The impact of industrialized countries' monetary policy on emerging economies

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VERY PRELIMINARY AND INCOMPLETE

Abstract

In this paper I study how the low interest rate policies adopted by industrialized countries after the 2018 financial crisis have impacted the economic performance of emerging countries. Although these policies may have reduced the outflows of capital from emerging countries to industrialized countries, the economic performance of emerging countries has deteriorated more than in industrial countries. I propose a model that captures the observed dynamics in capital flows and macroeconomic performance. Contrary to the more conventional view, lower interest rates in industrialized countries could have negative macroeconomic consequences for emerging countries even though the latter experience an increased inflows of capital.

Introduction

Following the 2008 financial crisis, many countries in the industrialized world pursued expansionary monetary policies that resulted in lower interest rates. Figure 1 plots the policy rates for the major industrialized countries and shows that, with the exception of Japan, they have all lowered the interest rates after 2008. Japan is an exception because the policy rate was already close to zero before the financial crisis.

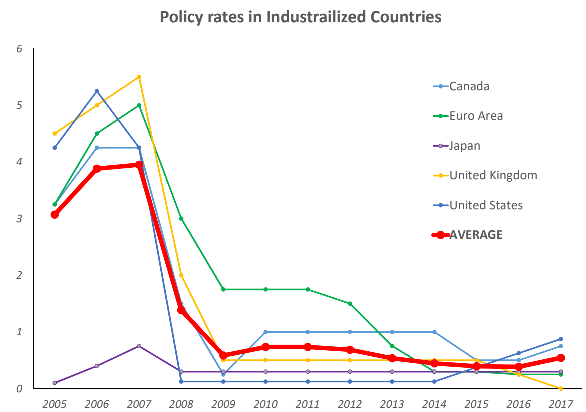


Figure 1: Policy interest rates in Industrialized Countries, 2005-2017. Sources: *International Financial Statistics, IMF*. The graph plots 'Policy-Related Interest Rates'. If unavailable it plots the 'Discount Rate'. The 'AVERAGE' is the un-weighted mean.

During the same period we observe a change in capital flows between industrialized and emerging countries as indicated by the current account (left panel of Figure 2). While emerging countries were net exporters of capital before the crisis (that is, they had positive current account balances), the post-crisis period shows a re-balancing of the current account. This is also noticeable by looking at the more liquid components of the financial account: the net flows of portfolio debt and international reserves plotted in the right panel of Figure 2.[1]

The fact that the capital flows re-balanced in conjunction with the lower interest rates in industrialized countries is consistent—although it is not a proof of it—with the view that loose monetary policies in industrialized countries increased the search for higher yields elsewhere. The goal of this paper is to study the macroeconomic consequences of these policies and the associated capital re-balancing for emerging countries.

The conventional view is that higher inflows of capital (or lower outflows)

[1] A recent study by the International Monetary Funds, IMF (2016), shows that the flows of capital to emerging countries has slowed down after the crisis. This study, however, uses only private flows which exclude foreign reserves. In my study, instead, I focus on the overall net flows of capital to emerging countries which, abstracting from errors and omissions, corresponds to the Current Account Balance.

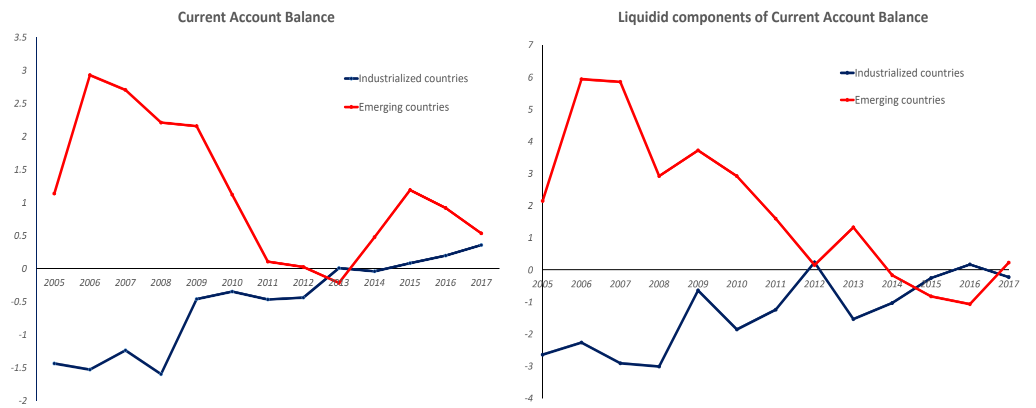


Figure 2: Net flows of capital as a percentage of GDP, 2005-2017. Sources: *Balance of Payments Analytic, IMF*. The liquid components of the current account shown in the second panel is the net flows of portfolio debt and foreign reserves. The aggregates are constructed using GDP at nominal exchange rates. **Emerging countries**: Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Industrialized countries**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States.

in emerging countries would lower the local interest rate and expand domestic credit, which in turn create the conditions for a macroeconomic boom. In the long-run it may also increase macroeconomic fragility since certain sectors of the economy become more leveraged. But, at least initially, it should stimulate growth in emerging countries. This, however, is not what happened to emerging countries after the financial crisis.

Figure 3 shows that GDP growth in emerging countries slowed down substantially after the crisis, while in industrialized countries the overall growth did not change much (besides the big drop during the crisis). As a result, the growth differential between emerging and industrialized countries dropped significantly. The second panel shows that this pattern is visible also for geographical sub-groups of emerging countries, although it is stronger for emerging counties in Latin American and Europe.

The stronger slow down of emerging countries is somewhat surprising because most of they did not face the financial turbulence experienced by industrialized countries, at least not to that extent. Nevertheless, the real sector of the economy did contract also in emerging countries during the crisis, which is not surprising given the high degree of global integration in real and financial markets. However, the fact that the post-crisis growth fell more than in industrialized countries suggests that the capital flows re-balancing has not been very helpful for the macro-economy of these countries. Again, this is contrary to the conventional view that lower interest rates and higher inflows of capital bring macroeconomic benefits to the receiving countries.[2]

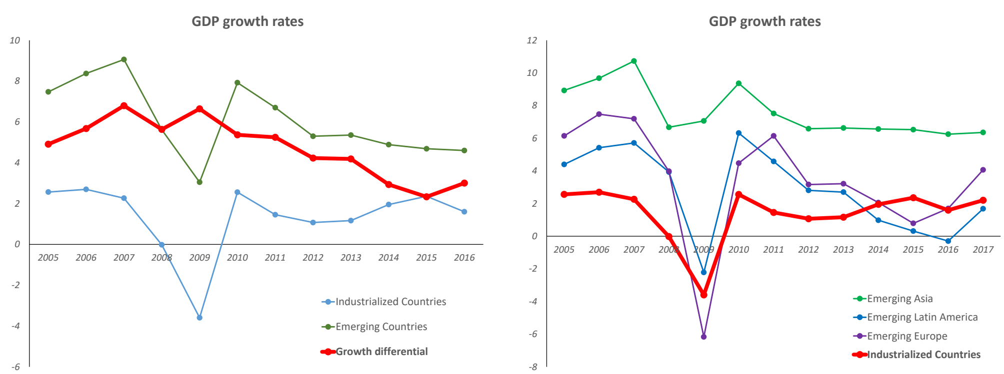


Figure 3: Growth rate of GDP in Industrialized and Emerging Countries, 2005-2017. Sources: *World Development Indicators, World Bank*. Aggregates growth rates are constructed by weighting individual real growth rates by GDP in PPP terms. Weighting by GDP at nominal exchange rates gives very similar pictures. **Emerging countries**: Argentina, Brazil, Bulgaria, Chile, China, Hong Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Industrialized countries**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States. **Emerging Asia**: China, Hong Kong, India, Indonesia, South Korea, Malaysia, Pakistan, Philippines, Thailand. **Emerging Latin America**: Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela. **Emerging Europe**: Bulgaria, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Russia, Turkey, Ukraine.

[2] An alternative interpretation of why the outflows of capital from emerging countries re-balanced is because the macroeconomic slowdown of industrialized countries made investments in these countries less attractive. This interpretation, however, is inconsistent with the fact that the growth rate of emerging countries slowed down even more than in industrialized countries. Therefore, in terms of growth prospects, industrial countries remained 'relatively' more attractive than emerging countries.

One limitation of the conventional view is that it focuses only on one of the mechanisms through which interest rates affect the macro-economy, that is, the lower cost of financing production and investment. However, there could be other mechanisms through which the interest rate affects macroeconomic activities. In particular, their impact on savings. When the interest rate drops, savers have less incentive to save and, as a result, they hold less financial assets. To the extent that the holding of financial assets affects real economic decisions, including investments, this may have a negative macroeconomic effects including lower growth.

I show this result with a model economy calibrated to emerging countries. There are two production sectors in the economy. The first sector produces output with a risky technology that uses labor as the only input of production. Risk derives from the fact that production is carried out by individual entrepreneurs and the production function is subject to an 'uninsurable' idiosyncratic shock. The idiosyncratic shock leads producers to save for precautionary reasons and when they hold more financial wealth, they are willing to take more risk by increasing the scale of production. The second sector, instead, produces output with a non-reproducible asset but with lower incidence of 'uninsurable' idiosyncratic shocks. Because of the lower (uninsurable) risk, producers in the second sector save less and, in equilibrium, they become net borrowers.

