Strike while the Iron is Hot:

Optimal Monetary Policy with a Nonlinear Phillips Curve

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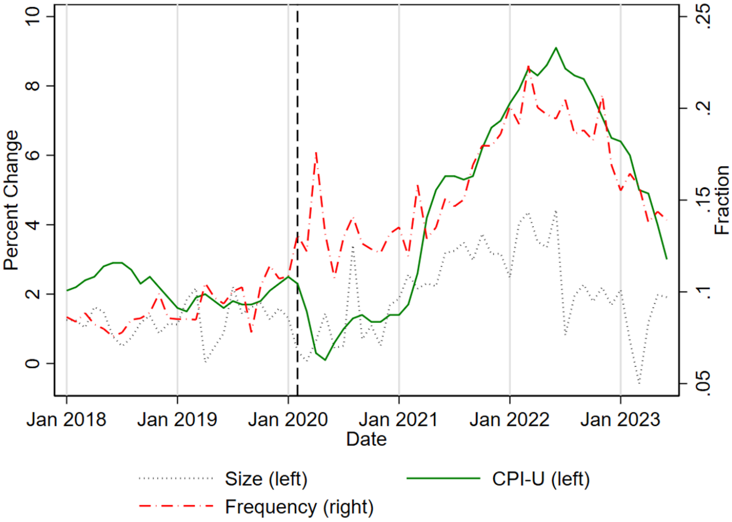
The views herein are those of the authors only, and do not necessarily reflect the views of the European Central Bank, the Bank of Spain or the Central Bank of Chile.

Motivation

- The recent inflation surge featured
  - Increase in the frequency of price changes (Montag and Villar, 2023) US
  - Increase in Phillips curve slope (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023) US
- Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant (Galí, 2008; Woodford, 2003)
- What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

CPI and frequency of price changes in the US, Montag and Villar (2023)



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What do we do?

- We use the standard state-dependent pricing model of Golosov and Lucas (2007)
  - Calibrate the model to match frequency and size of price changes in SS

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- Positive analysis under a Taylor rule
- Normative analysis: Ramsey optimal policy
  - Optimal long-run inflation
  - Characterize optimal responses to shocks

What we find

- Positive analysis:
  - The Phillips curve is non-linear: it gets steeper as frequency increases.

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  - When cost-push shocks are small, business as usual.
  - When cost-push shocks are large, more *hawkish* policy: "strike while the iron is hot."

What we find

- Positive analysis:
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- Normative analysis:
  - When cost-push shocks are small, business as usual.
  - When cost-push shocks are large, more *hawkish* policy: "strike while the iron is hot."
  - Divine coincidence holds for efficient shocks, either small or large.
  - Optimal long-run inflation is slightly positive.
  - The time-inconsistency problem is there, but weakened relative to standard framework.

Literature

- Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)
  - Microfounded by state-dependent price setting

    (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)
  - In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)

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  - In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)
- Optimal policy in a menu cost economy
  - Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)
  - Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)
  - Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of (our version of) the Golosov-Lucas model

= Textbook, Discrete-time New-Keynesian model with Calvo pricing (e.g. Galí, 2008)

– Calvo fairy [Calvoplus also includes this component]

+ fixed costs of price adjustments η

+ stochastic, idiosyncratic product quality $A_t(i)$

= Heterogeneous-firm NK DSGE model.

Sketch of the model

- Households consume a Dixit and Stiglitz (1977) basket of goods, work and save.
- Per-period utility of consumption is log and disutility of labor is linear.
- Idiosyncratic quality $A_t(i)$ implies that

$$C_t = \left\{ \int [A_t(i) C_t(i)]^{\frac{\epsilon - 1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon - 1}}.$$

- Monopolistic producers with $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, A_t is aggregate productivity.
- Firms face a fixed cost in labor units η to update prices and an employment subsidy τ_t.

Pricing decision

- Define $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log real price.
- Define $\lambda_t(p)$ be the price-adjustment probability. Value function is

$$\begin{aligned} V_t(p) &= \Pi(p, w_t, A_t, A_t(i), \tau_t) \\ &+ \mathbb{E}_t\left[(1-\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}))\Lambda_{t,t+1}V_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\right] \\ &+ \mathbb{E}_t\left[\lambda_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}(p') - \eta w_{t+1}\right)\right]. \end{aligned}$$

- The price adjustment probability is characterized by a (s,S) rule:

$$\lambda_t(p) = I[\max_{p'} V_t(p') - \eta w_t > V_t(p)].$$

Monetary Policy and shocks processes

- For positive analysis only, monetary policy follows a Taylor rule:

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

Aggregation and market clearing

- Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p) dp,$$

- Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p) dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

where $g_t(p)$ is endogenous object.

