Debt Sustainability and the Terms of Official Support

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Abstract

During the euro area crisis, the official loans by euro area institutions had longer maturities and lower rates relative to IMF standards. We study how the terms of official loans affect a government's decision to default, bond prices and market access. We show that longer maturities and lower rates facilitate consumption smoothing when countries face rollover risk, and raise the stock of safe debt countries can sustain. However, if this translates into higher debt hence lower consumption in the long run, official lending can make debt repayment less attractive during periods of persistent adverse shocks. Quantitatively, the model is able to replicate the Portuguese debt and rate dynamics through the crisis years.
Keywords: Sovereign debt, default, debt maturities, spread, rollover risk, bailouts, euro area crisis

JEL Codes: F33, F34, F45, H12

1 Introduction

In response to the sovereign debt crises that shook the euro area in the aftermath of the Great Recession, the governments of Cyprus, Greece, Ireland and Portugal received funds from both the International Monetary Fund (IMF) and newly created euro area bailout funds: the European Financial Stability Facility at first, then the European Stability Mechanism (ESM). The country programs in the euro area deviated significantly from consolidated international practices. In the IMF practice, programs are typically designed over a three to ten year horizon, with loans issued at significant premia over funding costs, rising over time. Relative to IMF standards, ESM loans were issued with longer maturities and lower interest rates. In addition, the size of the programs can far exceed the amount permissible under IMF rules.[1].

The scale and modalities of the official assistance provided by euro area institutions reignited a long-standing debate on sovereign debt sustainability and the design of international financial rescues (see Conesa and Kehoe (2014); Tirole (2015); D'Erasmo et al. (2016); Collard et al. (2015); Gourinchas and Martin (2017); Roch and Uhlig (2018) among others). The euro area crisis can indeed be seen as a large-scale experiment with international bailouts, creating an opportunity for reconsidering critical and understudied issues in international financial assistance.

This paper studies the effects of varying the terms of official lending on debt sustainability, market access conditions, and a country's vulnerability to crises. Our point of departure is the notion that official loans affect a government's incentives to issue, repay and default on debt, hence they matter for how much debt a country can sustain, just like tax capacity and default costs. The extent to which official loans impinge on a government fiscal decision may differ, however, depending on their maturity, spreads and size, raising questions in the underlying economic mechanisms, and the policy trade-off they may create.

To investigate these issues, we introduce official lending institutions in a model of sovereign debt and default after Conesa and Kehoe (2017). We use this framework to gain analytical insight on how official lending may restore debt sustainability, i.e. induce a government to choose higher surpluses over default, in the face of both non-fundamental (rollover) risk in the debt market, and fundamental (output) risk. Quantitatitely, we show that the model can account for the the dynamic of the sovereign risk crisis in Portugal in 2011 after the country received official lending. Based on this exercise, we carry out counter-factual exercises to explore the sensitivity of our results to changing the maturity and interest rates of loans.

Our main results are as follows. First, we show that the availability of official bailout

[1] For a detailed account of the evolution of official lending in Europe, see Corsetti et al. (2017)

funds raises the threshold below which debt is default-free, and helps high-debt countries facing rollover and fundamental risk to smooth consumption while reducing their liabilities towards this "safe-debt" threshold. However, in so far as the government has no welfare incentive to reduce its liabilities below this threshold, higher levels of safe debt translate into lower long-run consumption and thus lower welfare in states of the world where the sovereign chooses to repay. This means that while official debt can mitigate rollover risk during a deleveraging process, if lending is not properly structured and managed, it can also reduce the incentives to prefer repayment over default in response to adverse fundamental shocks. These contrasting effects of official assistance on debt sustainability unveil a critical trade-off, reflecting a basic form of "moral hazard" that arises independently of modelling agency problems (typically requiring the specification of costly effort/policy action by the government). The result also underscores the importance of modelling both rollover and output risk in the same analytical framework when addressing issues in debt sustainability.

Second, we carry out a quantitative assessment of official lending on borrowing costs and borrowing capacity during the euro area crisis, focused on Portugal. Without official lending, if a country accumulates more debt, default risk rises and drives up market rates. Upon entering a program, instead, the crisis countries in the euro area could raise their debt while simultaneously benefitting from a fall in market rates. Our model explains how this happened. As the new debt was largely absorbed by the official lenders, market had to finance only a small fraction of the government financing need. By offering long maturity loans, the bailout reduced rollover risk, driving down market rates.[2] In our quantitative section, we show that the model is able to replicate the dynamic evolution of debt and market interest rates in Portugal around the access of this country to the ESM and IMF loans. In our exercises, the debt stock and borrowing costs move in opposite directions, as the country substitutes market lending with official lending, according to the pattern in the data. From a theoretical perspective, our results call attention to the composition of debt (official versus market), as well as the terms of official lending, for debt sustainability. By mitigating borrowing costs and reducing the period-by-period financing need of the government, official financial programs of the kind Portugal received during the crisis reduced the need for severe austerity policies targeting fast deleveraging during a deep recession. However, in line with the trade-off discussed above, we also find evidence that the official loans increased the fragility to adverse fundamental developments—that is, it reduced the debt threshold at which the country defaults when facing output risk.

[2]In addition to the lowering refinancing needs, the long maturity of the loans reduced market concerns that the seniority of official loans would dilute privately-held sovereign claims. By extending repayment beyond the horizon fo market debt, the official sector effectively diluted its de jure senior status.

The extent to which official lenders can affect the sovereign's default decisions depends on their ability to offer a combination of rates that are favorable during times of crisis, and maturities that allow the sovereign borrower to roll over and pay down its debt. To gain insight on the role played by rates and maturity of official loans, having replicated the Portuguese experience, we use the model as a laboratory and assess debt sustainability under different counter-factual policies. Namely, we allow the ESM and IMF either to charge the same interest rate on their loans (first setting both to the IMF rate, before setting both to the ESM rate), or to issue loans with the same maturity (analogously, using first the IMF maturity and then its ESM counterpart). We find, while the ESM lending terms (lower rate and longer maturity) have the strongest effects on debt sustainability, differences in maturity matter more than differences in the interest rates.

On methodological grounds, our contribution consists of constructing and solving a comprehensive model of official lending that allows for both rollover and fundamental sources of sovereign risk.[3] We develop our theory in the framework developed by Conesa and Kehoe (2017), where both types of risk may induce a government to choose to default on its obligations to market creditors. Output risk reflects the fact that business cycles introduce uncertainty in the government's ability and willingness to raise taxes and generate surpluses when in a recession. Rollover risk manifests itself through market lenders' beliefs. For a high enough initial level of debt, the country is in a 'crisis zone:' if investors coordinate their expectations on the belief that the government will not repay, this belief becomes self-fulfilling and the inability to obtain funding drives the borrower to default. When in the crisis zone, a welfare-maximizing government has an incentive to reduce its debt, in order to eliminate rollover risk and so obtain a better price for newly issued debt; however, this incentive is mitigated and may be overturned in a recession, as the government may prefer to run up debt in anticipation of better times, so as to smooth consumption.

Without official assistance, the model identifies four debt "thresholds": two lower thresholds below which debt is default-free, one in an expansion, one in a recession; two upper thresholds beyond which default is the welfare dominant policy option because of weak fundaments, one in an expansion, one in a recession. These thresholds define a safe, a (rollover) crisis and a default zone for the debt stock. In our model, given the three different types of debt the borrower carries once it takes on official debt, the safe, crisis, and default zones become three-dimensional objects—since thresholds are now a function of the composition of debt and the terms of official lending. To gain insight, we characterize these thresholds

[3]The addition of official lending from two different sources is technically challenging as we must keep track of three different debt variables (one for market and two for official debt). This expansion in dimensionality carries over to the way we solve our model.

analytically under mild assumptions simplifying the structure of our model.

We take the model to bear on the dynamic of the Portuguese debt crisis in 2011 and subsequent years. At the height of the debt crisis, the Portuguese debt to GDP ratio was close to 100%. In the second quarter of 2011, borrowing costs were over 700 basis points (bps) higher than the German Bund, with an average maturity of market debt of six years. Throughout the euro area, default risk arose from a widespread fear of serial default across the euro area, arguably reflecting both non-fundamentals and fundamentals factors. [4] The Program with Portugal was signed in April 2001 and adjusted in June-July, as conditions further deteriorated. By July 2011, Portugal could count on an IMF loan with a spread of 300 bps over the SDR rate and a maturity of seven years, and a euro area loan with virtually no spread over funding costs and a maturity of 15 years. The substitution of risky high-yield, short-maturity market instruments with these safer low-yield, long-maturity instruments allowed Portugal to avoid default, extend the terms of engagement, and therefore reduce default risk and the premia that comes with it.[5] To replicate the dynamics of Portuguese debt, we make sure that the model accounts for fundamental and non-fundamental uncertainty. We posit that, initially, the country is in a recession and its debt is well inside the 'crisis zone,' so that the government can borrow from markets only at high rates. We then let the government access official loans up to a total of 25% of the total debt, with the same rates and average duration as IMF and ESM loans. We quantify the unobservable debt thresholds implied by model, contingent on the state of the economy. We are therefore able to provide an assessment of the impact of the rescue package on the values of these thresholds, for our benchmark as well as for a set of policy counterfactuals.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 reviews stylized facts of the sovereign crises in the euro area that motivate our study. Section 4 specifies the model. Section 5 derives analytical insights from a simplified version of it. Section 6 discusses our quantitative exercises. Section 7 concludes.

[4]The Portuguese official loans were funded the European Financial Stability Facility (operated through the Eurogroup) and the European Financial Stability Mechanism (operated directly by the European Commission), the prequels of the European Stability Mechanism (see Corsetti et al. (2017)). As the three funds operate using analogous maturity and pricing terms and the ESM is the one with a permanent character, we refer to the Portuguese bailout as an ESM bailout.

[5]Shortly after receiving these loans, indeed, market spreads on Portuguese bonds started to come down, to the point that, after a couple of years, Portugal started to repay its IMF debt by issuing market bonds at better terms.

2 Literature review

Our paper contributes to the vast body of literature on debt sustainability and sovereign default in at least two dimensions.[6]

First, our paper models official lenders offering different terms on their loans in a dynamic environment, where optimizing governments face both fundamental and rollover risks. Using the euro area crisis as motivating evidence, we are interested in understanding how the terms of lending can be used to enhance the effectiveness of official programs to address default risk. Close to our analysis, in a framework with output and roll-over risk and an official lender, Conesa and Kehoe (2014) shows that an official lender charging penalty rates would not have been able to resolve the euro area crisis, i.e., countries would have not avoided default. Roch and Uhlig (2018) show that even when optimal interventions are fickle, they may eliminate non-fundamental crises using actuarially fair lending prices.[7] The euro area crisis is also studied by Bocola et al. (2019) and Aguiar and Amador (2018) using the Eaton-Gersovitz model (see (Eaton and Gersovitz, 1981)). In this framework, Bocola et al. (2019) show that the observed in domestic public debt helps explain the increase in borrowing costs observed during the first years of the crisis. Aguiar and Amador (2018) argue that official loans that set a price floor for market prices can only be effective if combined with rules preventing official loans when debt is beyond safe levels. In line with our conclusions, these leading contributions (Conesa and Kehoe (2014); Roch and Uhlig (2018); Aguiar and Amador (2018); Bocola et al. (2019) argue that lending policies that result in lower interest rates or contain default costs lead governments to run up more debt, limiting the effects of official lending on sustainability.

The literature also analyzes the so-called catalytic effects of official lending—official lending is said to be catalytic if it enables sovereigns to re-access capital markets. Corsetti et al. (2006); Morris and Shin (2006) defines conditions under which official sector loans can prevent runs on sovereign debt markets. Broner et al. (2014) models how official loans can influence the participation of domestic investors and foreign private creditors in a country debt market, and the effect of portfolio decisions on investment and growth. Sandri (2015)

[6] See D'Erasmo et al. (2016); Aguiar and Amador (2014) for recent overviews. In this literature, a variety of theoretical analyses discuss sustainability issues specific to monetary unions (Aguiar et al., 2015, 2019), interactions with monetary policy and inflation (Aguiar et al., 2013) (Bacchetta et al., 2018), monetary backstop as non-conventional monetary policy (Corsetti and Dedola, 2016), the role of fiscal rules (Hatchondo et al., 2012) as well as of structural reforms accompanying official sector programs (Muller et al., 2015). While all these issues are at least indirectly relevant for our analysis, our model abstracts from them, and we do not discuss them here.

[7] Roch and Uhlig (2018) also show that, while the effects of interventions are stronger if official loans have longer maturities, sovereigns use the additional fiscal space and accumulate more debt, leaving default probabilities unchanged.

focuses on the role of official support in preventing spillovers from sovereign defaults. Dellas and Niepelt (2016) models heterogeneous private lenders and analyzes how the composition of debt shifts towards official sources as countries approach default. Our analysis contributes to this literature by showing how the extent of catalytic effects depends on the terms and conditions of official loans.[8]

Our second contribution is specific to the analysis of the role of debt maturity is determining sustainability and sovereign default. Our model is based on Conesa and Kehoe (2017), which extends Cole and Kehoe (2000), by analyzing self-fulfilling crises with severe recessions of uncertain recovery. In this environment, lengthening debt maturities reduces the region where debt is exposed to self-fulling roll-over crises.[9] Also building on Conesa and Kehoe (2017) framework, Bocola and Dovis (2018) models debt maturity management in the face of fundamental and liquidity risks, and evaluate the strategy of the Italian debt management office throughout the euro area crisis. Arellano and Ramanarayanan (2012) and Broner et al. (2013) show that, when interest rates rise, maturity shortens. In Arellano and Ramanarayanan (2012) this happens because, with imperfect commitment, short-term debt provides stronger incentives to repay and not to dilute current creditors through future debt issuance—with positive feedback effects on borrowing costs. When market rates are high, incentives to repay are important and governments willingly shift towards shorter debt maturities. In Aguiar et al. (2019), during a debt crisis, it is optimal for the government to switch to short-term financing, and only pay back longer debt as it matures. Chatterjee and Eyigungor (2012, 2015) argue that, to overcome the dilution problem that makes long-term debt more expensive, sovereigns should include an absolute priority rule clause on their bonds, giving seniority to earlier lenders.[10] In analyzing debt maturity strategy and default policies, these contributions governments focus on the need to overcome incentives problems and dilution concerns of creditors.

All of these contributions call attention to the trade-off between refinancing risks with borrowing costs—a trade-off that is center-stage in our analysis. Relative to these papers, first, we specifically model how the terms of official support impinges on the flow of liabilities over time, and can substantially alter the stock of debt a government finds it optimal to

[8]Mina and Martinez-Vazquez (2002); Saravia (2013), provides empirical evidence on the relation between sovereign debt maturity and official sector lending. Reinhart and Trebesch (2016) argues that providing longer maturity loans to increasingly stressed sovereigns eventually puts the role of the International Monetary Fund as a lender of last resort at risk.

[9]See Aguiar et al. (2019); Hatchondo and Martinez (2009); Hatchondo et al. (2016); Bai et al. (2015); Mihalache (2016, 2017); Sanchez et al. (2016) for related theoretical analyses of the role of debt maturities in containing the risk of liquidity-driven defaults.