I think of the first sector as the growth-enhancing sector while the second as the sector that produces services from less flexible inputs. An example is housing. Arguably, growth enhanced activities tend to be individually riskier than activities for which a higher component of income derives from rents. I refer to the first sector as 'growth-enhancing-sector' and the second as 'rent-seeking-sector'.

Within the model, the impact of an expansionary monetary policy in industrialized countries is captured by a reduction in the world interest rate. This has two consequences for emerging economies. First, the rent-seeking-sector borrows more because the cost of borrowing declines. This increases the demand for non-reproducible assets which in turn raises its market value. Therefore, a consequence of the lower interest rate is an asset price boom in the rent-seeking-sector of the economy. However, since the asset used in production is not reproducible, production does not change in this sector. Intuitively, cheaper credit increases the demand for house-like assets but the services from existing house-like assets remain the same. If we assume that new house-like assets can be produced, this would stimulate new constructions but slowly.

The impact of a lower interest rate in the growth-enhancing-sector is different. The lower interest rate reduces savings which in turn generates a decline in growth. Therefore, even though the change in external monetary policy generates an asset price boom in certain sectors (like real estates), it could decrease the overall economic growth of emerging countries.

In addition to lower growth, the higher leverage in the rent-seeking-sector increases future macroeconomic instability. Future internal or external shocks could created the conditions for larger re-adjustments when the economy is more leveraged, which in turns create larger macroeconomic contractions. A monetary policy reversal in industrialized countries could be once of the forces that induces a financial re-adjustment in emerging countries. The policy reversal would cause a price drop for non-reproducible assets, which could trigger default in the rent-seeking-sector. This, effectively, redistributes wealth away from savers (in the growth-enhancing-sector) to borrowers (in the rent-seeking-sector). The capital losses experienced by savers in the growth-enhancing-sector would then trigger a decline in real growth.

The paper is organized as follows. Section 1 develops the theoretical model. Section 2 calibrates the model and evaluate the impact of industrial countries' monetary policy on emerging countries. Section 3 extends the model by allowing for endogenous growth. Section 5 summarizes the results and concludes.

1 Model

I consider a small open economy representative of emerging countries. Modeling the group of emerging countries as a small opening economy is, obviously, a limitation since their contribution to the world economy is not small. However, by limiting the analysis to a small open economy I can treat the world interest rate as exogenous, which simplifies considerably the characterization of the equilibrium.

The economy has two domestic sectors: the entrepreneurial sector and the household sector. I start with the description of the entrepreneurial sector.

1.1 Entrepreneurial sector

In the entrepreneurial sector there is a unit mass of atomistic entrepreneurs, indexed by i, with lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t^i),$$

where c_t^i is the consumption of entrepreneur i at time t.

Entrepreneurs are business owners producing a single good. Entrepreneurial consumption should be thought as dividends paid by the firm and the concavity of the utility function captures the risk aversion of entrepreneurs or managers in the case of separation between ownership and management.

The production function for the single good is

$$y_t^i = z_t^i h_t^i,$$

where h_t^i is the input of labor supplied by households (as described below) at the market wage w_t, and z_t^i is an idiosyncratic shock to productivity. The shock is distributed independently and identically across entrepreneurs and time in the interval $[\underline{z}, \overline{z}]$ with probability distribution $\Gamma(z)$. The expected value of the shock is $\mathbb{E}z_t^i = A_t$, where A_t represents aggregate productivity.

As in Arellano, Bai, and Kehoe (2011), the input of labor h_t^i is chosen before observing the idiosyncratic component of productivity, π_t^i. This implies that the choice of labor is risky. To insure consumption smoothing, entrepreneurs have access to two types of bonds: domestic bonds, denoted by d_t^i, and foreign bonds, denoted by f_t^i. Domestic bonds are liabilities issued by households at price q_t^d while foreign bonds are liabilities issued by industrialized countries at price q_t^f.

There are two differences between domestic and foreign bonds. First, while the issuers of domestic bonds (households) could default on their liabilities, foreign bonds are always repaid. I will relax this assumption later in the paper. Second, while the supply of foreign bonds is perfectly elastic and the price q_t^f is exogenous (small open economy), the price of domestic bonds q_t^d is endogenous and reflects the probability of default. Since bonds, domestic or foreign, cannot be contingent on the realization of productivity, they provide only partial insurance.

An entrepreneur i enters period t with domestic and foreign bonds, d_t^i and f_t^i. In the event of a domestic financial crisis, the entrepreneur incurs financial losses that are proportional to the holding of domestic bonds. Denoting by δ_t the unit loss realized at the beginning of the period on domestic bonds, the residual value of the domestic bonds are $\tilde{d}_t^i = (1-\delta_t)d_t^i$ while the value of foreign issued bonds remains f_t^i. The unit loss δ_t is an endogenous stochastic variables and will be determined in equilibrium.[3]

Given the residual wealth after default, $\tilde{d}_t^i + f_t^i$, the entrepreneur chooses the input of labor h_t. Then, after the observation of the idiosyncratic productivity z_t^i, the entrepreneur chooses consumption c_t^i and purchase of new domestic bonds d_{t+1}^i at prices q_t^d, and new foreign bonds f_{t+1}^i at price q_t^f. The budget constraint, after the observation of productivity is

$$c_t^i + q_t^d d_{t+1}^i + q_t^f f_{t+1}^i = \tilde{d}_t^i + f_t^i + (z_t^i - w_t)h_t^i. \tag{1}$$

Because labor h_t^i is chosen before the observation of z_t^i, while the saving decision is made after observing z_t^i, it will be convenient to define the entrepreneur's wealth after production, that is,

$$a_t^i = \tilde{b}_t^i + f_t^i + (z_t^i - w_t)h_t^i.$$

Given the timing assumption, the input of labor h_t^i depends on $\tilde{d}_t^i + f_t^i$ while the portfolio decisions d_{t+1}^i and f_{t+1}^i depend on a_t^i. To further clarify the timing, it would be convenient to summarize the sequence of events in each period as taking place in three sequential stages:

1. **Stage 1**: The entrepreneur enters the period with financial assets d_t^i and f_t^i, and observes the repayment fraction δ_t. This brings the residual value of domestic bonds to $\tilde{d}_t^i = (1-\delta_t)d_t^i$.

2. **Stage 2**: Given $\tilde{d}_t^i$ and f_t^i, the entrepreneur chooses the input of labor h_t^i before knowing the idiosyncratic productivity z_t^i. Market clearing in the labor market determines the wage rate w_t.

3. **Stage 3**: Productivity z_t^i becomes known. The end-of-period wealth $a_t^i = \tilde{d}_t^i + f_t^i + (z_t^i - w_t)h_t^i$ is in part used for consumption, c_t^i, and in part (saved) to purchase new domestic and foreign bonds, $q_t^d d_{t+1}^i + q_t^f f_{t+1}^i$.

[3] For the moment I abstract from the possibility that a crisis could also arise in industrialized countries and focus only on the implications of interest rate policy chosen by industrialized countries.

The next step is to characterize the entrepreneur's policies before and after observing the idiosyncratic productivity.

Lemma 1.1 *The optimal entrepreneur's policies are*

$$\begin{aligned}
h_t^i &= \phi_t(\tilde{d}_t^i + f_t^i), \\
c_t^i &= (1-\beta)a_t^i, \\
q_t^d d_{t+1}^i &= \beta\theta_t a_t^i, \\
q_t^f f_{t+1}^i &= \beta(1-\theta_t)a_t^i.
\end{aligned}$$

where ϕ_t and θ_t satisfy

$$\mathbb{E}_z\left\{\frac{z-w_t}{1+(z-w_t)\phi_t}\right\}=0 \tag{2}$$

$$\mathbb{E}\left\{\frac{q_t^d}{(1-\delta_{t+1})q_t^f\theta_t+q_t^d(1-\theta_t)}\right\}=1 \tag{3}$$

Proof 1.1 *See Appendix A.*

The demand for labor, which is chosen before observing the realization of idiosyncratic productivity, is linear in financial wealth $\tilde{d}_t^i + f_t^i$. The proportional factor ϕ_t is determined by the first order condition for labor and takes the form specified in (2). Notice that this is the same for all entrepreneurs.

The factor ϕ_t captures the importance of risk aversion for determining the demand for labor. Because productivity is unknown when an entrepreneur chooses the scale of production, labor is risky and entrepreneurs require a positive profit over the cost of labor as a premium in compensation for risk. As a result, the expected marginal product of labor is higher than the wage rate, that is, $\mathbb{E}_t z_t^i > w_t$. Furthermore, higher is the expected unit profit, $\mathbb{E}_t z_t^i - w_t$, and higher is the scale of production ϕ_t. On the other hand, if we fix the expected unit profit, the scale of production decreases with the volatility of productivity (risk).

Since the distribution of z_t^i does not change over time, the only 'endogenous' variable that affects ϕ_t is the wage rate w_t. I make this dependence explicit by using the notation $\phi_t(w_t)$.

