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Agents' Preferences, the Equity Premium, and the Consumption-Saving Trade-Off: An Application to French Data

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Agents' Preferences, the Equity Premium, and the Consumption-Saving Trade-Off: An Application to French Data[1]

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Abstract

This paper aims to measure the risk premium on French equities during 1960–92 and to evaluate how well theoretical models based on various representations of agents' preferences can explain it. Aside from the standard, time-additive utility function with constant relative risk aversion, three other utility functions are reviewed: a recursive utility function, a habit formation utility function, and a utility function that accounts for the interdependence of preferences. Both calibration and econometric estimations show that none of the studied marginal changes in the representation of agents' preferences are sufficient to solve both the equity premium puzzle and the risk-free rate puzzle.

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I. Introduction

The equity premium is the difference between the return on equities (a risky asset) and the return on risk-free assets, such as passbook savings accounts or one-year treasury bills, which offer a guaranteed nominal rate of return. Mehra and Prescott (1985) first discussed the *equity premium puzzle*: the standard neoclassical economic model with rational expectations and market equilibrium cannot come close to generating a medium equity premium in line with the one observed in U.S. data, unless agents are assumed to be 10 times more risk averse than shown by the usual rough-and-ready calculations or in experiments and econometric research.[3] Whereas a risk aversion parameter between 1 and 5 seems plausible and in line with the findings of a great deal of research, it would take a risk aversion parameter of at least 40 to generate the equity premium observed in the U.S. data. Furthermore, and in apparently total contradiction with the preceding, econometric estimations of Euler equations from the consumption-based capital asset pricing model on U.S. data, as well as on Japanese (Hamori, 1992) and European data (Danish, German, Swedish, and British; Lund and Engsted, 1996), all reveal a risk aversion parameter that is not significantly different from zero. The estimated value is very small or even negative in some cases, and in any case far outside the range of plausible values. As this paper will attempt to show, the equity premium puzzle is actually a manifestation of the *risk-free rate puzzle*, first discussed by Weil (1989). The two puzzles are related: reproducing the equity premium seems to require very high risk aversion, and reproducing households' choices between consumption and saving seems to require a very high intertemporal elasticity of substitution. Yet the latter would seem to indicate very weak risk aversion in standard models, where the intertemporal elasticity of substitution is the reciprocal of risk aversion.

These puzzles are not mere academic curiosities; they show that standard models cannot capture one of the fundamental operating characteristics of a market economy. Some major macroeconomic and economic policy issues cannot be settled unless these puzzles are resolved. For example, R. Lucas (1987) rounds out his assertion that business cycle–oriented economic policies are ineffective with the contention that economic policies to stabilize the business cycle are useless. He supports this claim by calculating that the gain (in terms of a representative agent's welfare) from smoothing out fluctuations in consumption would be negligible compared with the gain to be derived from even the smallest increase in the consumption growth rate. The weakness of this demonstration lies in the premise that the measurement of the gain to be derived from eliminating fluctuations in consumption is valid under the very same set of assumptions that are incapable of explaining the equity premium. Yet it would be reasonable to expect that the premium demanded by stockholders for holding risky assets and the "welfare cost of fluctuations" would be based on the same economic mechanisms and the same characteristics of agents' preferences. Thus several solutions to these puzzles have been proposed since Mehra and Prescott published their article. Some solutions use a different formulation of agents' preferences; others cite financial market

[3] See Mankiw and Zeldes (1991) and D. Lucas (1994).

imperfections or the diversity of individual agents' situations and tastes. Kocherlakota (1996) and Cochrane and Hansen (1992) provide comprehensive and instructive reviews of the vast literature on this subject.

This paper aims to measure the risk premium on French equities and to evaluate how well theoretical models based on various representations of agents' preferences can explain it. One could argue that this is a waste of time: since we suspect that the French economy is even less well approximated than the U.S. economy by a model operating with flexible prices and rational expectations, the equity premium is bound to be even harder to explain from French data. Yet one could argue that the equity premium puzzle may be not a universal one. For the puzzle to be a relevant issue in macroeconomics, one has to assess its ubiquity. The second section of this paper, then, takes a look at the equity premium in France over the period 1960-92 and examines, through calibration and econometric estimations, whether both the equity premium puzzle and the risk-free rate puzzle are present in the data over that period. Having determined that they are, the next three sections look at how well certain marginal changes to the representative-agent model help solve these puzzles in the French data. We look at the case of agents' *recursive* preferences, the case where agents' utility depends on acquired consumption *habits,* and the case of agents' *interdependent* preferences.

II. THE FRENCH EQUITY PREMIUM: IS IT A PUZZLE?

A. Measuring the French Equity Premium

Relatively long data series are required to study the equity premium. The longest series we found on returns on assets in France are those recently published by Gallais-Hamonno and Arbulu (1995), which cover the period from 1950 to 1992. However, for the sake of consistency with the rest of the paper,[4] we restrict our study of the equity premium to the period from 1960 to 1992. Gallais-Hamonno and Arbulu propose four different indicators for measuring stock returns. The first considers only price changes, as if stock returns consisted solely of capital gains (or losses). The second indicator incorporates dividends into returns, and the third incorporates both dividends and tax credits. The fourth indicator subtracts the management costs that stockholders who reinvest dividends and receive tax credits are assumed to bear. Table 1 presents some descriptive statistics showing changes in these four indicators. In each case the returns are highly variable, because all of the indicators reflect the high volatility in stock prices during the period. The mean annual return on the stock portfolio is 7.08 percent before tax, when the tax credit and dividends are included. The lower part of Table 1 shows the real rates of return on two risk-free assets: a Livret A passbook savings account (-1.27 percent per year) and one-year treasury bills

[4] The data on household consumption used in the rest of the article only go back to 1960.

(-1.64 percent).[5] Both these rates of return are far below the rates of return on stocks, however measured. Obviously, the standard deviations of the rates of return on risk-free assets are much smaller than those for stock returns. In fact, they are little more than one-tenth as large.

Table 1. Real Returns on Risky and Risk-Free French Financial Assets

(In percent per year)

Type of Asset	Annual Mean Return	Mean of Annual Returns	Standard Deviation
Risky assets			
Capital gains	0.23	2.46	21.90
Capital gains + dividends	3.41	5.68	22.41
Capital gains + dividends + tax credit	4.80	7.08	22.69
Capital gains + dividends + tax credit – management costs	3.78	6.05	22.48
Risk-free assets			
Livret A passbook savings account	-1.26	-1.27	2.52
One-year Treasury bills	-1.65	-1.64	3.13

Source: Gallais-Hamonno and Arbulu (1995). Data are for the period 1960-92.

The equity premium is derived directly from the data in Table 1. Several different premiums could be calculated depending on the indicator chosen for stock returns and the risk-free asset used for comparison. Table 2 presents the various possibilities, showing an equity premium between 1.49 percent and 6.45 percent. This is a very wide range, but we feel the most plausible figure is the highest, which is the one derived from the difference between the stock return including the tax credit and dividends, and the yield on one-year treasury bills. This level of equity premium is fully comparable to the premiums found over longer periods. For example, Mehra and Prescott (1985) derived a 6.15 percent equity premium from U.S. secular data series (1889-1978), and Allais and Nalpas (1999) calculated an equity premium of 6.8 percent from French secular data series (1897-1996).

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[5] The nominal interest rate on a Livret A passbook savings account is taken from Table 11-08 of the *Comptes et indicateurs économiques, Rapport sur les comptes de la Nation 1997* (INSEE, 1998b). The nominal one-year treasury bill yield is taken from *Compte de la dette publique* (Ministère de l'économie, 1960-75) and *Compte de la dette publique—Année 1997*, (Ministère de l'économie, 1998, for 1976 to 1993). The deflator for both series is the consumer price index for nondurable goods and services.

Table 2. French Equity Premium for Two Different Risk-Free Assets and Various Definitions of the Return on Equities

(In percent per year)

Definition of equity return	Livret A passbook savings	One-year treasury bills
Capital gains	1.49	1.88
Capital gains + dividends	4.67	5.06
Capital gains + dividends + tax credit	6.06	6.45
Capital gains + dividends + tax credit – management costs	5.04	5.43

Source: Authors' calculations based on data from Gallais-Hamonno and Arbulu (1995). Data are for the period 1960-92.

