Fertility Policies and Social Security Reforms in China: Global Ramifications in the 21st Century

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Abstract

We analyse the domestic and international consequences of China's fertility policies and institutional reforms on global saving, interest rates, and social security programs. A key innovation is to allow for endogenous fertility responses and its aggregate feedback effect. The framework we develop consists of an open-economy overlapping generation model, in which fertility decisions, capital accumulation, and the evolution of social security is endogenously determined. We show that the impact of a move from a 'one-child policy' to a 'two-children' policy and social security reforms, along with other institutional developments, depend crucially on whether binding fertility constraints are in place. We demonstrate how financial integration and fast growth in China, along with a reduction in social security benefits, and/or an loosening of household credit constraints in China can potentially ease social security pressure elsewhere by stimulating fertility in ageing economies under pension sustainability duress.

JEL Classification: **F21, F32, F41**

Key Words: Household Credit Constraints, Age-Saving Profiles, International Capital Flows, Allocation Puzzle.

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1 Introduction

Challenges to the financial sustainability of social security programs around the world is at the forefront of a global policy debate. Aging in the industrialized economies and the imminent aging in China, combined with large government deficits in many of these countries have become a cause of general concern and a case for pension reform. In an increasingly globalized economy, the world interest rate and global demographics feature prominently in the fiscal sustainability of the pension system in individual countries. China may well play a key role in this process with its ever expanding importance in the world economy. While its contribution to global saving over the past two decades—and in turn to the steady decline in the world interest rate–has been plausibly significant, its ongoing and imminent array of policy reforms and institutional changes can continue to impinge on the world economy.

A unique and unprecedented program introduced in the early 1980s aimed at controlling China's population growth is the'one child policy'. The drastic reduction in fertility and ensuing large decline in the share of young workers has put substantial pressure on its own social security program. The one-child policy, as investigated in Chakoumane et al (2013), has been possibly an important factor causing the sharp rise in China's household saving rate. The gradual unwinding of these fertility restrictions, starting from the move towards a "two-children policy" recently, may reverse these effects on saving and potentially put upward pressure on global interest rates. In this paper, we analyse the effect of these policies on China's saving and social security program—and their potential global ramifications.

One of the central points argued in this paper is that the endogenous responses of fertility are particularly important when it comes to analysing social security reforms. Changes in social security (as well as other institutional changes) can affect fertility decisions which then feeds back onto social security taxes and/or benefits. At the same time, understanding the aggregate impact of fertility restrictions on China's economy requires knowledge on how its natural rate of fertility would have evolved along with its institutional and economic development. The past literature, however, has largely taken fertility and demographics as exogenous.

Thus the appropriate framework to analyze these issues, taking into account the important general equilibrium and feedbacks effects of fertility, saving and interest rates—and in an international setting, is hitherto absent. A main objective of this paper is to develop a tractable framework in which the underlying mechanism on how these variables interact are made transparent, and from which the consequences of policy reforms in China can be elucidated. Our generalized framework consists of a large-open economy, three-period overlapping generations model with endogenous capital accumulation and a social security system. Children support parents in old age—making intergenerational transfers a key factor in determining fertility decisions. The international interactions are centered around very different economies such as China and the U.S., and therefore we allow for basic asymmetries in the model—in fertility policies, social security programs, and financial development.

We show that the economy is characterised by three key relationships. The first is optimal saving and interest rate— which depend on fertility and social security taxes and benefits. Its implied relationship between interest rates and fertility is positive: a greater number of children is associated with higher expenditures and larger transfers in old-age—both of which lead to lower saving and higher interest rates. The second condition is based on optimal fertility choices—which depend on, among other things, the level of the interest rate. Higher interest rate to wage ratio lowers the benefits of children in terms of transfers and hence discourage fertility. The relationship between the interest rate and fertility under this second condition is thus negative. The third key relationship is determined by a a sustainable social security system, which embeds endogenous changes in social security parameters when fertility adjusts.

We perform the following policy experiments in China in a closed-economy setting: (1) a relaxation of the one-child policy to a 'two-children policy'; (2) social security reforms; (3) financial development; (4) intergenerational transfers subsides in importance. We show that the impact of these current and pending reforms crucially depends on whether China maintains its binding fertility constraint or not. For instance, a higher replacement ratio that increases social security benefits to the elderly under a PAYGO system would be partially offset by a hike in taxes—–if fertility can endogenously respond: lower saving that raise the interest rate would lead to a fall in the fertility rate. Under constrained fertility, however, agents would be able to reap the full benefits of the rise in social security, and the attendant decline in saving would be larger. Similarly, financial development in China —in particular, an easing of household borrowing constraints– may exert pressure on the social security system by reducing equilibrium birth rates—unless fertility constraints are in place.

As China embarks on a new two-children policy, the likely scenario is that its saving rate would fall considerably—not only because of direct conseqeunces of having more children (the expenditure effect and transfer effect), but also because higher fertility would ease the social security system and potentially raise social security benefits. Thus, the predicted decrease in saving is thus larger when taking into account the effect of fertility policies on the pension system.

The question of whether fertility constraints will still be binding or not, particularly as it moves to a two-children policy is an important one—determining not only the aggregate impact of the fertility policy or of a complete abolition of fertility controls, but also its indirect contribution to the impact of other reforms. Indeed, a framework that allows for endogenous adjustments to fertility demonstrate that the natural rate of fertility may fall substantially with institutional developments (such as financial development or cultural changes in which intergenerational transfers subsides in importance) and pension reforms. If the two-children policy is no longer binding, a complete lift of fertility control would have no effect on the economy.

We show that China's various reforms have international ramifications in an increasingly globalised economy, and the extent of these effects depend on the basic asymmetries that characterise countries like China and the U.S. Differences in fertility, financial development, and pension systems all play an important role, and any such policy changes can spillover onto the social security system in advanced economies. We revisit the policy reforms in China in an open economy setting—

in addition to the impact of further financial integration and China's ongoing growth or growth slowdown.

There are a number of likely reforms in China that can potentially relieve pressure on the sustainability of the social security system in the U.S.. For example, the continued integration of China and its ongoing fast growth can stimulate U.S. fertility by pushing down world interest rates. Binding fertility constraints, in addition, significantly contributes to this effect. Interestingly–thus—policies that aim at reducing fertility in China can inadvertently stimulate fertility elsewhere. By analogy, moving to a two children policy independent of other reforms in China may not bode well for U.S. pension going forward. Similar effects can arise from a social security reform that entails a reduction in pension benefits in China. The one child policy and the drastic reduction in the labor force has posed a challenge to the sustainability of China's own social security program— foreshadowing a hike in taxes and/or a reduction in replacement ratios in the future. The consequence of the attendant higher saving in China can then in turn positively affect U.S. fertility rates.

Section 2 describes the basic closed-economy framework, 3 builds intuition of the model in the absence of a social security system (laissez-faire), and Section 4 analyses China's various reforms under a PAYGO system in a closed-economy setting. Section 5 extends the model to a two-country setting and analyses the international impact of China's policy and institutional developments.

2 Model

2.1 Production

Let K_t denote the aggregate capital stock at the beginning of period t in country i, and $e_t L_{y,t} + L_{m,t}$ the total labor input employed in period t, where $L_{\psi,t}$ denotes the size of generation ψ and e_t the relative productivity of young workers ($e_t < 1$). The gross output in country i is

$$Y_t = (K_t)^{\alpha} \left[A_t \left(e_t L_{y,t} + L_{m,t}\right)\right]^{1-\alpha}, \tag{1}$$

where $0 < \alpha < 1$, and A_t is country-specific productivity. The capital stock in country i depreciates at a rate δ and is augmented by investment goods, I_t, with law of motion

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{2}$$

Factor markets are competitive so that each factor, capital and labor, earns its marginal product. Thus, the wage rates per unit of labor in youth and middle age for country i are

$$w^i_{y,t} = e_t(1-\alpha)A_t\left(k_t\right)^{\alpha}, \qquad w^i_{m,t} = (1-\alpha)A_t\left(k_t\right)^{\alpha}, \tag{3}$$

where $k_t \equiv K^i_t / [A_t(e_t L_{y,t} + L_{m,t})]$ denotes the capital-effective-labor ratio. The rental rate earned by capital in production equals the marginal product of capital, $r^i_{K,t} = \alpha\left(k^i_t\right)^{\alpha-1}$. The gross rate of return earned between period $t-1$ and t in country i is therefore $R_t = 1 - \delta + r_{K,t}$. We let $g_{A,t}$

and $g_{L,t}$ denote the growth rate of productivity and of the size of consecutive cohorts, respectively, so that $A_t = (1 + g_{A,t})A_{t-1}$ and $L_{y,t} = (1 + g_{L,t})L_{y,t-1}$.

2.1.1 The Social Security System

The social security system encapsulates a pay-as-you-go system (PAYGO), a fully-funded system, or more generally, some combination of the two. Each young agent in period t pays a Social Security tax in the amount of $\tau_t w_{y,t}$ in youth and $\tau_{t+1} w_{m,t+1}$ in period $t+1$ in middle-age. When reaching old age in period $t+2$, the agent receives social security benefits in the amount of $\sigma_{t+2} w_{m,t+1}$. A defined benefit system implies a constant σ.

The social security system can own a trust fund that invests in capital. Let K_t^s be the amount of capital held by the Social security system in the beginning of period t. Let $0 \leq \zeta_t < 1$ be the fraction of aggregate capital held in the trust fund so that

$$K_t^s = \zeta_t K_t.$$

The sources and uses of the funds for the social security system are thus given by

$$\tau_t w_{y,t} + \tau_t w_{m,t} L_{m,t} + R_t \zeta_t K_t = \sigma_t w_{m,t-1} L_{m,t-1} + \zeta_{t+1} K_{t+1} \tag{4}$$

The left hand side of the equation represents the sources of funds for the Social security system in period t, which consist of Social Security taxes plus the value of the trust fund, including capital income at the beginning of $t+1$. The right hand side represents the uses of funds by the Social Security system in the same period, including retirement benefits paid to old consumers and the purchase of capital to hold in the trust fund.