Lemma 1.1 also indicates that entrepreneurs allocate their end-of-period wealth between consumption and savings according to the fixed factor β. This is a property that derives from the logarithmic specification of the utility function. Finally, a fraction θ_t of savings are allocated to domestic bonds and the remaining fraction $1 - \theta_t$ to foreign bonds. The portfolio allocation is determined by (3), which is the first order condition for the foreign bonds f_{t+1}. Since this condition depends only on aggregate variables, θ_t is the same for all entrepreneurs, which explains the omission of subscript i. Notice that the portfolio allocation is well defined because foreign bonds are not risky while domestic bonds are defaultable. Therefore, domestic and foreign bonds are not perfectly substitutable.

The aggregate demand for labor is derived by aggregating individual demands and can be written as

$$H_t = \phi_t(w_t) \int_i \Big(\tilde{d}_t^i + f_t^i\Big) = \phi_t(w_t)\tilde{D}_t + F_t,$$

where capital letters denote aggregate variables (upon aggregation over all entrepreneurs).

The aggregate demand for labor depends negatively on the wage rate and positively on the aggregate financial wealth of entrepreneurs. The dependence of wealth does not derive from entrepreneurs being financially constrained. In fact, labor does not need to be financed since it is not paid in advance. Instead, the property derives from the fact that employment is risky and entrepreneurs are willing to hire more workers only if they have a higher wealth buffer.

1.2 Household sector

There is a unit mass of atomistic households with utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(c_t - \alpha A_t h_t\Big),$$

where c_t is consumption and h_t is employment. Households are homogeneous and they do not face idiosyncratic shocks.

The assumption that households have linear utility in consumption simplifies the characterization of the equilibrium and allows for some analytical results, without affecting the key properties of the model. As long as households do not face idiosyncratic risks (or the idiosyncratic risk is significantly lower than the risk faced by entrepreneurs), the model would display similar properties even if households were risk averse. The dependence of labor dis-utility from aggregate productivity A_t guarantees balanced growth.

The linear specification of the dis-utility from working can be justified with the indivisibility of labor, which is a common assumption in many business cycle models.

Households hold a non-reproducible asset which is available in fixed supply $\overline{K}$. Each unit of the asset produces A_t units of consumption goods to households but not to entrepreneurs. The asset is divisible and can be traded by households at the end of the period at the market price p_t. I will interpret the fixed asset as residential houses and its production as housing services.[4]

Debt and default. Households can borrow l_t/R_{t-1} at the end of period $t-1$ (R_{t-1} is the gross interest rate) with the promise to repay l_t in period t. At the beginning of period t, however, when the repayment l_t is due, the household could default on the debt.

In the event of default, creditors have the right to liquidate k_t and sell it at the liquidation price $\tilde{p}_t$. The liquidation price $\tilde{p}_t$ at the 'beginning of the period' could differ from the price p_t at the 'end of the period' when houses are traded. In particular, I assume that with some probability λ, the liquidation price drops to ξA_t. The parameter ξ is sufficiently small so that $\xi A_t < p_t$, that is, the liquidation price at the beginning of the period drops below the price at the end of the period.

Let ε_t be a random variable that takes the value of 0 with probability λ and 1 with probability $1-\lambda$. The liquidation price is

$$\tilde{p}_t = \begin{cases} \xi A_t, & \text{if } \varepsilon_t = 0 \\ \\ p_t, & \text{if } \varepsilon_t = 1 \end{cases} . \tag{4}$$

The micro-foundation for the price drop is described in details in Appendix C and it is based on self-fulfilling expectations. In that context, the

[4]In principle, I could allow entrepreneurs to hold and trade houses. However, if houses provide services only to households and renting them involves substantial agency problems, in equilibrium entrepreneurs would choose not to hold them.

stochastic variable ε_t is a sunspot shock and λ is the probability that the realization of the shock triggers negative self-fulfilling expectations.

Once $\tilde{p}_t$ becomes known at the beginning of period t, households could use the threat of default to renegotiate the outstanding liabilities l_t. Of course, the debt will be renegotiated only if the liabilities are bigger than the liquidation value, that is, $l_t > \tilde{p}_t k_t$. Under the assumption that households have the whole bargaining power, the debt will be renegotiated to the liquidation value. Thus, the post-renegotiation debt is

$$\tilde{l}(l_t, \tilde{p}_t k_t) = \min\Big\{ l_t \, , \, \tilde{p}_t k_t \Big\} \tag{5}$$

Renegotiation, however, also brings a convex cost that increases with the renegotiation size, that is,

$$\varphi\left(l_t, \tilde{p}_t k_t\right) = \frac{\chi \cdot \max\Big\{ 0 \, , \, l_t - \tilde{p}_t k_t \Big\}^2}{l_t} \tag{6}$$

The cost is zero if there is no renegotiation, that is, the liabilities are smaller than the value of the house ($l_t \leq \tilde{p}_t k_t$). It becomes positive if the borrower renegotiates the debt, that is, $l_t > \tilde{p}_t k_t$. Besides the renegotiation cost, there are no penalties and the borrower will re-enter the credit market immediately at the end of the period when the regular market for houses takes place (fresh-start). Under this assumption, the household's budget constraint after renegotiation is

$$\tilde{l}(l_t, \tilde{p}_t k_t) + \varphi\left(l_t, \tilde{p}_t k_t\right) + \Big(k_{t+1} - k_t\Big) p_t + c_t = \frac{l_{t+1}}{R_t} + w_t h_t + A_t k_t.$$

The gross interest rate R_t charged to the household is endogenously determined and depends on the amount borrowed. If the household borrows more, relatively to the value of the house, the expected repayment rate would be lower, which will be reflected in a higher interest rate R_t.

Denote by $\overline{R}_t$ the expected gross return from holding the debt issued in period t by 'all' households and repaid in period $t+1$. This is the market return which is taken as given in a single borrowing transaction. Since households are atomistic and financial markets are competitive, the expected return on the debt issued by an 'individual' household must be equal to the aggregate expected return $\overline{R}_t$. Thus, the interest rate on the debt issued by an individual household must satisfy

$$\frac{l_{t+1}}{R_t} = \frac{\mathbb{E}_t \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\overline{R}_t}. \tag{7}$$

The left-hand-side is the amount borrowed in period t while the right-hand-side is the expected repayment in period $t+1$, discounted by the market return $\overline{R}_t$. Since the household renegotiates in the next period if $l_{t+1} > \tilde{p}_{t+1} k_{t+1}$, the actual repayment could be lower than the original debt. Competition in financial intermediation requires that the left-hand-side of (7) is equal to the right-hand-side.

Equation (7) determines the interest rate R_t for an individual household. In equilibrium, of course, all households will make the same decisions and they all borrow at the same rate. However, in order to characterize the optimal decision of an individual household, we have to allow the individual household to deviate from other households, which in turn implies a deviation of the individual interest rate R_t as determined by equation (7).

First order conditions. As for entrepreneurs, households' decisions are made in three stages. In the first stage households decide whether to default on the debt. In the second stage, before the realization of aggregate productivity, they decide the supply of labor. In the third stage households choose investment in housing and the debt. Appendix B describes the households' problem and derives the following first order conditions

$$w_t = \alpha A_t, \tag{8}$$

$$\tfrac{1}{\overline{R}_t} = \beta + \Phi_t \left(\tfrac{l_{t+1}}{k_{t+1}} \right), \tag{9}$$

$$p_t = \beta \mathbb{E}_t \Big(A_{t+1} + p_{t+1} \Big) + \Psi_t \left(\tfrac{l_{t+1}}{k_{t+1}} \right). \tag{10}$$

The functions $\Phi_t(.)$ and $\Psi_t(.)$, derived in the appendix, are increasing in the ratio l_{t+1}/k_{t+1}. I refer to this ratio as leverage. Thus, according to equation (9), when the expected return on household debt declines, leverage increases. According to equation (10) this implies that an increase in leverage is associated with a higher return from houses which in turn increases their price. Thus, a decline in the (expected) interest rate will be associated with more leverage and a house price boom.

1.3 General equilibrium

I will use capital letters to denote aggregate variables. The states at the beginning of the period are aggregate productivity, A_t, domestic and foreign bonds held by entrepreneurs, D_t and F_t, liabilities issued by households, L_t, and exogenous shock ε_t. To use a compact notation I denote the vector of states by $\mathbf{s}_t \equiv (D_t, F_t, L_t, \varepsilon_t)$. The equilibrium is determined sequentially in three steps:

1. **Step 1**: Given the shock ε_t, the liquidation price is ξ if $L_t \geq \xi\overline{K}$ and $\varepsilon_t = 0$. Given the liquidation price, households choose whether to default. The renegotiated liabilities are

$$\tilde{L}_t = \min\left\{ L_t \,,\, \xi\bar{K} \right\}$$

   The post-renegotiation value of domestic bonds is $\tilde{D}_t = \tilde{L}_t$.

2. **Step 2**: Given the post-renegotiation wealth $\tilde{D}_t + F_t$, entrepreneurs choose the demand for labor and households choose the supply. At this stage the idiosyncratic productivity π_t^i is unknown.