The model in one slide

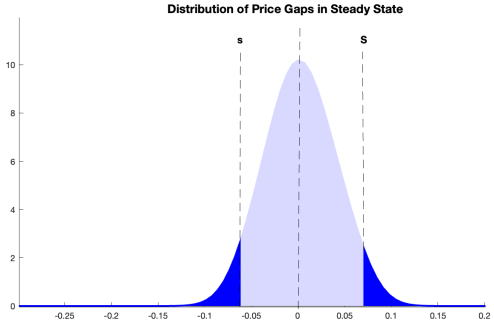
$$\max_{\left\{g_t^c(\cdot),g_t^0,V_t(\cdot),C_t,w_t,p_t^*,s_t,S_t,\pi_t^*\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \upsilon \frac{C_t}{A_t} \left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c\left(p\right) dx + g_t^0 e^{(p_t^*)(-\epsilon)} \right) - \upsilon\eta g_t^0 \right)$$

subject to

$$\begin{aligned}
1 &= \int e^{(x+p_t^*)(1-\epsilon)} g_t^c\left(x\right) dx + g_t^0 e^{(p_t^*)(1-\epsilon)}, \\
0 &= \Pi_t'(x) + \frac{1}{\sigma}\Lambda_{t,t+1} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial \phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x} dx' \\
&+ \Lambda_{t,t+1} \left(\phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right) \right) \left(V_{t+1}(0) - \eta w_{t+1}\right), \\
V_t\left(s_t\right) &= V_t\left(0\right) - \eta w_t, \\
V_t\left(S_t\right) &= V_t\left(0\right) - \eta w_t, \\
w_t &= \upsilon C_t^{\gamma}, \\
V_t(x) &= \Pi(x, p_t^*, w_t, A_t) + \Lambda_{t,t+1} \frac{1}{\sigma} \int_{s_t}^{S_t} \left[V_{t+1}(x') \phi\left(\frac{(x-x') - \pi_{t+1}^*}{\sigma} \right) \right] dx' \\
&+ \Lambda_{t,t+1} \left(1 - \frac{1}{\sigma} \int_{s_t}^{S_t} \left[\phi\left(\frac{(x-x') - \pi_{t+1}^*}{\sigma} \right) \right] dx' \right) \left[\left(V_{t+1}\left(0\right) - \eta w_{t+1} \right) \right], \\
g_t^c(x) &= \frac{1}{\sigma} \int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1}) \phi\left(\frac{x_{-1} - x - \pi_t^*}{\sigma} \right) dx_{-1} + g_{t-1}^0 \phi\left(\frac{-x - \pi_t^*}{\sigma} \right), \\
g_t^0 &= 1 - \int_{s_t}^{S_t} g_t^c(x) dx.
\end{aligned}$$

Model: Intuitive summary

- Each period, firm i chooses whether to reset its price and, if so, what new price to set
- The firm's optimality conditions define the reset price and the inaction region (S,s)



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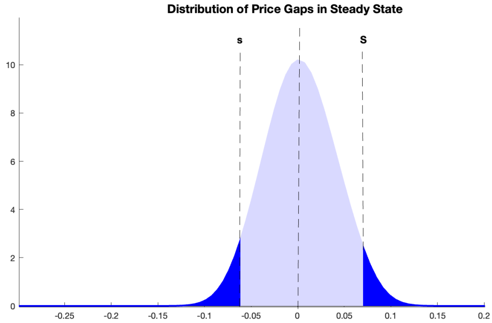

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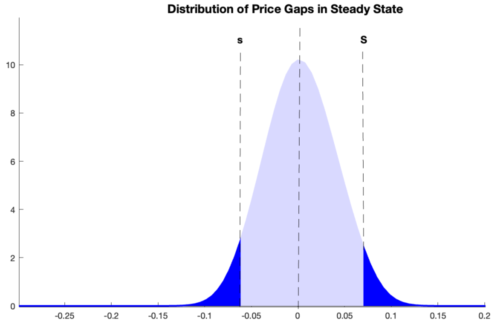
- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price



