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Barriers to Capital Accumulation and the Incidence of Child Labor

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Monetary and Financial Systems Department

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Abstract

This Working Paper should not be reported as representing the views of the IMF.

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The World Bank documents an inverse relationship between GDP per capita and child labor participation rates. We construct a life-cycle model with human and physical capital in which parents make a time allocation choice for their child. The model considers two features that have shown potential in explaining differences in states of development across nations. These are a minimum consumption requirement, and barriers to physical capital accumulation. We find the introduction of capital barriers alone is not enough to replicate the aforementioned observation by the World Bank. However, we find the interplay of a minimum consumption requirement and barriers to capital may enhance our understanding of child labor and the poverty of nations. Additionally, we find support for policies aimed at reducing barriers to capital accumulation as a means to reduce child labor.

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I. INTRODUCTION

The International Labor Office estimates there are some 206 million children between the ages of 5 and 14 working in developing countries (ILO 2002). Worldwide, the average child labor participation rate stands at a little over 10 percent, with estimated rates more than twice that for most of Africa and as high as 42 percent for Ethiopia (Ashagrie, 1998). Setting aside the ethical issues associated with this subject, the sheer number of children and hours worked suggest tremendous potential losses of human capital and output for these economies. Figure 1 summarizes some evidence from World Bank studies regarding child labor and world income inequality.²

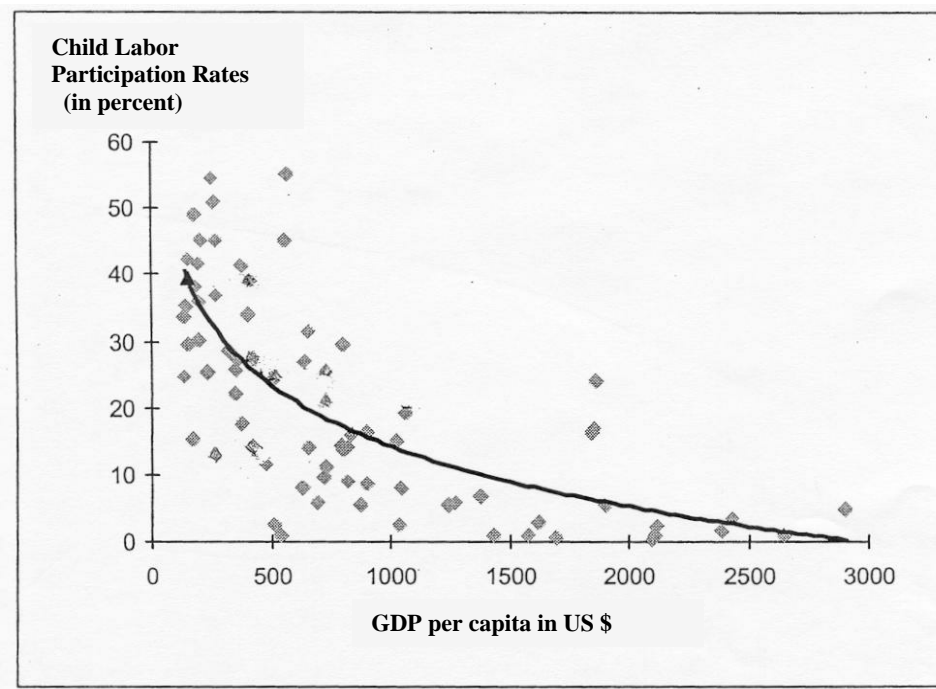


Figure 1. Child Labor and Output

This paper addresses the issue of child labor and its connection to the poverty of nations by introducing child labor and schooling into an otherwise conventional overlapping generations growth model, à la Diamond (1965). Our approach includes two key features which we believe are important in understanding differences in child labor participation rates across countries:

- Agents' preferences include a minimum consumption requirement (MCR).

² Source for Figure 1: World Bank (1997). Also see Helena Skyt Nielsen's website, <http://www.econ.au.dk/afn/phd-summary/hsnielsen.htm>.

- Agents face a barrier to (physical) capital accumulation.

The use of these in the literature has been somewhat limited, but independently, each has had some success in explaining several stylized observations on economic growth and transitional dynamics (Chatterjee and Ravikumar, 1997; Ngai, 2004; Álvarez-Peláez and Díaz, 2005), and world income inequality (Parente, Rogerson, and Wrigth, 2000; Restuccia, 2004). To the best of our knowledge, they have not been incorporated together in a neoclassical growth model. Do they play roles in development economics that extend beyond the scopes originally established in these papers? Our model suggests that in fact they do: low per capita income, child labor, and barriers to capital accumulation are ineluctably linked due to the presence of the MCR. We show that within this environment, the elimination of these barriers should be a priority: not only will it lead to greater physical capital accumulation and an increase in output, it will also reduce the incidence of child labor and increase human capital accumulation. Absent the elimination of such barriers, efforts to ban or restrict child labor may impoverish these nations even further.

In keeping with several recent papers on child labor and human capital, we assume parents make time allocations for their children (see for example, Baland and Robinson, 2000; Ranjan, 2001; Jafarey and Lahiri, 2002; Aiyagari, Greenwood, and Seshadri, 2002; Das and Deb, 2003). An implication underlying this assumption in many studies is that when credit markets prevent parents from committing their children to debt obligations incurred in the procurement of their education and there is an absence of two-sided altruism between parent and child, child labor is inefficient. While this inefficiency naturally carries over to our study any time there are binding constraints on the parents' ability to pass along the financial burden of their children's education, it is not a main thrust of our story, nor is it an especially important aspect of the model. Intertemporal trade restrictions of these sorts in and of themselves seem unlikely candidates for explaining the differences in output and child labor participation rates across developing countries, as shown in Figure 1. They are included, for example, in one version of our model, but that version cannot replicate the stylized relationship represented in Figure 1.³