2.2 Households

Consider an overlapping generations economy in which agents live for four periods, characterized by: childhood (k), youth (y), middle-age (m), and old-age (o). The measure of total population N_t at date t comprises the four co-existing generations: $N_t = N_{k,t} + N_{y,t} + N_{m,t} + N_{o,t}$.

An individual born in period $t-1$ does not make decisions on his consumption in childhood, $c_{k,t-1}$, which is assumed to be proportional to parental income. The agent supplies inelastically one unit of labor in youth and in middle-age, and earns a wage rate $w_{y,t}$ and $w_{m,t+1}$, which is used, in each period, for consumption and asset accumulation $a_{y,t}$ and $a_{m,t+1}$. At the end of period t, the young agent then makes the decision on the number of children n_t to bear. In middle-age, in $t+1$, the agent transfers a combined amount of $T_{m,t+1}$ to his n_t children and parents. In old-age, the agent consumes all available resources, which is financed by gross return on accumulated assets, $Ra_{m,t+1}$, and transfers from children $T_{o,t+2}$. A consumer thus maximizes the life-time utility including benefits from having n_t children:

$$U_t = \log(c_{y,t}) + v\log(n_t) + \beta\log(c_{m,t+1}) + \beta^2\log(c_{o,t+2})$$

where $v > 0$ reflects the preference for children, and $0 < \beta < 1$. The sequence of budget constraints for an agent born in $t-1$ obeys

$$\begin{aligned} c_{y,t} + a_{yt} &= (1-\tau_t)w_{y,t} \\ c_{m,t+1} + a_{m,t+1} &= (1-\tau_{t+1})w_{m,t+1} + R_{t+1}a_{y,t} + T_{m,t+1} \\ c_{o,t+2} &= R_{t+2}a_{m,t+1} + \sigma_{t+2}w_{m,t+1} + T_{o,t+2}. \end{aligned} \tag{5}$$

Without loss of generality, the cost of raising kids are assumed to be paid by parents in middle-age, in period $t+1$, for a child born at the end of period t. The total cost of raising n_t children falls in the mold of a time-cost that is proportional to current wages, $\phi n_t w_{m,t+1}$, where $\phi > 0$. These costs can be interpreted as "mouth-to-feed-costs" and education costs, which are substantial for Chinese households. Transfers made to the middle-aged agent's parents amount to a fraction $\psi n_{t-1}^{\varpi-1}/\varpi$ of current labor income $w_{m,t+1}$, with $\psi > 0$ and $\varpi > 0$. This fraction is decreasing in the number of siblings—to capture the possibility of free-riding among siblings sharing the burden of transfers. The combined amount of transfers made by the middle-aged agent in period $t+1$ to his children and parents thus satisfy

$$T_{m,t+1} = -\left(\phi n_t + \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right)w_{m,t+1}.$$

In old-age, agents become receivers of transfers from a total of n_t number of children:

$$T_{o,t+2} = \psi\frac{n_t^{\varpi}}{\varpi}w_{m,t+2}.$$

The life-time resource constraint thus requires

$$c_{y,t} + \frac{c_{m,t+1}}{R_{t+1}} + \frac{c_{o,t+2}}{R_{t+1}R_{t+2}} = w_{y,t} + \frac{w_{m,t+1}}{R_{t+1}}\left[1 - \tau_{t+1} - \phi n_t - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right] + \frac{\psi n_t^{\varpi}}{\varpi}\frac{w_{m,t+2}}{R_{t+1}R_{t+2}} + \frac{\sigma_{t+2}w_{m,t+1}}{R_{t+1}R_{t+2}}.$$

Assumption 1 *The young are subject to a credit constraint which is binding in all periods:*

$$a_{y,t+1} = -\theta\frac{w_{m,t+1}}{R_{t+1}}, \tag{6}$$

which permits the young to borrow up to a constant fraction θ of the present value of future wage income. This assumption is realistic in the case of China, and we introduce it to be able to analyse the impact of financial development on fertility and saving in China.

The assumption of log utility implies that the optimal consumption of the middle-age is a constant fraction of the present value of lifetime resources, which consist of disposable income—of what remains after the repayment of debt from the previous period—and the present value of transfers to be received in old-age, less current transfers to children and parents:

$$c_{m,t+1} = \frac{1}{1+\beta}\left[\left(1-\tau_{t+1}-\theta-\phi n_t-\frac{\psi n_{t-1}^{\varpi-1}}{\varpi}\right)w_{m,t+1}+\frac{\psi n_t^{\varpi}}{\varpi}\frac{w_{m,t+2}}{R_{t+2}}+\frac{\sigma_{t+2}w_{m,t+1}}{R_{t+2}}\right]$$

It follows from Eq. 5 that the optimal asset holding of a middle-aged individual is

$$a_{m,t+1} = \frac{\beta}{1+\beta}\left[\left(1-\tau_{t+1}-\theta-\phi n_t-\frac{\psi n_{t-1}^{\varpi-1}}{\varpi}\right)w_{m,t+1}-\frac{\psi n_t^{\varpi}}{\beta\varpi}\frac{w_{m,t+2}}{R_{t+2}}-\frac{\sigma_{t+2}}{\beta}\frac{w_{m,t+1}}{R_{t+2}}\right] \tag{7}$$

2.2.1 Fertility and Saving

Fertility decisions hinge on equating the marginal utility of bearing an additional child compared to the net marginal cost of raising the child:[1]

$$\frac{v}{n_t} = \frac{\beta}{c_{m,t+1}}\left(\phi w_{m,t+1}-\frac{\psi n_t^{\varpi-1}w_{m,t+2}}{R_{t+2}}\right) \tag{8}$$

where $g_{A,t+1} \equiv A_{t+2}/A_{t+1}-1$ is the growth rate of productivity. The right hand side is the net cost, in terms of the consumption good, of having an additional child. The net cost is the current marginal cost of rearing a child, $\partial T_{m,t+1}/\partial n_t$ less the present value of the benefit from receiving transfers next period from an additional child, $\partial T_{o,t+2}/\partial n_t$. In this context, children are analogous to investment goods—and incentives to procreate depend on the relative wage to interest rate ratio.

The market clearing condition for capital markets is[2]

$$L_{m,t+1}a_{m,t+1}+L_{y,t+1}a_{y,t+1} = K_{t+2}. \tag{9}$$

In what follows, we make the additional three assumptions, for analytical convenience:

Assumption 2 *Full depreciation:* $\delta = 0$

Assumption 3 *Transfers are not subject to decreasing returns in children:* $\varpi = 1$

Assumption 4 $e = 0$

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[1]Eq. 8 amounts to $\frac{v}{n_t} = \frac{\beta(1+\beta)(\phi k_{t+1}^{\alpha}-\frac{\psi}{\alpha}n_t^{\varpi-1}g_{A,t+2}k_{t+2})}{\left[(1-\tau_{t+1}-\theta-\phi n_t-\frac{\psi}{\varpi}n_{t-1}^{\varpi-1})k_{t+1}^{\alpha}+\frac{\psi}{\alpha\varpi}n_t^{\varpi}g_{A,t+2}k_{t+2}+\frac{\sigma_{t+2}}{\alpha}k_{t+2}^{1-\alpha}k_{t+1}^{\alpha}\right]}$.

[2]Eq. 9 amounts to:
$K_{t+2} = \frac{\beta}{1+\beta}\left(\frac{L_{m,t+2}}{n_t}\right)\left[\left(1-\tau_{t+1}-\theta-\phi n_t-\frac{\psi n_{t-1}^{\varpi-1}}{\varpi}\right)(1-\alpha)A_{t+1}k_{t+1}^{\alpha}-\frac{1-\alpha}{\alpha}\frac{\psi n_t^{\varpi}}{\beta\varpi}A_{t+2}k_{t+2}-\frac{1-\alpha}{\alpha}\frac{\sigma_{t+2}}{\beta}A_{t+1}k_{t+2}^{1-\alpha}k_{t+1}^{\alpha}\right]-\frac{1-\alpha}{\alpha}L_{m,t+2}\theta A_{t+2}k_{t+2}$. In capital per efficiency unit of labor terms, $k_{t+2} \equiv \frac{K_{t+2}}{A_{t+2}(L_{m,t+2}+e_{t+2}L_{y,t+2})}$, we have:
$k_{t+2} = \left(\frac{1}{1+e_{t+2}n_{t+1}}\right)\left\{\frac{\beta}{n_t(1+\beta)}\left[\left(1-\tau_{t+1}-\theta-\phi n_t-\frac{\psi n_{t-1}^{\varpi-1}}{\varpi}\right)(\frac{1-\alpha}{g_{A,t+2}})k_{t+1}^{\alpha}-\frac{1-\alpha}{\alpha}\frac{\psi n_t^{\varpi}}{\beta\varpi}k_{t+2}-\frac{\sigma_{t+2}}{\beta g_{A,t+2}}(\frac{1-\alpha}{\alpha})k_{t+2}^{1-\alpha}k_{t+1}^{\alpha}\right]-\frac{\theta(1-\alpha)}{\alpha}k_{t+2}\right\}$.

With four simplifying assumptions we arrive at the two key relationships between interest rates and fertility that characterise the economy in the long run when $k_{t+2} = k_{t+1} = k$; $n_t = n$; $g_{A,t} = g_A$; $\sigma_{t+2} = \sigma$; $\tau_{t+1} = \tau$. The first derives from the capital markets condition, Eq. 9, and captures saving dynamics:

$$R_{KK}(\underset{+}{n}) = \frac{n g_A \Phi + \sigma}{\beta\left(1 - \tau - \theta - \phi n - \psi\right)}. \tag{10}$$

This curve is upward sloping—reflecting the fact that a greater number of children raise costs and also raises total expected transfers—both of which tends to reduce saving and drive up the rate of return.

