Fiscal Inflation in the UK

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Fiscal policy and real interest rates

- We study inflation and output dynamics in the United Kingdom
- We employ a model with partially unfunded government debt (Bianchi, Faccini, and Melosi, QJE 2023)
  1. At any given point in time, part of the outstanding government debt is unfunded
  2. Unfunded debt is not backed by future fiscal adjustments $\Rightarrow$ Inflationary pressure accommodated by the central bank
- Debt stability achieved with a mix of fiscal adjustments and inflation
- With nominal rigidities, unfunded fiscal shocks cause persistent movements in inflation and in real interest rates $\rightarrow$ A fiscal theory of persistent inflation
- With respect to other shocks, policymakers follow a typical Monetary-led policy $\Rightarrow$ other shocks propagate as in standard business cycle models

Empirical analysis

We augment a TANK model with partially unfunded debt

Distinct implications of funded and unfunded shocks

1. Funded fiscal shocks: small **increase** in real interest rates
2. Unfunded fiscal shocks: large **decline** in real interest rates

Main results:

- Low-frequency movements in inflation driven by unfunded fiscal shocks
- **Pandemic**: Large fiscal stimulus and accommodative monetary policy
  1. Quick rebound in real activity counteracting adverse supply-side shocks
  2. Rapid increase in fiscal inflation
  3. While actual inflation has declined, fiscal inflation remains elevated as of 2024

Endowment economies

Fisherian model

Consider a linearized endowment economy:

$$\begin{aligned}
\hat{r}_{n,t} &= \mathbb{E}_t \hat{\pi}_{t+1}, && (1)\\
\hat{s}_{b,t} &= \beta^{-1}[\hat{s}_{b,t-1} + \hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\hat{\tau}_t], && (2)\\
\hat{r}_{n,t} &= \phi \hat{\pi}_t, && (3)\\
\hat{\tau}_t &= \gamma \hat{s}_{b,t-1} + \zeta_t. && (4)
\end{aligned}$$

Plugging the monetary rule into the Fisher equation leads to the monetary block:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t. \quad (5)$$

Combining the law of motion for debt with the fiscal rule yields the fiscal block:

$$\hat{s}_{b,t} = \beta^{-1}[1-(1-\beta)\gamma]\hat{s}_{b,t-1} + \beta^{-1}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\zeta_t\right]. \quad (6)$$

Equilibrium determinacy (Leeper 1991)

Two regions of the parameter space deliver a unique stationary solution (Leeper, 1991)

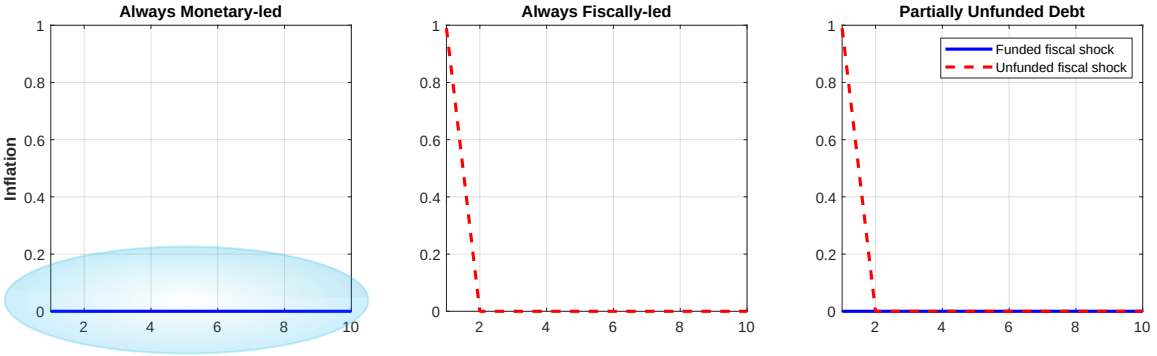
- Monetary-led policy mix: The fiscal authority is committed to implementing the necessary fiscal adjustments. Fiscal policy is passive ($\gamma > 1$) because it passively accommodates the behavior of the active monetary authority ($\phi > 1$).

  $\Rightarrow$ Inflation is insulated from the fiscal block.

- Fiscally-led policy mix: The fiscal authority is not committed to implementing the necessary fiscal adjustments. Monetary policy is passive ($\phi \leq 1$) because it passively accommodates the behavior of the active fiscal authority ($\gamma \leq 1$).

  $\Rightarrow$ Inflation is not insulated from the fiscal block.

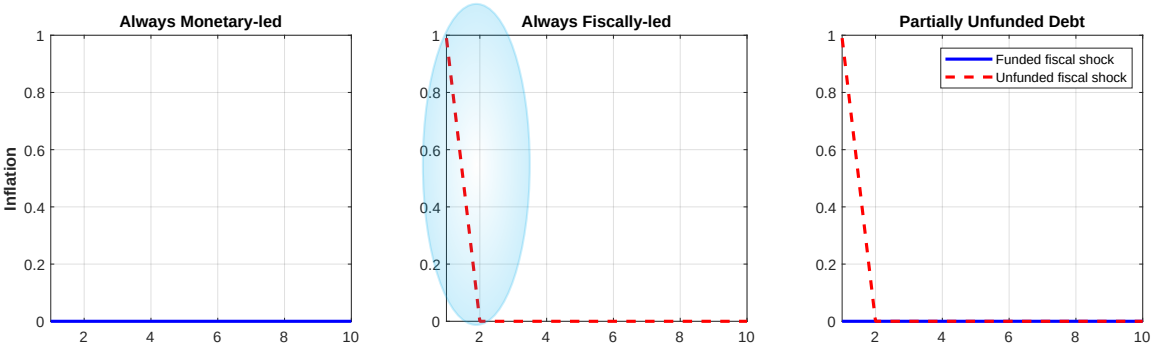
Inflation response to fiscal shocks



Impulse responses:

1. Inflation does not respond under the Monetary-led policy mix ($\phi = 2.0; \gamma = 0.2$)

Inflation response to fiscal shocks



Impulse responses:

2. Inflation responds under the Fiscally-led policy mix $(\phi = 0; \gamma = 0)$

Fisherian model with partially unfunded debt

We now introduce the notion of partially unfunded debt:

- We consider the following fiscal rule:

$$\hat{\tau}_t = \gamma^M \left(\hat{s}_{b,t-1} - \hat{s}^F_{b,t-1} \right) + \gamma^F \hat{s}^F_{b,t-1} + \zeta^M_t + \zeta^F_t . \tag{7}$$

  where ζ^M_t and ζ^F_t denote funded and unfunded fiscal shocks, respectively, and $\gamma^F < 1$, and $\gamma^M > 1$.

- The new monetary rule is:

$$\hat{r}_{n,t} = \phi^M \left(\hat{\pi}_t - \hat{\pi}^F_t \right) + \phi^F \hat{\pi}^F_t . \tag{8}$$

  where $\hat{\pi}^F_t$ denotes fiscal inflation, i.e., the amount of inflation that is tolerated by the central bank to stabilize the share of unfunded debt $\hat{s}^F_{b,t-1}$, $\phi^M > 1$ and $\phi^F \leq 1$.