[10]Niepelt (2014), sovereign risk leads sovereigns to issue short-maturity debt when debt issuance is high, output is low and cross-default is more likely.

sustain in the face of output and rollover risk.[11] Second, we explicitly model multiple official lenders with heterogeneous lending terms. This extension poses quantitative challenges, but allows us to study the effect of the terms of official lending on default incentives using euro-crisis countries, receiving assistance by both the IMF and the ESM, as a laboratory. We can thus study how the coexistence of different lenders, determining the size and time profile of debt repayment, impinges on borrowing costs and the government incentives to service its market debt. A notable result from the analysis is that, if official creditors are willing to increase the maturity of their exposure, the likelihood of repayment of market debt maturing in the short run also rises.[12]

3 Official Lending in the Euro Area: Stylized Facts

In this section, we provide a a synthetic account of the creation and evolution of the euro area crisis resolution framework, stressing a set of stylized facts that we use to motivate and discipline our theoretical model. First, we briefly describe the creation of this crisis resolution framework and the key elements of the various programs. Second, we compare and contrast the approach to official support followed by the IMF and the euro area official lenders, with a focus on their lending terms. Finally, we provide stylized facts relating the sovereign debt dynamics in the euro area to the terms of official support. In the Appendix, we provide additional details on the euro area programs by country.

3.1 A Brief Review of Euro Area Official Lending

In 2009 the Greek authorities admitted they had fiddled with the fiscal deficit figures and rapidly lost market access. The first reaction by the euro area authorities was to demand a significant fiscal adjustment. As this failed and the situation spun out of control, in March 2010 euro area governments, together with the IMF, agreed to provide financial assistance, setting up the Greek Loan Facility. This first program consisted of IMF credit and bilateral loans by other euro area members, for a total of 110 billion euros including a 30 billion euros

[11] From an empirical perspective, Dias et al. (2014); Bassanetti et al. (2016); Gabriele et al. (2017) all argue that understanding sovereign risk requires going beyond the analysis of the debt stock and recognize the importance of the cash-flows stream in defining the sustainability of a country's total liability. A leading instance is the case stuty of Greece by Weder di Mauro and Schumaker (2015); Zettelmeyer et al. (2017), focused on the implications of different amortization schedules and interest rates.

[12] One way to interpret this result is that the official lender dilutes itself through maturity. By lengthening the maturity of the official claims, the lender effectively counteracts debt dilution of private invetors as discussed by Arellano and Ramanarayanan (2012); Chatterjee and Eyigungor (2015); Hatchondo et al. (2016)

IMF loan, using an Stand-By Agreement with a maturity of five years.[13] Following IMF practice, the pricing of these loans was a step-wise function of their maturity.

When financial stress did spread to Ireland and Portugal, the reaction was to move away from a bilateral approach and create jointly managed institutions. In June 2010, the European Financial Stability Mechanism (EFSM) and the European Financial Stability Facility (EFSF) were created. The EFSM was designed as an emergency funding program, managed by the European Commission, with the authority to borrow up to 60 billion euros. In turn, the EFSF was created as a temporary rescue mechanism to provide financial assistance within the framework of an adjustment program.

In December 2010, Ireland became the first country to seek assistance of the new institutions. The Irish program provided a financing package of 85 billion euros, including contributions from the EFSM (22.5 billion euros), EFSF (17.7 billion euros) and bilateral loans from the UK, Sweden and Denmark (3.8, 0.6 and 0.4 billion euros, respectively). In addition, Ireland signed a 7-year Extended Fund Facility (EFF) agreement with the IMF for 22.5 billion euros. A few months later, in April 2011, it was Portugal's turn to seek support. In this case the financing of the 78 billion euros program fell on equal parts on the EFSM, EFSF and IMF. The IMF loan to Portugal was also disbursed through a 7-year EFF program. The conditions on the euro area loans were adjusted in June/July.

In June 2011, the European authorities agreed to set-up a permanent crisis-management institution, the European Stability Mechanism (ESM), to become operative by 2014. As the euro area problems did not abate, the authorities brought its inauguration forward in time, in 2012. With 500 billion euros lending capacity, supported by 700 billion euros in capital, the ESM provided assistance to Spain (December 201-February 2013), Cyprus (May 2013-October 2015) and Greece (August 2015-August 2018).

3.2 The Terms of Official Support: IMF versus Euro Area

The International Monetary Fund relies on two crisis-resolution credit lines, the Stand-By Agreement (SBA) and the Extended Fund Facility (EFF): SBA programs aim to help members address short-term balance of payments (BoP) problems; EFF programs instead aim to help countries overcome medium-term BoP problems. While SBA programs are typically structured over 3 years, with a repayment horizon up to 5 years, EFF programs are structured over 4 years, with a repayment horizon of up to 10 years. SBA and EFF programs apply identical borrowing limits and pricing structure. The lending rate is linked to the

[13]These were the quantities originally envisaged. In the end, the bilateral loans provided to Greece amounted to 73 billion euros and the IMF to 20.1 billion euros.

special drawing rights (SDR) interest rate. For loans below 145 percent of the member's quota, the IMF charges 100 bps. This increases to 200 bps for credit above 145 percent of the quota. Moreover, to discourage large use of IMF resources, the spread over the SDR rate increases during the period in which credit is outstanding. Additional 100 bps are charged on loans outstanding over 36 months, provided the loan size remains above 145 percent of quota, or if credit remains outstanding in excess of 51 months.[14]

Relative to the IMF, euro area official lending has a larger concessional element. As described in detailed in a companion paper (Corsetti et al., 2017), the euro area framework evolved significantly in time. Initially, euro area official loans were designed after the IMF blueprint. As new institutions were created, the terms of official lending changed. Euro area official lenders do not apply fixed loan maturity standards, and stand ready to extend maturities even beyond three decades. This is in contrast to IMF practice, where loans horizons are officially limited to 10 years. Similarly, while IMF spreads over SDR rates are set to grow both with the size of the loan and the repayment period and can vary between 100 and 300 basis points, euro area official lenders charge a lower margin over funding costs, between 10 bps for standard loans and 30 bps for loans directed to the banking system. As a result of its narrower margins, the interest rate paid for euro area official loans became significantly lower than that charged for an IMF loan. Another difference worth noting is that, through the long maturities, the euro area official lenders effectively diluted the seniority of their loans.

While euro area institutions and the IMF operated in coordination in each country program, their approach to official lending significantly differ. In the rest of this section, we highlight key implications for sovereign debt dynamics in the euro area.

3.3 Key Facts

Below, we present a set of stylized facts regarding both the terms and conditions of official sector lending and the dynamics of debt, borrowing costs and the sovereigns' creditor composition during the euro area crisis. We rely on three main data sources. Financial markets data (sovereign yields) is collected from Bloomberg. Data on the various official lending vehicles comes from Corsetti et al. (2017). Finally, quarterly data on debt stocks and refinancing needs come from the European Central Bank.

Fact 1: *Euro area official loans are larger in size, feature longer maturities, and imply*

[14] A country has access up to 145 percent of its quota for any 12-month period, and cumulative access up to 435 percent of quota over a program. The IMF can, on a case-by-case basis, lend above normal limits under the exceptional access policy framework.

lower borrowing costs that loans from the International Monetary Fund

As illustrated by Table 1, the financial terms offered by the euro area official lenders in the programs signed by Spain, Cyprus or Greece with the European Stability Mechanism were substantially more concessional than IMF programs. In Cyprus and Greece, the original maturity of the loans stood above 20 and 30 years, with 87 bps and 107 bps interest rates, respectively. This contrasts with the 7-year maturity loan Cyprus received from the IMF with an interest rate 20 bps higher, and the 8-year maturity EFF loan to Greece with a 406 bps interest rate.

Table 1 here

Fact 2: *Despite the sharp increase in borrowing costs, public debt kept increasing. Sovereigns financed further debt accumulation by switching from market financing to official sources.*

Figure 1 compares the dynamics of sovereign market yields with the creditor composition of the public debt stock. It shows that public debt in the euro area began to increase in 2008, but it took a while before market reacted negatively, especially in the market for peripheral countries debt. Despite a significant worsening in market access conditions, public debt accumulation proceeded unabated into the crisis. In fact, starting in 2010, when market access conditions significantly worsened, official creditors, especially from the euro area, played a key role in supporting debt expansion.

Figure 1 here

Fact 3: *When market access conditions improved, sovereigns returned to bond financing.*

Figure 2 compares the evolution of market and official borrowing costs for each country, over time. Sinve 2013, as market yields fell significantly (also by virtue of the ECB policies), Ireland and Portugal started to replace the by-then more expensive IMF debt, with cheaper bond issuance. In January 2014, the cost of the IMF credit to Portugal stood above 4 percent. Given that Portuguese market rates during 2014 were consistently below the IMF rate, the authorities decided to embark on an early repayment of the IMF loan, financed by issuing marketable securities. Similarly, in the Irish case, following large IMF disbursements since January 2011, borrowing from the Fund exceeded the 300 percent of the quota in early 2014. This implied that Ireland faced marginal interest payments of 4.05 percent on its IMF credit. In contrast, during the summer of 2014, prevailing market interest rates and the rates on the longer maturity ESM loans were far lower. This created an opportunity for Ireland to lower interest expenses by replacing the portion of IMF credit subject to surcharges with newly issued cheaper bonds.

Figure 2 here

Fact 4: *Official loans were designed to reduce public debt refinancing risks in program countries.*

Long-maturity loans shifted repayments into the future, containing the risk of roll-over crises and easing the constraints on further debt accumulation. Figure 3 compares the repayment profiles of IMF and euro area loans. The longer maturities of the euro area official loans clearly played an important role in smoothing the debt repayment schedule in the four program countries with joint IMF/ESM programs. The repayment of euro area loans kicks in only after the IMF loans are repaid in full.

Figure 3 here

From a different angle, Figure 4 plots one-year ahead roll-over needs—measured as debt maturing in the next 12 months as percentage of total debt—against different shares of official debt in total debt. The negative relation between the two is apparent from this figure. Euro area official loans significantly smooth the repayment flows over time, reducing the period-by-period refinancing needs of crisis countries and their vulnerability to rollover risk.

Figure 4 here

Fact 5: *Official loans, especially those from the euro area, significantly reduced the interest payments for the sovereign under programs.*

Figure 5 shows an assessment of the extent to which euro area reduced the interest bill of the crisis countries. The 'saving' on interest costs shown in the figure is calculated based on the difference between the rates charged by euro area official lenders in the period 2011-2016, relative to both the IMF and the markets. [15]

The effects of euro area official loans on interest payments, relative to market conditions, are quite significant. Even for Spain, whose program was the smallest in percentage of GDP

[15] In the Figure 5, "savings" are calculated by comparing each country's average sovereign market rates (or IMF rate corresponding to a loan with the same size and maturity as the euro area loan), matching the ESM maturity profile, with the equivalent ESM funding cost, and applying that difference to the actual euro area loans. Following ESM (2017), a cap of 6.4 percent is applied to the market rate

relative to other crisis countries, official assistance is estimated to have lowered the interest bill by a full percentage point of GDP.[16] While an order of magnitude smaller, savings are also non-negligible relative to the IMF lending conditions.[17]

Figure 5 here

4 The Model

Building on Conesa and Kehoe (2017), henceforth CK), we model an economic environment in which the country faces both rollover and fundamental (output) risk. Depending on the size and composition of its debt, the country can be in one of three zones, labeled 'safe', 'crisis' and 'default' zone. At sufficiently low levels of debt, the country is in the 'safe zone' where it never defaults, not even if it suffers a debt rollover crisis and loses market access. For high enough debt levels, the country is in the 'default zone,' where a sovereign crisis occurs for fundamental reasons (a persistent recession), regardless of the availability of market funding. At intermediate levels of debt, the country is in a 'crisis zone,' where it services its debt if market funding is available, but defaults if funding dries up, i.e., if the country suffers a rollover crisis. The domestic government and consumers have concave utility (the crisis zone would disappears with linear utility). International investors are risk neutral.

In this framework, we model the possibility that sovereigns obtain financing from international bailout agencies, which may offer different terms on their official loans. We specify two types of bailout agencies: an IMF-like agency which lends short at relatively high (still below market) rates; and an ESM-like agency which lends long on more generous terms.

In any given period, the state of the economy is given by the vector $s = (b, b_i, b_e, a, z_{-1}, \zeta)$. Here b is the level of government debt owed to international creditors, b_i is debt owed to the IMF, b_e is debt owed to the ESM; a is an indicator variable recording whether the economy is in a recession, $a = 0$, or in normal times, $a = 1$; z is an indicator variable recording whether default has occurred in the past $z_{-1} = 0$ or has not (yet) occurred $z_{-1} = 1$; finally, ζ is a sunspot coordinating agents' belief on the possibility of a rollover crisis. As in Conesa and Kehoe (2017), the country GDP is given by $GDP(a, z) = A^{1-a} Z^{1-z} y$.

[16] We note that Figure 5 does not include savings from the EFSM, GLF and other bilateral official loans. Given that the conditions of those vehicles were analogous to the ones applied to the EFSF loans, overall savings might be markedly larger.

[17] As Figure 5 does not to include the hedging costs of borrowing on SDR nor different fees the IMF charges, but it does include the fees charged by the euro area bailout funds, we see these figures as a lower bound on the amount of extra savings.

In our analysis, we assume that the economy starts out with $a = 1$ and $z = 1$ but is hit by a recessionary shock in period 0, $a = 0$. Every period thereafter, the economy recovers with probability $p < 1$, and, once recovered, never falls into recession again. By the same token, if and when the government chooses to default, the economy stays in default forever, $z = 0$. We posit a constant tax rate θ (calibrated to match the data) so that consumption is $c(a, z) = (1 - \theta)y(a, z)$. The government can sell new bonds b' at the price $q(b', s)$ to international investors, or seek a bailout either from the IMF b'_i at the price q_i, or from the ESM b'_e at price q_e. The country's government takes the official prices q_e and q_i as given. In accordance with the data we will set $q_e \leq q_i$, i.e. the ESM lends at more generous terms. Furthermore, to derive transparent analytical solutions, we will index the maturity of debt with the parameter δ, whereas $\delta = 0$ corresponds to the case of consols and $\delta = 1$ corresponds to the case of one period bond. A bond issued at $t-m$ is equivalent to $(1-\delta)^m$ bonds issued at t. Hence, the outstanding market bonds can be summarized by a single state variable B.

Denoting government expenditure with g, the government's budget constraint is

$$g + z(b + b_i + \delta b_e) = \theta y(a, z) + q(b', s)b' + q_i b'_i + q_e \big[b'_e - (1 - \delta)b_e\big] \tag{1}$$

As in Cole and Kehoe (2000) and Conesa and Kehoe (2017), rollover risk is modeled as a sunspot ζ drawn from a uniform distribution on $[0, 1]$. If $\zeta > 1 - \pi$, international creditors develop beliefs that a rollover crisis may occur, and, for debt levels high enough that a crisis is self-validating in equilibrium, refuse to lend to the government. The probability π determines the probability that a self-fulfilling rollover crisis materializes, for debt levels that are high enough for a speculative run on debt to induce the government to default.