The second derives from Eq.8 and captures fertility decisions:

$$R_{NN}(n) = \frac{n g_A \psi + \lambda_0 \sigma}{n\phi - \lambda_0\left(1 - \tau - \theta - \psi\right)}, \tag{11}$$

where we denote $\lambda_0 \equiv \left(\frac{v}{v+\beta(1+\beta)}\right)$ and $\Phi \equiv (1+\beta)\left(\frac{\alpha}{1-\alpha} + \theta + \frac{\psi}{1+\beta}\right)$.

This relationship captures how fertility choices respond to the interest rate. If one makes the reasonable assumption that $\frac{1-\tau-\theta-\psi}{\phi} > n > \lambda_0 \frac{1-\tau-\theta-\psi}{\phi}$, the NN curve is upward sloping, since:

$$\frac{\partial R_{NN}}{\partial n} = \frac{-\psi g_A v\left(1 - \tau - \theta - \psi\right)}{\left[n\phi - \lambda_0\left(1 - \tau - \theta - \psi\right)\right]^2} < 0$$

where $\lim_{\infty} R_{NN} = \left(\frac{\psi g_A}{\phi}\right)$. The intuition is that higher interest rates tend to lower the returns on children (relative to capital) and thus induces lower fertility–everything else constant.

The KK and NN relationship, along with the long-run analogue for the social security system (Eq. 4):

$$\tau = \frac{\sigma}{g_A n} - \frac{\alpha - k^{1-\alpha} g_A n}{1-\alpha}\zeta \tag{12}$$

together determine the long-run equilibrium of the economy, which can be expressed as

$$R_{NN} = \frac{n g_A}{n\phi - \lambda_0(1 - \theta - \tau - \psi)}\left[\psi + \lambda_0\left(\tau + \zeta\frac{\alpha - k^{1-\alpha} g_A n}{1-\alpha}\right)\right] \qquad (\mathbf{KK} \tag{13}$$

$$R_{KK} = \frac{n g_A}{\beta(1 - \tau - \theta - \phi n - \psi)}\left[\Phi + \left(\tau + \zeta\frac{\alpha - k^{1-\alpha} g_A n}{1-\alpha}\right)\right] \qquad (\mathbf{NN}) \tag{14}$$

3 Laissez Faire

In this section, we examine fertility, interest rate and saving dynamics under laissez-faire. There is no social security system so that $\tau_t = \sigma_t = \zeta_t = 0$ for all t.

Endogenous Fertility. Let $s_{t+1} \equiv K_{t+2}/Y_{t+1}$. In this closed economy with full depreciation, s_t is both the national saving rate and investment rate. Rewriting the capital markets condition Eq. 9 and the optimal fertility condition Eq. 8 in terms of saving and fertility choices, we have

$$s_{t+1} = \frac{\alpha\beta(1-\theta-\phi n_t-\psi)}{\Phi} \tag{15}$$

$$n_t = \frac{\lambda_0}{\phi}(1-\theta-\psi)+\frac{\psi}{\alpha\phi}s_{t+1}, \tag{16}$$

which leads to the following proposition:

Proposition 1 *Under laissez-faire and endogenous fertility, the saving rate and the fertility rate are constant in all periods, with*

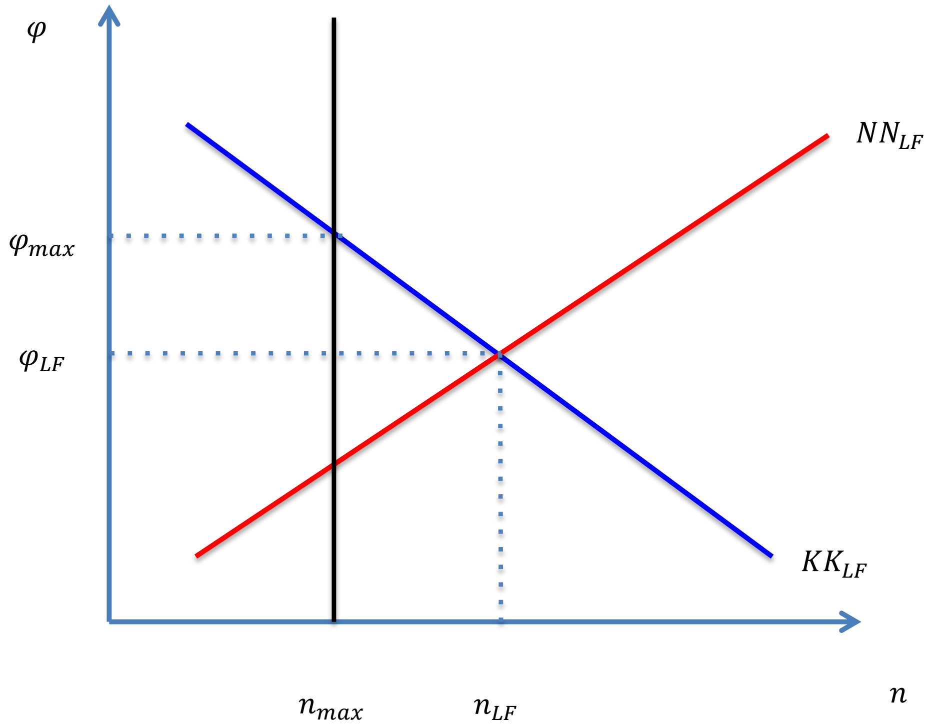
$$s_t = s^{LF} = \frac{\alpha\beta(1-\theta-\psi)(1-\lambda_0)}{\Phi+\beta\psi} \tag{17}$$

$$n_t = n^{LF} = \left(\frac{1-\theta-\psi}{\phi}\right)\left(\frac{\lambda_0\Phi+\beta\psi}{\Phi+\beta\psi}\right). \tag{18}$$

Constrained Fertility. Suppose that the economy starts from the steady state prior to period t, and fertility becomes binding from t onwards. The NN curve is no longer binding and is replaced with a vertical line with $n_t = n_{max}$ for all t. When fertility constraints are binding ($n_{max} < n^{LF}$), then the national saving rate is higher under constrained fertility than under unconstrained fertility: $\psi_{n_{max}} > \psi_{n^{LF}}$ (see Figure 1). An exogenous reduction in fertility thus raises the national saving rate as a consequence of a reduction in total expenditures on children. An additional channel through which changes in fertility affects saving is a transfer effect—which cancels out in this special case—with endogenous responses to the interest rate. A greater number of children raises transfers–which tend to induce less saving in parents, but the rise in the wage to interest rate ratio exactly offsets the reduction in n in this case, thus cancelling out an additional channel that would have lowered saving in response to a fall in the number of children.

Comparative Statics. We can compare the effect of changes in various parameters on the equilibrium fertility rate and national saving rate in a closed economy—under unconstrained fertility and binding fertility constraints.

Preference for Children.
Since $\partial n^{LF}/\partial v > 0$, an increase in v raises the desired number of children n_t, given any s_{t+1}. The KK curve is unaffected, thus resulting in a higher fertility rate and a lower equilibrium national saving rate. Changes to v thus exogenously changes the optimal rate of fertility, and affects the saving rate only through its impact on fertility.



Notes: This figure captures the KK curve—saving rate as a function of fertility rate, and the NN curve, the optimal fertility rate as a function of the saving rate under endogenous fertility and under constrained fertility—the vertical line $n = n_{max}$.

Figure 1: Laissez-Faire Saving and Fertility Rate

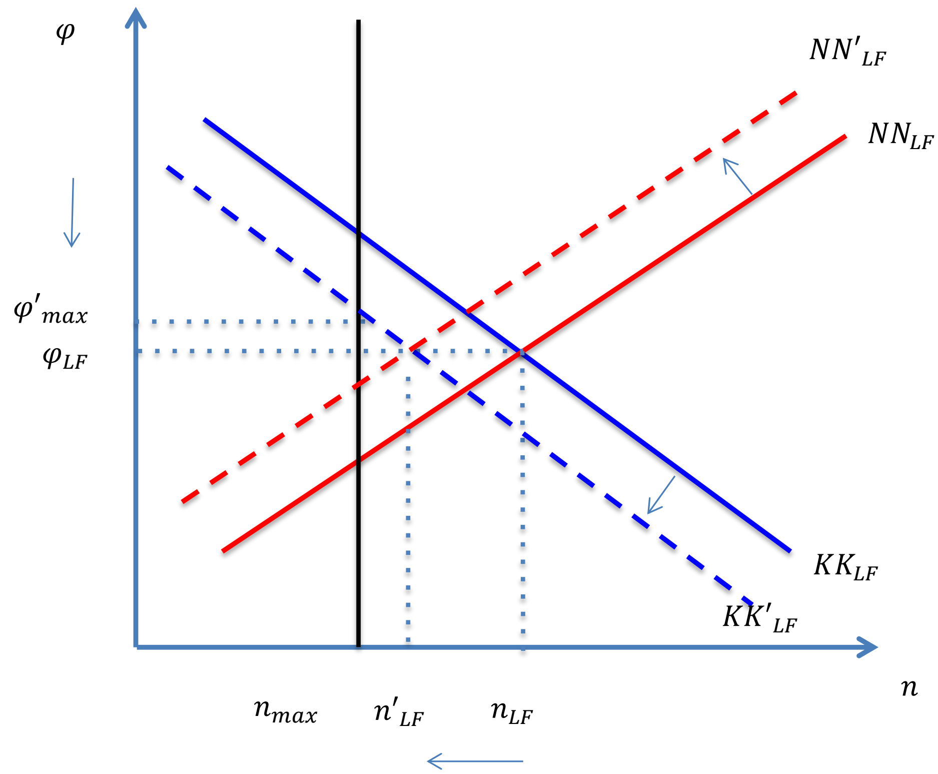
Costs to Children

The optimal laissez-faire fertility rate n^{LF} also shows that $\partial n^{LF}/\partial \phi < 0$. Higher costs to children leads to fewer desired children, given any s_{t+1}, and higher costs to children lowers saving rate, given any fertility rate n_t (Figure 2). This amounts to a leftward shift of the NN curve and a downward shift of the KK curve. The net effect is a lower fertility rate, and an unaltered national saving rate: while higher costs to children leads to a lower saving rate, fewer equilibrium number of children raises the saving rate; and the two effects exactly cancel out in determining national saving rate. In contrast, the effect of an increase in costs to children unambiguously lowers national saving rate under constrained fertility, $n_{max} < n^{LF}$.