Linearized model

- The monetary block:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F \right) + \phi^F \hat{\pi}_t^F .$$

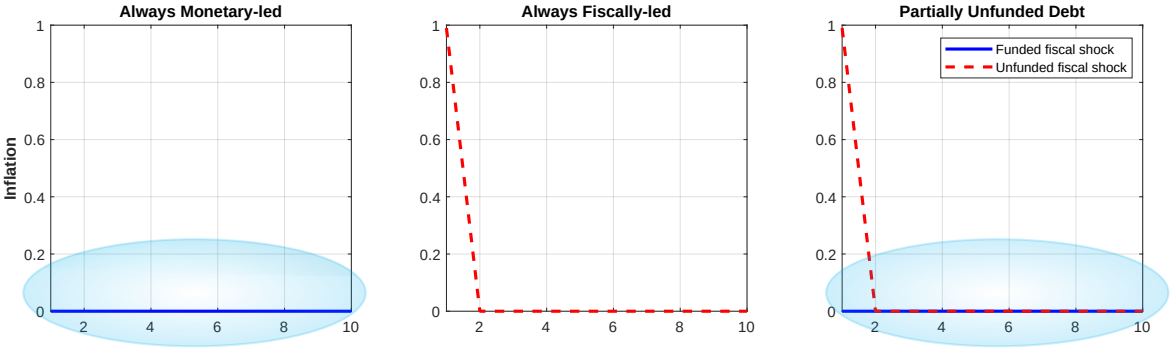
- The fiscal block ($\gamma_F = 0$)

$$\hat{s}_{b,t} = \beta^{-1}[1 - (1-\beta)\gamma^M]\hat{s}_{b,t-1} + \beta^{-1}[(1-\beta)\hat{s}^F_{b,t-1} + \hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)(\zeta_t^M + \zeta_t^F)]$$

- To close the model, we need to characterize the dynamics of fiscal inflation, $\hat{\pi}_t^F$, and of the associated amount of unfunded debt, $\hat{b}_t^F$.
- We construct a shadow economy in which the Fiscally-led policy mix is always in place and only shocks to unfunded spending ζ_t^F occur.

Subeconomies

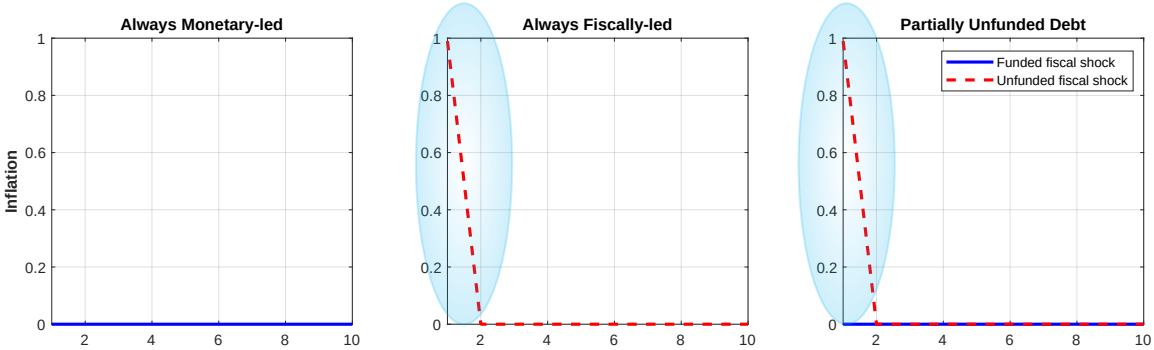
Inflation response to funded and unfunded fiscal shocks



Impulse responses $(\phi^M = 2.0; \gamma^M = 0.2)$; $(\phi^F = 0; \gamma^F = 0)$:

1. Inflation does not respond to a funded fiscal shock $(\phi = 2.0; \gamma = 0.2)$

Inflation response to funded and unfunded fiscal shocks



Impulse responses $(\phi^M = 2.0; \gamma^M = 0.2)$; $(\phi^F = 0; \gamma^F = 0)$:

2. Inflation responds to an unfunded fiscal shock $(\phi = 0; \gamma = 0)$

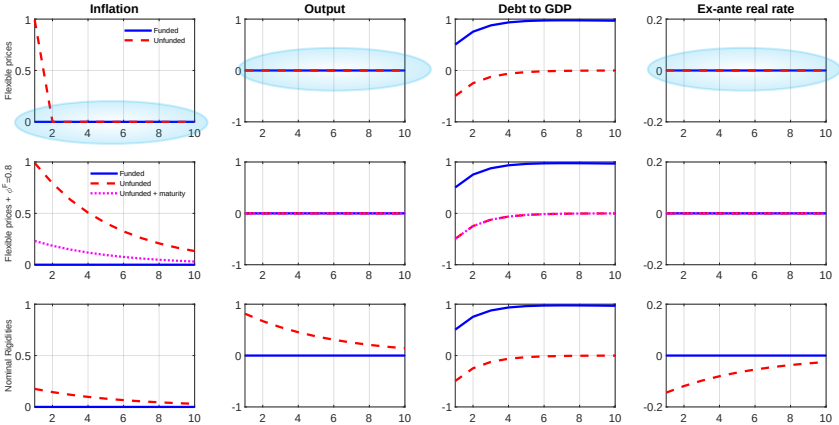
Production economies

Production economies

We now extend the analysis to a production economy.

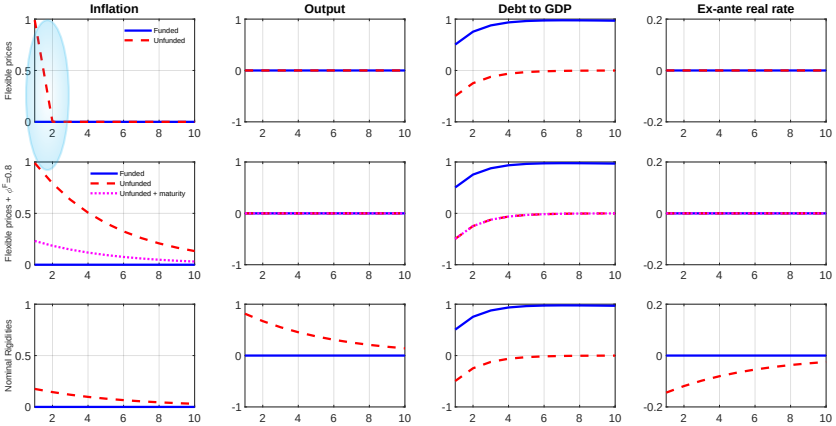
- Simple environment with no capital, but endogenous labor supply and production
- Two alternatives:
  1. Flexible prices ▸ equations
  2. Nominal rigidities ▸ equations
- Nominal rigidities and unfunded shocks deliver a fiscal theory of persistent inflation:
  1. Persistent movements in inflation
  2. Persistent movements in real interest rates
  3. Persistent movements in output (real effects)