B. Asset Returns and the Consumption-Saving Trade-Off

To determine the relationship between financial asset returns and agents' behavior, consider the program of an individual with an infinite life span who makes a trade-off between consumption and saving and who is free to choose the allocations of various assets carrying varying risks and paying varying returns in his or her portfolio. When this agent's utility function displays a constant relative risk aversion, the program can be solved sequentially, starting with the optimal allocation of wealth among the various assets according to their returns and risks. In this case the agent's wealth has no effect on the optimal portfolio allocation. One may then compute a mean expected return on the portfolio, on which the agent bases the trade-off between consumption and saving. The agent's program is written as:

$$Max\, U_0 = E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

subject to the wealth accumulation constraint:

$$W_{t+1} = (1+r_{m,t})(W_t - c_t) \tag{2}$$

where:

$$(1+r_{m,t}) = \sum_i \alpha_{i,t}(1+r_{i,t}) \tag{3}$$

with $r_{m,t}$ being the real market return, $r_{i,t}$ being the return on asset i, and $\alpha_{i,t}$ being the share of asset i in the representative agent's wealth. Obviously, $\Sigma_i\, \alpha_i = 1$.

Denoting the risk-free asset return $r_{1,t}$, the first-order conditions may be written as follows:

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}(1+r_{m,t})\right]=1 \tag{4}$$

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}(r_{i,t}-r_{1,t})\right]=0 \tag{5}$$

In the specific case where there are only two assets (one risk-free and the other risky), these equations can be understood straightforwardly: equation (4) shows the trade-off between consumption and saving. It postulates equality between the marginal rate of substitution of consumption and the market return on the asset portfolio between periods t and $t + 1$. Equation (5) shows the choice made between assets with different risk levels and shows the equilibrium equity premium ($r_{i,t} - r_{1,t}$) with regard to the agent's aversion to risk. The equity premium puzzle stems from the lack of empirical evidence to support this simple model.

C. From the Theoretical Model to the Empirical Equity Premium Puzzle

Data

Data on returns on the various assets were described previously. The theoretical model outlined above also incorporates the consumption growth rate. We took consumption of nondurable goods and services from annual data available from 1960 onward in publications of the Institut National de la Statistique et des Études Économiques (INSEE, 1993, 1996, 1999). We derive consumption per capita by using the population data taken from the same publications. The quarterly data do not break down the various components of household consumption by durability of goods. Therefore we worked with total household consumption data (INSEE, 1998b) for the quarterly time series. Tables A1 and A2 in the appendix show the descriptive statistics for the quarterly and annual data. Figure 1 shows the real growth of per capita consumption, along with the real returns on risky assets (capital gains + dividends + tax credits) and on risk-free assets (one-year treasury bills) derived from annual data.

Empirical methodology

The equity premium puzzle arises when it is impossible to come up with plausible values for the parameters of the representative agent's preferences to solve Euler equations, that is, values that can reproduce both the mean equity premium observed in financial markets and the rate of return on the risk-free asset. A more demanding condition for solving the risk premium puzzle would also require reproduction of the second moments of asset returns. Because it is hard enough just to reproduce the first moments (a necessary but not a sufficient condition for the model to be valid), we have restricted ourselves to that objective. Two different methods are used to derive the parameters of the agent's preferences: calibration of Euler equations and econometric estimations of Euler equations. The two methods complement each other since they require different sets of assumptions.

Calibration of Euler equations

For the calibration we apply the law of iterated expectations to the equations derived from the theoretical model. If the Euler equations derived from the theoretical model are true, then the equations obtained by applying the law of iterated expectations (referred to hereafter as transformed equations) are true, too. This gives us a method that is likely to invalidate the theoretical model, since we could conclude that the theoretical model is rejected if the transformed equations are not solved. On the other hand, solving the transformed equations empirically does not imply that the theoretical model is valid. The main problem with calibration is that it does not allow statistical inferences to be made.

Econometric estimations of Euler equations

The second method we use to compare the theoretical model with the data is econometric estimation of Euler equations, using appropriate econometric methods (the generalized method of moments). In this case we seek parameters for agents' preferences that minimize the value of an objective function that combines the mean squared errors of the Euler equations (4) and (5) above. Furthermore, the estimation of the parameters directly incorporates the assumption that agents' expectations are rational, since forecast errors must be orthogonal to all of the information available to the agent when making his or her consumption-saving choices and portfolio allocation decisions. Econometric estimations provide another way of comparing the theoretical model with the data and, potentially, another way of illustrating the equity premium puzzle. As in the calibration exercise conducted above, the estimators obtained for agents' preferences could point to a fundamentally poor fit between the theoretical model and the data, if, for example, the estimated parameter values are implausible or lie outside their defined ranges. In addition, econometrics offers the possibility of statistical inference that provides confidence intervals for the parameter values. Hansen's test provides us with an overall test to see whether the residuals are orthogonal to all the information or not. But most important, the variance-covariance matrix of the residuals provides a minimization criterion that takes account of both the Euler equations and the links between them. This is not possible when calibrating Euler equations, since we deal with each equation separately in this case. Finally, the instability test proposed by Andrews (1993) can be used to test for breaks in the preference parameters. In the case of a structural model with rational expectations, the stability tests for the coefficients could also be interpreted as tests of the validity of the theoretical model.

Other available methods

Other methods have been proposed for comparing the theoretical model with actual data. As a rule, these methods require more restrictive assumptions than those required for calibration or econometric estimation of Euler equations. Examples of the other methods in common use include calibration of the equity premium and the equilibrium risk-free interest rate (derived from Euler equations) as proposed by Mehra and Prescott (1985). This method requires assumptions about the form of the consumption process (its mean growth rate, autocorrelation of the growth rate). Hansen and Jagannathan (1991) propose the construction of bounds that define the admissible range of mean-variance pairs for intertemporal marginal rates of substitution of consumers to solve Euler equations. Kocherlakota (1996) shows that this method has weaker implications than the method used for calibrating equations with iterated expectations. In practice, this means that the rejection of the conditions represented by the bound would lead to the rejection of transformed equations and thus to the rejection of the theoretical model. Therefore, in terms of our strategy for evaluating the equity premium puzzle on French data, these two methods do not add anything to the contribution made by calibrating Euler equations.

Calibrating Euler equations

In line with Kocherlakota (1996), we identify the value of the relative risk aversion coefficient that reproduces the equity premium by simulating the residuals from the Euler equations for various values of this parameter. The idea of calibrating in this way is to test for a risk aversion value (γ) such that the mean of these residuals is equal to zero. More specifically, by applying the law of iterated expectations and denoting the rate of return on the risky asset *re* and that on the risk-free asset *rf*, the optimality conditions for the agent's decisions can be rewritten as:

$$E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}(1+r_{m,t+1})\right]=1 \qquad (6)$$

$$E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}(r_{i,t+1}-r_{1,t-1})\right]=0 \qquad (7)$$

We evaluate the left-hand sides of these equations for the period from 1960 to 1992. Figure 2 plots the means of these evaluations over this period as a function of the degree of risk aversion, with a discount rate set at 0.99.[6] We see that it is impossible to find a risk aversion value such that the means of the residuals from the two Euler equations (denoted *e*1 and *e*2 in Figure 2) both equal zero simultaneously.

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[6] Other values were considered for β but produced no substantial change in the results.

The mean of the residuals from the Euler equation that represents the consumption-saving trade-off ($e1$ in the figure) would be equal to zero for a negative or very weak level of risk aversion. This shows that agents would have to have a strong elasticity of substitution to be willing to save at such a low risk-free rate of return. Furthermore, the risk aversion that meets this Euler condition is an increasing function of the discount parameter (for $\beta = 1$, $\gamma = 0.8$; for $\beta = 0.90$, γ is negative).

Conversely, the equation representing the equity premium would be solved ($e2 = 0$) for a level of risk aversion greater than 40. This is too high to be plausible. To get an idea of the acceptable levels of relative risk aversion, we can calculate how much a risk-averse agent would be willing to pay to avoid a gamble in which the agent has a 50-50 chance of gaining or losing 10 percent of his or her wealth (or his or her consumption). A risk-neutral agent would not be willing to pay anything to escape making this gamble, whereas an infinitely risk-averse agent would be willing to pay up to 10 percent of wealth. For a relative risk aversion of 10, the agent would be willing to pay 4.4 percent of his or her wealth, and 8.4 percent for a relative risk aversion of 40, which seems much too high.

