Unconventional Fiscal Policy at the Zero Bound*

Isabel Correia Emmanuel Farhi Juan Pablo Nicolini Pedro Teles

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Abstract

When the zero lower bound on nominal interest rates binds, monetary policy cannot provide appropriate stimulus. We show that in the standard New Keynesian model, tax policy can deliver such stimulus at no cost and in a time-consistent manner. There is no need to use inefficient policies such as wasteful public spending or future commitments to inflate. We conclude that in the New Keynesian model, the zero bound on nominal interest rates is not a relevant constraint on both fiscal and monetary policy.

Key words: Zero Bound; Fiscal policy; Monetary policy; Sticky prices.

JEL classification: E31; E40; E52; E58; E62; E63.

1 Introduction

Arbitrage between money and bonds restricts nominal interest rates from becoming negative. One could imagine circumstances in which, in the event of a potential recession, it is desirable for the Central Bank to lower the policy rate. If the interest rate is very close to zero to begin with, the constraint may be binding. This is the "zero bound" problem of monetary policy.

But, is there a zero bound problem when policy is more generally considered to include both fiscal and monetary instruments? Is fiscal policy able to avoid a downturn when the zero bound constraint binds? In this paper we show that the zero bound on nominal interest rates is not a relevant constraint on both fiscal and monetary policy. If the nominal interest rate is zero, taxes can play the role that the nominal interest rate would play, could it be used without restrictions.

*Correia: Banco de Portugal, Universidade Catolica Portuguesa and CEPR. Farhi: Harvard University. Nicolini: FRB of Minneapolis and Universidad Di Tella. Teles: Banco de Portugal, Universidade Catolica Portuguesa and CEPR. This paper circulated with the title Policy at the Zero Bound. We thank Fernando Alvarez, Pierpaolo Benigno, Javier Garcia-Cicco, Patrick Kehoe, Narayana Kocherlakota, John Leahy, Kjetil Storesletten, Sam Schulhofer-Wohl, Harald Uhlig, Tao Zha, participants at the 8th Hydra Workshop, and at seminars at the University of Chicago, Princeton U., U.C. San Diego, Bank of Spain, the Federal Reserve Banks of Atlanta, Chicago, Minneapolis and St Louis, and the Board of Governors. Correia and Teles gratefully acknowledge financial support of FCT.

Considerable attention has been placed on this issue in recent times, following the outbreak of the 2008 and 2009 financial crisis. Nominal interest rates have indeed been very close to zero in the US, the EMU, the UK and other countries. Given the restrictions on monetary policy, attention has shifted to alternative policies. There has been work on public spending multipliers, showing that these can be very large at the zero bound (see Christiano, Eichenbaum, Rebelo (2009), Eggertsson (2009), Woodford (2010), Mertens and Ravn (2010)[1]). Eggertsson (2009) also considers different alternative taxes and assesses which one is the most desirable to deal with the zero bound. The zero bound is also a key component in the numerical work presented in the evaluation of the American Recovery and Reinvestment Plan by Romer and Bernstein (2009). It is also a main concern in Blanchard, Dell'Ariccia and Mauro (2010) who argue for a better integration between monetary and fiscal policy.

There is also earlier work on the implications of the zero bound for monetary and fiscal policy, motivated by the prolonged recession in Japan where overnight rates have been every close to zero for the last fifteen years, as well as by the low targets for the Fed funds rate in the US in 2003 and 2004.[2] Eggertsson and Woodford (2003 and 2004a) show that there may be downturns that could, and should, be avoided if it was not for the zero bound. They also show how monetary policy can be adjusted so that the costs of those downturns may be reduced. In particular they propose policies that keep the interest rate for a longer period at zero in order to generate inflation. Eggertsson and Woodford (2004b) consider both monetary and fiscal policy in a Ramsey taxation model with consumption taxes only.[3] All this work is done in the context of standard sticky price models, where the zero bound on interest rates can be a serious challenge to policy. That is indeed the general conclusion, justifying the use of inefficient policies, such as wasteful government spending, leading to undesirable inflation.