Persistent Fiscal Inflation



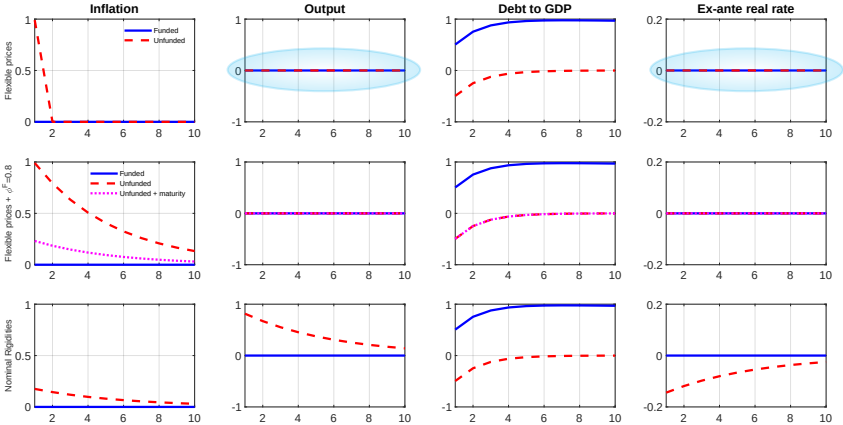
1. **Absent nominal rigidities**, macro-fiscal dichotomy holds for funded shocks

Persistent Fiscal Inflation



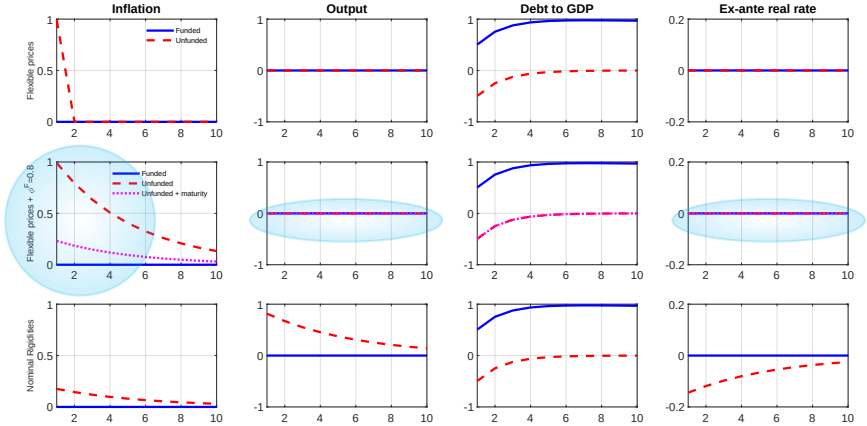
2. **Absent nominal rigidities**, price level increases after unfunded shocks as in the Fisherian model

Persistent Fiscal Inflation



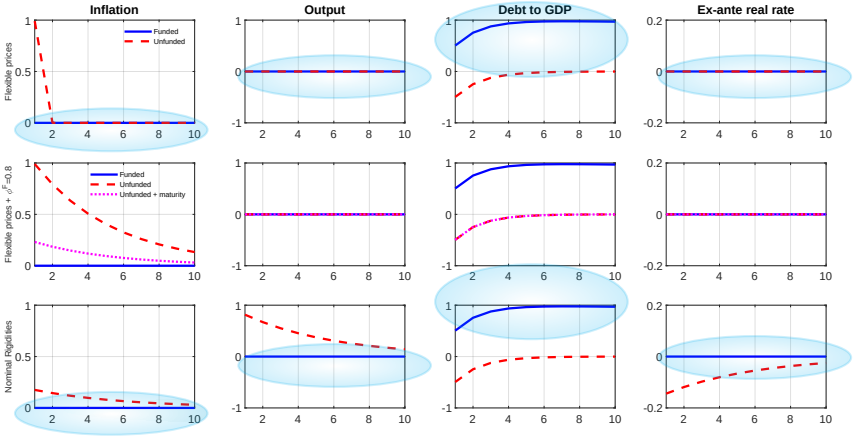
3. **Absent nominal rigidities**, real economy unaffected by unfunded shocks

Persistent Fiscal Inflation



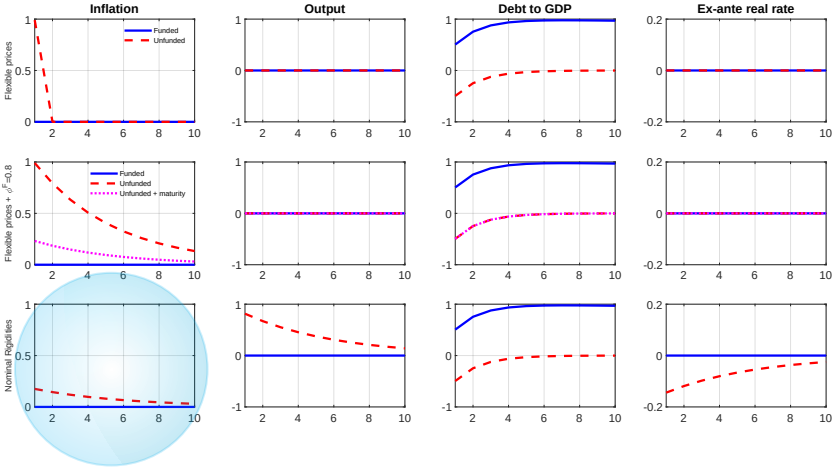
4. **With flexible prices and** $\phi_\pi^F > 0$, persistent inflation but no real effects in response to unfunded shocks

Persistent Fiscal Inflation



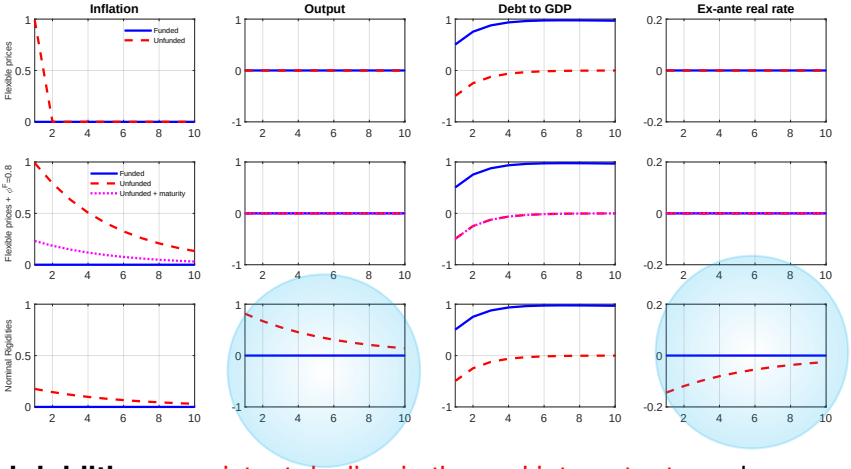
5 **Nominal rigidities:** No macro effects of funded shocks as in flex prices
→ macro-fiscal dichotomy

Persistent Fiscal Inflation



6 **Nominal rigidities:** persistent and moderate inflation response to unfunded shocks

Persistent Fiscal Inflation



- **Nominal rigidities:** persistent decline in the real interest rate and real effects of unfunded shocks

A Quantitative General Equilibrium Model

The Model

State-of-the-art TANK model

- Distortionary taxation on labor and capital income
- Price and wage rigidities
- Hand-to-mouth households
- Long-term government bonds
- **Typical set of business cycle shocks** plus **fiscal shocks** and a **shifter of the Phillips curve** capturing market and non policy forces such as globalization

