Narrative Sign Restrictions: A Premier

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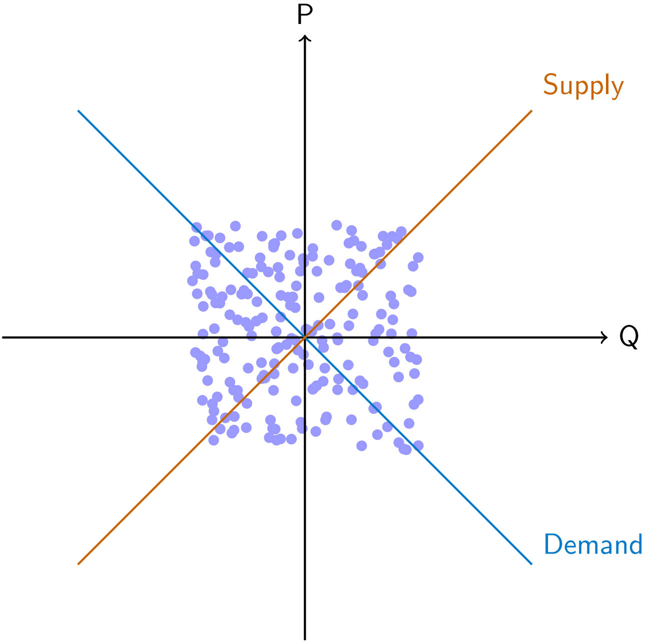
VAR: Structural Representation

Consider the structural vector autoregression (SVAR) with the general form

$$\mathbf{A}_0'\mathbf{y}_t = \sum_{\ell=1}^{p} \mathbf{A}_\ell'\mathbf{y}_{t-\ell} + \mathbf{d} + \varepsilon_t \quad \text{for } 1 \leq t \leq T,$$

- $\mathbf{y}_t$ is an $n \times 1$ vector of variables.
- $\mathbf{A}_\ell$ is an $n \times n$ matrix of parameters for $0 \leq \ell \leq p$.
- $\mathbf{A}_0$ invertible.
- $\mathbf{d}$ is a $1 \times n$ vector of parameters, p is the lag length.
- T is the sample size.
- The vector of structural shocks ε_t is Gaussian with mean zero and covariance matrix $\mathbf{I}_n$, the $n \times n$ identity matrix.


P
Supply
Q
Demand

VAR: reduced-form representation

Write the VAR in the reduced-form representation

$$\mathbf{y}_t = \sum_{\ell=1}^{p} \mathbf{B}'_\ell \mathbf{y}_{t-\ell} + \mathbf{c} + \boldsymbol{u}_t \text{ for } 1 \leq t \leq T,$$

- $\mathbf{B}_\ell = \mathbf{A}_\ell \mathbf{A}_0^{-1}$ is an $n \times n$ matrix of parameters.
- $\mathbf{u}_t = \mathbf{A}_0^{-1} \varepsilon_t$ are the reduced-form innovations.
- $\mathbb{E}\left[\mathbf{u}_t \mathbf{u}'_t\right] = \boldsymbol{\Sigma} = \left(\mathbf{A}_0 \mathbf{A}'_0\right)^{-1}$ is the covariance of innovations.
- The shocks ε_t are orthogonal and economic interpretation.
- The innovations $\mathbf{u}_t$ are, in general, correlated and no interpretation.

VAR: A Simple Supply and Demand Model

- Consider a system without lags:

$$\begin{aligned} Q_t &= \alpha P_t + \varepsilon_t^D, \\ P_t &= \beta Q_t + \varepsilon_t^S. \end{aligned}$$

- Then:

$$\underbrace{(P_t, Q_t)}_{\mathbf{y}_t'} \underbrace{\begin{pmatrix} -\alpha & 1 \\ 1 & -\beta \end{pmatrix}}_{\mathbf{A}_0} = \underbrace{\left(\varepsilon_t^D, \varepsilon_t^S\right)}_{\varepsilon_t'} \text{ for all } 1 < t < T.$$