Credit Constraints

A loosening of credit constraints (higher θ) leads to lower equilibrium fertility rate and a higher equilibrium saving rate: $\frac{dn^{LF}}{d\theta} < 0$, $\frac{ds^{LF}}{d\theta} > 0$. This implies that a country with tighter credit constraints have a higher desired fertility rate than countries with looser credit constraints. Increased borrowing of the young reduces disposable income in middle age, thus lowering the number of de-

Figure 2: Costs to Children: Fertility and Saving



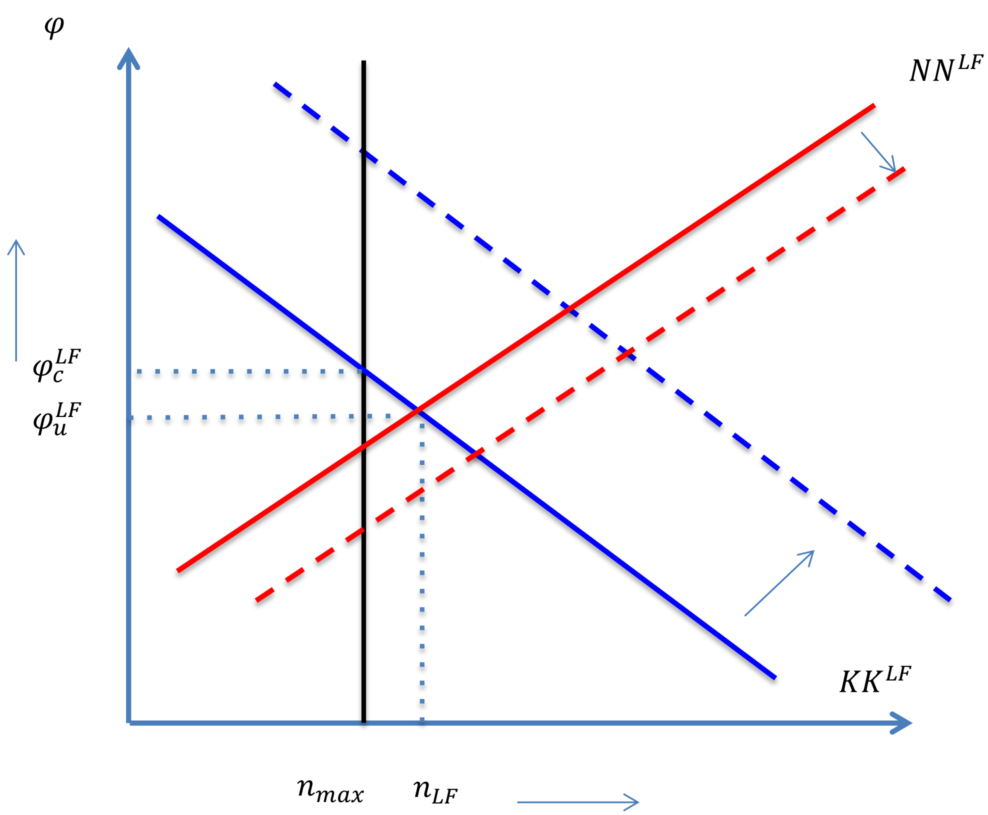
Notes: This figure plots the saving rate (y-axis) against the fertility rate (x-axis), and shows the impact of an increase in the cost to children, under both endogenous and constrained fertility. A rise in the cost of children under endogenous fertility shifts the NN curve to the left and the KK curve down—resulting in lower fertility and no changes in the saving rate. Under binding fertility constraints, the saving rate is unambiguously lowered.

sired children. While the direct effect of looser credit constraints is to lower the saving rate, the indirect effect on saving rate via a reduced number of children raises the saving rate and dominates the former effect.

4 PAYGO Social Security System

We next analyse the long-run economy under a PAYGO system, and the impact of likely reforms and institutional changes that are taking place or will take place in China. The impact of these changes on the economy hinge crucially on whether fertility can adjust or not—and thus exerts a very different effect under unconstrained fertility and fertility restrictions in China. In a PAYGO system, $\zeta = 0$, and the long-run social security system is characterised by

Figure 3: Credit Constraints: Saving and Fertility under Laissez-Faire



Notes: The figure plots the saving rate against the fertility rate under endogenous fertility (subscript u) and fertility constraints (subscript c).

$$\tau = \frac{\sigma}{ng_A}$$

which, combined with the saving and optimal fertility condition, yields the R_{KK} and R_{NN} curves:

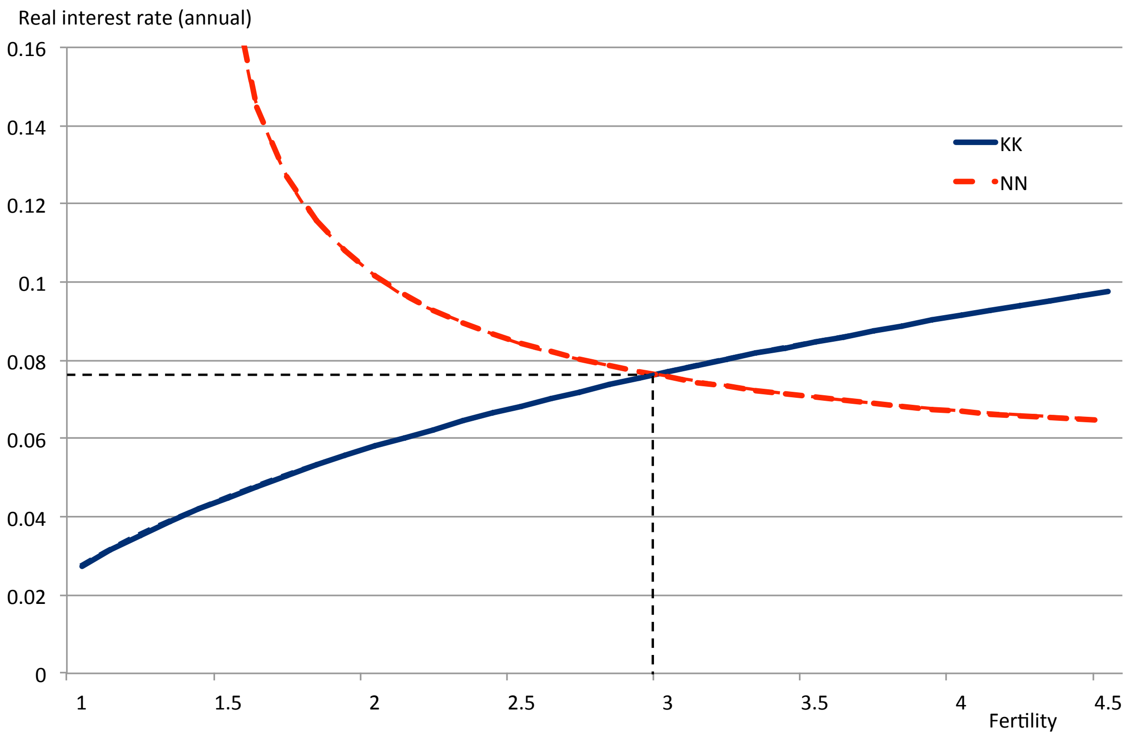
$$R_{KK} = \frac{ng_A}{\beta(1-\tau-\theta-\phi n-\psi)}\left(\Phi+\tau\right) \tag{19}$$

$$R_{NN} = \frac{ng_A}{n\phi-\lambda_0(1-\theta-\tau-\psi)}\left(\psi+\lambda_0\tau\right), \tag{20}$$

These two curves collapse to the laissez-faire curves when $\tau = 0$. With social security benefits

$\sigma > 0$, saving falls compared to the laissez-faire state—given any fertility rate. Both a reduced need to save in middle-age and lower after-tax disposable income implies a higher interest rate given any level of fertility rate. These two curves are illustrated in Figure 4.

Figure 4: PAYGO Benchmark



Notes: Benchmark parameters are described in Table 1. The KK curve describes the relationship between interest rate and fertility rate based on optimal saving decisions; the NN curve capture the relationship between these two variables based on optimal fertility decisions.

The equilibrium fertility rate under endogenous fertility that solve Eq. 19 and 20 is

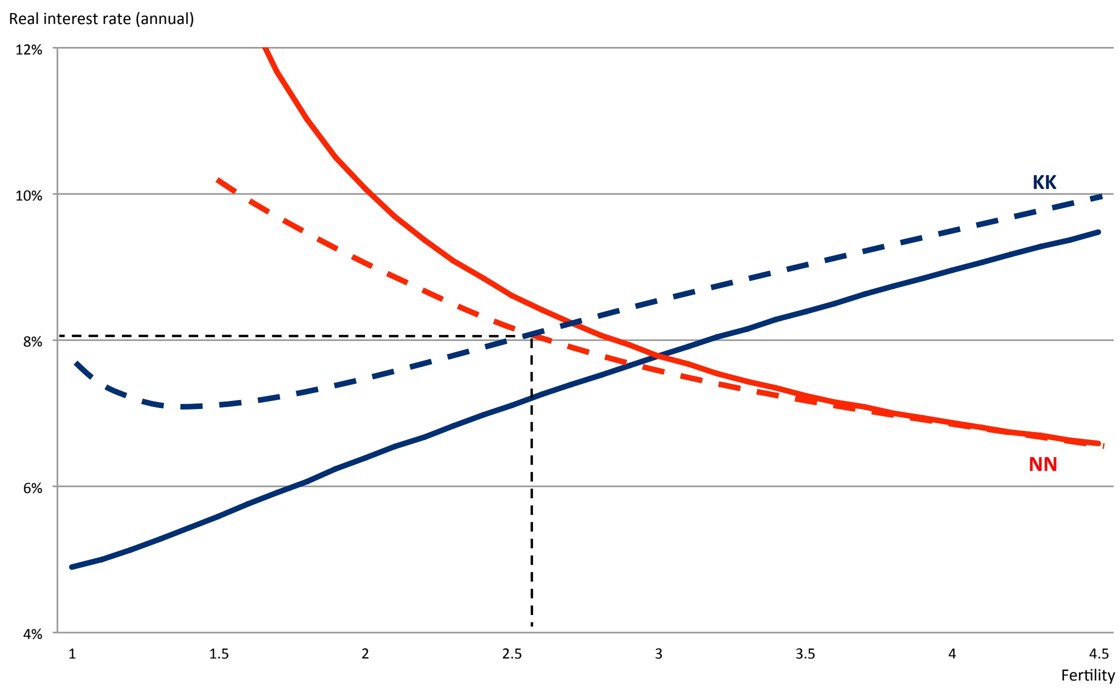
$$n_u = \frac{(1-\tau-\theta-\psi)}{\phi}\left(\frac{\psi\beta+\lambda_0\Phi+\lambda_0(1+\beta)\tau}{\psi\beta+\Phi+(1+\beta\lambda_0)\tau}\right) \tag{21}$$