Figure 2 also shows only a very weak dependence of the equity premium on risk aversion. This is explained by a characteristic of the French data that can be highlighted by a simplified version of the theoretical model. If we assume that the joint distributions of returns and the consumption growth rate are normal, using gc to denote the consumption growth rate and solving a Taylor development around $gc = 0$,[7] then equations (6) and (7) can be rewritten as:

$$\begin{aligned} &E(re_{t+1} - rf_{t+1}) - \gamma\left[E(re_{t+1} - rf_{t+1})E(gc_{t+1}) + \operatorname{cov}\left[(re_{t+1} - rf_{t+1}); gc_{t+1}\right]\right] \\ &+\frac{1}{2}\gamma(\gamma+1)\left[E(re_{t+1} - rf_{t+1})E(gc_{t+1}^2) + \operatorname{cov}\left[(re_{t+1} - rf_{t+1}); gc_{t+1}^2\right]\right] \approx 0 \end{aligned} \tag{8}$$

and:

$$\begin{aligned} &\beta E(1 + rf_{t+1}) - \beta\gamma\left[E(1 + rf_{t+1})E(gc_{t+1}) + \operatorname{cov}[rf_{t+1}; gc_{t+1}]\right] \\ &+\frac{1}{2}\gamma(\gamma+1)\left[E(re_{t+1} - rf_{t+1})E(gc_{t+1}^2) + \operatorname{cov}\left[rf_{t+1}; gc_{t+1}^2\right]\right] \approx 1 \end{aligned} \tag{9}$$

where gc_{t+1} is the consumption growth rate between periods t and $t + 1$. We derive the expressions of the equity premium and the risk-free rate from these equations:

[7] The highest consumption growth rate for the period is 4.3 percent, which means that this approximation looks acceptable.

$$E\left(re_{t+1}-rf_{t+1}\right)\approx\frac{\gamma\operatorname{cov}\left[\left(re_{t+1}-rf_{t+1}\right);gc_{t+1}\right]-0.5\gamma\left(\gamma+1\right)\operatorname{cov}\left[\left(re_{t+1}-rf_{t+1}\right);gc_{t+1}^{2}\right]}{1-\gamma E\left(gc_{t+1}\right)+0.5\gamma\left(\gamma+1\right)E\left(gc_{t+1}^{2}\right)}\qquad(10)$$

$$E\left(rf_{t+1}\right)\approx\frac{\gamma\operatorname{cov}\left[rf_{t+1};gc_{t+1}\right]-0.5\gamma\left(\gamma+1\right)\operatorname{cov}\left[rf_{t+1};gc_{t+1}^{2}\right]+\frac{1}{\beta}}{1-\gamma E\left(gc_{t+1}\right)+0.5\gamma\left(\gamma+1\right)E\left(gc_{t+1}^{2}\right)}-1\qquad(11)$$

Table 3 gives the expectations and covariances needed to calculate these expressions. The covariances, particularly between the equity premium and the consumption growth rate, are very weak, whereas the equity premium is high. This means it would take an enormously high level of risk aversion to solve Euler equation (10). On the other hand, Euler equation (11) would be true for a very weak aversion to risk. This is the risk-free rate puzzle highlighted by Weil (1989), who attempted to use calibration to reproduce the equilibrium equity premium and risk-free rate on U.S. data. By way of comparison, the covariance between the consumption growth rate and the rate of return on risky assets on U.S. data is 0.002 (see Mehra and Prescott, 1985). This is more than 200 times greater, yet still not enough to reproduce the equity premium. The equity premium puzzle seems to be even more striking in the French data than in the U.S. data.

Table 3. French Equity Premium and Consumption, Variances and Covariances

	re	*rf*	*re – rf*	*gc*	gc^2
Re	0.0569	0.0023	0.0546	40.0 10^{-6}	-4.5 10^{-6}
Rf		0.0009	0.0014	64.0 10^{-6}	2.5 10^{-6}
Re – rf			0.0568	8.0 10^{-6}	1.0 10^{-6}
Gc				200.0 10^{-6}	10.0 10^{-6}
Gc^2					1.0 10^{-6}

Source: Authors' calculations. Underlying data are annual French data for the period 1960-92.

Finally, an examination of the series of residuals obtained by calibrating equations (6) and (7) shows that the standard deviations of the mean residuals are very high. Thus, for a risk aversion of less than 12, the mean residuals might not be significantly different from zero (see Figure 2). French stock returns are twice as volatile as U.S. stock returns, which means that the equity premium puzzle is different in nature from that found in U.S. data.

Econometric estimations

Parameters β and γ are estimated by applying the generalized method of moments to annual French data from 1962 to 1992, and then to quarterly data from the first quarter of

1972 to the fourth quarter of 1993. Each data set has its advantages and drawbacks, and therefore we felt that it was prudent to use both. Although the annual data make for an easier comparison with results obtained from U.S. data, a major drawback is the small number of observations available in France. This is why we also performed estimations using quarterly data.

Lagged variables were used as instrumental variables. Thus the lagged rate of return and past consumption growth rates, along with a constant, constitute the set of instrumental variables in every case. The number of lags included was determined by increasing the number of lags until the overidentifying restrictions no longer rejected the model. We then kept on increasing the number of lags to make sure that the estimated parameter values were not sensitive to an increase in the number of instruments. This led us to choose two lags for the annual data and four lags for the quarterly data. At the same time, we corrected the autocorrelation of the residuals using the usual procedure first put forward by Newey and West (1987). We fixed the degree of autocorrelation at 1 for the annual data and 2 for the quarterly data.

The results are shown in the first two columns of Table 4. There are striking differences in the estimation results between the annual and the quarterly data. The discount coefficient on the annual data is much greater than 1, whereas that on the quarterly data is only slightly greater than 1. This type of estimation usually produces a discount parameter that is slightly greater than 1, even though this is not consistent with the theoretical model (for example, see Hansen and Singleton, 1982, using U.S. data, or Benabou, 1985, using French data). However, it is rare to obtain estimators that are so much greater than 1 on annual data, as is the case here.

The value of the risk aversion coefficient varies greatly from one estimation to the next. Whereas this coefficient is evaluated at about 3 (and significantly different from zero) on the annual data, the quarterly data produce a coefficient that is not significantly different from zero. One should keep in mind the calibration exercise conducted above when interpreting these results. The level of risk aversion needs to be low, or the preference for present consumption high, to fit the consumption-saving trade-off, but the relative risk aversion needs to be high to reproduce the equity premium. The consumption-saving trade-off represented by equation (4) predominates for both the annual and the quarterly data. We attempted to minimize a quadratic function of the residuals, and decreasing the risk aversion parameter has practically no effect on the residuals of the equity premium equation, even though it reduces the residuals from the consumption-saving trade-off equation. This leads directly to a very weak aversion to risk and a high intertemporal elasticity of substitution in the case of the quarterly data. In the case of the annual data, the pressure for a high intertemporal elasticity of substitution falls on the discount coefficient, which becomes much greater than 1. The predominant role of equation (4) is confirmed when the two equations are estimated separately.[8] Estimation of equation (4) gives a very weak aversion to risk on both

[8] Table 4 does not show these estimations, but they can be obtained from the authors.

the annual and the quarterly data, whereas estimation of equation (5) gives a very high aversion to risk.

Table 4. Econometric Estimations of Euler Equations

	Utility function			
	Additive		Recursive	
Parameter	Annual data, 1963-92	Quarterly data, 1972:1-93:4	Annual data, 1963-92	Quarterly data, 1972:1-93:4
NMA	1	2	1	2
β	1.11*	1.01*	1.03*	1.02*
	(0.027)	(0.002)	(0.017)	(0.002)
γ	3.03*	0.03	0.84*	0.83*
	(0.889)	(0.168)	(0.171)	(0.057)
ρ			0.52	0.82*
			(0.541)	(0.182)
Hansen's test				
N	12	24	11	22
$J(n)$	10.04	25.73	11.56	24.61
P	0.613	0.367	0.397	0.371
Test $\gamma = 1 - \rho$				
$\left[\chi^2(1)\right]$			0.10	1.49
P			0.751	0.222

Source: Authors' calculations.
Note: Estimation method is the generalized method of moments. Annual data consist of 30 observations (plus 2 lagged observations used for the instruments); instruments = $\{1, rf_{t-1}, re_{t-1}, c_{t-1}/c_{t-2}, rf_{t-2}, re_{t-2}, c_{t-2}/c_{t-3}\}$. Quarterly data consist of 88 observations (plus 4 lagged observations used for the instruments); instruments = $\{1, rf_{t-1}, re_{t-1}, c_{t-1}/c_{t-2}, rf_{t-2}, re_{t-2}, c_{t-2}/c_{t-3}, rf_{t-3}, re_{t-3}, c_{t-3}/c_{t-4}, rf_{t-4}, re_{t-4}, c_{t-4}/c_{t-5}\}$. Estimated standard deviations are given in parentheses; * indicates significantly different from zero at the 5 percent level. $J(n)$ is Hansen's test for overidentifying restrictions, which follows a χ^2 distribution with n degrees of freedom. The p-values for the χ^2 test are given in square brackets.