With a different, more general focus, Correia, Nicolini, and Teles (2008) show that fiscal policy can be used to neutralize the effects of price stickiness. They consider an optimal Ramsey taxation model without capital and with a monetary distortion, similar to the one in Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991), but with sticky prices. They show that under sticky prices it is possible to implement the same allocations as under flexible prices, and that it is optimal to do so. Since the zero bound is the optimal policy under flexible prices, it must also be the optimal one under sticky prices. In this sense, the zero bound is not a constraint to policy. These results and the pressing relevance of the policy question were the motivation for this work.

In this paper, we take the standard set up analyzed by most of the zero bound literature, allow for capital accumulation, and consider labor income, consumption, and capital income taxes. We show that whatever policy can do with the nominal interest rate can also be done with a combination of those three taxes.

[1] Mertens and Ravn show that multipliers can be low if the economy is close to an alternative, liquidity trap, steady state.

[2] In 2003 and 2004, the Fed funds rate fell down to 1%, and remained there for more than year.

[3] They also consider the case of two consumption taxes, such that prices are set after one and before the other. In that case it would be possible to implement the same allocation, as if the zero bound did not bind. They find those taxes to be highly unrealistic and move on to analyze the case of a single consumption tax.

Furthermore, there is no equivalent restriction to the zero bound on nominal interest rates, when policy uses taxes rather than interest rates. We conclude that, when fiscal policy is used, the zero bound on nominal interest rates does not restrict the set of implementable allocations. In the simple New Keynesian model, as in Eggertsson (2009), it is possible to achieve the first best allocation if the zero bound does not bind, or, alternatively, if taxes are used. This is an extreme result. In more general set ups, full efficiency cannot be attained. It is still the case, though, that the zero bound is irrelevant for both fiscal and monetary policy. We show this by considering an extension of the model where productivity shocks are firm specific or the initial distribution of prices across firms is non-degenerate.[4]

Suppose real rates ought to be negative. Since the nominal interest rate cannot be negative, the only way to achieve negative real interest rates is to generate inflation. This is precisely what the commitment to low future interest rates first suggested in Krugman (1998) achieves in Eggertsson and Woodford (2003 and 2004a). But producer price inflation is costly. Indeed, in the New Keynesian, sticky price, literature, price setting decisions are staggered. Producer price inflation then necessarily leads to dispersion in relative prices—a real economic distortion. Is it possible to achieve negative real interest rates without incurring this economic cost? We show that the answer to this question is affirmative if flexible tax instruments are available.

The intuition why tax policy can neutralize the effects of the zero bound constraint is simple. It turns out that the prices that matter for intertemporal decisions are consumer prices, which are gross of consumption taxes. The idea is to induce inflation in consumer prices, while keeping producer price inflation at zero. The result is negative real interest rates, and the distortions associated with producer price inflation are altogether avoided. This can be achieved by simultaneously adjusting consumption and labor taxes. Imagine first that producer price inflation is zero. Then a temporarily lower consumption tax generates inflation in consumer prices. The problem is that this change in consumption taxes introduces undesirable variations in the marginal cost of firms over time: a lower consumption tax reduces the marginal cost of firms. It also creates incentives for producers to reduce their prices. This effect must therefore be counteracted by temporarily raising the labor tax. Overall, this policy acts as a costless tax on money.[5] It essentially achieves a negative nominal interest rate in the consumer price numeraire.

In a model with capital, this policy must be supplemented with a temporary capital subsidy. This is because a path of consumption taxes which increases over time acts as a tax on capital. This tax on capital is undesirable and must be counteracted with a corresponding subsidy. The goal is to tax money, not capital.

Importantly, because our policy implements the efficient allocation, it is time-consistent: if a future planner were given an opportunity to revise this policy in the future, it would choose not to do so. This should be contrasted with the policy recommendations involving future commitments to low interest rates in Krugman (1998) and Eggertsson and Woodford (2003 and 2004a).

[4] Yun (2005) analyzes optimal monetary policy when the initial distribution of prices is non-degenerate.

[5] In conformity with the New Keynesian literature, we consider cashless economies. We therefore ignore the costs of inflation associated with the inflation tax resulting from deviations from the Friedman rule.

Our policy recommendation requires flexibility of taxes. It has been argued that fiscal instruments are not as flexible as monetary policy instruments. Whether this argument applies to stabilization policy during a "great moderation" period could be argued about. However, it certainly does not apply to exceptional circumstances such as the recent crisis or the Japanese stagnation in the nineties, precisely because the need to use fiscal instruments is exceptional. There have been recent policy proposals in this direction by Robert Hall and Susan Woodward[6], and earlier on, by Feldstein (2003), intended at Japan.[7] Both of them suggested lowering consumption taxes as a way to fight the crisis. Our model formalizes these proposals and highlights the way other taxes must be jointly used.

