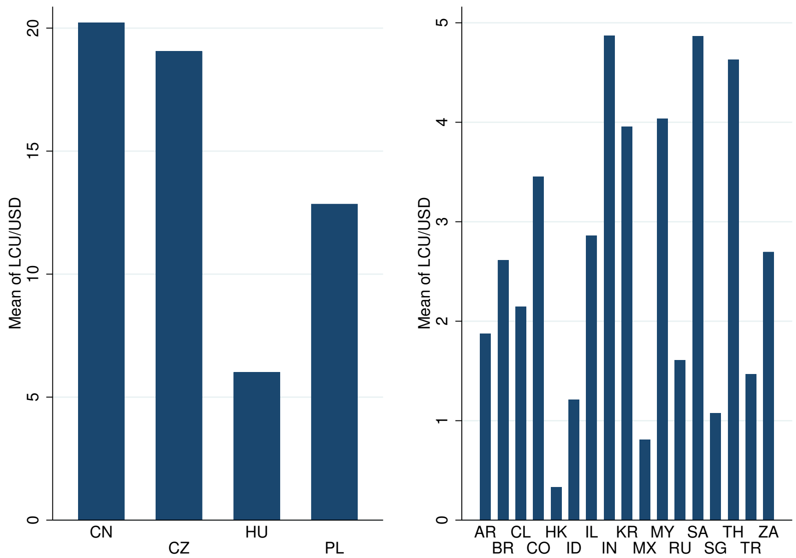
(a) the fraction $\frac{B_t}{B_t(\$)\mathcal{E}_{\$,i,t}}$ is monotone increasing in the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ if and only if $B_t \;\geq\; B_t(\$)\mathcal{E}_{\$,i,t}$;

(b) the fraction $\frac{B_t}{B_t(\$)\mathcal{E}_{\$,i,t}}$ is always monotone decreasing in $\sigma_{i,\varepsilon}$.

The intuition for the first theoretical result is that local currency debt partly replicates insurance properties of the dominant currency in downturns, while it is a better hedge against

[31] Data were obtained from the Institute for International Finance (IIF) for the period from 2005 Q1 to 2018 Q2. The countries in our sample are Argentina, Brazil, Chile, China, Colombia, Czechia, Hong Kong, Hungary, India, Indonesia, Israel, Republic of Korea, Malaysia, Mexico, Poland, Russian Federation, Saudi Arabia, Singapore, South Africa, Thailand and Turkey.

Figure 11: Mean of the local currency to USD debt ratio by country



Source: IIF, authors' calculations

domestic productivity shocks. The second result is that volatile inflation generates volatility of profits which the firms avoid by issuing less local currency debt.

Items (a)-(b) of Proposition A.1 directly translate into the testable empirical hypotheses. We test the two implications of our theory:

1. The local currency share of corporate debt is *higher* for countries in which domestic inflation correlates more with US inflation when controlling for relevant factors.

2. Firms in countries with more volatile domestic inflation tend to have less debt denominated in local currency.

We find statistically significant evidence for the first prediction. Our second test results in a coefficient with the predicted sign, yet statistically insignificant.

In order to test the first hypothesis, we proceed as follows. For each in our sample, we estimated the following time series regression:

$$\pi_t^i = \gamma_0 + \gamma_1 \cdot Ret\_MSCIACWorld_t + \Gamma \cdot Ret\_DomesticStockIndex_t^i + \pi_t^{res,i}\,, \tag{6}$$

where π_t^i is the domestic monthly inflation rate in and $Ret\_MSCIACWorld_t$ is the monthly return on the MSCI AC World Index. $Ret\_DomesticStockIndex_t^i$ is the monthly return on the domestic stock market index. $\pi_t^{res,i}$ are the residuals from this regression. We also run the following regression for the US:

$$\pi_t^{US} = \mu_0 + \mu_1 Ret\_MSCIACWorld_t + \pi_t^{res,US}\,, \tag{7}$$

We then run the following regression to compute a proxy for the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ between the residual domestic inflation and residual US inflation (see item (a) of Proposition A.1),

$$\pi_t^{res,i} = \alpha + \beta \pi_t^{res,US} + \epsilon_t\,,$$

where $\pi_t^{res,i}$ is the residual domestic monthly inflation rate in from (6) and $\pi_t^{res,US}$ is the residual monthly inflation rate in the US from (7). We denote the estimated slope coefficient by $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$.