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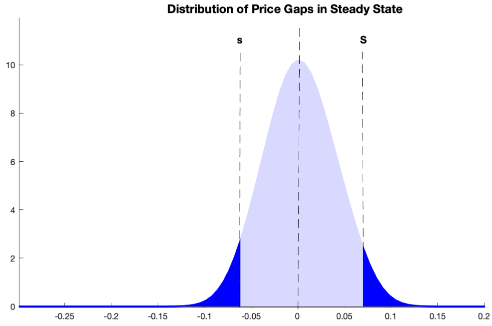
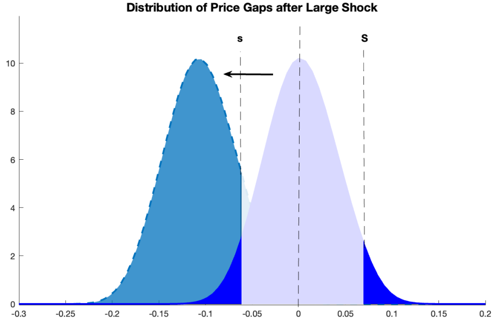
- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price
- Let $x_t(i) \equiv p_t(i) - p_t^*(i)$ be the difference of that price from the optimal price



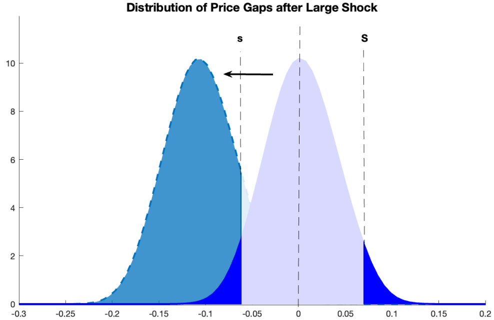
Model under large shock

- Large aggregate shock: shifts the distribution of price gaps for all firms
- Limited impact on the (s,S) bands



Model under large shock

- Large aggregate shock: shifts the distribution of price gaps for all firms
- Limited impact on the (s,S) bands
- Pushes a large fraction of firms outside of the inaction region
- Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (on top of "selection")

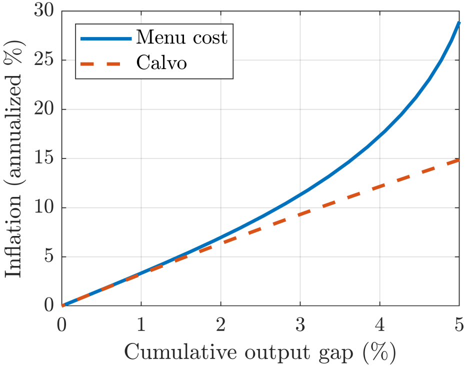


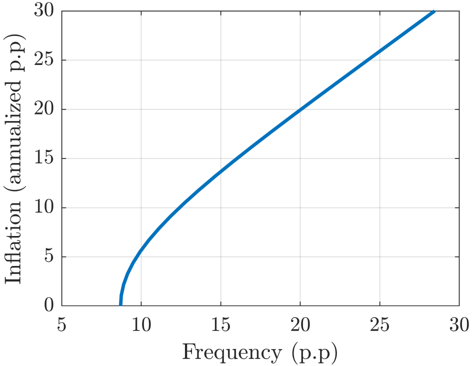
Calibration

Households			
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
υ	1	Utility weight on labor	Set so that $w = C$
Price setting targets			
Frequency	8.7%	Frequency of price changes	Nakamura and Steinsson (2008)
Size	8.5%	Absolute size of price changes	Nakamura and Steinsson (2008)
Monetary policy			
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.125	Output coefficient in Taylor rule	Taylor (1993)
Shocks			
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.25^{1/3}$	Persistence of the cost-push shock	

Main positive result: Non-linear Phillips curve

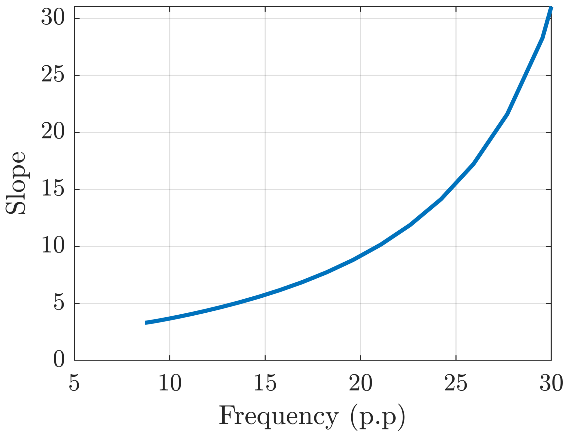
Small shocks: like *adjusted* Calvo; large shocks: non-linear. more





Corollary: State-dependent monetary policy

- P.C. slope determines the sacrifice ratio: the relative impact on inflation versus output gap of a marginal monetary policy tightening.
- Key: state-dependent monetary policy effects.