The paper proceeds as follows: We first describe the model, in section 2. In section 3, we characterize the first best allocation and show how it can be implemented, away from the zero bound using interest rate policy, and at the zero bound using tax policy. We consider the linearized model in section 4, so that the relation with the literature can be made more clear. We consider a model with capital in section 5. In section 6, we show that the results can be generalized to environments where it is not optimal (or feasible) to replicate flexible prices. In a model with firm specific productivity shocks and/or a non-degenerate distribution of initial prices, it is still the case that the zero bound constraint on nominal interest rates can be overcome using tax policy.

2 The Model

The model we analyze is a standard new-Keynesian model, similar to the one analyzed by Eggertsson and Woodford (2003) and (2004b), and Eggertsson (2009). As it has become standard in the New Keynesian literature, the economy is cashless.

The uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$, where S_t is the set of possible events at t, and the history of its realizations up to period t is denoted by $s^t \in S^t$. For simplicity we index by t the variables that are functions of s^t.

The preferences of the households are described by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t, \xi_t\right) \tag{1}$$

where

$$C_t = \left[\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{2}$$

where c_{it} is private consumption of variety $i \in [0,1]$, N_t is total labor, and ξ_t is a preference shock.

[6] An article by Justin Lahart in the Wall Street Journal, January 5, 2009, "State Sales-Tax Cuts: Get Another Look", comments on the proposals of Hall and Woodward in their blog. See also the speech by Narayana Kocherlakota, President of the Minneapolis Fed, "Monetary Policy Actions and Fiscal Policy Substitutes," November 18, 2010.

[7] "The Japanese government could announce that it will raise the current 5 percent value added tax by 1 percent per quarter and simultaneously reduce the income tax rates to keep revenue unchanged, continuing this for several years until the VAT reaches 20 percent." Feldstein (2003).

Aggregate government consumption G_t is exogenous. It is also a Dixit-Stiglitz aggregator of public consumption of different varieties g_{it},

$$G_t = \left[\int_0^1 g_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

The production function of each good i, uses labor, n_{it}, according to

$$c_{it} + g_{it} = A_{it} n_{it}, \tag{4}$$

where A_t is an aggregate productivity shock.

Total labor is

$$N_t = \int n_{it} di. \tag{5}$$

2.1 Government

The government minimizes the expenditure on the individual goods, for a given aggregate, and finances it with time varying taxes on consumption, τ_t^c, and labor income, τ_t^n. As is standard in the new-Keynesian literature, we also allow for lump-sum taxes, T_t, which is a residual variable that adjusts so that the government budget constraint is satisfied.

If we let

$$P_t = \left[\int_0^1 {p_{it}}^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \tag{6}$$

where p_{it} is the price of variety i, then, the minimization of expenditure on the individual goods, implies

$$\frac{g_{it}}{G_t} = \left(\frac{p_{it}}{P_t}\right)^{-\theta}. \tag{7}$$

2.2 Households

Households also minimize spending on aggregate C_t, by choosing the consumption of different varieties according to

$$\frac{c_{it}}{C_t} = \left(\frac{p_{it}}{P_t}\right)^{-\theta}, \tag{8}$$

The budget constraints of households can then be written in terms of the aggregates as

$$\begin{aligned} \frac{1}{1+i_t}\overline{B}_t^h + E_t Q_{t,t+1} B_{t,t+1} &= \overline{B}_{t-1}^h + B_{t-1,t}^h + (1-\tau_t^n) W_t N_t \\ &\quad + \left(1-\tau^d\right)\Pi_t - (1+\tau_t^c) P_t C_t - T_t \end{aligned} \tag{9}$$

together with a no-Ponzi games condition. $B_{t,t+1}$ represent the quantity of state contingent bonds that pay one unit of money at time $t+1$, in state s^{t+1} and $\overline{B}_t^h$ are risk free nominal bonds. $Q_{t,t+1}$ is the price of the state contingent bond, normalized by the probability of occurrence of the state at $t+1$, and $\frac{1}{1+i_t}$ is the price of the riskless bond—so $1 + i_t$ is the gross nominal interest rate. W_t is the nominal wage and Π_t are profits. We assume that profits are fully taxed, $\tau^d = 1$.[8]

