Macro-Financial Implications of the Surging Global Demand (and Supply) of International Reserves *

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Abstract

Research has shown that the unilateral accumulation of international reserves by a country can improve its own macro-financial stability. However, we show that when many countries accumulate reserves, the induced general equilibrium effects weaken financial and macroeconomic stability, especially for countries that do not accumulate reserves. The issuance of public debt by advanced economies has the opposite effect. We show these results with a two-region model where private defaultable debt has a productive use. Quantitative counterfactuals show that the surge in reserves (public debt) contributed to reduce (increase) world interest rates but also to increase (reduce) private leverage. This in turn increased (decreased) volatility in *both* emerging and advanced economies.

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1 Introduction

The foreign exchange (FX) reserves of emerging market economies (EMEs) increased significantly during the last three decades, as shown in the first panel of Figure 1. The sharp increase is especially notable after the 1990s Sudden Stops: FX reserves increased from 10 percent of GDP in 1997 to 30 percent in 2009.[1] Foreign reserves also increased in advanced economies but at a much slower pace.

Foreign reserves are mainly held in the form of short-term public debt issued by advanced economies (AEs), particularly U.S., Europe and Japan. The second panel of Figure 1 shows that the public debt of AEs rose sharply following the 2008 global financial crisis. It rose from about 60 percent of GDP in 2007 to about 95 percent in 2012.

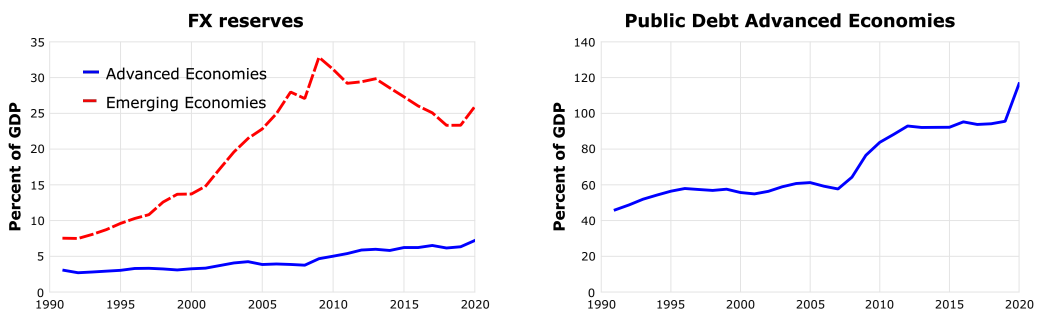

Figure 1: Foreign Exchange Reserves of Advanced and Emerging economies and Public Debt of Advanced economies.

Note: Data for FX reserves is from External Wealth of Nations database (Lane and Milesi-Ferretti (2018)). Advanced economies: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. Emerging economies: Algeria, Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Colombia, Estonia, Hong Kong, Hungary, India, Indonesia, Israel, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, Singapore, South Africa, Thailand, Turkey, Ukraine, Venezuela. Data on public debt is from IMF Global Debt Database. We use the series Central Government Debt which is available for thirteen countries: Canada, Finland, France, Germany, Italy, Japan, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. The Global Debt Database provides two series: 'Central Government Debt' and 'General Government Debt'. We use the former. Data for all years 1991-2020 are available only for thirteen of the advanced economies (listed above). Hence, our measure of debt-to-GDP ratio for advanced economies results from the aggregation of these thirteen countries.

[1] It is well-known that China played an important role in this increase, but other EMEs did too. As a share of global GDP, EME's (China's) reserves grew from roughly 1.2% (0.2%) in 1991 to 11.25% (4.8%) in 2010.

From a global perspective, these changes are important because the growth of FX reserves in EMEs raised the demand for risk-free financial assets (contributing to a lower world interest rate), while more issuance of public debt by AEs increased their supply (contributing to a higher interest rate). The goal of this paper is to understand how these changes affected world credit markets (the world interest rate and credit positions of EMEs and AEs) and impacted global financial and macroeconomic volatility.

To this end, we develop a quantitative model that features two regions, representing AEs and EMEs, respectively. In each region, there are borrowers (issuers of financial liabilities) and lenders (buyers of those liabilities). Two characteristics of these liabilities are central to our analysis. The first is that lenders in the private sector hold the liabilities because they provide a convenience yield by facilitating production. The second is that the liabilities issued by the private sector are defaultable.

A financial crisis occurs when private borrowers default and do not repay their debt in full. This happens in states in which the debt exceeds the liquidation value of the real assets owned by borrowers, and generates haircuts that redistribute wealth from creditors to debtors. This redistribution is critical for our findings, because it causes adverse real macroeconomic effects by wiping out some of the financial assets held by producers. The magnitude of these effects depends on the financial structure of the economy: When leverage is high, a financial crisis generates a larger redistribution of wealth and hence stronger macroeconomic effects.

How does the accumulation of foreign reserves by EMEs affect the magnitude of financial crises and global macroeconomic volatility? A sizable increase in EMEs' reserves relatively to the size of the world economy causes a reduction in the world interest rate, which in turn leads to higher private sector leverage in *both* emerging and advanced economies. Because of the higher leverage, financial crises cause larger wealth redistribution and stronger effects on the real economy (higher output volatility). Since volatility increases in both regions, the surge in EMEs' reserves has a negative spillover effect on AEs. An increase in the supply of public debt has the opposite implications: It increases the supply of assets, raising the interest rate and reducing the severity of financial crises.

In order to use the model to quantify the impact of reserves accumulation and public debt issuance on financial and real sectors globally, we need to set values for three key model parameters in each of the two regions: Total factor productivity; a parameter that affects the private demand for financial instruments (determining the extent to which they are needed as a productive asset); and a parameter that affects the private supply of financial assets (determining the liquidation value of borrowers' capital and thus their capacity to issue debt). We also need to set the initial values of foreign reserves in both AEs and EMEs and public debt in AEs.

We calibrate the six parameters together with the three initial values of foreign reserves and public debt so that the model matches nine data targets in year 1991. We then conduct counterfactual simulations over the period 1991-2020 comparing a scenario in which (detrended) reserves and debt are kept constant at their 1991 values with one in which they take the values actually observed in each year. The results show that the observed surge in reserves caused a sharp increase in macroeconomic and financial volatility while the increase in public debt reduced it.

We also consider the possibility that FX reserves may be used to provide liquidity and stabilize the economy in the event of a financial crisis. In particular, since the adverse real effects of a financial crisis in our model are due to the destruction of entrepreneurial wealth (i.e., the defaulted debt), we assume that FX reserves are used to bail out a fraction of the financial losses of entrepreneurs. This arrangement helps little to reduce the volatility of advanced economies, because they do not hold large stocks of FX reserves relatively to their size and hence bailouts are relatively small. However, aggregate volatility drops markedly for emerging economies that accumulate reserves.

Related literature. Our work is related to three strands of literature: (i) financial and macroeconomic implications of FX reserves; (ii) financial crises or Sudden Stops; (iii) scarcity of financial assets.

There is an extensive literature on the financial and macroeconomic implications of FX reserves. One branch focuses on foreign exchange interventions and their effects on exchange rates and financial stability (see the detailed survey by Popper (2022)). Interestingly, Kim, Mano, and Mrkaic (2020) found that firm-level leverage in EMEs increases in the aftermath of these interventions. Some studies focus on the implications of reserves for sovereign borrowing, vulnerability to financial crises, and design of macroprudential policy (e.g., Alfaro and Kanczuk (2009), Durdu, Mendoza, and Terrones (2009), Devereux and Wu (2022), Bianchi, Hatchondo, and Martinez (2018), Bianchi and Lorenzoni (2022), Bianchi and Sosa Padilla (2024), Kondo and Hur (2016)).

The above studies analyze the role of reserves in the context of small open economies, and thus treat the world risk-free interest rate as exogenous. In contrast, our analysis deviates from the small open economy assumption by proposing a mechanism that operates through general equilibrium changes in the world interest rate. This is also a feature of the model studied in Das, Gopinath, Kim, and Stein (2023). They show that central banks may over-accumulate reserves because they ignore the general equilibrium effects of their individual actions on the dollar interest rate. Our goal differs, however, in that we aim to quantify the impact of general-equilibrium changes in the world interest rate on global volatility rather than on the optimality of reserves accumulation.

The mechanism through which the world interest rate affects the economy is also different, because in our setup it operates through financial leverage rather than currency mismatch. In particular, we show that a *collective* increase in reserves by EMEs, which is exogenous in our model, contributed to the fall in the world real interest rate, the expansion of private credit, and the increase in global macroeconomic volatility since the 1990s. By contrast, most of the existing literature finds that unilateral increases in reserves by *individual* countries improve financial stability by reducing the likelihood of self-fulfilling sovereign debt crises or allowing countries to provide liquidity to the private sector in the eventuality of a crisis.

Various contributions in the Sudden Stops literature examine the role of financial globalization, credit booms and high leverage as causing factors of financial crises. Examples include Calvo and Mendoza (1996), Caballero and Krishnamurthy (2001), Gertler, Gilchrist, and Natalucci (2007), Edwards (2004), Mendoza and Quadrini (2010), Mendoza and Smith (2014), Fornaro (2018). See also Bianchi and Mendoza (2020) for a survey of the literature. Some of these studies emphasize mechanisms that cause financial crises because of equilibrium multiplicity due to self-fulfilling expectations as in Aghion, Bacchetta, and Banerjee (2001) and Perri and Quadrini (2018). Crises in our model also follow from periods of fast credit and leverage growth, and they are the result of self-fulfilling expectations.

Studies in the corporate finance literature document and provide explanations for the raising demand of financial assets. An example is the literature on the growing cash holdings of nonfinancial businesses (e.g., Riddick and Whited (2009), Busso, Fernández, and Tamayo (2016) and Bebczuk and Cavallo (2016)). Our model has a similar feature in that some businesses, but not all, hold positive positions in financial assets. Our focus, however, is on the macroeconomic implications. The increase in net demand for financial assets due to the increased accumulation of FX reserves, depresses the interest rate—a general equilibrium effect—which in turn strengthens incentives to leverage. While the higher leverage allows for sustained levels of financial intermediation and economic activity, it also makes both emerging and advanced economies more vulnerable to crises (global instability).

The remaining of the paper is organized as follows: Section 2 describes the model and characterizes the equilibrium. Section 3 calibrates the model and conducts counterfactual simulations to quantify the general equilibrium effects of FX reserves and public debt on the world interest rate, credit, leverage and net foreign assets (NFA). Section 4 analyzes the implications of changes in FX reserves and public debt for macro and financial volatility. Section 5 studies an extension of the model in which reserves are used to cover part of the entrepreneurs' losses in the eventuality of a crisis. Section 6 discusses the welfare implications of FX reserves accumulation and issuance of public debt. Section 7 concludes.

2 Model

Consider a world economy that consists of two regions indexed by $j \in \{1, 2\}$. Region 1 represents advanced economies and Region 2 emerging economies. In each region there are three sectors: (i) a business sector with two types of firms; (ii) a household sector that supplies labor; and (iii) a public sector that provides lump-sum transfers to households, holds financial assets in the form of FX reserves and, in Region 1, issues liabilities (public debt).

We model two types of firms as a means to generate private borrowing and lending within a region (in addition to cross-regional borrowing and lending). We distinguish the private demand for financial assets by firms with positive financial positions from the private supply by firms with negative positions. The public sector allows us to study how the issuance of public debt and accumulation of FX reserves affect the economies of the two regions.

Regions are heterogeneous in three key dimensions: (i) economic *size*, captured by differences in aggregate productivity, z_j; (ii) a financial parameter that affects directly the *demand* for financial assets, ϕ_j; and (iii) a financial parameter that affects directly the *supply* of financial assets, κ_j. Differences in economic size could be generated by other factors besides productivity (e.g., population, real exchange rates, etc.). For the questions addressed in this paper, however, they are isomorphic to productivity differences.

Regions also differ in their stocks of foreign reserves, $FX_{j,t}$, and public debt issued by the government of advanced economies, $D_{p,t}$. For simplicity we assume that emerging economies do not issue public debt. Foreign reserves $FX_{j,t}$ and public debt $D_{p,t}$, are time-varying but not stochastic. Thus, their evolution over time is fully anticipated. The only source of uncertainty in the model comes from "sunspot" shocks described later.

2.1 Household sector

In each region, there is a unit mass of households that maximize the following expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{j,t} - \mu_j \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

where $c_{j,t}$ is consumption, $h_{j,t}$ is the supply of labor, ν is the elasticity of labor supply, and μ_j is a parameter that scales labor disutility. The assumption that the utility is linear in consumption simplifies the characterization of the equilibrium. It allows us to derive analytic results without affecting, in important ways, the properties of the model that are central for the questions addressed in this paper. The scale parameter μ_j allows the model to support similar employment rates in the two regions even if there are larger regional differences in productivity.

The households' budget constraint is

$$c_{j,t} = w_{j,t} h_{j,t} + \text{div}_{j,t} + T_{j,t}.$$

Consumption is paid for with wage income, $w_{j,t} h_{j,t}$, dividends distributed by firms owned by households, $\text{div}_{j,t}$, and government transfers, $T_{j,t}$, or taxes when $T_{j,t} < 0$.

The only relevant decision made by households is the supply of labor, which is determined by this first-order condition:

$$\mu_j h_{j,t}^{\frac{1}{\nu}} = w_{j,t}. \tag{1}$$

Notice the linearity of utility in $c_{j,t}$ neutralizes the wealth effect on labor supply by making the marginal rate of substitution between labor and consumption (i.e., the left-hand-side of eq. (1))independent of the latter.

2.2 Business sector

There are two types of firms in the business sector: producers of intermediate goods and producers of final goods. The former are owned by households, and the latter are owned and operated by entrepreneurs. An important difference between them is that capital—which is pledgeable as collateral—is used only by intermediate-goods firms. Final-goods producers lack collateral assets. At equilibrium, then, the first type of firms are net borrowers and the second are net lenders (i.e., they have a positive position in financial assets). In this way, we generate borrowing and lending within the business sector.[2] We begin with the description of intermediate-goods producers.

2.2.1 Intermediate goods producers

Intermediate-goods firms produce inputs $x_{j,t}$ using labor, $l_{j,t}$, and capital, $k_{j,t}$, with the following Cobb-Douglas technology:

$$x_{j,t} = l_{j,t}^{\gamma} k_{j,t}^{1-\gamma}.$$

Firms solve a dynamic problem that maximizes the discounted value of dividends paid to households (see Appendix C), but the resulting labor demand decision is actually static. Given the stock of capital $k_{j,t}$, they choose labor demand to maximize profits $p_{j,t}x_{j,t} - w_{j,t}l_{j,t}$, where $w_{j,t}$ is the

[2]Differences in financial structure could reflect the tangibility of capital. Firms that are intensive in intangible capital do not have enough collateral assets to borrow and, as a result, accumulate financial assets or cash. Falato, Kadyrzhanova, Sim, and Steri (2022) show the importance of this mechanism for explaining the rising cash holdings of US corporations during the last four decades. These firms are captured in the model by the final-goods producers. However, the fact that in the model intermediate-goods producers are net borrowers and final-goods producers are net lenders should not be interpreted literally when mapping the model to the data. What really matters is that there is production complementarity between the two groups of firms, so that, when firms in one group cut production due to financial conditions, the other firms also cut their production due to lower demand. A similar property would arise if we assume that final output results from aggregating the production of the two types of firms.

wage rate and $p_{j,t}$ is the price at which they sell the intermediate goods to final producers in competitive markets (in units of final goods). The optimal demand for labor is then determined by the first-order condition that equates the marginal revenue product of labor to the wage rate,

$$\gamma p_{j,t} l_{j,t}^{\gamma-1} k_{j,t}^{1-\gamma} = w_{j,t}.$$

Capital is reproducible without adjustment costs. Thus, in normal conditions, the price of capital is 1. To keep the model tractable, however, we assume that capital evolves exogenously.

Borrowing and default. Intermediate-goods firms can also borrow. At the end of period $t-1$, firms borrow $d_{j,t}/R_{j,t-1}$, where $R_{j,t-1}$ is the gross interest rate and $d_{j,t}$ is the debt (promised repayment) due at time t. At the beginning of period t, when the debt is due, they could default. Under default, creditors have the right to liquidate the capital $k_{j,t}$. However, the liquidation value of capital could be insufficient to fully repay the debt $d_{j,t}$.

Denote by $\ell_{j,t}$ the liquidation price of capital at the beginning of period t. If the debt is bigger than the liquidation value, that is, $d_{j,t} > \ell_t k_{j,t}$, the debt is renegotiated. Under the assumption that borrowers have the whole bargaining power, the renegotiated debt is

$$\tilde{d}(d_{j,t}, \ell_{j,t} k_{j,t}) = \min\Big\{ d_{j,t} \ , \ \ell_{j,t} k_{j,t} \Big\}. \tag{2}$$

After renegotiation, the market for capital returns to normal at the end of the period (i.e., there is no market exclusion). Note also that liquidation never happens at equilibrium, it only acts as a threat to renegotiate the debt because neither party gains from liquidation, and so they settle for a lower repayment of the debt (for an amount equal to $\ell_t k_{j,t}$) with the physical capital remaining in place.

A key assumption we make is that there are states of nature in which the market for liquidated capital freezes and the liquidation price at the beginning of the period drops below its normal price of 1. More specifically, with probability $1-\lambda$ the liquidation price remains at its normal price $\ell_{j,t} = 1$. With probability λ, however, it drops to $\ell_{j,t} = \kappa_j$, where $\kappa_j < 1$. As we explain below, κ_j is a key parameter in the determination of the private supply of financial assets.

Appendix D describes the mechanism that generates a freeze in the market for liquidated capital as a result of self-fulfilling expectations about the liquidation price of capital. This depends on the borrowers' leverage. More specifically, when $d_{j,t} > \kappa_j k_{j,t}$, there are two equilibria. In one equilibrium, the market does not freeze and the liquidation price is 1. In the other, the market freezes and the liquidation price drops to $\kappa_j < 1$. The selection between the two equilibria is determined by the draw of a sunspot shock $\varepsilon_j \in \{0, 1\}$, and λ is the exogenous probability that the draw of the sunspot shock is the one associated with the market freeze.

Readers interested in the micro-foundation of the market freeze may wish to treat Appendix D as an integral part of the current section. Otherwise, the Appendix can be skipped. What is essential for the analysis that follows is that the liquidation price of capital $\ell_{j,t}$ takes the value of 1 with probability $1 - \lambda$ and $\kappa_j < 1$ with probability λ. The sunspot variables ε_1 and ε_2 are the only exogenous stochastic variables (shocks) in the model.[3]

The final assumption regarding intermediate-goods firms is that the issuance of new debt $d_{j,t+1}$ carries the convex cost

$$\varphi\left(d_{j,t+1}, \kappa_j k_{j,t+1}\right) = \eta \left[\frac{\max\{\,0\,,\; d_{j,t+1} - \kappa_j k_{j,t+1}\,\}}{d_{j,t+1}}\right]^2 d_{j,t+1}. \qquad (3)$$

Figure 2 provides a graphical illustration of this cost. As long as the debt repayment promised for the next period, $d_{j,t+1}$, does not exceed the minimum liquidation value, $\kappa_j k_{j,t+1}$, the cost is zero. Beyond that point, the cost rises at an increasing rate.