How do these pieces fit together? With an MCR, the elasticity of intertemporal substitution is an increasing function of agents' wealth—all else the same, agents save proportionally more of their wealth as their wealth increases. Higher capital barriers, on the other hand, lead to lower capital accumulation and lower wealth. Together, the two imply agents will allocate proportionally more resources to current consumption (save less) when confronted with a higher capital barrier. This in turn raises the equilibrium effective return on capital

³ We use one-sided altruism and restrictions on certain trades in credit to preserve the lifecycle properties of the underlying overlapping generations model. When such constraints are either absent or not binding, the model resembles an infinite-lived representative agent model: the steady-state effective return to capital is pinned down by the preference parameters of the model and capital barriers have no impact on the allocation of the child's time. This very last outcome also obtains (for different reasons) when intergenerational trades (bequests) are not permitted and the MCR = 0.

and reduces the present value of the child's future earnings, motivating parents to allocate less of their child's time to school and more to work.

Our treatment of the MCR and its relationship to child labor is quite different from Basu and Van (1997), in which children are sent to work in order to augment the household's income to prevent contemporaneous consumption falling below an MCR. By contrast, we focus on the effect the presence of an MCR has on parents' willingness to substitute intertemporally and the impact this has on the time allocation of children. Our study in fact complements theirs in that it spotlights how an MCR can impact child labor in cases not rooted in extreme poverty.⁴

We should stress that our analysis sidesteps other potential effects capital barriers may have on child labor. For example, Restuccia (2004) addresses the distortions capital barriers may have on the choice of technologies; these in turn impact the allocation of factors between modern and traditional sectors and lower aggregate productivity. Similarly, Parente, Rogerson, and Wright (2000) focus on how barriers impact on the size of an economy's formal and informal sectors. By simple extension of these studies, it is not difficult to see how higher capital barriers may lead to higher instances of child labor if traditional and/or informal sectors, as compared to more modern/conventional ones, find it easier to circumvent existing child labor laws.⁵ Our study explores a more subtle link between distortions in the market for physical capital and the choice of human capital and its implications for child labor, one that does not rely on the differential treatment of hiring practices across sectors of the economy.

A formal description of the model is provided in Section II. Here we derive the agents' consumption and savings, and characterize the decision rule for the allocation of a child's time. The section also contains a description of the model's market-clearing conditions and a definition of a competitive equilibrium. Section III contains the main results of the paper. We discuss the steady-state properties of the model when there is no minimum consumption requirement, establishing, in Proposition 3, that changes in barriers to capital have no effect on child labor allocations in this setting. We then reinstate the minimum consumption requirement, and show, under some mild assumptions on the steady-state returns, that an increase in the capital barrier reduces the time allocated for schooling

⁴ There is some evidence that a sizable portion of child labor is the result of families using a child as a means to augment family income and not simply to ensure a minimum sustainable consumption level. For example, recent surveys of ground transport, battery recharging-recycling, and welding establishments in Bangladesh report that nearly two-thirds of the children working in these industries live in a house owned by their families. Most report that only part of their earnings are given to their families. About half report spending their leisure time watching TV (Bangladesh Bureau of Statistics, 2004a, 2004b, 2004c).

⁵ There are some clear exceptions to these extensions. High child labor participation rates in agriculture, for example, are more likely due to inadequate school facilities in rural areas and to the skill levels required for many agricultural tasks. While recent bans by developed countries on imports of products made with child labor have made it more difficult for children in many developing countries to find employment in formal export sectors, the enforcement of child labor laws in other formal sectors is mixed.

(increases the amount of time the child works). These are accompanied by a decrease in the steady-state capital stock and a decrease in output. Section IV contains a few illustrative examples; Section V concludes.

II. THE MODEL

We use barriers to capital accumulation as one explanation for the differences in output and the incidence of child labor across countries. These barriers are meant to reflect bribes, bureaucratic red tape, or other capital market distortions common to many developing economies. As in Ngai (2004), our approach follows Parente, Rogerson, and Wrighth (2000) in that barriers reduce the efficiency of capital formation by driving a wedge between the return to capital and its marginal physical product. As a consequence, the capital stock depends negatively on the size of the barrier, and, so too, as it turns out in the main version of the model, will the amount of time children spend in school.

We assume an overlapping generations model populated with agents that live three periods. Each agent is endowed with a unit of time each period of life. Agents do not value leisure, and allocations of work-time, when middle aged and old, equal the time endowment of 1 . In the first period of life, children (members of generation t) live with their parents within a family unit which we refer to as a household. Middle-aged agents (parents) head these households, making contemporaneous consumption and time allocations for the family unit, including the amount of time the child spends at work and school.⁶

Schooling enhances the child's labor endowment at each stage of adulthood. The family is dissolved before the start of date $t+1$ when members of generation t become heads of households of their own. For simplicity, members of each generation are identical and the size of each generation is normalized to one.

A. Technologies

A single final good is produced each date using inputs of labor and capital. We assume

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \tag{1}$$

where Y_t denotes aggregate output at date t and K_t , L_t , inputs of capital and labor, respectively. Physical capital is assumed to depreciate fully in the production process.

The presence of a barrier to capital implies

⁶ For simplicity, we assume agents do not consume when young.

$$X_t / \pi = K_{t+1}, \quad (2)$$

where X_t is aggregate investment in physical capital at date t and $\pi \geq 1$ is the capital barrier.

Firms in the economy are competitive and factors are paid their marginal product. The wage rate and rental rate satisfy

$$w_t = (1 - \alpha)AK_t^\alpha L_t^{-\alpha} \quad (3)$$

and

$$r_t = \alpha AK_t^{\alpha-1} L_t^{1-\alpha}. \quad (4)$$

B. Preferences and the Agent's Problem

The head of a household at date t chooses current consumption for herself, c_{1t} , saving, s_t , and human capital for her child, $h(1 - l_t)$, where $h(\bullet)$ represents the human capital production function and l_t is the amount of time the child works. Transfers (bequests) b_t between parent and child are made when the parent is middle-aged and command the same effective return as capital.⁷

When old, the head of the household consumes c_{2t+1} .