Normative analysis: Computation

- Challenges
  - Price distribution $g_t(p_t)$ and value function $V_t(p_t)$ are infinite-dimensional objects
  - We need sufficient accuracy for optimal policy assessment
- New algorithm, in discrete time
  - Approximate distribution and value functions by piece-wise linear functions on grid.
  - Endogenous grid points: (S,s) bands and the optimal reset price.
  - Evaluate integrals analytically.
  - Solve non-linearly in the sequence space using Dynare's perfect foresight Ramsey solver.

Normative result 1: Optimal response to cost-push shocks is non-linear

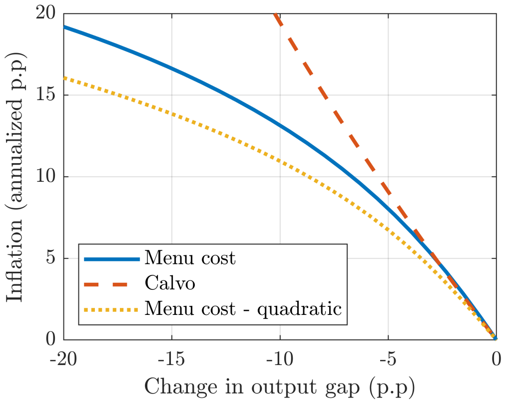
- In the textbook, LQ framework, optimal policy is a price-level targeting rule

$$\hat{p}_t = -\frac{1}{\epsilon}\tilde{y}_t^e$$

- For small cost-push shocks, optimal policy in the menu cost model is about the same.
- For large cost-push shock, strike while the iron is hot!

Nonlinear targeting rule

- Globally, the target rule is nonlinear Robust



Nonlinear targeting rule

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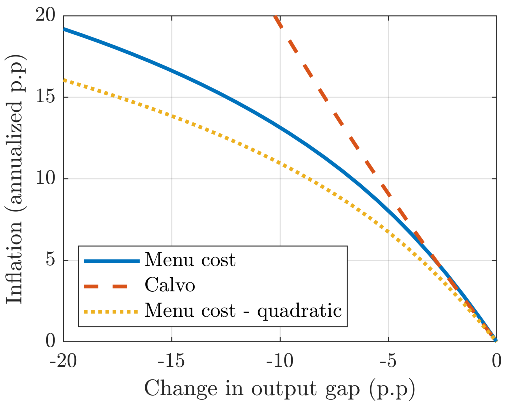
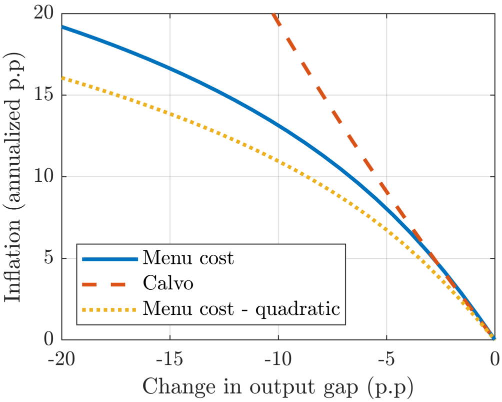
- After large shocks, the planner stabilizes inflation more relative to the output gap



Nonlinear targeting rule

- Globally, the target rule is nonlinear Robust
- After large shocks, the planner stabilizes inflation more relative to the output gap
- Why? Stabilizing inflation is cheaper due to the lower sacrifice ratio (higher freq.)
  - Similar results with quadratic objective
  - The nonlinearity of the targeting rule is due to the nonlinear Phillips curve



