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Measuring and Analyzing Sovereign Risk with Contingent Claims

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This paper develops a comprehensive new framework to measure and analyze sovereign risk. Contingent claims analysis is used to construct a marked-to-market balance sheet for the sovereign and derive a set of forward-looking credit risk indicators that serve as a barometer of sovereign risk. Applications to 12 emerging market economies show the approach to be robust, and the risk indicators are a significant improvement over traditional macroeconomic vulnerability indicators and accounting-based measures. The framework can help policymakers design risk mitigation strategies and rank policy options using a calibrated structural model unique to each economy. [JEL E61, G13, G15, H63]
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As economies have become more reliant on private capital flows, they have also become more vulnerable to the volatility of capital flows and associated market, liquidity, and credit risk. A comprehensive framework of

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risk identification and management is needed to analyze—and hopefully help prevent—large-scale capital account crises and associated financial distress. A useful approach that has been gaining popularity since the Asian crisis is to assess the risk posed by potentially unstable positions in sectoral balance sheets, including the corporate, financial, and public sectors. Shocks to interest rates, exchange rates, or market sentiment that bring about a deterioration in the value of a sector's assets compared to its liabilities lead to a reduction in net worth. In an extreme case, net worth turns negative and the sector may become insolvent, triggering widespread distress and transferring risk across balance sheets. Risk transfer can be "bottom-up" from the corporate sector to the banking system and ultimately to the sovereign balance sheet, as was the case during the Asian crisis, or it can be "top-down," as was seen more recently in Latin America. Developing an effective approach to detect and assess balance sheet vulnerabilities before they become severe is essential to minimize risks and protect the stability of the overall economy.

The main purpose of this paper is to show how modern contingent claims analysis can be used to measure and analyze risk stemming from the public sector balance sheet. Estimating risk using such an approach has a long tradition in modern financial theory. The approach has been widely applied in the analysis of corporate sector credit risk (for example, Black and Scholes, 1973; Merton, 1973 and 1998; McQuown, 1993; Sobehart and Stein, 2000; Crouhy, Galai, and Mark, 2000; and Cossin and Pirotte, 2001) and it is increasingly being used to estimate risk in the financial sector (for example, Merton, 1977; Kupiec, 2002; and Chan-Lau, Jobert, and Kong, 2004), but the analysis has yet to be broadly applied at the sovereign level. This paper represents an important step in this direction.

Contingent claims analysis is effective for risk management because it identifies existing balance sheet mismatches, incorporates uncertainty inherent in balance sheet components, and translates uncertainty into quantifiable risk indicators that reveal whether balance sheet risks are building or subsiding. The approach uses the structure and seniority of the consolidated sovereign balance sheet with market prices and uncertainty as key inputs to derive simple, forward-looking indicators of risk. In the process of measuring sovereign risk, the approach derives estimates for future sovereign asset value and asset volatility, which are otherwise not directly observable, and weighs these values against existing contractual liabilities to provide a market-based assessment of sovereign default risk. The incorporation of uncertainty and sovereign asset volatility is important in sovereign risk analysis because uncertain changes in future asset value relative to promised payments on debt obligations ultimately drive default risk. Therefore, the approach differs markedly from traditional macroeconomic vulnerability indicators and accounting-based measures that do not address uncertainty in a comprehensive or forward-looking manner given their reliance on static balance sheet ratios.

This paper develops a set of key credit risk indicators to measure sovereign balance sheet risk. These include the distance to distress, probability of default, credit spreads, and the market value of risky foreign currency–denominated debt. These indicators are closely related because they are all derived from the core contingent claims relationships. Associated with these risk indicators are sensitivity measures that report how responsive the credit risk indicators are to changes in underlying model parameters, such as changes in the value of sovereign assets and volatility. Importantly, the sensitivity measures capture nonlinear changes in value that are often observed during crisis periods and are crucial for a full understanding of risk.

The model credit risk indicators are subjected to robustness tests using observed market data for a sample of emerging market countries in order to illustrate the usefulness of the model output as a collective barometer of sovereign risk. The tests suggest a high degree of correlation between the credit risk indicators and the observed market data on sovereign bond spreads, spreads on sovereign credit default swaps (CDS), and implied default probabilities from CDS markets. Because this market information was not used as inputs in deriving the model risk indicators, the high correlation suggests that the risk indicators can be confidently used as reasonable measures of sovereign credit risk and lend support to the contingent claims structural model developed in this paper. The risk indicators can be examined in individual country cases to evaluate whether market expectations of sovereign vulnerabilities are increasing or decreasing over time, or they can be examined across countries to rank relative risk.

As a further demonstration of the applicability of contingent claims analysis in evaluating sovereign risk, the paper uses the model to evaluate how risk indicators change given specific scenarios. Through scenario analysis, policymakers can observe the extent to which negative economic shocks could worsen sovereign financial soundness through capital outflows, a depreciating exchange rate, or slower economic growth. As an additional step, Monte Carlo simulations are used to yield probability distributions and confidence intervals for the set of sovereign credit risk indicators. Because simulations allow for the assessment of many potential market scenarios, it provides for a more comprehensive risk analysis that includes probability distributions and value-at-risk (VaR) measures. Policymakers can use these tools to help them design and implement risk mitigation strategies to reduce balance sheet risk and to rank competing policy choices.

Finally, the paper points to two promising areas in which contingent claims analysis can be usefully applied in sovereign risk mitigation: reserve management and debt sustainability. The framework can be used to derive an appropriate target for reserve adequacy, where an adequate level of reserves is defined as the level of reserves that keep the credit risk indicators above a specified threshold (or below, in the case of default probability). The framework also offers several advantages over traditional debt sustainability analysis that has tended to focus on ratios of debt to GDP as the primary criterion for deciding whether public debt is on a sustainable path. In particular, the approach provides a structural framework that relates debt payments with the capacity to pay, linking reserve levels and fiscal policy with threshold levels for sovereign credit risk.

I. A Practical Approach to Sovereign Risk

Sovereign default is driven by the relationship between the level of sovereign assets, sovereign asset volatility, and leverage. Sovereign assets generally include current and future surpluses, international reserves, and other assets.[1] Because the value of sovereign assets is dependent on the country's future economic prospects and policy decisions, and these future economic prospects are uncertain, asset volatility captures the inherent uncertainty of future sovereign asset value. Leverage measures the size of the sovereign's contractual liabilities. Contractual liabilities are measured in book value terms because these are the amounts that the sovereign is obligated to pay.

Sovereign distress increases when the market value of sovereign assets declines relative to its contractual obligations on debt or when sovereign assets become volatile.[2] Default ultimately occurs when the sovereign assets fall below the contractual liabilities. Contractual liabilities, therefore, constitute a distress barrier, and sovereign distress is measured by the interplay between sovereign assets relative to this distress barrier. Default risk increases when the value of sovereign assets declines toward the distress barrier or when asset volatility increases (for example, the value of sovereign assets becomes more uncertain).[3] In either case the probability of the value of sovereign assets falling below the distress barrier increases.

In analyzing a database having more than 250,000 company-years of data and more than 4,700 incidents of bankruptcy or default, Crosbie and Bohn (2003) find evidence that the market value of firm assets can sometimes trade below the book value of total liabilities for a significant period of time. This is most often the case when the majority of liabilities are long term, allowing the firm to continue servicing debt payments while undertaking steps to improve the financial health of the firm. A similar argument can be applied to sovereign credit risk, whereby the probability of distress is increased when most of the liabilities are short term, or when rollover risk is highest. Therefore, the approach adopted in this paper follows the well-established procedure in estimating corporate default risk, namely, that the value of sovereign assets that triggers an incidence of sovereign distress lies

[1]See Buiter (1993) for a discussion of the many items on the balance sheet of the public sector, including nonmarketable items such as social overhead capital.

[2]Xiao (2007) shows that an increase in volatility dampens demand for sovereign bonds.

[3]Volatility of sovereign assets can differ across countries for many reasons, including, but not limited to, the level of international reserves on the government's balance sheet, the exchange rate, and variations in government revenue and expenditures. Countries with lower asset volatility are generally able to use larger amounts of leverage with relative comfort whereas countries with higher asset volatility would be better off taking on less leverage.

somewhere between the book value of total liabilities and short-term liabilities. This adjusted value of liabilities is used as the distress barrier.

The market value of sovereign assets in relation to the distress barrier is illustrated in Figure 1. The uncertainty in future sovereign asset value is represented by a probability distribution at the time horizon at period T. At the end of the period, the value of sovereign assets may be above the distress barrier, indicating that debt service can be made, or below the distress barrier, leading to default. The probability that sovereign assets will fall below the distress barrier is simply the area of the distribution that lies below the distress barrier. Any decline in expected sovereign asset value means more of the probability distribution lies below the unchanged distress barrier and thus there is a higher probability of default. If the lower expected sovereign asset value is also accompanied by an increase in asset volatility, then this will widen the probability distribution, leading to an even higher probability of default because more of the area under the probability distribution now lies below the distress barrier.

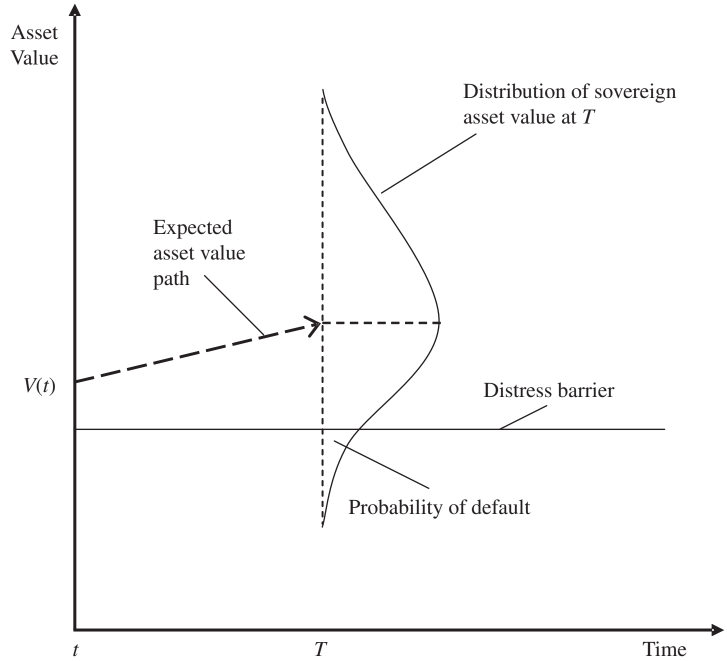
Estimating the Value of Sovereign Assets

Given the conceptual definition of sovereign distress, the main challenge lies in deriving an accurate estimate for the market value and volatility of sovereign assets. Although the levels and amounts of contractual liabilities are relatively easy to determine from balance sheet information, the same is not true when measuring the value of sovereign assets or its volatility.[4] The market value of sovereign assets is not directly observable and must therefore be estimated. One approach would be to determine value from observed market prices of all or part of the asset. This can be accomplished using a market price quote, through direct observation, or using bid-ask quotes or other similar direct measures. A second method would be to determine value using a comparable asset or adjusted comparable asset. A sophisticated version of obtaining a comparable value is the present value of discounted expected cash flows—such as the primary surplus—with an appropriate discount rate. Finally, a third method would be to determine an implied value where the balance sheet relationships between assets and liabilities allow the observed prices of liabilities to be used to obtain the implied value of the assets.

The three methods have different advantages and disadvantages. The first method is straightforward but difficult to apply because only a few components of sovereign assets have directly observable market prices. International reserves are observable and have a market value, yet the remaining

[4]Foreign currency debt in global markets is predominantly fixed-rate, "bullet" maturity debt that results in easily defined contractual flows. Some global debt is amortizing, but these payments are usually well specified. The main difficulties in estimating debt payments arise when the debt payments are linked to changes in interest rates, exchange rates, or inflation. These forms are more often found in domestic as opposed to global capital markets.