As regards timing, we adopt the same sequence as in Conesa and Kehoe (2017), save for the fact that in our model the sovereigns can appeal to international bailout agencies if international creditors refuse to lend. In particular, in each period, the time line is as follows. First, the shocks a and ζ are realized, and given the aggregate state $s = (b, b_i, b_e, a, z_{-1}, \zeta)$, the government chooses how much to borrow from international creditors b', the IMF b'_i, and the ESM b'_e. Second, each of a continuum of measure-one international bankers choose how much debt B' to purchase, and the IMF and ESM provide the funds $B'_i = b'_i$ and $B'_e = b'_e$ according to the sovereign's request but within the constraint of their standard loans. Lastly the government decides to repay or default z, thereby generating y, c, and g.

We write the problem of the government as follows:

$$
\begin{aligned}
V(s) = \max u(c, g) + \beta EV(s') \qquad (2)\\
c = (1-\theta)y(a, z)\\
g + z(b + b_i + rb_e) = \theta y(a, z) + q(b', s)b' + q_e b'_e + q_i b'_i\\
z = 0 \text{ if } z_{-1} = 0 \qquad g \geq \bar{g}; c \geq \bar{c}; = 0
\end{aligned}
$$

where we allow for the possibility that there are minimum levels of non defaultable public and private consumption, $\bar{g}$ and $\bar{c}$, respectively. As in CK we assume that, for any feasible (b, b_i, b_e), the following condition holds:

$$
u_g((1-\theta)Ay, \theta Ay - b - b_i - \delta b_e) > u_g((1-\theta)y, \theta y - b - b_i - \delta b_e). \qquad (3)
$$

This ensures that government has an incentive to raise debt and "gamble for redemption" during a recession, as the marginal benefit of government spending is higher in a recession than in normal times. This assumption is satisfied by standard concave utility functions like $\log(c + g - \bar{c} - \bar{g})$.

International creditors are risk neutral with discount factor β, so bond prices $q(b', s)$ are determined by probability of default next period. There is a continuum of such creditors, each solving

$$
\begin{aligned}
W(b, b', s) = \max x + \beta EW(b', b'', s') \qquad (4)\\
x + q(b', s)b' = w + z(b', s, q(b', s))b\\
x \geq 0,\ b \geq -A
\end{aligned}
$$

whereas we assume that investors have 'deep enough pocket', i.e., x is large enough to rule out corner solutions, and the condition on A rules out Ponzi schemes. The bailout agencies, IMF and ESM, solve similar problems, except that the prices q_i and q_e at which they lend are exogenously set and while the entire stock of IMF debt comes due in the next period, only a fraction δ of the ESM debt comes due every period.

Following Conesa and Kehoe (2017), we consider equilibria with a simple Markov structure. In an environment where output and rollover risk can only take on two values, the three zones (safe, crisis and default) can then be characterized by four debt thresholds: two above which default occurs in a recession—of which one without market financing $b(0)$ and the other with market financing $B(0)$; two in normal times, of which one without market financing $b(1)$ and the other with market financing $B(1)$. Since $b(0) < B(0)$ and $b(1) < B(1)$, the intervals between these thresholds define the crisis zone in normal times and in a recession, respectively. The safe zone, where there is no default, is for level of debt below $b(0)$ in a recession, and $b(1)$ in normal times. In the original contribution by Conesa and Kehoe, these thresholds are points on the real line. In our model, these are three-dimensional objects, as they depend on the composition of debt, i.e. the share of b_i and b_e in total debt. A qualifying feature of our analysis is indeed that sustainability will be assessed in relation to these four thresholds, hence conditional on the state of the economy and the investors' 'sentiment.'

Finally, as in the CK model, our model also features multiple equilibria. Given our interest in understanding the effect of official bailouts on sovereign incentives to run up or run down its debt due to output and rollover risk, we turn to the characterization of the equilibrium under these two types of risk next.

5 Official lending in the presence of liquidity and fundamental risks

The mechanism of a bailout relies on managing both the maturity and the price of official loans. To gain analytical insight, in this section we consider a slightly simplified version of the model. Namely, we let the country have access to only one official lending instrument, characterized by two parameters: maturity δ and price q_e. We model bailouts as a one-for-one exchange of (short-term) market debt with official debt, indexing official loans by δ and q_e.

Intuitively, by offering longer maturity loans, an official lender can boost sustainability by reducing the period-by-period cash flow the government has to generate to service and rollover its debt—a key advantage especially, but not exclusively, to address rollover risk. By offering a high price for the government bond (i.e., by charging an interest rate below market), the government essentially raises the borrowing capacity of the government against any given payment flows, an advantage relevant against both rollover and fundamental risk. In what follows, the analysis will highlight a set of key effects and implications of bailouts.

For tractability, in the rest of this section we also assume that a government in default suffers a sunk loss in output equal to τ and recessions are associated by a sunk loss in output equal to a (rather than fractions of output $1 - Z$ and $1 - A$ as in the full model).[18] We also assume no minimum consumption spending. Initially, the country has outstanding market debt, but zero official debt.

[18] For analytical tractability, without loss of generality we posit that governments do not abscond with current period borrowing (the proofs will go through even if they do, but with added complications).

To illustrate the economic forces at work, we consider two separate model environments, one with rollover risk only, the other with output risk only. As a matter of notation, below we will denote the thresholds in the rollover risk environment with a tilde, e.g., $\tilde{b}(1)$; the thresholds in the output risk environment will be denoted with a hat, e.g., $\hat{B}(1)$. When appropriate, we will denote (sustainable) debt conditional on official lending and no official lending with the subscripts 'ℓ' and 'nℓ' respectively.

5.1 Rollover Risk

Absent output risk, the government's only problem is to choose the path of debt reduction that brings the country out of the crisis zone, that is, the government needs to decide how fast it should run its debt down to safe levels. The key policy trade-off is between smoother consumption (a longer period in the crisis zone) and better borrowing costs (from reaching the safe zone earlier). We will see below that official lending can substantially ameliorate this trade-off, fostering consumption smoothing in the process of deleveraging from the crisis to the safe zone. Official lending indeed allows the government to sustain higher consumption along the transition and can (be structured to) ensure an early exit from the crisis zone.

When the only source of risk is the possibility of a rollover crisis, the equilibrium in our model can be characterized in terms of *two* debt thresholds: a lower threshold $\tilde{b}(1)$ beyond which the government defaults conditional on a rollover crisis, i.e., if market funding becomes unavailable; an upper threshold $\tilde{B}(1)$ conditional beyond which the government would default even if market funding were available. Note that, since there is no recession risk, hereafter we assess these thresholds conditional on "normal times".

5.1.1 The safe debt threshold conditional on a rollover crisis

As a reference, drawing on CK, we start by characterizing sustainability when the country loses access to market financing in the absence official lending available. Let $\tilde{b}(1)_{n\ell}$ denote the maximum level of debt that the country can sustain in such circumstances (with "nℓ" standing for "no official lending"). The debt limit that defines the safe zone is the level of debt at which the value to repayment is equal to the value of default (same as in CK):

$$u(y - \tilde{b}(1)_{n\ell}) + \beta \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \tag{5}$$

The effect of official lending. Suppose that, when facing a rollover crisis, the country can count on official support in the form of a short-term loan B' at the price q. Let $\tilde{b}(1)_{\ell=1}$ denote the new, sustainable debt limit (where "ℓ" stands for official lending and 1 for the maturity of the official loan). It is easy to see that $\tilde{b}(1)_{\ell=1}$ satisfies the following:

$$u(y - \tilde{b}(1)_{\ell=1} + qB') + \beta u(y - B') + \beta^2 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta}, \qquad (6)$$
$$qu'(y - \tilde{b}(1)_{\ell=1} + qB') = \beta u'(y - B').$$

where the latter condition defines a mapping $B' = G(b, q)$, essentially ensuring that B' is chosen optimally to satisfy the government's Euler equation. Let the official bailout be available for two periods instead of one. The conditions defining the debt threshold, now denoted with $\tilde{b}(1)_{\ell=2}$, become:

$$u(y - b + qB') + \beta u(y - B' + qB'') + \beta^2 u(y - B'') + \beta^3 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta}, \qquad (7)$$
$$qu'(y - b + qB') = \beta u'(y - B' + qB''), \quad qu'(y - B' + qB'') = \beta u'(y - B''')$$

For simplicity, posit log preferences and $q = \beta$, such that official lending is at the risk free rate. With a single period bailout, the optimal policy function is $B' = \frac{\tilde{b}(1)_{\ell=1}}{1+\beta}$. The sustainability condition simplifies to:

$$u\left(y - \tilde{b}(1)_{\ell=1} + \frac{\beta\tilde{b}(1)_{\ell=1}}{1+\beta}\right) + \beta u\left(y - \frac{\tilde{b}(1)_{\ell=1}}{1+\beta}\right) + \beta^2 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \qquad (8)$$

By concavity, it follows that $\tilde{b}(1)_{\ell=1} > \tilde{b}(1)_{n\ell}$ since

$$(1+\beta)u\left(y - \frac{\tilde{b}(1)_{\ell=1}}{1+\beta}\right) = u\left(y - \tilde{b}(1)_{\ell=1} + \frac{\beta\tilde{b}(1)_{\ell=1}}{1+\beta}\right) + \beta u\left(y - \frac{\tilde{b}(1)_{\ell=1}}{1+\beta}\right) = u(y - \tilde{b}(1)_{n\ell}) + \beta u(y)$$

Similarly, when the bailout is available for two periods, the sustainability condition becomes

$$(1+\beta+\beta^2)u\left(y - \frac{\tilde{b}(1)_{\ell=2}}{1+\beta+\beta^2}\right) + \beta^3 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \qquad (9)$$

Again, by concavity, $\tilde{b}(1)_{n\ell} < \tilde{b}(1)_{\ell=2}$. By the logic of the argument, it is easy to see that sustainability is increasing in the number of periods during which the bailout is available, and the proof holds for general preferences.

The following numerical example illustrates the results above. Let $\tau = 5$, $Y = 100$, $\beta = 0.95$. With log preferences, in the absence of official lending, sustainable debt would be up to $\tilde{b}(1)_{n\ell} = 64.15\%$ of GDP. Now let official lending become available for one period, at the price $q_e = 0.95$, up to 15% of GDP. The availability of official funds (not subject to rollover risk) for one period at the equilibrium default-free price raises the threshold to $\tilde{b}(1)_{\ell=1} = 72.4\%$ of GDP: any stock of debt between 64.15 and 72.4% of GDP is now safe. For any debt level below the latter threshold, private investors know that, even if they coordinated on not rolling over their credit to the country, access to official fund would allow the government to (optimally) avoid default. Hence no rollover crisis will occur in equilibrium. The difference between 72.4 and 64.15% of GDP is the additional private sector lending at default-free prices generated by the one-period bailout. When the country's debt exceeds the threshold of 72.4% of GDP, default is possible despite official financial assistance, and rollover risk is priced by the markets. The size of available official assistance is not enough to rule out rollover crises. However, the disbursement official loans still generate benefits, as it creates some limited room for smoothing consumption.[19]

The effect of varying maturity and price of official loans. Having established how lengthening the time horizon over which official loans are available always widens the safe zone, we now focus on the general case of an official loan with long maturity, parameterized by δ, and price q_e, and analyze how the threshold responds to changing these two parameters. For simplicity and tractability, in what follows we assume that the official loan is available on a permanent basis, and simply write $\tilde{b}(1)$ without specifying subscripts for δ or q_e .

With official lending, parameterized by (q_e, δ) and made available to the country in the event of a crisis, the conditions defining $\tilde{b}(1)$ is

$$u(y - \tilde{b}(1) + q_e b'_e) + \beta u(y - \delta b'_e + q_e[b''_e - (1-\delta)b'_e]) + \ldots = \frac{u(y-\tau)}{1-\beta} \tag{10}$$

To derive an analytical expression for the threshold, we posit $b'_e = b''_e = \ldots$, as required by optimality, so that:

$$\begin{aligned} &\tilde{b}(1) + q_e b'_e = -\delta b'_e + q_e[b'_e - (1-\delta)b'_e] \\ &\tilde{b}(1) = b'_e[\delta(1-q_e) + q_e] \\ &b'_e = \frac{\tilde{b}(1)}{[\delta(1-q_e)+q_e]} \\ &-\tilde{b}(1) + q_e b'_e = \frac{[-\delta(1-q_e) - q_e + q_e]\tilde{b}(1)}{[\delta(1-q_e)+q_e]} = \frac{[-\delta + q_e\delta]\tilde{b}(1)}{[\delta(1-q_e)+q_e]} \end{aligned} \tag{11}$$

[19] We should note that, when official lending widens the boundary of the safe zone as to include the initial debt stock, sustainability can be guaranteed without any need for actual disbursement of official funds. With rollover risk only, a credible commitment by an official lender to give credit to the country can eliminate the "bad equilibrium", hence calm market fears and allow countries to sustain market financing for higher levels of debt, commensurate to the size of the official program—see Corsetti and Dedola (2016) for an analysis of monetary backstops.

Given that the threshold condition implies $-b+q_e b'_e = -\tau$, we can write $\tilde{b}(1)$ as a function of price and maturity of official loans, as follows:

$$\tilde{b}(1) = \frac{\delta(1-q_e)+q_e}{\delta(1-q_e)}\tau. \tag{12}$$

From the above, it follows that

$$\frac{d\tilde{b}(1)}{d\delta} = -\frac{q_e}{\delta^2(1-q_e)^2} < 0 \tag{13}$$

$$\frac{d\tilde{b}(1)}{dq_e} = \frac{\delta(1-q_e)+q_e\delta}{\delta^2(1-q_e)^2} > 0 \tag{14}$$

This establishes that longer maturities (lower δ) and lower rates (higher q_e) both serve to raise the level of debt sustainable without market financing. Note that, as $\delta \nearrow 1$, and maturity gets shorter, the debt to be repaid in full the following period approaches $\tilde{b}(1) \searrow \frac{1}{1-q_e}\tau$.

5.1.2 The debt threshold conditional on market financing

The discussion of the upper threshold defining the crisis zone is more involved. Recall that beyond this limit, default is the preferred policy option even if there is no rollover crisis (and investors are in principle willing to finance government borrowing). The government weighs the advantage of repudiating debt, against the costs of a policy of debt reduction facing high borrowing costs, as investors price rollover risk as long as debt remains in the crisis zone. As in the previous subsection, we start by characterizing the upper debt limit in the absence of official lending, denoted $\tilde{B}_{n\ell}(1)$. With short-term market loans, this is defined by the following conditions:

$$\max\{V^1(\tilde{B}_{n\ell}(1)), V^2(\tilde{B}_{n\ell}(1)), \ldots, V^\infty(\tilde{B}_{n\ell}(1))\} = \frac{u(y-\tau)}{1-\beta} \tag{15}$$

$$V^T(B) = \frac{1-[\beta(1-\pi)]^T}{1-\beta(1-\pi)}u(g^T) + \frac{1-[\beta(1-\pi)]^{T-1}}{1-\beta(1-\pi)}\frac{\beta\pi}{1-\beta}u(y-\tau)+$$
$$+[\beta(1-\pi)]^{T-2}\frac{\beta u(y-[1-\beta]\tilde{B}_{n\ell}(1))}{1-\beta}$$
$$g^T_{nl} = y - \frac{1-\beta(1-\pi)}{1-[\beta(1-\pi)]^T}\left(B-[\beta(1-\pi)]^{T-1}\tilde{B}_{n\ell}(1)\right)$$

where T denotes the (endogenous) time of exit from the crisis zone, when debt is brought to the safe zone, and the last equation in the system above is the result of the series of equations determining the path of spending and debt in the transition. Starting at $t = 0$, this series of equation (running through T) is given by:

$$\begin{aligned} g^T_{nl} + B_0 &= y + \beta(1-\pi)B_1 \\ g^T_{nl} + B_1 &= y + \beta(1-\pi)B_2 \\ &\dots \\ g^T_{nl} + B_{T-2} &= y + \beta(1-\pi)B_{T-1} \\ g^T_{nl} + B_{T-1} &= y + \beta\tilde{b}_{n\ell}(1) \end{aligned} \tag{16}$$

wheres, as discussed by CK, it is optimal for the government to pursue a constant level of spending.