Hansen's test on both annual and quarterly data leads to acceptance of the null hypothesis that the residuals are orthogonal to all the information known, which supports the model. However, there are enough observations of quarterly data to conduct Andrews' (1993) instability tests, and these lead to rejection of the null hypothesis of stability of the coefficients. This result stems from the accuracy of the estimations rather than from the instability of the estimated coefficients. Moving regressions over 50 observations show a risk aversion parameter that ranges from 0 to 1.14.

As Summers (1991) has pointed out, the illustration of the equity premium puzzle is actually less clear here than in the case of calibration, unless we consider that the underlying theoretical model is negated by the failure of econometric estimations to identify a risk aversion parameter that is significantly different from zero or a discount coefficient that is less than 1, or that the Andrews instability tests, which can be seen as a test of the validity of the model in the case of the estimation of structural parameters, lead to the rejection of the null hypothesis of the stability of the parameters.

The theoretical model discussed above cannot account for the facts observed in the French data. Calibration of the equations showed that, for a positive present time preference, the risk aversion parameter has to be simultaneously very high to reproduce the observed equity premium and very low to reproduce the consumption-saving trade-off. In the econometric estimations, this tension leads to very low risk aversion values for the French data. The equity premium puzzle and the risk-free rate puzzle can both be resolved at the same time if the calibration of the consumption-saving trade-off equation requires a higher value and the portfolio allocation equation a lower value for the risk aversion parameter, and if the econometric estimations lead to a higher value for this parameter. For this purpose, we consider various modifications in agents' preferences. We look first at recursive utility functions, in which risk aversion is not necessarily the reciprocal of the intertemporal elasticity of substitution. We then look at the case in which the agents' preferences incorporate the effects of habit, and finally we look at the case in which agents' preferences are interdependent.

III. Recursive Utility and the Equity Premium

One of the usual criticisms of the standard additive utility function is that it does not distinguish between risk aversion and the intertemporal elasticity of substitution. Specifically, parameter γ represents both relative risk aversion and its reciprocal, the intertemporal elasticity of substitution. This criticism is particularly relevant when it comes to accounting for the risk-free rate of return and the equity premium. Intuitively,[9] the risk-free interest rate should depend on the agent's propensity to substitute between present and future consumption, whereas the equity premium depends on the agent's attitude toward risk.

Previous calibrations revealed a tension between the consumption-saving trade-off, on the one hand, which requires a low aversion to risk (meaning a fairly high intertemporal elasticity of substitution) and the trade-off between risky and risk-free assets, which requires a high aversion to risk. This is what Figure 2 shows. The idea, initially used by Weil (1989) and by Epstein and Zin (1991), is that the recursive utility functions proposed by Kreps and

[9] But this is also the result derived from a deterministic general-equilibrium model in which the equilibrium interest rate depends on capital supply and demand, and thus on the intertemporal elasticity of substitution.

Porteus (1978), which distinguish between risk aversion and the intertemporal elasticity of substitution, should make it possible to resolve the puzzle by matching a preference parameter to each behavior. However, the use of such a function does not help to resolve the equity premium puzzle in U.S. data. We examine the French case below by presenting the theoretical model and then the results from empirical investigation methods.

A. The Theoretical Model

Assume that the representative agent maximizes a recursive utility function, in which the reciprocity between risk aversion and the intertemporal elasticity of substitution is broken. To illustrate the agent's attitude with regard to intertemporal substitution alone, we write that the intertemporal utility at date t depends on consumption on that date and the certainty equivalent at date t of the agent's intertemporal utility at date $t + 1$ (denoted $\bar{U}_{t+1}$). These two are linked in a function where the elasticity of substitution between present consumption and the certainty equivalent of future utility is assumed to be constant and equal to $1/(1 - \rho)$:

$$U_t = \left[c_t^{\rho} + \beta\left(\overline{U}_{t+1}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{12}$$

It is helpful to use a constant relative risk aversion function to show the agent's attitude to risk:

$$V_t\left(\overline{U}_{t+1}\right) = U_{t+1}^{1-\gamma} \quad \Rightarrow \quad \overline{U}_{t+1} = \left[E\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}} \tag{13}$$

We end up with a *recursive utility function*:

$$U_t = \left[c_t^{\rho} + \beta\left[E\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{\beta}{1-\gamma}}\right] \tag{14}$$

In the special case where risk aversion is the reciprocal of the intertemporal elasticity of substitution ($\gamma = 1 - \rho$), this utility function takes a fairly standard form:

$$U_t = \left[E_t \sum_{j=0}^{\infty} \beta^j c_{t+j}^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{15}$$

The representative agent maximizes intertemporal utility in equation (14) subject to the constraints of changes in his or her wealth, from equation (2). The Euler conditions derived from solving the optimization program are written as follows:

$$E_t\left[E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{\gamma-1-\rho}{1-\gamma}} U_{t-1}^{1-\gamma-\rho}\left[\frac{c_{t+1}}{c_t}\right]^{\varphi-1}\left(1+rf_{t+1}\right)\right] = 1 \tag{16}$$

$$E_t\left[U_{t+1}^{1-\gamma-\rho}\left[\frac{c_{t+1}}{c_t}\right]^{\varphi-1}\left(re_{t+1}-rf_{t+1}\right)\right]=0 \tag{17}$$

Further assumptions are required to match these equations to actual data, since the unobserved future utility is part of the two Euler equations. There is a corresponding set of assumptions for each empirical method.

B. From the Theoretical Model to the Data

Calibrating Euler equations

It is helpful in calibrating the Euler equations below to assume that the consumption growth rate is independent of time. Under this assumption, the expected future utility is a multiple of future consumption that remains constant over time. By applying the law of iterated expectations, we can rewrite the Euler equations as:

$$E\left[\beta\left[\frac{c_{t+1}}{c_t}\right]^{-\gamma}\left(1+rf_{t+1}\right)\right]E\left[\left[\frac{c_{t+1}}{c_t}\right]^{1-\gamma}\right]^{\frac{\gamma-1+\rho}{1-\gamma}}-1=0 \tag{18}$$

$$E\left[\left[\frac{c_{t+1}}{c_t}\right]^{\varphi-1}\left(re_{t+1}-rf_{t+1}\right)\right]=0 \tag{19}$$

We can see that equation (19), which gives the equity premium, is identical to the one derived from the standard utility function—see equation (7)—and that it does indeed incorporate the risk aversion parameter. Therefore it is obvious from the outset that the equity premium puzzle cannot be resolved by this formalization of preferences. On the other hand, equation (18) incorporates both the intertemporal elasticity of substitution and risk aversion, which leaves an extra degree of freedom in the choice of preference parameters to solve the consumption-saving trade-off equation. We are only interested in the calibration of the first Euler equation in this case, since the second one has already been calibrated above. Recall that that calibration required high values of the risk aversion parameter, but the mean of the residuals was never significantly different from zero because of the high volatility of stock returns.

We calibrated equation (18) as follows: for all risk aversion values from 0 to 40, we sought values for ρ that would make the mean of the residuals from the equation equal to zero. The results are shown in Figure 3 for various values of the discount parameter β. For plausible discount parameter values (less than unity), solution of the equation for the risk-free rate requires negative values for the intertemporal elasticity of substitution ($\rho > 1$). The intertemporal elasticity of substitution only becomes positive for high values of the discount parameter (greater than unity). Thus, what we have is an illustration of the risk-free rate puzzle in French data. To reconcile households' consumption-saving trade-off with the low level of observed risk-free interest rates, either agents must have a preference for future consumption (in which case they wish to save for the future even at negative interest rates), or their marginal utility has to be increasing (which means it is optimal for them to save today even though interest rates are low and their consumption is expected to grow).