Figure 1. Distribution of Sovereign Asset Value and the Distress Barrier



Note: Sovereign default is driven by the relationship between the level of sovereign assets, sovereign asset volatility, and leverage. The uncertainty in future sovereign asset value is represented by a probability distribution, while contractual obligations on debt constitute a distress barrier. At the end of the time horizon *T*, sovereign assets may be above the distress barrier, indicating that debt service can be made, or below the distress barrier, indicating default. The probability of default is the area under the distribution that lies below the distress barrier.

items on the public sector balance sheet lack observable market prices. The second method, using comparables, is commonly used but also has shortcomings related to the difficultly of projecting future cash flows, deciding the appropriate discount rate, and determining all of the relevant components that underlie the cash flow projections for tangible and intangible items included in the asset value estimation. For example, determining the present value of the net fiscal asset requires estimates of future economic performance, the political commitment to a variety of programs including social security and other entitlement programs, and the use of an appropriate discount rate. Estimates for the value of other assets, such as the value of the public sector monopoly on money issuance, run into similar problems. Finally, it is unclear how asset volatility should be best measured under the first two methods.

The third method, which is the approach adopted in this paper, circumvents the problems in the first two methods by estimating sovereign asset value and volatility indirectly with observable information from the liability side of the balance sheet. Because liabilities are claims on current or future assets, this approach is often referred to as "contingent claims" analysis and yields an "implied" estimate for sovereign assets. The calculation of implied values is a very common technique in the finance world. The collective views of many market participants are incorporated into the observable market prices of liabilities, and changes in these market prices determine its volatility. Contingent claims analysis implicitly assumes that market participants' views on prices incorporate forward-looking information about the future economic prospects of the sovereign. This does not imply that the market is always correct about its assessment of sovereign risk, but that the prices reflect the best available collective forecast of the expectations of market participants.

Because contingent claims analysis relies on the balance sheet relationship between assets and liabilities, implementation of the approach requires constructing a contingent claims sovereign balance sheet that explicitly links observable liabilities to assets. This process requires several steps and assumptions that are discussed in the next section.

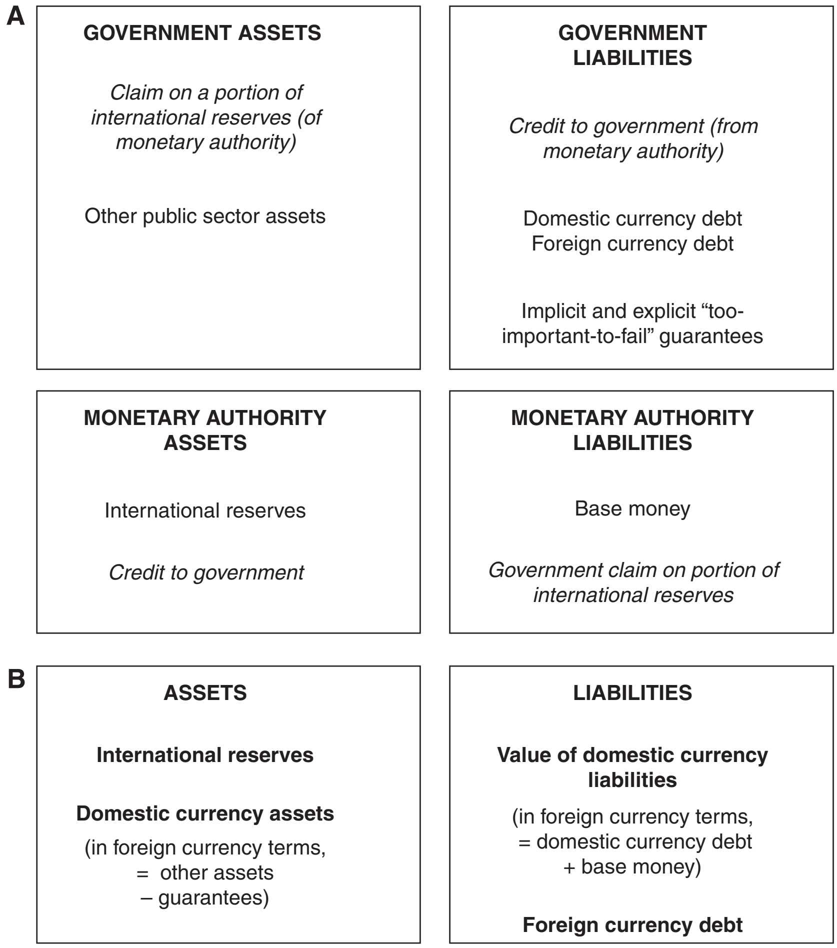
II. Contingent Claims Analysis of the Sovereign Balance Sheet

The contingent claims sovereign balance sheet is constructed from the basic accounting balance sheet of the government and monetary authorities. Panel A of Figure 2 shows the balance sheets of the government and monetary authorities as two segregated yet linked balance sheets. Government liabilities include foreign currency debt, domestic currency debt, obligations owed by the government to the monetary authorities, and guarantees to "too-important-to-fail" entities. Government assets include a claim on a portion of the foreign currency reserves held by the monetary authority and other public sector assets such as the present value of the primary fiscal surplus. The balance sheet of the monetary authority has assets consisting of international reserves (net foreign assets) and credit to government (net domestic assets). Liabilities of the monetary authority are base money and the government's claim on a portion of foreign currency reserves.

The two segregated balance sheets of the government need to be consolidated so that every entry on the liability side can be traced to observable data and each item is denominated in a common currency. Balance sheets for the country case studies presented in this paper are measured in U.S. dollars for ease of comparison, but the analysis holds even if they are valued in domestic currency.[5] Through the consolidation process the government claim on foreign currency reserves and credit to government net out (the italicized items in Panel A of Figure 2) and guarantees to too-

[5]Measuring the balance sheet in U.S. dollars results in variable sovereign assets vs. a fixed distress barrier. Measuring the balance sheet in domestic currency will result in both variable sovereign assets and a variable distress barrier. In either configuration, the contingent claim formulas will produce the same results.

Figure 2. The Consolidated Contingent Claims Public Sector Balance Sheet



Note: Panel A shows the balance sheets of the government and monetary authorities as two segregated yet linked balance sheets. These balance sheets are consolidated in Panel B so that every entry on the liability side can be traced to observable data and denominated in a common currency. Government claims on foreign currency reserves and credit to government net out (the italicized items in Panel A), and guarantees to too-important-to-fail entities are subtracted from the sovereign asset.

important-to-fail entities are subtracted from the sovereign asset.[6] Panel B of Figure 2 shows the consolidated sovereign balance sheet denominated in a common foreign currency. All the entries on the liability side of the

[6]The implicit guarantees to the financial sector, or other entities, could remain on the liability side of the consolidated public sector balance sheet and modeled as implicit put options. For more details, see Merton (1977); Gray, Merton, and Bodie (2002, 2006); Gapen and others (2004); and Van den End and Tabbae (2005). These papers link the sovereign to the contingent claim balance sheets of the banking or corporate sectors. The detailed analysis of the links to other sectors is beyond the scope of this paper.

contingent claims sovereign balance sheet are now directly observable from market prices.

In order to use this contingent claims balance sheet to estimate the value and volatility of sovereign assets, two additional steps are needed. First, assumptions on the seniority of sovereign liabilities need to be defined before applying standard contingent claims relationships. Second, and given the seniority structure assumed, option pricing techniques are used to formalize the relationship between assets and liabilities.

Seniority of Consolidated Balance Sheet Liabilities

Seniority of sovereign liabilities is not defined through legal status, as is the common practice in the private sector, but may instead be inferred from examining the behavior of government policymakers during periods of stress. In such periods, governments often make strenuous efforts to remain current on their foreign currency debt, efforts that effectively make such debt senior to domestic currency liabilities. Because the payment of foreign currency debt requires the acquisition of foreign currency, which the government has a more limited capacity to produce, governments sometimes introduce capital controls to prevent convertibility and preserve remaining international reserves to service sovereign external debt obligations.[7] In contrast, the government has much more flexibility to issue, repurchase, and restructure local currency debt. Governments, for example, may insist on the mandatory rollover or restructuring of domestic currency debt during periods of distress without simultaneously engaging foreign currency creditors. In these circumstances, holders of domestic currency liabilities will see the value of their claim greatly reduced because sovereign distress is often accompanied by instances of exchange rate depreciation, which reduces the value of the domestic currency liability in terms of foreign currency.

Several recent examples of sovereign debt restructuring illustrate this implicit seniority structure. Ariyoshi and others (2000) detail how Russia introduced capital controls, forced a lengthening of maturities on domestic currency government debt, and declared a unilateral moratorium on private sector external debt obligations in 1998–99 while still publicly stating their intention to honor sovereign external debt. According to Gulde and others (2003) and IMF (2002), Ecuador froze all checking, savings, and time deposits in March 1999 to limit further exchange rate depreciation. In August 1998, Ukraine imposed convertibility restrictions in the foreign exchange market and selectively restructured domestic debt held by banks (see, for example, IMF, 1999 and 2002). Other examples include government restructuring of debt held by domestic banks or pension funds, thereby reducing their present value, prior to the restructuring of foreign currency–denominated external debt.

[7]Support for viewing foreign currency debt as senior can also be found in the literature on "original sin" in Eichengreen, Hausmann, and Panizza (2002).

The assumption that domestic currency government debt liabilities are junior claims with equity-like properties (for example, unbounded behavior) is not inconsistent with the structure of fixed income contracts. Sims (1999) supports the modeling of domestic currency liabilities as junior claims, arguing that domestic currency debt has many similarities to equity issued by firms. In this setting, domestic currency debt becomes an important absorber of fiscal risk, just as equity is a cushion and risk absorber for firms. Sims claims that as long as there is some probability that the government will run a primary surplus in the future and/or will engage in the repurchase of domestic currency debt, then such debt has value.

These equity-like properties are clearly revealed through examination of the change in the price and quantity of domestic currency liabilities in U.S. dollars over time, just as the value of the firm is dependent on the number of shares and market price of equity (Merton, 1990, p. 368). In a pure floating exchange rate environment, for example, the quantities of domestic currency liabilities remain constant while the exchange rate bears the adjustment of changing economic conditions. A favorable economic environment that leads to an appreciation of the domestic currency results in an increase in the U.S. dollar value of domestic currency liabilities. In contrast, capital flight under floating exchange rates that severely depreciates the currency results in a much lower U.S. dollar value of domestic currency liabilities. This depreciation could, theoretically, proceed without bound until the point where domestic currency liabilities become nearly worthless in terms of foreign currency. Consequently, the exchange rate acts like an equity price in the corporate setting when the exchange rate floats freely.

In a fully fixed exchange rate environment, however, the government adjusts the quantities of domestic currency liabilities to maintain the set exchange rate. In the limit under capital flight and a currency board, the monetary authority would have to allow complete reabsorption of base money in order to maintain the fixed exchange rate. Capital inflows that tend to appreciate the currency, however, necessitate an increase in domestic currency liabilities to hold the exchange rate constant. Therefore, the quantities of domestic currency liabilities can change dramatically, resulting in quasi-unbounded behavior.[8] These situations are similar to share buybacks or new issuance, which produce changes in firm value through changes in quantity, and stock splits or reverse-splits, which produce changes in quantity and price but not necessarily an immediate change in firm value.

[8]The analysis also holds in the case of large developed markets such as the United States. While the balance sheet is already measured in dollars, the domestic bondholder is subject to (1) increased debt issuance that dilutes existing bondholders' claims on sovereign assets and (2) an increase in base money, or unexpected inflation risk, which reduces the real value of domestic currency debt. Even though developed markets are not thought to exhibit problems of "original sin" and currency mismatches, domestic currency liabilities of developed countries exhibit equity-like properties through dilution risk.

For these reasons, foreign currency debt is modeled as a senior claim and domestic currency liabilities as junior claims.[9] Sovereign default in this setting, therefore, means a default on foreign currency debt and is assumed to occur when the implied value of sovereign assets declines below the distress barrier. The seniority structure also leads directly to concepts of valuation. Although senior, foreign currency debt is risky because sovereign asset value may not be sufficient to meet promised payments. The value of senior claims, therefore, can be seen as having two components, the default-free value (promised payment value) and the expected loss associated with default when assets are insufficient to meet the promised payments. The value of junior domestic currency liabilities is dependent on the level of sovereign assets above and beyond what is necessary to service senior foreign currency debt, or the residual value of sovereign assets after the promised payments to senior claims have been made. Thus, in financial terminology, the value of domestic currency liabilities can be modeled as an implicit call option on sovereign assets, whereas the value of risky foreign currency debt can be modeled as default-free value of debt (for example, the distress barrier) minus the expected loss given default.