- The IRF:

$$\mathbf{L}_0 = (\mathbf{A}_0^{-1})'$$

The Identification Problem

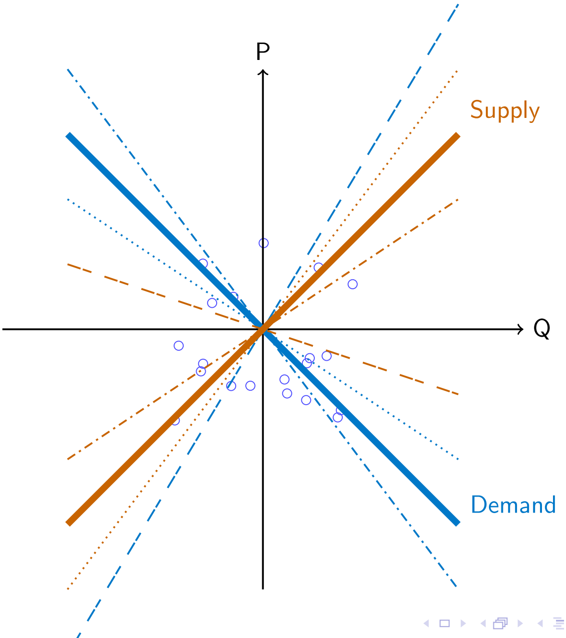
- Then:

$$(P_t, Q_t) = \left(\varepsilon_t^D, \varepsilon_t^S\right) \underbrace{\frac{\begin{pmatrix} -\beta & -1 \\ -1 & -\alpha \end{pmatrix}}{\beta\alpha - 1}}_{\mathbf{A}_0^{-1}} = \boldsymbol{u}_t'$$

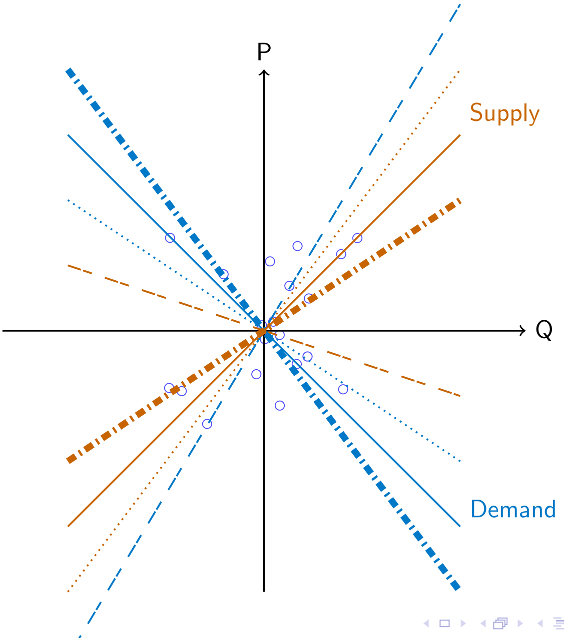
- Data $\rightarrow$ $\boldsymbol{\Sigma}$ (1 free parameters) $\rightarrow$ $\mathbf{A}_0$ (2 free parameters)

- Additional identifying assumptions required.

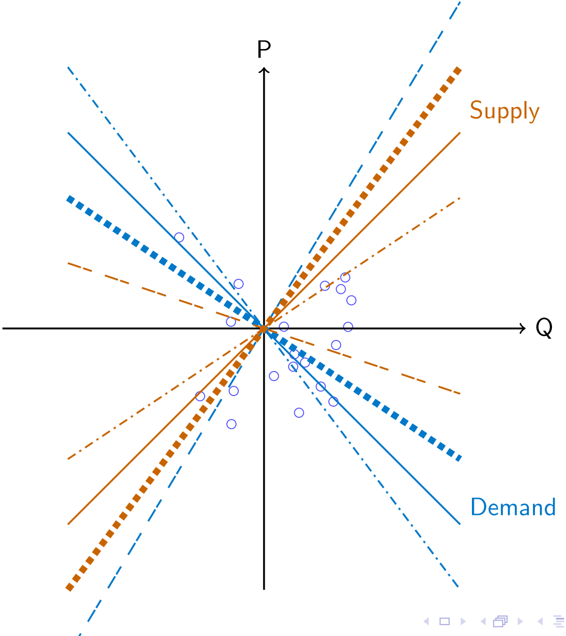
The Identification Problem



The Identification Problem

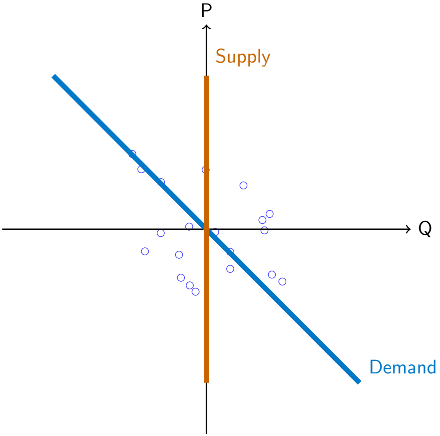


The Identification Problem



Traditional Zero Restrictions

- A *perfectly inelastic supply curve* uniquely identifies the model.
- Problem: Not always justifiable a priori.