This calibration shows that separating risk aversion from the intertemporal elasticity of substitution in a functional form like equation (14) does not provide an answer to either the equity premium puzzle or the risk-free rate puzzle. This leaves the possibility that this failure stems from the assumption that the consumption growth rate is independent of time. Thus, the econometric estimations, which do not require us to make this assumption, are a complement to calibration.

Econometric estimations

We proceed in exactly the same way for the econometric estimations as we did for the estimation of the model with the additive utility function. The estimated Euler equations are obtained following some manipulation of the first-order conditions (see Epstein and Zin, 1991):

$$E_t\left[\beta^{\frac{1-\gamma}{\rho}}\left[\frac{c_{t+1}}{c_t}\right]^{\frac{(1-\gamma)(1-\rho)}{\rho}}\left(1+r_{m,t+1}\right)^{\frac{1-\gamma-\rho}{\rho}}\left(1+r_{i,t+1}\right)\right]-1=0 \quad i=e,f \tag{20}$$

$$E_t\left[\beta^{\frac{1-\gamma}{\rho}}\left[\frac{c_{t+1}}{c_t}\right]^{\frac{(1-\gamma)(1-\rho)}{\rho}}\left(1+r_{m,t+1}\right)^{\frac{1-\gamma}{\rho}}\right]\frac{\rho}{1-\gamma}=0 \tag{21}$$

and we easily show that, in the special case where risk aversion is the reciprocal of the intertemporal elasticity of substitution ($1 - \gamma = \rho$), we get the same Euler equations (4) and (5) as before. One difficulty arises with these specifications, however. We need to know both the mean return on the representative household's portfolio and the share of the portfolio invested in the risky asset. French data provide the share of equities in French households' assets.[10] Yet this is only a proxy for the allocation of wealth between risky and risk-free assets. This means we have to substitute an approximation of households' portfolio allocation for the assumption of time independence required for calibration.

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[10] See *Comptes et Indicateurs Economiques: Rapport sur les Comptes de la Nation*, Table 11-13 (INSEE, 1998b).

The last two columns of Table 4 show the results of the econometric estimations. As was the case for the estimations of the model with an additive utility function, the Hansen test does not reject the orthogonality conditions required for estimation of the structural parameters when the set of instrumental variables incorporates two lags on the annual data and four lags on the quarterly data. The discount coefficient β is greater than unity, as was also the case for the estimation of the model with an additive utility function. This is also a fairly common result of recursive utility functions: see Epstein and Zin (1991) using U.S. data, Bufman and Leiderman (1990) using Israeli data, and Koskievic (1999) using French data. The estimated risk aversion coefficient is the same for both the quarterly and the annual data at about 0.8. Parameter ρ ranges from 0.5 (and not significantly different from zero) on annual data to 0.8 on quarterly data, which puts the intertemporal elasticity of substitution at between 2 and 5, both of which are acceptable values.

In all cases, the likelihood ratio test of the assumption $\gamma = 1 - \rho$ does not reject this restriction. This means that the model with recursive utility is not preferable to the one with additive utility. This result is in contrast to the results that Epstein and Zin (1991) obtained with U.S. data and that Bufman and Leiderman (1990) obtained with Israeli data, which accepted the model with recursive utility as superior to the model with additive utility. Similarly, Koskievic (1999), working with quarterly French data but in another context that did not involve explaining the equity premium and risk-free interest rate, found a much lower level of risk aversion than our estimate ($\gamma = 0.098$) and $\rho = 0.685$ (close to our result here) and ultimately rejected the equation $\gamma = 1 - \rho$.

The econometric estimations and the calibration exercises lead to the conclusion that the use of a recursive utility function is not in itself enough to resolve the equity premium puzzle. Nor does such a utility function make it possible to solve the risk-free rate puzzle. For a better explanation of the equity premium, we can seek formulations of agents' preferences that change the consumption-saving trade-off as well as the allocation between risky and risk-free assets. We consider two utility functions: one that incorporates habit formation and one that accounts for the interdependence of preferences.

IV. Habit Formation and the Equity Premium

A utility function with habit formation is one in which high consumption in the recent past reduces the instantaneous utility of a given level of current consumption. Constantinides (1990) was the first to propose solving the equity premium puzzle by using this characteristic of utility functions. Habit formation has two major implications. First, like the recursive utility function, it makes it possible to weaken the assumption of time separability of the utility function, which causes problems in reproducing the risk-free interest rate. Second, since current consumption decisions affect future utility, the marginal intertemporal utility of current consumption accounts for habit formation, which changes the optimality conditions and, more particularly, the expression of the equity premium. The idea is to see whether there are plausible values for the preference parameter pair (risk aversion, habit formation) that can reproduce both the risk-free rate and the equity premium.

A. The Theoretical Model

We construct a very simple habit formation model by taking a decreasing function of consumption from the previous period as the instantaneous utility. The higher the previous period's consumption, the less satisfaction a given level of consumption procures at a given date. For the sake of simplicity, values with more than one consumption-period lag are not taken into account, contrary to what Constantinides (1990) proposes:

$$U_t = E_t\left[\sum_{s=0}^{\infty} \beta^s \frac{\left(c_{t+s} - \theta c_{t+s-1}\right)^{1-\gamma}}{1-\gamma}\right] \quad 1 > \theta > 0 \tag{22}$$

The habit parameter θ represents the impact of past consumption on present utility. It is positive and less than unity. A negative θ could actually signify a consumption durability effect. Braun, Constantinides, and Ferson (1993) considered this possibility, but their econometric estimations (using Canadian, French, Japanese, British, U.S., and German data) show that the habit formation effect prevails over the durability effect.

When we introduce the utility function in equation (22) into the consumption-saving trade-off and the portfolio allocation model, the two Euler equations are written as follows:

$$E_t\left[\beta\left(\frac{MU_{t+1}}{MU_t}\right)(1 + r_{m,t+1})\right] = 1 \tag{23}$$

$$E_t\left[\beta\left(\frac{MU_{t+1}}{MU_t}\right)(r_{i,t+1} - r_{1,t+1})\right] = 0 \quad i = 2,..N \tag{24}$$

where MU_t is the marginal utility of consumption in period t, so that:

$$MU_t = \left(c_t - \theta c_{t-1}\right)^{-\gamma} - \beta\theta E_t\left[\left(c_{t+1} - \theta c_t\right)^{-\gamma}\right] \tag{25}$$

Clearly, the Euler equation (24) that accounts for the equity premium depends on the habit parameter θ. It is here that this Euler equation differs from the corresponding equation for models with a recursive or an additive utility function, which leaves us with a possible solution to the equity premium puzzle.

We still have to determine what risk aversion is when habit formation is taken into account. In line with Constantinides (1990), a distinction is usually made between consumption-risk aversion (which is sometimes taken to be the reciprocal of the intertemporal elasticity of substitution) and wealth-risk aversion. Consumption-risk aversion represents the effect of consumption risk at a given date on intertemporal utility. Wealth-risk aversion represents the effect on intertemporal utility of a risk to the agent's wealth (meaning the risk to his or her entire consumption program). Both aversions depend on the agent's set of parameters (β, γ, θ). When consumption habits are formed, the consumption risk at a given date has a negative effect on the instantaneous utility at that date, as well as on the instantaneous utility on the following date. This consumption-risk aversion (denoted η) is then greater than γ, and we simply calculate:

$$\eta = -\frac{c_t \partial^2 U_t / \partial c_t^2}{\partial U_t / \partial c_t} = \gamma c_t \frac{(c_t - \theta c_{t-1})^{-\gamma-1} + \beta\theta^2 E_t\left[(c_{t+1} - \theta c_t)^{-\gamma-1}\right]}{(c_t - \theta c_{t-1})^{-\gamma} - \beta\theta E_t\left[(c_{t+1} - \theta c_t)^{-\gamma}\right]} \tag{26}$$

Explicit calculation of wealth-risk aversion requires analytical resolution of the agent's intertemporal program and the value function associated with that program. This is what Constantinides (1990) proposes using a different habit model. The functional form used here does not enable us to calculate wealth-risk aversion that is independent of the initial level of consumption on the basis of a complete analytical resolution of the model. To overcome this difficulty, Lettau and Uhlig (1997) calculate a wealth-risk aversion coefficient by placing very strong constraints on the consumption process. We did not wish to use these restrictive assumptions, which do not lend much significance to the resulting wealth-risk aversion coefficient. Instead, we preferred to focus solely on the consumption-risk aversion coefficient.