Calculating Implied Sovereign Asset Value and Volatility

Because domestic and foreign currency liabilities are observable through publicly available data, and because the value of domestic currency liabilities can be modeled as an implicit call option on sovereign assets, standard option pricing techniques can be applied to derive implied estimates for sovereign asset value and volatility. The option pricing formulas employed to estimate sovereign asset value and volatility rely on a few select variables: the value and volatility of domestic currency liabilities, the distress barrier, the risk-free interest rate, and time.

The distress barrier is assumed to equal to the book value of short-term external debt and one-half of long-term external debt plus interest. As mentioned earlier in the conceptual discussion on default, the default point lies somewhere between the book value of total liabilities and short-term liabilities. Using half of long-term liabilities for estimating the sovereign distress barrier mirrors what has proven successful in the estimation of

[9]See Cossin and Pirotte (2001) for a discussion on how the framework can handle multiple layers of liabilities or default sequences. The use of two layers of sovereign liabilities is a reasonable approximation given the observed robustness of the model and the behavior of spreads during periods of stress. Assuming that *all* money and local currency debt are senior and all foreign currency debt is junior leads to inconsistencies. Crises resulting in depreciation of the exchange rate would cause the "foreign currency junior claim" to grow large compared to domestic currency debt. This is inconsistent with the observation that credit default swap spreads on foreign currency debt increase with sharp depreciations. In situations of large exchange rate appreciation, usually considered beneficial from a credit risk perspective, the value of the "foreign currency debt junior claim" would be very small relative to domestic currency debt, indicating a large expected loss is associated with the domestic currency debt.

private sector credit risk.[10] Furthermore, the assumption also makes implementation tractable because these sovereign debt figures are readily available from public sources. An alternative procedure would be to use all long-term external debt discounted by the risk-free rate. This latter approach would require knowing the entire maturity structure of the debt, not just what is classified between short term and long term.

The volatility of domestic currency liabilities is derived from the volatility of exchange rates and variations in quantities of domestic currency debt and base money issued. The volatility of the exchange rate process is relatively more important in a floating exchange rate environment, whereas the quantities of domestic currency liabilities may vary substantially under a fixed or heavily managed exchange rate system. The preferred method for estimating exchange rate volatility is to compute an implied value from foreign exchange options markets (for example, Malz, 1997). If derivative markets are not present or data are unavailable, estimates can be obtained using a rolling window of historical daily data from the nondeliverable forward market.

The time horizon for the estimate of default risk can vary, but the convention in credit risk models is to estimate default risk over a one-year-ahead horizon, or $t = 1$. The estimate of the risk-free rate would, therefore, be equal to the one-year treasury bill rate or the London interbank offered rate. Given these inputs, the Black-Scholes option pricing formula (Black and Scholes, 1973; and Merton, 1973 and 1974) for the value of domestic currency liabilities as a call option on sovereign assets is

$$V_{DCL} = V_A N(d_1) - DBe^{-r_f t} N(d_2), \tag{1}$$

where V_{DCL} is the value of domestic currency liabilities, V_A is the value of sovereign assets, DB is the distress barrier or value of default-free debt, r_f is the risk-free interest rate, and t is the time to maturity on a default-free bond in years.[11] $N(d)$ is the cumulative probability distribution function for a standard normal variable (that is, the probability that a random draw from a standard normal distribution will be below d) where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{V_A}{DB}\right) + \left(r_f + \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}}, \\ d_2 &= d_1 - \sigma_A\sqrt{t}, \end{aligned} \tag{2}$$

and σ_A is the standard deviation of return on sovereign assets.

Applying option pricing techniques also results in a second formula, relating the volatility of sovereign assets to the volatility of domestic currency

[10]This definition of the distress barrier is identical to that used by Moody's KMV in corporate sector default risk analysis (Crosbie and Bohn, 2003). Short-term is defined as one year or less by residual maturity. See Sobehart and Stein (2000) and Sobehart, Keenan, and Stein (2000) for evidence that this approach outperforms other models in estimation of corporate sector credit risk.

[11]See Hull (1993) and Baxter and Rennie (1996) for discrete-time representations.

liabilities according to

$$V_{DCL} = \frac{\sigma_A}{\sigma_{DCL}} V_A N(d_1), \tag{3}$$

where σ_{DCL} is the standard deviation of domestic currency liabilities.[12] Equations (1) and (3) are two equations in (V_A, σ_A, V_{DCL}, σ_{DCL}, DB, t, r_f) and can be used to solve for the implied market value and volatility of sovereign assets.[13] Thus the information embedded in the value and volatility of domestic currency liabilities (in units of foreign currency) and the distress barrier derived from the book value of foreign currency debt yield estimates of implied sovereign asset value and implied asset volatility over a one-year-ahead time horizon.

III. Sovereign Credit Risk Indicators

Having derived the estimates of implied asset value and volatility, this section details how they can be used to develop useful indicators of sovereign risk. These risk indicators are the distance to distress, the risk-neutral probability of default, the value of senior foreign currency debt, and the sovereign risk-neutral credit spread.[14] Price and spread information may be easily observable from market data, but the market information itself does not reveal the rationale underlying the risk premium nor does it reveal what is often the most valuable piece of information in risk analysis—how much risk exposures could change as the health of the sovereign improves or declines on the margin. Contingent claims analysis links the credit risk premium to the balance sheet framework, allowing for an evaluation of the structural determinants of credit risk.

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[12]Here, $N(d_1)$ is the change in the price of domestic currency liabilities with respect to a change in sovereign assets, or $\partial V_{DCL}/\partial V_A$. This ratio is also referred to as the option *delta*. See Hull (1993, p. 38).

[13]The main difficulty in applying Equations (1) and (3) lies in the computation of the cumulative normal distribution. Numerical integration methods can be used to evaluate the distribution directly, computing a finite number of evaluations of the integrand and then summing over these values. Judd (1998) provides a menu of available integration methods, including the Gauss-Hermite quadrature that is often used in conjunction with normally distributed random variables. Alternatively, the distribution can be approximated using a high-order polynomial approximation, as is done in Hull (1993, pp. 226–27). Standard prepackaged routines in Matlab can then be implemented to find the zero roots of the nonlinear equations using iterative methods. A sample Matlab program can be found in Miranda and Fackler (2002, pp. 382–85). Using either of these techniques, Equations (1) and (3) can be solved for the implied value of sovereign assets and sovereign asset volatility.

[14]Risk-neutral valuation is an important factor underlying the derivation of the Black-Scholes option pricing formula whereby the value of the option can be derived by forming a riskless hedge portfolio. Thus, option values do not depend on the investor's or decision maker's attitude toward risk, which is a major benefit of this approach. Alternative balance sheet approaches based on discounted cash flows are subject to serious error not only from errors in cash flow projections but also from errors in choosing the discount rate. See Hull (1993, pp. 221–22) and Chriss (1997, pp. 190–93) for additional discussion of risk-neutral valuation.

Distance to Distress and Risk-Neutral Probability of Default

The distance to distress computes the difference between the implied future market value of sovereign assets and the distress barrier scaled by a one-standard-deviation move in sovereign assets. The distance to distress is defined conceptually as

$$\frac{\text{Implied market value of sovereign assets} - \text{Distress barrier}}{\text{Implied market value of sovereign assets} \times \text{Sovereign asset volatility}}.$$

The numerator above measures the distance between the expected one-year-ahead market value of sovereign assets and the distress barrier. This amount is then scaled by a one-standard-deviation move in sovereign assets. The distance to distress therefore yields the number of standard deviations sovereign asset value is from distress. Lower market value of sovereign assets, higher levels of foreign currency debt, and higher levels of sovereign asset volatility all serve to decrease the distance to distress. The precise measure of distance to distress is d_2 from the Black-Scholes option pricing formula,

$$d_2 = \frac{\ln\left(V_A \exp\left(\left(r_f - \frac{1}{2}\sigma_A^2\right)t\right)\right) - \ln(DB)}{\sigma_A\sqrt{t}}. \tag{4}$$

Distance to distress can be translated into a measure of probability of default, commonly referred to as the risk-neutral default probability, which determines the likelihood that future sovereign asset value will fall below the distress barrier. The option pricing formula used in this analysis assumes that future sovereign asset value is distributed log-normally and the risk-neutral probability of default is therefore the shaded area that lies below the distress barrier as shown in Figure 1. The risk-neutral default probability (*RNDP*) is

$$RNDP = N(-d_2), \quad \text{where } d_2 = f(V_A, DB, r_f, \sigma_A, t). \tag{5}$$

Value of Foreign Currency Liabilities and the Sovereign Credit Risk Premium

Two other useful sovereign risk indicators that can be obtained are the sovereign credit spread or credit risk premium, and the market value of foreign currency liabilities. The value of risky senior foreign currency liabilities, V_{FCL}, can be derived using Equation (1) and the standard balance sheet identity that assets equal liabilities, $V_A = V_{DCL} + V_{FCL}$,[15]

$$V_{FCL} = V_A(1 - N(d_1)) + DBe^{-r_f t}N(d_2). \tag{6}$$

[15]Merton (1974) derives similar measures for the pricing of corporate debt. The value of senior foreign currency liabilities can also be obtained using the implicit put option in risky debt (Gapen and others, 2004; Gray, 2004; Chacko and others, 2006; and Gray, Merton, and Bodie, 2006), or $V_{FCL} = DBe^{-r_f t} - (DBe^{-r_f t}N(-d_1) - V_A N(-d_2))$.

Manipulating Equation (6) results in an estimate of the risk-neutral credit spread (*RNS*),

$$RNS = y - r_f = \frac{-1}{t}\ln\left(\frac{V_A}{DBe^{-r_f t}}N(-d_1) + N(d_2)\right), \tag{7}$$

where $y = -(1/t)\ln(V_{FCL}/DB)$. The left-hand side of Equation (7) represents the yield to maturity on risky foreign currency debt less the risk-free rate of interest and is therefore equivalent to a credit risk premium, or risk-neutral credit spread. In addition to the risk-free rate and time, the sovereign risk premium is a function of only two variables: the volatility of sovereign assets and the ratio of the value of sovereign assets to the distress barrier. Increases in the ratio of sovereign assets to foreign currency liabilities and decreases in sovereign asset volatility both reduce the sovereign risk premium. Conversely, as the ratio of sovereign assets to foreign currency liabilities decreases or sovereign asset volatility increases, the risk premium widens.

It is useful to note that no market information on foreign currency–denominated debt—namely, bond spreads or CDS spreads—has been used while computing the value of risky foreign currency liabilities and credit risk premium in the model. Only information on the book value of payments on existing foreign currency debt is used in constructing the distress barrier. This is combined with market information from domestic currency liabilities and the exchange rate to estimate the value of foreign currency liabilities and the credit risk premium. This feature is noteworthy because the model output can be then compared with readily available market information to evaluate the robustness of this approach. If the model output corresponds to available market data, then the remaining credit risk indicators should also prove robust.

Sensitivity Measures

Associated with the sovereign risk indicators are sensitivity measures that reveal how responsive the set of risk indicators are to changes in model parameters, namely, changes in the value of sovereign assets and asset volatility. This paper focuses on eight sensitivity measures.[16] The first four are the changes in distance to distress, risk-neutral default probability, risk-neutral credit spreads, and value of foreign currency debt from a 1 percent change in the value of sovereign assets. The second four are changes in the same risk indicators from a 1 percent change in sovereign asset volatility.

[16]Variations in the derivative asset price with respect to changes in the underlying parameters that enter the option formula are known as Greek-letter risk measures. Frequently used measures are the option delta, gamma, and vega. See Briys and others (1998, pp. 124–28) for these and other measures of option sensitivities that are used in managing exposures.

Sensitivity measures are critical in risk analysis because they capture nonlinear changes in value. The inclusion of nonlinearities in option pricing relationships accounts for much of the improved predictive power of contingent claims credit models of corporate sector default over traditional linear credit models. Equally important, the sensitivity measures allow practitioners to look beyond the current level of distance to distress, spreads, or probability of default to provide an indication of the potential risk exposure of the sovereign. The sensitivity measures are highest when sovereign asset value is in the neighborhood of the distress barrier, reflecting magnified default risk. In this instance, small changes in underlying asset value in either direction will have proportionately larger impacts on the balance sheet risk indicators. In sum, the credit risk indicators yield current estimates of sovereign balance sheet risk, and the sensitivity measures point to how sovereign risk could further change if the balance sheet improves or weakens on the margin.