▸ Equations

Unfunded debt and monetary and fiscal coordination

- Changes in transfers ($\zeta^M_{z,t}$, $\zeta^F_{z,t}$) and purchases ($\zeta^M_{g,t}$, $\zeta^F_{g,t}$) determine the share of funded and unfunded debt
- Funded debt $\tilde{b}^M_t$ is stabilized by fiscal instruments
- Unfunded debt $\tilde{b}^F_t$ is stabilized by fiscal inflation $\hat{\pi}^F_t$, which the monetary authority accommodates
- No fiscal response to unfunded debt ($\gamma^F = 0$)
- No monetary response to fiscal inflation ($\phi^F = 0$) $\rightarrow$ endogenous inflation target

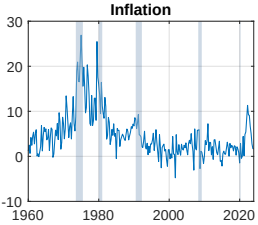
Empirical Analysis

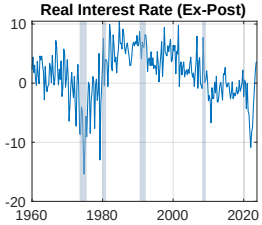
Estimation

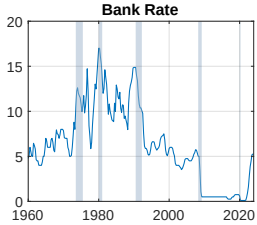
- The model is estimated using a data set of 20 macro and fiscal variables
  1. Real GDP growth
  2. Real consumption growth
  3. Real investment growth
  4. Hours worked
  5. Inflation (Household consumption deflator)
  6. Growth rate of real average weekly earnings
  7. Real transfers payments growth rate
  8. Real government consumption and investment growth rate
  9. Debt to GDP ratio
  10. Bank Rate

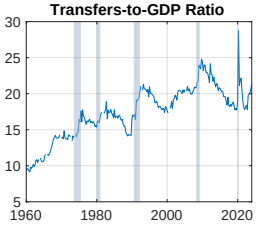
  11-20. 1Q-10Q ahead expected market path of the Bank Rate (OIS data)
- Sample periods: 1960q1-2007q4 and 2008q1-2024q1 Estimated parameters
- Second sample includes all the 20 observables; re-estimation of standard deviations of fiscal shocks and the factor model governing the forward guidance shocks (Campbell et al. 2012)

A look at the data

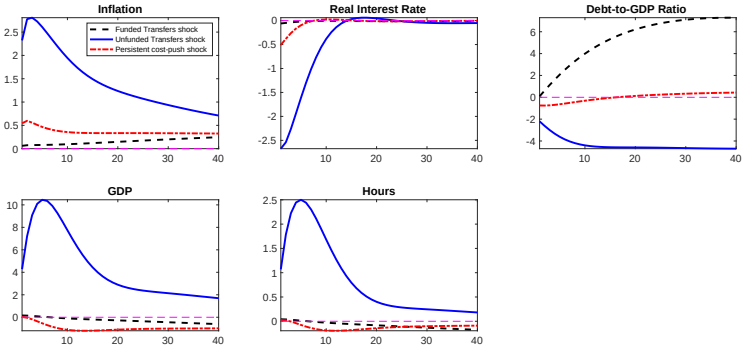






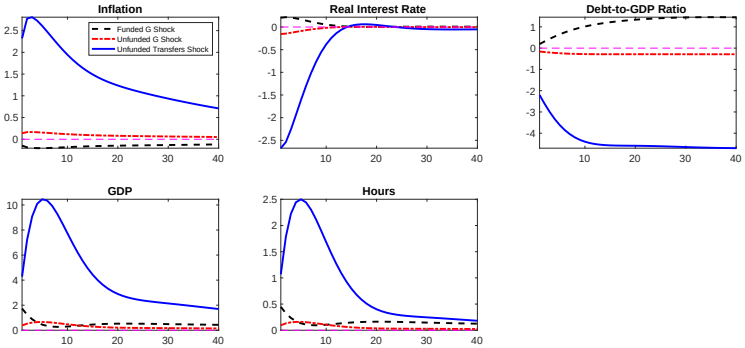


Funded and unfunded transfer shocks



- **Funded transfers**: Modest impact on the macroeconomy, debt increases
- **Unfunded transfers**: Persistent inflation increase, real rate and debt decline

Funded and unfunded G shocks



- **Funded G**: Modest impact on the macroeconomy, debt increases
- **Unfunded G**: Modest impact on inflation and real interest rate

Government transfers, purchases, and real interest rate

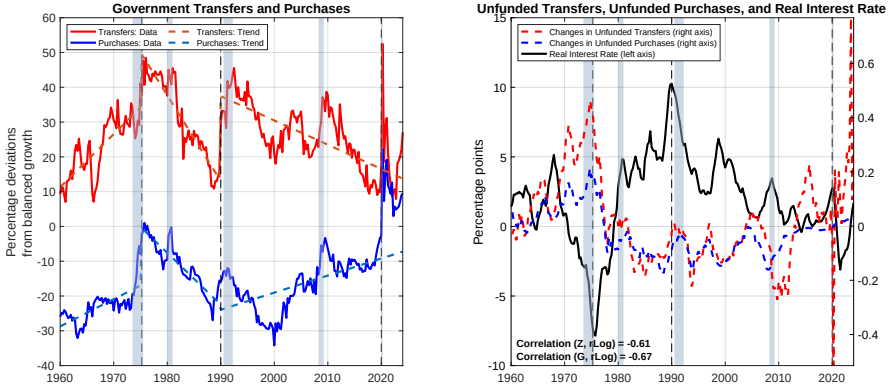


Figure VIII - Government Transfers, Purchases, and Real Interest Rate.

Decomposition of total government transfers

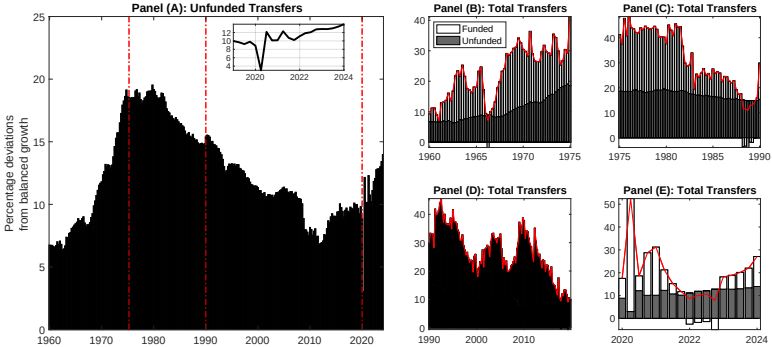


Figure IX.1 – Estimated Decomposition of Total Government Transfers into their Funded and Unfunded Components.