The effect of official lending. Now introduce official support, with long-term loans with maturity indexed by δ_e, issued at rates not higher than the market, that is, $q_e \geq q$. For analytical tractability, it is useful to consider the limit case in which, during the transition from the risky to the safe debt zone, the country relies almost exclusively on official loans. Under this simplifying assumption, the sequence of budget constraints in (16) above can be rewritten as follows:

$$\begin{aligned} g^T + B_0 &= y + q_{e,T-1}B_1 \\ g^T + \delta_e B_1 &= y + q_{e,T-2}[B_2 - (1-\delta_e)B_1] \\ &\dots \\ g^T + \delta_e B_{T-2} &= y + q_{e,1}[B_{T-1} - (1-\delta_e)B_{T-2}] \\ g^T + \delta_e B_{T-1} &= y + q_{e,0}[b(1) - (1-\delta_e)B_{T-1}] \end{aligned} \tag{17}$$

whereas the key difference from market lending is that the government may find it optimal to revise spending plans, and the time of exit T needs not be the same. The key equation characterizing g^T can be written as follows:

$$\left[1 + \frac{q_{e,T-1}}{\delta_e + q_{e,T-2}(1-\delta_e)} + \frac{\Pi_{t=T-2}^{T-1} q_{e,t}}{\Pi_{t=T-3}^{T-2}\delta_e + q_{e,2}(1-\delta_e)} + \dots + \frac{\Pi_{t=1}^{T-1} q_{e,t}}{\Pi_{t=0}^{T-2}\delta_e + q_{e,2}(1-\delta_e)}\right] g^T + B_0 = \tag{18}$$

$$\left[1 + \frac{q_{e,T-1}}{\delta_e + q_{e,T-2}(1-\delta_e)} + \frac{\Pi_{t=T-2}^{T-1} q_{e,t}}{\Pi_{t=T-3}^{T-2}\delta_e + q_{e,2}(1-\delta_e)} + \dots + \frac{\Pi_{t=1}^{T-1} q_{e,t}}{\Pi_{t=0}^{T-2}\delta_e + q_{e,2}(1-\delta_e)}\right] y +$$

$$\frac{\Pi_{t=1}^{T-1} q_{e,t}}{\Pi_{t=0}^{T-2}[\delta_e + q_{e,t}(1-\delta_e)]} q_{e,0}\tilde{b}_(1)$$

The functional specification of the value function V^T is unchanged save for the g^T argument—by definition $B(1)$ is the maximum amount that can be sustained provided the country is not in a rollover crisis, to which the country is still vulnerable (with probability π).[20] Note that it is certainly possible that a change in debt maturity and/or price affects the optimal time to exit—that is, it is possible that $dT/d\delta_e \neq 0$ or $dT/d_e \neq 0$. Indeed, by revising its current debt and spending decision upon receiving official support, the country chooses how long it remains in the crisis zone. For easy of exposition, in the rest of this subsection we will carry out our analysis keeping T as given—we extend our argument and results to the general case in an appendix.

The effect of varying maturity and price of official loans. As before, we provide insight on the role of official lending by splitting the analysis into the case when official loans differ from market loans in maturity only, and in the price only. First, consider official lending with maturity $\delta_e < 1$, while setting $q_{e,0} = \beta$, $q_{e,1} = q_{e,1} = \ldots = q_{e,T-1} = \beta(1-\pi)$. In this case the equation for g^T specializes to

$$g^T = y - \frac{1-x}{1-x^T}\left[B_0 - x^{T-1}\beta\tilde{b}(1)\right], \tag{19}$$

where

$$x = \frac{\beta(1-\pi)}{\delta_e + \beta(1-\pi)(1-\delta_e)} \tag{20}$$

Suppose that the optimal time to exit is unaffected by the change in δ_e. In this case, we can write:

$$\frac{dV^T}{d\delta_e} = \underbrace{\frac{1-[\beta(1-\pi)]^T}{1-\beta(1-\pi)}u'(g^T)\underbrace{\frac{dg^T}{d\delta_e}}_{<0}}_{\uparrow V^T \text{ as } \downarrow \delta_e} + \underbrace{[\beta(1-\pi)]^{T-2}\frac{\beta u'(y-[1-\beta]\tilde{b}(1))}{1-\beta}\cdot -(1-\beta)\underbrace{\frac{d\tilde{b}(1)}{d\delta_e}}_{<0}}_{\downarrow V^T \text{ as } \downarrow \delta_e} \tag{21}$$

[20]Note that in the special case where $q_{e,0} = \beta$, $q_{e,1} = q_{e,1} = \ldots = q_{e,T-1} = \beta(1-\pi)$ and $\delta_e = 1$, we get back the equation that obtains in the absence of official lending (with $\tilde{b}_(1)$ coinciding with $\tilde{b}_{n\ell}(1)$).

where

$$\frac{dg^T}{d\delta_e} = \underbrace{-\underbrace{\frac{d\frac{1}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0}\underbrace{\frac{dx}{d\delta_e}}_{<0}B_0 - \beta\underbrace{\frac{d\frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0}\underbrace{\frac{dx}{d\delta_e}}_{<0}\tilde{b}(1)}_{\uparrow \text{ consumption for } \downarrow \delta_e} + \underbrace{\beta\frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}}\underbrace{\frac{d\tilde{b}(1)}{d\delta_e}}_{<0}}_{\uparrow \text{ consumption for } \downarrow \delta_e} \tag{22}$$

The decomposition on the right hand side of $\frac{dg^T}{d\delta_e}$ in (22) highlights the effects of lengthening debt maturity on government spending. The first two terms show that a longer maturity ($d\delta_e < 0$) raises government consumption in the transition by increasing the effective discount rate on the amount to be repaid (which lowers the present value of any given flow of payments). The last terms highlights that longer maturity also affects transitional government spending by raising the level of debt that the government can sustain in the long run, $\tilde{b}(1)$. In other words, the government has less debt to run down or repay, so that it can consume more.

Having established that government consumption rises with longer maturities, we turn our attention to the value function V^T. There are two opposing forces at play that determines whether V^T rises or falls with the maturity parameter δ_e. On the one hand, the value function rises with higher transitional government consumption—this is shown in the first term in the decomposition of $\frac{dV^T}{d\delta_e}$ on the RHS of (21) . On the other hand, a higher level of debt $\tilde{b}(1)$ in the safe zone decreases the value function, as a higher steady-state debt lowers long-term consumption—the last term in (21).

Which of these two forces dominates crucially depends on the probability that market financing dries up, π, and the time to exit the crisis zone T. To appreciate this point, note that the second term in (21) disappears—and the value function unambiguously rises with longer maturities—both when (exogenously) $\pi \to 1$, so that $\beta(1-\pi) \to 0$, and when (endogenously) $T \to \infty$. In the first case, a rollover crisis and hence the default are almost sure; in the second case, the country would stay in the crisis zone indefinitely, as long as market lenders remain willing to lend (so that, again, a crisis will occur almost surely at some point). Intuitively, in either case longer maturities allows the government to consume more while still trying to adjust debt down in the face of an impending rollover crisis.

This result has stark implications for the level of sustainable debt $\tilde{B}(1)$. Consider the indifference condition defining this upper debt threshold:

$$\max\{V^1(\tilde{B}(1)), V^2(\tilde{B}(1)), \ldots, V^\infty(\tilde{B}(1))\} = \frac{u(y-\tau)}{1-\beta} \tag{23}$$

The key observation is that, while the right hand side of remains unchanged, the left hand side of this condition changes this debt maturity. Consider an initial upper threshold defined conditional on some given debt maturity, and ask what happens when this maturity is lengthened. We have established above that, for high enough probability of a rollover crisis and a long enough expected transition to the safe zone, V^T will increase. Since this means that the value of repayment is now higher than the value of default, the government will choose to default at a higher debt threshold $\tilde{B}(1)$. Conversely, if the probability of a rollover crisis is sufficiently low, or/and the exit time to the safe region is near, lengthening debt maturity will decrease the value function. With a lower value of repayment, default will occur at a lower debt threshold.

Just as we have shown how debt maturity δ affects $B(1)$, we can examine the effect of lowering the spread on (i.e. raising the price of) official debt, q_e. The expressions for $\frac{dV^T}{dq_e}$ and $\frac{dg^T}{dq_e}$ are as follows:

$$\frac{dV^T}{dq_e} = \underbrace{\frac{1-[\beta(1-\pi)]^T}{1-\beta(1-\pi)} u'(g^T) \underbrace{\frac{dg^T}{dq_e}}_{>0}}_{\uparrow V^T \text{ as } \uparrow q_e} + \underbrace{[\beta(1-\pi)]^{T-2} \frac{\beta u'(y-[1-\beta]b(1))}{1-\beta} \cdot -(1-\beta) \underbrace{\frac{db(1)}{dq_e}}_{>0}}_{\downarrow V^T \text{ as } \uparrow q_e} \tag{24}$$

and

$$\frac{dg^T}{dq_e} = \underbrace{-\underbrace{\frac{d\frac{1}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0} \underbrace{\frac{dx}{dq_e}}_{>0} B - \beta \underbrace{\frac{d\frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0} \underbrace{\frac{dx}{dq_e}}_{>0} b(1)}_{\uparrow \text{ consumption for } \uparrow q_e} + \underbrace{\beta \frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}} \underbrace{\frac{db(1)}{dq_e}}_{>0}}_{\uparrow \text{ consumption for } \uparrow q_e} \tag{25}$$

The impact of the higher price on government consumption and the value function is analogous to that of a longer maturity. Namely, a higher debt price increases both the discount rate and the debt limit absent market funding—both serve to increase government consumption in the transition (as shown by (25)). Higher transitional government consumption g^T in turn raises the value function for a given level of $B(1)$ (the first term on the RHS of (24)). This positive effect is however countered by the higher steady-state level of $b(1)$, which lowers the value function (the last terms on the RHS of (25)). With $\pi \to 1$ or $T \to \infty$, the second force vanishes and $V^T(B(1))$ increases with higher q_e. Given that the optimal time to exit remains unchanged, this in turn implies that $B(1)$ rises with higher q_e.

5.2 Output Risk

In an environment with fundamental output risk only, there are again only two debt thresholds, defining the maximum level of sustainable debt in normal times and in a recession. Let $\hat{B}(1)$ and $\hat{B}(0)$ denote these two thresholds, where a hat indicates that there is no rollover risk. Recall that, under the simplifying assumptions of our model, once the economy is out of a recession, the expansion is an absorbing state: if there is no rollover risk, the economy becomes deterministic.

5.2.1 The debt threshold in normal times

As above, we start our analysis by characterizing the debt threshold with short-term market lending and no official lending. In the absence of rollover risk, the condition defining $\hat{B}_{n\ell}(1)$ is:

$$\frac{u(y-[1-\beta]\hat{B}_{n\ell}(1))}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \Rightarrow \hat{B}_{n\ell}(1) = \frac{\tau}{1-\beta}. \tag{26}$$

Let $\hat{B}_\delta(1)$ define the threshold with long-term borrowing, at this point without specifying whether from investors or from an official lender. When the country issues bonds with maturity $\delta < 1$ at the risk-neutral price q, the threshold (denoted by $\hat{B}_\delta(1)$) satisfies:

$$\frac{u(y-\delta\hat{B}_\delta(1)+q[\hat{B}_\delta(1)-(1-\delta)\hat{B}_\delta(1)])}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \Rightarrow \hat{B}_\delta(1) = \frac{\tau}{\delta(1-q)} \tag{27}$$

It is easy to verify that $\hat{B}_\delta(1) > \hat{B}_{n\ell}(1)$ since, with bonds traded at the risk neutral price,

$$q = \frac{\beta\delta}{1-\beta(1-\delta)} \Rightarrow \delta(1-q) = \frac{\delta(1-\beta)}{1-\beta(1-\delta)} < 1-\beta \text{ for } \delta \in (0,1) \tag{28}$$

To gain insight, it is useful to decompose the ratio $\hat{B}_\delta(1)/\hat{B}_{n\ell}(1)$ into a maturity and a price term:

$$\frac{\hat{B}_\delta(1)}{\hat{B}_{n\ell}(1)} = \underbrace{\frac{1}{\delta}}_{\text{maturity effect} > 1} \cdot \underbrace{\frac{1-\beta}{1-q}}_{\text{price effect} > 1} \tag{29}$$

an expression which is increasing in maturity (that is, decreasing in δ) and in bond prices. The maturity effect is intuitive. Suppose that with one period debt (all debt had to be repaid in each period), the maximum sustainable level is 100. If only a quarter has to be repaid in each period, a given cash flow can sustain 400. The price effect is subtler. Essentially, for any given credible repayment flow, a lower price for debt allows the government to consume more (and hence have a higher value function when it repays); so, to make the government indifferent between repaying and defaulting, it must be allowed to borrow more.

We can think of the effects of a bailout, with an official lender replacing short-term market debt with long-term official debt, as a combination of the cases above: the threshold will be higher than in the case of short-term debt with no official support, but lower than the case in which debt is long-term to start with. Assuming once again that the lending regime is permanent and approximating average maturity and prices with the q_e δ_e, the condition defining the threshold $\hat{B}_\ell(1)$ is:

$$u(y-\hat{B}_\ell(1)+q_e\hat{B}_e)+\beta u(y-\delta\hat{B}_e+q_e[B'_e-(1-\delta)\hat{B}_e])+\beta^2\frac{u(y-\delta\hat{B}'_e+q_e[\hat{B}'_e-(1-\delta)\hat{B}'_e])}{1-\beta}=\frac{u(y-\tau)}{1-\beta} \tag{30}$$

where we allow for the switch to occur in one period, so that the economy is in its steady state from period 2 on. In particular, setting $q_e = q$, and $\hat{B}_e = \hat{B}'_e = \hat{B}_\delta$, we can see that $\hat{B}_{n\ell}(1) < \hat{B}_\ell(1) < \hat{B}_\delta(1)$, since $\hat{B}_\ell(1) = (q + \delta(1-q))\hat{B}_\delta = \frac{\delta(1-\beta)}{1-\beta(1-\delta)}\hat{B}_\delta < \hat{B}_\delta(1)$.