The debt issuance cost plays a similar role as a borrowing limit ensuring that borrowing is bounded at equilibrium. The parameter η determines, for a given stock of capital, the speed at which the cost rises with debt. Thus, it captures the flexibility with which borrowing responds to changing market conditions (e.g., the interest rate). For very high η we have, effectively, a standard borrowing limit, that is, $d_{j,t+1} \leq \kappa_j k_{j,t+1}$. At lower values of η, however, the model allows for an endogenous response of debt to changes in the interest rate. With a hard borrowing limit, instead, the interest rate would not impact the equilibrium debt (unless the limit also changes).

The budget constraint for intermediate-goods firms, after the renegoti-

[3] Benhabib, Dong, Wang, and Xu (2024) develop an interesting model of self-fulfilling default cycles. The mechanism generating multiple equilibria in their model relies on the survival of active firms, a number that changes with crises. Our mechanism, instead, relies on the liquidation value of collateral.

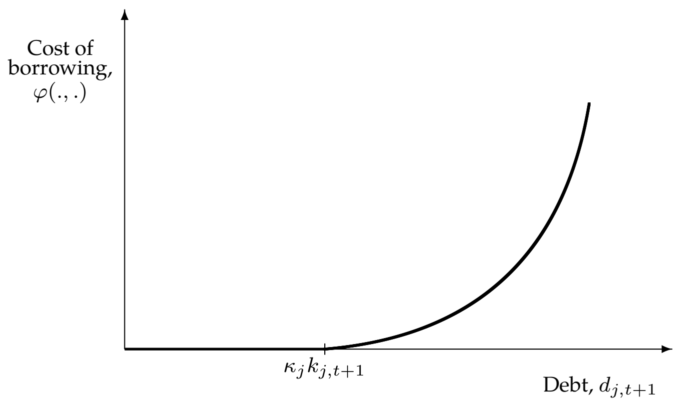

Figure 2: Convex cost of borrowing as a function of debt.

ation of the debt, is

$$\text{div}_{j,t} = p_{j,t} l_{j,t}^{\gamma} k_{j,t}^{1-\gamma} - w_{j,t} l_{j,t} - i_{j,t} - \tilde{d}(d_{j,t}, \ell_{j,t} k_{j,t}) + \frac{d_{j,t+1}}{R_{j,t}} - \varphi(d_{j,t+1}, \kappa_j k_{j,t+1}). \quad (4)$$

where $i_{j,t} = k_{j,t+1} - (1-\tau)k_{j,t}$ is investment and τ the depreciation rate.

The gross interest rate $R_{j,t}$ depends on individual borrowing decisions. If the firm borrows more, relatively to the ownership of capital, the expected repayment will be lower in the next period. This will be reflected in a higher interest rate on the newly issued debt.

Denote by $\overline{R}_{j,t}$ the *expected* gross return from buying a diversified portfolio of debt issued by all intermediate-goods firms in Region j at time t. Since firms are atomistic and financial markets are competitive, the expected return on the debt issued by an 'individual' firm must be equal to the expected return from the diversified portfolio, that is,

$$\frac{d_{j,t+1}}{R_{j,t}} = \frac{1}{\overline{R}_{j,t}} \mathbb{E}_t \tilde{d}(d_{j,t+1}, \ell_{j,t+1} k_{j,t+1}). \quad (5)$$

The left-hand-side is the amount borrowed in period t while the right-hand-side is the expected repayment in period $t+1$, discounted by the market return $\overline{R}_{j,t}$. Since an intermediate-goods firm renegotiates the debt when $d_{j,t+1} > \ell_{j,t+1} k_{j,t+1}$, the actual repayment $\tilde{d}(d_{j,t+1}, \ell_{j,t+1} k_{j,t+1})$ could be lower than $d_{j,t+1}$. Competition in financial markets requires that the left-hand-side equals the right-hand-side.

Equation (5) determines the interest rate $R_{j,t}$ for an individual borrower. It can also be viewed as determining the borrowing spread paid by the borrower, $R_{j,t}/\overline{R}_{j,t} = d_{j,t+1}/\mathbb{E}_t\tilde{d}(d_{j,t+1}, \ell_{j,t+1}k_{j,t+1})$. For a firm expected to fully repay with certainty, the spread is zero ($R_{j,t}/\overline{R}_{j,t} = 1$). For a firm that is expected to repay in full only with some probability, $R_{j,t}$ exceeds $\overline{R}_{j,t}$. The higher rate depends on how much the contracted repayment, $d_{j,t+1}$, falls below the expected repayment after renegotiation, that is, $\mathbb{E}_t\tilde{d}(d_{j,t+1}, \ell_{j,t+1}k_{j,t+1})$. At equilibrium, all firms make the same decisions and they all borrow at the same rate. In order to characterize the optimal borrowing, however, we need to allow for individual deviations.

Firms' decisions. Intermediate-goods firms make decisions sequentially. At the beginning of the period they decide whether to default and renegotiate the debt. After that, they choose the input of labor $l_{j,t}$ and produce $x_{j,t}$. Finally, they choose the new debt $d_{j,t+1}$. Since the default and production decisions have already been characterized, we focus here on the optimality condition for the choice of the new debt.

Appendix C presents the optimization problem solved by an individual firm. The first-order condition for $d_{j,t+1}$ is

$$\frac{1}{\overline{R}_{j,t}} = \beta + \Phi\left(\frac{d_{j,t+1}}{\kappa_j k_{j,t+1}}\right). \tag{6}$$

The function $\Phi(.)$ is an endogenous object that embeds expectations of future variables, with the explicit functional form provided in Appendix C. The only source of uncertainty in the model is the realization of sunspot shocks. Since future repayments conditional on default and the probability of default are known in advance, we can calculate analytically the expected repayment, which is incorporated in $\Phi(.)$.

The function $\Phi(.)$ is increasing in the ratio $d_{j,t+1}/\kappa_j k_{j,t+1}$, mirroring the increasing cost of borrowing showed in Figure 2.[4] This ratio is a measure of *effective* leverage: debt over the minimum liquidation value of capital. Because $\Phi(.)$ is an increasing function, condition (6) posits a *negative* relationship between the expected cost of the debt (the interest rate) and effective leverage. This relationship is central to our finding that lower interest

[4]It corresponds to the ratio of the marginal issuance cost to the expected marginal change in repayment of the new debt. When the cost is zero, i.e., $\eta = 0$, $\Phi(.) = 0$ and the debt issuance decision becomes undetermined in equilibrium.

rates, resulting from the surge in FX reserves, increase leverage and worsen financial instability. A reverse mechanism applies when public debt increases.

2.2.2 Final goods producers (entrepreneurs)

In each region, there is a unit mass of atomistic entrepreneurs that produce final goods. They maximize logarithmic expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c^e_{j,t}),$$

where $c^e_{j,t}$ denotes the entrepreneur's consumption in Region j at time t. As we will see, the concavity of the utility is helpful because implies that entrepreneurs will hold a diversified portfolio of financial assets.

Entrepreneurs are business owners producing homogeneous goods that can be traded internationally. Although they resemble privately-owned firms, we should think of them more broadly and including also some publicly-traded companies. Entrepreneurial consumption, then, can be interpreted as dividend payments and the concavity of the utility function could derive from the risk aversion of managers and/or major shareholders. Although not explicitly modeled, the concavity could also reflect, in reduced form, the cost associated with financial distress: even if shareholders and managers are risk-neutral, a convex cost of financial distress would make the objective of the business concave. Since there are no idiosyncratic shocks in the model, we can focus on the representative entrepreneur.

The production function of final-goods producers is linear:

$$y_{j,t} = z_j x_{j,t}, \tag{7}$$

where $y_{j,t}$ is production, z_j is region-specific productivity, and $x_{j,t}$ is the input of intermediate goods purchased from intermediate-goods firms.

Working capital and accumulation of financial resources. Production of final goods also requires financial resources that increase with intermediate goods used in production. For this purpose, entrepreneurs accumulate financial wealth $m_{j,t}$ in order to satisfy the constraint

$$m_{j,t} \geq \phi_j p_{j,t} x_{j,t}. \tag{8}$$

A narrow interpretation of this constraint is that it represents advanced payment of a fraction ϕ_j of the cost of production (working capital). However, we give it a broader interpretation since, in reality, there are other channels through which financial wealth affects production. For example, financial wealth provides insurance against earning risks, increasing the willingness to operate larger firms (Angeletos (2007)). Also, firms with more favorable financial positions may find easier to attract new workers (Monacelli, Quadrini, and Trigari (2023)) or to retain existing workers (Baghai, Silva, Thell, and Vig (2021)).

The parameter ϕ_j plays an important role in determining the demand for financial assets. The higher the value of ϕ_j, the higher the need for those assets and, hence, the larger the holdings of $m_{j,t}$.

The financial wealth of entrepreneurs is in the form of liabilities issued by intermediate-goods firms (either domestic or foreign) and liabilities issued by the government of advanced economies. Even though we are assuming perfect capital mobility, public and private liabilities have different prices because they have characterized by different repayment risks. While private bonds are defaultable, public bonds issued by advanced economies are always repaid in full. We denote by $q_{j,t}$ the price of bonds issued by intermediate-goods firms in Region j, and by $q_{p,t}$ the price of public bonds issued by Region 1 (advanced economies).

Entrepreneurial decisions. The representative entrepreneur in Region j enters period t with bonds issued by firms in Region 1, $b_{1,j,t}$, bonds issued by firms in Region 2, $b_{2,j,t}$, and government bonds issued by advanced economies, $b_{p,j,t}$. The first subscript denotes the issuer (Region 1 or Region 2 for private bonds, and p for public bonds), while the second subscript denotes the residence of the holder. In the event of default, entrepreneurs incur financial losses proportional to their ownership of private bonds (but not public bonds since they are risk-free).

Denote by $\delta_{1,t}$ and $\delta_{2,t}$ the fractions of private bonds repaid, respectively, by Region 1 and Region 2. The post-default values of the two bonds are then $\delta_{1,t}b_{1,j,t}$ and $\delta_{2,t}b_{2,j,t}$. The repayment fractions $\delta_{1,t}$ and $\delta_{2,t}$ are endogenous variables that are determined in the general equilibrium. After their realization at the beginning of the period, the entrepreneur's wealth is

$$m_{j,t} = \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}.$$

This is the variable that enters the financial constraint (8).

After production, the end-of-period wealth is

$$a_{j,t} = m_{j,t} + (z_j - p_{j,t})x_{j,t}.$$

The end-of-period wealth is in part allocated to consumption and in part to new bonds, in accordance to the budget constraint

$$c^e_{j,t} + q_{1,t}b_{1,j,t+1} + q_{2,t}b_{2,j,t+1} + q_{p,t}b_{p,j,t+1} = a_{j,t}. \tag{9}$$

While the production scale depends on $m_{j,t}$ (through constraint (8)), portfolio decisions, $b_{1,j,t+1}$, $b_{2,j,t+1}$ and $b_{p,j,t+1}$, depend on $a_{j,t}$. The following lemma characterizes the production decision.

Lemma 2.1 *If constraint (8) binds, $p_{j,t} < z_j$ and the demand for intermediate goods chosen by final-goods producers is*

$$x_{j,t} = \left(\frac{1}{\phi_j p_{j,t}}\right) m_{j,t}.$$

If (8) does not bind, $p_{j,t} = z_j$ and the demand for intermediate goods is determined by the supply from intermediate-goods firms.

Proof 2.1 *Appendix A.*

When the marginal productivity of the intermediate input exceeds its cost, that is, $z_j > p_{j,t}$, the firm makes a profit on each unit of final output (see Appendix A). It is then optimal for the entrepreneur to expand the scale of production until the financial constraint binds, that is, $m_{j,t} = \phi_j p_{j,t} x_{j,t}$. Solving the binding constraint for $x_{j,t}$ returns the expression reported in Lemma 2.1.

For the financial constraint not to be binding, profits must be zero, that is, $z_j = p_{j,t}$. In this case, the financial wealth $m_{j,t}$ and the financial parameter ϕ_j are irrelevant for the final production chosen by an individual firm. Only the aggregate production is determined in equilibrium (by the supply of intermediate-goods firms).

Under what conditions is constraint (8) binding? In general, the constraint is binding when entrepreneurs have low financial wealth ($m_{j,t}$ is small), the production input requires more funds (ϕ_j is high), and entrepreneurial firms are more productive (z_j is high). As shown in Appendix 2.1, when this constraint binds, entrepreneurs earn positive profits that are proportional to $m_{j,t}$. This implies that bond holdings have a convenience yield—the profit—over and above the contracted market yield.

The next step is to characterize the optimal saving and portfolio choices made at the end of the period.

Lemma 2.2 *The entrepreneur allocates the end-of-period wealth $a_{j,t}$ as follows:*

$$\begin{aligned} c^e_{j,t} &= (1-\beta)a_{j,t}, \\ q_{1,t}b_{1,j,t+1} &= \beta\theta_{1,t}a_{j,t}, \\ q_{2,t}b_{2,j,t+1} &= \beta\theta_{2,t}a_{j,t}, \\ q_{p,t}b_{p,j,t+1} &= \beta(1-\theta_{1,t}-\theta_{2,t})a_{j,t}, \end{aligned}$$

where $\theta_{1,t}$ and $\theta_{2,t}$ solve the first-order conditions

$$\begin{aligned} \mathbb{E}_t\left\{\frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,t}\frac{\delta_{1,t+1}}{q_{1,t}}+\theta_{2,t}\frac{\delta_{2,t+1}}{q_{2,t}}+(1-\theta_{1,t}-\theta_{2,t})\frac{1}{q_{p,t}}}\right\} &= 1, \\ \mathbb{E}_t\left\{\frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,t}\frac{\delta_{1,t+1}}{q_{1,t}}+\theta_{2,t}\frac{\delta_{2,t+1}}{q_{2,t}}+(1-\theta_{1,t}-\theta_{2,t})\frac{1}{q_{p,t}}}\right\} &= 1. \end{aligned}$$

Proof 2.2 *Appendix B.*

Lemma 2.2 establishes that entrepreneurs split the end-of-period wealth between consumption and saving according to the fixed factor β. This derives from the logarithmic specification of the utility function. A fraction $\theta_{1,t}$ of saved wealth $\beta a_{j,t}$ is then allocated to private bonds issued by Region 1, a fraction $\theta_{2,t}$ to private bonds issued by Region 2, and the remaining fraction $1-\theta_{1,t}-\theta_{2,t}$ to public bonds issued by Region 1 (advanced economies). As stated earlier, the three bonds are not perfect substitutes because they face different probabilities of default. Thus, there is a gain from diversification that explains why the optimal portfolio shares are well defined.

The portfolio shares $\theta_{1,t}$ and $\theta_{2,t}$ change over time as recovery rates and bond prices vary. However, they are the same for entrepreneurs in Region 1 and Region 2. This is indicated by the fact that $\theta_{1,t}$ and $\theta_{2,t}$ do not have the region subscript j. Thus, entrepreneurs in both regions choose the same portfolio composition.[5] This is the case because the three types of bonds are

[5]It is important to emphasize that, because $\theta_{1,t}$ and $\theta_{2,t}$ do not have the j subscript, the last three conditions in Lemma 2.2 are not just accounting identities.

freely traded internationally and default by a country's borrowers reduces equally the repayment to foreign and domestic holders of that debt.

2.3 Public sector

The government of Region 1 issues risk-free bonds (public debt), and the governments of both regions hold some of these bonds as FX reserves. Governments also pay lump-sum transfers to (or raise taxes from) households in order to balance their budgets.

The reason we focus on public debt issued by advanced economies is in part related to data limitations for emerging economies. More importantly, however, our choice is motivated by considerations related to two key differences between the public debt issued by the two regions. First, sovereign default in advanced economies is rare and public bonds issued by countries like Germany, Japan, the United Kingdom, and the United States are considered to be risk-free. This makes the public debt of these countries very different from their private debt, which is not risk-free. Because of their negligible repayment risk, these government bonds are important for liquidity and accumulation of FX reserves. U.S. public debt, in particular, represents roughly 60% of the assets held as FX reserves worldwide (see Ito and McCauley (2020)). Also, because the public debt of advanced economies is large relatively to the size of the world economy, it could have important general equilibrium implications.

Another reason we focus on public debt issued by advanced economies is as follows. While governments in emerging economies do issue public debt, their debt is not risk-free and sovereign default arises often in conjunction with private default. Hence, from the perspective of an investor, there may be less significant differences between private and public debt issued by emerging economies. The public debt issued by emerging economies is also much smaller than the public debt of advanced economies.

The budget constraint of the government of Region 1 (AEs) is

$$FX_{1,t} + q_{p,t}D_{p,t+1} = q_{p,t}FX_{1,t+1} + D_{p,t} + T_{1,t}. \tag{10}$$

The left-hand-side includes the sources of funds, and contains two terms. The first is the stock of FX reserves accumulated in the previous period, $FX_{1,t}$. The second is the funds raised with the issuance of new debt $D_{p,t+1}$ sold at price $q_{p,t}$. The right-hand-side contains the uses of funds. The first term is the purchase of new reserves. The second is the repayment of the public debt issued in the previous period. The third is the transfer $T_{1,t}$ to domestic households (or taxes if negative). Notice that reserves are only in the form of public bonds issued by Region 1. Therefore, what matters for the government of Region 1 is the net debt, that is, $D_{p,t} - FX_{1,t}$.[6]

The budget constraint for the government of Region 2 (EMEs) is

$$FX_{2,t} = q_{p,t} FX_{2,t+1} + T_{2,t}. \tag{11}$$

The variables $D_{p,t}$, $FX_{1,t}$ and $FX_{2,t}$ are time varying but exogenous. In the quantitative exercise, these variables match the observed time-series of public debt in advanced economies and FX reserves in both advanced and emerging economies.

2.4 General equilibrium

Using capital letters to denote aggregate variables, the aggregate states include the bonds held by entrepreneurs, $B_{1,1,t}$, $B_{2,1,t}$, $B_{p,1,t}$, $B_{1,2,t}$, $B_{2,2,t}$, $B_{p,2,t}$, and the sunspot shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. Knowing these variables we can determine the aggregate debt issued by intermediate-goods firms in the previous period as $D_{1,t} = B_{11,t} + B_{12,t}$ and $D_{2,t} = B_{21,t} + B_{22,t}$.

The sequences of public debt and reserves—$D_{p,t}$, $FX_{1,t}$ and $FX_{2,t}$— and capital stocks—$K_{1,t}$ and $K_{2,t}$—are also relevant for the equilibrium. Since these variables are exogenous and perfectly anticipated, their full sequence going into the future is part of the state space. We denote the sequence of a variable starting at time t and going to infinity with subscript t and superscript ∞. For example, $K^{\infty}_{j,t}$ represents the sequence of capital in Region j from time t to ∞. To use a compact notation, we denote the state vector as

$$\begin{aligned}\mathbf{s}_t \equiv (& D^{\infty}_{p,t}, FX^{\infty}_{1,t}, FX^{\infty}_{2,t}, K^{\infty}_{1,t}, K^{\infty}_{2,t}, \\ & B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).\end{aligned}$$

Figure 3 sketches the steps to define an equilibrium by dividing the period in three sub-periods.

[6]Technically, the reserves of Region 1 are foreign assets, not the repurchase of its own public debt. However, since Region 1 is the aggregation of all advanced economies, it is not possible to clearly distinguish $D_{p,t}$ from $FX_{1,t}$. In reality, the reserves held by some advanced economies (for example European countries) could be in bonds issued by other advanced economies (for example, the US). Once we aggregate all advanced economies, without netting out the reserves from the public debt, it looks like advanced economies issue public bonds and then repurchase the same bonds as FX reserves.