Model Risk

Because the calibrated inputs of the distress barrier and volatility of domestic currency liabilities are estimates, uncertainty surrounding their values may inject noise or bias into model outputs. Contingent claims analysis, like any model framework, therefore, contains the possibility of model risk. For example, overestimating the distress barrier will lead to an overestimate of the probability of default. Use of a historical window may underestimate exchange rate volatility and credit risk in periods where volatility is rising or is expected to rise, and it may overestimate credit risk in periods where volatility is declining. The option implied method of estimating exchange rate volatility is also not without problems. Jorion (1995) examines implied volatility from the Black-Scholes model for several currency options and finds that although implied volatility estimates are more accurate than time-series estimates, they are biased upward. Systematic upward bias in calibrated exchange rate volatility would create upward bias in model credit risk. Consequently, estimates of the distress barrier or volatility of domestic currency liabilities may contain error or bias, which will reduce the accuracy of model indicators. Although the parameter uncertainty in applying the model is inevitable, it can be addressed to a certain degree in simulations, as shown in Section V.

IV. Robustness of Sovereign Credit Risk Indicators

Validation of corporate credit risk models (Sobehart, Keenan, and Stein, 2000; Sobehart and Stein, 2000; and Bohn, Arora, and Korablev, 2005) often involves testing the model against a database of actual defaults, whereby models are assessed according to type 1 (model indicates low risk when risk is in fact high) and type 2 (model indicates risk when none is present) errors. Because no equivalent database of sovereign defaults is available, the degree to which the contingent claims risk indicators closely parallel actual market data may serve as an indicator of their robustness and as early warning indicators of sovereign risk.[17] Robustness of the sovereign credit risk indicators is examined through their correlation and relationship with actual data and through regression analysis. To this end, a historical time series of the risk indicators is compared with actual market data for a number of emerging market countries.[18]

Correlation with Market Data

If the model output is robust, distance to distress should be negatively correlated with actual sovereign credit spreads. As distance to distress increases, credit risk should decline and be reflected in lower CDS spreads. Figure 3 displays the relationship between the distance to distress indicator for 12 emerging market sovereign balance sheets and that country's observed CDS spread.[19] Table 1 reports the Spearman's rank correlation between distance to distress and risk-neutral spreads with the observed sovereign CDS spreads and JPMorgan Emerging Market Bond Index (EMBI+) spreads.[20]

The data reveal a very high degree of correlation for most countries between the two risk indicators and the observed market data from January 2003 to August 2004. The reported correlations confirm the expected negative relationship between distance to distress and both CDS and EMBI+ spreads.[21] The correlations also display a high degree of significance: 29 of

[17]Mapping of risk-neutral default probability into actual default probabilities is necessary for rating agencies in the ratings process but is not necessary for valuation purposes. The Merton framework substitutes the risk-free interest rate for the actual expected return in the asset-probability distribution. Because the actual expected return is greater than the risk-free return, the risk-neutral probability of default is higher than the actual probability of default. However, expected returns are not necessary for valuation purposes. The relationship between the risk-neutral spreads and risk-neutral default probability and actual spreads and market-implied default probability, as undertaken in this paper, is examined mainly for robustness purposes. See Merton (1990) for additional discussion.

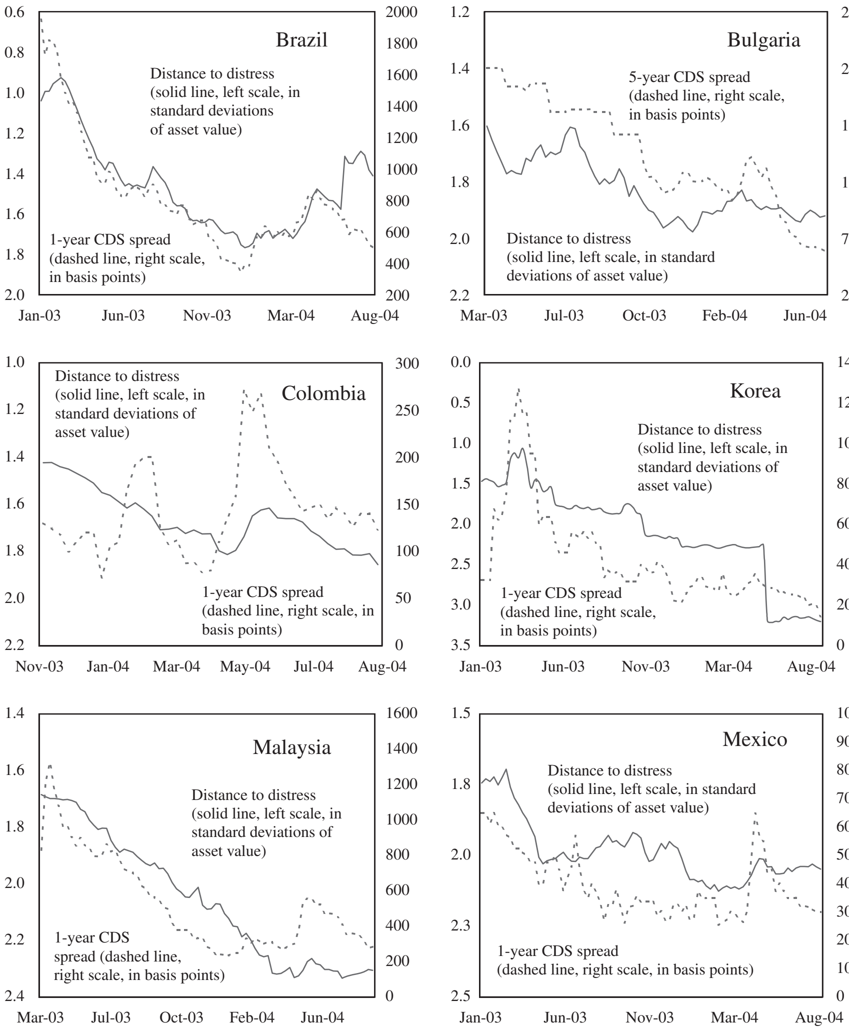
[18]The historical data for the sovereign risk indicators in Equations (3) to (6) were obtained from the Macrofinancial Risk (M*f*Risk) model, which applies the contingent claims methodology as described in this paper. The model was developed under a joint research effort between Moody's and Macro Financial Risk, Inc., and applied to 17 countries. At the time of the writing of this paper, access to M*f*Risk was available only through subscription.

[19]Reported output in Figure 3 is limited to the 12 countries for which credit default swap data were available.

[20]The reported correlations in Table 1 were computed using Spearman's rank correlation instead of conventional correlation. Conventional correlation is inappropriate in this case because it implicitly assumes linear relationships among variables, an assumption that contradicts the nonlinear relationship between variables as found in this paper. Spearman's rank correlation is a less restrictive measure to gauge relationships among variables because it does not impose any linearity assumptions.

[21]The JPMorgan EMBIG index has replaced the EMBI+ index as the preferred index for tracking emerging market credit spreads, but historical EMBIG index data were not available at the time of the writing of this paper.

Figure 3. Distance to Distress and CDSs

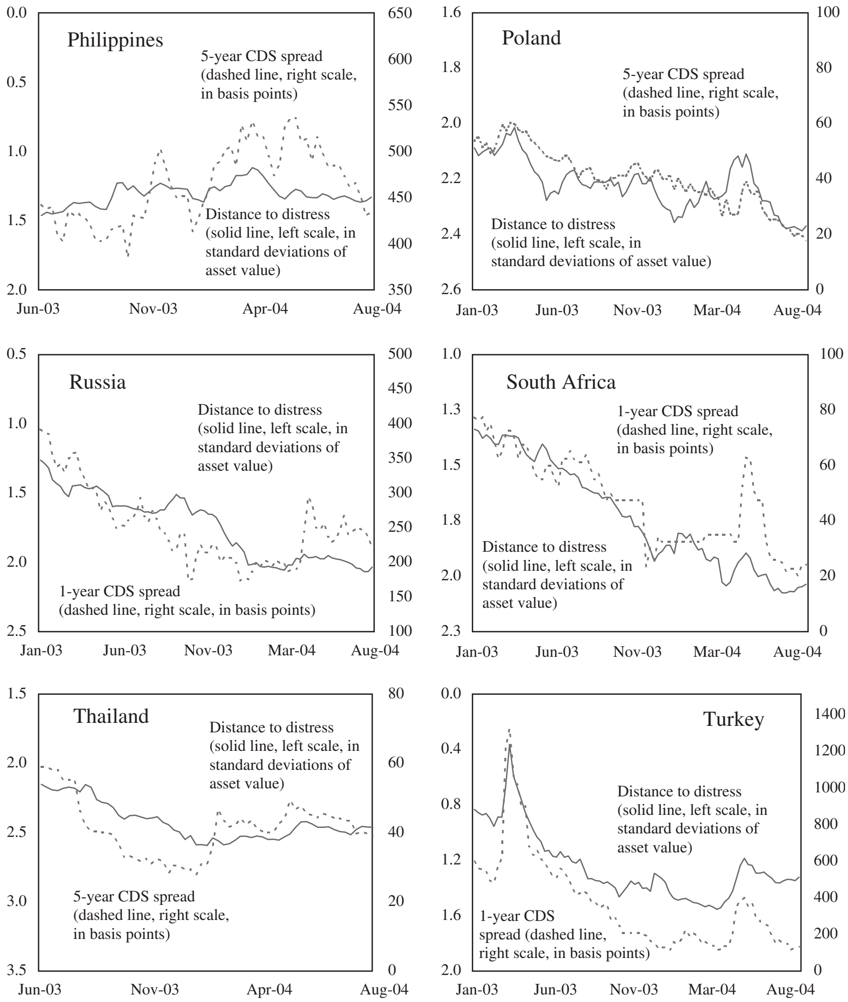


Sources: Bloomberg L.P.; and authors' calculations.

Note: The panels display distance to distress in the number of standard deviations that asset value is above the distress barrier for 12 emerging market sovereign balance sheets vs. each country's observed CDS spread in basis points. CDS data were obtained from Bloomberg L.P. The distance to distress scale is inverted, meaning that increases in distance to distress correspond with lower spreads on CDSs. Because the model distance to distress is computed as a one-year-ahead measure, one-year CDS spreads are used if available. Otherwise, five-year CDS spreads are used. Data are weekly observations from January 2003 through August 2004, though data limitations for some countries shorten the sample.

the 34 reported correlations between distance to distress and CDS spreads are significant at the 95 or 99 percent level. In many cases, correlation is highest with the five-year CDS spread, which likely reflects the greater liquidity in this market relative to the shorter maturity CDS market. A similar level of

Figure 3. *(concluded)*



significance is found between distance to distress and country EMBI+ spreads with eight of the nine reported correlations significant at the 99 percent level.

As a second check on robustness, the risk-neutral sovereign credit spread for each country is compared with the EMBI+ spread and CDS spread. Figure 4 displays the expected positive relationship between the risk-neutral sovereign credit spread and each EMBI+ country spread for nine emerging markets for the sample period from January 2003 to August 2004. The correlation between the risk-neutral sovereign credit spread and the respective EMBI+ spread during the same time period is also reported in Table 1. The correlations show the expected positive relationship between the risk-neutral credit spread and EMBI+ country and CDS spreads. The correlations

Table 1. Correlation Between Sovereign Risk Indicators and Market Data

Country	Distance to Distress and 1-year CDS spread	3-year CDS spread	5-year CDS spread	Country EMBI+ spread	Risk Neutral Spread and 1-year CDS spread	3-year CDS spread	5-year CDS spread	Country EMBI+ spread
Brazil	−0.68**	−0.79**	−0.80**	−0.81**	0.70**	0.82**	0.82**	0.83**
Bulgaria	NA	−0.72**	−0.91**	NA	NA	0.72**	0.83**	NA
Korea	−0.83**	−0.85**	−0.88**	NA	0.84**	0.85**	0.89**	NA
Malaysia	−0.72**	−0.73**	−0.14	−0.36**	0.72**	0.73**	0.15	0.39**
Mexico	−0.44**	−0.62**	−0.73**	−0.72**	0.44**	0.62**	0.73**	0.73**
Philippines	−0.33*	−0.43**	−0.53**	−0.20	0.33*	0.43**	0.54**	0.17
Poland	−0.16	−0.68**	−0.69**	−0.44**	0.06	0.67**	0.69**	0.45**
Russia	−0.29**	−0.54**	−0.66**	−0.47**	0.30**	0.54**	0.67**	0.47**
South Africa	−0.80**	−0.76**	−0.75**	−0.47**	0.86**	0.77**	0.75**	0.64**
Thailand	−0.29	NA	−0.28*	NA	0.41*	NA	0.27*	NA
Turkey	−0.83**	−0.84**	−0.84**	−0.85**	0.82**	0.83**	0.83**	0.85**
Venezuela	−0.29*	−0.22	−0.20	−0.89**	0.33*	0.27	0.22	0.90**

Sources: Bloomberg L.P.; and authors' calculations.