B. From the Theoretical Model to the Data

Up until now, we have tested the usefulness of the various assumptions about the utility function for explaining the equity premium and the risk-free interest rate by means of calibration and econometric estimation of Euler equations. In dealing with the habit formation assumption, however, we shall limit ourselves to calibration only, because econometric estimation gives rise to specific problems: it is difficult to identify the various parameters, and it requires fairly laborious treatment. Allais (2000) establishes appropriate econometric methods for French data. Below we compare the results of our calibrations with Allais's econometric results.

As before, we apply the law of iterated expectations to write the equations to be calibrated:

$$E\left[\beta\left(\frac{MU_{t+1}}{MU_t}\right)(1 + r_{1,t+1})\right] = 1 \tag{27}$$

$$E\left[\beta\left(\frac{MU_{t+1}}{MU_t}\right)(r_{i,t+1} - r_{1,t+1})\right] = 0 \quad i = 2,..N \tag{28}$$

and we calculate the marginal utility growth rate, assuming that the future consumption growth rate does not depend on current economic conditions or information currently known:

$$\frac{MU_{t+1}}{MU_t} = \frac{(c_{t+1} - \theta c_t)^{-\gamma} - \beta\theta E\left[(c_{t+2} - \theta c_{t+1})^{-\gamma}\right]}{(c_t - \theta c_{t-1})^{-\gamma} - \beta\theta E\left[(c_{t+1} - \theta c_t)^{-\gamma}\right]} \tag{29}$$

The purpose of this calibration is to identify the set(s) of values for the three parameters (β, γ, θ) that make the mean residuals of the two Euler equations equal to zero. In practice, we seek such values by using the three parameters as variables. For each set of values for the three parameters considered, we first calculate the series $(C_t + 1 - \theta c_t)^{-\gamma}$. The mean of this series gives $E[(C_t + 1 - \theta c_t)^{-\gamma}]$. We proceed in the same manner to calculate the expectation in the numerator of equation (29). In a second step, we can calculate the series $\frac{MU_{t+1}}{MU_t}$, along with the residuals of the two Euler equations. The calibration results are shown in Figure 4 for three different habit parameter values ($\theta = 0$, 0.65, and 0.7), for γ ranging from 0 to 20 and for a discount parameter β of 0.99.[11]

The mean of the residuals from the equity premium equation is nearly independent of the consumption habit formation parameter (the three curves are combined in Figure 4), and it would take a very high value for γ to make the mean of the residuals from the equity premium equation equal to zero. Thus habit formation does not enable us to solve the equity premium puzzle. However, it could explain some of the risk-free rate puzzle. Figure 4 shows that, when the habit formation parameter is high enough ($\theta = 0.7$), the mean of the residuals from the Euler equation representing the consumption-saving trade-off is equal to zero when the value for γ is near 15. In terms of the representative agent's optimal behavior, the mechanism is as follows: when there is no uncertainty, habit formation encourages the agent to consume less and save more in the present in order to ensure consumption growth over time, which is the only source of utility. It is easy to see how this behavior can reconcile the observed positive consumption growth rates, and the underlying saving behavior, with the low level of the risk-free interest rate. This leaves us with the problem of understanding the link, in the case of habit formation, between γ and risk aversion so that we can assess how acceptable a value of γ of around 15 is.

Table 5 shows the values of the relative consumption-risk aversion coefficient recalculated on the basis of equation (26), using the same methods as for calibration of the Euler equations, in order to account for habit formation.

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[11] Other values were considered for β but produced no substantial change in the results.

Table 5. Consumption-Risk Aversion with Habit Formation

	Relative consumption-risk aversion		
γ	$\theta = 0$	$\theta = 0.65$	$\theta = 0.7$
1	1	10.2	14.1
2	2	19.5	26.8
5	5	43.5	59.5
10	10	76.7	106.8
15	15	106.5	156.0
20	20	136.5	226.5

Note: γ is the relative risk aversion parameter and θ the habit formation parameter.

Since there is no obvious way to interpret these values in the case of habit formation, we shall give the same fairly instructive example that we used to illustrate the additive utility function. This is the amount that an agent would be willing to pay to get out of a gamble in which he or she has a 50-50 chance of winning (or losing) 10 percent of consumption at a given date, without this gamble having any effect on other consumption. A simple calculation of the equivalent variation shows that we cannot answer this question without an idea of consumption levels on other dates and without postulating a rate of preference for present consumption. We deemed that the consumption growth rate in the certainty case was 2.7 percent per period, which corresponds to the mean from the sample, and that the agent had no preference for present consumption. Of course, if the habit formation parameter is 0, one would get the same result as in the case of the additive utility function in which the agent with a γ of 10 would be willing to pay 4.4 percent of his or her consumption at a given date to get out of the gamble. With a habit formation parameter of 0.7 and the same value for γ, the agent would be willing to pay 8.8 percent of consumption to get out of the gamble. Similarly, with the value of γ at 5 and no habit formation, the agent would be willing to pay 2.4 percent of consumption at a given date to get out of the gamble, and 6.3 percent if the habit formation parameter value is 0.7.

It seems as if consumption habits intensify the agent's aversion to consumption risk. The calibration of the Euler equations led to a high value for γ and therefore a consumption-risk aversion that seems much too high to be acceptable. Allais (2000) also reached this conclusion after estimating an even higher habit formation parameter (0.7 to 0.86) on French quarterly data and a value for γ on the order of 2, thus obtaining a consumption-risk aversion that he deemed too high. Habit formation (at least with the functional form used here) does not help us make any significant progress in resolving the equity premium puzzle.

V. Preference Interdependence and the Equity Premium

Another way to break down the time separability of the utility function is to consider that an agent is sensitive to the consumption of others as well as his or her own. This means that consumption by others around the agent either increases his or her utility (in the case of an altruistic agent) or decreases it (an envious agent). We refer to this as preference interdependence. In practice, we consider that the "others" that the agent considers include all other individuals in society. The agent's utility function thus incorporates society's mean consumption as well as his or her own. It is easy to see that an agent who is sensitive to society's mean consumption will also be averse to risk and to changes in this mean consumption. Thus the equity premium will be explained by the agent's aversion to fluctuations in society's mean consumption as well as in his or her own consumption.

A. The Theoretical Model

In line with Abel (1990) and Gali (1994), we use an instantaneous agent's utility that is a function of the agent's own current consumption and the current and past levels of society's mean consumption. The intertemporal utility function can be written as follows:[12]

$$U_t = E_t\left[\sum_{j=0}^{\infty}\beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma} C_{t+j}^{\tau} C_{t+j}^{\tau} - 1\right] \tag{30}$$

where C_{t+j} is the level of consumption in the economy. We can rewrite the instantaneous utility function to show the relative levels of consumption in order to make the significance of this utility function clearer:

$$u(c_t, C_{t-1}, C_t) = \frac{1}{1-\gamma}\left[\frac{c_{t+j}}{C_{t+j}^{-\frac{\tau}{1-\gamma}} C_{t+j}^{-\frac{\tau}{1-\gamma}}}\right]^{1-\gamma} \tag{31}$$

Present or past mean consumption increases the individual's welfare (the agent is altruistic) if $-\tau/(1-\gamma)$ is negative. The marginal utility of mean consumption is then positive if $\tau > 0$ and $(1-\gamma) > 0$.

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[12] Another way of modeling preference interdependence is to write, as Campbell and Cochrane (1999) did: $U_t = E_t\left[\sum_{s=0}^{\infty}\beta^s \frac{(c_{t+s} - X_{t+s})^{1-\gamma}}{1-\gamma}\right]$, where χ_t is society's mean level of consumption, which depends on mean consumption levels in the past. The agent takes this past consumption as an exogenous variable.