We then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_1 + \beta_1 \hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}} + X_i + \eta_i\,. \tag{8}$$

Here, $\frac{L\bar{C}U}{USD}_i$ is the average ratio of debt denominated in local currency to debt denominated in dollars for corporates in the countries of the dataset; X_i denotes other control variables.[32]

[32]See **??** for averages across countries.

Item (a) of Proposition A.1 predicts that the coefficient β_1 in the regression (8) should be positive.

To test the second hypothesis, we calculate the standard deviation of $\pi_t^{res,i}$ as a proxy for $\sigma_{\varepsilon,i}$ in Proposition A.1, and then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_2 + \beta_2 \sigma_i^{\pi_t^{res,i}} + X_i + \eta_i \,. \tag{9}$$

Proposition A.1, item (b) predicts that $\beta_2 < 0$.

In column (1), we run univariate regressions In column (2), we add an additional control variable $ka\bar{o}pen_i$: a financial openness index obtained from Chinn and Ito (2006). In column (3), we take the predictions of the model literally as they appear in item (a) of the Proposition A.1: $\beta_1 > 0$ for countries where $\frac{L\bar{C}U}{USD}_i > 1$ and we exclude the two countries where $\frac{L\bar{C}U}{USD}_i < 1$, namely, Hong Kong and Mexico. In all three columns, regressions corroborate *Hypothesis CS-1.* [33] The first three columns are in line with the predictions of our theory. Column (4) of Table 3 shows the results of regression (9). Although the result is lacking statistical significance, the sign of the coefficient is indeed consistent with our theoretical prediction.

[33] All our results are qualitatively and quantitatively similar when we use raw domestic and US inflation rates, instead of residuals. Moreover, all results remain valid if we use the share of local currency debt in total debt instead of the ratio of local currency debt to USD debt.

Table 3: The cross-section of the local currency to dollar debt ratio

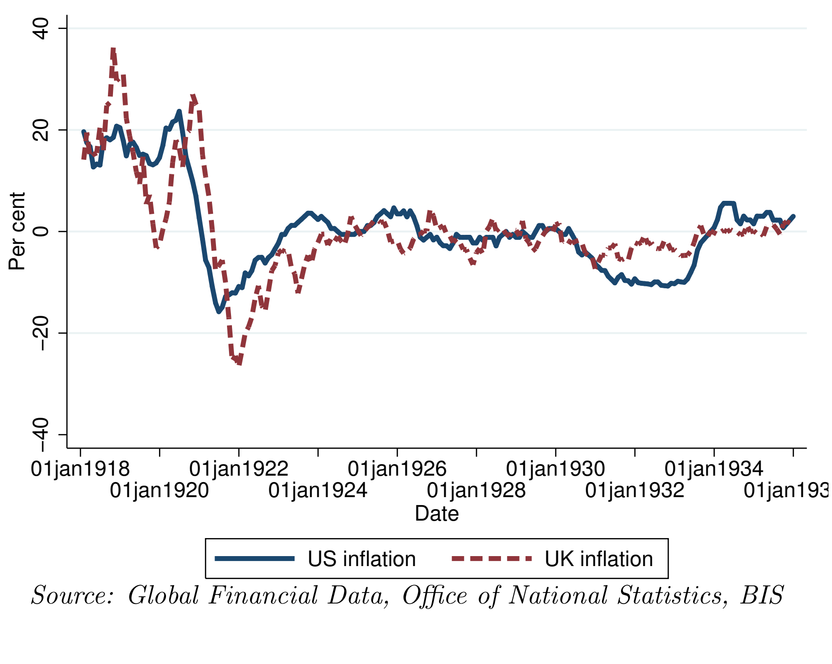
	(1)	(2)	(3)	(4)
	$\frac{\bar{LCU}}{USD}_i$	$\frac{\bar{LCU}}{USD}_i$	$\frac{\bar{LCU}}{USD}_i$	$\frac{\bar{LCU}}{USD}_i$
$\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$	3.951***	3.930***	3.713***	
	(0.680)	(0.640)	(0.775)	
$ka\bar{o}pen_i$		-0.0108	0.102	-0.334
		(0.327)	(0.413)	(0.349)
$\sigma_i^{\pi_t^{res,i}}$				-2.218
				(1.306)
Observations	17	17	15	17
R-squared	0.537	0.537	0.409	0.217