Note: The table reports the correlation between model indicators of risk and market data on credit default swap (CDS) spreads and spreads on external debt (EMBI+) for 12 emerging market countries between January 2003 and August 2004. Data for country CDS spreads and JPMorgan Emerging Market Bond Index (EMBI+) spreads were obtained from Bloomberg, L.P. Correlations were computed using Spearman's rank correlation. Significance at the 5 and 1 percent levels is indicated by*and**, respectively. NA = not available.

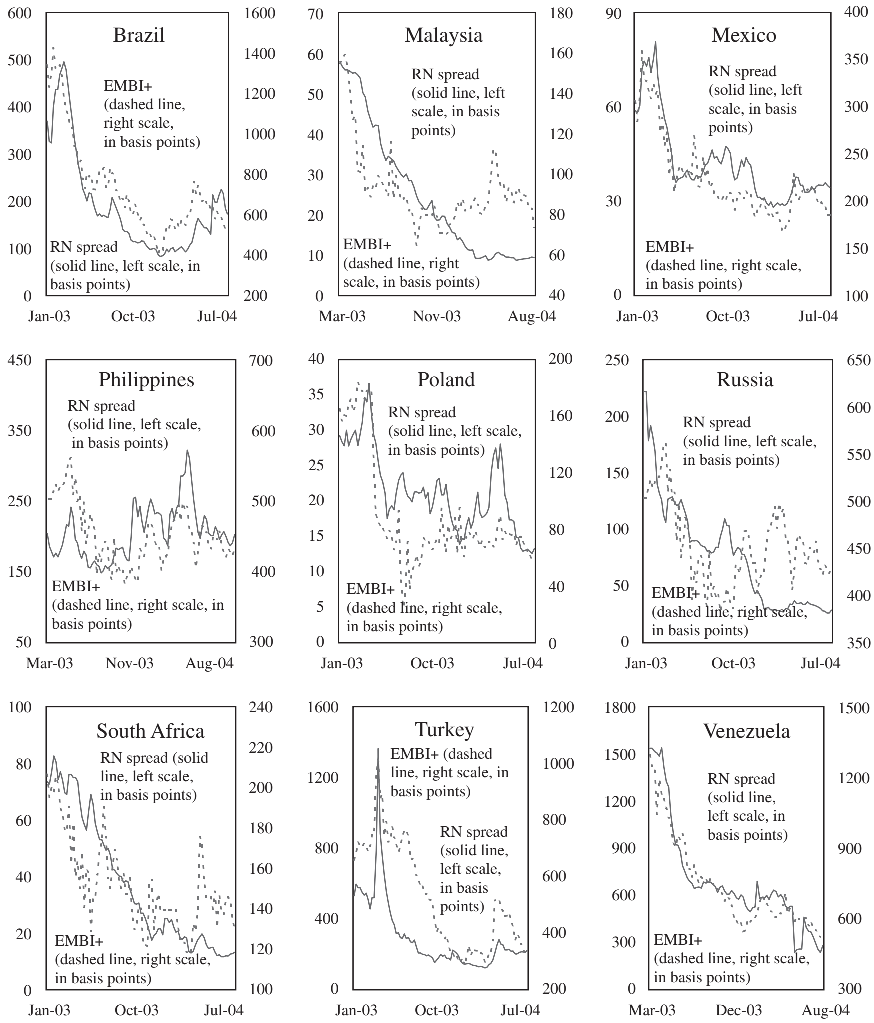
between the risk-neutral credit spread and the CDS spread display a high degree of significance at the 95 and 99 percent levels for 30 out of 34 reported correlations. The correlations between the risk-neutral credit spread and EMBI+ spread display significance at similar levels in eight of nine cases.

Regression Analysis

Two fixed effects panel regressions are applied to a combined cross-country sample of 981 observations from April 2002 to August 2004 to estimate the relationship between risk-neutral spreads and CDS spreads and EMBI+ country spreads.[22] The fixed effects model treats differences across countries in the sample as parametric shifts of the regression function (that is, differences across countries are captured in differences in the constant term). Results for CDS spreads, which are reported in Table 2, indicate that the coefficient and constants are highly significant at all confidence intervals and

[22]The countries in the sample include Brazil, Colombia, Korea, Malaysia, Mexico, Philippines, Poland, Russia, South Africa, Turkey, and Venezuela.

Figure 4. Model Spreads vs. Market Data *(In basis points)*



Sources: Bloomberg L.P.; and authors' calculations.

Note: The panels display the risk-neutral spread (RN spread) from the model for nine emerging markets vs. each country's observed spread on external bonds, based on the JPMorgan Emerging Market Bond Index (EMBI+) from Bloomberg L.P. Data are weekly observations during the sample period from January 2003 to August 2004, though data limitations for some countries shorten the sample.

the R^2 from the panel regression is 88 percent. The relationship between risk-neutral credit spreads and EMBI+ spreads is reported in Table 3 and indicates that the coefficient and constants are highly significant at all confidence intervals and the R^2 from the panel regression is 96 percent.

Given the goodness of fit of the above regressions, the individual country panel equations can be used to map sovereign risk-neutral credit spreads into (1) actual CDS spreads and (2) actual EMBI+ spreads. For example, the

Table 2. Regression: Risk-Neutral Spreads and Credit Default Swap Spreads

Independent Variables/Country	Constant	Coefficient	Standard Error	*t*-Statistic	Probability
ln(*RNS*)		0.52	0.02	24.53	0.00
Brazil	3.43		0.12	28.09	0.00
Colombia	2.54		0.11	22.34	0.00
Korea	2.59		0.07	38.50	0.00
Malaysia	1.54		0.08	18.57	0.00
Mexico	1.72		0.09	18.42	0.00
Philippines	2.53		0.12	20.50	0.00
Poland	1.20		0.08	14.55	0.00
Russia	2.72		0.10	26.94	0.00
South Africa	2.09		0.09	23.63	0.00
Turkey	2.98		0.13	23.04	0.00
Venezuela	2.94		0.15	20.14	0.00
	R^2		0.88		
	Adjusted R^2		0.88		
	Log likelihood		−645.02		
	F-statistic		640.78		
	Prob (*F*-statistic)		0.00		

Sources: Bloomberg L.P.; and authors' calculations.

Note: This table presents the results from the following cross-country panel regression: $\ln(CDS_t) = i\alpha_i + \beta \ln(RNS_i) + \varepsilon_i$, where CDS_i is the credit default swap spread on external debt and RNS_i is the risk-neutral spread on external debt from the model. The regression is a fixed-effect ordinary least-squares panel regression that treats differences across countries in the sample as parametric shifts in the regression function. Data for country CDS spreads were obtained from Bloomberg L.P. and include weekly observations between April 2002 and August 2004 for a total of 981 observations.

estimated equation for Mexico used to map sovereign risk-neutral credit spread into the actual spread on CDSs is

$$\ln(CDS_t) = 1.72 + 0.52\ln(RNS_t). \tag{8}$$

The similar estimated equation used to map risk-neutral credit spreads into actual EMBI+ spreads for Mexico is

$$\ln(EMBI_t) = 4.78 + 0.15\ln(RNS_t). \tag{9}$$

As a numerical example, suppose that applying Equation (7) results in risk-neutral spreads on foreign currency debt for Mexico of 200 basis points. Inserting this value into Equations (8) and (9) results in a CDS spread of 88 basis points and an EMBI+ spread of 263 basis points, respectively.

The relationship between sovereign risk-neutral default probability and estimated actual default probability can also be examined for robustness purposes. To implement this robustness test, some estimate of actual default

Table 3. Regression: Risk-Neutral Spreads and JPMorgan EMBI+ Spreads

Independent Variables/Country	Constant	Coefficient	Standard Error	t-Statistic	Probability
ln(*RNS*)		0.15	0.02	6.30	0.00
Brazil	5.69		0.11	50.86	0.00
Colombia	4.68		0.08	60.98	0.00
Korea	5.41		0.11	48.94	0.00
Malaysia	4.04		0.07	60.30	0.00
Mexico	4.78		0.08	58.74	0.00
Philippines	5.29		0.12	42.88	0.00
Poland	3.79		0.08	47.83	0.00
Russia	5.05		0.09	54.28	0.00
South Africa	4.52		0.07	61.41	0.00
Turkey	5.26		0.12	43.85	0.00
Venezuela	5.61		0.14	39.41	0.00
	R^2		0.96		
	Adjusted R^2		0.96		
	Log likelihood		340.48		
	F-statistic		1548.00		
	Prob(F-statistic)		0.00		

Sources: Bloomberg L.P.; and authors' calculations.

Note: This table presents the results from the following cross-country panel regression: $\ln(EMBI_i) = i\alpha_i + \beta \ln(RNS_i) + \varepsilon_i$, where $EMBI_i$ is the JPMorgan EMBI+ spread on external debt and RNS_i is the risk-neutral spread on external debt from the model. The regression is a fixed-effect ordinary least-squares panel regression that treats differences across countries in the sample as parametric shifts in the regression function. JPMorgan EMBI+ spreads were obtained from Bloomberg L.P. and include weekly observations between April 2002 and August 2004 for a total of 981 observations.

probability is needed. Without a large data set of sovereign defaults, a second-best approach is to use estimates of actual default probability, or market-implied default probabilities (MIDP), from CDS markets. Such probabilities can be obtained from CDS spreads assuming a specific loss given a default and time horizon (a recovery rate of 30 percent was used in this analysis).[23] Using this approach, a fixed effects panel regression was applied to a cross-country sample of 935 observations from January 2003 to August 2004 to examine the relationship between risk-neutral default probabilities and MIDP.[24] The fixed

[23]MIDP can be obtained from CDS spreads through the following equation: $MIDP = \frac{1 - \exp(-spread t)}{1 - R}$, where *spread* is the net one-year credit default swap spread, t is the time horizon (equal to 1 in this case), and $R = 30$ percent is the recovery rate. If the one-year CDS spread is 180 basis points, the implied default probability is 2.5 percent.

[24]The countries in the sample include Brazil, Colombia, Korea, Malaysia, Mexico, Philippines, Poland, Russia, South Africa, Turkey, and Venezuela.

effects panel regression displays high explanatory power with an R^2 of 93 percent. Results are reported in Table 4.

However, close examination of Table 4 reveals that the regression equations for Korea, Malaysia, Mexico, Poland, and South Africa result in MIDP that are higher than risk-neutral default probability, which is a contradiction. This problematic result is likely due to two factors: (1) the assumption of a constant loss given default for all countries regardless of credit risk and (2) lack of a sufficiently long-time series for CDSs. In practice, loss given default may change as probability of default and CDS spreads change. More sophisticated methods of estimating MIDP from CDS data are therefore needed, including methods that allow the recovery rate to vary with probability of default. These advanced methods are beyond the scope of this paper and are left for future research.

Table 4. Regression: Default Probability

Independent Variables/Country	Constant	Coefficient	Standard Error	*t*-Statistic	Probability
ln(*RNDP*)		0.23	0.03	7.38	0.00
Brazil	0.61		0.08	7.94	0.00
Colombia	0.24		0.07	3.37	0.00
Korea	−0.77		0.04	−21.63	0.00
Malaysia	−1.44		0.03	−49.84	0.00
Mexico	−0.90		0.04	−25.61	0.00
Philippines	0.50		0.09	5.52	0.00
Poland	−1.65		0.04	−38.67	0.00
Russia	0.16		0.04	3.82	0.00
South Africa	−0.79		0.05	−17.13	0.00
Turkey	0.64		0.08	8.39	0.00
Venezuela	1.08		0.08	12.76	0.00
	R^2		0.93		
	Adjusted R^2		0.93		
	Log likelihood		−157.18		
	F-statistic		770.00		
	Prob(*F*-statistic)		0.00		

Sources: Bloomberg L.P.; and authors' calculations.