   The aggregate demand for labor is $\phi_t(w_t)(\tilde{D}_t + F_t)$, which depends negatively on the wage rate w_t and positively on the aggregate wealth of entrepreneurs, $\tilde{D}_t + F_t$. The supply of labor is derived from the households' first order condition (8). Market clearing will then determine the wage rate w_t and employment H_t.

3. **Step 3**: Idiosyncratic productivities z_t^i are realized. The aggregate wealth of entrepreneurs becomes $\tilde{D}_t + F_t + (\mathbb{E}z - w_t)H_t$, which is in part consumed and in part saved in new bonds, $q_t^d D_{t+1}$ and $q_t^f F_{t+1}$. Households choose the new loans, L_{t+1}, and houses, $K_{t+1} = \overline{K}$.

   Market clearing in financial assets gives rise to condition $D_{t+1} = L_{t+1}$. The net foreign asset position of the country is $q_t^f F_t$. Competition implies that the price paid by entrepreneurs to purchase households' debt is consistent with the interest rate charged to households, that is, $q_t^d = 1/R_t$. Since $\overline{R}_t = R_t(1 - \mathbb{E}_{t+1}\delta_{t+1})$, we also have $q_t^d = (1 - \mathbb{E}_{t+1}\delta_{t+1})/\overline{R}_t$, where $\delta_{t+1} = \tilde{L}_{t+1}/L_t$.

   As shown in Lemma 1.1, the optimal savings of entrepreneurs take the form $q_t^d d_{t+1}^i + q_t^f f_{t+1}^i = \beta a_t^i$, where a_t^i is the end-of-period wealth.

Aggregating over all entrepreneurs we obtain aggregate savings which are allocated to domestic and foreign bonds, that is,

$$q_t^d D_{t+1} + q_t^f F_{t+1} = \beta \int_i a_t^i. \tag{11}$$

The demand for domestic bonds is determined by the fraction θ_t of savings allocated to these bonds, that is,

$$q_t^d D_{t+1} = \theta_t \beta \int_i a_t^i. \tag{12}$$

The supply of domestic bonds, instead, is derived from the borrowing decisions of households. From the first order condition (9) we have

$$\frac{1}{\overline{R}_t} = \beta + \Phi_t\left(\frac{L_{t+1}}{\overline{K}}\right).$$

Since in equilibrium $\overline{R}_t = R_t(1 - \mathbb{E}\delta_{t+1})$ and $q_t^d = 1/R_t$, the first order condition can be rewritten as

$$q_t^d = \beta\Big(1 - \mathbb{E}\delta_{t+1}\Big) + \Phi_t\left(\frac{L_{t+1}}{\overline{K}}\right). \tag{13}$$

Given the end-of-period wealth held by entrepreneurs, $\int_i a_t^i$, we can solve for θ_t, q_t^d, D_t, F_t, L_t using the market clearing condition in domestic bonds, $D_t = L_t$, equations (12) and (13), and entrepreneurs' first order conditions for the choice of domestic and foreign bonds,

$$\mathbb{E}_t\left\{\frac{(1-\delta_{t+1})q_t^f}{(1-\delta_{t+1})q_t^f\theta_t + q_t^d(1-\theta_t)}\right\} = 1$$
$$\mathbb{E}_t\left\{\frac{q_t^d}{(1-\delta_{t+1})q_t^f\theta_t + q_t^d(1-\theta_t)}\right\} = 1$$

The following proposition characterizes a property of the equilibrium.

Proposition 1.1 *Suppose that q_t^f is constant and greater than β. The equilibrium is characterized by $\overline{R} < 1/\beta$ and households borrow from entrepreneurs.*

Proof 1.1 *See Appendix* **??**

The reason entrepreneurs hold domestic bonds even if their expected return is lower than the intertemporal discount rate is because they face uninsurable risks and bonds provide consumption insurance.

The determination of the equilibrium can be illustrated with Figures 4 and 5. The economy starts with domestic and foreign assets held by entrepreneurs, D_t and F_t, household liabilities, L_t, and the realization of ε_t. The latter will then determine whether households default on their liabilities and the entrepreneurs' wealth becomes $\tilde{D}_t + F_t$. Given post-default wealth, the labor market equilibrium determines the wage rate and the employment through the intersection of demand and supply as shown in Figure 4. The supply of labor is perfectly elastic given the linear specification of households' utility. The demand depends negatively on the wage rate. Its position depends on entrepreneur's wealth $\tilde{D}_t + F_t$.

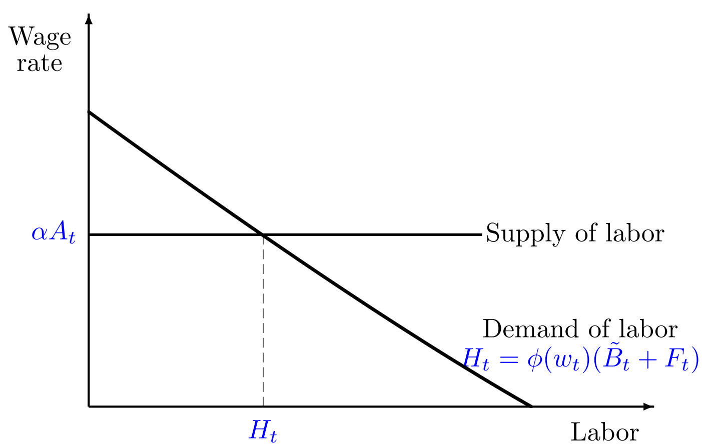


Figure 4: Labor market equilibrium

The labor market equilibrium determines aggregate production and profits which then determine the aggregate wealth of entrepreneurs at the end of the period, that is, $\tilde{D} + F_t + (\mathbb{E}z - w_t)H_t$. This will then determine the demand for financial assets. Together with the supply of financial assets this determines the financial market equilibrium as illustrated in Figure 5.

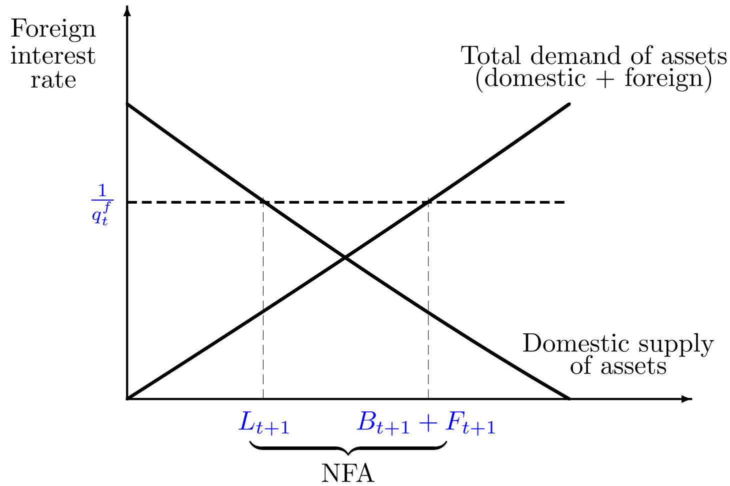


Figure 5: Financial market equilibrium

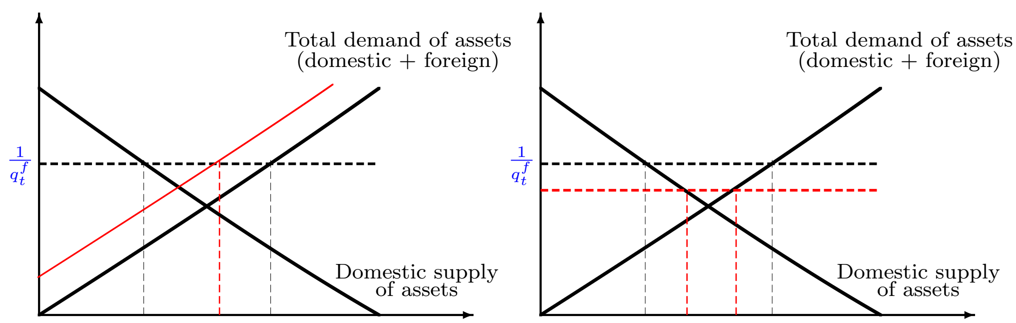
The demand of assets from entrepreneurs, $D_{t+1}+F_{t+1}$, is increasing in the foreign interest rate, $1/q_t^f$. The domestic supply L_{t+1}, instead, is decreasing in the foreign interest rate. Even thought the domestic and foreign interest rates are not equal, the domestic rate $1/q_t^d$ increases with the foreign interest rate, $1/q_t^f$. This explains why the demand of assets is upward sloping in $1/q_t^f$ while the supply is downward sloping.

The dashed thicker line denotes the foreign interest rate which is exogenous in the model. Effectively, this is the supply of foreign assets which is perfectly elastic since the country is a small open economy. The intersection of this line with the demand and domestic supply of assets determines the equilibrium. The difference is the net foreign asset position of the country.

1.4 Effects of financial crises and lower interest rates

We can now use the above graphs to show the effects of a domestic financial crisis and a reduction in the foreign interest rate.