From the above expression, we have:

$$\frac{\partial n_u}{\partial \tau} > 0$$

Under weak conditions, a higher tax rate reduces optimal fertility rates, as socials security serves as a substitute for children in providing for old age. The equilibrium interest rate is ambiguous as the reduced number of desired children indirectly raises saving and thus offsets some of the direct fall in saving due to higher social security benefits. Figure 5 illustrates the impact of a rise in the replacement ratio from 28% to 50%. The KK curve shifts up and the NN curve shifts left, resulting in a decrease in fertility—from 3 to 2.4 and the interest rate rises slightly from 7.5% to 8%. Social security benefits σ rises by less than one for one with τ as fertility rate falls under endogenous fertility rates.

Figure 5: A Rise in Replacement Ratios (PAYGO)



Notes: This figure illustrates the impact of raising the replacement ratio from $\sigma = 0.3$ to $\sigma = 0.5$. All other parameters are taken from the benchmark parameters in Table 1.

Constrained Fertility. The impact of a rise in tax rates under binding fertility constraints $n_c = n_{max}$ where $n_{max} < n_u$ differs from that under endogenous fertility. Under constrained fertility, the R_{NN} curve is not binding, and comparative statics performed on Eq.19 yield the following proposition:

Proposition 2 *Under a PAYGO system and binding fertility constraints ($n_{max} < n_u$ and $n_{max} < n'_u$), both a partial relaxation of fertility constraint, or a rise in tax rates (social security benefits) will lower the aggregate saving rate and raise the interest rate. That is,*

$$\frac{\partial R_{KK}}{\partial \tau} \quad | \quad {}_{n=n_{max}} > 0$$
$$\frac{\partial R_{KK}}{\partial n_{max}} \quad | \quad {}_{\tau} > 0.$$

where $n_u = \frac{(1-\tau-\theta-\psi)}{\phi}\left(\frac{\psi\beta+\lambda_0\Phi+\lambda_0(1+\beta)\tau}{\psi\beta+\Phi+(1+\beta\lambda_0)\tau}\right)$.

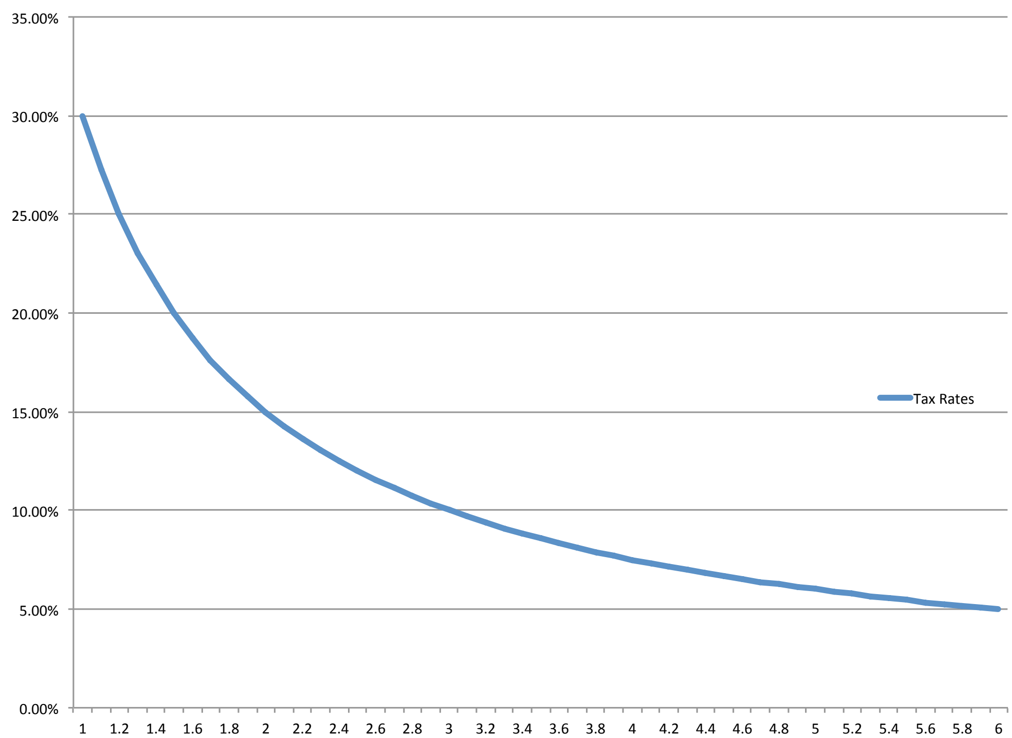
There are two channels through which a relaxation of fertility restrictions reduce private saving: first, a rise in n raises the replacement ratio σ, given τ (Eq. 19), and hence reduces the need for saving. The second channel is associated with a direct increase in the number of children: rising expenditures will tend to lower saving. There is one additional potential channel that turns out to cancel out in this special case that relates to the transfer effect. Since the total amount of transfers received by parents from children are $\phi n \frac{w_{t+2}}{R_{t+2}}$, an increase in n would tend to raise transfers, everything else constant. The general equilibrium effect of changes in the wage-interest ratio tends to offset the rise in n–and in this case—perfectly. With a smaller endogenous response to the interest rate—for example, in a small open economy where the interest rate is exogenous—then an increase in the number of children will raise transfers, and thus induce an additional reduction in saving. Thus, moving from a one child policy to a two-children policy in China will tend to raise domestic interest rates and lower domestic savings.

Under binding fertility constraints, agents will be able to reap the full benefits of a rise in social security benefits financed by an increase in taxes, without the endogenous reduction in fertility that tends to offset some of the rise in σ. In this case, the saving rate falls and the interest rate rises unambiguously. Figure 5 shows that under the one child policy, the interest rate rises by 1.3% and by 0.5% under a two-children policy. A policy reform in China that entails simultaneously increasing the replacement ratio and moving to a two-children policy would reduce the saving rate and raise the interest rate by about 3.6%.

Other Policy Experiments. We next explore the consequences of a series of reforms that are relevant for China— occurring at present or likely to take place in the future. We examine the consequences on saving, fertility and interest rates, and the sustainability of the social security system–comparing the case under endogenous fertility and under exogenous fertility. In a defined benefit system, where σ is constant, changes in fertility, given constant productivity growth, engenders proportional changes in the tax rate. Figure 6 illustrates the amount of taxes, as a function of fertility, needed to keep the social security system sustainable under a constant replacement ratio of about 28%.

The case in which financial development allows for a loosening of credit constraints will tend to put pressure on a PAYGO social security system by reducing the equilibrium natural rate of fertility. The case in which θ rises from 0.02 is shown in Figure 7. Looser credit constraints occasions less saving, for any given fertility rate, and a higher interest rate (KK curve shifts up). Optimal fertility falls, given any interest rate (NN curve shifts left) and the equilibrium fertility rate falls from 3 to a little more than 2 children per family, with the interest rate rising from 7.3% to about 8%. In a defined benefit system where σ is constant, a reduction in fertility amounts to a rise in taxes, from 10% to 15%. A two-children policy would still be binding in this scenario, and thus under constrained fertility of $n_{max} = 2$, the interest rate would rise by more —about 2 %, and by 2.5% under a one child policy. Simultaneously changing fertility policies from 1 to 2 and loosening credit constraints would significantly reduce saving, and raise the interest rate from 3 to 7.8%.

Figure 6: Tax Rates under PAYGO



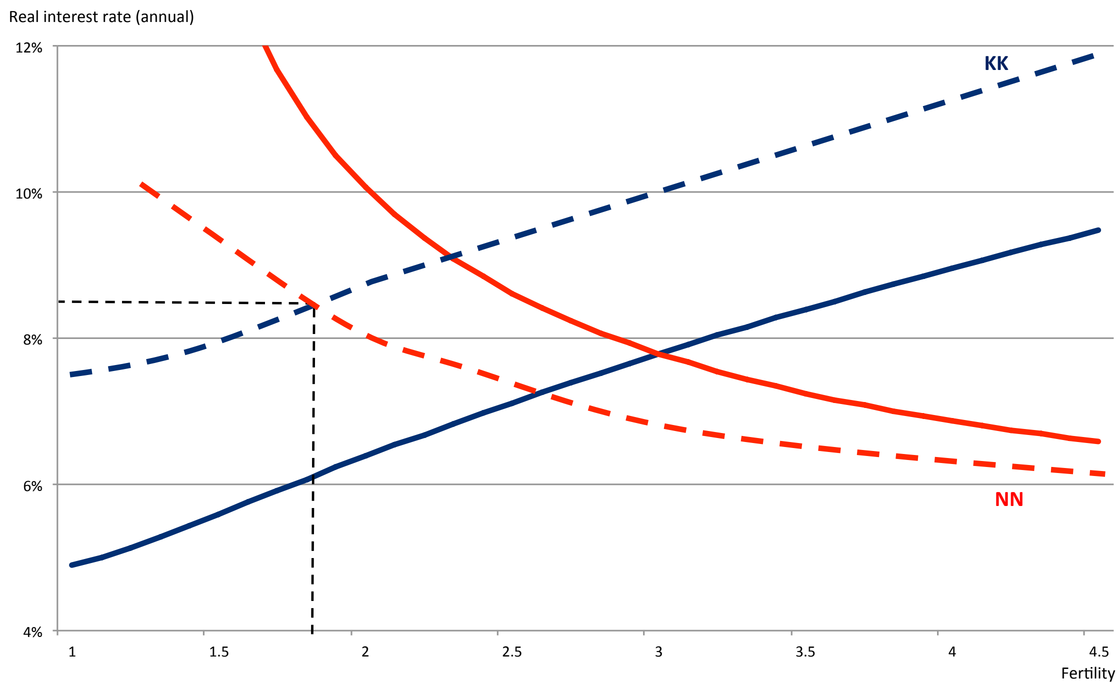
Notes: This figure shows how taxes vary with fertility, given constant replacement ratio $\sigma = 0.3$ and productivity growth $g_A = 0.025$. All other parameters are taken be the benchmark parameters.