5.2.2 The debt threshold in recessions

The last threshold is for the case of an economy in a recession—while current output equal to $y - a$, the economy recover with probability equal to p. Absent official lending, the indifference condition is, for the case of short-term borrowing:[21]

$$u(y-\hat{B}_{n\ell}(0)-a+\beta p\hat{B}_{n\ell}(1))+\beta p\frac{u(y-[1-\beta]\hat{B}_{n\ell}(1))}{1-\beta}+\beta(1-p)\frac{u(y-\tau-a)}{1-\beta}= \tag{31}$$
$$u(y-\tau-a)+\beta p\frac{u(y-\tau)}{1-\beta}+\beta(1-p)\frac{u(y-\tau-a)}{1-\beta}$$

where from the discussion above, we know that $\hat{B}_{n\ell}(1) = \frac{\tau}{1-\beta}$. It follows that:

$$\hat{B}_{n\ell}(0)+a-\beta p\hat{B}_{n\ell}(1)=\tau+a \Rightarrow \hat{B}_{n\ell}(0)=\tau+\beta p\frac{\tau}{1-\beta}=\frac{\tau(1-\beta+\beta p)}{1-\beta}<\hat{B}_{n\ell}(1) \tag{32}$$

It is useful to clarify from the start what drives apart the thresholds contingent on a recession $\hat{B}_{n\ell}(0)$ and in normal time $\hat{B}_{n\ell}(1)$ is default risk in future recession. To see this most clearly, take the ratio of the two, using (31) and (31):

$$\frac{\hat{B}_{n\ell}(0)}{\hat{B}_{n\ell}(1)} = 1-\beta+\beta p \leq 1 \tag{33}$$

[21] In writing this condition, we assume that the government issues the level of debt that is sustainable if the country is out of the recession in period 1. This is optimal for a probability p large enough.

Under our assumptions, once in a recession, the economy only recovers with finite probability p and thus remains exposed to risk of default. The debt price at which investors are willing to lend is therefore βp and not β, as it would be if the economy were expected to recover for sure. Indeed, as $p \nearrow 1$, debt becomes less risky and the two thresholds converge—they actually coincide in the limiting case in which the probability of remaining in a recession (hence default risk) vanishes.

As in the previous section, consider an economy with long-term borrowing. With self-explanatory notation we can write the equations defining the threshold $\hat{B}_\delta(0)$ as follows:

$$\begin{aligned}
&u(y - \delta\hat{B}_\delta(0) - a + q_0[\hat{B}_\delta(1) - (1-\delta)\hat{B}_\delta(0)])+\\
&\beta p\frac{u(y - \delta\hat{B}_\delta(1) + q[\hat{B}_\delta(1) - (1-\delta)\hat{B}_\delta(1)])}{1-\beta} + \beta(1-p)\frac{u(y-\tau-a)}{1-\beta} =\\
&u(y-\tau-a) + \beta p\frac{u(y-\tau)}{1-\beta} + \beta(1-p)\frac{u(y-\tau-a)}{1-\beta}.
\end{aligned}$$

where from risk-neutral pricing and the previous analysis we know that

$$q_0 = \frac{\beta p\delta}{1-\beta p(1-\delta)}, \quad \hat{B}_\delta(1) = \frac{\tau}{\delta(1-q)}, \quad q = \frac{\beta\delta}{1-\beta(1-\delta)} \tag{34}$$

After some algebraic manipulation, we can further obtain an expression for the stock of sustainable debt, $\hat{B}_\delta(0)$:

$$\delta\hat{B}_\delta(0) - q_0\hat{B}_\delta(1) + q_0(1-\delta)\hat{B}_\delta(0) = \tau \Rightarrow \hat{B}_\delta(0) = \frac{\tau + q_0\hat{B}_\delta(1)}{\delta + q_0(1-\delta)} \Rightarrow \hat{B}_\delta(0) = \frac{1-\beta+\beta p\delta}{\delta(1-\beta)}\tau \tag{35}$$

Note that the ratio between the two debt thresholds in the above expression, $\hat{B}_\delta(1)$ and $\hat{B}_\delta(0)$, given by

$$\frac{\hat{B}_\delta(0)}{\hat{B}_\delta(1)} = \frac{1-\beta+\beta p\delta}{1-\beta+\beta\delta}, \tag{36}$$

is converging to 1 as $p \nearrow 1$—whereas the rate of convergence reflects the fact that only a fraction δ of debt is repaid every period.

We also find it insightful to compare the thresholds with short and long-term borrowing we just derived, by decomposing their ratio as follows:

$$\frac{\hat{B}_\delta(0)}{\hat{B}_{n\ell}(0)} = \underbrace{\frac{\hat{B}_\delta(0)}{\hat{B}_\delta(1)}}_{\text{default}\cdot\text{maturity effect}} \cdot \underbrace{\frac{\hat{B}_\delta(1)}{\hat{B}_{n\ell}(1)}}_{\text{pure maturity+price effect}} \cdot \underbrace{\frac{\hat{B}_{n\ell}(1)}{\hat{B}_{n\ell}(0)}}_{\text{pure default price effect}} = \frac{1-\beta+\beta p\delta}{\delta(1-\beta+\beta p)} > 1. \tag{37}$$

The first component is what we have just discussed (see (36)): it tells us that the two thresholds are different because the price effect induced by default differs when interacted with longer maturity debt. The second component reflects our earlier discussion, for the case of no recessions and hence no default. In this case, long-term debt has both maturity and price effects. The third component is a pure 'default price effect', that account for the change in the sustainable level of debt brought about by the mere possibility of default in future recessions, with no interaction with debt maturity (see (33)). Of these three components, only the first one is smaller than one. Overall, the ratio 37 is greater than 1.

We conclude with an assessment of the effects of a bailout, with official lender replacing short-term market debt with long-term official debt. The implications for sustainability of a change in the composition of debt, from short-term market loans to long-term official loans, are synthesized by the threshold $\hat{B}(0)$. The indifference condition is:

$$u(y-\hat{B}(0)-a+q_{e,0}[\hat{B}_\delta(1)-(1-\delta)\hat{B}(0)]) + \beta p \frac{u(y-\delta\hat{B}_\delta(1)+q_e[\hat{B}_\delta(1)-(1-\delta)\hat{B}_\delta(1)])}{1-\beta}$$
$$+\beta(1-p)\frac{u(y-\tau-a)}{1-\beta} = u(y-\tau-a)+\beta p\frac{u(y-\tau)}{1-\beta}+\beta(1-p)\frac{u(y-\tau-a)}{1-\beta} \tag{38}$$

where for comparison with the case of long-term borrowing we hold constant the debt policy $\hat{B}_\delta(1)$. Setting $q_e = q$ and $q_{e,0} = q_0$, this yields the following result

$$\hat{B}(0) = \frac{\tau+q_0\hat{B}_\delta}{1+q_0(1-\delta)} < \frac{\tau+q_0\hat{B}_\delta}{\delta+q_0(1-\delta)} = \hat{B}_\delta(0) \tag{39}$$

where the inequality is strict because the initial stock of market debt is short term. Under risk-neutral pricing, a shift in the debt composition due to official lending does improve sustainability—although less debt is sustainable relative to the case where the debt was already long-term, $\hat{B}_\delta(0)$.

5.3 Discussion

A key lesson from our analysis is that official lending is always desirable conditional on an rollover crisis (as shown in subsection 5.1.1); but is not necessarily welfare enhancing when market financing is available (as shown in subsection5.1.2). Without output risk, results are quite stark in this respect. The safe zone unambiguously widens with official support, and responds positively to both maturity and prices of official loans. The maximum level of sustainable debt however does not necessarily rise, due to the fact that the country ends up with a high stock of risk-free debt in the long run, and this reduces the value of repayment relative to that of default at the threshold.

A discussed in Subsection 5.2, relative to the case of rollover risk only, output risk raises both the value of financial support in the transition to the safe zone, and the value of a larger safe zone—a key potential benefit from the latter is that the government has more "fiscal space" to borrow at low rates in downturns. The interaction between output and rollover risk thus creates an additional policy trade-off, that plays in favor of official support. Yet, it does not eliminate the limit problem discussed above, due to the effect of official lending on the steady state level of debt.

In this respect, it is worth stressing that, in our model, we assume that official institutions dictate the terms of official loans to the country, but impose no "conditionality", which would raise a host of issues in the strategic interaction between national policymakers and the official lenders. One such issue could indeed concern the size of deleveraging—i.e., the level at which the country can be expected to bring its debt in the long run. A condition on deleveraging would make the transition more costly, but would eliminate the long-run fall in consumption and utility that weigh on the decision to default at the upper threshold.

The analytical characterization presented in this section aim to illustrate the forces at work in a simple environment. A few comments are in order concerning our simplifying assumptions. For tractability, we have focused on the case in which bailout loans are available at all times and for an infinite number of periods. Not all our results can be expected to go through if this is not the case—i.e., if the availability of bailout funds is on a temporary basis. As official loans affect exit times and repayment/default decisions, it is possible that bailouts over finite horizons can paradoxically have adverse effects on sustainability. In particular, to the extent that a few years with official lending induce countries to service debt in a persistent recession, the eventual withdrawal of support may actually result in a lower threshold for the initial stock of debt that the country is willing to sustain—relative to the case where the default and repayment states are unaffected by the bailout terms. Also, to ease exposition, we have assumed that a country utility in default (the right-hand side of the indifference conditions) is unchanged following the introduction of the bailout. This need not be the case.

In addition, we have analyzed rollover and output risk separately, but these have significant interactions in the model (as well as in the data). Similarly, while we have considered only one type of bailouts, debt dynamics can potentially be very different if various types of official debt are used in conjunction with market lending. We will allow for this possibility in the quantitative analysis to follow.

As will be apparent from below, our analytical discussion in this section provides key key insight in the results from our calibrated exercises. In particular, the safe debt thresholds in a recession and in normal times move as predicted by theory. A smoother and easier transition reflects on a fall of market rates. Even the possibility of a fall in the upper debt threshold conditional on official support turns out to be quantitatively relevant for the Portugal calibration. Namely, abundant official lending ends up narrowing the 'crisis zone' not only from below (raising the level of safe debt), but also from above (lowering the debt threshold $B(1)$ at which the government defaults even absent rollover crisis).

6 Quantitative Analysis

In this section, we use the model in its full specification described at the beginning of Section 4 for a quantitative exercise focusing on the case of Portugal in July 2011—preferring it over Ireland, which did not face a recession but a banking crisis, and over Greece, as Portugal did not restructure its debt the way Greece did.[22] We then use the model to carry out some counterfactual exercises, to gain insight on the sensitivity of our results to key features of official lending—maturity and price.[23]

6.1 Calibration

We show the list of parameters and targets in Table 2. We normalize output Y to 100 so that the units in our model can be interpreted as percentage of GDP: e.g. $B = 120$ means debt that is 120% of GDP. We set the default cost at 5% of GDP, consistent with Cole and Kehoe (1996). This default cost is low relative to the literature (e.g. Mendoza and Yue (2012)), on the grounds that we assume this cost to be permanent—while others assume this to be temporary. We set the probability of recovery p equal to 0.33, in line with evidence of recovery in Portugal (and other euro area countries that received official support) where the economy bounced back after 3 years. Similarly, we set the fraction of

[22] See the Appendix for a summary of the Portuguese program.

[23] In Corsetti et al. (2017) we study the effect of the terms of the official loans on Ireland's market access conditions, and find results in line with the ones here.

output lost in a recession in line with the realized output drop in the 2011 recession, equal to seven percent. The target for the level of 'essential' government expenditure is average government consumption (21% of GDP). The presence of this non-homothetic term allows us to have a discount factor closer to standard business cycle or growth models than in standard quantitative sovereign default models (where the discount factor can be as low as 0.8; see also Bocola and Dovis 2015). To match model with data, the probability of a market closure π is calibrated in conjunction with the discount factor and the probability of recovery to target the market interest rates on Portuguese bonds in July 2011. Government revenue as a fraction of output is used to parameterize the tax variable θ. We follow Conesa and Kehoe (2017) in setting the relative weight of c and g in the utility function equal to 0.5; sensitivity analysis shows that this particular parameter is unimportant for our results.

The parameters discussed thus far are relatively standard and found in other models in this literature; we depart from the literature by introducing two types of official debt instruments into the government's decision problem. In accordance with their empirical counterparts, these two instruments have different maturities and rates. To capture debt maturity in a parsimonious way, we model long-term debt in the same way as most models in this literature (Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009) where the borrowing country repays a fraction δ of debt each period, and old and new debt are treated alike. Given this assumption, ESM debt is parameterized to reflect initial lending conditions to Portugal: 15 years maturity and no spread over the Bund rate and likewise for IMF debt (7 years maturity and a spread of 300 bps over the SDR rate). The market rate is endogenously determined; debt maturity is set to 6 years, consistent with the average maturity of Portuguese debt.[24]

6.2 The Portugal case study

In our calibration, the model economy is initially well within the crisis zone, conditional on the level of debt and the recessionary state of the economy. The country can only issues market debt at the high interest rates implied by both rollover and output uncertainty; as the cost of borrowing from the market is prohibitive, the government then chooses to borrow from the IMF and ESM instead.

We find that, with the parameterization of Table 2, our model replicates both the initial state of the Portuguese economy and the dynamic evolution of debt and market rates following the access to euro area official lending. In Figures 6 through 8 we show the evolution of

[24]We note that our stylized specification of maturity captures the key difference between IMF- and ESM-style lending, but falls short of accounting for the specific management of repayment flows that ESM programs feature in reality.

the Portuguese debt distinguishing market, IMF and ESM debt. In particular, as the IMF and ESM debt accumulates in the model as it does in the data, consistent with empirical evidence, market rates fall after the economy recovers in the model—holding constant the sequence of shocks to output and for given market financing. Remarkably, in the model, total debt as a fraction of output is upward sloping, although only mildly so. In this dimension, the model is quantitatively less effective: the total-debt-to-GDP ratio rises by more than 20% in the data. In other words, while our model and parameterization replicate the shifts in debt composition towards official loans, the substitution into (cheaper) official debt and away from market debt hinders the model's ability to replicate the overall rising trajectory observed in the data (where market debt stays almost flat as official debt rises).

It should be stressed that the model is able to capture not only most of the change in dynamics of official debt and the associated evolution of borrowing costs (especially the endogenous response of market rates), but also the Portugal's transition from the 'crisis' to the 'safe' zone—all these generated endogenously and not targeted in the calibration. If anything, the model achieves too much, in that our calibration does not factor in key policy initiatives that, especially after 2012, weighed on market conditions—such as the introduction of the Outright Monetary Transactions program by the ECB, five quarters after the start of the rescue program.

6.3 Counterfactuals

We now use the model as a lab to shed light on how the terms of official lending may affect debt sustainability. In particularly, theory suggests that we can think of sustainability in terms of four debt limits or thresholds—separating a safe from a crisis zone for debt, conditional on the economy being in a recession or in normal time. To gain insight on sustainability, we can evaluate quantitatively how these four limits respond to changing the terms of official assistance in three dimensions: size, price and maturity.