The utility U_t of a middle-aged decision-maker at date t is given by

$$U_t = \frac{(c_{1t} - \gamma)^{1-\sigma}}{1-\sigma} + \frac{\beta(c_{2t+1} - \gamma)^{1-\sigma}}{1-\sigma} + \beta \lambda U_{t+1},$$

where $\gamma > 0$ is a minimum consumption requirement and $0 < \beta < 1$ is the discount factor. The parameter, λ , $0 < \lambda < 1$, measures the degree of the parent's altruism.

The choices for c_{1t} , s_t , c_{2t+1} , l_t , and b_t conform to the constraints

$$(C-1) \quad c_{1t} + s_t + b_t \leq w_t [h(1 - l_{t-1}) + l_t] + r_t b_{t-1} / \pi.$$

$$(C-2) \quad c_{2t+1} \leq w_{t+1} h(1 - l_{t-1}) + r_{t+1} s_t / \pi.$$

$$(C-3) \quad c_{1t} \geq \gamma; c_{2t} \geq \gamma; 0 \leq l_t \leq 1.$$

⁷ Our treatment of intergenerational transfers follows Rangazas (2000). A more standard approach assumes transfers between parent and child are made directly (not via the capital market) when the parent is old. This alternative, however, does not change the paper's main results.

$$(C-4) \quad b_t \geq 0.$$

Let $\theta_t \equiv \{l_{t-1}; b_{t-1}\}$ and Ω_t denote the set of state variables at date t beyond the control of the agent. The agent chooses consumption, capital, transfers, and time allocations for the child to solve

$$V(\theta_t; \Omega_t) = \max \left\{ \frac{(c_{1t} - \gamma)^{1-\sigma}}{1-\sigma} + \frac{\beta(c_{2t+1} - \gamma)^{1-\sigma}}{1-\sigma} + \beta \lambda V(\theta_{t+1}; \Omega_{t+1}) \right\}$$

subject to the constraints (C-1) - (C-4).

The household's optimal consumption allocations satisfy

$$(F-1) \quad (c_{1t} - \gamma)^{-\sigma} = \beta(r_{t+1} / \pi)(c_{2t+1} - \gamma)^{-\sigma}$$

The decisions for bequests and child labor time allocations, respectively, satisfy:

$$(F-2) \quad (c_{1t} - \gamma)^{-\sigma} \geq \beta \lambda (r_{t+1} / \pi)(c_{1t+1} - \gamma)^{-\sigma}$$

$$(F-3) \quad w_t (c_{1t} - \gamma)^{-\sigma} \geq w_{t+1} h'(1-l_t) \beta \lambda (c_{1t+1} - \gamma)^{-\sigma} + w_{t+2} h'(1-l_t) \beta^2 \lambda (c_{2t+2} - \gamma)^{-\sigma}$$

When $b_t > 0$ and $0 < l_t < 1$, (F-3) becomes (using (F-2) and (F-3) with equality, along with (F-1) for date $t+1$):

$$w_t = w_{t+1} h'(1-l_t) (\pi / r_{t+1}) + w_{t+2} h'(1-l_t) (\pi / r_{t+1}) (\pi / r_{t+2}) \quad (5)$$

Alternatively, when the nonnegativity constraint on bequests binds and $0 < l_t < 1$, we have

$$w_t (c_{1t} - \gamma)^{-\sigma} = h'(1-l_t) \beta \lambda (c_{1t+1} - \gamma)^{-\sigma} [w_{t+1} + w_{t+2} (\pi / r_{t+2})] \quad (6)$$

Equations (5) and (6) have a standard interpretation. When allocating the child's time, the middle-aged decision-maker equates the marginal utility lost to the household from sending the child to school in the current period, $w_t u'(c_{1t})$, to the marginal utility gained from the increment in the child's lifetime income of an additional unit of schooling,

$\beta \lambda (w_{t+1} + w_{t+2} / (r_{t+1} / \pi)) h'(1-l_t) u'(c_{1t+1})$. If $b_t > 0$, Condition (F-2), with equality, links the marginal rate of substitution between periods for any decision-maker, $u'(c_{1t}) / \beta u'(c_{2t+1})$, and the marginal rate of substitution across consumption of middle-aged decision-makers, $u'(c_{1t}) / \beta \lambda u'(c_{1t+1})$ and both equal the effective return on capital, r_{t+1} / π . Equation (5) incorporates this link. In the steady state, this return equals $1 / \beta \lambda$ and the capital barrier has no impact on child labor. When the nonnegativity constraint on bequests binds, the link is severed, as in (6). Without this link, the fact the parent makes the choice of human capital

for the child very much matters.⁸ Assumption 1 ensures the non-negativity constraint binds in the steady state.

Equation (6) also illustrates why our assumption that an investment in human capital lasts the entire lifetime of the child (and not just to middle age) is critical to the model. If agents work only in middle age, barriers to capital do not distort directly (6)—the reason of course being that the only marginal rate of substitution that matters in the child labor decision is that involving the consumptions of the two middle-aged decision-makers, $u'(c_{1t})/\beta\lambda u'(c_{1t+1})$, which is independent of π and equals $1/\beta\lambda$ in the steady state. On the other hand, as seen in equation (6), the capital barrier makes a difference in this decision when schooling affects the child's human capital when middle aged and old. The child (when it becomes a middle-aged adult) discounts future (date $t+2$) wages by the effective return r_{t+2}/π . A change in the equilibrium return changes the date $t+1$ value of these wages, affecting both the value of these earnings to the child and the benefit to the parent of educating that child.