We next consider a fall in ψ (Figure 9), so that children become less altruistic towards their parents. This will lead to both a reduced desire to have children (NN curve shifts left), and higher saving needs (KK curve shifts down). Both the equilibrium interest rate and fertility rate fall. Thus, when the traditional mode of intergenerational support subsides in importance, the social security system will become more strained under endogenous fertility. Keeping replacement ratios constant would require a tax rate increase of 2%. The impact is larger under endogenous fertility than under constrained fertility, as parents not only have to save more to substitute for transfers provided but children, they also have to save more as a result of reduced social security benefits. The interest rate falls by 1.6 percentage points under natural fertility, compared to 0.4 percent under a one child policy.

5 The Open Economy

Consider two large open economies $i = \{H, F\}$. Under financial integration, capital flows across borders until the rate of return is equalised across countries. Financial integration implies that $R_t^i = R_t$ and $k_t^i = k_t$ for all i. The first order conditions on consumption and fertility remain the

Figure 7: A Loosening of Credit Constraints (PAYGO)



Notes: This figure illustrates the effect of increasing $\theta = 0.02$ to $\theta = 0.2$. All other parameters are benchmark parameters. We keep the replacement ratio $\sigma = 0.3$ constant and allow τ to vary.

same as the one in each country's respective autarky state, and the world capital markets clearing condition becomes

$$\sum_i \left(L^i_{m,t+1} a^i_{m,t+1} + L^i_{y,t+1} a^i_{y,t+1}\right) = \sum_i K^i_{t+2} \tag{22}$$

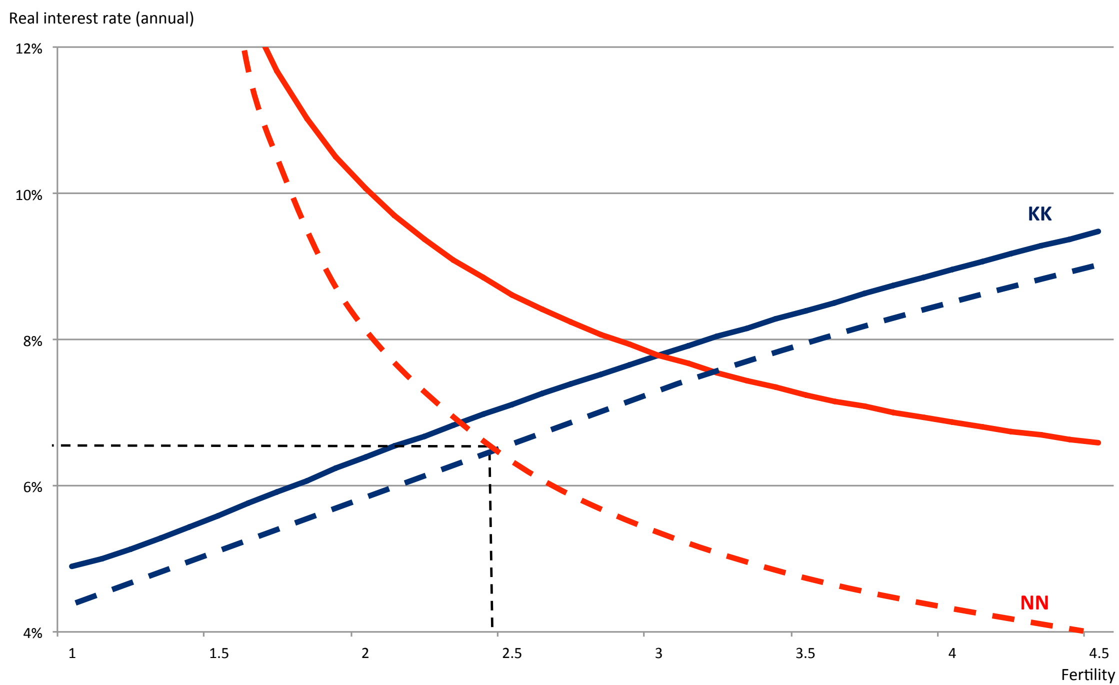
Substituting in country-specific $a^i_{m,t+1}$ and $a^i_{y,t+1}$ given by Eq. 6 and 7 gives the evolution of the capital-effective labor ratio.

Fertility decisions are the same as before, except that the common world interest rate replaces the autarky interest rates in a closed economy:

$$\frac{v}{n^i_t} = \frac{\beta}{c^i_{m,t+1}} \left(\phi w^i_{m,t+1} - \frac{\psi^i n_t^{\varpi - 1} w_{m,t+2}}{R_{t+2}} \right). \tag{23}$$

Let $\lambda_{i,t} = \frac{L^i_{m,t} A^i_t}{\sum_i L^i_{m,t} A^i_t}$ denote country i' effective size. Under Assumptions 1-4, the equivalent interest rate and fertility relationships deriving from the saving and optimal fertility decisions are

Figure 8: PAYGO: A Fall in Intergenerational Transfers



Notes: This figure illustrates the effect of a fall in ψ from 10% to 5%. All other parameters are benchmark parameters. We keep the replacement ratio $\sigma = 0.3$ constant and allow τ to vary.

determined by the following two equations:

$$k_{t+1}^{\alpha} = \frac{1+\beta}{\beta} \frac{\sum_i \lambda_{i,t+2} \left(1 + \theta_i(\frac{1-\alpha}{\alpha}) + \frac{\psi^i}{1+\beta}(\frac{1-\alpha}{\alpha})\right) k_{t+2}}{\sum_i \lambda_{i,t+2} \frac{(1-\alpha)}{n_t^i g_{A,t+2}^i} \left[\left(1 - \tau_{t+1}^i - \theta^i - \phi n_t^i - \psi^i\right) - \frac{\sigma_{t+2}^i}{\beta}(\frac{1-\alpha}{\alpha}) k_{t+2}^{1-\alpha}\right]}$$

$$\frac{v}{n_t^i} = \frac{\beta(1+\beta)\left[\phi k_{t+1}^{\alpha} - (\frac{\psi^i}{\alpha}) g_{A,t+2}^i k_{t+2}\right]}{\left[\left(1 - \tau_{t+1}^i - \theta^i - \phi n_t^i - \psi^i\right) k_{t+1}^{\alpha} + \frac{\psi^i}{\alpha} n_t^i g_{A,t+2}^i k_{t+2} + \frac{\sigma_{t+2}^i}{\alpha} k_{t+1}^{\alpha} k_{t+2}^{1-\alpha}\right]}.$$

Steady State. Consider the long run where the relative size of countries are constant, with $\lambda_{i,t} = \lambda_i$; $k_t = k$; $n_t^i = n^i$ and $g_{At}^i = g_A^i$. Under a PAYGO system, $\zeta = 0$, and the KK curve and the NN curve deriving from Eq. 22 and 23, for each country i, become:

$$R_{KK} = \frac{\sum_i \lambda_i \left(\Phi^i + \tau^i\right)}{\beta \sum_i \lambda_i \left(\frac{1-\tau^i-\theta^i-\phi n^i-\psi^i}{n^i g_A^i}\right)}$$

where $\Phi^i \equiv (1+\beta)\left(\frac{\alpha}{1-\alpha} + \theta^i + \frac{\psi^i}{1+\beta}\right)$.

The world interest rate can be, alternatively, expressed as a weighted average of the autarky interest rates,

$$R_{KK} = \sum_i \left(\frac{\lambda_i \left(\frac{1-\tau^i-\theta^i-\phi n^i-\psi^i}{n^i g_A^i} \right)}{\sum_i \lambda_i \left(\frac{1-\tau^i-\theta^i-\phi n^i-\psi^i}{n^i g_A^i} \right)} \right) R_{KK}^i,$$

where R^i is given by Eq. 19. The weights depend on the relative country size λ_i and also on the country-specific intuitional parameters. The common world R_{KK} curve thus illustrates how growth in the presence of basic asymmetries across countries can influence the world interest rate. For example, take a developing country such as China, with tighter credit constraints (low θ) and lower fertility compared to an advanced economy —such as the U.S. An increase in the weight of China—a rise in λ^{China} –say, brought about by a period of fast growth—tends to *lower* the world interest rate: more weight placed on China (with the higher saving and lower autarky interest rate) as a consequence of either China's tighter credit constraints or its lower fertility[3] thus tends to put downward pressure on the world interest rate. Faster growth in the U.S., on the other hand, has the opposite effect and would raise the world interest rate.

The R_{NN} curve becomes:

$$R_{NN}^i = \frac{n^i g_A^i (\psi^i + \lambda_0 \tau^i)}{\phi n^i - \lambda_0 \left(1 - \tau^i - \theta^i - \psi^i\right)},$$

which together with the common world saving curve, and the country specific social security system $\sigma^i = \tau^i n^i$ determine country-specific fertility rates.