Financial crisis. The first panel of Figure 6 shows the new equilibrium after a domestic financial crisis. The first effect is to reduce entrepreneurs' wealth, $\tilde{B}_t + F_t$. Due to default, entrepreneurs incur losses in the domestic component of their wealth, that is, $\tilde{D}_t < D_t$. This reduces the demand for assets held in the next period, which is shown in the graph by the shift to the left of the demand function. As a result, the next period wealth of entrepreneurs also drops. This, in turn, implies that the demand for labor declines not only in the current period (due to the lower value of $\tilde{D}_t$) but also in the next period. Eventually, in absence of further crises, the demand for assets will return to the initial position as entrepreneurs rebuild their wealth through savings. However, this takes time. In the meanwhile, the net foreign asset position of the country (the difference between $D_{t+1} + F_{t+1}$ and L_{t+1}) declines.



(a) Financial crisis

(b) Lower interest rate

Figure 6: Equilibrium after a financial crisis and after a lower interest rate.

In 2018, however, the global financial crisis was somewhat external to emerging countries. The global crisis also induced capital losses for the holders of liabilities issued by industrialized countries. This can be captured in the model with an unanticipated drop in the value of foreign assets, that is, F_t. The macroeconomic consequences for emerging countries are similar to the domestic financial crisis just described: lower entrepreneurial wealth would generate a persistent decline in employment and economic activity. The difference is that, while in a domestic crisis there is internal redistribution (from entrepreneurs to households), in an external crisis the redistribution is external (from entrepreneurs to foreign borrowers). Therefore, in aggregate, the welfare impact of a foreign crisis could be worse.

Lower interest rates. The second panel of Figure 6 illustrates the effect of a lower interest rate in industrial countries. This is captured by a shift to the bottom of the ticker horizontal line.

The consequence of the lower interest rate is to decrease the demand for assets from entrepreneurs and to increase the supply of domestic assets from households. The net foreign asset position of the country declines, meaning lower capital outflows in emerging countries. However, since the lower outflows are associated with higher domestic borrowing from households and lower savings from entrepreneurs (so that entrepreneurs end up holding less financial wealth) the demand for labor declines and the country experiences a macroeconomic contraction.

2 Quantitative analysis

The model is calibrated annually using data for the period 1991-2005. Starting in 2005, I simulate the model until 2017. The list of industrialized and emerging countries is provided in Figure 2.

2.1 Calibration

The discount factor is set to $\beta = 1/1.07 = 0.9346$, implying an annual intertemporal discount rate close to 7%.

Total production is the sum of entrepreneurial output, $A_t H_t$, and housing services, $A_t \overline{K}$. Thus, aggregate output is $Y_t = A_t(H_t + \overline{K})$. Because in the model there is no capital accumulation, the empirical counterpart of aggregate output is Gross Domestic Product minus Investment. I start with the assumption that A_t is constant and normalized to 1. This should be interpreted as the de-trended value of aggregate productivity for the group of emerging countries during the period 1991-2005.

To pin down the value of $\overline{K}$ I use the share of housing services in net GDP (net of investment), which in the model is equal to $\overline{K}/(H_t + \overline{K})$. Unfortunately, data for the share of housing services is not available for many countries. To obviate this problem, I impose that emerging countries have the same share of housing services in output (GDP minus investment in the data) and use the US share as the calibration target. Based on NIPA data, the average share of housing services in net GDP over the period 1991-2005 is 12.1%. Thus, I calibrate $\overline{K}$ using the condition

$$\frac{\overline{K}}{\overline{H} + \overline{K}} = 0.121,$$

where $\overline{H}$ is the average employment-to-population ratio over the period 1991-2005 for emerging countries. Using data from World Development Indicators (WDI) I set $\overline{H} = 0.441$.

The probability that the liquidation price drops to ξA_t, which I interpret as a crisis, is set to $\lambda = 0.02$. Thus, crises are very low probability events. On average, one every fifty years. Similar numbers have been used in the literature. See for example Bianchi and Mendoza (2013).

The stochastic process for the uninsurable idiosyncratic productivity z_t^i follows a truncated normal distribution with mean $A_t = 1$ and standard deviation σ_z. The standard deviation determines the 'demand' of assets (in the spirit of Mendoza, Quadrini, and Ríos-Rull (2009)). Higher values of σ_z increase the demand for domestic and foreign bonds. I set the standard deviation of the idiosyncratic shock to 0.1 which in consistent with empirical measures of firm-level volatility. The parameter ξ, instead, determines the recovery value of loans when the housing market drops and there is default. This in turns determines the incentive of households to borrow and, therefore, the ability of the country to create financial assets (in the spirit of Caballero, Farhi, and Gourinchas (2008)). Another parameter that affects the creation of financial assets is the cost of renegotiation captured by the parameter χ. Unfortunately, I do not have direct information to calibrate this parameter and I set it to $\chi = 5$. To calibrate ξ I then use the ratio of domestic credit to net GDP, which in the model corresponds to L_{t+1}/Y_t. For the group of emerging countries during the period 1991 to 2005 this ratio is equal to 0.753.

Finally the price of foreign bonds is chosen so that the net foreign asset position of emerging countries is equal to the 1991-2005 average. The average, as a percentage of net GDP, is equal to 17.4. The required value is $q_t^f = 0.9421$ which corresponds to an interest rate of about 4 percent. The full list of parameter values are shown in Table 1.

2.2 Quantitative results

Simulations are based on a sequence of random draws for the variable ε_t (sunspot shock). With probability $\lambda = 0.02$ the random draw is $\varepsilon_t = 0$ and the liquidation price is $\tilde{p}_t = \xi$; with probability $1 - \lambda = 0.98$ the random draw is $\varepsilon_t = 1$ and the liquidation price is $\tilde{p}_t = p_t$.

The focus of the simulations is over the period 2005-2017. However to alleviate the impact of initial conditions determined by the initial states, I start each simulation 100 periods before 2005. Thus, each simulation is

Table 1: Parameter values.

Description	*Value*
Intertemporal discount factor	$\beta = 0.9346$
Price of foreign bonds	$q^f = 0.9421$
Mean and standard deviation idiosyncratic shock	$A = 1$, $\sigma_z = 0.1$
Fixed supply of houses	$\overline{K} = 0.0607$
Labor dis-utility	$\alpha = 1.007$
Probability of crisis	$\lambda = 0.020$
Liquidation price in crisis	$\xi = 0.385$
Renegotiation cost parameter	$\chi = 5.000$

comprises 113 periods with the last 13 periods corresponding to 2005-2017. In each simulation I keep the price of the foreign bond constant at the calibrated value of $q^f = 0.9421$ until 2008. After 2008 the price increases by 2 percentage points (which corresponds to a 2 percent drop in the foreign interest rate).

In absence of the sunspot shock ε_t, the dynamics of the economy would be solely driven by changes in the foreign interest rate. The sunspot shock adds another source of macroeconomic dynamics. The dynamics would then depend on the actual realizations of the shock. To illustrate how the sunspot shock affects the stochastic properties of the model, I repeat the simulation 1,000 times, with each simulation conducted over 113 years (the first 100 periods to reduce the effects of initial conditions and the last 13 periods corresponding to 2005-2017).

Simulation results Figure 7 plots the average as well as the 5th and 95th percentiles of the 1,000 repeated simulations. The range of variation between the 5th and 95th percentiles indicates volatility at any point in time.

Let's focus first on the average of the repeated simulation (continuous line). As a result of the interest rate drop in industrialized countries, households borrow more. Entrepreneurs, however, have less incentive to save and their wealth starts to decline. Since households borrow more while entrepreneurs save less, the country exports less capital. As can be seen in the fourth panel, the net foreign asset position of the country switches from positive to negative. Entrepreneurs are now borrowing from abroad, while at the same time they hold the domestic debt issued by households. Therefore, part of the households' debt is now funded, indirectly, by foreigners.

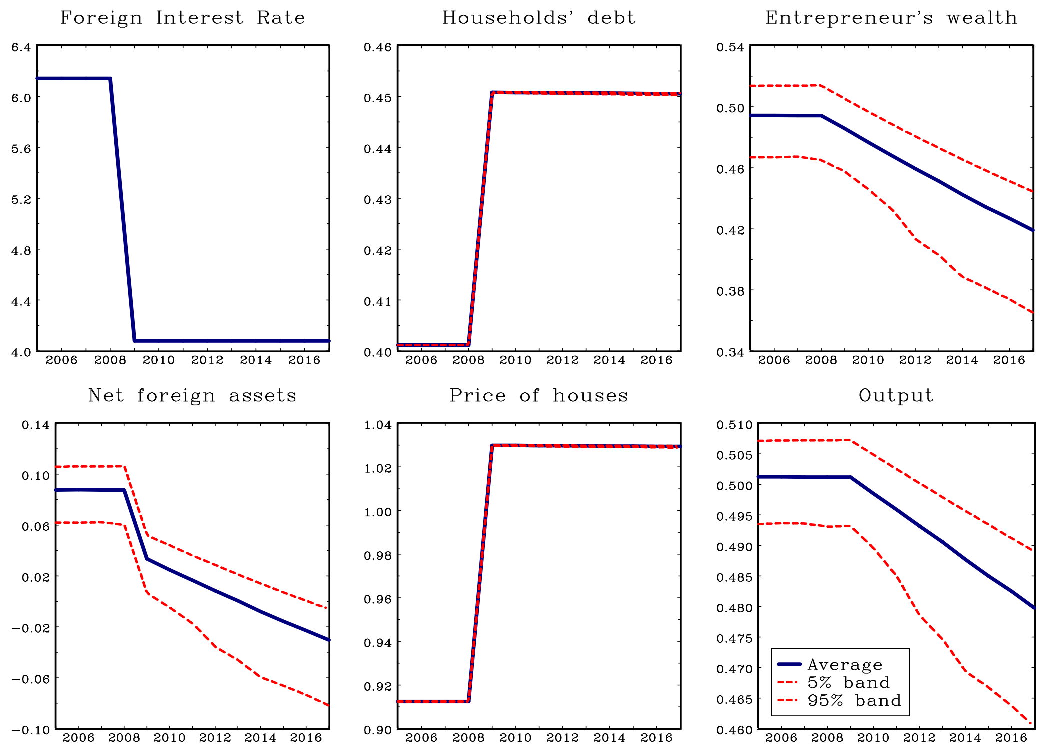


Figure 7: Response to low interest rates in Industrialized Countries, 2005-2017.