Notes: Robust standard errors in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. $\frac{\bar{LCU}}{USD}_i$ is the mean share of local currency debt obtained from the IIF for each of the 17 emerging market economies between 2005 Q1 and 2018 Q2. $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$ is the estimated regression coefficient for a linear regression of residuals of monthly domestic inflation rate from (6) on the residuals of the US inflation rate from (7). $ka\bar{o}pen_i$ is the mean of the Chinn-Ito financial openness index for each country. $\sigma_i^{\pi_t^{res,i}}$ is the standard deviation of the residuals of the monthly domestic inflation rate obtained from (6). In column (3), Hong Kong and Mexico are excluded.

A.5 Pound vs. dollar in the interwar years

One can use the expectations channel to shed light on the history of multiple, repeated switches between the pound and the dollar and their roles in the main reserve currencies during the inter-war period (see Chiţu, Eichengreen and Mehl (2014)). Consider the two currencies (the pound and the dollar) with sufficiently similar indices $\phi_{\$,t} \approx \phi_{GBP,t}$. Our model predicts that shocks to expectations about the difference $\phi_{\$,t} - \phi_{GBP,t}$ can lead to quick switches back and forth between different dominant currency debt equilibria, wherein one currency repeatedly gains and then loses the dominant currency role to its competitor. Consistent with our theory, the pound started losing its role as the dominant currency after the negative inflation surprise at the beginning of the 1920s during the 1920-21 recession (Figure 12). At the same time, the dollar faced greater deflation during the Great Depression, (1929-1939) with a subsequent partial regaining of dominance by the pound, based on the evidence provided by Chiţu, Eichengreen and Mehl (2014).

Figure 12: Historical Inflation Rates



Source: Global Financial Data, Office of National Statistics, BIS

B The fall and the rise of the dollar: Results for non-banks and banks

Table 4: QRP, IRP, debt currency choice: Sample restricted to banks

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Banks	Banks	Banks	Banks	Banks	Banks
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP_{€/\$,t}^{2Y}$	-4.606***	-0.559	-3.785***			
	(0.683)	(1.081)	(0.534)			
$IRP_{\$,t}^{2Y}$				-0.0127	0.00173	0.0764***
				(0.0254)	(0.0328)	(0.0274)
$IRP_{€,t}^{2Y}$				-0.281***	-0.318***	-0.221***
				(0.0576)	(0.0720)	(0.0515)
Trend		X			X	
Control			X			X
Period	09q4-15q3	09q4-15q3	09q4-15q3	99q1-18q3	99q1-18q3	99q1-18q3
Observations	24	24	24	79	79	79
R-squared	0.489	0.629	0.772	0.238	0.242	0.502

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes only banks. Latest observed values of $QRP_{€/\$,t}^{2Y}$, $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ in a given quarter are used. $QRP_{€/\$,t}^{2Y}$ data come from Kremens and Martin (2019), and $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