As mentioned, one feature of this sort of preferences is the fact that the elasticity of intertemporal substitution depends on the agent's wealth. Using (F-1), we obtain

$$\varepsilon = (1/\sigma)(c_{2t+1} - \gamma)/c_{2t+1}.$$

This elasticity is increasing in consumption and lies in the interval $(0, 1/\sigma)$. Of course, when the MCR $\gamma = 0$, ε is constant and equal to $1/\sigma$.

The proposition below provides the savings and bequest decision rules.⁹ Note that if $b_t > 0$, the saving and bequest decisions (though not their sum) are indeterminate.

Proposition 1 *Given bequest b_{t-1} and the parent's human capital, $h(1-l_{t-1})$, the parent's bequest-saving decision is*

$$b_t + s_t = \frac{\gamma [1 - (\beta r_{t+1}/\pi)^{1/\sigma}]}{r_{t+1}/\pi + (\beta r_{t+1}/\pi)^{1/\sigma}} + \frac{(\beta r_{t+1}/\pi)^{1/\sigma} [w_t L_{1t} + r_t b_{t-1}/\pi] - w_{t+1} L_{2t+1}}{r_{t+1}/\pi + (\beta r_{t+1}/\pi)^{1/\sigma}} \quad (7)$$

where $L_{1t} \equiv l_t + h(1-l_{t-1})$ is the effective labor input of the child and the parent and $L_{2t+1} \equiv h(1-l_{t-1})$ is the labor input of the parent in old age.

⁸ Rangazas (2000) make a similar point. In our specific case, it is easy to show the parent underinvests in the child's education (as compared with a similar problem which allows the child to choose to invest earnings w_t or to forgo work in favor of schooling) provided condition (F-2) holds with a strict inequality.

⁹ The consumption decision rules are listed in the Appendix I.

Aggregate investment in physical capital, $X_t = b_t + s_t$. For much of Section III and the remainder of the paper thereafter, we assume the nonnegativity constraint on bequest binds (i.e., $b_t = 0$ for all t). We refer to the steady-state counterpart to (7) as the function $X(r/\pi; w; l)$, where

$$X(r/\pi; w; l) = \frac{\gamma [1 - (\beta r/\pi)^{1/\sigma}]}{r/\pi + (\beta r/\pi)^{1/\sigma}} + \frac{w [(l + h(1-l))(\beta r/\pi)^{1/\sigma} - h(1-l)]}{r/\pi + (\beta r/\pi)^{1/\sigma}}$$

When the nonnegativity constraint binds, (6) summarizes the child labor decision. Solving for the marginal product of schooling,

$$h'(1-l_t) = \frac{((c_{t+1} - \gamma)/(c_t - \gamma))^\sigma w_t}{\beta \lambda (w_{t+1} + w_{t+2}/(r_{t+2}/\pi))}$$

In the steady state, we have:

$$h'(1-l) = \frac{1}{\beta \lambda (1 + \pi/r)}. \quad (8)$$

Equation (8) can be compared with the marginal condition for child schooling in Baland and Robinson (2000). In their two-period model, there is no time discounting, no capital barriers, and the return to capital equals one, so an interior solution for l in their model requires $h'(1-l) = 1$.

In our model it is possible that the barrier to capital π may affect the allocation of child labor, as evident from (8). As mentioned, this result stems from the fact that different values of the barrier may affect the present value of the child's returns to schooling, through changes in the equilibrium effective return r/π . Unlike models of capital barriers with infinitely lived agents, such as Parente, Rogerson, and Wright (2000), the steady-state effective return in a life-cycle model such as this one need not be pinned down by the preference parameter $1/\beta$. It remains to show, then, under what conditions changes in the barrier will affect the equilibrium return r/π , and how the child labor decision changes; both are addressed in Section III.

C. Competitive Equilibrium

Given the initial stock of physical capital, K_1 , and human capital $h(1-l_i)$ for $i = 0, -1$, an equilibrium for this economy consists of sequences for factor payments $\{w_t, r_t\}_{t=1}^\infty$, consumptions $\{c_{1t}, c_{2t}\}_{t=1}^\infty$, and investment, capital, and labor, $\{X_t, K_{t+1}, L_t, l_t\}_{t=1}^\infty$ such that

1. Given the factor payments, the allocations for consumption, investment, and child labor solve the agent's optimization problem.
2. Factor payments at each date given by equations (3) and (4).
3. All markets clear:

$$\mathbf{Labor:} \quad L_t = l_t + h(1 - l_{t-1}) + h(1 - l_{t-2}).$$

$$\mathbf{Capital:} \quad X_t / \pi = K_{t+1}.$$

$$\mathbf{Goods:} \quad c_{1t} + c_{2t} + X_t = Y_t.$$

III. PROPERTIES OF THE STEADY STATE

Our primary focus is on the comparative statics properties of the model's steady state, assuming an interior solution for the child's time allocation.¹⁰ The model's steady state can be summarized by four equations:

the factor payments

$$w = (1 - \alpha)AK^\alpha L^{-\alpha} \tag{3}$$

$$r = \alpha AK^{\alpha-1} L^{1-\alpha}, \tag{4}$$

the marginal condition for the child's schooling,

$$h'(1-l) = \frac{1}{\beta \lambda (1 + \pi / r)}, \tag{8}$$

and the steady-state clearing condition for the capital market,

$$\frac{\gamma [1 - (\beta r / \pi)^{1/\sigma}]}{r / \pi + (\beta r / \pi)^{1/\sigma}} + \frac{w [(l + h(1-l) + rb / \pi)(\beta r / \pi)^{1/\sigma} - h(1-l)]}{r / \pi + (\beta r / \pi)^{1/\sigma}} = \pi K \tag{9}$$

A. Nonbinding Nonnegativity Constraint on Bequests

For most of Section III, we assume the nonnegativity constraint on bequests binds. Our strategy is to show that the model with barriers to capital alone cannot replicate the stylized observations of Figure 1. We then demonstrate that by adding an MCR into the mix, capital barriers can yield a negative relationship between output and child labor, similar in spirit to the World Bank observations. In making our central argument, we rely on a few ancillary steps. These are used to show how the inputs K and L respond to a change in the barrier π .