We report our results in Figure 9 through 12. Each figure includes 4 panels, one for each debt limit. Each panel shows 16 histograms, that are static representations of the full dynamic model. Debt thresholds can be read on the y-axis, corresponding to the height of each histogram. The color pattern of the histogram represents the composition of borrowing. This changes from market only (the first block to the left of each figure) to markets plus official lending—allowing the IMF- and ESM-type loans to increase in steps, each with size 5% of GDP. Going from left to right: Block 1 is the no-assistance case. Block 2 through 4 shows what happens to the debt limit when the model economy receives IMF-type assistances in tickets of 5, 10 or 15% of GDP. In block 4 through 8, we consider the effect of an official loan with ESM characteristics equal to 5% of GDP—first on its own, then added to the sequence of tickets from the IMF. In blocks 9 through 12, we repeat the same experiment with ESM loans up to 10% of GDP. Blocks 13 through 16 repeat the same with an ESM ticket up to 15% of GDP.

Consider Figure 9 first, referred to our benchmark calibration. Focus on b(0), on the upper left-hand side of the figure. This threshold measures how much debt free of default risk the country can sustain when the economy is in a recession and suffers a market rollover crisis. A key result suggested by the panel is clearly in line with our analytical result—that the size of the safe zone, inside which there is no vulnerability to either rollover or fundamental risk, is increasing in official assistance. Our quantitative analysis confirms that this goes true whether assistance comes in the form of either IMF or ESM loans, or both. Quantitatively, the first panel suggests that, in a recession, the economy could only sustain about 80% GDP of safe debt, had it to rely exclusively on borrowing from the market. But the stock of sustainable safe debt can go up to 90% when the country holds a portfolio of ESM and IMF loans, each measuring up to 15% of GDP. Again, this is consistent with our theory where official debt unambiguously raises the level that is sustainable without market financing.

Comparing the two graphs in the first row of the figure further suggests that the effect of official lending is similar on b(0) and b(1)—the latter is the debt limit when the economy is not in a recession. This limit, higher than b(0), is also monotonically rising in assistance—the response is stronger.

The picture is however quite different, and much richer, for B(0) and B(1), the debt limit in a recession or in normal times, respectively, beyond which the government defaults whether or not subject to a rollover crisis (debt is not sustainable for fundamental reasons). Consider the case in which the country has access to market financing while in a recession B(0). Here, a moderate amount of official loans is good—as it can raise debt levels from 175% to 180%. But higher levels of official debt turn out to be counterproductive, in the sense that it ends up *decreasing* the total amount of sustainable debt. This is consistent with the theory as well: we have shown that, to the extent that a wider safe zone translates into an increase in long-term steady-state debt (shown in the first panel) lowers consumption in the future; this raises the incentives to default, hence lowers the amount of sustainable debt. Similar conclusions can be drawn looking at the panel for B(1).

There are four key variables underlying our benchmark result: ESM debt maturity, ESM lending rate, IMF debt maturity, and IMF lending rate. To examine the role of each of these variables separately, we run four counterfactuals: a counterfactual where both the ESM and IMF lend at the ESM rate, keeping everything else constant; a counterfactual where both the ESM and the IMF lend at the same maturity, ceteris paribus; and two counterfactual switching maturity and rates across lending institutions.

The results for these counterfactuals are shown in Figures 10 through 13, using the same format as Figure 9. Figure 10 illustrates the effects of these four types of policies on the level of sustainable debt for a country in a recession with no access to market financing—the b(0) threshold. In the top row of the figure, the IMF and the ESM loans have the same long maturity as ESM debt (15 years) in the panel to the left—the same shorter maturity (7 years) as IMF debt in the panel to the right. In the bottom row, the loans of both the ESM and IMF have the same low interest rate (150 bps above risk-free) as charged by the ESM in the panel to the left—the same higher interest rate (350 bps over risk-free) as charged by the IMF in the panel to the right. A key result is that, *ceteris paribus*, a longer ESM-like debt maturity has a stronger impact on the threshold than a lower ESM-like interest rate. Holding the debt composition constant, sustainability is highest in the case when both lending institutions structure their bailouts with ESM-debt maturity, and are lowest when both bailouts have IMF-debt maturity.

The result that changing debt maturity has a stronger effect on thresholds than varying interest rates is confirmed in the other counterfactuals. These however also lend quantitative support to a key theoretical prediction from Section 5. Namely, debt limits are increasing in official debt conditional on no market financing, but not necessarily so conditional on market financing. In particular, Figures 12 and 13 show that the debt limits conditional on market financing at first increase with official debt, but eventually drop. They actually drop to levels lower than conditional on no official support at all. As explained in Section 5, underlying this result is the fact that the the upper limit with market financing is a function of the lower (safe debt) threshold—and a higher stock of safe debt raises the average level of debt that the government finds it optimal to maintain in the long run. Since a rise in the stock of long run debt translates into lower long run consumption, every thing equal this reduces the incentive to repay debt relative to defaulting. This is indeed a key message of our paper: depending on how official debt is structured, the maximum stock of debt that is sustainable in the crisis zone (when markets keep pricing rollover risk) can be higher or lower than the amount that is sustainable without official lending—the way in which public support is structured and made available to country matters significantly.

In closing, we should note that the somewhat strong effect of official lending on sustainability that we find in our calibration can be moderated by introducing some (political) uncertainty in the access to official lenders, something that we abstract from but would be straightforward to model. Similarly, more nuanced results could follow from an important development of the model, to include program conditionality specifying reforms and policies the government should follow in exchange for official support.

Nevertheless, there is an important lesson from our exercises. In our calibration, sustainable debt can range from low (80% GDP) to very high (180% GDP) levels, depending on the state of the economy (output and market access) as well as on the availability, size spreads and maturities (debt composition) of official loans. The wide range of variation in the thresholds and across them predicted by the model is a healthy reminder that, per se, the stock of outstanding debt is a very imperfect indicator of sustainability.

7 Conclusion

In this paper we have explored the extent to which the terms of official lending affect debt sustainability, by impinging on the behavior of both governments and investors. Official support modifies the incentives for a country to borrow, repay, and eventually reduce its debt to a sufficiently low level, such that the government can borrow at default-risk free prices. Using our model, we have explored the mechanisms by which official loans improve risk-sharing in periods of market turbulences and recessions—gaining insight into various countervailing forces that official lenders may want to consider when designing a bailout package. Indeed, if the official lending is structured as to induce higher levels of debt and thus higher interest bill in the long run, the upper threshold for sustainable debt can actually fall. This is because a high interest bill in the long long run lowers long run consumption, and thus raises the cost of servicing the debt relative to defaulting. Repayment may become less likely in states of the world where the economy has been subject to a sufficiently long sequence of negative shocks to output.

We take our model to the data and show that it can replicate the debt and interest rate dynamics in the recent debt crisis in Portugal (2011-2015). Consistent with our theory, we find in the quantitative analysis that official debt is a crucial component in spurring the recovery of the Portuguese economy and lowering market rates. While both quantitatively significant, longer maturities have a stronger effect on sustainability than lower spreads. Different terms of official lending give rise to significantly different thresholds for sustainable debt—quantitatively, per effect of the bailout, the safe debt threshold varies between 80 and about 100% of GDP; the fundamental default threshold varies between about 150 to over 180% of GDP. While political uncertainty about the access to and the terms of the program could moderate the effects of the bailout, the implications are bound to remain significant.

Among directions for future research, an important one concerns the problem official lenders solves when offering bailouts. The key issue is how to model the objective function(s) of the lenders, possibly reflecting welfare-relevant distortions and spillovers from a country default, as well as the constraints under which they operate. Relatedly, the analysis in this paper abstracts from possible adverse consequences of a bailout on the government incentives to undertake costly but beneficial reforms or policies. These incentives may cause the government to respond differently to bailouts addressing self-fulfilling risk as opposed to fundamental risk. A richer framework needs to be developed to gain insight on the policy trade-offs that arise as they two risks interact significantly before and during crises.

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8 Appendix A: Country Programs

8.1 Greece

When in late 2009, the Greek Government approached its Euro area partners, the first reaction was to impose on Greece a significant fiscal adjustment. As this strategy failed and the situation spin out of control, in March 2010, Euro area governments together with the IMF agreed to provide a 110 billion EUR financial assistance package, composed of an IMF credit and bilateral loans by euro area members (Greek Loan Facility, GLF). The GLF contributed with 80 EUR billion. The maturity of this loan was 5 years, with a 3-year grace period. Its pricing, following IMF practice, was organized in steps. For the first 3 years the interest rate was set at 6-month Euribor with a 300bps surcharge. Thereafter, the cost increased by 100bps. From its side, the IMF contributed with a 30 EUR billion Stand-by-Agreement with a 3-years duration, a maturity of five years, the standard 200 bps for credit above 300 percent of the quota, and additional 100 bps for credit outstanding after 3 years.

By mid-2011, despite that the first program reviews spoke of an impressive start, the situation took a turn for the worst. According to Pisani-Ferry et al. (2013) the reasons for the set-back were excessively optimistic projections, initial official indecision, weak program implementation and the excessive cost of initial funding conditions. The reaction of the authorities was to provide additional support by modifying the terms of the GLF. In June 2011, GLF maturities were extended 5 years, and spreads reduced, by 100 bps, to those of the IMF loan. Despite these additional measures, by early 2012, it became clear that Greece would not make it without a contribution from its shrinking private-sector creditor base. In March 2012, Greece signed a second program.[25] The new program, signed with the EFSF and the IMF, envisioned EUR 130 bill of additional funding, which were to be added to 34.5 bill undisbursed Funds from the GLF. From the EUR 130 bill, 25 EUR bill. came from a new 4-year IMF EFF program. The rest (EUR 104 bill) was provided by the EFSF, with a 20 year maturity and 150 bps margin. Simultaneously, the GLF rates and maturities were modified to match the EFSF conditions. The terms of the EFSF loan were further softened in December 2012, when Greece received: a reduction on some fees and margins to zero, an extension of maturities up to 30 years and a 10 years interest deferral. At that time, the maturities of the GLF were extended further, to 30 years, and the spread lowered to 50 bps. In the final step thus far, in September 2016, Greece entered into a new 3-year 86 EUR bill program with the ESM.

[25] The MoU of the new program included a private-sector debt restructuring. The exercise brought 100 billion of NPV relief (Gulati et al., 2013).

8.2 Ireland

On December 2010, overburdened by the housing bubble burst and the subsequent bail out its banking system, Ireland became the first client of the EFSF and the EFSM. The Irish program, designed to re-establish a sound economic and financial situation and to restore its capacity to finance itself on the financial markets, provided a financing package of EUR 85 bill, to be disbursed between 2010 and 2013. It included contributions by the EFSM (22.5 bill) and EFSF (17.7 bill), and bilateral loans from UK, Sweden and Denmark (3.8 bill, 0.6 bill and 0.4 bill, respectively).[26] The maturity of the loan was set at 7.5 years and the margin at 250 bps. Additionally, Ireland signed a 7 years EFF agreement with the IMF for 22.5 bill.

Despite the official support, by mid-2011, market rates had crept up to unsustainable levels. The bad performance was the result of the effect on public debt of excessively rapid deleveraging and the bail-out of the banks junior creditors. In attempt to provide further support to Ireland, the terms of the financial agreement were modified in July 2011. In addition to fully eliminating the margin for both EFSM and EFSF loans, the maturity of both loans was extended by seven years, to a maximum of 15 years. A final change in the financing terms occurred in April 2013, when authorities provided additional breathing space by deciding that EFSF and EFSM loan maturities would be extended by 7 and a half years, to 22 years.

8.3 Portugal

In April 2011, it was the turn of Portugal to request support to re-establish a sound economic situation and restore its capacity to finance itself on the financial markets. In this case the financing of the 78 bill program fell on equal parts on the EFSM, EFSF and IMF. While the maturity was set to 7.5 years, as in Ireland, the margin was lower, about 210 bps. The lower Portuguese spread might have reflected the fact that by that time both the Greek and Irish programs were performing below expectations, and the authorities were already discussing cutting the borrowing costs for Greece. In turn, Portugal signed a 26 EUR billion 3-year EFF program with the IMF.

The program relied on the timely implementation of structural reforms. It was soon apparent that these reforms could not be expected to materialize over the relevant horizon. In reaction to these negative developments, the EFSF and the EFSM granted to the Portuguese authorities an improvement in the conditions of the loan similar to those provided to the

[26]The program also included an Irish contribution of EUR 17.5 bill.

Irish. As early as July 2011, the euro area governments decided to fully eliminate the margin for both EFSM and EFSF loans and to extend the maturity of EFSM and EFSF loans to a maximum of 15 years.[27] In order to maintain identical conditions in Portugal and Ireland, identical to what was done with the Irish loan, a final change in the terms of the EFSF and EFSM programs occurred in April 2013. On that date, authorities decided that EFSF and EFSM loan maturities would be extended by additional 7 and half years, to 22 years.

8.4 Spain

On June 2012, Spain made a request for a Bank Recapitalization Facility to finance the recapitalization of its domestic financial institutions. Initially, it was envisaged that this Financial Assistance was to be provided by the EFSF until the ESM became available. Eventually, the ESM became operational in time to address the assistance from the onset. The program designed by the ESM for Spain was not oriented to tackle a balance of payments problem, but a structural problem on the banking system. Reflecting the specific focus of the program, the attached conditionality addressed financial sector issues only. As a result, the program design and implementation deviated so much from the IMFs program template that the Fund could not participate financially in that program. In this way, despite the Fund participated in the program by providing technical assistance and performing monitoring tasks, Spain became the first Euro area country to be treated by the euro area sovereign bail-out funds in financial solitude.

The ESM program to Spain granted the authorities access to up to 100 EUR bill. Eventually, only 40 EUR bill were actually disbursed. Following the pricing guidelines of the ESM, Spain is charged a 50 bps margin. The loan had at its inception a 15 years maturity.

8.5 Cyprus

Cyprus addressed a request for stability support to the ESM and the IMF on June 2012. The economic adjustment program was intended to address short and medium-term financial, fiscal and structural challenges facing Cyprus. The key program objectives were to restore the soundness of the Cypriot banking sector and rebuild market confidence by restructuring and downsizing financial institutions.

[27] As the initial EFSF loan to Portugal featured a lower margin, the June 2011 margin cut was 50 bps larger for Ireland than for Portugal.

In Cyprus the template replicated previous EFSF programs, and both the ESM and the IMF contributed to the program. The ESM contributed with EUR 9 bill and the IMF with 1 EUR 1 bill respectively. The IMF provided support under a 7-year EFF agreement with the usual pricing structure. In turn, the ESM loan to Cyprus had a 15 years maturity, extending up to 2030. The margin charged by the ESM is 10 bps.

Appendix B: Endogenous exit time, T

The analysis of the upper threshold $B_(1)$ in the text abstracts from the effect of varying the terms of the official loans on the time of exit from the crisis zone. In this appendix, we complete our argument by examining the general case, discussing how the optimal time to exit T changes with a (sufficiently large) fall in δ. To do so, observe that $\frac{dV^T}{d\delta} < 0$ as $T \to \infty$, since the steady-state term disappears, and $\frac{dV^T}{d\delta} > 0$ as $T \to 0$, since the transitional term disappears. Thinking of T as a continuous variable and using the fact that $\frac{dV^T}{d\delta}$ is continuous and decreasing in T, by the Intermediate Value Theorem, we have that $\exists T^*$ s.t. $\frac{dV^T}{d\delta} > 0$ for all $T < T^*$ and $\frac{dV^T}{d\delta} \leq 0$ for all $T \leq T^*$.