Table 5: QRP, IRP, debt currency choice: Sample restricted to non-banks

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Non-Banks	Non-Banks	Non-Banks	Non-Banks	Non-Banks	Non-Banks
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP^{2Y}_{€/\$,t}$	-3.082***	-2.077*	-3.110***			
	(0.467)	(1.092)	(0.609)			
$IRP^{2Y}_{\$,t}$				-0.0129	-0.0249	0.0123
				(0.0204)	(0.0264)	(0.0253)
$IRP^{2Y}_{€,t}$				-0.182***	-0.151***	-0.188***
				(0.0316)	(0.0404)	(0.0274)
Trend		X			X	
Control			X			X
Period	09q4-15q3	09q4-15q3	09q4-15q3	99q1-18q3	99q1-18q3	99q1-18q3
Observations	24	24	24	79	79	79
R-squared	0.566	0.588	0.566	0.272	0.279	0.345

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes only non-banks. Latest observed values of $QRP^{2Y}_{€/\$,t}$, $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ in a given quarter are used. $QRP^{2Y}_{€/\$,t}$ data come from Kremens and Martin (2019), and $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

C Leverage

Proposition C.1 *Suppose that firms have a possibility of hedging foreign exchange risk by acquiring $h_t \geq 0$ units of a financial derivative contract with a payoff of $X_{t+1} \geq 0$ and a price of $E_t[M_{t,t+1}X_{t+1}]$ to be paid at time t. The firm always chooses $h_t = 0$.*

The intuition behind this result is straightforward. Hedging effectively plays a role of investment, and the firm only gets the payoff X_{t+1} from this investment in good (survival) states, while paying the market price at time t to get the payoff in all states. Thus, hedging is just a transfer of funds from shareholders to debt-holders, and firms optimally decide to minimize this transfer.[34]

Proof of Proposition C.1. The maximization problem is

$$\max_{h_t} \Bigg\{ - E_t[M_{t,t+1} X_{t+1}] h_t$$
$$+ E_t \left[M_{t,t+1} \int_{\Omega_{t+1} Z_{t+1} > \mathcal{B}_{t+1}(B_t) - h_t(1-\tau)X_{t+1}} (\Omega_{t+1} Z_{t+1} - \mathcal{B}_{t+1}(B_t) + h_t(1-\tau)X_{t+1}) \phi(Z_{t+1}) dZ_{t+1} \right] \Bigg\}.$$

The derivative of this objective function with respect to h_t is given by

$$-E_t[M_{t,t+1} X_{t+1}] \,+\, (1-\tau) E_t \left[M_{t,t+1} X_{t+1} \left(1 - \Phi\left(\frac{\mathcal{B}_{t+1}(B_t) - h_t(1-\tau) X_{t+1}}{\Omega_{t+1}} \right) \right) \right] \,<\, 0\,,$$

and hence $h_t = 0$ is optimal. Q.E.D.

Proof of Theorem 1.1. Firm's problem is to maximize

$$\sum_j E_t \left[M_{t,t+1} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \right) (1+c) \mathcal{E}_{j,t+1} \right] \right] B_{j,t} (1 - q(j))$$
$$+\; E_t \left[M_{t,t+1} \left[-\mathcal{B}_{t+1}(B_t) \left(1 - \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \right) + \Omega_{t+1} \ell (\ell+1)^{-1} \left(1 - \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell+1} \right) \right] \right]$$

[34] There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (2018): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." Du and Schreger (2016) also provide evidence that firms do not fully hedge their currency risk exposures. See also Niepmann and Schmidt-Eisenlohr (2017), Bruno and Shin (2017). While it is known that costly external financing makes hedging optimal (see, for example, Froot, Scharfstein and Stein (1993) and Hugonnier, Malamud and Morellec (2015)), Rampini, Sufi and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less.