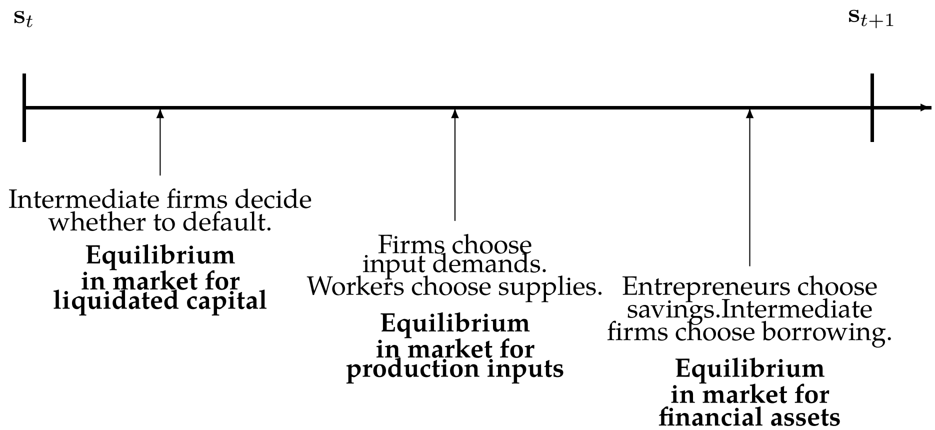

Figure 3: Timing within period t.

1. **Subperiod 1**: Given the realization of the sunspot shocks $\varepsilon_{j,t}$, intermediate-goods firms choose the fraction of debt to repay

$$\delta_{j,t} = \begin{cases} \frac{\kappa_j K_{j,t}}{D_{j,t}}, & \text{if} \quad D_{j,t} \geq \kappa_j K_{j,t} \text{ and } \varepsilon_{j,t} = 0 \\ 1, & \text{otherwise} \end{cases} .$$

A financial crisis, which arises when $\delta_{j,t} < 1$, has a fundamental cause—the level of debt or leverage—and a self-fulfilling cause driven by sunspot shocks.

Figure 4 plots the probability of a crisis as a function of the debt, $D_{j,t}$. Given the aggregate stock of capital $K_{j,t}$, the probability is zero when the debt $D_{j,t}$ is below the threshold $\kappa_j K_{j,t}$. Above this threshold, the crisis probability becomes λ, which corresponds to the probability of drawing the sunspot shock $\varepsilon_{j,t} = 0$. For values of $D_{j,t}$ greater than $K_{j,t}$ the crisis probability becomes 1 because the liquidation value of capital is always smaller than the debt. This shows that a financial crisis is not just the result of a negative sunspot shock but also the consequence of high leverage (the fundamental cause).

After default, the aggregate wealth of entrepreneurs becomes

$$M_{j,t} = \delta_{1,t} B_{1j,t} + \delta_{2,t} B_{2j,t} + B_{p,j,t}.$$

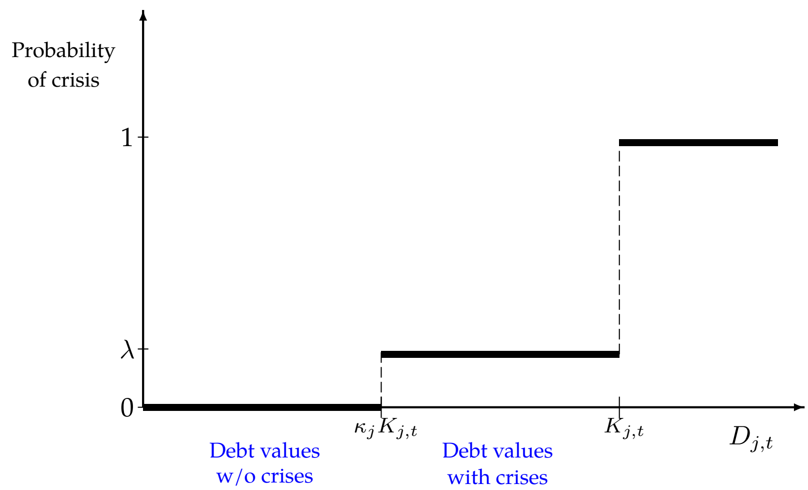

Figure 4: Probability of crisis: debt values with and without crises.

2. **Subperiod 2**: Intermediate-goods firms choose labor demand, entrepreneurs choose their demand for intermediate goods, and households choose the supply of labor. The demand for labor is

$$L_{j,t} = \left(\frac{\gamma p_{j,t}}{w_{j,t}}\right)^{\frac{1}{1-\gamma}} K_{j,t}. \tag{12}$$

Lemma 2.1 established that the demand for intermediate goods depends on whether constraint (8) is binding or nonbinding. When binding, the aggregate demand for intermediate goods is

$$X_{j,t} = \left(\frac{1}{\phi_j p_{j,t}}\right) M_{j,t}.$$

If constraint (8) is not binding, the demand is determined by the supply chosen by intermediate producers, that is,

$$X_{j,t} = L_{j,t}^{\gamma} K_{j,t}^{1-\gamma}.$$

The aggregate supply of labor is derived from the household's firs-order condition (1), which we can re-arrange as

$$H_{j,t} = \left(\frac{w_{j,t}}{\mu_j}\right)^{\nu}. \tag{13}$$

The stock of capital evolves exogenously. Market-clearing in the labor market and in the intermediate goods market determine the wage rate $w_{j,t}$ and the price for intermediate goods $p_{j,t}$, respectively.

3. **Subperiod 3**: The end-of-period wealth of entrepreneurs is

$$A_{j,t} = M_{j,t} + z_j X_{j,t} - p_{j,t} X_{j,t}.$$

According to Lemma 2.2, a fraction $1 - \beta$ is consumed while the remaining fraction β is saved in new bonds: A fraction $\theta_{1,t}$ in private bonds issued by Region 1, a fraction $\theta_{2,t}$ in private bonds issued by Region 2, and the remaining fraction $1 - \theta_{1,t} - \theta_{2,t}$ in public bonds issued by the government of Region 1. Intermediate firms choose the new debt $D_{j,t+1}$.

Market-clearing in the three financial markets requires

$$B_{1,1,t+1} + B_{1,2,t+1} = D_{1,t+1}, \tag{14}$$

$$B_{2,1,t+1} + B_{2,2,t+1} = D_{2,t+1}, \tag{15}$$

$$B_{p,1,t+1} + B_{p,2,t+1} + FX_{1,t+1} + FX_{2,t+1} = D_{p,t+1}. \tag{16}$$

Because of capital mobility and cross-region heterogeneity, the net foreign asset positions could be different from zero. Formally, in Region 1 we could have $B_{1,1,t+1} + B_{2,1,t+1} + B_{p,1,t+1} + FX_{1,t+1} - D_{1,t+1} - D_{p,t+1} \neq 0$, and in Region 2 we could have $B_{1,2,t+1} + B_{2,2,t+1} + B_{p,2,t+1} + FX_{2,t+1} - D_{2,t+1} \neq 0$.

Appendix F derives the region-specific and world resource constraints implied by the market-clearing conditions for labor and financial markets, and the budget constraints of households, firms, entrepreneurs and governments. From these conditions we can derive the region-specific trade balance, current account and NFA positions.

Competition also implies that the price paid by entrepreneurs to purchase private debt is consistent with the interest rate, that is,

$$q_{j,t} = \frac{\mathbb{E}_{t+1}\delta_{j,t+1}}{\overline{R}_{j,t}}. \tag{17}$$

This relates the price of private bonds $q_{j,t}$ to their expected return. A similar condition applies to public bonds, that is, $q_{p,t} = \frac{1}{\overline{R}_{j,t}}$.

The supply of private bonds is derived from the borrowing decisions of intermediate-goods firms (equation (6)),

$$\frac{1}{\overline{R}_{j,t}} = \beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_j K_{j,t+1}}\right).$$

Using equation (17), we can rewrite the condition as

$$q_{j,t} = \left[\beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_j K_{j,t+1}}\right)\right] \mathbb{E}\delta_{j,t+1}. \tag{18}$$

Since intermediate-goods firms could default, the economy displays stochastic dynamics driven by the sunspot shocks. The sunspot shocks can take two values: $\varepsilon_{j,t} = 0$ (with *possible* market freeze) and $\varepsilon_{j,t} = 1$ (no market freeze). The realization $\varepsilon_{j,t} = 0$ could generate a drop in the liquidation value of capital (if the leverage of the region is sufficiently high), which in turn leads to a financial crisis where bonds are only partially repaid. This redistributes wealth from lenders (final-goods firms) to borrowers (intermediate-goods firms). The decline in entrepreneurs' wealth $M_{j,t}$, then, reduces the demand for intermediate goods which in turn lowers its price $p_{j,t}$. Intermediate-goods firms respond to the price drop by reducing their demand for labor and, at equilibrium, there is lower employment and production. This is the mechanism through which financial crises have real macroeconomic consequences.

2.5 Sequential property of the equilibrium

The particular structure of the model allows us to solve for the equilibrium at time t independently of future equilibria as if the model were static. More precisely, given the states s_t, we can find the values of all equilibrium variables at time t by solving the system of nonlinear equations listed in Appendix E. This allows us to solve the model sequentially. For example, in the quantitative application, we will solve for the sequence of equilibria from $t = 1991$ to $t = 2020$. To do so we first solve for the equilibrium at $t = 1991$, then at $t = 1992$, and continue until $t = 2020$. This property would not hold if investment were endogenous and households were risk-averse. Thus, these two simplifying assumptions are key for making the analytical characterization of the model possible.

The sequential property of the equilibrium allows us to reduce the sufficient set of state variables. In general, the equilibrium depends on the full time-varying sequences $FX_{j,t}$, $D_{p,t}$, $K_{j,t}$ from t to infinity. However, thanks to the sequential property, the date-t equilibrium depends only on $FX_{j,t+1}$, $D_{p,t+1}$, $K_{j,t}$ and $K_{j,t+1}$. Therefore, we redefine the sufficient set of state variables as

$$\begin{aligned} \mathbf{s}_t \equiv & \big(FX_{1,t+1}, FX_{2,t+1}, D_{p,t+1}, K_{1,t}, K_{2,t}, K_{1,t+1}, K_{2,t+1}, \\ & B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}\big). \end{aligned}$$

2.6 Anatomy of a crisis

The structure of the model allows us to illustrate analytically how a crisis impacts the economy. Substituting the demand and supply of labor (equations (12) and (13)) in the market-clearing condition for the labor market ($L_{j,t} = H_{j,t}$), we derive the wage rate and employment,

$$\begin{aligned} w_{j,t} &= \mu_j^{\frac{\nu(1-\gamma)}{1+\nu(1-\gamma)}} \left(\gamma p_{j,t} K_{j,t}^{1-\gamma}\right)^{\frac{1}{1+\nu(1-\gamma)}} \\ L_{j,t} &= \left(\frac{\gamma p_{j,t} K_{j,t}^{1-\gamma}}{\mu_j}\right)^{\frac{\nu}{1+\nu(1-\gamma)}} \end{aligned}$$

Both variables depend positively on the price of intermediate goods $p_{j,t}$: if the price rises, intermediate firms hire more labor which in turn leads to a higher wage rate.

We can now use the above equations to eliminate $L_{j,t}$ in the intermediate-goods production (where $X_{j,t} = L_{j,t}^{\gamma} K_{j,t}^{1-\gamma}$), and then use the resulting expression in the production function for final-goods ($Y_{j,t} = z_j X_{j,t}$). This allows us to write production in Region j as

$$Y_{j,t} = z_j \left(\frac{\gamma p_{j,t}}{\mu_j}\right)^{\frac{\nu\gamma}{1+\nu(1-\gamma)}} K_{j,t}^{\frac{1+\nu(1-\gamma)-\gamma}{1+\nu(1-\gamma)}}. \tag{19}$$

Given $K_{j,t}$, final-goods output depends positively on the intermediate-goods price $p_{j,t}$. This dependence on the price has the same intuition outlined above for employment: a higher $p_{j,t}$ increases the production of intermediate goods and, therefore, final production.

The dependence of final output on the price of intermediate goods is the key for understanding the effects of financial frictions on the real economy.

Recall that final-goods firms choose $x_{j,t}$ to maximize profits $\pi_{j,t} = z_j x_{j,t} - p_{j,t}x_{j,t}$, while facing the working capital constraint $m_{j,t} \geq \phi_j p_{j,t} x_{j,t}$. Appendix C shows that the first-order condition for the demand of intermediate goods yield the following condition:

$$z_j = (1 + \hat{\xi}_{j,t}\phi_j)p_{j,t}.$$

The variable $\hat{\xi}_{j,t}$ is the Lagrange multiplier associated with the working capital constraint, expressed in units of final goods. This condition shows that, given current productivity z_j, the price of intermediate goods is inversely related to the tightness of the working capital constraint, the multiplier $\hat{\xi}_{j,t}$. Moreover, since profits of final-goods producers can be expressed as $\pi_{j,t} = \hat{\xi}_{j,t}m_{j,t}$, they are *positively* related to the tightness of the constraint, and thus negatively related to $p_{j,t}$.

Intuitively, low intermediate-goods prices, relatively to z_j, increase profits. Higher profits increase the incentive of final-goods firms to expand production by purchasing more intermediate goods $x_{j,t}$. However, the quantity of intermediate goods that can be acquired is limited by the working capital constraint $m_{j,t} \geq \phi_j p_{j,t} x_{j,t}$. Provided that profits are positive, entrepreneurs expand production until the working capital constraint binds. Then, relaxing the working capital constraint faced by an individual entrepreneur with an increase in financial wealth allows more profits. The increase in profits allowed by the increase in wealth is bigger when the price $p_{j,t}$ is low. Thus, relaxing the working capital constraint when $p_{j,t}$ is low, has a higher value for the entrepreneur. This is captured by a higher value of the multiplier $\hat{\xi}_{j,t}$.

Consider now what happens at equilibrium when the wealth of all entrepreneurs, denoted by $M_{j,t}$, declines. The working capital constraint implies that the demand for intermediate goods falls, which in turn reduces $p_{j,t}$. This, of course, makes the entrepreneurs' wealth even more valuable, which is captured by a higher value of $\hat{\xi}_{j,t}$. But the lower $p_{j,t}$ also means that aggregate production drops as we can see from equation (19).

This is exactly what happens in a financial crisis. Default implies that some of the bonds held by entrepreneurs are not repaid. As a result of the lower repayment, $M_{j,t}$ declines, causing the macroeconomic impact we just described. Importantly, default by itself does not have any direct macroeconomic effect. It only redistributes wealth from final producers to intermediate producers (and, ultimately, households who are the owners of intermediate firms). It is the destruction of entrepreneurial wealth associated with this redistribution that causes the macroeconomic downturn.

To summarize, a financial crisis is associated with a macroeconomic downturn and a tighter financial condition captured by the multiplier $\hat{\xi}_{j,t}$. This multiplier is the analog of the interest rate spread that plays an important role in models used to study business cycles in emerging markets, for example Neumeyer and Perri (2005) and Uribe and Yue (2006). In these models, a higher spread increases factor costs and causes a recession because of the need to finance working capital. Similarly, here, a higher $\hat{\xi}_{j,t}$ increases the factor costs and causes a macroeconomic contraction.[7]

2.7 Additional remarks

A property of the equilibrium worth noting is that the risk-free interest rate is on average lower than the rate of time preference (or, equivalently, the price of a risk-free bond is higher than the subjective discount factor β). In models with precautionary savings, this property derives from the self-insurance incentive. In our model, instead, it derives from the willingness of entrepreneurs to hold private and public debt because of its inside money-convenience yield property: it is a financial asset that facilitates production. When constraint (8) is binding, entrepreneurs receive a benefit from holding bonds in addition to the market yield. This becomes evident by noting that, with a binding working capital constraint, entrepreneurs's profits are positive and given by $\pi_{j,t} = \hat{\xi}_{j,t} m_{j,t}$. Thus, the tightness of the constraint—captured by $\hat{\xi}$—measures the convenience yield on $m_{j,t}$.

The equilibrium property by which final-goods firms are net savers and intermediate-goods firms are borrowers is important for the macroeconomic consequences of a financial crisis. Because final-goods producers have a positive financial position, a crisis redistributes wealth away from them and toward intermediate-goods producers. The drop in entrepreneurial net worth causes a decline in the demand for intermediate goods which, in turn, reduces the demand for labor and generates a macroeconomic contraction. In an environment in which final-goods producers

[7] The analogy would be even clearer if instead of imposing a strict working capital constraint, we allow entrepreneurs to relax the working capital constraint by borrowing at a cost that increases with the size of the loan. Then, when the entrepreneur's wealth drops, he/she borrows more, increasing the interest rate spread endogenously.

are net borrowers, the lower repayments of debt associated with a financial crisis would increase the net worth of these firms and would have the opposite macroeconomic consequence.[8]

Having some producers with a positive financial position is consistent with the recent changes in the financial structure of US corporations characterized by higher holdings of financial assets. As a result of this, the proportion of financially dependent firms has declined over time as documented in Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2016).

The large accumulation of financial assets by producers—often referred to 'cash'—is related to the significance of business savings. Although the rising savings of US corporations has attracted considerable attention in the literature (see, for example, Riddick and Whited (2009) and Begenau and Palazzo (2021)), this is not just a US phenomenon. Busso et al. (2016) document the share of savings done by firms both in advanced and emerging economies and present evidence that in Latin America this share is even larger than in advanced economies. The importance of business savings is also documented in Bebczuk and Cavallo (2016). This study uses data for 47 countries over 1995–2013 and shows that the contribution of businesses to national savings is more than 50%.

The increase in corporate cash suggests that more and more firms borrow less than what could be available to them. Our entrepreneurial sector captures the growing importance of these firms. It also captures the significant heterogeneity among corporate firms as many of them are net borrowers and have become more leveraged over time. Most likely, those are firms that own substantial tangible assets. In our model, they are represented by intermediate-goods producers while corporations that own large amounts of cash are represented by final-goods producers.[9]

[8] It is possible to rewrite the model so that intermediate-goods firms are net lenders and final-goods firms are net borrowers. What matters, however, is that (i) a crisis redistributes wealth from units that have a higher marginal value of wealth to those with a lower marginal value of wealth, and (ii) the productions of the two units are complementary. If the productions of the two units were substitutable, the contraction of adversely affected firms could be offset by the expansion of firms that were positively affected.

[9] See Kalemli-Ozcana, Sorensen, and Yesiltas (2012) for stylized facts about bank and firm leverage using international micro data.

3 Quantitative analysis

In this section, we assess quantitatively how the *observed* accumulation of FX reserves and issuance of public debt affected credit-market conditions and impacted financial and macroeconomic volatility. To do so, we start with a calibration of the model in which we target empirical moments observed in the early 1990s, including FX reserves and public debt.

For this quantitative application, the theoretical model is viewed as a detrended version of the world economy. We assume that there is exogenous long-run balanced growth driven by standard labor-augmenting technological change. Productivity grows at the common rate g in both regions, and the implied long-run balanced-growth rate of macro variables, except labor, is $(1+g)^{1/\gamma}-1$.[10] Consistently, empirical variables that display secular growth are detrended by their long-run growth rate, which we proxy with the average growth rate of GDP in advanced economies over the period 1991-2020.