¹⁰ Throughout, we assume the existence of the steady state.

From there we can infer how output changes with π , since any change in output can be decomposed into changes in the two inputs, $dY/Y = \alpha dK/K + (1 - \alpha)dL/L$. First, however, we state formally a proposition regarding the model's steady state when the nonnegativity constraint on bequests does not bind.

Proposition 2 *Assume steady state bequests $b > 0$. Then $r/\pi = 1/\beta\lambda$ and barriers to capital accumulation have no effect on child labor in the steady-state.*

The first part of the proposition follows from (F-2) with equality, as noted in the previous section. The second part is implied by the marginal condition (8).

B. Binding Nonnegativity Constraint on Bequests

We next address the magnitude of the steady-state effective return r/π in our showcase model with no bequests, as well as the properties of the human capital production function $h(\bullet)$.

Assumption 1. *The steady-state return satisfies $1/\beta < r/\pi < 1/\beta\lambda$.*

Assumption 2. *The function h is increasing, strictly concave, and twice continuously differentiable, with $h(0) = 1$, $h'(0) \geq 1/[\beta\lambda(1 + \beta\lambda)]$ and $h'(1) \leq 1/[\beta\lambda(1 + \beta)]$.*

The lower and upper bounds for r/π in Assumption 1 do two things. First, the lower bound, along with Assumption 2, ensures that the effective aggregate supply of labor is decreasing in child labor in the steady-state (see Result 1 below). It can be shown that $1/\beta < r/\pi$ must hold in any steady-state with a positive capital stock, provided γ is small enough. Second, as we have indicated, the upper bound ensures the nonnegativity constraint for bequests binds in the steady-state. Regarding this, we follow an approach similar to Rangazas (2000); we assume the steady-state return satisfies Assumption 1 and verify in fact it obtains given values for the primitives.

The assumption $h(0) = 1$ means an adult worker with no schooling provides the same quality labor input as a child. The assumptions regarding the marginal conditions of h at the corners, along with the concavity of h and the continuity of its first derivative ensure an interior solution to (8) exists whenever Assumption 1 prevails.

For the remainder of the paper, we assume Assumption 1 holds, so $b = 0$.

We tackle next the issue of how each of the factors changes with a change in the effective return. The first two results address how the effective aggregate labor supply changes, first in response to a given change in child labor, and second, how child labor changes in response to a change in r/π . The third result addresses how r/π affects capital.

Result 1 (*Child labor and effective aggregate labor input*). Suppose $dl > 0$. Then $dL < 0$, where $L = l + 2h(1-l)$ is the effective labor input in the steady state.

Result 2 (*Policy-induced changes in child labor*). $dl \gtrless 0$ whenever $dr/r \gtrless d\pi/\pi$.

Result 3 (*Capital and r/π*). Suppose $dr/r > d\pi/\pi$. Then $dK < 0$.

Proofs of these results are contained in the Appendix I.

Taken together, Results 1-3 establish the point that if an increase in the barrier π raises the equilibrium return r/π , differences in capital barriers across economies will yield a negative cross-sectional relationship between child labor and per capita output, similar to what is shown in Figure 1. Our next proposition illustrates why the minimum consumption requirement is important in understanding the impact of capital barriers on child labor.

Proposition 3 *Suppose $\gamma = 0$. Then $dr/r = d\pi/\pi$.*

Proof: With a MCR $\gamma = 0$, investment is given by

$$X(r/\pi, w, l) = w \left[(l + h(1-l))(\beta r/\pi)^{1/\sigma} - h(1-l) \right] / \left[r/\pi + (\beta r/\pi)^{1/\sigma} \right].$$

Substituting, $w = ((1-\alpha)r/\alpha)(K/L)$ the equilibrium factor price r satisfies

$$\frac{(1-\alpha)(r/\pi) \left[(l + h(1-l))(\beta r/\pi)^{1/\sigma} - h(1-l) \right]}{a \left(r/\pi + (\beta r/\pi)^{1/\sigma} \right) L} = 1,$$

using the market-clearing condition (9). The left-hand side of this expression is homogenous of degree 0 with respect to an equiproportional change in r and π , using Results 1 and 2.

How then does an increase in the capital barrier impact on the economy in this instance? Its impact on output is felt solely through the impact on the steady-state capital stock. Since $dK/K = -1/(1-\alpha) d\pi/\pi$ in this case, $dY/Y = -\alpha/(1-\alpha) d\pi/\pi$. Note that this is the same outcome, if say, the nonnegativity constraint on bequests did not bind (for in this case, the steady-state return satisfies $r/\pi = 1/\beta\lambda$, by constraint (F-3)).

Our main result follows.

Proposition 4. *Let $\gamma > 0$ and $d\pi > 0$. If $dr/r > 0$, then $dr/r > d\pi/\pi$.*

Proof: See the Appendix I.

The difference between Propositions 3 and 4 rests largely on how investment responds to a change in steady-state wages w . In our baseline case, with $\gamma = 0$, the wage elasticity of X is 1. An increase in π decreases K ; from (3), wages fall by $\alpha dK / K$ in this baseline case, and so too will X . With the market-clearing condition (2), we have

$$dK / K = dX / X - d\pi / \pi ,$$

so $dK / K = -1/(1 - \alpha) d\pi / \pi$. From (4), it follows $dr / r = -(1 - \alpha) dK / K = d\pi / \pi$.

By contrast, when $\gamma > 0$, X falls proportionally more than wages. For ease of discussion, suppose the labor supply is constant. Then, as steady-state wages (and wealth) fall, the elasticity of intertemporal substitution falls—the relative consumption allocation of agents shifts in favor of consuming when young. Since $dK / K = dX / X - d\pi / \pi$ and $dX / X < \alpha dK / K$, it follows that $dK / K < -1/(1 - \alpha) d\pi / \pi$. By (4), $dr / r > d\pi / \pi$.