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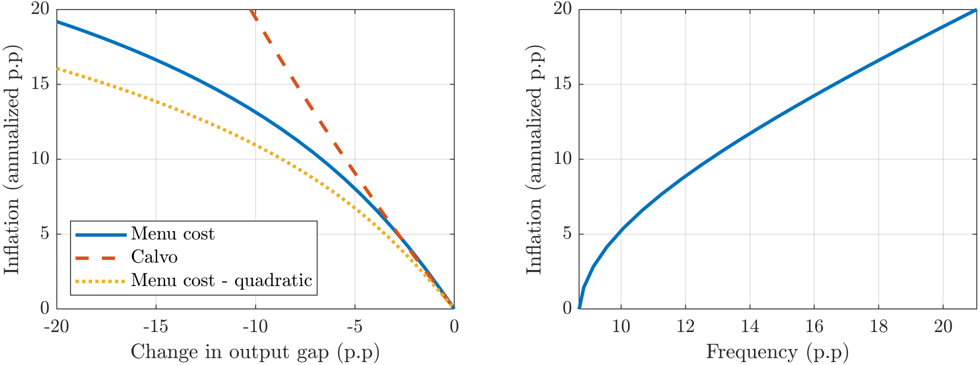
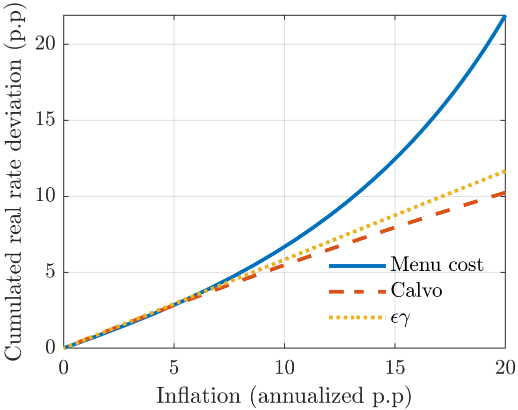
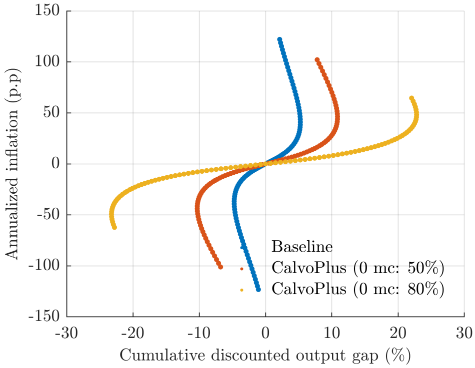


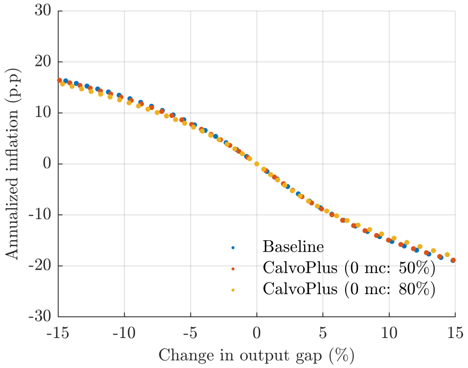
Nonlinear targeting rule for the real interest rate



Normative result 1.1: Calvoplus

Calvo plus: very different Phillips curve slope, almost the same optimal monetary policy.





Normative result 2: “Divine coincidence” holds

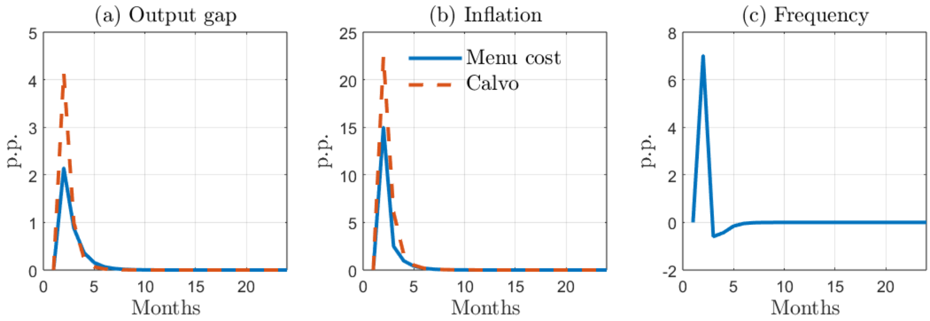
- In the standard NK model with Calvo pricing: divine coincidence holds after shocks affecting the efficient allocation: TFP (A_t) [also true for a discount rate shock].
- Optimal policy stabilizes inflation and closes the output gap.
- Same result holds in menu-cost models, regardless shocks are small or large.

Normative result 3: Optimal long-run inflation rate

- The steady-state Ramsey inflation rate is slightly above zero: $\pi^* = 0.3\%$
- Why not zero?
  - Asymmetric profit function: negative price gaps more harmful $=>$ Asymmetric (S,s) bands.
  - At zero inflation, more mass around the lower than higher threshold.
  - Slightly positive inflation raises p^* and pushes the mass of firms upwards.
  - $=>$ Lower frequency $=>$ less waste of resources paying for the menu cost.