Criticism of Zero Restrictions

- In recent years zero restrictions have come under attack.
- Often researchers use zero impact restrictions simply because they are easy to implement via Choleski decomposition, not because there is a strong theoretical motivation for imposing zeros.
- Zero-impact restrictions awkward when VAR contains financial prices. Zero-long-run restrictions usually not very robust.
- Alternative: *Sign Restrictions*.

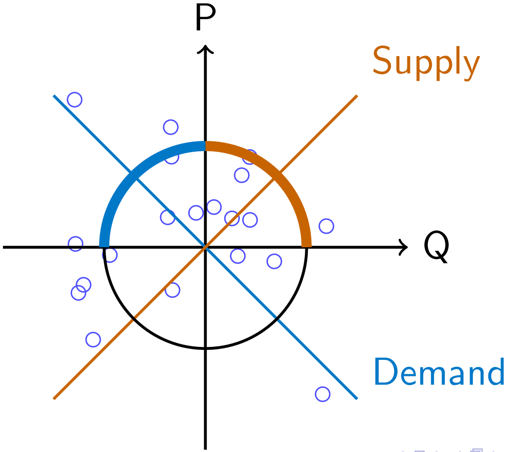
Identification with Traditional Sign Restrictions

- Based on a handful of uncontroversial sign restrictions on either the IRFs or the structural parameters.
- Likely to be agreed upon by a majority of researchers.
- Robust across the set of SVARs that satisfy the restrictions.

Traditional Sign Restrictions

- Traditional Sign Restrictions:

$$\mathbf{L}_0 = \mathbf{A}_0^{-1} = \begin{pmatrix} + & + \\ + & - \end{pmatrix} \qquad \text{or} \quad \beta < 0 \quad \text{and} \quad \alpha > 0.$$



Problems with Traditional Sign Restrictions

- The small number of Traditional Sign Restrictions results in a large set of structural parameters with very different implications.
- Best case: difficult to arrive at meaningful conclusions.
- Worst case: retain in the admissible set structural parameters with implausible implications.

Problems with Traditional Sign Restrictions

- The small number of Traditional Sign Restrictions results in a large set of structural parameters with very different implications
- Best case: difficult to arrive at meaningful conclusions
- Worst case: retain in the admissible set structural parameters with implausible implications
- Challenge: find additional uncontentious sign restrictions that shrink the set of admissible structural parameters.

Understanding: Narrative Sign Restrictions

- Suppose that historical sources tell us something about what happened that particular month.
- Example: In August 1990 there was a positive Supply shock.
- Given data, shocks are a function of structural parameters:

$$\varepsilon_t^D = Q_t - \alpha P_t,$$
$$\varepsilon_t^S = P_t - \beta Q_t \quad \text{for all } 1 < t < T.$$

- The Narrative Sign Restriction:

$$\varepsilon_{t_1}^S > 0 \quad \text{for } t_1 = 1990M8.$$

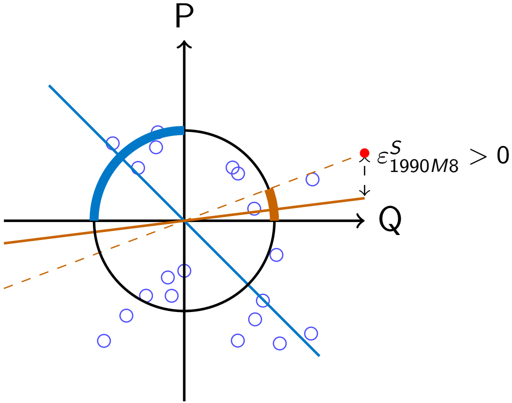
- Imposes the following restriction on elements of $\mathbf{A}_0$:

$$P_{t_1} - \beta Q_{t_1} > 0 \rightarrow \beta < \frac{P_{t_1}}{Q_{t_1}}$$

- Notice that Q_{t_1} and P_{t_1} is given.

Let's Look at the Data Again

- The restriction $\varepsilon^{S}_{1990M8} > 0$ restricts the set of admissible β!



Understanding: Narrative Sign Restrictions

- Example: In August 1990 the Supply shock was the most important contributor to the unexpected movement of prices.
- The contribution of the S shock to the unexpected change in the P for the period t_2:
- The Narrative Sign Restriction:

$$|l_{0,P,S}\varepsilon_{t_2}^{S}| > |l_{0,P,D}\varepsilon_{t_2}^{D}|$$

- Imposes the following restriction on elements of $\mathbf{A}_0$:

$$|\frac{-\beta\left(P_{t_2} - \beta Q_{t_2}\right)}{\beta\alpha - 1}| > |\frac{-\left(Q_{t_2} - \alpha P_{t_2}\right)}{\beta\alpha - 1}|$$

- The restriction jointly restricts the set of admissible β and α!