Changes in the labor supply and in the effective return r / π mitigate some of the impact of the fall in wealth on X . The overall impact of a change in π on K , when l is allowed to adjust, lies between the two polar cases with fixed labor inputs, with $l = 0$ and $l = 1$.

Note that the conditioning *if* in Proposition 4 does not appear too restrictive—by (4), $dr / r = -(1 - \alpha)(dK / K - dL / L)$. For the capital return to fall, in this instance, the increase in the capital barrier would need to have a proportionally larger impact on the labor supply than on the capital stock. This does not appear likely, though it cannot be ruled out based solely on our comparative statics.¹¹

Discussion

Our results suggest that child labor participation rates and capital barriers should be positively related. Figure 2 uses observations on child labor participation rates from the World Bank (2000) and relative capital prices from Chad Jones's Website.¹² It depicts a positive relationship between the two variables, consistent with the model's prediction.

¹¹ We could find no numerical example where equilibrium r falls with an increase in π .

¹² <http://elsa.berkeley.edu/~chad/RelPrice.asc>.

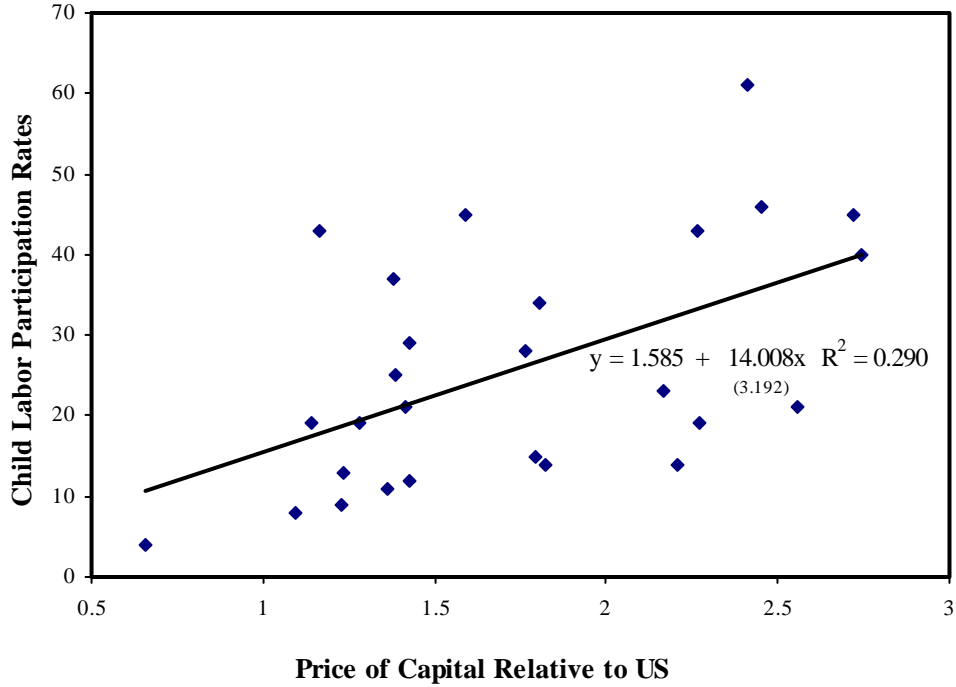


Figure 2. Child Labor and Relative Capital Prices

IV. SOME ILLUSTRATIVE EXAMPLES

A. Barriers, Child Labor, and Output

We illustrate some of the qualitative properties of our featured model using a simple numerical example. Set $\beta = .9$, $\sigma = .9$, $\alpha = .3$, $A = 3$, $\lambda = .3$ and the minimum consumption requirement, $\gamma = .3$.¹³

We assume $h(x) = 1 + v(1 - e^{-\rho x})$. Note that $h(0) = 1$ and $h'(0) = v\rho$ and $h'(1) = v\rho e^{-\rho}$. Setting $v = 6.50$ and $\rho = .45$, Assumptions 1 and 2 and Result 2 are satisfied for $1 \leq \pi \leq 50$. Figure 3 provides a hypothetical cross-sectional plot of child labor and output for economies with the same primitives, varying the capital barrier π from 1 to 50. All observations in Figure 3 are relative to the no-barrier ($\pi = 1$) economy.

¹³ The elasticity of intertemporal substitution ε in this example ranges from .940 to .456; by contrast, when the MCR = 0, the elasticity is $1/.90 \approx 1.111$.