As before, it is easy to calculate the agent's wealth-risk aversion, which is equal to his or her own consumption-risk aversion in this case (γ) and a mean consumption-risk aversion (τ – 1). This gives:

$$-\frac{c_t \partial^2 U_t / \partial c_t^2}{\partial U_t / \partial c_t} = \gamma$$

$$-\frac{C_t \partial^2 U_t / \partial C_t^2}{\partial U_t / \partial C_t} = \tau - 1$$

Solving the agent's maximization program means considering the consumption of the rest of society as an exogenous variable, meaning that the agent does not take account of the marginal effect of his or her own consumption on society's mean consumption in the maximization program. The Euler equations for the agent's program are as follows:

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} C_{t+1}^{\tau} C_{t-1}^{-\tau}(1+rf_{t+1})\right] = 1$$

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} C_{t+1}^{\tau} C_{t-1}^{-\tau}(re_{t+1}-rf_{t+1})\right] = 0$$

In symmetrical equilibrium, where all agents are identical, we get $C_t = c_t$ ex post, and the Euler equations are rewritten as follows:

$$\beta E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-(\gamma-\tau)}\left(\frac{c_t}{c_{t-1}}\right)^{\tau}(1+rf_{t+1})\right] = 1 \tag{32}$$

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-(\gamma-\tau)}\left(\frac{c_t}{c_{t-1}}\right)^{\tau}(re_{t+1}-rf_{t+1})\right] = 0 \tag{33}$$

We can see that, as in the case of habit formation, the equation representing the equity premium is very different from the one in the model with an additive utility function. This leaves an extra degree of freedom for calibrating the equity premium, which now depends on a preference interdependence parameter as well.

B. From the Theoretical Model to the Data

As we did with the recursive utility function, we calibrated the Euler equations before making the econometric estimations.

Calibrating Euler equations

After applying the law of iterated expectations, the Euler equations are written as follows:

$$\beta E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-(\gamma-\tau)}\left(\frac{c_t}{c_{t-1}}\right)^{\tau}(1+rf_{t+1})\right]=1 \tag{34}$$

$$E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-(\gamma-\tau)}\left(\frac{c_t}{c_{t-1}}\right)^{\tau}(re_{t+1}-rf_{t+1})\right]=0 \tag{35}$$

As above, we calculated the mean of the residuals of these equations on the sample for different values of the preference parameter. The value of the discount parameter β was fixed at 0.99, and several values for the preference interdependence parameter (τ) and the individual consumption-risk aversion parameter (γ) were tried.[13] Figure 5 shows the results. Our conclusion here is similar to that arrived at with habit formation: the mean of the residuals of the Euler equation representing the equity premium ($e2$ in Figure 5) cannot be equal to zero for plausible values of the individual consumption-risk aversion parameter γ. On the other hand, it is possible to find values for the preference parameters that make the mean of the residuals from the Euler equation representing the consumption-saving trade-off ($e1$ in Figure 5) equal to zero. For example, for $\tau + 2$, the mean of the residuals from equation (34) is equal to zero for an individual consumption-risk aversion of 3. Such preferences lead us to assume that agents are envious. Furthermore, a simple calculation, similar to the ones already performed for the other utility functions, shows that an agent is willing to give up 0.5 percent of his or her own consumption to avoid a gamble of 10 percent of society's mean consumption. This looks like a reasonable figure. These calibrations lead us to the conclusion that, on the one hand, preference interdependence does not resolve the equity premium puzzle in French data and, on the other hand, the risk-free rate puzzle can be resolved with plausible values for the preference parameters.

Econometric estimations

Unconstrained estimation of equations (32) and (33) leads to negative values for the individual consumption-risk aversion parameter on both quarterly and annual data. Rather than include penalties in the algorithm to avoid this type of result, we decided to fix the preference interdependence parameter at the values derived from calibration, leaving the other two parameters (individual consumption-risk aversion and the discount rate) to be estimated freely. The results are shown in Table 6 for $\tau = 2$ and $\tau = 5$. The Hansen's tests accept the null hypothesis of errors that are orthogonal to all of the information. When we

[13] Other values were considered for β, but with no substantial change in the results.

compare these results with those obtained under the assumption of an additive utility function (Table 4), we see that preference interdependence enables us to obtain lower values for the discount parameter and higher values for the individual consumption-risk aversion parameter. However, this effect is only marginal, and the consumption-saving trade-off equation still prevails in econometric estimation, since the equity premium equation is not very sensitive to changes in the preference parameters. It should be remembered that the consumption-saving trade-off requires a strong intertemporal elasticity of substitution, which led to very low values for risk aversion in the case of an additive utility function. In the case of preference interdependence, the introduction of aversion to risk to society's mean consumption makes higher values possible for individual consumption-risk aversion. This leads to the same conclusion as that arrived at through calibration: preference interdependence makes it possible to resolve the risk-free rate puzzle. In this case it gives acceptable values, which are not too low, for the estimation of the agent's relative consumption-risk aversion.

Table 6. Econometric Estimations of Euler Equations: Preference Interdependence

	Annual data, 1963-92		Quarterly data, 1972:1-93:4	
Parameter	(1)	(2)	(3)	(4)
NMA	1	1	2	2
τ	2	5	2	5
β	1.07*	1.06*	1.00*	0.99
	(0.023)	(0.033)	(0.002)	(0.004)
γ	5.77*	11.72*	2.45*	6.57*
	(0.818)	(1.246)	(0.247)	(0.660)
Hansen's test				
N	12	12	28	28
$J(n)$	9.80	9.72	23.44	22.66
P	0.633	0.640	0.711	0.750

Source: Authors' calculations.
Note: See table 4 for details of the data, the estimation method, and statistical tests.

VI. Conclusion

Using the the additive and isoelastic utility function that is standard in macroeconomics, one cannot identify a risk aversion parameter from either French or U.S. data that makes it possible to reproduce both the risk-free rate and the equity premium. Reproducing the risk-free rate requires a very low aversion to risk (or, in other words, a very strong intertemporal elasticity of substitution), whereas the equity premium is a very slightly increasing function of risk aversion, because of the weak observed covariance between the consumption growth rate and risky asset returns. However, the variability of returns on the risky asset under consideration (French equities) is so great (twice as great as that calculated for U.S. data) that the equity premium puzzle is not statistically significant. This is the conclusion arrived at through econometric estimation of the optimality equations for the representative agent. Risk aversion is estimated at a very low value, which makes it possible to reproduce the rate of return on risk-free assets practically independently of the rate-of-return equation for risky assets.

The recursive utility function, which distinguishes between risk aversion and its reciprocal, the intertemporal elasticity of substitution, does not provide a satisfactory solution to the equity premium puzzle or to the risk-free rate puzzle, nor does the utility function with habit formation. In the case of preference interdependence, calibration and econometric estimations lead to the assumption that agents are envious, in that the consumption of others reduces their welfare. This does not enable us to reproduce the equity premium, but it does let us reproduce the consumption-saving trade-off with plausible values for the preference parameters and thus resolve the risk-free rate puzzle.

This paper, which focuses on agents' preferences, does not consider some other avenues for resolving the equity premium puzzle. Kocherlakota (1996) shows that, even though it is tempting to imagine that the source of the equity premium puzzle is the incompleteness of financial markets, and more particularly the difficulties that agents have in covering themselves against income risk, taking this incompleteness into account has not yet made it possible to resolve the puzzle. Similarly, including transactions costs on financial assets does not help to resolve the equity premium puzzle, unless we assume that there are great differences in these costs for risky assets and risk-free assets. Mankiw and Zeldes (1991) tried to restrict the application of the consumption-based capital asset pricing model to stockholders only. They conclude that this restriction gives a lower value for the relative risk aversion coefficient, but it is not enough to resolve the equity premium puzzle. Jorion and Goetzmann (1999) take up the idea put forward by Rietz (1998) that the theoretical model of Mehra and Prescott (1985) does not account for the possibility of a stock market crash, which may explain why it cannot account for the equity premium at an acceptable level of risk aversion. Jorion and Goetzmann turn this idea around by asserting that the historical data series most commonly used to measure the equity premium omit the losses recorded on markets that experienced such extreme events as temporary suspensions or that disappeared altogether. These data lead to overestimation of the equity premium, which Jorion and Goetzmann demonstrate on the basis of data combining space and time dimensions and including financial markets that suffered such extreme events.

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Appendix

Table A1. Descriptive Statistics for Annual Data, 1960–92

(In percent)

	Mean	Standard Deviation	Minimum	Maximum
Real consumption growth rate	2.7	0.9	0.7	4.3
Treasury bill rate	-1.7	3.1	-10.5	3.0
Risky asset return	6.7	24.2	-34.4	58.8
Equity premium	8.4	23.4	-29.2	60.0

Source: Authors' calculations based on data from Gallais-Hamonno and Arbulu (1995).