The lower interest rate has also a positive effect on housing prices since it lowers the cost to finance houses. The macroeconomic impact, however, is negative. As shown in the last panel of Figure 7, output declines on average. This is a direct consequence of the lower entrepreneurial savings shown in the third panel: as entrepreneurs hold less wealth, they choose a smaller scale of production in order to reduce the risk.

We can now look at the dashed lines of Figure 7 which denote the 5th and 95th percentiles of the repeated simulations. The distance between these two lines provides a measure of the underlying macroeconomic volatility. As can be seen in the last panel, the distance widens. This shows that lower interest rates not only reduce the level of economic activity but also increase volatility (higher fragility). This is because the economy becomes more leveraged and when a crisis materializes, the wealth losses incurred by entrepreneurs (due to default) are larger. Larger capital losses then cause larger effects on output.

3 Endogenous growth

So far I have assumed that the productivity variable A_t is exogenous and constant. I now extend the model by allowing A_t to change endogenously over time.

Following the endogenous growth literature, I introduce a production externality that depends on the aggregate inputs of production. In the simpler version of the endogenous growth model—the AK model—the production function takes the form

$$y_t = A_t k_t^{\alpha},$$

where k_t is the 'individual' input of capital and productivity $A_t = K_t^{1-\alpha}$ depends on the 'aggregate' input of capital K_t. As the economy accumulates more capital, productivity increases and this leads to persistent growth.

In the model used in this paper, the production input is labor. Therefore, I assume that the externality is in the aggregate input of labor instead of capital. Furthermore, since labor is not reproducible, I assume that the input of labor affects the 'growth rate' of aggregate productivity rather than its 'level', that is,

$$\frac{A_{t+1}}{A_t} = \kappa H_t.$$

One way to interpret this formulation is that there is learning-by-doing: higher labor (hours or employment) increases human capital which in turn affects productivity. Reinterpreting K_t as the stock of human capital, we would then have $A_t = K_t$ with human capital evolving according to $K_{t+1} = \kappa K_t H_t$. The only difference with the more standard AK model is that the increase in (human) capital is not determined by savings but by the number of workers and/or time spent in the working place.

Simulation. Compared to the previous version of the model we have only one additional parameter, κ. We set this parameter so that the average growth rate pre-2005 is zero. This should be interpreted as the detrended version of the model. Keeping all other parameters with the same values as in the previous simulations, the required calibration value is $\kappa = 2.268$.

Figure 8 plots the growth rate of output in response to the lower interest rate in industrialized countries. By making the growth rate of productivity endogenous, the model generates a slow down in growth as a result of the lower interest rates. The mechanism leading to the slow down operates through the reduction in savings. Since financial assets have a lower return, entrepreneurs choose to hold less financial wealth. But when entrepreneurs hold less financial wealth they are less willing to take on production risk and reduce the scale of production. Through the externality, then, the lower production scale translates in lower productivity growth.

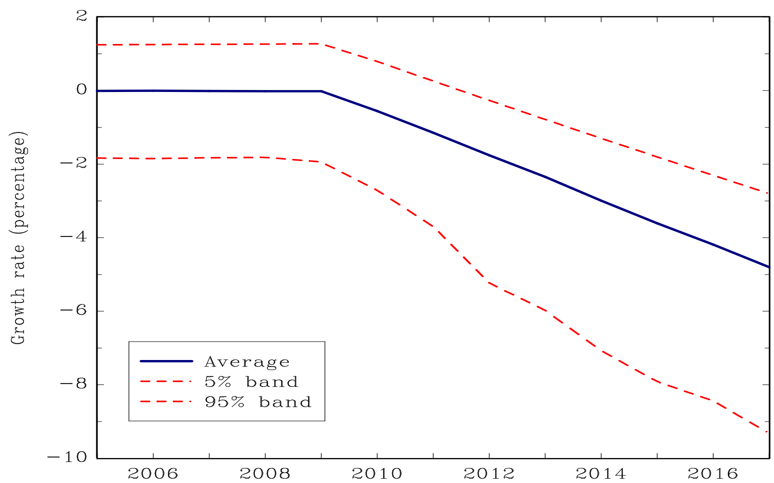


Figure 8: Response to low interest rates in Industrialized Countries, 2005-2017. Model with endogenous growth.

Although the growth dynamics shown in Figure 8 does not prove that the slow down experienced by emerging countries was caused by the lower interest rates in industrialized countries, it is consistent with the theory proposed in this paper.

Figure 8 also shows that the volatility of growth increases. This is shown by the widening band between the 5th and 95th percentiles for the 1,000 repeated simulations. Thus, lower interest rates cause slower and more volatile growth.

4 Spill over of crises to emerging countries

The current account surplus experienced by emerging countries before the financial crisis allowed an accumulation of financial assets issued by industrialized countries. These assets, however, lost market value during the financial crisis, which translated in significant capital losses for emerging countries.

An example is given by mortgage-based securities sold to investors in many countries including emerging economies. In some instances, the losses were experienced by financial institutions such as banks in addition to firms, given their international interconnectivity. Llaudes, Salman, and Chivakul (2010) find that the impact of the global crisis on emerging market economies was more pronounced in countries with greater financial and trade linkages. Using the model I can explore how the capital losses experienced by emerging countries as a result of the financial linkage affected the macroeconomic performance of these economies.[5]

Figure 9 shows the response of the growth rate of output in emerging countries after a 50 percent drop in the value of foreign assets as a consequence of the 2008 financial crisis. The left panels assume that the foreign interest rate does not change (industrial countries do not react to the crisis by changing monetary policy). The right panels, instead, assume that industrialized countries change monetary policy in response to the crisis.

The foreign crisis and the associated capital losses lead to a decline in the growth rate of output which is quite persistent. This is because it takes a long time for entrepreneurs to rebuilt their wealth through savings. Without the reduction in the foreign interest rate, growth recovers over time (although slowly). With loose monetary policy in industrialized countries, instead, the growth rate of output continues to decline after the crisis.

The impact monetary policy predicted by the model contrasts with the more conventional view that lower interest rates were important to lessen the negative effects of the global crisis on emerging countries. Although it is beyond the scope of this paper, it would be interesting to investigate the significance and relative importance of these two mechanisms.

5 Conclusion

In this paper I have shown that low interest rate policies adopted by industrialized countries may have impacted negatively the economic performance of emerging countries. Although lower interest rates in industrialized countries could cause a reduction in net outflows of capital in emerging countries, lower interest rates also reduce savings, a channel ignored by the conventional view about the transmission of monetary policy.

[5]Of course, given the absence of a significant trade linkage in the model, I cannot explore this additional channel which could have also played an important role.