Differentiating, we get from the standard Kuhn-Tucker conditions that borrowing only in dollars is optimal if and only if

$$
\begin{aligned}
&E_t\left[M_{t,t+1}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,t+1}\right]\right](1-q(j)) \\
&+E_t\left[M_{t,t+1}\left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell-1}\Omega_{t+1}^{-1}\right)(1+c)\mathcal{E}_{\$,i,t+1}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right]\right]B_{\$,t}(1-q(\$)) \\
&-(1+c(1-\tau))E_t\left[M_{t,t+1}\mathcal{E}_{j,t+1}\right] \\
&+E_t\left[M_{t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right. \\
&\left.-\ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right]\leq 0
\end{aligned}
$$

for all j with the identity for $j=\$$. This inequality can be rewritten as

$$
\begin{aligned}
&E_t[M_{t,t+1}\mathcal{E}_{j,t+1}]((1-q(j))(1+c)-(1+c(1-\tau))) \\
&\leq\ E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{j,t+1}\right]((1-\rho)(1+c)[(1-q(j))+\ell(1-q(\$))]-(1+c(1-\tau)))
\end{aligned}
$$

At the same time, for the dollar debt we get

$$
\begin{aligned}
&E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]((1-q(\$))(1+c)-(1+c(1-\tau))) \\
&=\ E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{\$,i,t+1}\right]((1+\ell)(1-\rho)(1+c)(1-q(\$))-(1+c(1-\tau)))
\end{aligned}
$$

implying that

$$
B_{\$,t}(1+c(1-\tau))\ =\ \left(\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\Omega_{t+1}^{-\ell}\mathcal{E}_{\$,i,t+1}^{1+\ell}\right]}\right)^{\ell^{-1}},
$$

and we get the Kuhn-Tucker conditions

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} \frac{E_t[M_{t,t+1}\mathcal{E}_{j,t+1}]}{E_t\left[M_{t,t+1}\Omega_{t+1}^{-\ell}\mathcal{E}_{j,t+1}\mathcal{E}_{\$,i,t+1}^{\ell}\right]} \;\le\; \frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\Omega_{t+1}^{-\ell}\mathcal{E}_{\$,i,t+1}^{1+\ell}\right]}$$

Q.E.D.

Proof of Theorem 1.3. follows from the following known result.

Lemma C.2 *Suppose that f, g are monotone decreasing and bounded. Then,*

$$\mathrm{Cov}_t(f(X), g(X)) \;\ge\; 0$$

for any bounded random variable X.

Q.E.D.

Proof of Proposition 3.1. We need to compute

$$IRP_t \;=\; \frac{e^{r_t}\mathrm{Cov}_t(M_{t,t+1}, \mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t+1}]}\,.$$

For simplicity, we will assume that all idiosyncratic shocks are identically zero. Define $\tilde{a}_t = -\log S_t$. Our goal is to prove that

$$\begin{aligned} IRP_t + 1 \;&=\; \frac{E_t[M_{t,t+1}\mathcal{P}_{i,t,t+1}]}{E_t[M_{t,t+1}]E_t[\mathcal{P}_{i,t,t+1}]} \\ &=\; \frac{E_t[e^{(\phi+\gamma)\tilde{a}_{t+1}}]}{E_t[e^{\gamma\tilde{a}_{t+1}}]E_t[e^{\phi\tilde{a}_{t+1}}]} \end{aligned}$$

is monotone increasing in ϕ. We have

$$\frac{\partial}{\partial\phi}\log(IRP_t(\phi) \;+\; 1) \;=\; \frac{E_t[e^{\tilde{a}_{t+1}(\phi+\gamma)}\tilde{a}_{t+1}]}{E_t[e^{\tilde{a}_{t+1}(\phi+\gamma)}]} \;-\; \frac{E_t[e^{\tilde{a}_{t+1}\phi}\tilde{a}_{t+1}]}{E_t[e^{\tilde{a}_{t+1}\phi}]}$$

Making a change of measure $d\tilde{P} = e^{\tilde{a}_{t+1}\phi}/E_t[e^{\tilde{a}_{t+1}\phi}]$, we can rewrite the required inequality as

$$\frac{\tilde{E}_t[e^{\gamma\tilde{a}_{t+1}}\tilde{a}_{t+1}]}{\tilde{E}_t[e^{\gamma\tilde{a}_{t+1}}]} > \tilde{E}_t[\tilde{a}_{t+1}]\,,$$

which is a direct consequence of Lemma C.2. Q.E.D.