Table A2. Descriptive Statistics for Quarterly Data, 1970:2–93:4

(In percent)

	Mean	Standard Deviation	Minimum	Maximum
Real consumption growth rate	0.5	0.8	-1.3	2.7
Treasury bill rate (real)	-1.2	1.0	-3.9	0.4
Risky asset return (real)	2.2	10.4	-33.0	27.0
Equity premium	3.3	10.3	-32.8	30.1

Source: Authors' calculations based on data from Gallais-Hamonno and Arbulu (1995).

Table A3. Equity Premium and Consumption, Variances and Covariances

	re	*Rf*	*re – rf*	*gc*
Re	0.0107	0.0002	0.0105	$-5.8\ 10^{-5}$
Rf		0.0001	$6.6\ 10^{-5}$	$1.2\ 10^{-5}$
re – rf			0.0105	$-7.0\ 10^{-5}$
gc				$6.0\ 10^{-5}$

Source: Authors' calculations.

Figure 1. Asset Return Rates, Equity Premium, and Consumption Growth Rate

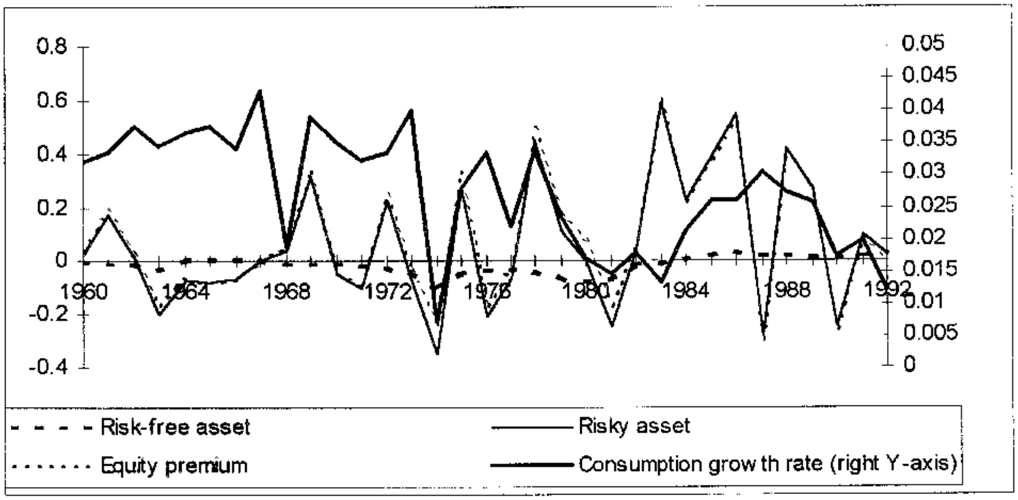
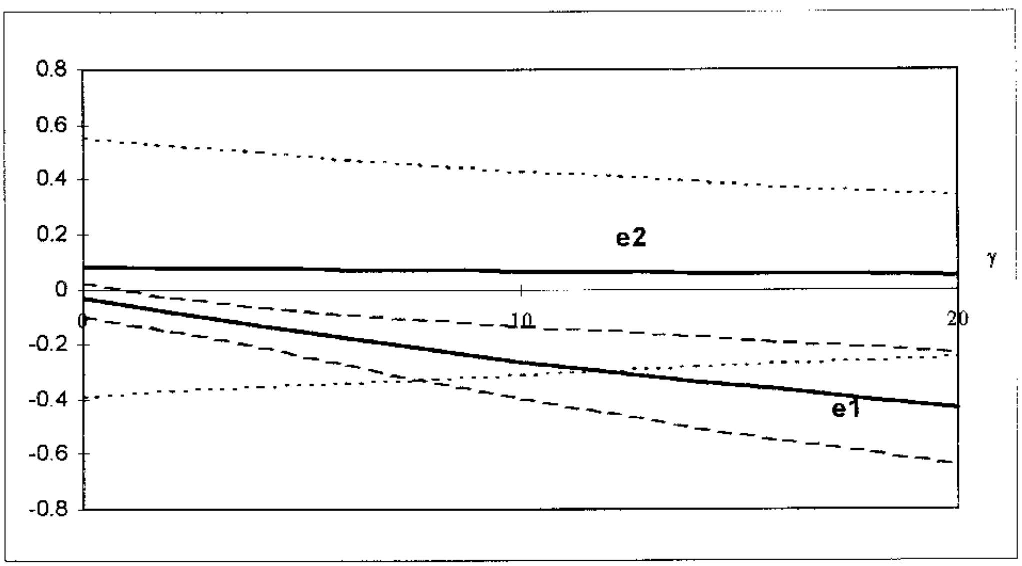


Figure 2. Means and Confidence Intervals
of the Residuals from Euler Equations as a Function of Risk Aversion[1]



[1]Confidence intervals are plus or minus 2 standard deviations.

Figure 3. (γ,ρ) Pairs That Solve the Euler Equation for the Consumption-Saving Trade-Off

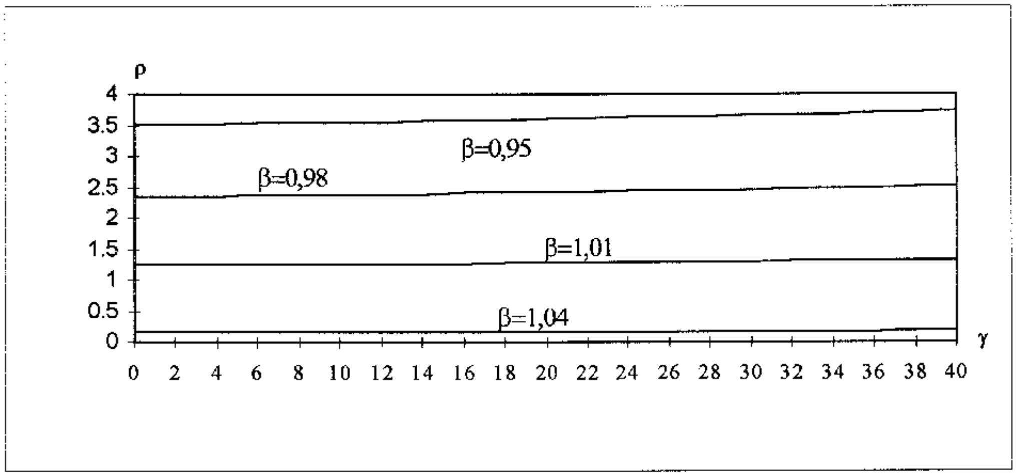


Figure 4. Means of the Residuals of the Euler Equations as a Function of γ for Different Values of the Habit Formation Parameter

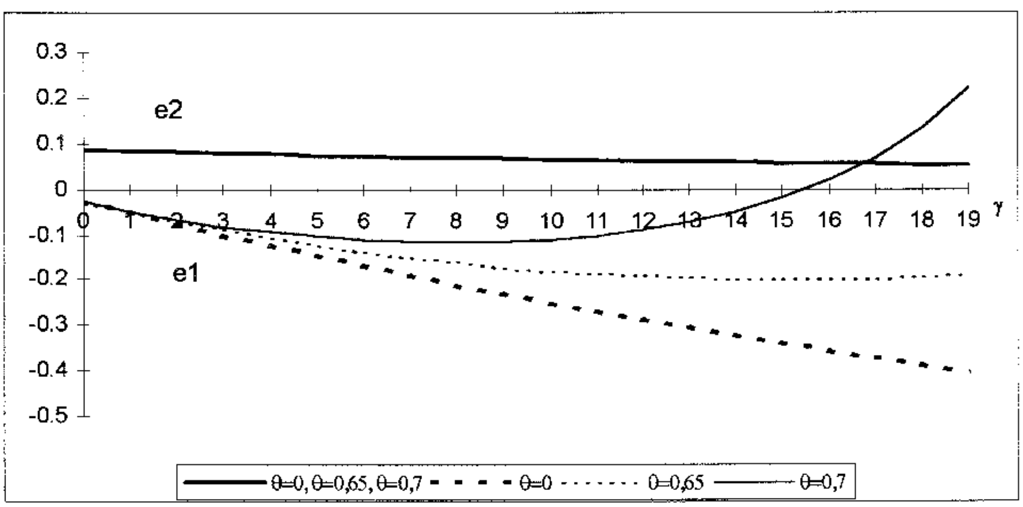


Figure 5. Means of the Residuals of the Euler Equations as a Function of γ for Different Values of the Preference Interdependence Parameter