Note: This table presents the results from the following cross-country panel regression: $\ln(MIDP_i) = i\alpha_i + \beta \ln(RNDP_i) + \varepsilon_i$, where $MIDP_i$ is the market implied default probability and $RNDP_i$ is the risk-neutral default probability from the model. Market implied default probabilities were obtained by transforming country credit default swap spreads through the following equation: $MIDP = \frac{1 - \exp(-spread_t)}{1 - R}$, where *spread* is the net one-year credit default swap spread, t is the time horizon (equal to 1 in this analysis), and $R = 30$ percent is the assumed recovery rate in the event of default. The regression is a fixed-effect ordinary least-squares panel regression that treats differences across countries in the sample as parametric shifts in the regression function. Credit default swap spreads were obtained from Bloomberg L.P. and include weekly observations between January 2003 and August 2004 for a total of 935 observations.

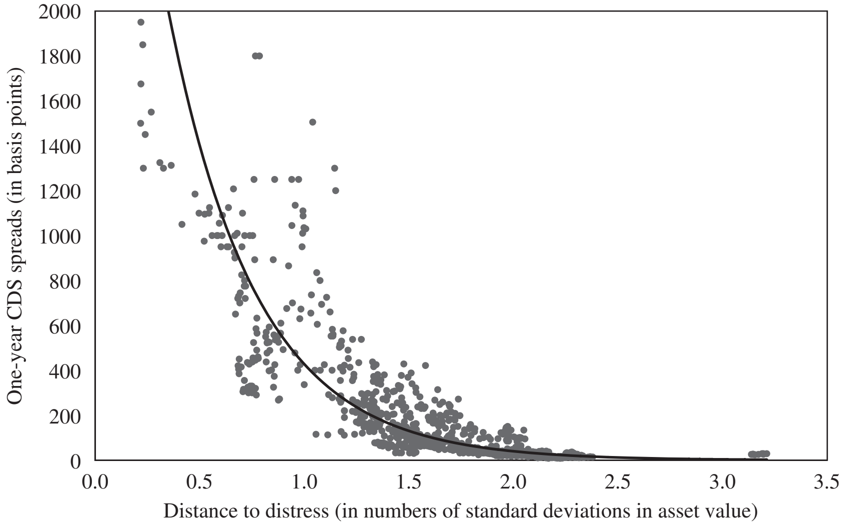
An alternative approach is to pool the individual country data into one regression to increase the number of observations, while maintaining the assumption of a constant loss given default (for example, Crouhy, Galai, and Mark, 2000). The estimated equation using data from the pooled countries in the sample is

$$\ln(MIDP_t) = \underset{(-15.54)}{-1.24} + \underset{(23.23)}{1.01} \underset{R^2=0.735}{\ln(RNDP_t)}, \tag{10}$$

where the numbers in parentheses represent the relevant t-statistic. Although the explanatory power of this pooled regression falls slightly relative to the panel regression reported in Table 4 (R^2 declines from 0.93 to 0.74), the level of explanatory power remains high and the relationship is highly significant. Furthermore, Equation (10) produces a market-implied probability of default that is lower than the risk-neutral probability of default. For example, suppose that application of Equation (5) results in a risk-neutral probability of default equal to 8 percent. Inserting this value into Equation (10) results in an MIDP of 2.3 percent. In other words, actual probability of default is approximately one-third of risk-neutral probability of default.

Finally, Figure 5 plots observed market spreads (one-year CDS spreads) vs. model distance to distress for each country in the sample (889 data pairs

Figure 5. Market CDS Spreads and Model Distance to Distress



Sources: Bloomberg L.P.; and authors' calculations.

Note: Observed one-year CDS spreads vs. model distance to distress for Brazil, Colombia, Korea, Malaysia, Mexico, the Philippines, Poland, Russia, South Africa, Turkey, and Venezuela from mid-2002 to mid-2004, depending on data availability. The data points represent the weekly observations across countries for a total of 889 data pairs, and the solid line represents the line of best fit. The figure reveals the nonlinear relationship between sovereign spreads and distance to distress and confirms the importance of applying nonlinear option pricing relationships in assessing risk.

from the period mid-2002 to mid-2004).[25] The figure reveals the nonlinear relationship between sovereign spreads and distance to distress and simultaneously confirms the importance of applying nonlinear option pricing relationships. The sensitivity of changes in spreads for a given change in distance to distress is much lower for countries with a high distance to distress. As the distance to distress declines from 1.5 to 1.4, the spread increases on average by 35 basis points. However, if the distance to distress drops from 0.5 to 0.4 the spread increases on average by 375 basis points. Use of a linear model would miss the substantial increase in risk as sovereign assets approach the distress barrier.

The robustness checks in this section suggest that the distance to distress, risk-neutral credit spread, and risk-neutral probability of default are useful for evaluating sovereign vulnerabilities. The evidence indicates that the book value of foreign currency liabilities along with market information from domestic currency liabilities and the exchange rate contain important information about changes in the value of foreign currency liabilities and credit risk premium. The nonlinearities and inclusion of volatility in the option pricing relationship used in this analysis contribute to the high degree of explanatory power and correlation with actual data. Finally, the robustness checks used to estimate the relationships in Equations (8)–(10) allow for straightforward transformation of model outputs into estimates of observable market data if desired.

V. Scenario and Simulation Analysis: Hypothetical Sovereign

With robustness verified, the structural models calibrated using the contingent claims framework unique to each economy can be used with scenario and simulation analysis to evaluate shocks and policies. The goal of this exercise is to estimate the potential effects of changes in economic conditions and impact of government policies on sovereign credit risk and sensitivity indicators. To begin with, a baseline balance sheet for a hypothetical sovereign is calibrated and the resulting baseline risk indicators and sensitivity measures are computed. This example reflects a hypothetical sovereign that has a relatively high level of debt, issues liabilities in external and local markets, and operates in a floating exchange rate environment. In this case, volatility in the value of domestic currency liabilities in U.S. dollars is mainly determined by exchange rate volatility.

Scenario analysis is then conducted using two capital flow examples, and the resulting point estimates for the credit risk indicators and sensitivity measures are compared to the baseline set of indicators. Next, the scenario analysis is extended using Monte Carlo simulations, which is a method of generating a large number of market outcomes and yields probability distributions for each risk indicator. Unlike the scenario analysis that is intended to investigate the

[25]The solid line in the figure represents the line of best fit, $y = 4597.3\exp(-2.3743x)$, with $R^2 = 0.7957$.

effects of a specific market outcome, the Monte Carlo simulation draws randomly from sample interest rate and exchange rate distributions to compute probability distributions and confidence intervals for a set of market outcomes. This process allows for the "stress testing" of the sovereign risk indicators to derive what are commonly known as VaR measures.

The Calibrated Baseline Sovereign Balance Sheet

The starting point is the baseline balance sheet as displayed in the middle column of Table 5. The distress barrier is assumed to be \$100 billion, comprising short-term foreign currency debt plus interest of \$40 billion and one-half of long-term debt of \$60 billion. The value and volatility of domestic currency liabilities in dollar terms are calibrated at \$82 billion and 0.76 (76 percent), respectively. Using Equations (1) and (2), the implied value of sovereign assets is \$175 billion and the implied volatility of sovereign assets is 0.38 (38 percent). Foreign currency reserves are assumed to make up \$40 billion of implied sovereign assets.

Given these calibrated inputs and implied values, the resulting distance to distress under the baseline is 1.4 standard deviations, which is equivalent to an MIDP of 2.3 percent. The value of risky foreign currency debt is \$95 billion and the estimated market credit spread is 356 basis points.[26] Finally, the market value of risky foreign currency debt implies a present value expected loss of \$1 billion. This value is derived from the difference between the discounted present value of the distress barrier (an annualized risk-free rate of 4 percent yields a present value distress barrier of \$96 billion) and the implied market value of foreign currency debt.

The baseline sensitivity measures are calculated from a 1 percent change in sovereign asset value and volatility. For example, Table 5 shows that when the value of sovereign assets decreases by 1 percent, the distance to distress falls by 0.03 standard deviations (that is, from 1.4 to 1.37 standard deviations), MIDP increases by 0.12 (that is, from 2.3 to 2.42 percent), estimated market credit spreads increase by 13 basis points, and the expected loss on foreign currency debt increases by \$70 million. Sensitivity measures are also reported for a 1 percent change in sovereign asset volatility.

Scenario Analysis

Two scenarios are examined and compared with the baseline in Table 5. Scenario 1 represents the potential negative effects associated with capital outflows, and Scenario 2 illustrates the positive effects from capital inflows. First, suppose that economic conditions deteriorate so that capital outflows

[26]Since the scenario and Monte Carlo simulations are based on this hypothetical sovereign balance sheet, we use the results from a panel regression between risk-neutral spreads and EMBI+ spreads applied to the countries in Table 3. The estimated equation between risk-neutral spreads and market credit spreads is $\ln(EMBI_t) = 2.97 + 0.61\ln(RNS_t)$.

Table 5. Alternative Scenarios and Contingent Claim Sovereign Risk Indicators

	Scenario 1	Baseline	Scenario 2
	(US$ billions, unless otherwise indicated)		
Contingent Claim Sovereign Balance Sheet			
Value of sovereign assets (implied)	155	175	195
Foreign reserves (observed value)	35	40	45
Sovereign assets less reserves (implied)	120	135	150
Value of risky foreign currency debt	93	95	96
Distress barrier[1]	100	100	100
Present value of distress barrier[1]	96	96	96
Present value of expected losses (= implicit put option)	3	1.5	0.5
Value of local currency liabilities[1]	64	82	100
Volatility of assets (implied)	43%	38%	37%
Credit Risk Indicators			
Distance to distress[2]	1.0	1.4	2.1
Market implied default probability (MIDP)	5.1%	2.3%	1.2%
Estimated market spread (EMBI)[3]	672	356	239
Sensitivity Measures[4]			
Change in distance to distress/1 percent change in assets[2]	−0.02	−0.03	−0.03
Change in distance to distress/1 percent change in asset volatility[2]	−0.03	−0.05	−0.06
Change in MIDP/1 percent change in assets	0.19	0.12	0.07
Change in MIDP/1 percent change in asset volatility	0.25	0.21	0.15
Change in estimated market spreads/1 percent change in assets[3]	20	13	7
Change in estimated market spreads/1 percent change in asset volatility[3]	35	30	21
Change in present value of expected loss/1 percent change in assets	0.15	0.07	0.03
Change in present value of expected loss/1 percent change in asset volatility	0.26	0.15	0.08

Source: Authors' calculations.

Note: This table presents the results of the calibrated baseline sovereign balance sheet and changes in the balance sheet values and risk indicators from two alternative scenarios. The baseline balance sheet, indicated in the middle column, reflects a hypothetical sovereign that has a relatively high level of debt, issues liabilities in external and local markets, and operates in a floating exchange rate environment. Given the calibrated values for the baseline balance sheet, risk indicators and sensitivity measures are calculated. Scenario 1 then represents the potential negative effects associated with capital outflows. Sovereign asset value is assumed to fall by $25 billion, with reserves falling from $40 billion to $35 billion. Sovereign asset volatility increases from 38 to 43 percent. Scenario 2 illustrates the positive effects from capital inflows. The value of sovereign assets is assumed to rise to $195 billion while its volatility drops to 37 percent. Reserves are assumed to rise by $5 billion and the increase in the dollar value of domestic currency liabilities is a reflection of both sterilization and exchange rate appreciation.

[1]Model inputs. Remainder are model outputs.

[2]In standard deviation of sovereign asset value.

[3]Spread in basis points.

[4]Based on a 1 percent change in sovereign asset value (for example, from 175 to 176.75) and sovereign asset volatility (for example, from 38 to 39 percent).

occur. Capital outflows are normally associated with some combination of an exchange rate depreciation, a drop in domestic debt prices (possibly associated with a rise in domestic interest rates), and an increase in volatility of both debt prices and the exchange rate. The impact of capital outflows on the sovereign balance sheet risk indicators depends in part on the response of policymakers. The assumption in this example is that policymakers accommodate some, but not all, of the shock. This would include some loss of international reserves, tighter interest rate policy, and an increase in the net fiscal asset. Under this scenario, sovereign asset value is assumed to fall by \$25 billion to \$155 billion, with international reserves falling from \$40 to \$35 billion. Sovereign asset volatility increases from 38 to 43 percent. The left column in Table 5 (Scenario 1) displays the new contingent claims sovereign balance sheet, balance sheet risk indicators, and sensitivities after capital outflows.