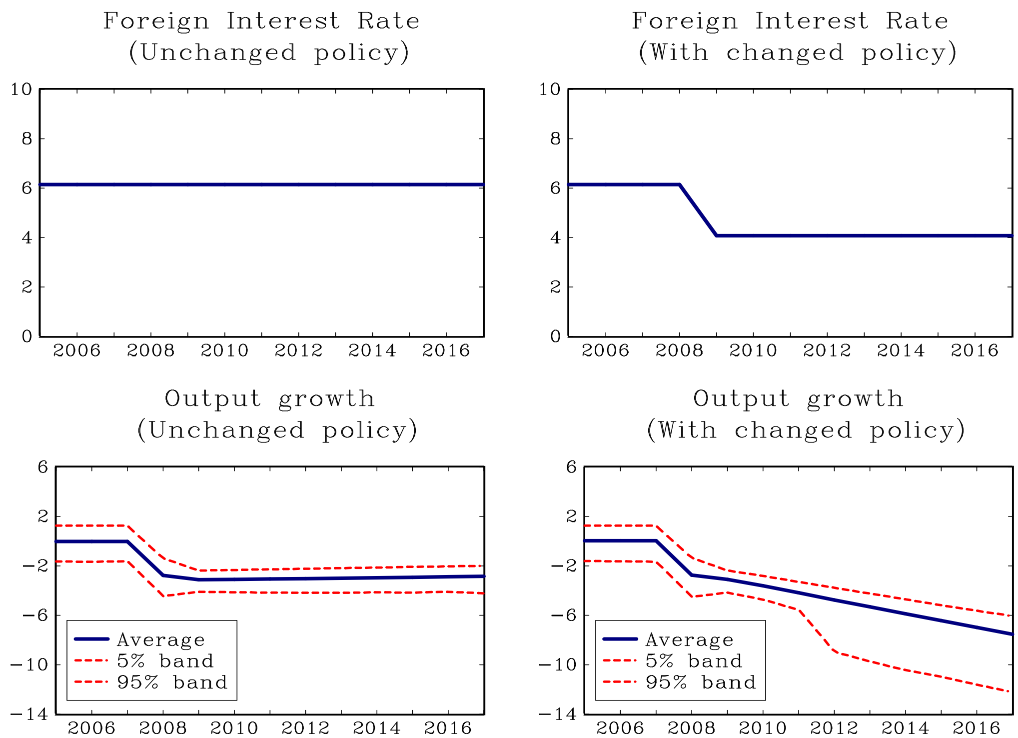


Figure 9: Response to foreign financial crisis leading to a loss in f_t in 2008.

Although the theory proposed in this paper emphasizes the negative consequences of capital inflows, this should not be interpreted as suggesting that capital controls are desirable. In this paper I only showed that capital inflows could have negative consequences for macroeconomic stability and growth if the inflows are caused by external factors. In particular, I focused on external monetary policy. However, if the inflows are driven by higher growth prospects for emerging countries, they could be beneficial as they accelerate their growth by funding investments. As far as the post-crisis period is concerned, however, it does not appear that the higher inflows to (or lower outflows from) emerging countries were caused by higher growth prospects, at least not ex-post.

Appendix

A Proof of Lemma 1.1

Ignoring the agent superscript i, the optimization problem of an entrepreneur can be written recursively as

$$\begin{aligned} \Omega_t(f_t, b_t) &= \max_{h_t} \mathbb{E}_t \tilde{\Omega}_t(a_t) \\ & \text{subject to} \\ & a_t = f_t + \tilde{b}_t + (z_t - w_t)h_t \\ & \tilde{b}_t = (1-\delta_t)b_t \end{aligned} \tag{14}$$

$$\begin{aligned} \tilde{\Omega}_t(a_t) &= \max_{f_{t+1}, b_{t+1}} \Big\{ \ln(c_t) + \beta \mathbb{E}_t \Omega_{t+1}(f_{t+1}, b_{t+1}) \Big\} \\ & \text{subject to} \\ & c_t = a_t - q_t^f f_{t+1} - q_t^b b_{t+1} \end{aligned} \tag{15}$$

Since the information set changes from the beginning of the period to the end of the period, the optimization problem has been separated according to the available information. In sub-problem (14) the entrepreneur chooses the input of labor before knowing the productivity z_t. The variable δ_t is an aggregate stochastic variable that denotes the possible losses incurred by the entrepreneur at the beginning of the period. This is taken as given by an individual entrepreneur. In sub-problem (15) the entrepreneur allocates the end of period wealth in consumption and savings after observing z_t.

The first order condition for sub-problem (14) is

$$\mathbb{E}_t \frac{\partial \tilde{\Omega}_t}{\partial a_t}(z_t - w_t) = 0.$$

The envelope condition from sub-problem (15) gives

$$\frac{\partial \tilde{\Omega}_t}{\partial a_t} = \frac{1}{c_t}.$$

Substituting in the first order condition we obtain

$$\mathbb{E}_t \left(\frac{z_t - w_t}{c_t} \right) = 0. \tag{16}$$

At this point we proceed by guessing and verifying the optimal policies for employment and savings. The guessed policies take the form:

$$\begin{aligned}
h_t &= \phi_t(f_t + \tilde{b}_t) && (17)\\
c_t &= (1-\beta)a_t && (18)\\
q_t^b b_{t+1} &= \theta_t \beta a_t && (19)\\
q_t^f f_{t+1} &= (1-\theta_t)\beta a_t && (20)
\end{aligned}$$

Since $a_t = f_t + \tilde{b}_t + (z_t - w_t)h_t$ and the employment policy is $h_t = \phi_t(f_t + \tilde{b}_t)$, the end of period wealth can be written as $a_t = [1 + (z_t - w_t)\phi_t](f_t + \tilde{b}_t)$. Substituting the guessed consumption policy we obtain

$$c_t = (1-\beta)\Big[1 + (z_t - w_t)\phi_t\Big](f_t + \tilde{b}_t). \qquad (21)$$

This expression is used to replace c_t in the first order condition (16) to obtain

$$\mathbb{E}_t\left[\frac{z_t - w_t}{1 + (z_t - w_t)\phi_t}\right] = 0, \qquad (22)$$

which is the condition stated in Lemma 1.1.

To complete the proof, we need to show that the guessed policies (17) and (18) satisfy the optimality condition for the choice of consumption and saving. This is characterized by the first order conditions of sub-problem (15), which is equal to

$$\begin{aligned}
-\frac{q_t^f}{c_t} + \beta \mathbb{E}_t \frac{\partial \Omega_{t+1}}{\partial f_{t+1}} = 0,\\
-\frac{q_t^b}{c_t} + \beta \mathbb{E}_t \frac{\partial \Omega_{t+1}}{\partial b_{t+1}} = 0.
\end{aligned}$$

From sub-problem (14) we derive the envelope conditions

$\partial\Omega_t/\partial f_t = 1/c_t$ and $\partial\Omega_t/\partial b_t = \mathbb{E}_t[(1-\delta_t)/c_t]$ which can be used in the first order conditions to obtain

$$\begin{aligned}
\frac{q_t^f}{c_t} &= \beta \mathbb{E}_t \frac{1}{c_{t+1}},\\
\frac{q_t^b}{c_t} &= \beta \mathbb{E}_t \frac{1-\delta_{t+1}}{c_{t+1}}.
\end{aligned}$$

We have to verify that the guessed policies satisfy this condition. Using the guessed policy (18) and equation (21) updated one period, the first order conditions can be rewritten as

$$\frac{q_t^f}{a_t} = \beta \mathbb{E}_t \frac{1}{[1+(z_{t+1}-w_{t+1})\phi_{t+1}](f_{t+1}+\tilde{b}_{t+1})},$$
$$\frac{q_t^b}{a_t} = \beta \mathbb{E}_t \frac{1-\delta_{t+1}}{[1+(z_{t+1}-w_{t+1})\phi_{t+1}](f_{t+1}+\tilde{b}_{t+1})}.$$

Since z_{t+1} is independent of δ_{t+1} and $\tilde{b}_{t+1}$, the first order conditions can be rewritten as

$$\frac{q_t^f}{a_t} = \beta \mathbb{E}_t \left\{ \frac{1}{1+(z_{t+1}-w_{t+1})\phi_{t+1}} \right\} \mathbb{E}_t \left\{ \frac{1}{f_{t+1}+\tilde{b}_{t+1}} \right\},$$
$$\frac{q_t^b}{a_t} = \beta \mathbb{E}_t \left\{ \frac{1}{1+(z_{t+1}-w_{t+1})\phi_{t+1}} \right\} \mathbb{E}_t \left\{ \frac{1-\delta_{t+1}}{f_{t+1}+\tilde{b}_{t+1}} \right\}.$$

Condition (22) implies that the first term on the right-hand-side is 1. Therefore, we can rewrite the first order conditions as

$$\frac{q_t^f}{\beta a_t} = \mathbb{E}_t \left\{ \frac{1}{f_{t+1}+\tilde{b}_{t+1}} \right\},$$
$$\frac{q_t^b}{\beta a_t} = \mathbb{E}_t \left\{ \frac{1-\delta_{t+1}}{f_{t+1}+\tilde{b}_{t+1}} \right\}.$$

Now we can use $q_t^f f_{t+1} = (1-\theta)\beta a_t$ and $q_t^b b_{t+1} = \theta\beta a_t$ in the two conditions to obtain

$$\begin{aligned}
\mathbb{E}_t \left\{ \frac{q_t^b}{(1-\delta_{t+1})q_t^f\theta + q_t^b(1-\theta)} \right\} &= 1, \\
\mathbb{E}_t \left\{ \frac{(1-\delta_{t+1})q_t^f}{(1-\delta_{t+1})q_t^f\theta + q_t^b(1-\theta)} \right\} &= 1,
\end{aligned}$$

where the first equation corresponds to the second condition reported in Lemma 1.1. *Q.E.D.*

B First order conditions for households

The optimization problem of a household is

$$W_t(l_t, k_t) = \max_{h_{t+1}, l_{t+1}, k_{t+1}} \Big\{ c_t - A_t h_t + \beta \mathbb{E}_t W_{t+1}(l_{t+1}, k_{t+1}) \Big\}$$

subject to

$$c_t = \frac{\mathbb{E}_t \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\overline{R}_t} + w_t h_t + (A_t + p_t) k_t - \tilde{l}(l_t, \tilde{p}_t k_t) - \varphi\left(\frac{l_t}{\tilde{p}_t k_t}\right) l_t - p_t k_{t+1}.$$