Once the model is calibrated, we simulate the model over the period 1991-2020 under two scenarios (keeping all structural parameters, productivity and capital unchanged):

> **Scenario I:** Detrended FX reserves and public debt are kept constant (i.e., their ratios relative to GDP remain at their 1991 values).[11]
>
> **Scenario II:** Detrended FX reserves and/or public debt take the values observed in the data in each year over the period 1991-2020.

The comparison of these two scenarios addresses an important and well-defined question: how were the macroeconomic dynamics of advanced and emerging economies affected by the observed increase in FX reserves and public debt?

Of course, during this period, both advanced and emerging economies experienced many structural changes that contributed, in different ways,

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[10]The model's variables are detrended in standard fashion. Given $\bar{g}=(1+g)^{1/\gamma}-1$ the long-run growth of macroeconomic variables, we define the variable $G_t=(1+\bar{g})^t$. We then divide output, capital and financial variables including FX reserves and Public debt at time t by G_t. In addition, we set $\mu_j=z_j^{\frac{1}{\gamma}}$.

[11]The model displays stochastic fluctuations driven by the sunspot shocks around an exogenous balanced-growth path, hence these GDP ratios represent detrended reserves and public debt, respectively.

to the observed dynamics of the two regions. Thus, we do not expect the model to replicate the exact dynamics observed in the data when we only allow FX reserves and public debt to change. The exercise, however, provides a way to quantify the *contribution* of FX reserves and public debt to some of the changes observed in the world economy during this period. It is also a way to quantify the impact of FX reserves and public debt on financial and macroeconomic volatility.

3.1 Calibration of structural parameters

The model is calibrated at an annual frequency and the discount factor is set to $\beta = 0.93$, implying an annual intertemporal discount rate of about 7%. We set the elasticity of labor supply to $\nu = 1$, the labor share parameter in production to $\gamma = 0.6$, and the depreciation rate to $\tau = 0.08$.

The probability that the liquidation price of capital drops to $\kappa_j < 1$ (i.e., the probability of a realization of the sunspot shock $\varepsilon_j = 0$) is $\lambda = 0.04$. This is within the range of crisis probabilities used in the literature (see, for example, Bianchi and Mendoza (2018)). It implies that crises are low probability events, every twenty-five years on average. Since sunspot shocks are region-specific and independent across regions, a *global* financial crisis is an even rarer event, with a probability of $0.04 \times 0.04 = 0.0016$. Still, a financial crisis that originates in one region only affects the other region through the international diversification of portfolios.

The parameter η determines the sensitivity of the borrowing cost to the debt. Unfortunately, we have limited information to pin down this parameter. We set it to $\eta = 0.1$ but we will conduct a sensitivity analysis to gauge its relevance in Appendix G.

The stock of capital K_j in each region is set to its 1991 value. To construct the capital stock measures, we apply the perpetual inventories method using investment and depreciation data.[12]

[12] We apply the perpetual inventories method using an iterative procedure with data going back to 1980. We have data on investment, $I_{j,t}$, and depreciation, $DEP_{j,t}$, from the *World Development Indicators*. We start the iterations in 1980 with the guess of the initial value of capital, $K_{j,1980}$. We then compute $K_{j,1981} = K_{j,1980} - DEP_{j,1980} + I_{j,1980}$. Given the calculated value of $K_{j,1981}$, we move to the second stage and compute $K_{j,1982} = K_{j,1981} - DEP_{j,1981} + I_{j,1981}$. We continue until 2020. At this point we check whether the capital-GDP ratio displays no trend over the whole iteration period 1980-2020. If it does, we change the initial guess for $K_{j,1980}$ and repeat the iteration. The capital stocks in the calibrated model are set to the 1991 values obtain from this procedure.

The parameter values that remain to be set are z_j, ϕ_j, κ_j, and, for Scenario I, the 1991 values of reserves and public debt, FX_j, and D_p. Thus, we need to set the values of nine items, six structural parameters and three government variables. We choose these values so that the baseline model replicates nine empirical moments observed in 1991.

It is important to note that even if FX reserves and public debt remain constant, the model does not converge to a deterministic steady state because there are sunspot shocks. This also implies that the moments generated by the model are stochastic. We will then use the *averages* generated by the model to target the nine empirical targets. We compute the model's averages by repeating the stochastic simulation of the model 10,000 times. Each simulation is for 130 years and the simulated data is the response to randonmly drawn sunspot shocks. The first 100 years are needed to eliminate the impact of initial states, while the remaining 30 years represent the 1991-2020 period. This is the period in which we evaluate the impact of FX reserves and public debt. The simulation is repeated 10,000 times in order to generate an approximation to the invariant distribution.[13]

Table 1 lists the sources for the nine empirical moments, as well as their values in 1991. We use four data sources: the World Bank's *World Development Indicators*, the International Monetary Fund's *Global Debt Database*, the *External Wealth of Nations* database from Lane and Milesi-Ferretti (2018), and the FRED database from St. Louis Fed. Aggregate variables for Advanced and Emerging Economies are constructed by aggregating individual country variables. To construct aggregate GDP and other variables for advanced economies, we sum the values of the specific variable for all countries included in AEs. We do the same to compute aggregate EMEs variables. The countries included in AEs and EMEs are listed at the bottom of Table 1.

Although the values of the six parameters and the three government variables all contribute, interdependently, to the nine moments, some parameters have a more direct impact than others on a specific moment. Productivity z_j is important for determining aggregate output (the first two moments). The value of z_j is determined by inverting the aggregate production function evaluated at the calibration year 1991, after consolidating

[13]The moments generated by the model that are matched to the empirical targets are the averages of the 10,000 realizations from the repeated simulations in period 101, which corresponds to year 1991.

Table 1: Parameter values.

Moment	*Value in 1991* (*Trillions USD at 2015 prices*)	*Source*
Aggregate GDP AEs	26.03	*World Development Indicators*
Aggregate GDP EMEs	7.74	*World Development Indicators*
Private domestic credit AEs	28.51	*World Development Indicators*
Private domestic credit EMEs	3.62	*World Development Indicators*
Net foreign asset position AEs	-0.72	*Wealth of Nations*
FX reserves AEs	0.79	*Wealth of Nations*
FX reserves EMEs	0.58	*Wealth of Nations*
Public debt AEs	11.97	*Global Debt Database*
World interest rate	3.93%	*FRED*

Advanced economies: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. **Emerging economies**: Algeria, Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Colombia, Estonia, Hong Kong, Hungary, India, Indonesia, Israel, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, Singapore, South Africa, Thailand, Turkey, Ukraine, Venezuela. Data on public debt is from IMF Global Debt Database. We use the series Central Government Debt which is available for thirteen countries: Canada, Finland, France, Germany, Italy, Japan, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. The Global Debt Database provides two series: 'Central Government Debt' and 'General Government Debt'. We use the former. Data for all years 1991-2020 are available only for thirteen of the advanced economies (listed above). Hence, our measure of public debt issued by AEs is derived from data on debt-to-GDP ratio for of these thirteen countries.

the intermediate and final sectors. Since intermediate-goods production is $X_{j,1991} = L^{\gamma}_{j,1991} K^{1-\gamma}_{j,1991}$, replacing X_{1991} in the final-goods production yields

$$Y_{j,1991} = z_j L^{\gamma}_{j,1991} K^{1-\gamma}_{j,1991}.$$

Hence, we can compute z_j using a procedure analogous to that used to construct Solow residuals. This requires measures of production inputs and outputs for 1991. For output, we use GDP at nominal exchange rates, not PPP. Since nominal exchange rates affect the purchasing power of a country in the acquisition of foreign assets, our productivity measure should also reflect the exchange rates. Another factor that contributes to generate differences in aggregate GDP is the size of population. Since population is not explicitly modeled, productivity will also capture population differences.

Denote by $P_{j,1991}$ the nominal price index for country j expressed in US dollars. This is calculated by multiplying the price in local currency with the dollar exchange rate. We can then define the nominal (dollar) aggregate output of country j as

$$P_{j,1991}Y_{j,1991} = P_{j,1991}\hat{z}_j L_{j,1991}^{\gamma} K_{j,1991}^{1-\gamma} N_{j,1991},$$

where $\hat{z}_j$ is actual productivity, $L_{j,1991}$ is per-capita employment, $K_{j,1991}$ is the per-capita stock of capital, and $N_{j,1991}$ is population.

If we deflate the nominal GDP in both regions by the price index in region 1, we obtain

$$Y_{1,1991} = \hat{z}_{1,1991} L_{1,1991}^{\gamma} K_{1,1991}^{1-\gamma} N_{1,1991},$$

$$\frac{P_{2,1991}Y_{2,1991}}{P_{1,1991}} = \left(\frac{P_{2,1991}\hat{z}_{2,1991}}{P_{1,1991}}\right) L_{2,1991}^{\gamma} K_{2,1991}^{1-\gamma} N_{2,1991},$$

Thus, aggregate productivities in the model correspond to

$$z_1 = \hat{z}_1 N_{1,1991},$$
$$z_2 = \hat{z}_2 (P_{2,1991}/P_{1,1991}) N_{2,1991}.$$

Since $P_{2,1991}$ is the dollar price of output in emerging economies, the ratio $P_{2,1991}/P_{1,1991}$ corresponds to the real exchange rate. Thus, the above expressions show that z_1 and z_2 reflect cross-regional differences in real exchange rates and population, in addition to actual TFP.

The productivity values for the model are calculated from the data as

$$z_1 = \frac{Y_{1,1991}}{L_{1,1991}^{\gamma} K_{1,1991}^{1-\gamma}}, \tag{20}$$

$$z_2 = \frac{(P_{2,1991}/P_{1,1991})Y_{2,1991}}{L_{2,1991}^{\gamma} K_{2,1991}^{1-\gamma}}. \tag{21}$$

To construct these values, we use empirical counterparts for $Y_{1,1991}$, $L_{1,1991}$, $(P_{2,1991}/P_{1,1991})Y_{2,1991}$, $L_{2,1991}$, $K_{1,1991}$, and $K_{2,1991}$ from the *World Development Indicators*. $Y_{1,1991}$ and $P_{2,1991}Y_{2,1991}/P_{1,1991}$ are computed by aggregating the GDP of countries in advanced and emerging economies, respectively, both expressed in constant US dollars. For labor input $L_{j,1991}$ we use the employment-to-population ratio (population over 15 years of age). The values of $K_{j,1991}$ were constructed as described earlier in footnote 12.

The parameters ϕ_j, and κ_j are important for determining private domestic credit, net foreign assets and the interest rate. More specifically, ϕ_j

has a direct impact on the *demand* for financial assets: Higher values of ϕ_j increase the demand because more financial assets are needed for production (working capital, etc.). The parameter κ_j has a direct impact on the *supply* of financial assets: Higher values of κ_j strengthen the incentive for intermediate firms to borrow. In general, higher values of these two parameters lead to higher volume of domestic private credit. However, they have different effects on the interest rate and NFA. A higher value of ϕ_j (higher supply of credit) lowers the interest rate and increases NFA of country j. This is because the country demands more assets than those created domestically. The difference must then be filled with foreign assets. An increase in the value of κ_j (higher demand for credit) raises the interest rate and decreases NFA of country j. Since the country creates more assets than demanded domestically, part of it is sold to foreigners. Data for private domestic credit, NFA and interest rate were obtained from, respectively, *World Development Indicators*, *Wealth of Nations*, and *FRED*.

The values of foreign reserves are obtained from the *Wealth of Nations* and data for public debt issued by advanced economies are from the IMF *Global Debt Database*. Table 2 provides the full list of calibrated parameters.

Table 2: Parameter values.

Description	*Parameter*	*Value*
Discount factor	β	0.930
Share of labor in production	γ	0.600
Depreciation rate	τ	0.080
Elasticity of labor supply	ν	1.000
Probability of crises (low sunspot shock)	λ	0.040
Cost of borrowing	η	0.100
Long-run growth rate of productivity	g	0.010

The precise equations that link the nine empirical targets to the corresponding variables in the model are as follows:

$$
\begin{aligned}
\text{GDP AEs} &= Y_{1,1991} = z_1^{\frac{1-\nu\gamma}{1+\nu(1-\gamma)}} \left(\gamma p_{1,1991}\right)^{\frac{\nu\gamma}{1+\nu(1-\gamma)}}, && (22)\\
\text{GDP EMEs} &= Y_{2,1991} = z_2^{\frac{1-\nu\gamma}{1+\nu(1-\gamma)}} \left(\gamma p_{2,1991}\right)^{\frac{\nu\gamma}{1+\nu(1-\gamma)}}, && (23)\\
\text{Private Credit AEs} &= q_{1,1991} D_{1,1991}, && (24)\\
\text{Private Credit EMEs} &= q_{2,1991} D_{2,1991}, && (25)\\
\text{NFA in AEs} &= q_{1,1991} B_{1,1,1991} + q_{2,t} B_{2,1,1991} + q_{p,1991} B_{p,1,1991} + && (26)\\
&\qquad q_{p,1991} FX_{1,1991} - q_{1,1991} D_{1,1991} - q_{p,1991} D_{p,1991}, && (27)\\
\text{US real interest rate} &= \frac{1}{q_{p,1991}} - 1. && (28)\\
\text{FX reserve AEs} &= q_{p,1991} FX_{1,1991}, && (29)\\
\text{FX reserve EMEs} &= q_{p,1991} FX_{2,1991}, && (30)\\
\text{Public Debt AEs} &= q_{p,1991} D_{p,1991}, && (31)
\end{aligned}
$$

The terms in the right-hand-side are equilibrium objects we compute from the model, given the values of z_j, ϕ_j, κ_j, FX_j and D_p. Thanks to the sequential property of the equilibrium (see Section 2.5), we find the equilibrium in period t by solving the system of equations listed in Appendix E. We solve for ϕ_j and κ_j by applying two nested nonlinear solvers. The inner solver finds the equilibrium given the values of ϕ_j and κ_j. The outer solver uses the inner solution to check whether the equilibrium associated with the particular values of ϕ_j and κ_j satisfies conditions (24)-(28). We then update the values of ϕ_j and κ_j until conditions (24)-(28) are satisfied.

3.2 Simulation results

We are now ready to address the question of how the observed increase in FX reserves and public debt affected the macroeconomic dynamics of advanced and emerging economies. As noted earlier, we do this by comparing the simulations of the calibrated model under two scenarios:

> **Scenario I:** Detrended FX reserves and public debt remain constant at their 1991 values. Since the calibration matches the GDP values of 1991, this means that the GDP ratios of reserves and public debt are kept constant at the 1991 values shown in Figure 1.
>
> **Scenario II:** Detrended FX reserves and/or public debt take the values observed in the data during the period 1991-2020. In this case, we

use the observed time-series of FX reserves and public debt over that sample period, allowing their ratios to GDP to vary endogenously.[14]

In Scenario I, we impose that detrended FX reserves and public debt remain constant over the full 130 years of each of the 10,000 simulations. In Scenario II, detrended FX reserves and detrended public debt remain constant during the first 100 years, and after that they take the values observed in the data in the remaining 30 years. Since the simulation over 130 years is repeated 10,000 times, we have a cross-section of 10,000 simulated values of each variable of interest for each year. We can then calculate summary statistics for each variable, in each year, using these 10,000 points.

The impact of FX reserves. Figure 5 plots the time-series of the means of the simulated variables (over the 10,000 repeated simulations for each year) under the two scenarios. The dashed line is for scenario I, where FX reserves and public debt remain constant. The continuous line is for scenario II, where FX reserves take the values observed in the data over the period 1991-2020. In both scenarios, the public debt issued by AEs remains constant. For the first four variables, the figure also plots the empirical data (dotted-dashed line).

The comparison between the continuous and dashed lines illustrates the impact of the increased accumulation of FX reserves by EMEs. Panels (a) and (b) show that private credit, as a percentage of output, increases in response to the surge in FX reserves. The increase is lower than in the data but it is not negligible (roughly 20 and 10 percentage points of GDP between 1991 and 2020 in AEs and EMEs, respectively).

Panel (d) shows why: with growing FX reserves, the cost of borrowing (the interest rate) falls and the private sector borrows more. Panels (e) and (f) then show that effective leverage—the ratio of private debt to the liquidation value of capital during crises—rises. This is key for understanding the implications of higher FX reserves on macro volatility, as we will discuss shortly.

Looking now at panel (c), we observe that the accumulation of foreign reserves leads to a large decline in NFA of advanced economies. This is because most of the increase in FX reserves comes from emerging economies

[14]The detrended values of FX reserves and public debt enter the model as exogenous variables. However, since output is endogenous, the ratios are endogenous in the model.

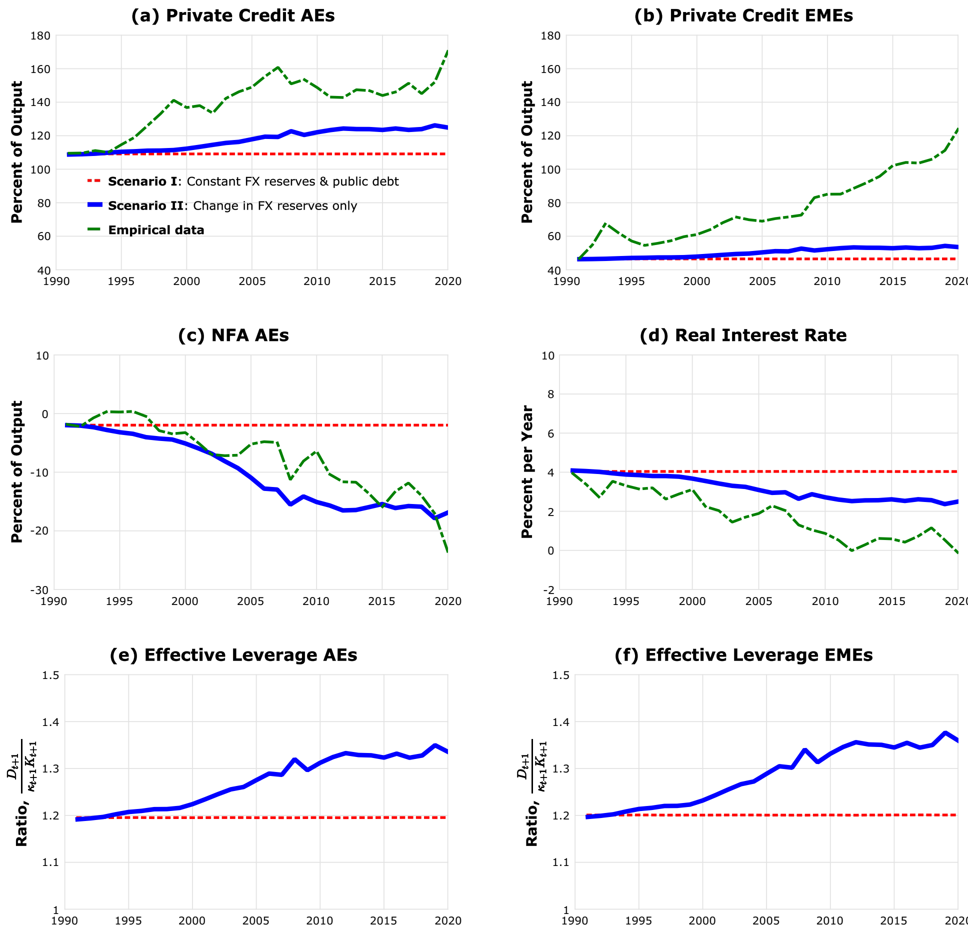

Figure 5: Simulation with changing FX reserves, 1991-2020.

as foreign demand for (public) assets issued by AEs. It is a consequence of higher public savings in EMEs. While the decline in the interest rate induced by the surge in FX reserves captures only part of the observed interest rate decline, the drop in NFA is of a similar magnitude as in the data.