D Mixture of local currency and dominant currency: Proof of Proposition A.1

We first state the following extension of the Theorem **??** for the case of firms borrowing both in local currency and in dollars.

Theorem D.1 *Suppose that $q = q(\$)$. Then, issuing in a mixture of local currency and dollars is optimal if and only if*

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \le \frac{\mathrm{Cov}_t^{\$}\left(\left(\frac{\Omega_{t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[\left(\frac{\Omega_{t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]}$$

for all $j = 1, \cdots, N$.

Proof of Theorem D.1 and Proposition A.1. The standard Kuhn-Tucker conditions that borrowing only in LC and dollars is optimal if and only if

$$
\begin{aligned}
& E_t\left[M_{t,t+1}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,t+1}\right]\right](1-q(j)) \\
& + E_t\left[M_{t,t+1}\left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell-1}\Omega_{t+1}^{-1}\right)(1+c)\mathcal{E}_{j,t+1}\right]\mathcal{B}_{t+1}(B_t)\right](1-q(\$)) \\
& -(1+c(1-\tau))E_t\left[M_{t,t+1}\mathcal{E}_{j,t+1}\right] \\
& + E_t\left[M_{t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right. \\
& \left. -\ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right] \leq 0
\end{aligned}
$$

for all j with the identity for $j = i, \$$. This inequality can be rewritten as

$$
\bar{q}(j,\$)\frac{E_t[M_{t,t+1}\mathcal{E}_{j,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{j,t+1}\right]} \;\leq\; 1 \;=\; \bar{q}(\$)\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{\$,i,t+1}\right]}
$$

and the first claim follows.

For the LC-\$ mixture, we assume for simplicity that $\ell = 1$. Then, we get the system

$$
\begin{aligned}
1 &= \bar{q}(\$)\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)\mathcal{E}_{\$,i,t+1}\right]} \\
1 &= \bar{q}(\$)\frac{E_t[M_{t,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)\right]}
\end{aligned}
$$

whereby

$$
\mathcal{B}_{t+1}(B_t) \;=\; (1+c(1-\tau))\left(B_t \;+\; B_t(\$)\mathcal{E}_{\$,i,t+1}\right)
$$

Thus, we get the system

$$
\begin{aligned}
E_t[M_{t,t+1}\Omega_{t+1}^{-1}]B_t \ + \ E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(\$) \ &= \ \tilde{q}(\$)E_t[M_{t,t+1}] \\
E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t \ + \ E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]B_t(\$) \ &= \ \tilde{q}(\$)E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]
\end{aligned}
$$

where we have defined

$$
\tilde{q}(\$) = \bar{q}(\$)/(1+c(1-\tau))\,.
$$

Thus,

$$
\begin{pmatrix} B_t \\ B_t(\$) \end{pmatrix} \ = \ \tilde{q}(\$)\Delta_t^{-1} \begin{pmatrix} E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2] & -E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}] \\ -E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}] & E_t[M_{t,t+1}\Omega_{t+1}^{-1}] \end{pmatrix} \begin{pmatrix} E_t[M_{t,t+1}] \\ E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}] \end{pmatrix}.
$$

where

$$
\Delta_t \ = \ E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]E_t[M_{t,t+1}\Omega_{t+1}^{-1}] - (E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}])^2
$$

Thus,

$$
\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \ = \ \frac{-\mathrm{Cov}_t^{\$}(\Omega_{t+1}^{-1}\mathcal{E}_{t,t+1,\$,i},\ \mathcal{E}_{t,t+1,\$,i}^{-1})}{\mathrm{Cov}_t^{\$}(\Omega_{t+1}^{-1},\ \mathcal{E}_{t,t+1,\$,i}^{-1})}\,.
$$

Thus,

$$
\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \ = \ \frac{-\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}{\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}\,.
$$

Let now $\tilde{a}_{i,t+1} \ \equiv \ \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}) - \beta\tilde{a}_{\$,t+1}$ where $\tilde{a}_{\$,t+1} = \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{\$,t+1}})$ and where β is such that $\tilde{a}_{i,t+1}$ and $\tilde{a}_{\$,t+1}$ are uncorrelated.