5.1 Open-Economy Policy Experiments

Financial Integration. The consequences of financial integration on fertility, saving and the social security system can be different for countries like China and the U.S., and can also be different for a country with binding fertility constraints. Integration between China (with a lower autarkic interest rate) and the U.S. (with a higher autarkic interest rate) will result in a rise in the interest rate for China and a reduction in the interest rate in the U.S., as the world interest rate is a weighted average of the two autarky interest rates. The U.S. will in turn see a rise in fertility rate, in response to a rising interest rate, while China will see the opposite. If, however, fertility constraints are indeed binding in China, the absence of adjustment on the fertility front will imply a larger rise in the interest rate in China than under unrestrained fertility .

Figure **??** displays the autarkic interest rate in each economy and the world interest rate after integration, in the absence of fertility controls and under the one child policy in China. The autarkic interest rates in each economy is the intersection between the autarkic saving and fertility curves. Financial integration implies that for any given fertility rate in the U.S., the world interest rate is lower under integration than under autarky, so the R_{KK} curve shifts down for the U.S. The

[3] This could be due to a binding fertility constraint or because China has a lower preference for children (low v)—both of which can exogenously reduce fertility.

opposite is true for China. Since lower nterest rates tends to stimulate U.S. fertility—financial integration interestingly helps sustain U.S. social security system.

Financial integration also tends to reduce U.S. saving—through three channels. The first is a fall in the cost of borrowing that allows the young to borrow more in the U.S., and the second is that higher social security benefits (holding constant the tax rate) through a rise in fertility also tends to reduce saving.

The opposite is true in China—unless binding fertility constraints in place. It is possible that the rise in the interest rate will reduce the natural rate of fertility to the extent that the fertility constraint is no longer binding. But, in the case that it still is, the upward pressure on saving rate through an endogenous fall in fertility rate and a fall in social security benefits are absent, and thus the saving rate would not fall by as much as under endogenous fertility.

Social Security Policies. Consider a social security reform that entails an increase in the replacement ratio in China. The impact on the world interest rate, global fertility and social security systems hinge crucially on whether binding fertility constraints are in place or not. Under binding fertility constraints, the world interest rate rises—as social security benefits increase one for one with taxes–causing a drop in saving in China, and a rise in the interest rate that is larger than under endogenous fertility. That is, for any given level of fertility, higher σ^{China} shifts up the KK^w. The NN^{US} curve is unaffected, thus causing the equilibrium fertility rate to fall in the U.S.. To maintain sustainability of the social security system, the U.S. would therefore need to raise taxes or reduce the replacement ratio.

However, if fertility constraints are absent in China, the world interest rate may not rise by as much, or may even fall. The reason is that under weak conditions, higher social security benefits are associated with a fall in the desired number of children, given any interest rate. In this case, the NN curve in China will shift leftward, leaving the world interest rate ambiguous. The U.S. fertility rate would respond by significantly less (and may even increase). Thus, binding fertility constraints have an impact on how social security reforms in China affect fertility and social security elsewhere in the world.

Fertility Policies. One can examine the global ramifications of the one child policy in China, and consequently, a partial relaxation of the policy to a two-children policy. Implementing a fertility constraint in China amounts to $n = n_{max}$ replacing the R_{NN} curve in China. In China, this effect amounts to a movement along the R_{KK} curve, from n to n_{max}. In the U.S., for any given fertility n^{US}, a fall in n^{China} reduces the interest rate, thus shifting the R_{KK} curve down. The world interest rate falls: lower fertility raises China's saving both because of lower social security benefits (or higher taxes if keeping σ constant) and of reduced spending on children. In response, the U.S. fertility rate rises. The one child policy is thus favourable for the social security system in the U.S. Moving to a two-children policy thus has adverse effects on the U.S. social security system.

Table 1: Benchmark Parameter Values

Parameter	Value	Target/Description (Data source)
β (annual basis)	0.99	/
g_A (annual basis)	2.5%	steady-state productivity growth rate
v		Targeted to match Fertility in 1966-1970 $n_{ss} = 3/2$ (Census)
$\theta^{China}, \theta^{US}$	$\{2\%, 18\%\}$	Coeurdacier et al. (2013)
α	0.36	Capital Share
ϕ	$\{0.06\}$	Average Educ. expenditures (UHS/RUMICI)
ψ^{China}, ψ^{US}	$\{10\%, 2\%\}$	Chackoumane et al. (2013)
$\sigma^{China}, \sigma^{US}$	$\{30\%, 45\%\}$	Replacement ratios in China and the U.S.

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6 Appendix: the Golden Rule

In the long run with constant values of Social Security parameters, the value of s_{t+1} is a constant. The Golden rule value of s, denoted as s^{GR}, is the constant value of s_t that maximises the long-run value of consumption per augmented unit of labor,

$$\tilde{c}_t \equiv \frac{C_t}{A_t N_{m,t}}.$$

Using the definition of s_{t+1}, and the resource clearing constraint, we can write aggregate consumption C_t as

$$C_t = (1 - s_t) K_t^\alpha (A_t N_{m,t})^{1-\alpha},$$

from which

$$\tilde{c}_t = (1 - s)\tilde{k}_t^\alpha$$

immediately follows. The value of s that maximises consumption per effective unit of labor in the long run is one such that

$$\frac{\partial \tilde{c}}{\partial s} = \left(\alpha \frac{1 - s}{s} \frac{\partial ln \tilde{k}_t}{\partial lns} - 1 \right) k_t^\alpha$$

is set to 0. Using

$$\frac{\partial ln \tilde{k}_t}{\partial lns} = \frac{1}{1 - \alpha},$$

we have the following proposition:

Proposition 3 *The Golden Rule, which maximises consumption per effective unit of labor in the long run, is attained by a saving rate of $s^{GR} \equiv \alpha$ and a rate of return of $R^{GR} = g_A n^{GR}$.*

This result can be interpreted as a criterion for dynamic efficiency. According to Abel, Mankiw, Summers, and Zeckhauser (1989), if in every period, the contribution of capital to output is larger than the flow of resources used to create capital, the economy is dynamically efficient. Here, the contribution of capital to the production of output is αY_t, and the flow of resources diverted from consumption to the creation of capital is I_t. Thus, the contribution of capital exceeds its use of resources by $\alpha Y_t - I_t = (\alpha - s) Y_t$ in period t. Thus, the economy is dynamically efficient if $s < \alpha$, and dynamically inefficient if $s > \alpha$, and attains the Golden Rule if $s = \alpha$.

Golden Rule Social Security Policies

Equation 4 says that in any economy that has attained the Golden Rule, with $s_{t+1} = s_t = \alpha$, constant values of social security parameters require that $\tau = \sigma / (g_A \cdot n)$, given any n. Plugging this relationship into the R_{KK} curve and the R_{NN} curves (Eq. 10 and 11), we obtain:

$$R_{KK} = n g_A \frac{\Phi + \tau}{\beta(1 - \tau - \theta - \phi n - \psi)} \quad (24)$$

$$R_{NN} = n g_A \frac{\psi + \lambda_0 \tau}{n\phi - \lambda_0 (1 - \tau - \theta - \psi)}. \quad (25)$$

The Golden Rule is attained when $s = \alpha$ and $R = \alpha(n \cdot g_A)$. Therefore, the social security parameters that implement the Golden Rule when fertility rate is endogenous is given by the following proposition:

Proposition 4 *Under endogenous fertility, the Golden Rule is attained in the long run with constant values of social security parameters when* $\sigma_u^{GR} = (n_u^{GR} \cdot g_A)\tau_u^{GR}$ *and*

$$\tau_u^{GR} = \left[s^{LF} - (1-\zeta)\alpha\right] \frac{\Phi}{\alpha\,(1+\beta)} \tag{26}$$

$$n_u^{GR} = \frac{(1-\theta-\psi)\lambda_0 + \psi}{\phi} \tag{27}$$

The subscript u denotes unconstrained fertility and c denotes constrained fertility. In a PAYGO system, $\zeta = 0$. Under this scenario, if $s^{LF} < \alpha$– so that the laissez-faire economy is dynamically efficient—the Golden Rule is attained by negative values of τ and σ. Negative values imply that young consumers receive a subsidy and old consumers pay a tax. Since subsidising the income of young consumers and increasing the taxes paid by old consumers both increase aggregate private saving, this tax/subsidy scheme will increase national saving and investment toward the Golden Rule. On the other hand, if the laissez-faire economy is dynamically inefficient, national saving and investment must be reduced to attain the Golden Rule.

The consequence of a tax/subsidy scheme that brings an economy with an under-accumulation of capital closer to the golden rule is a *higher* equilibrium fertility rate. This tax/subsidy scheme itself does not affect fertility choices—and hence the NN curve is unchanged. However, this scheme which increases national saving given any fertility rate—and hence pushes up the KK curve. Higher capital accumulation leads to higher expected transfers and thus bids people to have more children.

In a pay-as-you-go system, the taxes/subsidies that can implement the golden rule are smaller under binding fertility constraints than under unrestricted fertility. That is, $\tau_{n_max} < \tau_{n^{LF}}$ if $n_{max} < n^{LF}$.

This case of $s^{LF} < \alpha$.can be illustrated in Figure... In this scenario, there is an under-accumulation of capital and a fertility that is higher than the Golden Rule fertility rate. The implied negative tax/subsidy scheme thus will push the KK curve to the point at which the equilibrium is the Golden Rule fertility-capital pair. The NN curve, however, is not affected by such social security schemes. show graph that under the condition that $s^{LF} < s$, this requires an upward shift of KK curve through social security taxes—NN curve doesn't move. This would require a lowering of social security taxes and a benefits; on the same graph show n^{GR} compared to n_{max} and n_{ss}. Fertility is another tool to push economy towards GR. requires less taxes.