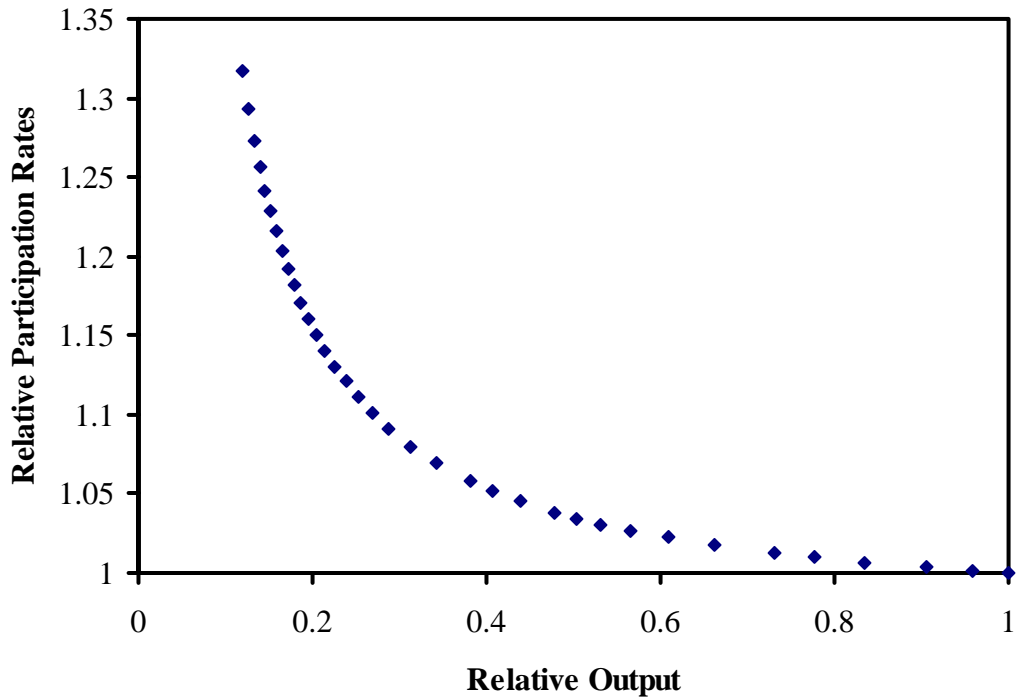


Figure 3. An Illustrative Example of Child Labor and Output

B. Banning Child Labor

Next, we provide an example that illustrates our assertion that unless capital barriers are removed or reduced, a ban on child labor may impoverish a nation further. We assume the same parameter values as above, with the exceptions $\nu = 2.95$ and $\rho = .35$.

Set $\pi = 1$. In this instance, the time allocation is a corner, $l = 1$, since the returns to human capital are so low. Without a ban on child labor, output is 6.498 and steady-state utility is 28.296; with the ban, output increases to 6.743 and utility is 28.334

Now suppose the barrier $\pi = 8$. Without a ban on child labor, output is 2.573 and steady-state utility is 25.274; with the ban, output drops to 2.566 and utility is 25.155.

Why doesn't a ban on child labor increase steady-state output unless accompanied by a reduction in the capital barrier? In this case, the low productivity of schooling means the ban will have a small impact on the overall effective labor supply. The ban also reduces the relative labor supply ratio L_1 / L_2 (from 2 to 1 in this case). This shift in the ratio reduces capital investment, and, since the marginal product of capital is fairly high (due to the high barrier and its effect on K), output can fall, as illustrated in this example. On the other hand, for lower barriers, the marginal product of capital will be lower, and the increase in the

effective labor supply L accompanying the ban more than offsets the impact of a lower capital stock on output, and output rises.¹⁴

V. CONCLUSIONS

This paper provides a general equilibrium model of child labor and physical and human capital. We embed child labor into a standard neoclassical growth model using the assumption the parent makes the schooling/labor decision for the child. The paper focuses on the critical role a minimum consumption requirement may play in explaining observed child labor differences across developing economies. We make the argument that under certain conditions, higher capital barriers can “deepen” child labor participation along an intensive margin. The model also suggests that by reducing capital barriers, developing countries can reduce child labor. Without a reduction in capital barriers, it is not evident that imposing stricter barriers to child labor participation will improve the lot of a country.

¹⁴ In the case with a high barrier, a partial ban on child labor would provide greater output and higher utility than obtained under no ban or under a total ban.

PROOFS AND OTHER DERIVATIONS

A1 Optimal Consumption Rules

$$c_{1t} = \frac{-\gamma \left[1 - (\beta r_{t+1} / \pi)^{1/\sigma}\right]}{r_{t+1} / \pi + (\beta r_{t+1} / \pi)^{1/\sigma}} + \frac{(r_{t+1} / \pi) [w_t L_{1t} + r_t b_{t-1} / \pi + w_{t+1} L_{2t+1} / r_{t+1} / \pi]}{r_{t+1} / \pi + (\beta r_{t+1} / \pi)^{1/\sigma}}$$

$$c_{2t+1} = \frac{\gamma (r_{t+1} / \pi) \left[1 - (\beta r_{t+1} / \pi)^{1/\sigma}\right]}{r_{t+1} / \pi + (\beta r_{t+1} / \pi)^{1/\sigma}} + \frac{(\beta r_{t+1} / \pi)^{1/\sigma} (r_{t+1} / \pi) [w_t L_{1t} + r_t b_{t-1} / \pi + w_{t+1} L_{2t+1} / r_{t+1} / \pi]}{r_{t+1} / \pi + (\beta r_{t+1} / \pi)^{1/\sigma}}$$

A2 Proofs of Results 1-3

Result 1. Using (8) and Assumptions 1 and 2, $\partial L / \partial l = 1 - 2h'(1-l) \leq 1 - 2/[\beta\lambda(1+\beta)] < 0$.

Result 2. Differentiate (8) totally with respect to r , π , and l and solving for dl :

$$dl = - \left((\pi / r) (dr / r - d\pi / \pi) \right) / \left(\beta\lambda(1 + \pi / r)^2 h''(1-l) \right).$$

Since $h(\bullet)$ is strictly concave, $h''(1-l) < 0$. Result 2 then follows.

Result 3. From (4), we have $dK / K = -(1/(1-\alpha))dr / r + dL / L$. If $dr / r > d\pi / \pi$, $dL / L < 0$, from Results 1 and 2. Result 3 then follows, since $dr / r > d\pi / \pi$ implies $dr / r > 0$ whenever $d\pi / \pi > 0$.

A3 Proof of Proposition 4

Begin by totally differentiating X :

$$\frac{dX}{X} = \eta_0 \left[\frac{dr}{r} - \frac{d\pi}{\pi} \right] + \eta_1 \frac{dl}{l} + \eta_2 \frac{dw}{w} \tag{A-1}$$

where $\eta_0 \equiv (\partial X / \partial r)(r / X)$, $\eta_1 \equiv (\partial X / \partial l)(l / X)$, and $\eta_2 \equiv (\partial X / \partial w)(w / X)$. All variables are set at their steady-state levels. Note, for future reference, $0 < \eta_2 \leq 1$, and $\eta_2 = 1$ if $\gamma = 0$, since, in this case, X is linear in w .