The first order conditions of the household problem that maximizes utility (1) subject to the budget constraint (9) with respect to the aggregates are

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{(1+\tau_t^c) P_t}{(1-\tau_t^n) W_t} \tag{10}$$

and

$$Q_{t,t+1} = \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{u_C(C_t, N_t, \xi_t)} \frac{P_t(1+\tau_t^c)}{P_{t+1}\left(1+\tau_{t+1}^c\right)}, \tag{11}$$

$$\frac{u_C(C_t, N_t, \xi_t)}{P_t(1+\tau_t^c)} = \beta(1+i_t) E_t \frac{u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{P_{t+1}\left(1+\tau_{t+1}^c\right)}. \tag{12}$$

2.3 Firms

Each variety is produced by a monopolist. Prices are set as in Calvo (1983). Every period, a firm is able to revise the price with probability $1-\alpha$. The lottery that assigns rights to change prices is *i.i.d.* over time and across firms. Since there is a continuum of firms, $1-\alpha$ is also the share of firms that are able to revise prices. Those firms choose the price p_t to maximize profits

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left[p_t y_{t+j} - W_{t+j} n_{t+j}\right]$$

where $Q_{t,t+j}$ is the nominal price at t of one unit of money at a particular state in period $t+j$, output $y_{t+j} = c_{t+j} + g_{t+j}$ must satisfy the technology constraint and the demand function

$$y_{t+j} = \left(\frac{p_t}{P_{t+j}}\right)^{-\theta} Y_{t+j}.$$

obtained from (8) and (7), where $Y_{t+j} = C_{t+j} + G_{t+j}$.

The optimal price set by these firms is

$$p_t = \frac{\theta}{(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}},$$

where

$$\eta_{t,j} = \frac{(\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} (P_{t+j})^{\theta-1} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} (P_{t+j})^{\theta-1} Y_{t+j}}.$$

The price level can be written as

$$P_t = \left[(1-\alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{13}$$

[8] This assumption is irrelevant for the results.

2.4 Equilibria

Using the demand functions (8), (7), it follows that

$$C_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} A_t N_t.$$

An equilibrium for $\{C_t, N_t\}$, $\{p_t, P_t, W_t\}$, and $\{i_t, \tau_t^c, \tau_t^n\}$ is characterized by

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{(1+\tau_t^c) P_t}{(1-\tau_t^n) W_t}, \tag{14}$$

$$\frac{u_C(C_t, N_t, \xi_t)}{(1+\tau_t^c) P_t} = E_t \left[(1+i_t) \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{\left(1+\tau_{t+1}^c\right) P_{t+1}} \right], \tag{15}$$

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}}, \tag{16}$$

$$P_t = \left[(1-\alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{17}$$

$$C_t + G_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t} \right)^{-\theta} \right]^{-1} A_t N_t. \tag{18}$$

In addition, an equilibrium condition is that the zero bound on nominal interest rates be verified so that $i_t \geq 0$.

Here ϖ_j is the share of firms that have set prices j periods before, $\varpi_j = (\alpha)^j (1-\alpha)$, $j = 0, 2, ..., t$, and $\varpi_{t+1} = (\alpha)^{t+1}$, which is the share of firms that have never set prices so far. We assume that they all charge an exogenous price p_{-1}.[9]

For now we abstract from the particular way in which monetary policy is conducted, whether it follows a standard feedback rule, a target rule or a simple target for the sequence of nominal interest rates. In what follows we characterize the efficient allocation and the policy variables and prices that are consistent with it. In Section 4, we explicitly consider an interest rate rule as well as fiscal policy rules and discuss uniqueness of equilibria.

3 Efficient allocations

The first best allocation is the one that maximizes utility (1) subject to the technology constraints (2), (3), (4) and (5), above.