Constrained Fertility. The Golden rule can be attained with a combination of fertility and social security policies. Under certain conditions, it can be implemented with a binding fertility constraint in the absence of any social security system:

Proposition 5 *Suppose that $s^{LF} < \alpha$. Under the condition that $\theta < \frac{\beta(1-\psi)-\psi}{1+\beta} - \frac{\alpha}{1-\alpha}$, one can implement the Golden Rule under laissez-faire social security policies by implementing*

$$n_{max}^{GR} = \frac{1-\theta-\psi}{\phi} - \frac{\Phi}{\beta\phi} > 0$$

This fertility rate is the one that solves $s = \alpha$ in Eq. 16. If $s^{LF} < \alpha$ then it must be that $n^{LF} > n_{max}^{GR}$—that is, the fertility constraint that implements the Golden Rule —-in the absence of any social security policies—must be binding. The condition on θ guarantees that this constrained fertility is greater than 0.

More generally, a combination of fertility and social security policies can be implemented:

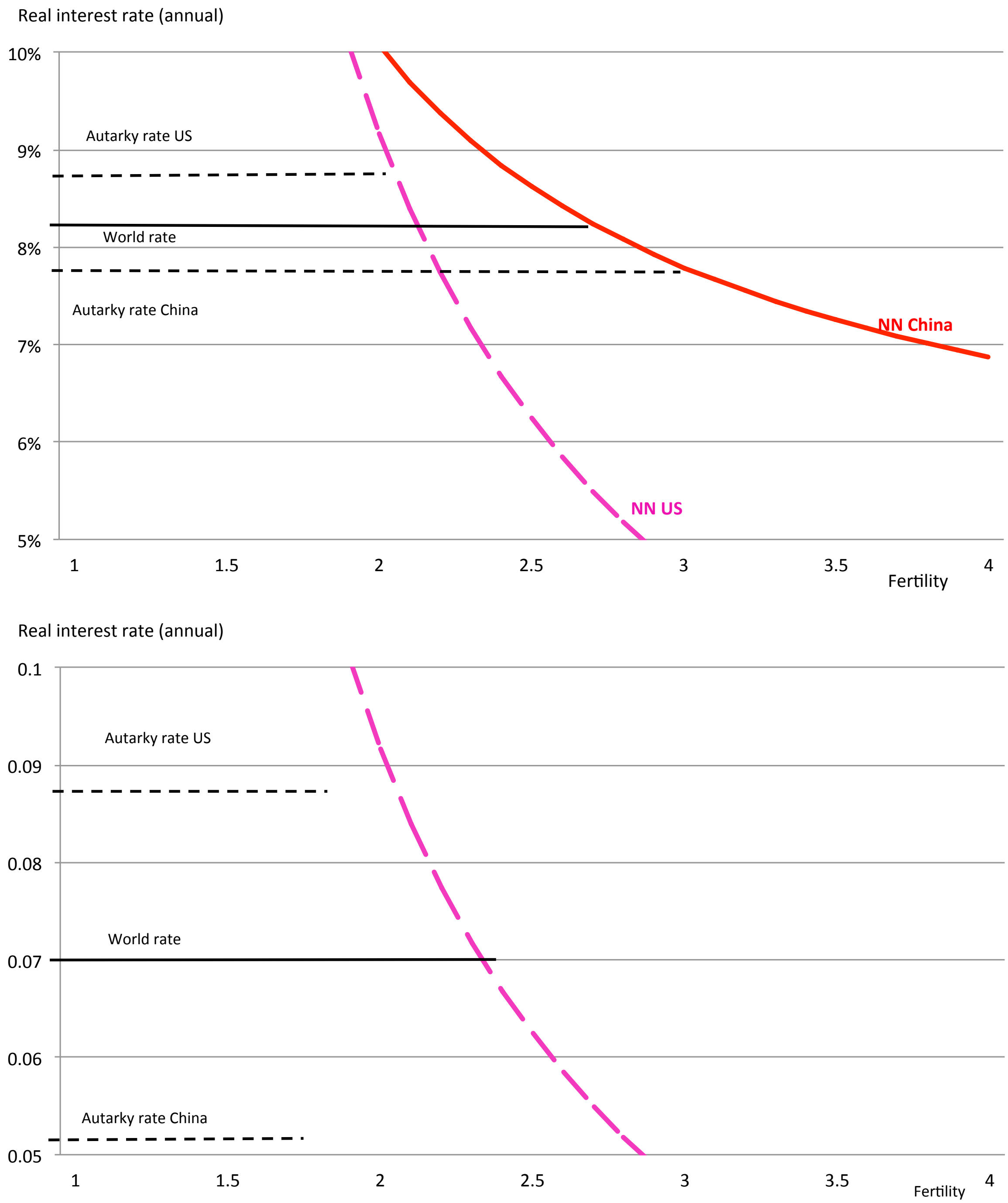
Corollary 6 *The Golden Rule can be attained by a combination of fertility policies $n = n_{max}$ such that $0 < n_{max} < n_u'$ and social security policies under constrained fertility $\tau_c^{GR}, \sigma_c^{GF}$ such that*

$$\sigma_c^{GR} = \tau_c^{GR}(n_{max} \cdot g_A)$$

$$\tau_c^{GR} = \left[s_c^{LF}(n_{max}) - (1-\zeta)\alpha\right] \; \frac{\Phi}{\alpha(1+\beta)}$$

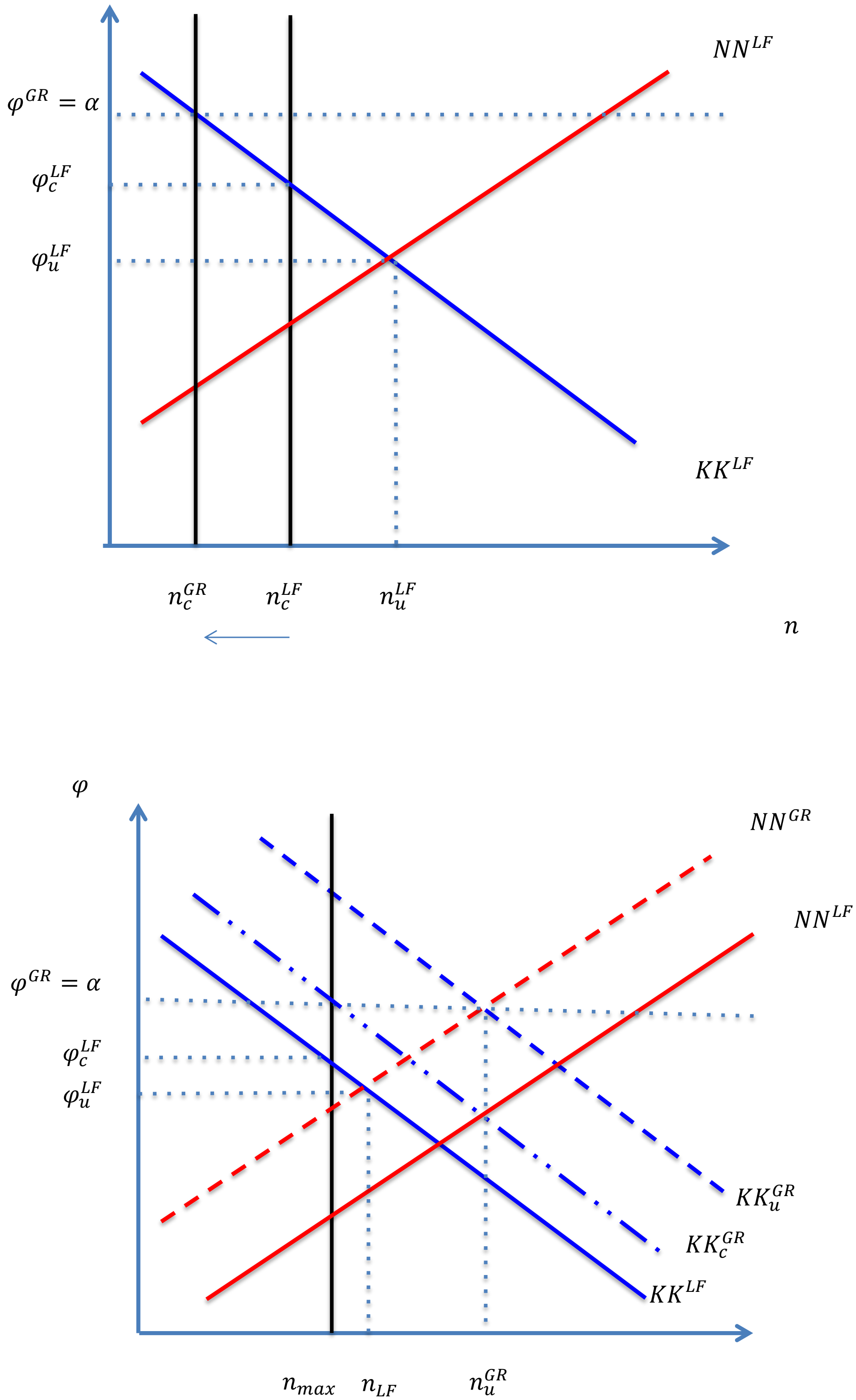
where the laissez-faire saving rate under any binding fertility constraint is $s_c^{LF}(n_{max}) = \frac{1}{1-\zeta}\frac{\alpha\beta(1-\theta-\phi n_{max})}{\Phi}$, given by Eq.16 when $n = n_{max}$. The fertility constraint is binding if $n < n_u'$ where n_u' is the natural rate of fertility that corresponds to the new set of social security rules—-i.e. the n that solves Eq. 10 and 11 when $\tau = \tau_c^{GR}$ and $\sigma = \sigma_c^{GR}$.

Figure 9: PAYGO: Integration



Notes: The top panel shows the autarky and world interest rate when fertility constraints are absent. The bottom panel shows the case under one chid policy in C/hina. Benchmark parameters provided in Table 1.

Figure 10: Golden Rule Fertility and Social Security Policies



Notes: The figure plots the saving rate against the fertility rate under endogenous fertility (subscript u) and fertility constraints (subscript c).