In sum, capital outflows worsen the credit risk indicators, and risk exposure of the sovereign has increased relative to the baseline. Distance to distress falls from 1.4 to 1.0 standard deviations and market-implied probability of default increases from 2.3 to 5.1 percent. Estimated market credit spreads on foreign currency debt rise to reflect the increased risk of nonrepayment because the expected loss has increased from \$1.5 billion to \$3 billion. In addition to a worsening of the credit risk indicators, the sensitivity measures have increased because implied sovereign asset value is fewer standard deviations from the distress barrier. For example, a 1 percent decline in sovereign asset value under the baseline scenario increased MIDP by 0.12 and estimated market credit spreads by 13 basis points, whereas in this capital outflow scenario these values are now 0.19 and 20, respectively. The higher sensitivities reflect the higher degree of nonlinearity within the option pricing formula as sovereign assets move closer to the distress barrier. This is indicative of observed nonlinear value changes in actual credit events.

A similar procedure can be applied to illustrate the opposite effects of capital inflows. Sustained capital inflows typically result in some exchange rate appreciation, improvement in domestic debt prices, and lower financial market volatility. Capital inflows may also provide space for an increase in international reserves that may necessitate sterilization operations. Based on this scenario, the value of sovereign assets is assumed to rise to \$195 billion while its volatility drops to 37 percent. Also, international reserves are assumed to rise by \$5 billion and the increase in the dollar value of domestic currency liabilities is a reflection of both sterilization and exchange rate appreciation. The right column of Table 5 (Scenario 2) displays the contingent claims sovereign balance sheet, credit risk indicators, and sensitivities after capital inflows. The increase in sovereign asset value and reduction in volatility yield the expected decrease in credit risk and sensitivity relative to the baseline. Distance to distress rises above two standard deviations and market-implied probability of default decreases by half to 1.2 percent. Estimated market spreads on foreign currency debt decline as the value of risky foreign currency debt approaches its default-free value. Each sensitivity measure decreases relative to the baseline from the improved sovereign asset value and volatility with respect to the distress barrier.

Monte Carlo Simulation

The scenario analysis above yields three related point estimates for the credit risk indicators, one from the baseline calibration and two from a negative and a positive shock. Although such scenario analysis may be useful in examining a specific event, it reveals only a very small view of the possible set of market disturbances. Scenario analysis to recreate a specific event is always subject to the criticism that market stress scenarios, in fact, rarely repeat themselves. In contrast, Monte Carlo simulation methods can be used to systematically deal with multiple scenarios, yielding probability distributions for risk indicators and VaR measures.[27] In addition to their use in stress testing, Monte Carlo simulations and the resulting distributions of each risk indicator are useful in understanding model uncertainty given the estimated inputs.[28] The Monte Carlo procedure implemented in this section takes random draws from hypothetical forward distributions for domestic interest rates and the exchange rate (for example, heir implied future distributions from options market data). The details of this process are discussed further in Box 1.

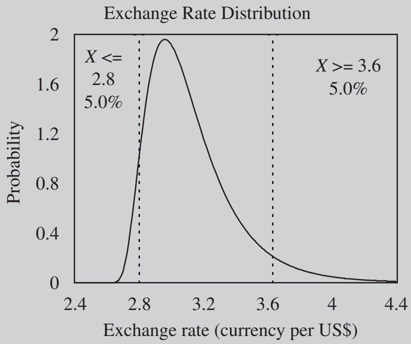
Probability distributions for distance to distress, risk-neutral default probability, risk-neutral spreads, and the value of sovereign assets resulting from the simulation are reported in Figure 6. Although the mean distance to distress remains 1.4 standard deviations, the same value as reported in Table 5 for the baseline calibration exercise, the distribution reveals a confidence interval for distance to distress based on the sample exchange rate

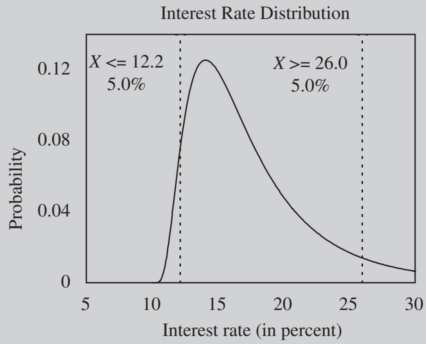
[27]Although Monte Carlo simulations are able to handle many thousand possible events, they produce a random set of outcomes based on the market characteristics assumed, which may or may not predict potential shocks. The simulation process will only produce as many extreme events as dictated by the distribution assumption of the market variables. To be comprehensive, simulation procedures should be combined with various scenario assumptions to produce a set of stress outcomes.

[28]As discussed in the previous section regarding the computation of implied sovereign assets and volatility, the calibrated inputs of the distress barrier and volatility of domestic currency liabilities are estimates. That is, $\hat{\sigma}$ is an estimate of the true volatility of domestic currency liabilities, $\hat{\sigma}$, and as such will contain standard error, leading to possible model risk. The presence of standard error otherwise results in confidence intervals around the point estimates of risk for each risk indicator. However, the traditional practices used to compute such estimates in the finance literature as described in this paper do not involve empirical regression estimation, making the construction of standard error bounds problematic. Instead of computing a confidence interval around the expected value of the risk indicator given standard error in $\hat{\sigma}$, the construction of probability distributions using Monte Carlo analysis is equivalent to confidence intervals around the expected value of the risk indicator given the estimate of $\hat{\sigma}$. The issue is further complicated by the fact that introducing standard errors on $\hat{\sigma}$ would result in error bands on the entire distribution of the risk indicators in the Monte Carlo simulation, greatly complicating the exercise.

Box 1. Implementing the Monte Carlo Simulation

The Monte Carlo simulation procedure applied in this section takes random draws from hypothetical forward distributions for both domestic interest rates and exchange rates and calculates the effects of these variables on balance sheet values and risk indicators. The one-year-forward exchange rate is assumed to be 3 units of the domestic currency to $1 and the one-year-forward interest rate is assumed to be 17 percent. Lognormal distributions for each were constructed based on recently observed market patterns in several emerging market economies, as shown in the figures below. The correlation between exchange rates and interest rates was set at 0.6, meaning that the Monte Carlo simulation conducts sample draws such that exchange rate depreciations are generally associated with higher interest rates and vice versa.





The Monte Carlo simulation procedure then selects random draws from these hypothetical distributions. The sample forward exchange rate is applied to the contingent claims sovereign balance sheet in the translation of domestic currency assets and liabilities into their respective U.S. dollar values. In contrast to exchange rate variations, simulating the effect of interest rate changes requires additional assumptions. Broad money is assumed to comprise half of domestic currency liabilities with the remainder in interest rate–linked domestic debt. The interest rate draw is applied to the existing domestic currency debt for a period of three years and then is assumed to return to 17 percent. If the realization of interest rates in the random draw is above the assumed 17 percent forward interest rate, the discounted marginal increase in interest costs is subtracted from the value of sovereign assets to reflect higher debt service costs. Alternatively, if the interest rate draw is below the assumed forward interest rate, then this discounted decrease in debt service costs is added to the value of sovereign assets.

The resulting sovereign balance sheet values from each random draw are then used to compute the new set of risk indicators. In contrast to the point estimates for the balance sheet risk indicators that result from scenario analysis, the process of conducting random samples from distributions of exchange rates and interest rates results in probability distributions for the relevant risk indicators.

and interest rate distributions. The lower 5 percent probability for distance to distress is 0.9 standard deviations, the upper 5 percent probability for MIDP is 5.4 percent, and the upper 5 percent probability for estimated credit spreads is 739 basis points. In other words, given the assumed exchange rate and interest rate distributions and correlation, distance to distress remains above 0.9 standard deviation, MIDP remains below 5.4 percent, and estimated credit spreads remain below 739 basis points 95 percent of the time. Finally, the 5 percent lower bound on sovereign assets is $160 billion, making the implied sovereign asset VaR equal to $15 billion.

VaR measures are often used to evaluate both market and credit risk in the financial sector.[29] In the financial sector, VaR typically defines a level of capital that is an upper bound on the amount of gains or losses to a portfolio from market or credit risk for a high degree of confidence. On the sovereign balance sheet, VaR by corollary could be defined as the upper bound on the amount of gains or losses to implied sovereign asset value from market risk.[30] Just as a bank or asset manager is required to hold capital in reserve to protect against market or credit loss, governments often identify a need to acquire sufficient levels of foreign currency reserves or insurance arrangements to protect against adverse market developments. The years following the Asian financial crisis have witnessed an increased desire by countries to hold reserves, with many of today's largest holders of reserves concentrated in developed and developing Asia. At end-2005, China topped the list with $946 billion in reserves, followed by Japan ($834 billion), Taiwan Province of China ($253 billion), Korea ($210 billion), India ($132 billion), and Singapore ($116 billion).[31] The sovereign VaR measure can be used as a tool to gauge whether the level of reserves is sufficient to protect against the risk of "sudden stops" or to maintain debt sustainability against adverse economic shocks.

Evaluating Policy Design

Using the Monte Carlo baseline simulation as a starting point, potential policy choices, such as changes in the level of reserves, alternative debt structures, or the use of risk mitigation instruments like insurance contracts, can be tested. Any change in policy modifies the sovereign balance sheet, and simulations using draws from the same interest rate and exchange rate distributions will reveal new distributions of risk indicators that can be evaluated against the original baseline configuration.[32] The example of debt

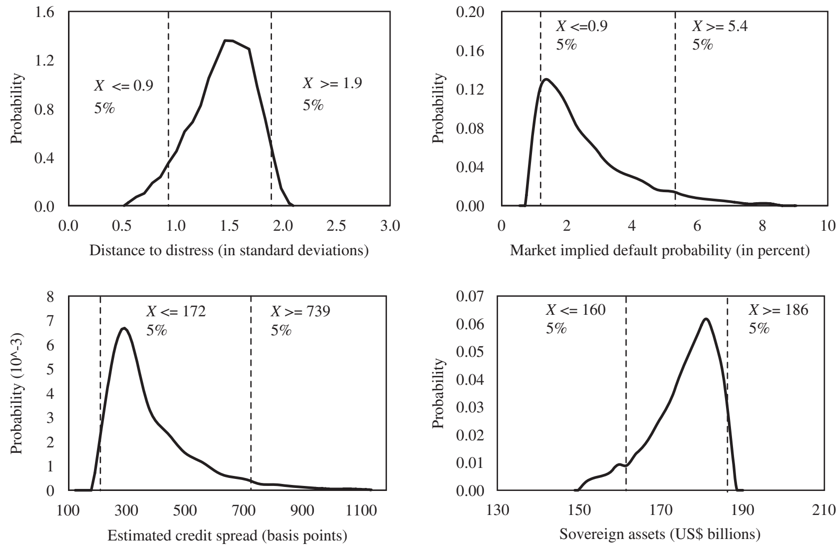
[29]See Jorion (2000). VaR models estimate the exposure of a portfolio, or the equivalent set of positions, to market risk. The measure captures the expected maximum loss and is usually expressed within a confidence interval.

[30]Two other sovereign VaR measures can be calculated. The first, sovereign capital-at-risk, is an extension of sovereign VaR for the central bank. The probability distribution of the residual value of "capital" or junior claim of the monetary authority is calculated and a confidence level attached to the risk that the monetary authority cannot meet its commitments. Blejer and Schumacher (1999) use a similar construction. The second, sovereign credit-at-risk, is the upper bound on gains or losses due to credit risk, which in this case is the value of the guarantee to the banking system. See Gapen and others (2004) for an example of how this could be modeled.

[31]See Gapen and Papaioannou (2007) for the various motives and implications of reserve accumulation throughout the Asian region.

[32]Simulating the adjusted sovereign balance sheet under the same exchange rate and interest rate distributions and correlation is subject to the critique that these distributions and correlations are derived from market expectations that are likely to change with the shift in policy. The simulations conducted in this paper should be viewed only as illustrating potential impacts from policy changes.

Figure 6. Monte Carlo Simulations: Hypothetical Sovereign



Source: Authors' calculations.

Note: The Monte Carlo simulation procedure takes random draws from hypothetical forward distributions for interest rates and exchange rates (as discussed in detail in Box 1) and applies this pair to the contingent claims balance sheet, resulting in a new set of one-year-ahead risk indicators. Repeating the process of random draws results in probability distributions and VaR measures for the relevant risk indicators.

management with alternative debt structures is examined first followed by a strategy for reserve accumulation. Finally, a combination of debt and reserve management is considered.