From (4) and (5), it follows that the marginal rate of transformation between any two varieties is equal to one. Because the marginal rate of substitution is $\left(\frac{c_{it}}{c_{jt}}\right)^{-\frac{1}{\theta}}$, it must be that an efficient allocation satisfies

$$c_{it} = C_t, \text{ all } i,\ t.$$

[9] We do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

A similar argument applies to public consumption of the different varieties, so that

$$g_{it} = G_t, \text{ all } i,\ t.$$

The efficiency conditions for the aggregates (C_t, N_t) are fully determined by:

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{1}{A_t}, \tag{19}$$

and

$$C_t + G_t = A_t N_t. \tag{20}$$

By comparing the efficiency conditions with the equilibrium conditions we can describe the prices and policy variables that are consistent with the efficient allocation.

We now show that there are policies and prices that support the efficient allocation, both away from and at the zero bound. At the zero bound, those policies involve state and time varying taxes. We do this by showing that there are policies and prices satisfying all the equilibrium conditions, above, for the efficient allocation, taking into account the zero bound constraint on the nominal interest rate.

3.1 Policy away from the zero bound.

In this section, we review how monetary policy can implement the efficient allocation with constant taxes on consumption τ^c and labor τ^n.

First, in order to achieve production efficiency, conditions (8) and (7) imply that prices must be the same across firms $\frac{p_{t-j}}{P_t} = 1$. That can only be the case if firms start at time zero with a common price, p_{-1},[10] as we assume, and if firms that can subsequently change prices choose that common price, so that $p_t = P_t = p_{-1}$. This means that the price level must be constant across time and states. The reason is simple. Because price setting decisions are staggered, inflation necessarily comes at the cost of dispersion in relative prices. This represents an economic distortion. Avoiding this distortion requires that inflation be zero.

It therefore follows that the aggregate resource constraint (18) becomes (20). From Calvo's price setting condition (16), it follows that

$$p_t = \eta_{t,0} \frac{\theta}{(\theta - 1)} \frac{W_t}{A_t} + \left(1 - \eta_{t,0}\right) E_t p_{t+1},$$

This implies that

$$p_t = p_{-1} = P = \frac{\theta}{(\theta - 1)} \frac{W_t}{A_t}, \tag{21}$$

as under flexible prices. Thus, the nominal wage must move with productivity so as to maintain the nominal marginal cost constant.

From (15), with constant consumption taxes, we have

$$u_C\left(C_t, N_t, \xi_t\right) = \left(1 + i_t\right) E_t \left[\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\right]$$

[10] This is the standard assumption. Yun (2005) analyzes the case with initial price dispersion.

so the nominal interest rate must equal the natural rate of interest—the real interest rate that prevails at the efficient allocation.

From (14) and (21), it must be that

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)}=\frac{\left(1+\tau^c\right) \frac{\theta}{(\theta-1)}}{\left(1-\tau^n\right) A_t}, \tag{22}$$

implying that $1-\tau^n=\left(1+\tau^c\right) \frac{\theta}{\theta-1}$.

One possibility is to set consumption taxes to zero, $\tau^c=0$. Therefore labor must be subsidized at the rate $1-\tau^n=\frac{\theta}{\theta-1}$. This labor subsidy is necessary to neutralize the mark up distortion. Note that the subsidy is constant over time and states.

As long as the natural rate of interest is nonnegative, $u_C\left(C_t, N_t, \xi_t\right) \geq E_t\left[\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\right]$, the zero bound constraint is not binding and the efficient allocation is implemented with constant taxes and flexible monetary rate policy. In this model, in normal times, monetary policy achieves perfect economic stabilization. We now look at the more interesting case where the natural rate of interest is negative.

3.2 Policy at the zero bound

We have seen that, in order to implement the efficient allocation with constant taxes, the nominal interest rate must equal the natural rate of interest, and prices must be constant. This implementation breaks down when the natural rate of interest turns negative, because of the zero lower bound on the nominal interest rate. With constant taxes, this failure is unavoidable and optimal monetary policy can only achieve a second best allocation. We start by reviewing the policy trade-offs confronting the design of monetary policy when the zero lower bound is binding. We then move on to explain how flexible taxes can be used to completely circumvent the zero lower bound and implement the efficient allocation.

One strategy is to then set the nominal interest rate to zero as long as the natural rate of interest is negative, and to start raising the nominal interest rate again when the natural rate of interest turns positive. This strategy results in deflation and hence positive real interest rates when the zero bound is binding, precisely when the natural rate of interest is negative. This deflation comes together with a contraction in output compared to the efficient allocation.