Let T_{nl} solve

$$V^{T_{nl}}(B_{nl}(1)) = \max\{V^1(B_{nl}(1)), V^2(B_{nl}(1)), \ldots, V^\infty(B_{nl}(1))\} = \frac{u(y-\tau)}{1-\beta}$$

so that T_{nl} is the optimal time to exit absent official lending, with market debt indexed by maturity parameter $\delta_{nl} = 1$. Define T_l to be the optimal time to exit when there is official lending characterized by maturity parameter $\delta_e < 1$ and q_e. Suppose $T_{nl} \leq T^* - 1$. Then we know that, as maturity lengthens with $\delta_e \searrow 0$, we have $\frac{dV^T}{d\delta} > 0$ for all $T \leq T_{nl}$ and $\frac{dV^T}{d\delta} \leq 0$ for all $T > T^*$. So, for a sufficiently large drop in δ_e it is possible that the optimal time to exit switches to $T_l > T_{nl}$.

Now let $B_{T_l}(\delta_e)$ and $B_{T_{nl}}(\delta_e)$ denote the level of sustainable debt under official lending associated with, respectively, the time to exit T_l and the time to exit T_{nl}. Similarly, let $B_{nl}(1) = B_{T_{nl}}(\delta_{nl})$ denote the level of sustainable debt in the absence of official lending with time to exit T_{nl}. We then have that the threshold $B(1)$ obeys the following: $B_{T_l}(\delta_e) > B_{T_{nl}}(\delta_e) > B_{T_{nl}}(\delta_{nl}) = B_{nl}(1)$. The first inequality follows from the value function being decreasing in debt, and T_l being the optimal time to exit (so $V^{T_l}(\delta_e) > V^{T_{nl}}(\delta_e)$);[28] and the second inequality follows from the earlier observation that the level sustainable rises

[28]The result follows from equating these values with the value of default.

with longer maturity provided π or T is sufficiently large. As the change in maturity transpires into a lengthening of the time spent in the crisis zone, this raises the equilibrium value function (and hence the level of sustainable debt)—since the utility gains from higher government consumption while in the crisis zone outweigh the losses in the safe zone.

Tables and Figures

Table 1: IMF versus ESM Lending Terms

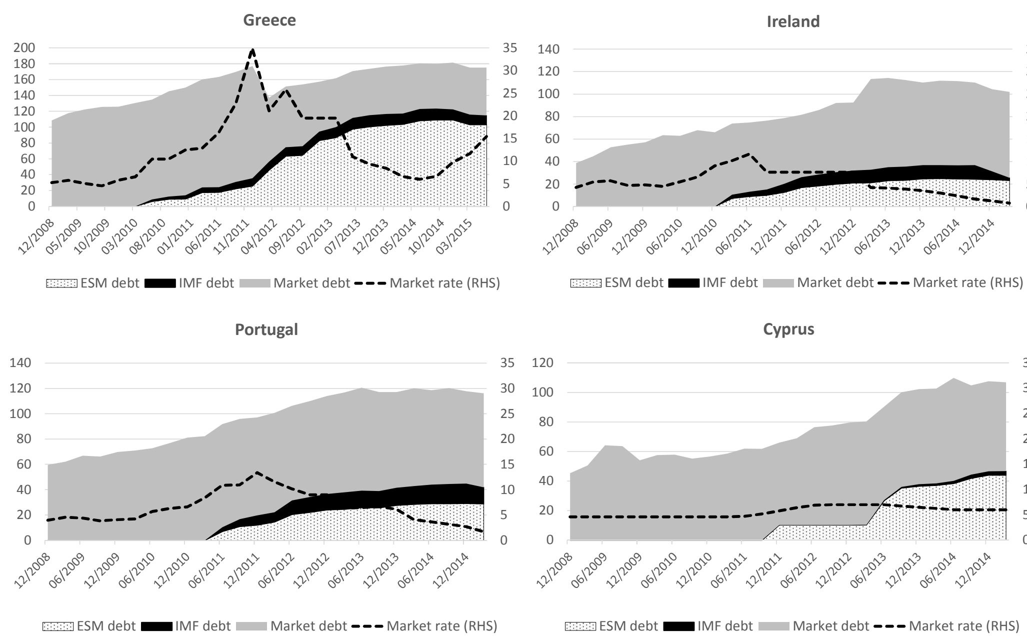
	EFSF/ESM Support		IMF Support	
	Maturity	**Interest rate**	**Maturity**	**Interest rate**
Greece	30 years	1.07	8 years	4.06
Ireland	22 years	2.45	7 years	3.07
Portugal	22 years	2.25	7 years	3.07
Spain	12.5 years	0.5	-	-
Cyprus	15 years	0.87	4 years	1.07

Sources: International Monetary Fund, European Commission, European Financial Stability Facility and European Stability Mechanism. Interest rates are computed as of June 2013.

Table 2: Calibration

Y	Output	100
Z_d	Default cost	0.95
A	Fraction of output during recession	0.93
β	Discount factor	0.98
π	Real interest rate in crisis	7%
θ	Goverment revenue as a share of output	0.4
$\overline{g}$	Level of essential government expenditure	25
γ	Relative weight of c and g in the utility function	0.5
p	Probability of leaving the recession	0.33
δ	Amortization of market borrowing	0.1667
δ_i	Amortization of IMF loan	0.1429
q_i	Interest on the IMF loan	0.9483
δ_e	Amortization of ESM loan	0.067
q_e	Interest on the ESM loan	0.9662

Figure 1: Market Spreads and Sovereigns' Creditor Structure



Sources: European Commission, European Stability Mechanism, Central Banks and Bloomberg. Debt is measured as percentage of GDP. The market rate, measured on the right hand side axis, refers to the spread on the benchmark 10 year sovereign bond. ESM debt refers to any debt issued by any of the various European vehicles (Greek Loan Facility, EFSF, EFSM, ESM) and to bilateral loans provided by European Governments.

Figure 2: Interest Rates on Market and Official Financing

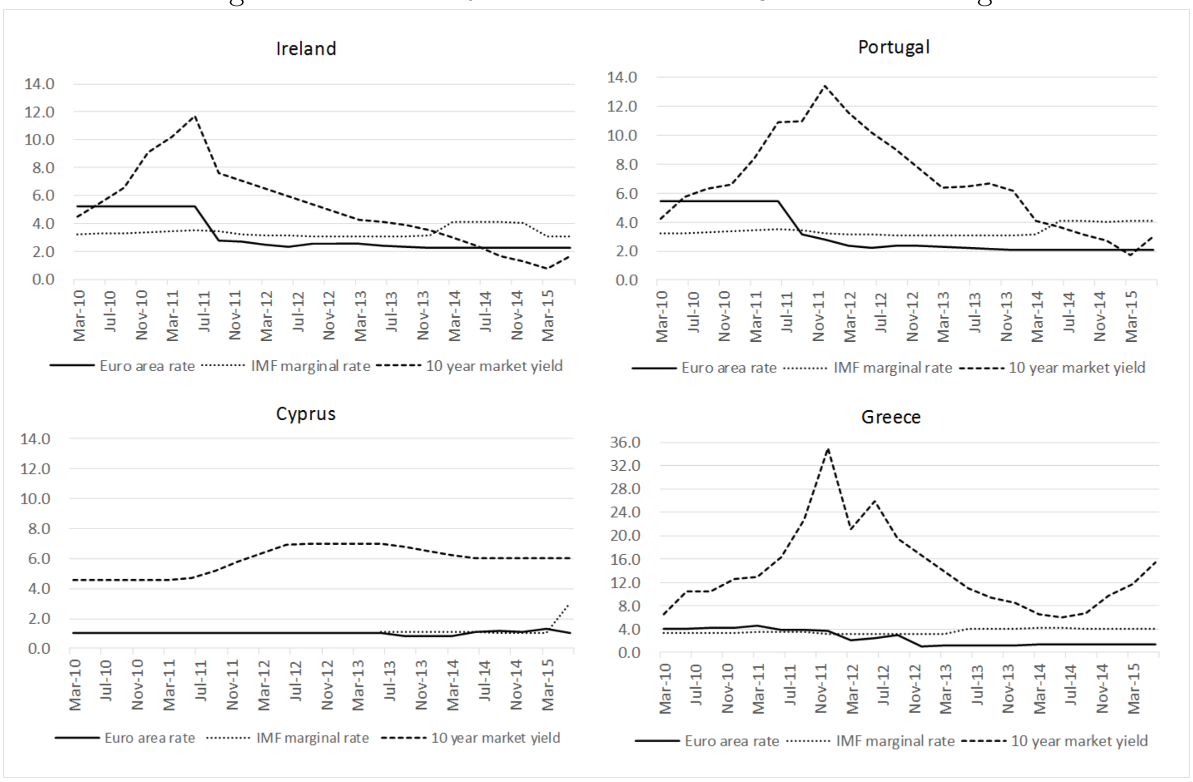
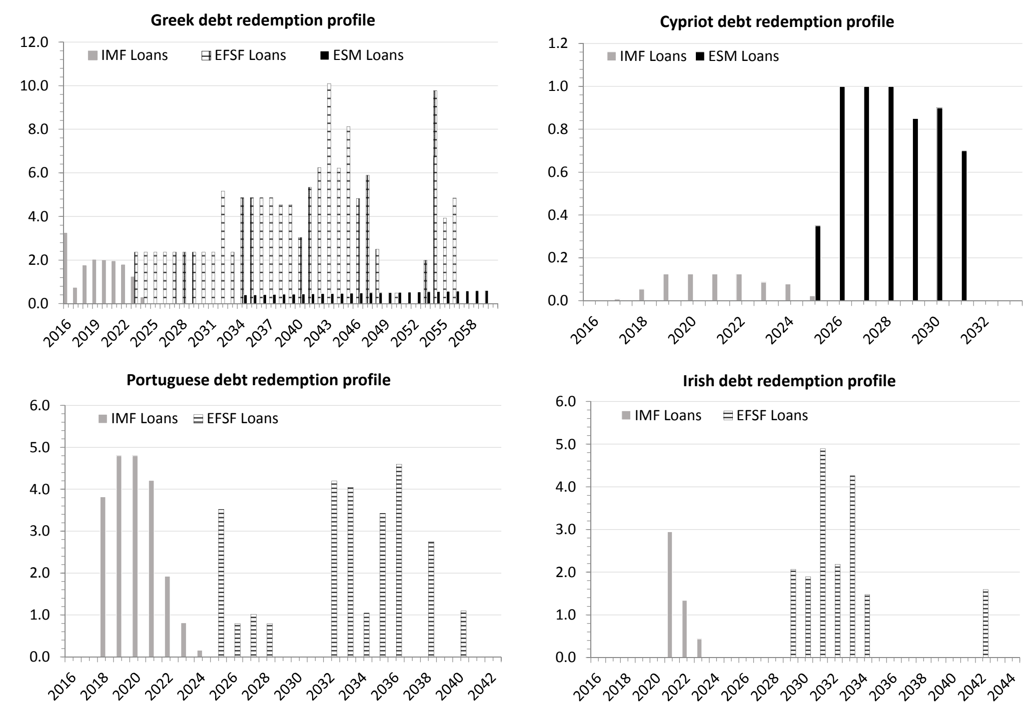


Figure 3: Repayment Profiles: IMF versus Euro-area Institutions



Sources: European Commission, European Stability Mechanism and International Monetary Fund. Debt repyaments measured in billion euros.

Figure 4: Roll-over needs and Official Lending

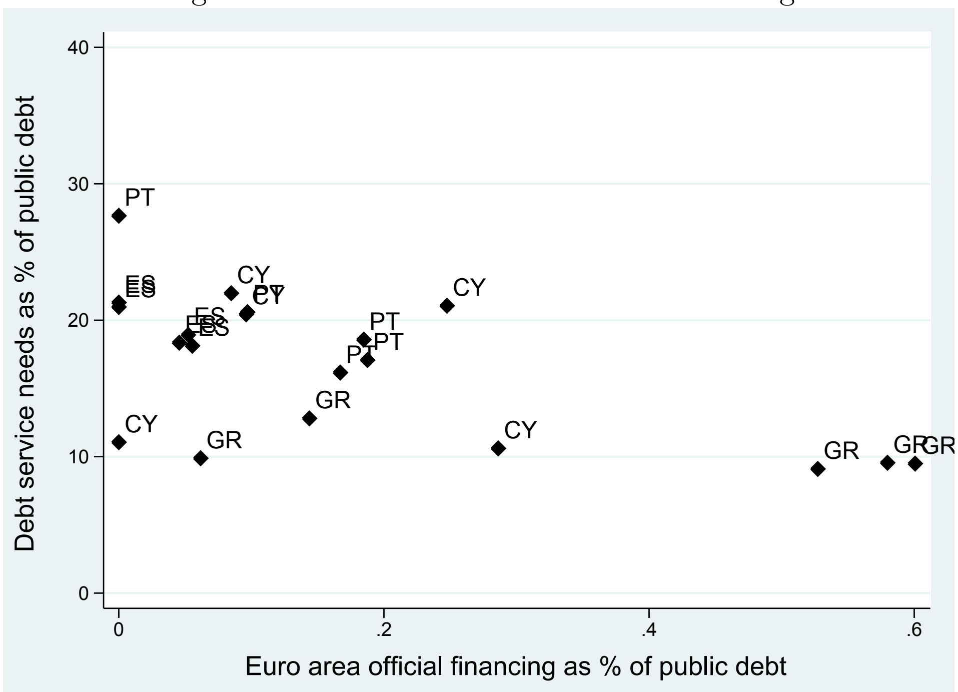
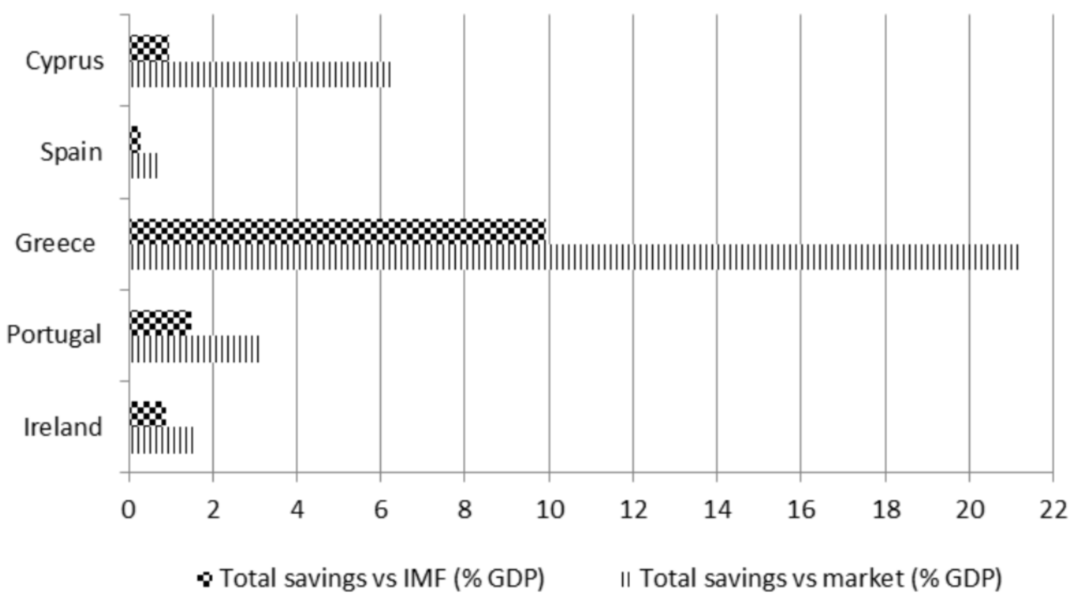


Figure 5: ESM vs. Market/IMF - Interest savings (as percentage of 2016 GDP)



Source: European Stability Mechanism, International Monetary Fund and Bloomberg.