Clearly, the results in Figure 5 also indicate that considering only the surge in FX reserves still leaves a non-trivial part of the observed rise in private credit of both regions and the drop in the world real interest rate unexplained. As noted earlier, however, the aim of this experiment is only to isolate the contribution of FX reserves, and sets aside other factors that contributed to the observed data dynamics (e.g., productivity growth and changes in financial structure).

The impact of public debt. Figure 6 plots the simulated variables under two scenarios. In the first scenario (dashed line) both FX reserves and public debt remain constant. In the second (continuous line) public debt takes the values observed in the data during the 1991-2020 period, while FX reserves remain constant.

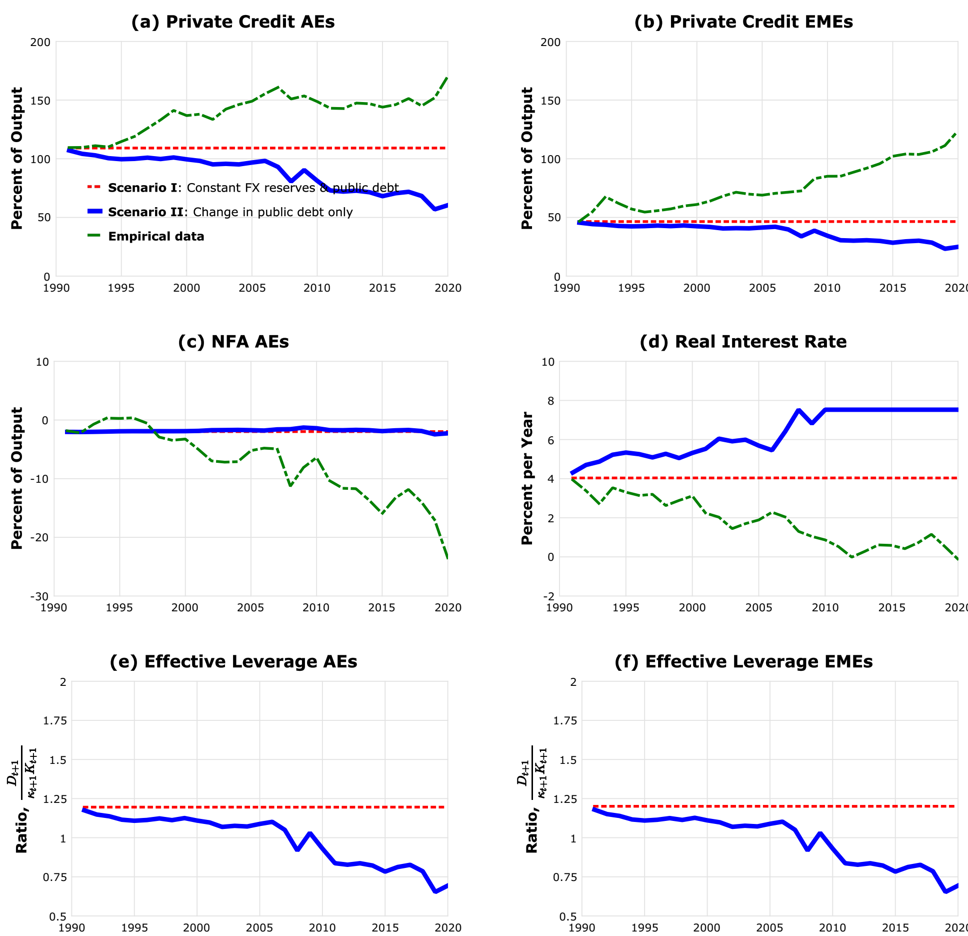

Figure 6: Simulation with changing public debt issued by AEs, 1991-2020.

Panels (a) and (b) show that private credit, as a percentage of output, declines as the public debt issued by AEs rises. This is the consequence of the increase in the interest rate shown by the continuous line in Panel (d). Notice that the equilibrium interest rate is bounded above by the intertemporal discount rate $1/\beta - 1 \approx 0.075$. This is because intermediate firms are risk neutral and when the interest rate equals the intertemporal discount rate, they become indifferent between borrowing and lending.

Following the decline in private credit, Panels (e) and (f) show that effective leverage decreases. As we will see, this is important for understanding the consequences of higher public borrowing for aggregate volatility.

The increase in public debt issued by advanced economies has a negligible impact on the NFA position of AEs (see Panel (c)). Even though part of the newly issued public debt is purchased by entrepreneurs in emerging economies, the size of the financial portfolio held by entrepreneurs in EMEs' is relatively small. As a consequence, the contribution of EMEs to the net foreign asset position of advanced economies is negligible.

As was the case for the FX reserves experiment, we see that the growing public debt of advanced economies leaves sizable shares of the actual movements of all variables unexplained. This is because we measure only the contribution of the AEs' public debt (i.e., the supply of risk free assets).

The combined impact of FX reserves & public debt. Figure 7 plots the simulated variables when, in scenario II, both FX reserves and public debt take the values observed in the data.

With the exception of the NFA in advanced economies, the impact of the growing public debt dominates the impact of higher FX reserves. As a result, private credit declines in both regions (Panels (a) and (b)), the world interest rate increases (Panel (d)), and effective leverages shrinks in both regions (Panels (e) and (f). These results follow from the market-clearing condition for public debt issued by AEs: because the total debt issuance is larger than the EMEs reserves component, with the remainder going to private sector demand from both regions (which weakens as the interest rises) and AEs reserves (which did not increase much).

4 Financial and macroeconomic volatility

We now explore the main question addressed in this paper: how the surge of FX reserves by EMEs and the issuance of public debt by AEs impacted macroeconomic volatility during the last three decades. Focusing on ag-

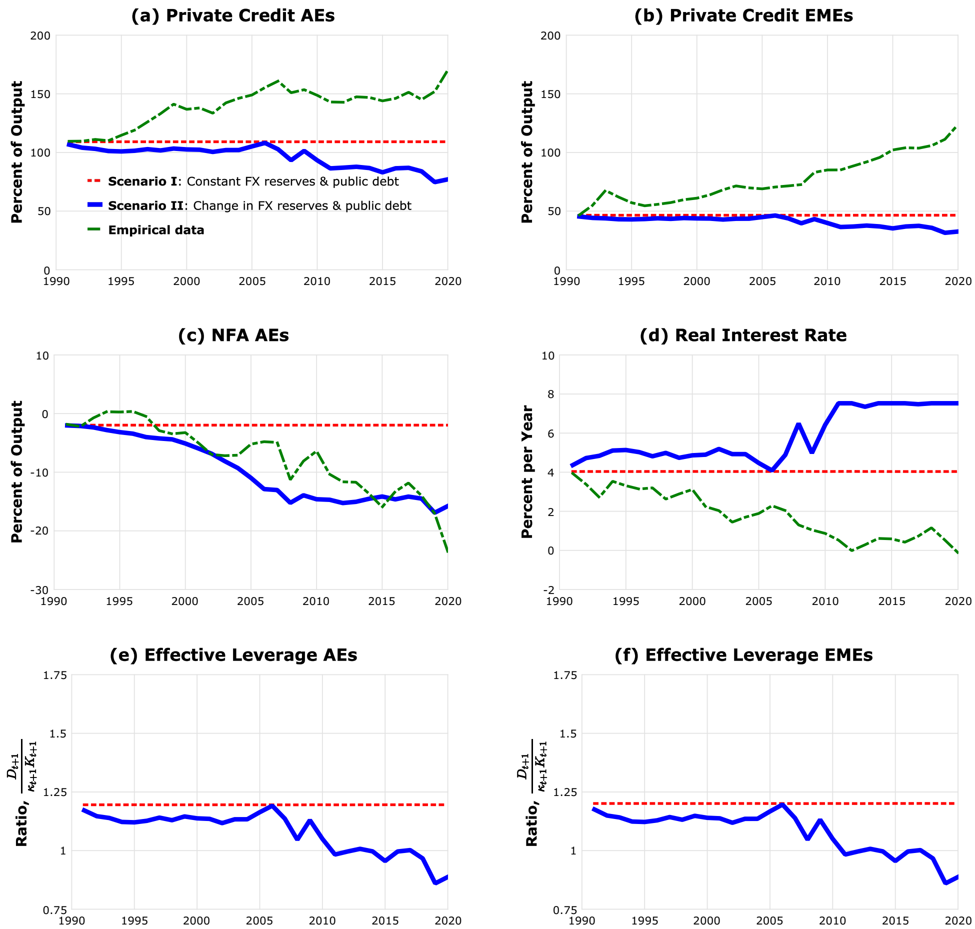

Figure 7: Simulation with changing FX reserves and public debt, 1991-2020.

gregate output, we measure volatility using the following indicator:

$$VOL_{j,t} = \left(\frac{P_{j,t}(95) - P_{j,t}(5)}{\overline{Y}_{j,t}} \right) \times 100. \tag{32}$$

The variables $P_{j,t}(5)$ and $P_{j,t}(95)$ are, respectively, the 5th and 95th percentiles of the 10,000 cross-sectional output values generated by the repeated simulations for year t in region j. Formally, the percentiles are the values of $P_{j,t}(5)$ and $P_{j,t}(95)$ that solve the equations

$$\frac{1}{10,000} \sum_{i}^{10,000} \left(1 \middle| Y_{j,t}^{i} < P_{j,t}(5) \right) = 0.05, \qquad \frac{1}{10,000} \sum_{i}^{10,000} \left(1 \middle| Y_{j,t}^{i} < P_{j,t}(95) \right) = 0.95.$$

The variable $\overline{Y}_{j,t}$ is the arithmetic average of the 10,000 data values for output obtained from the repeated simulations, that is,

$$\overline{Y}_{j,t} = \frac{1}{10,000} \sum_{i=1}^{10,000} Y_{j,t}^{i}.$$

The time-varying index of output volatility is the difference between the 5th and 95th percentiles, normalized by the mean.

The volatility index for years 1991-2020 is plotted in Figure 8. The dashed line is for the first scenario in which FX reserves and public debt remain constant throughout the whole simulation period. The continuous line is for the second scenario, where FX reserves and/or public debt take the values observed in the data.

Panels (a) and (b) illustrate the impact of FX reserves on output volatility in AEs and EMEs, respectively. As can be seen, the surge in FX reserves contributed to increase volatility in both regions, especially in AEs where it nearly doubled. To understand why output volatility increases with the surge in FX reserves, we should look back at Figure 5.

The increase in FX reserves represents an increase in the demand for assets which raises their price and, therefore, lowers the interest rate (Panel (d)). With a lower interest rate, it is cheaper to borrow and, as a result, private leverage increases (Panels (e) and (f)). With higher leverage, financial crises have bigger macroeconomic effects because they generate larger redistributions of wealth from lenders to borrowers and, since financial assets have a productive use for lenders, larger redistributions cause larger falls in demand for labor and output.

Consider now the effect of public debt. The continuous line in Panels (c) and (d) shows how output volatility changes when the public debt issued by advanced economies takes the values observed in the data. As can be seen, volatility drops significantly, especially after the 2008 financial crisis, that is, after AEs expanded substantially their public borrowing.

The intuition for why the increase in public debt leads to lower volatility is simple and can be explained by looking at the previous Figure 6. When the governments of AEs issue more debt, the increased supply of debt is followed by a price drop, which results in higher interest rates (Panel (d)). Intuitively, AEs' governments have to pay a higher interest rate to attract investors. With a higher interest rate, it becomes more costly for the private sector to borrow and leverage declines (Panels (e) and (f)). With lower

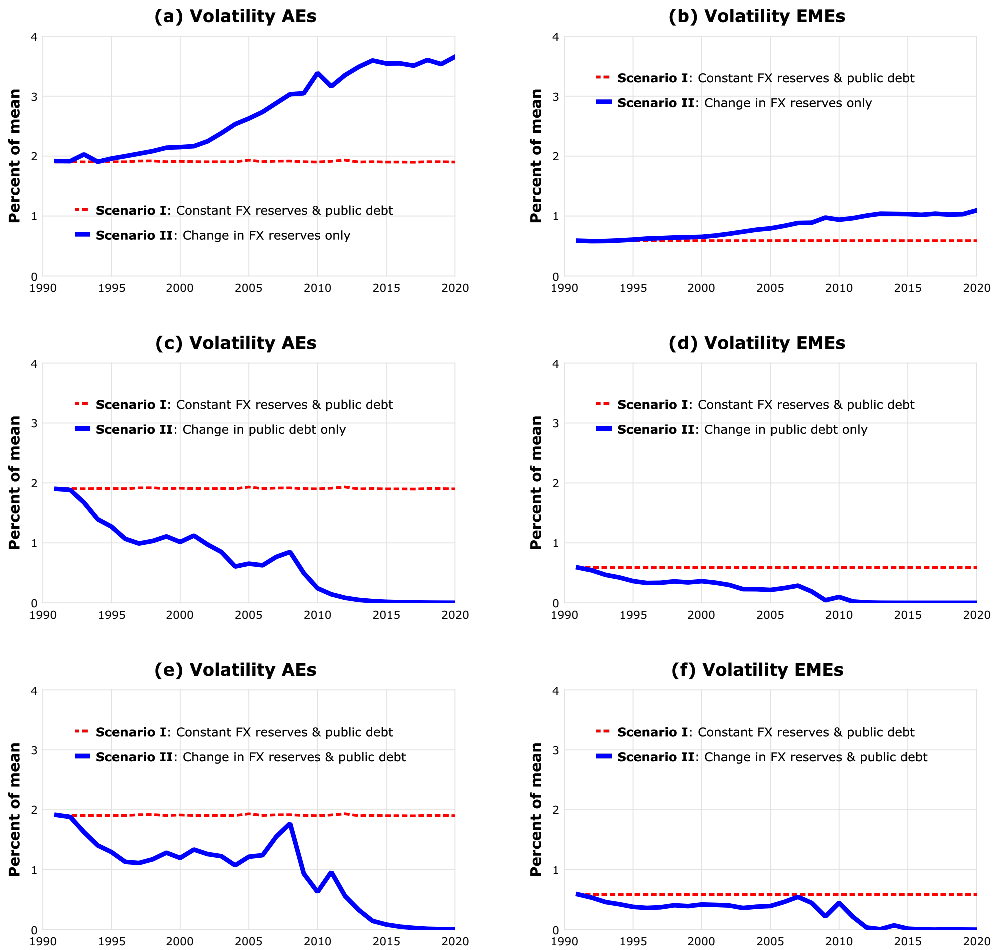

Figure 8: Output volatility over the period 1991-2020. The volatility measure is the difference between the 5th and 95th percentiles of output, as a percentage of average output, computed from the values generated by 10,000 repeated simulations.

leverage, financial crises have a smaller macroeconomic impact due to the lower redistribution of wealth.

The reason volatility becomes zero in the later period is related to the upper bound on the equilibrium interest rate. As observed earlier, the interest rate cannot be higher than the subjective discount rate. Otherwise, intermediate firms would become lenders as opposed to borrowers. With a sufficiently large supply of public debt, the equilibrium interest rate becomes equal to the upper bound (see Panel (d)). At his point, intermediate firms do not borrow and entrepreneurs do not hold private debt. But in absence of private debt there is not default, and the economy becomes immune to crises. We should keep in mind, however, that the model abstracts from many other consequences of a larger public debt, such as tax distortions, that can be detrimental to the economy even for AEs.

The last two panels of Figure 8 show the impact of the combined changes in both FX reserves and public debt. As can be see, the impact of public debt again dominates the impact of FX reserves and, as a result, output volatility declines in both regions.

It is important to note that there are sizable cross-country spillovers in all of these experiments. In the case of the increase in reserves of EMEs, we see a sharp increase in the volatility of AEs. In the case of the increase in the public debt of AEs, we see a decline in the volatility of EMEs. These spillovers occur because of the general equilibrium effects induced by the changes in the world real interest rate, which causes the changes in credit and leverage we already discussed.

5 Government bailout policies

Thus far, we have examined a setup in which FX reserves do not have any direct impact on the macroeconomic performance of the region that accumulates them. Their impact is only through general equilibrium effects. But, of course, FX reserves are a form of publicly-owned liquidity that could facilitate government interventions when needed. Financial crises are examples of situations in which the use of FX reserves could be especially desirable.

In this section, we extend the model by assuming that governments use FX reserves to provide liquidity and thereby contribute to stabilize the economy. In particular, since the main channel through which a financial crisis has negative real effects is by depleting entrepreneurial wealth, we assume that the government uses FX reserves to bail out a fraction of the financial losses incurred by entrepreneurs. For simplicity, we do not attempt to characterize the *optimal* accumulation of reserves and bailout policy. Instead, we specify the bailout mechanism as an exogenous rule.

5.1 Bailout mechanism

With the bailout mechanism, the government of region j makes transfers to domestic entrepreneurs, which we denote by $Bail_{j,t}$. With the bailout transfers, the government's budget constraint in Region 1 (AEs) becomes

$$FX_{1,t} + q_{p,t}D_{p,t+1} = q_{p,t}FX_{1,t+1} + D_{p,t} + T_{1,t} + Bail_{1,t}. \tag{33}$$

This is the same budget constraint as the one specified in equation (10) but with the additional variable $Bail_{1,t}$ on the right-hand-side as a new use of funds. A similar modification applies to the budget constraint of the government of Region 2 (EMEs),

$$FX_{2,t} = q_{p,t}FX_{2,t+1} + T_{2,t} + Bail_{2,t}. \tag{34}$$

To specify the determination of the bailout transfers, consider first the aggregate losses incurred by entrepreneurs in region j,

$$Loss_{j,t} = (1 - \delta_{1,t})B_{1,j,t} + (1 - \delta_{2,t})B_{2,j,t}. \tag{35}$$

The government of region j uses part of its FX reserves to cover the losses according to the following rule:

$$Bail_{j,t} = Loss_{j,t} \cdot \left[1 - e^{-\alpha\left(\frac{FX_{j,t}}{Loss_{j,t}}\right)}\right]. \tag{36}$$

The term in square brackets is the fraction of losses covered by the bailout. This fraction is always smaller than 1 and converges to 1 as $FX_{j,t}$ converges to infinity. The overall bailout spending is zero when either the losses are zero or the reserves are zero. The parameter α captures the easiness with which the region can use the accumulated reserves for a bailout. Provided that $\alpha \leq 1$, the size of the bailout transfer, $Bail_{j,t}$, is always smaller than the reserves, $FX_{j,t}$. When $\alpha = 0$, we get back to the model without bailouts.

The bailout transfers are paid to entrepreneurs in proportion to their after-default wealth. Denote by $\chi_{j,t}$ the transfer rate. The transfer received by an individual entrepreneur in region j is $\chi_{j,t}[\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}]$. The transfer rate $\chi_{j,t}$ is then determined so that the total funds allocated to a bailout, $Bail_{j,t}$, are equal to the total transfers paid to entrepreneurs, $\chi_{j,t}[\delta_{1,t}B_{1,j,t} + \delta_{2,t}B_{2,j,t} + B_{p,j,t}]$. Equalizing these two terms we obtain the transfer rate,

$$\chi_{j,t} = \frac{Bail_{j,t}}{\delta_{1,t}B_{1,j,t} + \delta_{2,t}B_{2,j,t} + B_{p,j,t}}. \tag{37}$$

Notice that the total bailout transfers, $Bail_{j,t}$, are zero if there is no default, that is, $\delta_{1,t} = \delta_{2,t} = 1$.