Panel A in Figure 7 illustrates an example of liability management, whereby $10 billion in foreign currency debt is replaced with an equal amount of interest rate–linked domestic currency debt to examine the impact of reduced exchange rate exposure. As a result, the distress barrier falls to $90 billion while domestic currency liabilities increase by $10 billion. The new Monte Carlo simulations on this adjusted balance sheet yield improvements in the risk indicators. The mean values and confidence intervals for distance to distress, MIDP, and estimated credit spreads all improve.

Panel B of Figure 7 illustrates the example of reserve accumulation financed with an equal amount of domestic currency debt such that the level of sovereign assets and interest rate–linked domestic currency liabilities both increase by $10 billion. This scenario could be viewed as a proactive strategy to accumulate reserves or reflect capital inflows and, consequently, the increase in domestic currency debt is the result of sterilization. The operation yields improvements in the risk indicators, although the margin of improvement is less than that found in the example on debt management.

The reserve and debt management strategies above can also be implemented simultaneously, as shown in Panel C of Figure 7. The distress barrier declines to $90 billion, the amount of domestic currency debt increases by $20 billion, and the level of foreign currency reserves increases by $10 billion. The effect of simultaneously enacting both strategies yields improvements in the risk indicators by an amount equal to more than the sum of the two strategies individually, reflecting model nonlinearity. Combining the two strategies is advantageous because the debt management operation reduces the distress barrier while the reserve accumulation strategy leaves the mean value of sovereign assets nearly unchanged relative to the baseline.

Deciding on the efficacy of any of the above strategies involves a systematic weighting of the trade-offs inherent in each case while taking into account the inherent limits of such an exercise. There are clear elements in each of the three alternative strategies that are beneficial from a policy perspective (the reduction in exchange rate exposure and increase in reserves) that need to be balanced against the clear negatives from a balance sheet perspective (the increase in domestic currency interest-bearing obligations). Contingent claims analysis can therefore be useful in guiding policy design given its ability to compare different policy options using quantifiable risk indicators.

VI. Next Steps

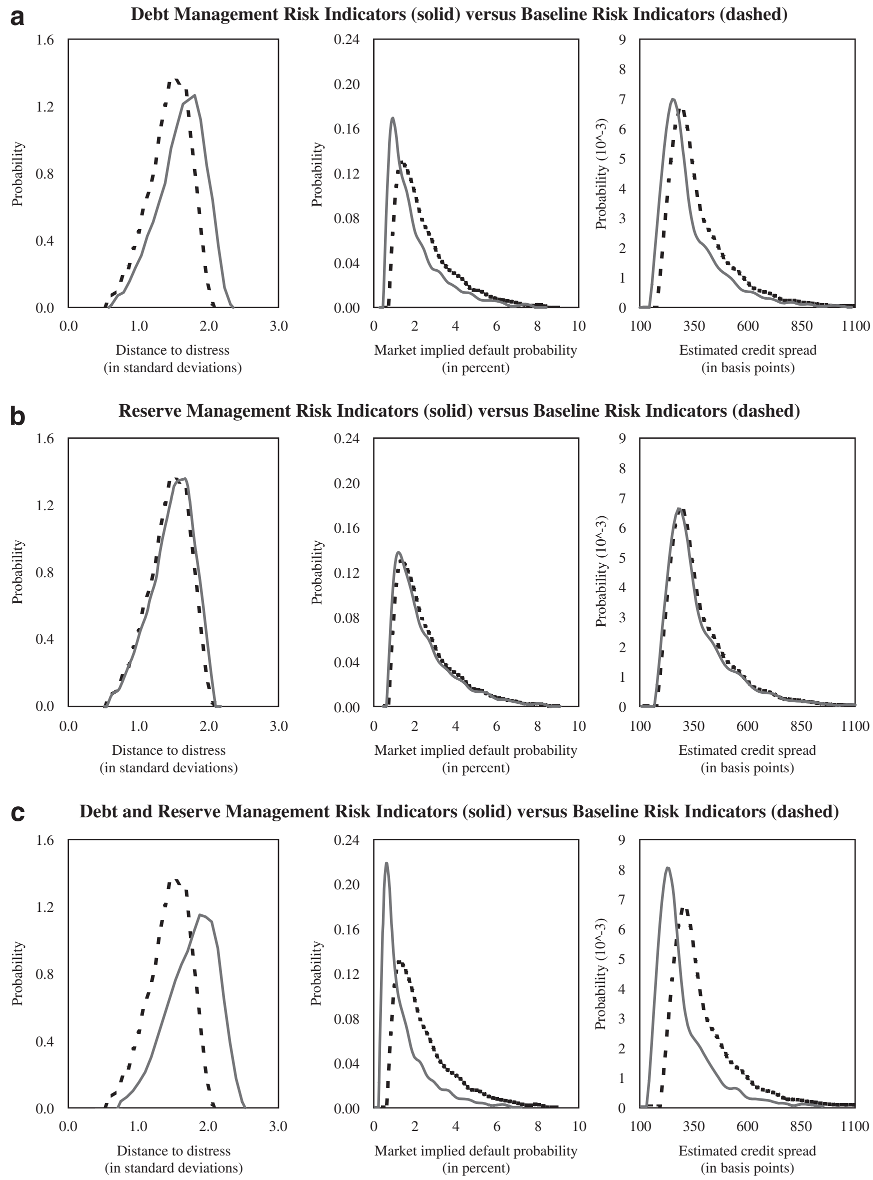
The contingent claims framework can be adapted and extended in several important directions. The framework can be used to estimate an appropriate target for reserve adequacy, where adequacy could be defined as the level of reserves that minimizes the probability of distress. It is also well suited for a more robust analysis of debt sustainability as compared with the widely used debt-to-GDP ratio, which is a static, backward-looking indicator.

A Robust Framework for Reserve Management

The application of contingent claims analysis and sovereign VaR to reserve management is a stark departure from accounting indicators commonly used for reserve management. One widely used indicator of reserve adequacy is the ratio of foreign currency reserves to total public and private short-term foreign currency debt. Both public and private sector debt is included because reserves of the public sector must facilitate transactions related to economy-wide financing requirements. However, the simple accounting ratio of reserves to total short-term foreign currency debt is deficient when it comes to risk analysis because it does not take uncertainty of balance sheet risks into account. Applying a broad-based rule for an appropriate ratio of reserve coverage uniformly across countries implicitly assumes all sovereign balance sheet risks are similar, and neglects cross-country differences in sovereign balance sheet risk.[33]

[33]IMF (2000) examines three ratios: reserves to imports, reserves to monetary aggregates, and reserves to public and private short-term foreign currency debt by residual maturity. The report concludes that reserves to short-term foreign currency debt is a superior measure and recommends that a ratio of 1 be a lower bound for adequate reserve coverage.

Figure 7. Evaluation of Policy Options



Source: Authors' calculations.

Note: Monte Carlo simulations are conducted using alternative balance sheet structures to assess the effect of alternate policies on one-year-ahead estimates of risk.

In contrast, an appropriate target for reserve adequacy could be based on a level of reserves that minimizes instances of distress using the contingent claims risk indicators. For example, an adequate level of reserves could be defined as the level of reserves that keeps distance to distress above a desired standard deviation 95 percent of the time based on the likely exchange rate process. Adequate reserve coverage could also target a basket of credit risk indicators by setting reserve levels to maintain the combined set of indicators at target levels for a specific confidence interval. In sum, reserve management using this framework examines the impact of the level and volatility of reserves as a component of the wider sovereign asset value and volatility with a link to the balance sheet risk indicators. The application of contingent claims in analyzing sovereign credit risk can be adapted to include many different aspects of reserve management, including the currency composition of reserves, or various other risk mitigation techniques.[34]

A Robust Framework for Debt Sustainability

In addition to providing a framework for reserve management, the use of contingent claims to analyze sovereign risk is well suited for robust debt sustainability analysis. Traditional debt sustainability analysis has focused on ratios of current and forecasted debt to GDP as the primary criterion for deciding whether the public sector debt remains on a sustainable path, usually without explicitly incorporating uncertainty in a systematic, coherent framework. The following elements indicate why the approach in this paper could provide the basis for a more robust framework for debt sustainability analysis:

- The contingent claims sovereign balance sheet translates balance sheet risks into quantifiable risk indicators. In this framework, debt sustainability could be defined as the debt structure that keeps key credit risk indicators below (or above) certain threshold levels for a given confidence level. In contrast, the debt-to-GDP ratio identifies an element of sovereign risk but is not part of a structural framework that measurably relates debt payments with the capacity to pay. For example, the contingent claims structural framework is able to assess the impact of changes in the level of reserves on sovereign risk, whereas the debt-to-GDP ratio remains invariant to such changes.
- The quantitative sovereign credit risk indicators described in this paper incorporate uncertainty and volatility. Higher market uncertainty is often translated into higher interest rate and exchange rate volatility, widening the forward distributions on both variables and increasing the volatility of sovereign assets. Distance to distress will fall, probability of default will rise along with spreads on foreign currency debt, and the expected loss on risky foreign currency debt will increase. However, the debt-to-GDP ratio

[34]See Gray (2007) for applications to sovereign wealth management and Gray and Malone (forthcoming) for additional examples. Caballero and Panageas (2005) also examine various instruments and risk mitigation strategies that policymakers could implement in addition to traditional reserve accumulation in a model of sudden stops in capital flows.

does not change with an increase in sovereign asset volatility and would therefore miss an important component of risk analysis.

- The contingent claims sovereign balance sheet includes an assessment of maturity or rollover risk through construction of the distress barrier.[35] The debt-to-GDP ratio does not change if a decrease in short-term foreign currency debt is matched by an equal book value increase in long-term foreign currency debt. The use of the contingent claims sovereign balance sheets reflects this change by signaling a decrease in sovereign risk owing to the more favorable debt profile.
- Finally, contingent claims analysis incorporates nonlinear value changes. The use of nonlinear modeling in a structural framework captures complex relationships and more accurately conveys the nonlinear nature of credit events. During periods of stress, small changes in interest rates, exchange rates, and/or volatility can result in large changes in sovereign risk on the margin. An accounting ratio such as debt to GDP is not capable of this level of complexity, nor is it released with enough frequency to enable its use during periods of stress where vulnerabilities may build or subside rapidly.

Using contingent claims to model sovereign credit risk therefore offers several advantages over the traditional debt-to-GDP analysis. Additional research in this direction could prove useful and would require extension of the framework to a medium-term setting while incorporating the outlook for relevant economic and policy variables.

VII. Conclusions

This paper develops a comprehensive new framework to measure and analyze sovereign risk by applying contingent claims analysis to the balance sheet of the combined government and monetary authorities. A marked-to-market balance sheet is constructed that provides a structural framework that identifies balance sheet risks, incorporates uncertainty, and yields quantifiable risk indicators. The main outputs of this framework include sovereign credit risk indicators, sensitivity measures, and sovereign VaR. These sovereign risk indicators incorporate both forward-looking market prices and nonlinear changes in values and should consequently have greater predictive power in estimating sovereign credit risk than would

[35]This is true whether one uses the simplified distress barrier in this paper (short-term foreign currency debt and interest plus one-half long-term foreign currency debt) or a more sophisticated approach (short-term foreign currency debt and interest plus the present discounted value of long-term foreign currency debt and interest). Both approaches would reflect an increase in sovereign risk if long-term foreign currency debt was traded for equal book value amounts of short-term foreign currency debt. The distress barrier under the second approach, however, would be more sensitive to near-term repayment humps that would carry a higher weight in the distress barrier than a similar payment profile further out on the maturity scale.

traditional macroeconomic vulnerability indicators or accounting-based ratios.

Application to a sample of emerging market economies shows the risk indicators to be robust and significant when compared with market observed credit spreads on foreign currency debt, even though the spreads were not used as inputs. This lends support for the approach as well as illustrates that the level and variation of forward exchange rates and other market variables contain valuable information for analyzing sovereign credit risk.

Using contingent claims to analyze sovereign risk has several merits from a policy perspective. The ability of the approach to provide a structural interpretation of the sovereign balance sheet, unique to each economy, is a valuable contribution in the area of policy design and risk management, translating policy choices and changing economic conditions directly into quantitative indicators of financial soundness. The ability of contingent claims analysis to measurably assess the potential policy mix can be an important element of strategic planning and offers policymakers the valuable opportunity to rank policy options.

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