The first order conditions with respect to h_t, l_{t+1}, k_{t+1} are, respectively,

$$w_t = A_t,$$

$$\frac{1}{\overline{R}_t} \frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial l_{t+1}} + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial l_{t+1}} = 0,$$

$$\frac{1}{\overline{R}_t} \frac{\partial \mathbb{E} \tilde{l}(k_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial k_{t+1}} - p_t + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial k_{t+1}} = 0.$$

The envelope conditions are

$$\frac{\partial W_t(l_t, k_t)}{\partial l_t} = -\frac{\partial \tilde{l}(l_t, \tilde{p}_t k_t)}{\partial l_t} - \frac{\partial \varphi\left(\frac{l_t}{\tilde{p}_t k_t}\right)}{\partial l_t} l_t - \varphi\left(\frac{l_t}{\tilde{p}_t k_t}\right),$$

$$\frac{\partial W_t(l_t, k_t)}{\partial k_t} = A_t + p_t - \frac{\partial \tilde{l}(l_t, \tilde{p}_t k_t)}{\partial k_t} - \frac{\partial \varphi\left(\frac{l_t}{\tilde{p}_t k_t}\right)}{\partial k_t} l_t.$$

Updating by one period and substituting in the first order conditions for l_{t+1} and k_{t+1} we obtain

$$\frac{1}{\overline{R}_t} = \beta \left[1 + \frac{\frac{\partial \mathbb{E} \varphi\left(\frac{l_{t+1}}{\tilde{p}_{t+1} k_{t+1}}\right)}{\partial l_{t+1}} l_{t+1}}{\frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial l_{t+1}}} + \frac{\mathbb{E} \varphi\left(\frac{l_{t+1}}{\tilde{p}_{t+1} k_{t+1}}\right)}{\frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial l_{t+1}}} \right] \tag{23}$$

$$p_t = \beta \mathbb{E}\Big(A_{t+1} + p_{t+1} \Big) + \beta \left[\left(\frac{1}{\beta \overline{R}_t} - 1 \right) \frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial k_{t+1}} - \frac{\mathbb{E} \varphi\left(\frac{l_{t+1}}{\tilde{p}_{t+1} k_{t+1}}\right)}{\partial k_{t+1}} l_{t+1} \right] \tag{24}$$

Let's focus on $\tilde{l}(l_{t+1}, \tilde{p}_{t+1}k_{t+1})$ and $\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1})$ defined in (5) and (6). Assuming that the optimal choice of l_{t+1} and k_{t+1} satisfy $l_{t+1} < p_{t+1}k_{t+1}$ and $l_{t+1} > \xi A_t k_{t+1}$, we have

$$
\begin{aligned}
\frac{\partial \mathbb{E}\tilde{l}(l_{t+1}, \tilde{p}_{t+1}k_{t+1})}{\partial l_{t+1}} &= 1 - \lambda \\
\frac{\partial \mathbb{E}\tilde{l}(l_{t+1}, \tilde{p}_{t+1}k_{t+1})}{\partial k_{t+1}} &= \lambda \xi A_t \\
\mathbb{E}\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1}) &= \lambda\chi \left(1 - \frac{\xi A_t k_{t+1}}{l_{t+1}}\right)^2 \\
\frac{\partial \mathbb{E}\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1})}{\partial l_{t+1}} l_{t+1} &= 2\lambda\chi \left(1 - \frac{\xi A_t k_{t+1}}{l_{t+1}}\right) \frac{\xi A_t k_{t+1}}{l_{t+1}} \\
\frac{\partial \mathbb{E}\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1})}{\partial k_{t+1}} l_{t+1} &= -2\lambda\chi \left(1 - \frac{\xi A_t k_{t+1}}{l_{t+1}}\right) \xi A_t
\end{aligned}
$$

We can see that the first two terms do not depend on l_{t+1} and k_{t+1}, while the last three terms are functions of the ratio l_{t+1}/k_{t+1}. Therefore, we can express the first order conditions (23) and (24) as,

$$
\frac{1}{\overline{R}_t} = \beta + \Phi_t \left(\frac{l_{t+1}}{k_{t+1}}\right) \tag{25}
$$

$$
p_t = \beta \mathbb{E}\Big(A_{t+1} + p_{t+1}\Big) + \Psi_t \left(\frac{l_{t+1}}{k_{t+1}}\right). \tag{26}
$$

It can be verified that the functions $\Phi_t(.)$ and $\Psi_t(.)$ are both increasing for $l_{t+1}/k_{t+1} > \xi A_t$. The time subscript takes into account the dependence on the aggregate state A_t. Conditions (25) and (26) are the equivalent of (9) and (10). *Q.E.D.*

C Market for liquidated houses

The functioning of the market for liquidated houses is characterized by two assumptions.

Assumption 1 *Houses can be sold either to other domestic households or domestic entrepreneurs. If sold to entrepreneurs, houses lose their functionality and must be converted to consumption goods at rate ξA_t.*

This assumption formalizes the idea that houses may lose value when reallocated to owners that do not use them directly. In the model this is proxied by assuming that entrepreneurs convert houses in consumption goods at rate ξA_t, which is typically lower than the price of houses in normal times.[6]

The parameter ξ determines the liquidation price of houses when the housing market freezes. It is important to point out that, in order for houses not to lose their functionality, they need to be purchased by domestic households. The question is then whether households have the capability of purchasing liquidated houses. This is established in the next assumption.

Assumption 2 *Households can purchase liquidated houses only if $l_t < \tilde{p}_t k_t$.*

If a household starts with liabilities that are bigger than the liquidation value of its own house, that is, $l_t > \tilde{p}_t k_t$, the household will be unable to raise additional funds to purchase the liquidated houses of other households. Potential lenders know that the new loan (as well as the outstanding liabilities) is not collateralized and the household will renegotiate immediately after taking the new loan. I refer to a household for which $l_t > \tilde{p}_t k_t$ as 'illiquid' since it cannot raise any funds.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating, $l_t \leq p_t k_t$. Furthermore, assume that $p_t > \xi A_t$, that is the price of houses in normal time, p_t is bigger than the value of houses for entrepreneurs. If this condition is satisfied, households have the ability to raise funds to purchase the house of a defaulting household. This insures that the market price for the liquidated house is p_t. However, if $l_t > \xi A_t k_t$ for all households, there will be no household capable of participating in the market. As a result, the liquidated house can only be sold to entrepreneurs at price $\tilde{p}_t = \xi A_t$.

This shows that the value of liquidated houses depends on the financial decision of households, which in turn depends on the price. This interdependence creates the conditions for multiple self-fulfilling equilibria.

Proposition C.1 *There exists multiple equilibria only if $l_t > \xi A_t$.*

When multiple equilibria are possible, the equilibrium is selected through the random draw of sunspot shocks.

Let ε_t be a variable that takes the value of 0 with probability λ and 1 with probability $1 - \lambda$. If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price $\tilde{p}_t = \xi A_t$ when $\varepsilon_t = 0$. Thus, the probability distribution of the low liquidation price is

$$\Upsilon_{t-1}\Big(\tilde{p}_t = \xi A_t\Big) = \begin{cases} 0, & \text{if} \quad l_t \leq \xi A_t k_t \\ \\ \lambda, & \text{if} \quad \xi A_t k_t < l_t \end{cases}$$

[6] Since the supply of houses $\overline{K}$ is fixed, while the services from houses depend on productivity, the price of houses grows with productivity.

If leverage is sufficiently small ($l_t/\xi A_t k_t < 1$), households remain liquid even if the (expected) liquidation price is ξA_t. But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when the leverage is high, the liquidity of households depends on the price. In this case the realization of ε_t becomes important for selecting one of the two equilibria. When $\varepsilon_t = 0$—which happens with probability λ—the market expects the liquidation price to be ξA_t, making the household's sector illiquid. On the other hand, when $\varepsilon_t = 1$—which happens with probability $1 - \lambda$—the market expects the liquidation price to be the one that prevails in the market with the participation of households, validating the expectation of the high liquidation price.[7] If the leverage is very large, however, households are always illiquid and the equilibrium price is ξA_t.

Notice that the argument is based on the assumption that ξ is sufficiently low (implying that $\xi A_t < p_t$). Also, the equilibrium value of houses at the end of the period, $p_t k_t$, is always bigger than the debt, l_t. Condition (7) determining the interest rate guarantees that this will always be satisfied in equilibrium. Furthermore, assuming that l_t is always bigger than ξA_t, the liquidation price $\tilde{p}_t$ fluctuates between p_t and ξA_t as assumed in (4).

[7] The assumption that houses lose their functionality if sold to foreign households, in addition to entrepreneurs, allows me to have equilibrium in which the default happens only in one country. If houses could maintain their functionality when sold to foreign entrepreneurs, implies that default in one country could arise only if the other country also defaults. Nevertheless, even if default takes place only in one country, we will see that it impacts the macro-economy of the other country because of the portfolio diversification of entrepreneurs.

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