Normative result 4: Time inconsistency is weakened by endogenous frequency

- Optimal policy without precommitment (time-0)
- Inefficient steady state
- Weaker time inconsistency in GL than in Calvo: costlier to increase output gap

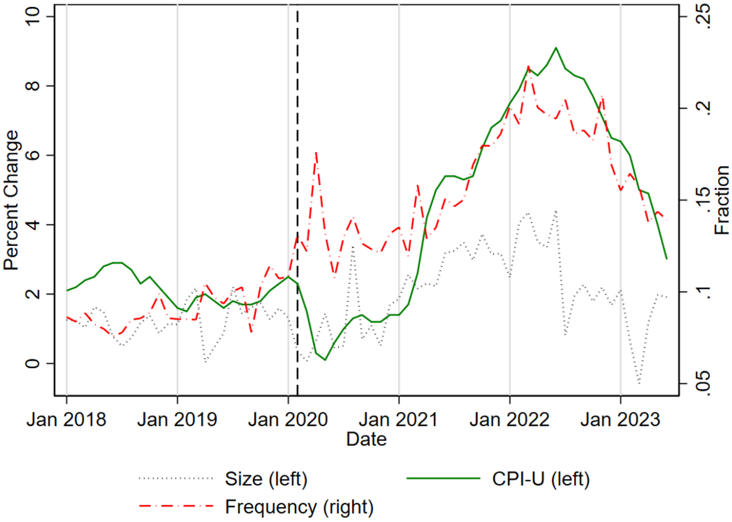


Conclusion

We study optimal policy in a menu cost model delivering a non-linear Phillips curve.

- Optimal response to small cost shocks similar to Calvo (1983).
- Lean against frequency for large cost-push shocks: strike while the iron is hot!
- Divine coincidence holds for efficient shocks, either small or large.
- Optimal long-run inflation is near zero.
- Time-inconsistency is there although weakened.

CPI and frequency of price changes in the US, Montag and Villar (2023)



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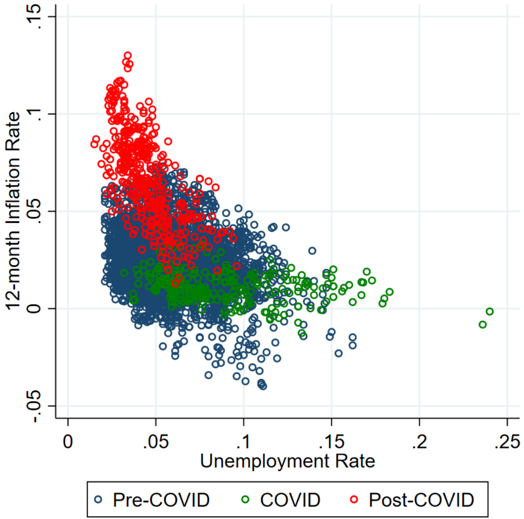
Phillips correlation across US cities, Cerrato and Gitti (2023)



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Modified Phillips correlation time, Benigno and Eggertsson (2023)

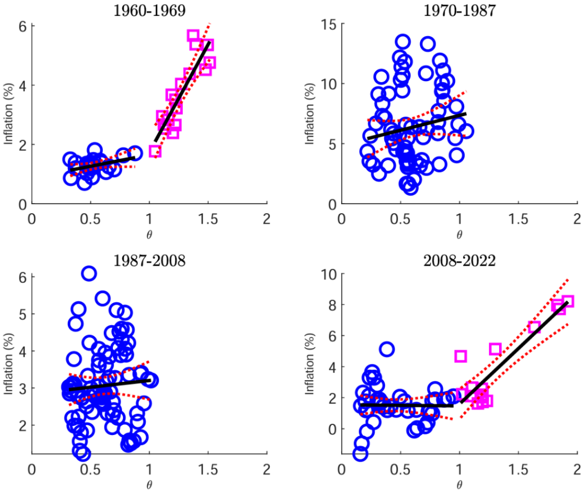
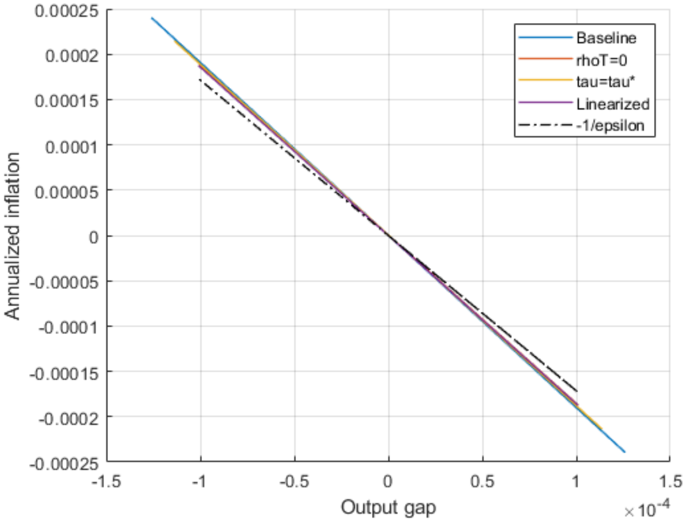