Differentiating totally, (8), we have

$$\frac{dl}{l} = \theta_0 \left[\frac{dr}{r} - \frac{d\pi}{\pi} \right] \quad (\text{A-2})$$

where $\theta_0 \equiv -(\beta\lambda\pi/r)(h'(1-l))^2 / (lh''(1-l)) > 0$.

From the factor payments, $K/L = (r/\alpha)^{-1/(1-\alpha)}$; therefore

$$\frac{dK}{K} = \theta_1 \frac{dr}{r} + \theta_2 \frac{dl}{l} \quad (\text{A-3})$$

with $\theta_1 \equiv -1/(1-\alpha) < 0$, $\theta_2 \equiv l(1-2h'(1-l))/(l+2h(1-l)) < 0$, provided $r/\pi > \beta/(2-\beta)$, which is the case if Assumption 1 holds.

From factor payments, wages can be written $w = (1-\alpha)(r/\alpha)^{\alpha/(\alpha-1)}$, so

$$\frac{dw}{w} = \alpha\theta_1 \frac{dr}{r}. \quad (\text{A-4})$$

From the market clearing condition, $X = \pi K$; we have

$$\frac{dX}{X} = \frac{d\pi}{\pi} + \frac{dK}{K}. \quad (\text{A-5})$$

Solving for dr/r , using (A-1) - (A-5), we have

$$\frac{dr}{r} = \left[\frac{1 + \eta_0 + \theta_0(\eta_1 - \theta_2)}{-\theta_1(1 - \alpha\eta_2) + \eta_0 + \theta_0(\eta_1 - \theta_2)} \right] \frac{d\pi}{\pi} \quad (\text{A-6})$$

We need to show the term in the square brackets in (A-6) is greater than 1 when $\gamma > 0$ and $dr/r > 0$.

If $dr/r > 0$, the numerator and denominator of (A-6) must be of the same sign. Since $1 > -\theta_1(1 - \alpha\eta_2) = (1 - \alpha\eta_2)/(1 - \alpha)$ by the fact that $0 < \eta_2 < 1$ when $\gamma > 0$, the numerator of (A-6) is greater in absolute value than its denominator. Hence $dr/r > d\pi/\pi$.

Note that the proof of Proposition 3 is also evident, from (A-6). When $\gamma = 0$, the term in brackets in (A-6) is 1, since, in this case, $\eta_2 = 1$, and $-\theta_1(1 - \alpha\eta_2) = 1$.

References

- Aiyagari, S. Rao, Jeremy Greenwood, and Ananth Seshadri, 2002, "Efficient Investment in Children," *Journal of Economic Theory*, Vol. 102, pp. 290–321.
- Álvarez-Peláez, M.J., and A. Díaz, 2005, "Minimum Consumption and Transitional Dynamics in Wealth Distribution," *Journal of Monetary Economics*, Vol. 52, pp. 633–67.
- Ashagrie, Kebebew, 1998, "Statistics on Child Labour and Hazardous Child Labour in Brief," (unpublished manuscript; Geneva: Bureau of Labor Statistics, International Labour Organization).
- Baland, Jean-Marie, and James A. Robinson, 2000, "Is Child Labor Inefficient?" *Journal of Political Economy*, Vol. 108, pp. 663–79.
- Bangladesh Bureau of Statistics, 2004a "Baseline Survey on Child Workers in Battery Recharging/Recycling Sector, 2002–03" (unpublished manuscript; Dhaka: Planning Division, Ministry of Planning).
- _____, 2004b, "Baseline Survey on Child Workers in Welding Establishments 2002–03" (unpublished manuscript; Dhaka: Planning Division, Ministry of Planning).
- _____, 2004c, "Report of the Baseline Survey of Child Workers in the Road Transport Sector 2003" (unpublished manuscript; Dhaka: Planning Division, Ministry of Planning).
- Basu, Kausik, and Pham Hoang Van, 1998, "The Economics of Child Labour," *American Economic Review*, Vol. 89, pp. 1386–88.
- Chatterjee, Satyajit, and B. Ravikumar, 1997, "Minimum Consumption Requirements: Theoretical and Quantitative Implications for Growth and Distribution," *Macrodynamics*, Vol. 3, pp. 482–505.
- Das, Satya P., and Rajat Deb, 2003, "Policy Dynamics of Child Labor," (unpublished manuscript; Dallas, Texas: Southern Methodist University).
- Diamond, Peter, 1965, "National Debt in a Neoclassical Growth Model," *American Economic Review*, Vol 55, pp.1126–50.
- International Labour Organization, 2002, "Every Child Counts: New Global Estimates on Child Labour," (Geneva: ILO).
- Jafarey, Saqib, and Sajal Lahiri, 2003, "Will Trade Sanctions Reduce Child Labor? The Role of Credit Markets," *Journal of Development Economics*, Vol. 68, pp. 137–56.

Ngai, Liwa Rachel, 2004, "Barriers and the Transition to Modern Growth," *Journal of Monetary Economics*, Vol. 51, pp. 1353–83.

Parente, Stephen L., Richard Rogerson, and Randall Wright, 2000, "Homework in Development Economics: Household Production and the Wealth of Nations," *Journal of Political Economy*, Vol. 108 (4), pp. 680–87.

Rangazas, Peter, 2000, "Schooling and Economic Growth: A King-Rebelo Experiment with Human Capital," *Journal of Monetary Economics*, Vol. 46, pp. 397–416.

Ranjan, Priya, 2001, "Credit Constraints and the Phenomenon of Child Labor," *Journal of Development Economics*, Vol. 64, pp. 81–102.

Restuccia, Diego, 2004, "Barriers to Capital Accumulation and Aggregate Total Factor Productivity," *International Economic Review*, Vol. 45, pp. 225–38.

World Bank, 1997, *World Development Indicators, 1997* (Washington, D.C.).

_____, 2002, *World Development Indicators, 2000* (Washington, D.C.).