Decomposition of total government purchases

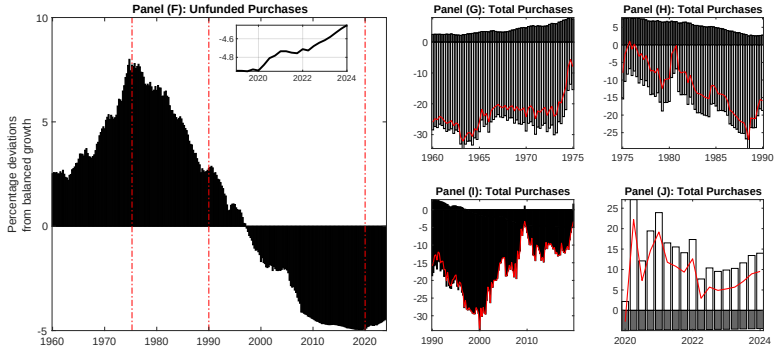


Figure IX.2 – Estimated Decomposition of Total Government Purchases into their Funded and Unfunded Components.

Drivers of inflation

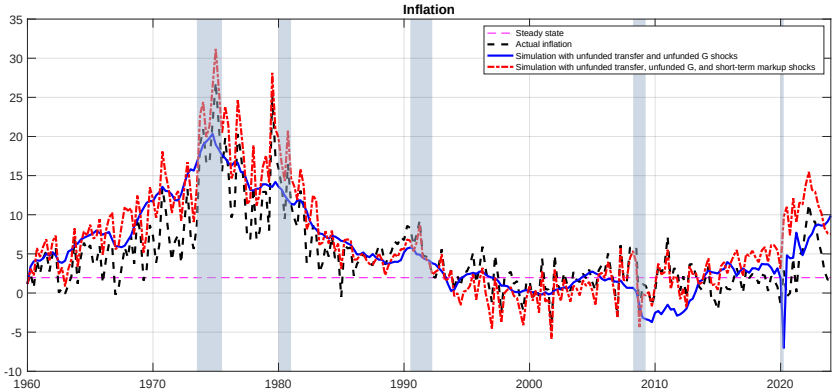


Figure X.1 – Drivers of Inflation

Fiscal inflation (solid blue line) explains low-frequency movements in **Inflation**

Drivers of hours gap

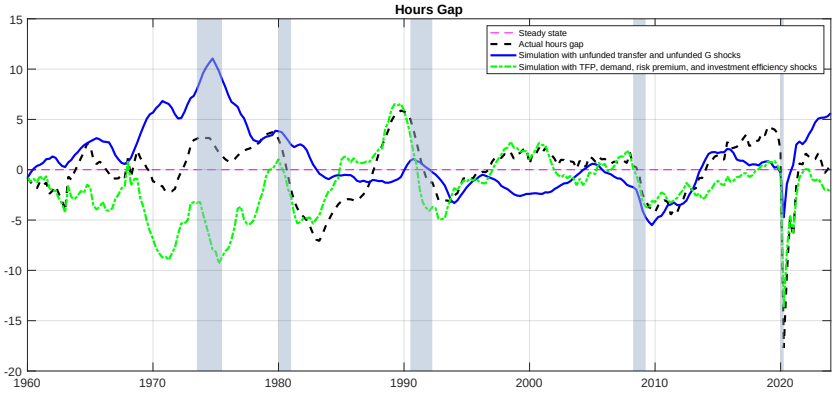
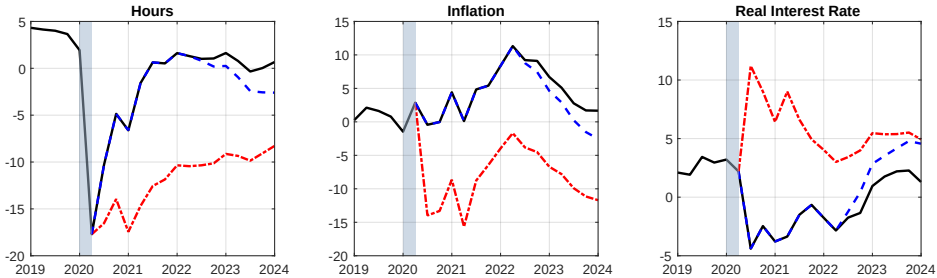


Figure X.2 – Drivers of Hours Gap

Unfunded fiscal shocks (solid blue line) counteract productivity slowdown in the 1960s and 1970s **+** trigger a quick rebound from the pandemic

Unfunded fiscal shocks and post-pandemic inflation



1. First counterfactual simulation, all fiscal shocks starting from post-pandemic expansion (2020:Q3) are assumed to be funded (red dash-dotted line).
2. Second counterfactual simulation, fiscal shocks starting from Premier Truss's Mini budget (2022:Q3) are assumed to be funded (blue dashed line)

Conclusions

- Low frequency movements in inflation related to fiscal policy:
  1. Unfunded spending critically affects inflation dynamics and real interest rates
  2. Funded spending has a small impact on real interest rates
  3. UK fiscal inflation in 1960s-1980s + pandemic similar to US, different in 1990s-2000s
- **Pandemic**: A large fiscal stimulus **and** accommodative monetary policy $\Rightarrow$ Large increase in fiscal inflation
  1. Quick rebound in real activity
  2. Debt to GDP **below** pre-pandemic level
- **Post pandemic**: Fiscal inflation remains elevated despite the decline in actual inflation $\Rightarrow$ soft landing, but also inflationary pressure

Appendix

Subeconomies

The linearized model economy can decomposed into two additive sub-economies

1. A sub-economy in which policymakers always follow the monetary-led policy mix and unfunded fiscal shocks are shut down $\rightarrow \hat{\pi}_t^M$ and $\hat{b}_t^M$
2. A sub-economy in which policymakers always follow the fiscally-led policy mix and all shocks except the unfunded fiscal shocks are shut down $\rightarrow \hat{\pi}_t^F$ and $\hat{b}_t^F$

It can be shown that

$$\begin{aligned} \hat{\pi}_t &= \hat{\pi}_t^M + \hat{\pi}_t^F \\ \hat{b}_t &= \hat{b}_t^M + \hat{b}_t^F \end{aligned}$$

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Flexible Price Economy

Euler equation

$$E_t \hat{y}_{t+1} = \hat{y}_t + \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) \tag{10}$$

Labor supply

$$\frac{n}{1-n} \hat{n}_t = \hat{y}_t + \widehat{w}_t^r \tag{11}$$

Labor demand

$$\hat{w}_t^r = -\alpha \hat{n}_t \tag{12}$$

Production function

$$\hat{y}_t = (1-\alpha) \hat{n}_t \tag{13}$$

Real rate

$$\hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{14}$$

Flexible Price Economy (cont'd)

Taylor rule

$$\hat{R}_t = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F \right) + \phi^F \hat{\pi}_t^F \tag{15}$$

Evolution of debt

$$b\hat{b}_t = -\tau \hat{\tau}_t + \beta^{-1} b \left(\hat{y}_{t-1} - \hat{y}_t + \hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1} \right) \tag{16}$$

Fiscal rule

$$\hat{\tau}_t = \gamma^M \left(\hat{b}_{t-1} - \hat{b}_{t-1}^F \right) + \gamma^F \hat{b}_{t-1}^F - \hat{b} + \varepsilon_t^F + \varepsilon_t^U \tag{17}$$

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New Keynesian model

Euler equation

$$E_t \hat{y}_{t+1} = \hat{y}_t + \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) \tag{18}$$