Figure 4: Inflation: CPI inflation rate at annual rates. θ: vacancy-to-unemployed ratio.

Slope of the target rule for small shocks



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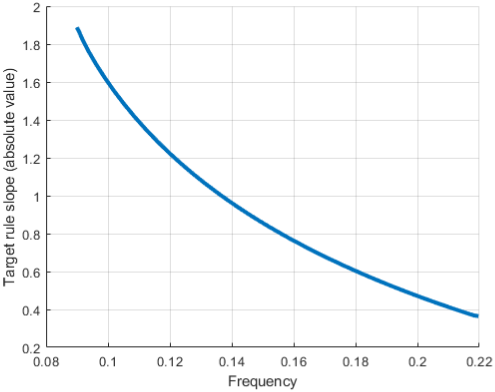
State-dependent inflation-output tradeoff

- Inflation-output tradeoff varies with frequency



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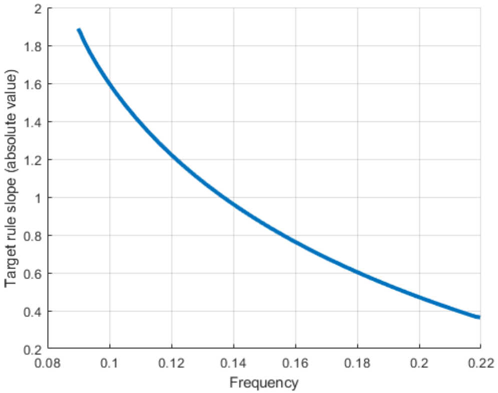
State-dependent inflation-output tradeoff

- Inflation-output tradeoff varies with frequency
- After large shocks, the planner stabilizes inflation relative to the output gap on the margin more Analogy with Calvo, 1983



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State-dependent inflation-output tradeoff

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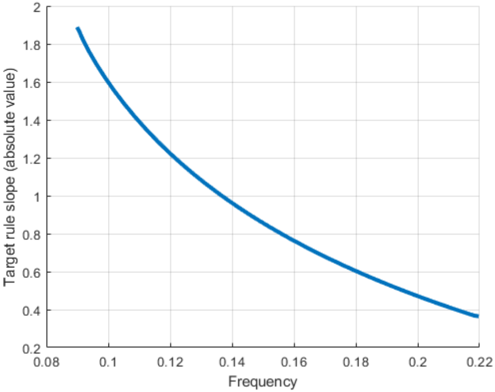
- Reduction in sacrifice ratio dominates decline in relative welfare weight of inflation



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Frequency and optimal policy in Calvo (1983)

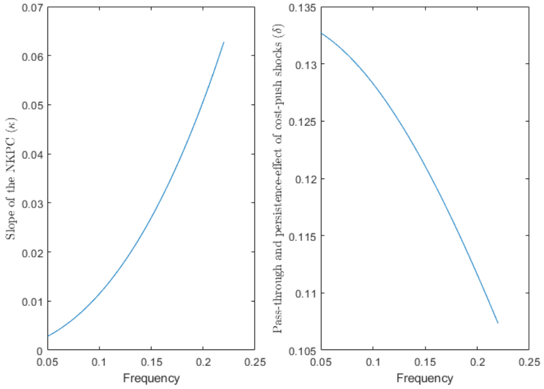
- Optimal response to an iid cost-push shock (u_t)

$$\hat{p}_t = \delta \hat{p}_{t-1} + \delta u_t$$
$$x_t = \delta x_{t-1} + \delta \epsilon u_t,$$

where $\hat{p}_t \equiv p_t - p_{-1}$ is the change in the price level and x_t is the output gap