In the design of the bailout mechanism we made two assumptions for analytical tractability. The first is that bailout transfers are proportional to the 'individual' wealth of entrepreneurs. The second is that the transfer rate $\chi_{j,t}$ does not depend on 'individual' composition of portfolios. Under these assumptions, entrepreneurs in the two regions would continue to choose the same portfolio compositions.[15]

The variables $FX_{1,t}$, $FX_{2,t}$ and $D_{p,t}$ are time varying but exogenous. Instead, the bailout transfers $Bail_{1,t}$ and $Bail_{2,t}$ are endogenously determined by condition (36). The households' transfers $T_{j,t}$ are determined by the two budget constraints specified in equations (33) and (34).

This setup can be justified by assuming that in subperiod 1—when default occurs and entrepreneurs are bailed out—the government uses $FX_{j,t}$ to provide the required resources (recall that the assumed policy rule implies $Bail_{j,t} \leq FX_{j,t}$). Then, in subperiod 3, the government adjusts $T_{j,t}$ as needed so that the exogenous $FX_{j,t+1}$ is still attained at the end of the period (i.e., reserves are only used within-the-period to finance the bailout). See Figure 3 for the definition of the three subperiods.

The specification of what happens in subperiod 1 and subperiod 3 formalizes the idea that changing $T_{j,t}$ requires time. By the time governments succeed in raising funds, bailouts may no longer be needed. By holding liquid reserves, instead, governments have the flexibility to intervene in a timely fashion. More generally, we could envisage a situation more akin to reality in which the change in $T_{j,t}$ (i.e., the tax hike needed to fund bailouts) occurs over time, so that the stock of FX reserves drops in the short run after the government intervention. The specification proposed here is a limiting case of this scenario in which taxes cannot adjust in subperiod 1 but they can adjust in subperiod 3.[16]

[15] An alternative assumption would be that the entrepreneurs' losses are covered with lump-sum bailout transfers. Under this assumption, however, entrepreneurs in different regions would choose different portfolio compositions, which complicates significantly the characterization of the equilibrium. Another possible assumption is that the transfers are proportional to the bond holdings that generated the losses. Again, this would lead to non-symmetric portfolio choices with significant analytical complications.

[16] We could assume that households' transfers $T_{j,t}$ are unchanged and $FX_{j,t+1}$ responds endogenously after the bailout. Although we did not adopt it for simplicity, the assumption raises the question of why FX reserves are needed and whether the government could

5.2 The portfolio choice of entrepreneurs

Recall that the representative entrepreneur in Region j enters period t with bonds issued by firms in Regions 1 and 2, $b_{1,j,t}$ and $b_{2,j,t}$, respectively, and government bonds issued by advanced economies, $b_{p,j,t}$. In the original setup, default by intermediate-goods producers causes entrepreneurs to incur financial losses proportional to their ownership of private bonds, with the post-default values given by $\delta_{1,t}b_{1,j,t}$ and $\delta_{2,t}b_{2,j,t}$. In the extension considered here, however, the government bails out entrepreneurs by covering some of their losses with transfers $\chi_{j,t}[\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}]$. Thus, the entrepreneur's wealth after bailout is

$$m_{j,t} = \Big[\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}\Big](1 + \chi_{j,t}).$$

Besides this, all the conditions that define the entrepreneur's problem remain unchanged, including the end-of-period wealth $a_{j,t} = m_{j,t} + (z_j - p_{j,t})x_{j,t}$. We can also show that Lemmas 2.1 and 2.2 remain unchanged. This is also true for all equilibrium conditions derived earlier.

5.3 Quantitative results

We simulate the extended model under the scenarios considered earlier. We use the same parameter values but we need to calibrated the new bailout parameter α. Since we do not have direct empirical evidence about this parameter, we show results for alternative values of α.

Simulation results. Figure 9 plots the output volatility measures for advanced and emerging economies. The top panels are for the scenario in which FX reserves take the values observed in the data with public debt constant. The bottom panels are for the scenario in which public debt takes the values observed in the data with FX reserves constant.

The continuous line is for the baseline case where FX reserves are not used for bailout interventions. This is the same as the continuous line shown in the previous Figure 8. The dashed lines, instead, are for the model with bailout, when $\alpha = 0.1$ and the dash-dotted lines for the case with $\alpha = 0.3$. As explained earlier, α captures the extent to which the government uses FX reserves to bail out entrepreneurs during financial crises.

not just reduce transfers (or raise taxes) directly to fund the bailout.

Given the accumulated FX reserves, the higher the value of α, the lerger the size of the bailout.

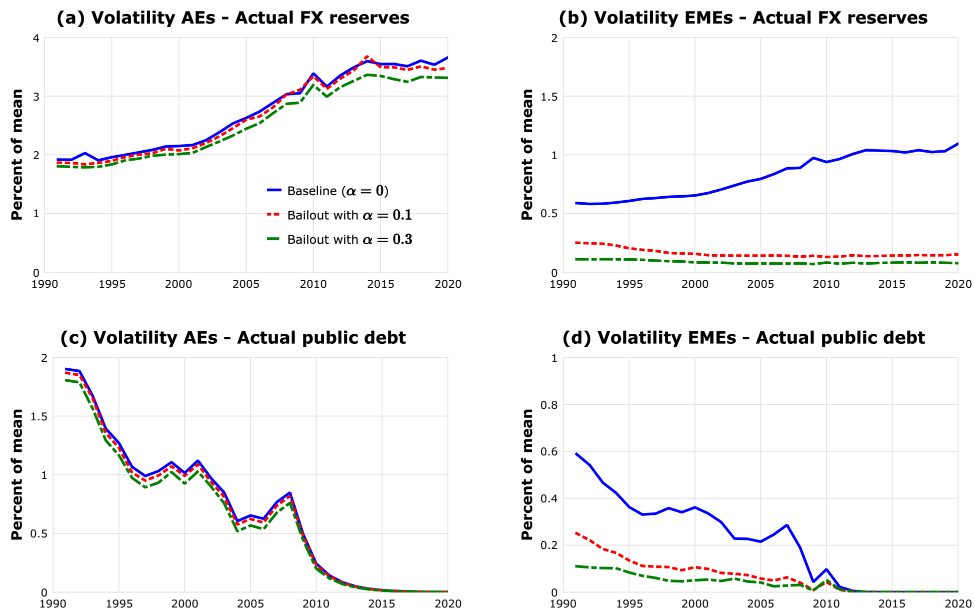

Figure 9: Counterfactual simulation when FX reserves are used for bailouts, 1991-2020.

Panels (a) and (c) show that, for advanced economies, the volatility measure is only marginally affected by the parameter α. This is a straightforward result because AEs do not hold large stocks of reserves relatively to the size of their economy. Therefore, bailouts are relatively small.[17]

For emerging economies, however, the picture is quite different (see Panels (b) and (d)). Even with $\alpha = 0.1$ (dashed line), aggregate volatility drops visibly. With $\alpha = 0.3$ (dash-dotted line), volatility drops to less than half of what it was in the baseline. The larger FX reserves held by EMEs give them a bigger liquidity buffer for stabilization policies than AEs.

[17] One could consider alternative means by which AEs bail out entrepreneurs, for example, swapping defaulted private obligations for newly-issued (risk-free) public debt paid for by future taxes (akin to the 2008 TARP program of the U.S. treasury).

5.4 Bailout policies and moral hazard

Although bailout policies could alleviate the consequences of crises, their anticipation could create undesired distortions. The standard argument is that the anticipation of a bailout, that is the anticipation that some of the investment losses will be covered by government, may induce investors to demand a lower expected return from borrowers. This reduces the cost of borrowing and creates the conditions for higher leverage which, in turn, makes financial crises more damaging.

We can explore the possible effects of this mechanism in the context of our model. The main question is whether the anticipation of bailouts affects equilibrium borrowing.

It turns out that in our model the anticipation of bailouts has a small effect on the interest rate, and therefore, on the equilibrium debt. In part, this derives from the fact that bailout subsidies are conditional on the materialization of a financial crisis, which is a very low probability event. But there is also another reason.

When a crisis materializes, entrepreneurs receive extra funds, part of which are saved to the next period. This should reduce the interest rate, at least after a crisis. However, because entrepreneurs have more funds after the bailout, they can purchase more intermediate inputs (this was the intent of the bailout), which raises the price of the intermediate inputs. The higher price reduces entrepreneurs' profits and, therefore, the end-of-period net worth that can be saved. It turns out that the two effects (transfers from the government and lower profits per unit of wealth) almost cancel each other out. As a result, the impact on the equilibrium interest rate is negligible.

6 Welfare analysis

In this section we explore the welfare consequences of FX reserves accumulation and issuance of public debt. Since in each region there are two types of agents (households who own intermediate-goods firms, and entrepreneurs who own final-goods firms), the welfare gains or losses are computed separately for each type.

We compare the expected lifetime utilities in year 1991 for agents living in two alternative scenarios. In scenario I, FX reserves and public debt remain constant for the full simulation period. In scenario II, FX reserves and/or public debt take the values observed in the data during the 1991-2020 period. Welfare is measured by standard compensating variations in consumption. Hence, the welfare gain is the percent increase in each period's consumption when living in scenario I that makes an agent's welfare equal to the welfare achieved in scenario II. Specifically, for households living in region j, the welfare gain is the value of g_j that solves

$$\mathbb{E}_0 \sum_{t=1991}^{\infty} \beta^t U\Big((1+g_j)c^I_{j,t}\,,\; h^I_{j,t}\Big) = \mathbb{E}_0 \sum_{t=1991}^{\infty} \beta^t U\Big(c^{II}_{j,t}\,,\; h^{II}_{j,t}\Big).$$

The superscripts I and II denote whether the variables are from the equilibrium in the first or second scenarios. The term in the left-hand-side is the expected lifetime utility in scenario I when consumption is raised proportionally by g_j in all periods. The term on the right-hand-side is the expected lifetime utility in scenario II.

The welfare gain for entrepreneurs is the value of g_j that solves

$$\mathbb{E}_0 \sum_{t=1991}^{\infty} \beta^t \ln\Big((1+g_j)c^{e,I}_{j,t}\Big) = \mathbb{E}_0 \sum_{t=1991}^{\infty} \beta^t \ln\Big(c^{e,II}_{j,t}\Big),$$

where, again, the left-hand-side is the expected lifetime utility in scenario I when consumption is raised at rate g_j. The right-hand-side is the expected lifetime utility in scenario II.

It is important to note that, to compute welfare as of 1991, it is not enough to simulate the model until 2020. This is because lifetime utilities are sums over the infinite future. To account for the subsequent future post-2020, we proceed as follows. Beyond 2020, FX reserves and public debt are assumed to remain constant at the values reached in 2020. Thus, if these variables grew during the 1991-2020 period, they remain at the higher values in all subsequent periods. Under this assumption, we extend the simulation of the model past 2020 for an additional 100 years (that is, until 2120). However, past 2070, we assume that there are not crises, so that the model converges (approximately) to a steady state by year 2120. Once we reach the steady state, the utility flow is constant and we can compute the lifetime utility in year 2120 analytically.

Since in the first part of the simulation the economy is stochastic (by assumption until 2070), to compute the expected lifetime utility in 1991, we repeat the simulation 10,000 times, each associated with a randomnly drawn sequence of shocks. More specifically, we simulate the model for 101 periods, with period 101 corresponding to year 1991. Then, from year 1991, we continue the simulation until we reach year 2120. The simulation that starts in year 1991 is repeated 10,000 times, always starting with the same states. For each simulation, we calculate the subjectively discounted value of realized utility flows. The 'expected' lifetime utility in year 1991 is then calculated as the arithmetic average of the lifetime utilities computed in each of the 10,000 repeated simulations.

The above computation of expected utility depends, however, on the particular states reached from the pre-simulation that precedes year 1991. To eliminate the dependence from the initial states, we repeat the whole procedure 100 times and then we average the expected lifetime utilities in year 1991 obtained in each of 100 repetitions.

Table 3 shows the welfare gains calculated by comparing different scenarios. The first row compares the scenario with constant FX reserves and public debt to the scenario in which FX reserves take the empirical values, but public debt remains constant. This case captures the welfare benefits (or costs if negative) when the two regions accumulate FX reserves as in the data, rather than keeping them constant at their 1991 levels.

In advance economies, households gain while entrepreneurs lose, and the losses of the latter are over five times larger than the gains of the former. This is primarily the result of a reduced tax burden on households. By reducing the interest rate, the increase in reserves implies that AEs make lower interest payments to EMEs. Since the AEs government pays the interest on public debt by taxing domestic households, lower interest rates imply lower tax payments which result in higher households' welfare.

The losses of entrepreneurs in AEs derive from two contrasting effects. On one hand, the reduction in the interest rate leads to lower financial wealth held by entrepreneurs. This is beneficial for them because it reduces the demand for intermediate inputs, which in turn reduces their price. Lower price of intermediate goods, then, raises entrepreneurial profits. The negative effect comes from the fact that now entrepreneurs earn lower interest on their financial wealth and the economy is more volatile (which reduces welfare given the concave utility of entrepreneurs). The second (negative) effect dominates the first (positive) effect.

For EMEs we see the opposite and the losses experienced by households are about 7 times larger than the gains of entrepreneurs. Households experience welfare losses because the accumulation of reserves is essentially forced household savings: the government taxes households to purchase the FX reserves. Since the interest rate is lower than the rate of time preference, saving is not desirable for households. Entrepreneurs in EMEs experience a gain but it is relatively small. This is the result of the two contrasting effects described above. In this case, however, the positive effect dominates. The reason is that, because the holdings of financial wealth by entrepreneurs in EMEs is lower than in AEs, the impact on profits is stronger and the negative impact is smaller.

These results, together with the volatility results, show that while the surge in EMEs reserves increased their own output volatility and caused global spillovers that increased volatility in AEs as well, the welfare implications are ambiguous. Households are better off and entrepreneurs worse off in AEs, while in EMEs the opposite is true. The magnitudes of the welfare gains and losses are sizable, keeping in mind the classic Lucas result about the cost of U.S. business cycle being about 0.1% or estimates of the welfare gains of fully eliminating distortionary capital taxes at about 1-2%.

Table 3: Welfare gains of going from Scenario I to Scenario II in absence of bailout ($\alpha = 0$). Gains are in percentage of consumption in Scenario I.

	AEs		**EMEs**	
	Hous.	*Entr.*	*Hous.*	*Entr.*
Impact FX Reserves	0.34	-1.82	-0.71	0.10
Scenario I: Constant Res. & Debt				
Scenario II: Actual Res & constant Debt				
Impact Public Debt	0.03	0.19	2.12	-0.30
Scenario I: Constant Res & Debt				
Scenario II: Constant Res & actual Debt				
Impact FX Reserves & Public Debt	0.28	-0.62	1.75	-0.23
Scenario I: Constant Res & Debt				
Scenario II: Actual Res & Debt				
Impact FX Reserves (actual Public Debt)	0.24	-0.81	-0.37	0.06
Scenario I: Constant Res & actual Debt				
Scenario II: Actual Res & Debt				
Impact Public Debt (actual Reserves)	-0.06	1.22	2.48	-0.33
Scenario I: Actual Res & constant Debt				
Scenario II: Actual Res & Debt				

Consider next the impact of higher public debt (second row in Table 3). In AEs, households' gains are very close to zero. On the one hand, the increase in debt is beneficial because, indirectly, it allows households to borrow at an interest rate that is lower than the rate of time preference. However, it also implies that the existing debt now pays a higher interest rate (which implies higher taxes paid by households). There is also a benefit due to higher production. The positive and negative effects, however, almost offset each other.

Entrepreneurs experience moderate gains. Also in this case there are two effects. The negative effect derives from lower profits induced by the higher demand for intermediate inputs. The positive effect derives from the higher interest earned on financial assets. The second effect dominates the first. Hence, welfare of households and entrepreneurs in AEs is marginally affected by the increase in their public debt.

In EMEs, however, households experience large gains, 2.12%, and nearly 7 times larger than the losses of entrepreneurs. This is because the increase in the interest rate raises the return from FX reserves that are paid back to households as transfers. The increase in production is also beneficial for households because they earn higher wages. Entrepreneurs, instead, experience welfare losses because they accumulate more financial wealth, which increases the demand for intermediate inputs and lowers profits. Entrepreneurs also earn more interests on bonds and benefit from reduced output volatility. The first effect, however, is stronger.

For the increase in public debt of AEs, we find then that the resulting welfare effects on AEs' residents are negligible (both for households and entrepreneurs). In EMEs, because of the sizable spillovers, we obtain a large welfare gain for households and a small loss for entrepreneurs.

The third case is when both FX reserves and public debt increase. The welfare effects are now the combination of the welfare effects shown in the previous two cases. The gain of households of EMEs remains markedly larger than both the loss of EME entrepreneurs and the gains and losses of AEs' households and entrepreneurs, respectively.

The last two cases (fourth and fifth rows) are similar to the first two. The difference is that the change in FX reserves arises when public debt is at the higher 2020 level instead of the 1991 level. Similarly for the fifth case: the change in public debt arises when FX reserves are at the higher 2020 level.

The signs of the welfare gains are similar but the magnitudes change. For example, the losses associated with the increase in reserves is now smaller for entrepreneurs in AEs. This is because the increase in FX reserves lead to a smaller decline in the interest rate. The same argument explains why the losses experienced by households in EMEs are smaller.

When public debt increases with a higher stock of FX reserves, the welfare gains for entrepreneurs in AEs are bigger. This is because the interest rate increases more due to the fact that, with higher FX reserves, the initial interest rate is smaller, allowing for a higher interest rate increase. The larger increase in the interest rate also explains the larger gain experienced by households in EMEs (since the higher interest rate translates in higher transfers to households).

Table 4 in Appendix H reports the welfare gains when the bailout policy is active, that is, $\alpha > 0$. Overall, bailout policies have a modest impact on welfare. Of course, these are average numbers. Conditional on a crisis, the welfare impact of bailouts is much bigger.

7 Discussion and conclusion

An implication of the increased size of emerging economies is that, collectively, they play a more influential role in driving global capital markets and macroeconomic dynamics. The view that emerging markets are a collection of small open economies with negligible impact on advanced economies is no longer a valid approximation. One way in which emerging economies affect the world economy is through financial markets. In this paper, we focused on one channel: the accumulation of foreign reserves.

Since the 1990s, emerging economies have sharply increased their reserves as a percentage of both their own GDP and, importantly, of global GDP. This represents a large increase in world demand for financial assets (typically government bonds issued by advanced economies). Through a counterfactual analysis, we showed that this surge in reserves contributed to the observed fall in the world interest rate. As the cost of borrowing fell, the private sector became more leveraged, and this increased financial and macro volatility *globally*.

While the accumulation of reserves by EMEs contributed to lower interest rates and greater global volatility, it also provided these economies with liquidity usable for stabilization purposes. The end result in the model is that the significant accumulation of FX reserves by EMEs reduced their financial and macroeconomic volatility but increased the volatility of advanced economies since they did not accumulate reserves as EMEs did.

During the same period, we also observed that governments in advanced economies increased public borrowing, raising the supply of financial assets. This had the opposite effect from the surge in EMEs' reserves: it propped up the world interest rate, which in turn discouraged private borrowing (crowding out). Lower private leverage, then, contributed to reducing global economic instability.