Labor supply

$$\frac{n}{1-n} \hat{n}_t = \hat{y}_t + \hat{w}_t^r \tag{19}$$

New Keynesian Phillips Curve

$$\hat{\pi}_t = \kappa \hat{w}_t^r + \beta E_t \hat{\pi}_{t+1} \tag{20}$$

Production function

$$\hat{y}_t = (1-\alpha) \hat{n}_t \tag{21}$$

Real rate definition

$$\hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{22}$$

New Keynesian model (cont'd)

Taylor rule

$$\hat{R}_t = \phi^M \left(\hat{\pi}_t - \hat{\pi}_t^F \right) + \phi^F \hat{\pi}_t^F \tag{23}$$

Evolution of debt

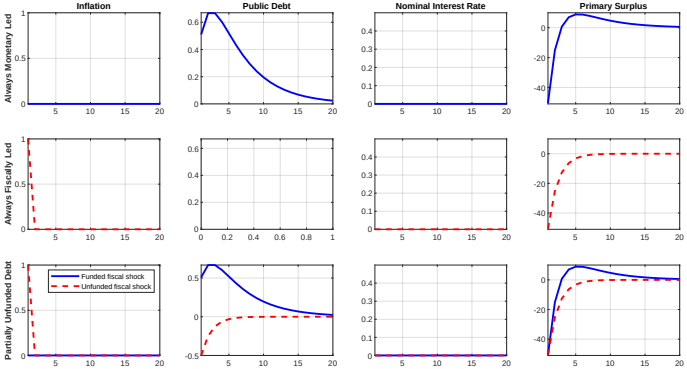
$$b\hat{b}_t = -\tau \hat{\tau}_t + \beta^{-1} b \left(\hat{y}_{t-1} - \hat{y}_t + \hat{R}_{t-1} - \hat{\pi}_t + \hat{b}_{t-1} \right) \tag{24}$$

Fiscal rule

$$\hat{\tau}_t = \gamma^M \left(\hat{b}_{t-1} - \hat{b}_{t-1}^F \right) + \gamma^F \hat{b}_{t-1}^F - \hat{b} + \varepsilon_t^F + \varepsilon_t^U \tag{25}$$

Back

Production Economy



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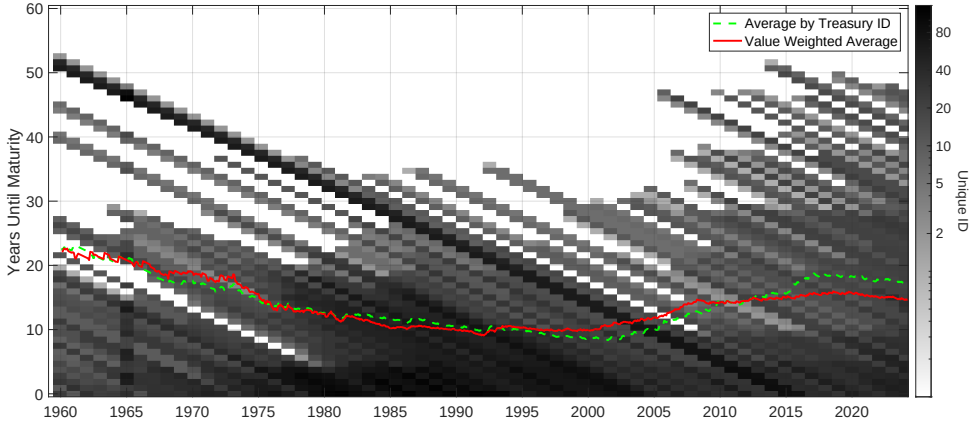
Maturity structure of UK government debt



Calibrated Parameters

Parameters Fixed in Estimation		
	Parameters	Values
Discount factor	β	0.9900
Average duration of debt	ρ	56.000
Capital depreciation rate	δ	0.0250
Elasticity of output to capital	α	0.3000
Wage markup	η_w	0.1200
Price markup	η_p	0.1200
Government expenditures to GDP ratio	s_{gc}	0.1200
Steady state tax rate on labor income	τ_L	0.2900
Steady state tax rate on capital income	τ_K	0.2900
Steady state tax rate on consumption	τ_C	0.2000

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First Sample Estimates

Prior and Posterior Distributions for the Structural Parameters

		Posterior Distribution				Prior Distribution		
Param	Description	Mode	Median	5%	95%	Type	Mean	Std
s_b	Debt to GDP annualized	1.3425	1.3446	1.2748	1.4168	N	1.21	0.05
$100\varkappa$	Steady state growth rate	0.3875	0.3657	0.2935	0.4375	N	0.50	0.05
$100\ln\Pi$	Steady state inflation	0.4892	0.4988	0.4143	0.5481	N	0.50	0.05
ξ	Inverse Frisch elasticity	1.9419	1.8870	1.8366	1.9319	G	2.00	0.25
μ	Share of hand-to-mouth	0.0147	0.0111	0.0052	0.0203	B	0.1	0.05
ω_w	Wage Calvo param	0.6845	0.6765	0.6424	0.7131	B	0.50	0.10
ω_p	Price Calvo param	0.8089	0.8233	0.7924	0.8508	B	0.50	0.10
ψ	Capital utilization cost	0.4300	0.3676	0.3276	0.4413	B	0.50	0.10
s	Investment adjust. cost	4.7373	4.6815	4.5973	4.7368	N	4.00	1.50
χ_w	Wage infl. indexation	0.2592	0.2628	0.2296	0.2965	B	0.30	0.15
χ_p	Price infl. indexation	0.2067	0.1442	0.1048	0.1855	B	0.30	0.15
θ	Habits in consumption	0.8439	0.8421	0.8278	0.8554	B	0.70	0.10
α_G	Subs. private/gov. cons.	-0.0089	0.0229	-0.0702	0.0822	N	0.00	0.10

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First Sample Estimates

Prior and Posterior Distributions for the Structural Parameters

		Posterior Distribution				Prior Distribution		
Param	Description	Mode	Median	5%	95%	Type	Mean	Std
ϕ_y	Interest response to GDP	0.2674	0.2543	0.2223	0.2872	N	0.11	0.05
ϕ_π	Interest response to infl.	1.7874	1.6919	1.6124	1.7789	N	1.87	0.10
ϕ_{zy}	Transfers response to GDP	0.0021	0.0027	0.0003	0.0095	G	0.50	0.50
ϕ_{gy}	G response to GDP	0.0016	0.0020	0.0002	0.0084	G	0.50	0.50
γ_Z	Transfers response to debt	0.2316	0.2469	0.2220	0.2995	N	0.20	0.10
γ_G	G response to debt	0.0008	0.0014	0.0002	0.0051	N	0.20	0.10
γ_K	Capital tax response to debt	0.0007	0.0015	0.0002	0.0052	N	0.20	0.10
γ_L	Labor tax response to debt	0.1100	0.1137	0.1040	0.1258	N	0.20	0.10
γ_C	Cons. tax response to debt	−0.0243	−0.0219	−0.1018	0.0364	N	0.20	0.10
ρ_r	AR coeff. monetary rule	0.9092	0.9013	0.8850	0.9166	B	0.50	0.10
ρ_G	AR coeff. gov. cons. rule	0.3898	0.4288	0.3864	0.5090	B	0.50	0.10
ρ_Z	AR coeff. transfers rule	0.5017	0.5269	0.4840	0.5840	B	0.50	0.10