Recall also that we assume that

$$\log \mathcal{P}_{i,t,t+1} \; = \; -\hat{\alpha}_i \tilde{a}_{\$,t+1} - \alpha_i \tilde{a}_{i,t+1} \; + \; \varepsilon_{i,t+1}, \; \log \mathcal{P}_{\$,t,t+1} \; = \; -\tilde{\alpha}_{\$} \hat{a}_{\$,t+1} + \varepsilon_{\$,t+1}$$

where $\varepsilon_{i,t+1} \sim N(0, \sigma^2_{\varepsilon,i})$. We also allow $\sigma_{\varepsilon,i,\$} \; \equiv \; \mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1}) \; \neq 0$. Then, to the first order in variance, the measure change is irrelevant and

$$\begin{aligned} & - \; \mathrm{Cov}_t^{\$} \left(\left(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}} \mathcal{P}_{\$,t,t+1} \right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1} \mathcal{P}_{\$,t,t+1} \right) \\ & \approx \; -\mathrm{Cov}_t(-\tilde{a}_{i,t+1} - \beta \tilde{a}_{\$,t+1} \; + \alpha_{\$} \tilde{a}_{\$,t+1} \; - \; \varepsilon_{\$,t+1}, -\alpha_{\$} \tilde{a}_{\$,t+1} \; + \; \varepsilon_{\$,t+1} + \alpha_i \tilde{a}_{i,t+1} + \hat{\alpha}_i \tilde{a}_{\$,t+1} \; - \; \varepsilon_{i,t+1}) \end{aligned}$$

whereas

$$\begin{aligned} & \mathrm{Cov}_t^{\$} \left(\left(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}} \mathcal{P}_{i,t,t+1} \right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1} \mathcal{P}_{\$,t,t+1} \right) \\ & \approx \; \mathrm{Cov}_t(-\tilde{a}_{i,t+1} - \beta \tilde{a}_{\$,t+1} + \alpha_i \tilde{a}_{i,t+1} + \hat{\alpha}_i \tilde{a}_{\$,t+1} \; - \; \varepsilon_{i,t+1}, \\ & - \alpha_{\$} \tilde{a}_{\$,t+1} \; + \; \varepsilon_{\$,t+1} + \alpha_i \tilde{a}_{i,t+1} + \hat{\alpha}_i \tilde{a}_{\$,t+1} \; - \; \varepsilon_{i,t+1}) \end{aligned}$$

In the small variance approximation, we that's get

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \; \approx \; \frac{\sigma^2_{\varepsilon,\$} - \sigma_{\varepsilon,i,\$} + \alpha_i \sigma_c^2 + \alpha_{\$}^2 \sigma_c^2(\$) - (\alpha_{\$} + \alpha_i \alpha_{\$}) \sigma_c(i,\$)}{\sigma^2_{\varepsilon,i} - \sigma_{\varepsilon,i,\$} + (1-\alpha_i)(\alpha_{\$} \sigma_c(i,\$) - \alpha_i \sigma_c^2)}$$

where $\sigma_c^2 = \mathrm{Var}_t[\log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}})]$ and $\sigma_c(i,\$) = \mathrm{Cov}_t[\log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}}), \log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{\$,t+1}})]$. The claims (monotonicity in $\sigma_{\varepsilon,i,\$}$ and $\sigma^2_{\varepsilon,i}$) follow then by direct calculation. Q.E.D.