In our counterfactual exercises, we used changes in FX reserves and public debt as exogenous inputs. Since we also found that these changes have non-negligible welfare effects, it would be interesting to explore how governments choose these policies. In an integrated world economy, these policies depend on the size of the country. For example, if a country is small compared to the world economy and chooses to increase its FX reserves, the economy of that country may become more stable. However, if many countries implement a similar policy, the world interest rate would fall, inducing more leverage and higher macroeconomic instability (as shown in the paper). This suggests that there could be over-accumulation of reserves.

The idea that emerging countries could over-accumulate reserves is consistent with the theoretical analysis of Das et al. (2023). However, there is also another side to the story. Low interest rates may encourage the governments of advanced economies to issue more public debt, which would move the world interest rate in the opposite direction. Thus, the study of the welfare implications and optimality of FX accumulation should also consider how the issuance of public debt by advanced economies responds to the demand from emerging economies.

Appendix

A Proof of Lemma 2.1

The optimization problem of an entrepreneur in region j is

$$\max_{\{x_{j,t},c^e_{j,t},b_{1,j,t+1},b_{2,j,t+1},b_{p,j,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c^e_{j,t}) \tag{38}$$

subject to

$$\begin{aligned} m_{j,t} &= \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}, \\ m_{j,t} &\geq \phi_j p_{j,t}x_{j,t}, \\ a_{j,t} &= m_{j,t} + z_j x_{j,t} - p_{j,t}x_{j,t}, \\ c^e_{j,t} &= a_{j,t} - q_{1,t}b_{1,j,t+1} - q_{2,t}b_{2,j,t+1} - q_{p,t}b_{p,j,t+1}. \end{aligned}$$

The first-order condition for $x_{j,t}$ is

$$z_j = (1 + \hat{\xi}_{j,t}\phi_j)p_{j,t}, \tag{39}$$

where $\hat{\xi}_{j,t} \equiv \xi_{j,t}/u'(c^e_{j,t}) = \xi_{j,t}c^e_{j,t}$ and $\xi_{j,t}$ is the Lagrange multiplier associated with the working capital constraint in the above optimization problem.

When the financial constraint is binding we have that $\xi_{j,t} > 0$. Then condition (39) implies that $z_j > p_{j,t}$ and the entrepreneurs' profits, $\pi_{j,t} = (z_j - p_{j,t})x_{j,t}$, are positive. When the constraint is not binding, instead, $\xi_{j,t} = 0$ and the first-order condition becomes $z_j = p_{j,t}$. Profits are then zero, that is, $\pi_{j,t} = 0$.

Using the financial constraint $m_{j,t} = \phi_j p_{j,t}x_{j,t}$ and condition (39), we can write the profits as

$$\pi_{j,t} = \hat{\xi}_{j,t}m_{j,t}. \tag{40}$$

The lender's wealth is $a_{j,t} = m_{j,t} + \pi_{j,t}$. Using (40) it can be rewritten as

$$a_{j,t} = (1 + \hat{\xi}_{j,t})m_{j,t}$$

This shows how the multiplier $\hat{\xi}_{j,t}$ captures the notion of a convenience yield. When the working capital constraint binds, bonds yield a return over and above the yield implicit in their prices at rate $\hat{\xi}_{j,t}$ per unit of financial wealth $m_{j,t}$.

The entrepreneur's optimality conditions can also be used to express the above results in terms of factor prices instead of the shadow value $\hat{\xi}_{j,t}$ as follows:

$$p_{j,t}x_{j,t} = \frac{m_{j,t}}{\phi_j}$$

$$y_{j,t} = z_j \frac{m_{j,t}}{p_{j,t}\phi_j}$$
$$\pi_{j,t} = \frac{m_{j,t}}{\phi_j}\left(\frac{z_j}{p_{j,t}} - 1\right)$$
$$a_{j,t} = m_{j,t}\left[1 + \frac{1}{\phi_j}\left(\frac{z_j}{p_{j,t}} - 1\right)\right] \geq m_{j,t}$$

The results for profits then imply that the shadow value of the financial constraint satisfies this condition:

$$\hat{\xi}_{j,t} = \frac{1}{\phi_j}\left(\frac{z_j}{p_{j,t}} - 1\right)$$

This demonstrates that end-of-period wealth is linear in initial financial wealth, with a slope of 1 if the working capital constraint does not bind and with a slope of $1+\hat{\xi}_{j,t}$ when it binds. In the latter case, the slope coefficient is a nonlinear function of productivity, factor prices and ϕ_j. This linearity of wealth will be used in Appendix B to solve for the entrepreneur's portfolio allocation problem.

B Proof of Lemma 2.2

Given that at the optimum of the entrepreneur's problem $a_{j,t} = (1+\hat{\xi}_{j,t})m_{j,t}$ and since $m_{j,t} = \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}$, we can write the end-of-period wealth at time t and at $t+1$ as

$$\begin{aligned} a_{j,t} &= (1+\hat{\xi}_{j,t})(\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}), \\ a_{j,t+1} &= (1+\hat{\xi}_{j,t+1})(\delta_{1,t+1}b_{1,j,t+1} + \delta_{2,t+1}b_{2,j,t+1} + b_{p,j,t+1}). \end{aligned}$$

We derive next the first-order conditions for Problem (38) with respect to $b_{1,j,t+1}$, $b_{2,j,t+1}$ and $b_{p,j,t+1}$,

$$\frac{q_{1,t}}{c^e_{j,t}} = \beta \mathbb{E}_t\left(\frac{(1+\hat{\xi}_{j,t+1})\delta_{1,t+1}}{c^e_{j,t+1}}\right), \quad (41)$$
$$\frac{q_{2,t}}{c^e_{j,t}} = \beta \mathbb{E}_t\left(\frac{(1+\hat{\xi}_{j,t+1})\delta_{2,t+1}}{c^e_{j,t+1}}\right). \quad (42)$$
$$\frac{q_{p,t}}{c^e_{j,t}} = \beta \mathbb{E}_t\left(\frac{(1+\hat{\xi}_{j,t+1})}{c^e_{j,t+1}}\right). \quad (43)$$

The right-hand-sides of these three Euler equations reflect again the convenience yield of financial wealth. The marginal benefit of buying bonds at t to carry over to $t+1$ increases by $(1+\hat{\xi}_{j,t+1})$ if the working capital constraint binds. This is because holding additional bonds relaxes the constraint, which is in addition to the contractual yield of each bond (the reciprocal of their prices). As shown earlier, this convenience yield is equal to profits per unit of financial wealth, but now in terms of expected profits at $t+1$.

We now guess that optimal consumption is a fraction $1-\beta$ of wealth,

$$c^e_{j,t} = (1-\beta)a_{j,t}.$$

The saved wealth is allocated to private bonds issued by region 1 and by region 2 and public debt issued by region 1. Denoting by $\theta_{1,j,t}$ and $\theta_{2,j,t}$ the portfolio shares allocated to private bonds issued by region 1 and region 2, respectively, we have

$$q_{1,t}b_{1,j,t+1} = \theta_{1,j,t}\beta a_{j,t}, \tag{44}$$

$$q_{2,t}b_{2,j,t+1} = \theta_{2,j,t}\beta a_{j,t}, \tag{45}$$

$$q_{p,t}b_{p,j,t+1} = (1-\theta_{1,j,t}-\theta_{2,j,t})\beta a_{j,t}. \tag{46}$$

We now multiply equation (41) by $b_{1,j,t+1}$, equation (42) by $b_{2,j,t+1}$, and equation (43) by $b_{p,j,t+1}$. Adding the resulting expressions and using the equations that define consumption and next period wealth, we obtain

$$q_{1,t}b_{1,j,t+1} + q_{2,t}b_{2,j,t+1} + q_{p,t}b_{p,j,t+1} = \beta a_{j,t}.$$

This is clearly satisfied given (44)-(46). Since we have derived this condition from the Euler equations (41)-(43), we have proved that, if consumption is a fraction $1-\beta$ of wealth, the three Euler equations are satisfied. This verifies our guess.

We now replace the guess for $c^e_{j,t}$ into equations (41) and (42), to obtain

$$\mathbb{E}_t\left\{\frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,j,t}\frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,j,t}\frac{\delta_{2,t+1}}{q_{2,t}} + (1-\theta_{1,j,t}-\theta_{2,j,t})\frac{1}{q_{p,t}}}\right\} = 1. \tag{47}$$

$$\mathbb{E}_t\left\{\frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,j,t}\frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,j,t}\frac{\delta_{2,t+1}}{q_{2,t}} + (1-\theta_{1,j,t}-\theta_{2,j,t})\frac{1}{q_{p,t}}}\right\} = 1. \tag{48}$$

These two conditions determine the shares of savings invested in the private bonds of the two regions. Since the conditions are the same for entrepreneurs in both regions, it must be that $\theta_{1,1,t} = \theta_{1,2,t} = \theta_{1,t}$ and $\theta_{2,1,t} = \theta_{2,2,t} = \theta_{2,t}$. ■

The above results show that the convenience yield plays two roles: First, in subperiod 2 of the lender's problem, it takes the form of profits as we showed in Appendix A (if the financial constraint binds, the ex-post payoff of a bond increases above its actual payout, inclusive of any haircut, because of the profits that are allowed by the bonds used as working capital). Second, in subperiod 3, if the financial constraint is expected to bind at date-t+1, the expected marginal return of the bonds purchased at date-t rises because of the expected convenience yield at t+1 (the expected profits the new portfolio of bonds will yield). To put it differently, the financial constraint induces both an atemporal wedge between market factor prices and their corresponding marginal products, and an intertemporal wedge between marginal costs and benefits of saving into bonds. The following proposition, however, establishes that the logarithmic utility neutralizes the intertemporal wedge.

Proposition B.1 *The intertemporal wedge of the working capital constraint does not enter the entrepreneur's Euler equations. In particular, the marginal benefit of saving into each of the three bonds in the right-hand-side of (41)-(43) is independent of $\hat{\xi}_{j,t+1}$.*

Proof B.1 *Consider the marginal benefit of buying an extra unit of $b_{1,j,t+1}$ with logarithmic utility, as expressed in the right-hand-side of (41):*

$$\beta \mathbb{E}_t \left(\frac{(1+\hat{\xi}_{j,t+1})\delta_{1,t+1}}{c^e_{j,t+1}} \right)$$

Since $c^e_{j,t+1} = (1-\beta)a_{j,t+1}$ and $a_{j,t+1} = (1+\hat{\xi}_{j,t+1})m_{j,t+1}$, the above expression can be re-written as:

$$\beta \mathbb{E}_t \left(\frac{(1+\hat{\xi}_{j,t+1})\delta_{1,t+1}}{(1-\beta)(1+\hat{\xi}_{j,t+1})m_{j,t+1}} \right).$$

Using $m_{j,t+1} = \delta_{1,t+1}b_{1,j,t+1} + \delta_{2,t+1}b_{2,j,t+1} + b_{p,j,t+1}$ and conditions (44)-(46), we obtain:

$$\mathbb{E}_t \left\{ \frac{\delta_{1,t+1}}{(1-\beta)a_t \left(\theta_{1,j,t}\frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,j,t}\frac{\delta_{2,t+1}}{q_{2,t}} + (1-\theta_{1,j,t}-\theta_{2,j,t})\frac{1}{q_{p,t}} \right)} \right\}.$$

This is independent of $\hat{\xi}_{j,t+1}$ because $\delta_{1,t+1}$ and $\delta_{2,t+1}$ are taken as given by the entrepreneur and the portfolio shares that solve (47) and (48) are independent of $\hat{\xi}_{j,t+1}$. A similar argument applies to the marginal benefit of saving into $b_{2,j,t+1}$ and $b_{p,j,t+1}$. ■

C Optimization problem of intermediate goods producers

Producers of intermediate goods maximize the present value of the dividends they pay to households. Their optimization problem can be written recursively as

$$V(d,k) = \max_{l,d'} \left\{ \text{div} + \beta \mathbb{E} V(d',k') \right\},$$

subject to

$$\tilde{d}(d,\ell k) + \text{div} + \varphi(d',\kappa k') \;=\; p l^{\gamma} k^{1-\gamma} - wl - k' + (1-\tau)k + \frac{1}{R}\mathbb{E}\tilde{d}(d',\ell' k'),$$

where k and the law of motion for k' are exogenous, $\tilde{d}(d,\ell k)$ is defined in (2) and $\varphi(d',\kappa k')$ in (3). The firm discounts dividends at rate β, which corresponds to the households' discount factor. Because households have linear utility in c, the marginal utility of consumption is always 1.

The first-order conditions with respect to l and d' are:

$$\gamma k^{1-\gamma} l^{\gamma-1} = w,$$

$$\frac{1}{R}\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\ell' k')}{\partial d'}\right\} - \frac{\partial \varphi(d',\kappa k')}{\partial d'} + \beta \mathbb{E}\left\{\frac{\partial V(d',k')}{\partial d'}\right\} = 0$$

The envelope condition for debt is

$$\frac{\partial V(d,k)}{\partial d} \;=\; -\frac{\partial \tilde{d}(d,\ell k)}{\partial d}.$$

Updating this condition by one period and substituting in the first-order condition for debt, we obtain

$$\frac{1}{R} = \beta + \frac{\frac{\partial \varphi(d',\kappa k')}{\partial d'}}{\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\ell' k')}{\partial d'}\right\}} \tag{49}$$

We now derive the analytical expressions for the derivatives included in the right-hand-side of the above expression. To do so we use the functional forms for $\tilde{d}(d,\ell k)$ and $\varphi(d',\kappa k')$ defined, respectively, in (2) and (3):

$$\frac{\partial \tilde{d}(d,\ell k)}{\partial d} = \begin{cases} 0, & \text{if} \quad d \geq \ell k \\ 1, & \text{otherwise} \end{cases}$$

$$\frac{\partial \varphi(d',\kappa k')}{\partial d'} = \begin{cases} 2\eta\left(1-\frac{\kappa k'}{d'}\right)\frac{\kappa k'}{d'} + \eta\left(1-\frac{\kappa k'}{d'}\right)^2, & \text{if} \quad d' \geq \kappa k' \\ 0, & \text{otherwise} \end{cases}$$

If $d' > \kappa k'$, the liquidation price ℓ is equal to κ with probability λ (probability of default). If $d' \leq \kappa k'$, the liquidation price ℓ is always equal to 1. Using this, we can rewrite the expected value of the derivative of $\tilde{d}$ as

$$\mathbb{E}\left\{\frac{\partial \tilde{d}(d', \ell' k')}{\partial d'}\right\} = \begin{cases} 1-\lambda, & \text{if } d' \geq \kappa k' \\ \\ 1, & \text{otherwise} \end{cases}$$

Using the above expressions in the first-order condition (49) we obtain

$$\frac{1}{R} = \beta + \Phi\left(\frac{d'}{\kappa k'}\right), \tag{50}$$

where

$$\Phi\left(\frac{d'}{\kappa k'}\right) = \begin{cases} \left(\frac{1}{1-\lambda}\right)\eta\left[1-\left(\frac{\kappa k'}{d'}\right)^2\right], & \text{if } \frac{d'}{\kappa k'} \geq 1 \\ \\ 0, & \text{if } \frac{d'}{\kappa k'} < 1 \end{cases}$$

The function $\Phi(.)$ is strictly increasing for $\frac{d'}{\kappa k'} \geq 1$. In addition, for $\frac{d'}{\kappa k'} \geq 1$, taking derivatives we can verify that it is increasing in d' and decreasing in both k' and κ. Note also that with $\eta = 0$ (costless debt issuance), the debt Euler equation collapses to $\frac{1}{R} = \beta$ and hence debt and leverage would be indeterminate. ■

D Market for liquidated capital and equilibrium multiplicity

In the main body of the paper, we assumed that the liquidation price $\ell_{j,t}$ can be either κ_j or 1 with constant probabilities λ and $1-\lambda$. In this section, we describe the market structure that provides the micro-foundation for the determination of ℓ_t. The specification admits two self-fulfilling equilibria and λ represents the probability of a sunspot shock that selects one of two equilibria.

The market for liquidated capital meets at the beginning of the period. We make two important assumptions about the operation of this market.

Assumption 1 *Capital can be sold to domestic intermediate-goods firms or final-goods firms (entrepreneurs). However, if sold to entrepreneurs, capital loses its functionality as a productive asset and it is converted to consumption goods at rate* $\kappa_j < 1$.

This assumption formalizes the idea that capital may lose value when reallocated to another sector or region. The assumption that capital loses its functionality also when reallocated abroad implies that a crisis could be local. However, even if a crisis takes place only in one region, it will have real economic consequences for the other region due to the cross-country diversification of bond portfolios.

Assumption 2 *Intermediate-goods firms can purchase liquidated capital only if the liquidation value of their own capital exceeds the debt obligations,* $d_{j,t} < \ell_{j,t} k_{j,t}$.

If an intermediate-goods firm starts with liabilities bigger than the liquidation value of the owned assets, that is, $d_{j,t} > \ell_{j,t} k_{j,t}$, it will be unable to raise additional funds to purchase the capital liquidated by other firms. Potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized, and the debt will be renegotiated immediately after taking the new debt. We refer to an intermediate-goods firm with $d_{j,t} < \ell_{j,t} k_{j,t}$ as 'liquid' since it can raise extra funds at the beginning of the period. Instead, a firm with $d_{j,t} > \ell_{j,t} k_{j,t}$ is 'illiquid'.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating, $d_{j,t} \leq \ell_{j,t} k_{j,t}$. If this condition is satisfied, intermediate-goods firms have the ability to raise funds to purchase additional capital. This ensures that the liquidation price is $\ell_{j,t} = 1$. If $d_{j,t} > \kappa_j k_{j,t}$ for all intermediate-goods firms, however, there will be no firms capable of buying the liquidated capital. Then, the liquidated capital can only be purchased by entrepreneurs at price $\ell_{j,t} = \kappa_j$.

This shows that the market price for liquidated capital depends on the financial decision of firms, $d_{j,t}$, which in turn depends on the liquidation price. This interdependence is critical for generating self-fulfilling equilibria.

Proposition D.1 *There exists multiple equilibria only if* $d_{j,t} > \kappa_j k_{j,t}$.

Proof D.1 *At the beginning of the period, firms choose whether to renegotiate the debt. Given the initial states* d_t *and* k_t*, renegotiation boils down to a take-it or leave-it offer made to creditors for the repayment of the debt.*

Denote by $\tilde{d}_t = \psi(d_t, k_t, \ell_t)$ *the offered repayment. This depends on the individual liabilities,* d_t*, individual capital,* k_t*, and the price for liquidated capital,* ℓ_t*. The liquidation price is the price at which the lender could sell capital after rejecting the offer from the borrower. The best offer made by the intermediate-goods firm is*

$$\psi(d_t, k_t, \ell_t) = \begin{cases} d_t, & \text{if} \quad d_t \leq \ell_t k_t \\ \ell_t k_t, & \text{if} \quad d_t > \ell_t k_t \end{cases}, \tag{51}$$

which is accepted by creditors if they cannot sell at a price higher than ℓ_t.

We assume, for the moment, that the equilibrium is symmetric, that is, all intermediate-goods firms start with the same ratio d_t/k_t. At this stage this is only an assumption. However, we will show below that firms do not have an incentive to deviate from the choice of other firms.