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First Sample Estimates

Prior and Posterior Distributions for the Exogenous Processes

		Posterior Distribution				Prior Distribution		
Param	Description	Mode	Median	5%	95%	Type	Mean	Std
ρ^{M}_{eG}	AR coeff. funded G	0.9951	0.9949	0.9932	0.9964	B	0.995	0.001
ρ^{F}_{eG}	AR coeff. unfunded G	0.9953	0.9951	0.9933	0.9965	B	0.995	0.001
ρ_g	AR coeff. short-term G	0.4899	0.5068	0.4195	0.6079	B	0.500	0.100
ρ^{M}_{eZ}	AR coeff. funded trans.	0.9950	0.9948	0.9929	0.9964	B	0.995	0.001
ρ^{F}_{eZ}	AR coeff. unfunded trans.	0.9949	0.9946	0.9930	0.9960	B	0.995	0.001
ρ_z	AR coeff. short-term trans.	0.4958	0.5060	0.4663	0.5663	B	0.500	0.100
ρ_a	AR coeff. technology	0.6537	0.6487	0.6034	0.7098	B	0.500	0.100
ρ_b	AR coeff. preference	0.3163	0.3408	0.2868	0.4174	B	0.500	0.100
ρ_m	AR coeff. mon. policy	0.3615	0.3442	0.2827	0.4141	B	0.500	0.100
ρ_i	AR coeff. investment	0.2923	0.3306	0.2675	0.4477	B	0.500	0.100
ρ_{rp}	AR coeff. risk premium	0.8986	0.9010	0.8751	0.9272	B	0.500	0.100
$\rho_{\mu^{NKPC}}$	AR coeff. pers. cost push	0.9955	0.9954	0.9936	0.9967	B	0.995	0.001

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First Sample Estimates

Prior and Posterior Distributions for the Exogenous Processes

		Posterior Distribution				Prior Distribution		
Param	Description	Mode	Median	5%	95%	Type	Mean	Std
σ_G^M	St.dev. funded G	2.0509	2.1676	1.9866	2.3428	IG	0.500	0.200
σ_G^F	St.dev. unfunded G	0.4919	0.4745	0.4499	0.5029	IG	0.500	0.200
σ_g	St.dev. short-term G	0.3793	0.3968	0.3650	0.4348	IG	0.500	0.200
σ_Z^M	St.dev. funded transfers	3.6981	3.7122	3.5927	3.8053	IG	0.500	0.200
σ_Z^F	St.dev. unfunded transfers	0.4536	0.4618	0.4239	0.5172	IG	0.500	0.200
σ_z	St.dev. short-term trans.	0.3920	0.4527	0.3886	0.5231	IG	0.500	0.200
σ_a	St.dev. technology	1.9050	1.9671	1.8086	2.0547	IG	0.500	0.200
σ_b	St.dev. preference	4.9845	4.9841	4.9626	4.9976	IG	0.500	0.200
σ_m	St.dev. mon. policy	0.2572	0.2588	0.2359	0.2843	IG	0.500	0.200
σ_i	St.dev. investment	1.4014	1.3206	1.1878	1.4281	IG	0.500	0.200
σ_w	St.dev. wage markup	0.6416	0.6469	0.5842	0.7192	IG	0.500	0.200
σ_p	St.dev. price markup	0.5958	0.6271	0.5794	0.6875	IG	0.500	0.200
σ_{rp}	St.dev. risk premium	0.4426	0.4165	0.3501	0.4722	IG	0.500	0.200
$\sigma_{\mu_{NKPC}}$	St.dev. persistent cost push	0.4164	0.4444	0.4010	0.4863	IG	0.500	0.200
σ_{GDP}^m	Measur. error GDP	0.9447	0.9492	0.8872	1.0197	IG	0.500	0.200
σ_{by}^m	Measur. error Debt/GDP	0.3777	0.2644	0.2378	0.3607	IG	0.500	0.200

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Second Sample Estimates

Prior and Posterior Distributions for the Exogenous Processes

		Posterior Distribution				Prior Distribution		
Param	Description	Mode	Median	5%	95%	Type	Mean	Std
σ_G^M	St.dev. funded G	4.2383	4.2439	4.2377	4.2493	IG	2.0509	0.500
σ_G^F	St.dev. unfunded G	0.2978	0.2991	0.2971	0.3009	IG	0.4919	0.500
σ_g	St.dev. short-term G	0.2004	0.1997	0.1987	0.2007	IG	0.3793	0.500
σ_Z^M	St.dev. funded transfers	7.5499	7.5556	7.5501	7.5604	IG	3.6981	0.500
σ_Z^F	St.dev. unfunded transfers	1.5653	1.5649	1.5632	1.5667	IG	0.4536	0.500
σ_z	St.dev. short-term trans.	0.2098	0.2097	0.2089	0.2112	IG	0.3920	0.500
σ_{GDP}^m	Measur. error GDP	0.7531	0.7538	0.7520	0.7561	IG	0.9447	0.200
σ_{by}^m	Measur. error Debt/GDP	0.2702	0.2697	0.2684	0.2706	IG	0.3777	0.200

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Production function:

$$\hat{y}_t = \frac{y + \Omega}{y} \left[\alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \right]. \tag{26}$$

Capital-labor ratio:

$$\hat{r}_t^K - \hat{w}_t = \hat{L}_t - \hat{k}_t. \tag{27}$$

Marginal cost:

$$\widehat{mc}_t = \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t. \tag{28}$$

Phillips curve:

$$\hat{\pi}_t = \frac{\beta}{1 + \chi_p \beta} E_t \hat{\pi}_{t+1} + \frac{\chi_p}{1 + \chi_p \beta} \hat{\pi}_{t-1} + \kappa_p \widehat{mc}_t + \kappa_p \hat{\eta}_t^p, \tag{29}$$

where $\kappa_p = \left[(1 - \beta \omega_p)(1 - \omega_p) \right] / \left[\omega_p (1 + \beta \chi_p) \right]$.

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Saver household's FOC for consumption:

$$\hat{\lambda}_t^S = \hat{F}_t^b - \frac{\theta}{e^\gamma - \theta}\hat{F}_t^a - \frac{e^\gamma}{e^\gamma - \theta}c_t^{*S} + \frac{\theta}{e^\gamma - \theta}c_{t-1}^{*S} - \frac{\tau^C}{1+\tau^C}\hat{\tau}_t^C, \tag{30}$$

where $\hat{F}_t^a = u_t^a - \gamma$.
Public/private consumption in utility:

$$\hat{c}_t^* = \frac{c^S}{c^S + \alpha_G g}\hat{c}_t^S + \frac{\alpha_G g}{c^S + \alpha_G g}\hat{g}_t. \tag{31}$$

Euler equation:

$$\hat{\lambda}_t^S = \hat{R}_t + E_t\hat{\lambda}_{t+1}^S - E_t\hat{\pi}_{t+1} - E_t\hat{F}_{t+1}^a. \tag{32}$$

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Maturity structure of debt:

$$\hat{R}_t + \hat{P}^B_t = \frac{\rho}{R} E_t \hat{P}^B_{t+1}. \tag{33}$$

Saver household's FOC for capacity utilization:

$$r^k_t - \frac{\tau^K}{1-\tau^K}\hat{\tau}^K_t = \frac{\psi}{1-\psi}\hat{v}_t. \tag{34}$$