Figure 6: Evolution of Portuguese Debt - Data

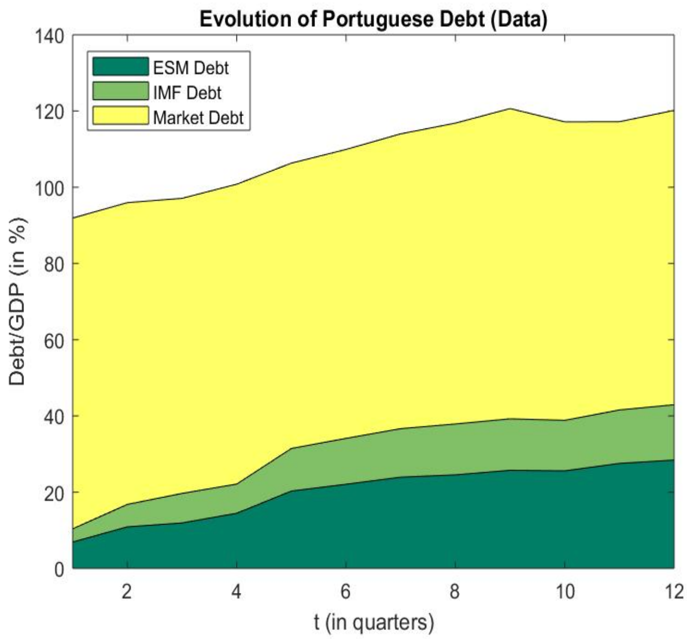


Figure 7: Evolution of Portuguese Debt - Model

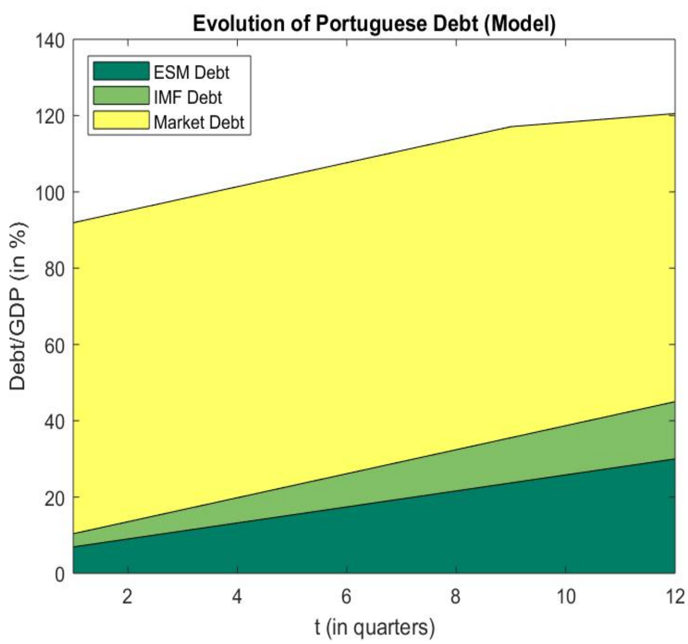


Figure 8: Market rates - Model vs Data

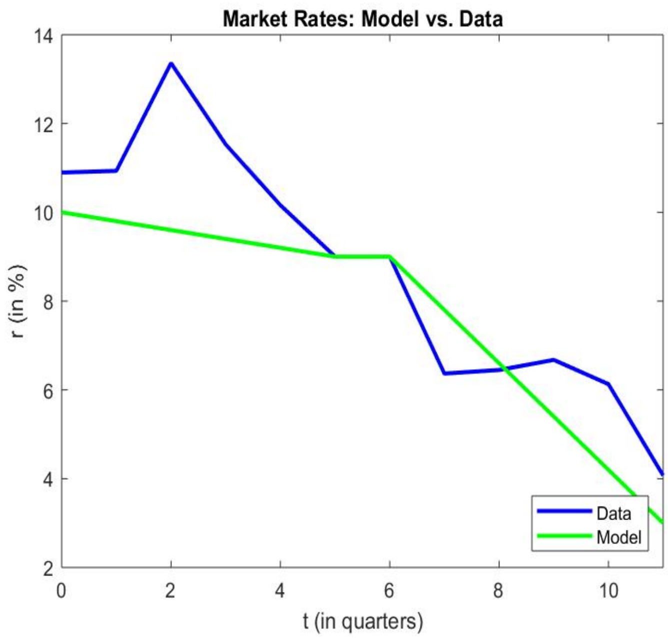


Figure 9: Benchmark

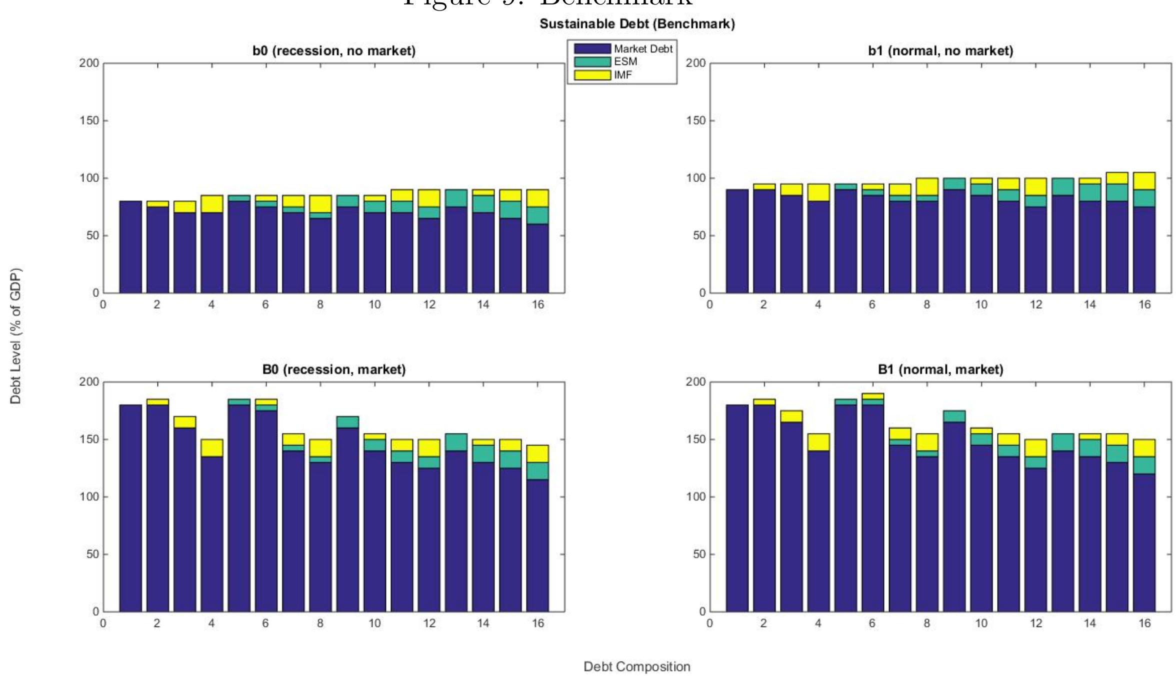


Figure 10: Safe zone threshold in a recession and no market financing $b(0)$

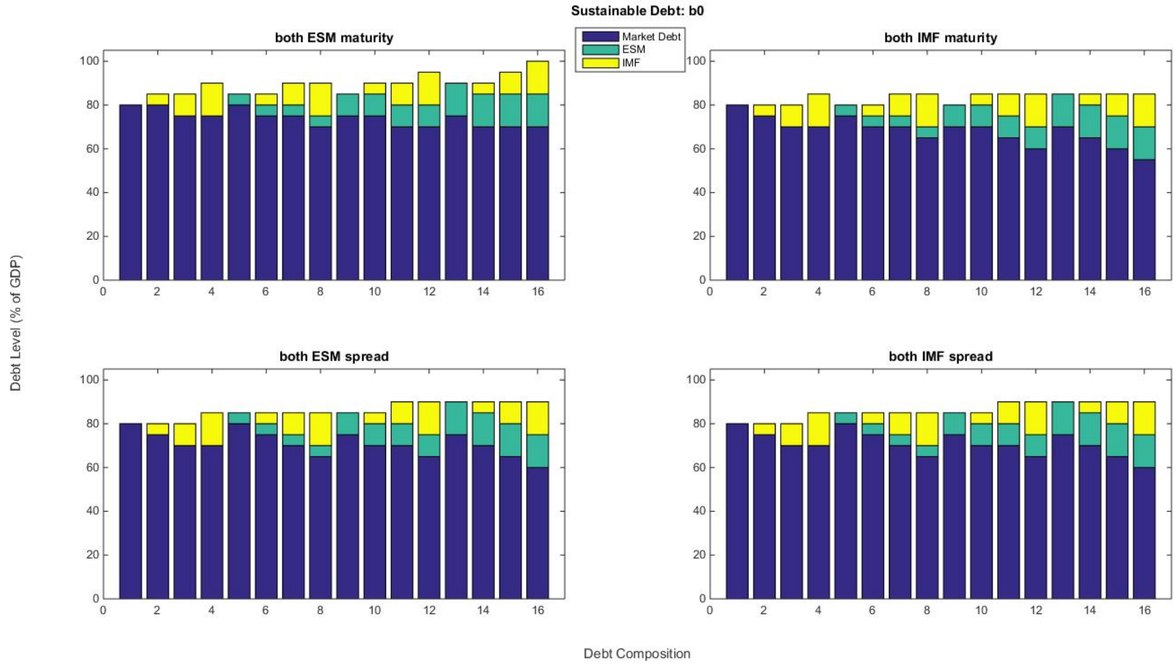


Figure 11: Safe zone threshold in normal time but no market financing $b(1)$

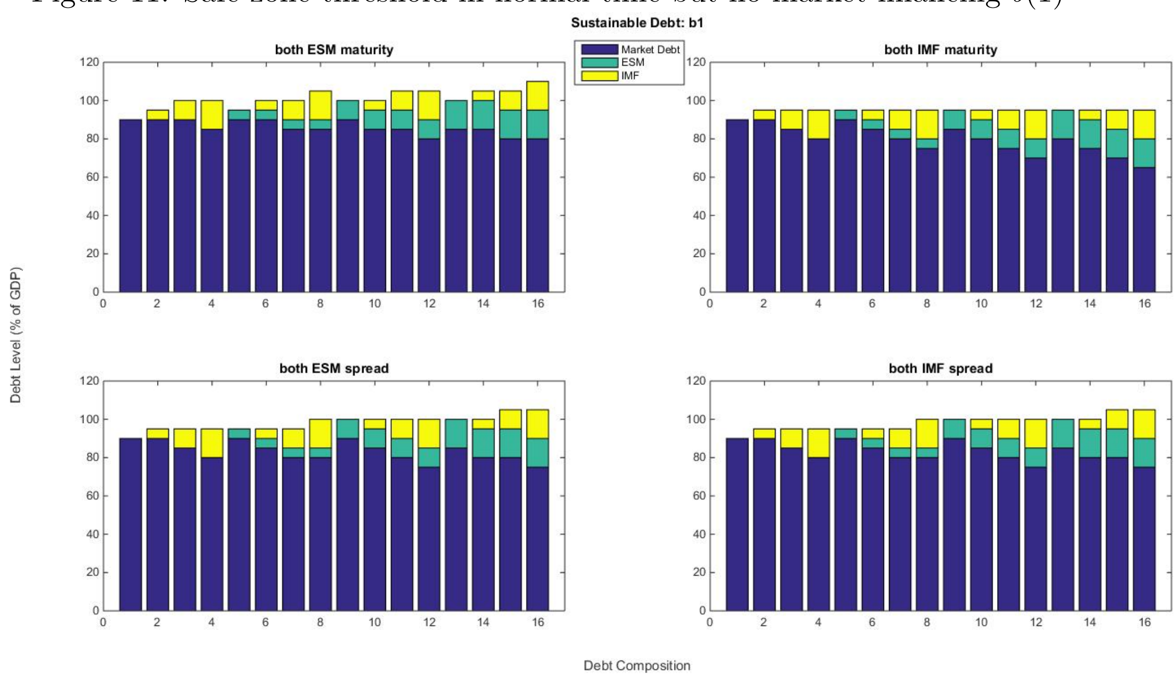


Figure 12: Upper crisis zone threshold in a recession $B(0)$

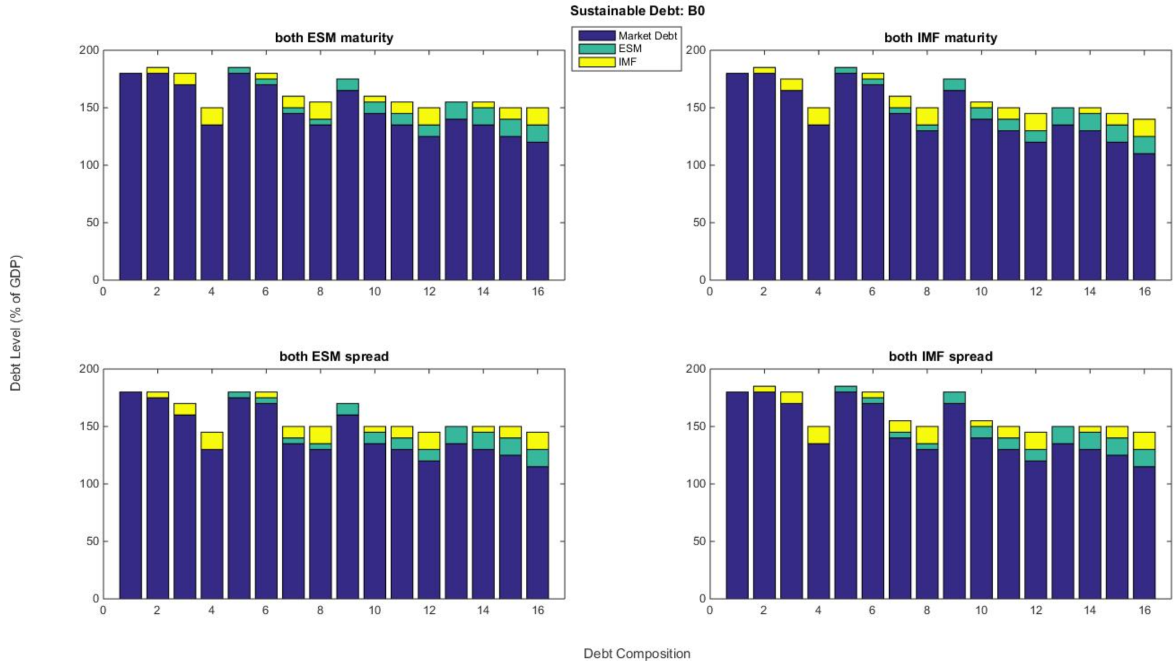


Figure 13: Upper crisis zone threshold in normal times $B(1)$