Given the assumption that the equilibrium is symmetric, multiple equilibria arise if $d_t/k_t \in [\kappa, 1)$. If the market expects that the liquidation price is $\ell_t = \kappa$, all firms are illiquid and they choose to renege on their liabilities (given the renegotiation policy (51)). As a result, there will be no firms that can purchase the liquidated capital of other firms. The only possible liquidation price that is consistent with the expected price is $\ell_t = \kappa$. On the other hand, if the market expects $\ell_t = 1$, intermediate-goods firms are liquid and, if one firm reneges, creditors can sell the liquidated capital to other intermediate-goods firms at the liquidation price $\ell_t = 1$. Therefore, it is optimal for firms not to renegotiate.

To complete the proof we need to show that an individual firm does not have an incentive to deviate from the symmetric equilibrium and choose a different ratio d_t/k_t at $t-1$. Specifically, we want to show that, in the anticipation that the liquidation price could drop to $\ell_t = \kappa$, an intermediate-goods firm does not find optimal to borrow less at $t-1$ so that it will be able to purchase the liquidated capital at t.

The first point to consider is that, in equilibrium, capital is never liquidated. The low liquidation price κ represents the threat value for creditors. Since creditors accept the renegotiation offer, no capital is ever liquidated. What would happen if there is an intermediate-goods firm that is liquid and has the ability to purchase the capital at a price higher than κ? Debtors know that their creditors could liquidate the capital and sell it at a higher price than κ. Knowing this, debtors will offer a higher repayment and, as a result, capital will not be liquidated. The liquidation price, then, could be driven to 1. This shows that an intermediate-goods firm cannot gain from remaining liquid. Thus, there is no incentive to deviate from the symmetric equilibrium. ■

The proof of the proposition establishes that the equilibrium is symmetric and all intermediate-goods firms choose the same ratio d_t/k_t. Then, multiple equilibria determined by self-fulfilling expectations about the liquidation price exists if $d_t/k_t \in [\kappa, 1)$. On the one hand, if the market expects a liquidation price $\ell_t = \kappa$, all intermediate-goods firms are illiquid and choose to renege on their liabilities. As a result, there are no intermediate-goods firms that can purchase the liquidated capital and, therefore, the only liquidation price consistent with the expected price is $\ell_t = \kappa$. On the other hand, when the market expects $\ell_t = 1$, intermediate-goods firms are liquid and, if one firm reneges, creditors can sell the liquidated capital to other firms at price $\ell_t = 1$, which makes it optimal not to renege.

When multiple equilibria are possible, that is, when we have $d_{j,t} > \kappa_j k_{j,t}$, the equilibrium is selected by random draws of sunspot shocks. Let $\varepsilon_{j,t}$ be a variable that takes the value of 0 with probability λ and 1 with probability $1-\lambda$. If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price κ_j when $\varepsilon_{j,t} = 0$. This implies that the probability distribution of the low liquidation price is

$$f_{j,t}\Big(\ell_{j,t} = \kappa_j\Big) = \begin{cases} 0, & \text{if} \quad d_{j,t} \leq \kappa_j k_{j,t} \\ \\ \lambda, & \text{if} \quad d_{j,t} > \kappa_j k_{j,t} \end{cases}$$

The ratio $d_{j,t}/\kappa_j k_{j,t}$ is the relevant measure of leverage. When it is sufficiently small, intermediate-goods firms remain liquid even if the (expected) liquidation price is κ_j. But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when leverage is high, firms' liquidity depends on the liquidation price. The realization of the sunspot shock $\varepsilon_{j,t}$ then becomes important for selecting one of the two equilibria. When $\varepsilon_{j,t} = 0$—which happens with probability λ—the market expects that the liquidation price is κ_j, making the intermediate-goods sector illiquid. On the other hand, when $\varepsilon_{j,t} = 1$—which happens with probability $1 - \lambda$—the market expects that intermediate-goods firms are capable of participating in the liquidation market, validating the expectation of a higher liquidation price.

The above argument is based on the assumption that κ_j is sufficiently low (implying a low liquidation price if the capital freezes). Also, the value of capital without a freeze, $k_{j,t}$, is always bigger than the debt $d_{j,t}$. Otherwise, firms would be illiquid with probability 1 and the equilibrium price would be always κ_j. Condition (5) guarantees that this does not happen at equilibrium: if the probability of default is 1, the anticipation of the renegotiation cost increases the interest rate, which deters intermediate-goods firms from borrowing too much.

E Equilibrium system of equations at time t

Given the state vector

$$\begin{aligned} \mathbf{s}_t \quad \equiv \quad & \big(FX_{1,t+1}, FX_{2,t+1}, D_{p,t+1}, K_{1,t}, K_{2,t}, K_{1,t+1}, K_{2,t+1}, \\ & \qquad B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}\big), \end{aligned}$$

we can find the values of $\delta_{j,t}$, $M_{j,t}$, $L_{j,t}$, $X_{j,t}$, $w_{j,t}$, $p_{j,t}$, $q_{j,t}$, $q_{p,t}$, $A_{j,t}$, $B_{j,1,t+1}$, $B_{j,2,t+1}$, $B_{p,j,t+1}$, $D_{j,t+1}$, $\theta_{1,t}$ and $\theta_{2,t}$, by solving the following nonlinear system of equations (using also the assumption that $\mu_j = z_j^{1/\gamma}$):

$$
\begin{aligned}
\delta_{j,t} &= \begin{cases} \min\left\{1 \, , \, \frac{\kappa_j K_{j,t}}{D_{j,t}}\right\}, & \text{if} \quad \varepsilon_{j,t} = 0 \\ \\ 1, & \text{if} \quad \varepsilon_{j,t} = 1 \end{cases} & (52) \\
M_{j,t} &= \delta_{1,t} B_{1,t} + \delta_{2,t} B_{2,t} + B_{p,t} & (53) \\
L_{j,t} &= \left(\frac{\gamma p_{j,t}}{w_{j,t}}\right)^{\frac{1}{1-\gamma}} K_{j,t}, & (54) \\
L_{j,t} &= \left(\frac{w_{j,t}}{z_j^{1/\gamma}}\right)^{\nu}, & (55) \\
X_{j,t} &= L_{j,t}^{\gamma} K_{j,t}^{1-\gamma}, & (56) \\
p_{j,t} &= \begin{cases} \frac{M_{j,t}}{\phi_j X_{j,t}}, & \text{if} \quad M_{j,t} < \phi_j X_{j,t} \\ \\ 1, & \text{if} \quad M_{j,t} = \phi_j X_{j,t} \end{cases} & (57) \\
A_{j,t} &= M_{j,t} + z_j X_{j,t} - p_{j,t} X_{j,t}, & (58) \\
B_{1,j,t+1} &= \frac{\theta_{1,t} \beta A_{j,t}}{q_{1,t}}, & (59) \\
B_{2,j,t+1} &= \frac{\theta_{2,t} \beta A_{j,t}}{q_{2,t}}, & (60) \\
B_{p,j,t+1} &= \frac{(1 - \theta_{1,t} - \theta_{2,t}) \beta A_{j,t}}{q_{p,t}}, & (61) \\
1 &= \mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,t} - \theta_{2,t}) \frac{1}{q_{p,t}}} \right\}, & (62) \\
1 &= \mathbb{E}_t \left\{ \frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,t} - \theta_{2,t}) \frac{1}{q_{p,t}}} \right\}, & (63) \\
D_{j,t+1} &= B_{j1,t+1} + B_{j2,t+1}, & (64) \\
q_{j,t} &= \left[\beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_j K_{j,t+1}}\right)\right] \mathbb{E}_t \delta_{j,t+1}. & (65) \\
D_{p,t+1} &= F X_{1,t+1} + F X_{2,t+1} + B_{p,1,t+1} + B_{p,2,t+1}. & (66)
\end{aligned}
$$

Equation (52) defines the optimal renegotiation strategy (the fraction of the debt repaid). Equation (53) defines entrepreneurial wealth after default. Equation (54) is the demand for labor from intermediate-goods firms. Equation (55) is the supply of labor from households. Equation (56) is the production of intermediate goods and (57) defines its price $p_{j,t}$, which depends on whether the working capital constraint is binding or not binding. Equation (58) defines the end-of-period wealth of entrepreneurs after production. This is allocated to private bonds issued by the two regions and public bonds issued by region 1 as indicated in equations (59)-(61). Equations (62) and (63) are the conditions that determine the investment shares $\theta_{1,t}$ and $\theta_{2,t}$. They are the Euler equations derived from the optimization problem of entrepreneurs. Equation (64) is equilibrium in the bond market. Equation (65) is the Euler equation for intermediate-goods firms for the issuance of debt. This determines the price of bonds. The final equation (66) is the market equilibrium for public bonds.

The list includes 15 equations. However, since 12 of them are for $j \in \{1, 2\}$, the total system has 27 equations. The number of unknown variables is also 27: $\delta_{j,t}$, $M_{j,t}$, $L_{j,t}$, $X_{j,t}$, $w_{j,t}$, $p_{j,t}$, $q_{j,t}$, $A_{j,t}$, $B_{j,1,t+1}$, $B_{j,2,t+1}$, $B_{p,j,t+1}$, $D_{j,t+1}$ for $j \in \{1, 2\}$, plus $q_{p,t}$, $\theta_{1,t}$ and $\theta_{2,t}$.

F Resource constraints & balance-of-payments accounting

Combining the budget constraints of households, producers, and governments, plus the market-clearing conditions for financial and labor markets, we obtain the following resource constraints for Country 1 and Country 2:[18]

$$\begin{aligned} C_{1,t} + C^e_{1,t} + I_{1,t} + \varphi(D_{1,t+1}, \kappa_1 K_{1,t+1}) \\ = z_1 L^{\gamma}_{1,t} K^{1-\gamma}_{1,t} - [q_{2,t} B_{2,1,t+1} - q_{1,t} B_{1,2,t+1} - q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})] \\ + [\delta_{2,t} B_{2,1,t} - \delta_{1,t} B_{1,2,t} - (B_{p,2,t} + FX_{2,t})] \end{aligned} \quad (67)$$

$$\begin{aligned} C_{2,t} + C^e_{2,t} + I_{2,t} + \varphi(D_{2,t+1}, \kappa_2 K_{2,t+1}) \\ = z_2 l^{\gamma}_{2,t} K^{1-\gamma}_{2,t} - [q_{1,t} B_{1,2,t+1} - q_{2,t} B_{2,1,t+1} + q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})] \\ + [\delta_{1,t} B_{1,2,t} - \delta_{2,t} B_{2,1,t} + (B_{p,2,t} + FX_{2,t})] \end{aligned} \quad (68)$$

The uses of resources in the left-hand-sides of these conditions represents domestic absorption, which includes final-goods consumption of households and final-goods producers (entrepreneurs), investment expenditures $I_{j,t} = K_{j,t+1} - (1 - \tau)K_{j,t}$, and borrowing costs. The sources in the right-hand-sides include GDP and cross-border capital flows related to the three bonds traded by the two regions.

[18]In deriving these results, we should note that the bond prices satisfy $q_{j,t} = 1/R_{j,t}$ and that $\delta_{j,t} D_{j,t} - \tilde{d}(D_{j,t}, \kappa_j K_{j,t}) = 0$ always (when $D_{j,t} < \kappa_j K_{j,t}$, we have $\delta_{j,t} = 1$ and $\tilde{d}(\cdot) = D_{j,t}$, and when $D_{j,t} \geq \kappa_j K_{j,t}$, we have $\delta_{j,t} = \kappa_j K_{j,t}/D_{j,t}$ and $\tilde{d}(\cdot) = \kappa_j K_{j,t}$).

Adding the above constraints yields the world resource constraint, which simply states that global absorption must equal global output (because the cross border asset positions are an asset for one country and a liability for the other):

$$\begin{aligned} C_{1,t} + C^e_{1,t} + I_{1,t} + \varphi(D_{1,t+1}, \kappa_1 K_{1,t+1}) + C_{2,t} + C^e_{2,t} + I_{2,t} \; + \\ \varphi(D_{2,t+1}, \kappa_2 K_{2,t+1}) \;=\; z_1 L^{\gamma}_{1,t} K^{1-\gamma}_{1,t} + z_2 L^{\gamma}_{2,t} K^{1-\gamma}_{2,t} \end{aligned}$$

The country resource constraints can be re-written to show that the balance-of-payments accounting condition holds in each country:

$$\begin{aligned} NX_{1,t} &\equiv z_1 L^{\gamma}_{1,t} K^{1-\gamma}_{1,t} - \Big[C_{1,t} + C^e_{1,t} + I_{1,t} + \varphi\left(D_{1,t+1}, \kappa_1 K_{1,t+1}\right)\Big] \\ &= \Big[q_{2,t} B_{2,1,t+1} - q_{1,t} B_{1,2,t+1} - q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})\Big] - \\ &\qquad \Big[\delta_{2,t} B_{2,1,t} - \delta_{1,t} B_{1,2,t} - (B_{p,2,t} + FX_{2,t})\Big] \end{aligned} \tag{69}$$

$$\begin{aligned} NX_{2,t} &\equiv z_2 L^{\gamma}_{2,t} K^{1-\gamma}_{2,t} - \Big[C_{2,t} + C^e_{2,t} + I_{2,t} + \varphi(D_{2,t+1}, \kappa_2 K_{2,t+1})\Big] \\ &= \Big[q_{1,t} B_{1,2,t+1} - q_{2,t} B_{2,1,t+1} + q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})\Big] - \\ &\qquad \Big[\delta_{1,t} B_{1,2,t} - \delta_{2,t} B_{2,1,t} + (B_{p,2,t} + FX_{2,t})\Big] \end{aligned} \tag{70}$$

In these expressions, $NX_{j,t}$ denotes the trade balance (exports minus imports) which is equal to the gap between GDP and domestic absorption. The second equality in each expression shows that the trade balance equals the current account $CA_{j,t}$ minus net factor payments to the rest of the world. For instance, in Country 1, $\delta_{2,t} B_{2,1,t} - \delta_{1,t} B_{1,2,t} - (B_{p,2,t} + FX_{2,t})$ is the beginning of period net foreign asset position (NFA), after the borrower's default decision is made, and $[q_{2,t} B_{2,1,t+1} - q_{1,t} B_{1,2,t+1} - q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})]$ is the end-of-period NFA position minus net factor payments ($NFP_{j,t}$), implicit in the fact that the bonds have a zero coupon so that the final holdings of each bond are discounted by the corresponding yield (i.e., the implied interest payment is netted out). Hence, $NX_{j,t} = CA_{j,t} - NFP_{j,t}$.

The above expressions are useful for quantifying the effects of the parameter changes we study on international trade and financial flows. In addition, they can be used to calculate gross and net foreign asset positions for each country.

G Sensitivity to the cost of borrowing, η

In this section we conduct a sensitivity analysis with respect to the parameter η. The parameter determines the elasticity with which the cost of borrowing increases with debt. In all simulations presented in the paper, we used the value of

$\eta = 0.1$. We now show how the results change when we double the value of this parameter, that is, we set $\eta = 0.2$.

After changing η, we repeat all quantitative exercises, including the construction of the parameters z_j, ϕ_j and κ_j to replicate the same empirical targets (domestic credit, NFA and interest rate).

Figure 10 plots the volatility measure when $\eta = 0.2$. The corresponding plots for the baseline model with $\eta = 0.1$ were shown in Figure 8. Both graphs use the same scale so they are easily comparable.

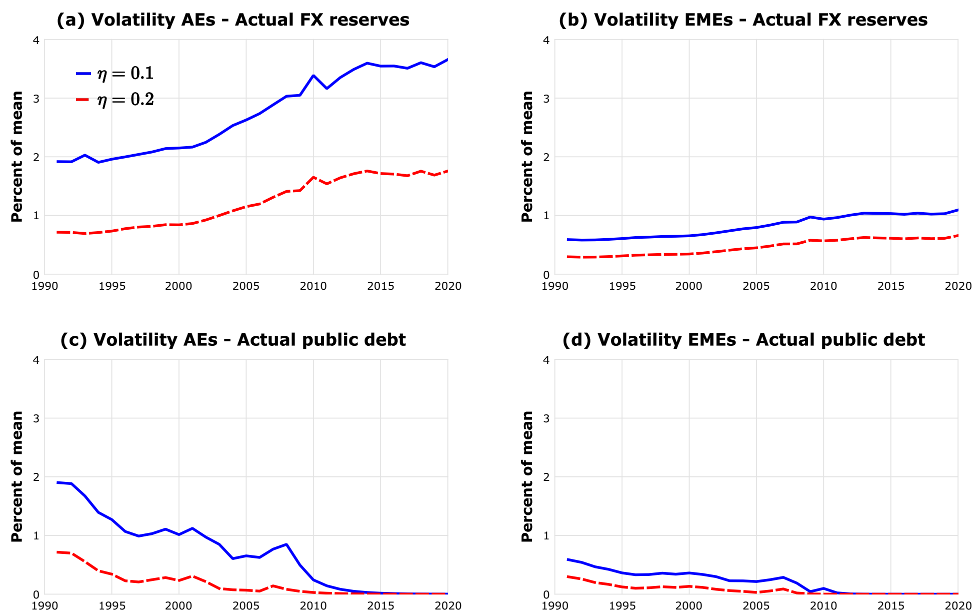

Figure 10: Sensitivity to cost of borrowing parameter η in Advanced Economies.

With a higher value of η, the cost of borrowing increases more rapidly with the stock of debt, and leverage responds less to the increase in reserves and public debt. As a result, the increase in output volatility is smaller. Qualitatively, however, the predictions of the model do not change. The impacts of the growth in FX reserves and public debt (difference between dashed and continuous lines) are smaller in absolute value but the proportional changes are similar.

H Welfare gains with bailout

Table 4: Welfare gains going from Scenario I to Scenario II with bailout. Gains are in percentage of consumption in Scenario I.

(a) Bailout with $\alpha = 0.1$

	AEs		EMEs	
	Hous.	*Entr.*	*Hous.*	*Entr.*
Impact FX Reserves	0.34	-1.67	-0.75	0.08
Scenario I: Constant Res. & Debt				
Scenario II: Actual Res & constant Debt				
Impact Public Debt	0.02	0.18	2.11	-0.29
Scenario I: Constant Res & Debt				
Scenario II: Constant Res & actual Debt				
Impact FX Reserves & Public Debt	0.27	-0.56	1.70	-0.24
Scenario I: Constant Res & Debt				
Scenario II: Actual Res & Debt				
Impact FX Reserves (actual Public Debt)	0.25	-0.74	-0.40	0.05
Scenario I: Constant Res & actual Debt				
Scenario II: Actual Res & Debt				
Impact Public Debt (actual Reserves)	-0.07	1.12	2.47	-0.32
Scenario I: Actual Res & constant Debt				
Scenario II: Actual Res & Debt				

(b) Bailout with $\alpha = 0.3$

	AEs		EMEs	
	Hous.	*Entr.*	*Hous.*	*Entr.*
Impact FX Reserves	0.36	-1.57	-0.77	0.06
Scenario I: Constant Res. & Debt				
Scenario II: Actual Res & constant Debt				
Impact Public Debt	0.00	0.13	2.13	-0.29
Scenario I: Constant Res & Debt				
Scenario II: Constant Res & actual Debt				
Impact FX Reserves & Public Debt	0.26	-0.60	1.72	-0.24
Scenario I: Constant Res & Debt				
Scenario II: Actual Res & Debt				
Impact FX Reserves (actual Public Debt)	0.25	-0.73	-0.40	0.05
Scenario I: Constant Res & actual Debt				
Scenario II: Actual Res & Debt				
Impact Public Debt (actual Reserves)	-0.10	0.98	2.51	-0.30
Scenario I: Actual Res & constant Debt				
Scenario II: Actual Res & Debt				

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