Saver household's FOC for capital:

$$\hat{q}_t = E_t\hat{\pi}_{t+1} - \hat{R}_t + \beta e^{-\gamma}\left(1-\tau^K\right) r^k E_t \hat{r}^k_{t+1} - \beta e^{-\gamma}\tau^K r^k E_t \hat{\tau}^K_{t+1} + \beta e^{-\gamma}\left(1-\delta\right) E_t \hat{q}_{t+1}. \tag{35}$$

Saver household's FOC for investment:

$$\hat{\imath}_t + \frac{1}{1+\beta}\hat{F}^a_t - \frac{1}{\left(1+\beta\right) se^{2\gamma}}\hat{q}_t - \hat{F}^i_t - \frac{\beta}{1+\beta}E_t\hat{\imath}_{t+1} - \frac{\beta}{1+\beta}E_t\hat{F}^a_{t+1} = \frac{1}{1+\beta}\hat{\imath}_{t-1}. \tag{36}$$

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Effective capital:

$$\hat{k}_t = \hat{v}_t + \hat{\bar{k}}_{t-1} - \hat{F}^a_t. \tag{37}$$

Law of motion for capital:

$$\hat{\bar{k}}_t = (1-\delta)\, e^{-\gamma}\left(\hat{\bar{k}}_{t-1} - \hat{F}^a_t\right) + \left[1-(1-\delta)\, e^{-\gamma}\right]\left[(1+\beta)\, se^{2\gamma} + \hat{\imath}_t\right]. \tag{38}$$

Hand-to-mouth household's budget constraint:

$$\tau^C c^N \hat{\tau}^C_t + \left(1+\tau^C\right) c^N \hat{c}^N_t = \left(1-\tau^L\right) wL\left(\hat{w}_t + \hat{L}_t\right) - \tau^L wL\hat{\tau}^L_t + z\hat{z}_t. \tag{39}$$

Aggregate households' consumption

$$c\hat{c}_t = c^S\,(1-\mu)\,\hat{c}^S_t + c^N \mu \hat{c}^N_t. \tag{40}$$

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Wage equation:

$$\begin{aligned}\hat{w}_t &= \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - \xi\hat{L}_t + \hat{\lambda}_t^S - \frac{\tau^L}{1-\tau^L}\hat{\tau}_t^L\right] \\ &+ \frac{\chi^w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\chi^w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{\chi}{1+\beta}\hat{F}_{t-1}^a - \frac{1+\beta\chi-\rho_a\beta}{1+\beta}\hat{F}_t^a + \kappa_w\hat{\mu}_t^w \quad (41)\end{aligned}$$

where $\kappa_w \equiv \left[\left(1-\beta\omega_w\right)\left(1-\omega_w\right)\right] / \left[\omega_w\left(1+\beta\right)\left(1+\frac{(1+\eta^w)\xi}{\eta^w}\right)\right]$.

Aggregate resource constraint:

$$y\hat{y}_t = c\hat{c}_t + i\hat{i}_t + g\hat{g}_t + \psi'\left(1\right)k\hat{v}_t. \quad (42)$$

Government budget constraint:

$$\begin{aligned}&\frac{b}{y}\hat{b}_t + \tau^K r^k\frac{k}{y}\left[\hat{\tau}_t^K + \hat{r}_t^k + \hat{k}_t\right] + \tau^L w\frac{L}{y}\left[\hat{\tau}_t^L + \hat{w}_t + \hat{L}_t\right] + \tau^C\frac{c}{y}\left(\hat{\tau}_t^C + \hat{c}_t\right) \\ &= \frac{1}{\beta}\frac{b}{y}\left[\hat{b}_{t-1} - \hat{\pi}_t - \hat{P}_{t-1}^B - \hat{F}_t^a\right] + \frac{b}{y}\frac{\rho}{e^\gamma}\hat{P}_t^B + \frac{g}{y}\hat{g}_t + \frac{z}{y}\hat{z}_t. \quad (43)\end{aligned}$$

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Fiscal Rules

$$\hat{\tau}_{J,t} = \rho_J \hat{\tau}_{J,t-1} + (1-\rho_J)\,\gamma_J \tilde{b}^*_{t-1}, \quad J \in \{K, L, C\} \tag{44}$$

$$\hat{z}^b_t = \rho_Z \hat{z}^b_{t-1} - (1-\rho_Z)\left[\gamma_Z \tilde{b}^*_{t-1} + \phi_{zy}\hat{y}_t\right] + \hat{\zeta}_{z,t} \tag{45}$$

$$\hat{z}_t = \hat{z}^b_t + \hat{\zeta}^M_{z,t} + \hat{\zeta}^F_{z,t} \tag{46}$$

$$\hat{g}^b_t = \rho_G \hat{g}^b_{t-1} - (1-\rho_G)\left[\gamma_G \tilde{b}^*_{t-1} + \phi_{gy}\hat{y}_t\right] + \hat{\zeta}_{g,t} \tag{47}$$

$$\hat{g}_t = \hat{g}^b_t + \hat{\zeta}^M_{g,t} + \hat{\zeta}^F_{g,t} \tag{48}$$

Monetary Rule:

$$\hat{R}_t = \max\left(-\ln R_*, \rho_r \hat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi \hat{\pi}^*_t + \phi_y \hat{y}_t\right]\right) + \epsilon_{R,t} \tag{49}$$

The variables with the * superscript in equations (44) to (49) above belong to the shadow economy. ▸ Back

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The block of equations that characterize the shadow economy consists in an additional set of equations (26) to (43), where any variable that refers to the actual economy x_t is replaced by the same variable in the shadow economy x_t^*, plus the rule for the monetary authority

$$\hat{R}_t^* = \max\left(-\ln R_*, \rho_r \hat{R}_{t-1}^* + (1-\rho_r)\left[\phi_\pi \hat{\pi}_t^* + \phi_y \hat{y}_t^*\right]\right) + \epsilon_{R,t} \tag{50}$$

and the rules for the fiscal authority,

$$\hat{\tau}_{J,t}^* = \rho_J \hat{\tau}_{J,t-1}^* + (1-\rho_J)\,\gamma_J \tilde{b}_{t-1}^*, \quad J \in \{K, L, C\} \tag{51}$$

$$\hat{z}_t^{b*} = \rho_Z \hat{z}_{t-1}^{b*} - (1-\rho_Z)\left[\gamma_Z \tilde{b}_{t-1}^* + \phi_{zy} \hat{y}_t^*\right] + \hat{\zeta}_{z,t} \tag{52}$$

$$\hat{z}_t^* = \hat{z}_t^{b*} + \hat{\zeta}_{z,t}^M \tag{53}$$

$$\hat{g}_t^{b*} = \rho_G \hat{g}_{t-1}^{b*} - (1-\rho_G)\left[\gamma_G \tilde{b}_{t-1}^* + \phi_{gy} \hat{y}_t^*\right] + \hat{\zeta}_{g,t} \tag{54}$$

$$\hat{g}_t^* = \hat{g}_t^{b*} + \hat{\zeta}_{g,t}^M \tag{55}$$

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