- Parameter δ decreasing in frequency

- Reduction in sacrifice ratio dominates



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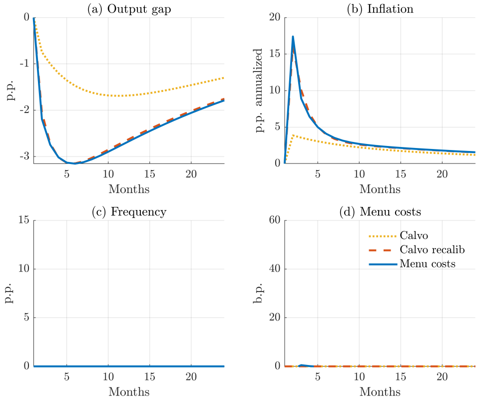
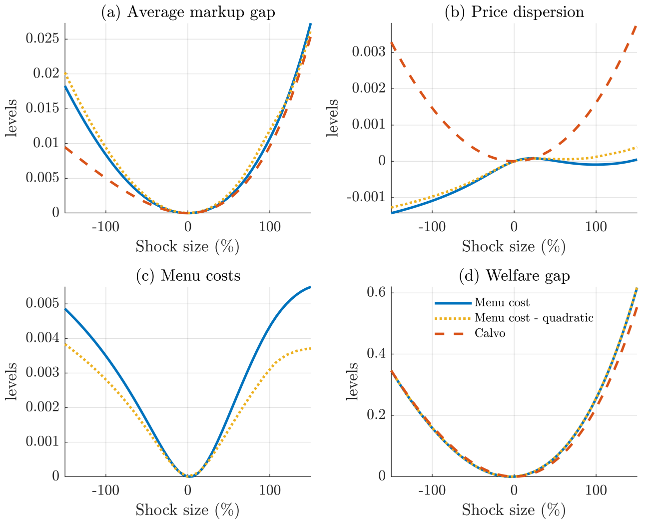
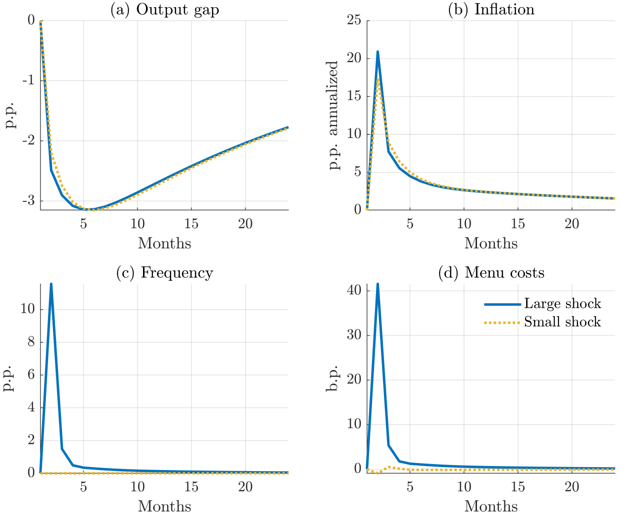
Response to a cost-push shock under a TR (Calvo vs. Golosov-Lucas)



Welfare decomposition

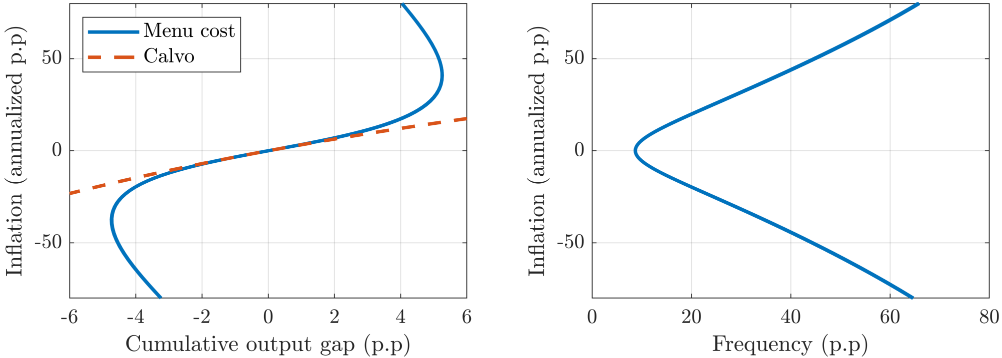


Response to a cost-push shock (large vs. small shock in Golosov-Lucas)



Main positive result: Non-linear Phillips curve

Small shocks: like *adjusted* Calvo; large shocks: non-linear, even bending backwards.



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