With constant taxes, the only way to achieve a negative real interest rate is to generate inflation. Because price setting decisions are staggered, this necessarily generates dispersion in relative prices. This represents a real distortion and implies that the efficient allocation cannot be implemented. These distortions have to be weighted against the stimulation benefits of lower real interest rates in the form of higher output and consumption.

Recognizing this trade-off leads to another strategy whose premise is to supplement zero nominal interest rates with a commitment to keeping nominal interest rates below the natural rate of interest even when the natural rate turns back positive. This commitment to stimulate the economy in the future raises demand today through a wealth effect. Both higher present and future demand induces firms to raise their prices. This in turn generates inflation, which lowers the real interest rate today and further stimulates the economy. In fact, following Krugman (1998), Eggertsson and Woodford (2003 and 2004a) show that the optimal monetary policy (with constant taxes) precisely follows this strategy. It is important to emphasize that this strategy does not implement the efficient allocation.

Instead if taxes are used, the efficient allocation can be implemented at the zero bound. To see this, we set the nominal interest rate to the natural rate of interest whenever the latter is positive, and to zero otherwise. The intertemporal condition (15), repeated here with constant prices,

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right) P} = \left(1+i_t\right) E_t\left[\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right) P}\right]$$

can be satisfied with the appropriate choice of consumption taxes over time. Similarly, the intratemporal condition (22), repeated here,

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^n\right)} \frac{\theta}{(\theta-1)} \frac{1}{A_t}$$

can then be satisfied by the choice of the labor income tax, so that

$$\frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^n\right)} \frac{\theta}{(\theta-1)} = 1$$

and the first best is achieved.

As long as consumption and labor income taxes are flexible instruments, the zero bound is not a constraint to policy. Notice that the tax policies that implement the first best at the zero bound do not have to respond to contemporaneus information. Consumption and labor income taxes can be predetermined.

The tax policy that implements the efficient allocation does not involve net taxing or subsidizing. Notice that the present value budget constraint of the households, can be written, replacing prices and taxes from the households marginal conditions (10) and (11), as

$$E_0 \sum_{t=0}^{\infty} \frac{\beta^t \xi_t}{\xi_0}\left[-u_N(t) N_t - u_C(t) C_t\right] - E_0 \sum_{t=0}^{\infty} \frac{\beta^t \xi_t}{\xi_0} u_C(t) \frac{T_t}{P_t} = \frac{\mathbb{W}_0 u_C(0)}{P_0\left(1+\tau_0^c\right)}.$$

The efficient allocation is such that $C_t = A_t N_t - G_t$, and $-u_N(t) = A_t u_C(t)$. This implies that

$$E_0 \sum_{t=0}^{\infty} \frac{\beta^t \xi_t}{\xi_0} u_C(t)\left(G_t - \frac{T_t}{P_t}\right) = \frac{\mathbb{W}_0 u_C(0)}{P_0\left(1+\tau_0^c\right)}.$$

Notice that τ_0^c is unrestricted by the implementation of the efficient allocation whether at the zero bound, or away from it. It is a lump sum tax on the initial nominal wealth of households. The present value of lump sum taxes is equal to the present value of government spending plus the value of initial liabilities. The present value of the other taxes, used to implement the efficient allocation, is zero. This is the case whether the allocation is implemented with interest rates away from the zero bound, or with consumption and labor income taxes. In this sense, tax policy that implements the efficient allocation at the zero bound is revenue neutral.

We now consider a special case of the model—the same considered by Eggertsson (2009) and Christiano, Eichenbaum and Rebelo (2009)—and describe optimal tax policy following a shock that lowers the natural rate of interest to the point where the zero bound constraint would be binding. The discussion on alternative policies in this context has focused on the role of government purchases.[11] This is not without a, possibly major, resource loss. Instead, the policy we characterize below deals with the zero bound constraint on monetary policy at no cost.

3.3 Using fiscal policy to avoid a recession

As in Eggertsson (2009) and Christiano et al. (2009) we consider specific preferences as

$$u\left(C_t, N_t, \xi_t\right) = u\left(C_t, N_t\right) \xi_t \tag{23}$$

In this way, the preference shock does not affect the marginal rate of substitution between consumption and leisure. It will, however, affect the marginal rate of substitution between consumption at time t and consumption at time $t+1$. We also assume that $G_t = G$, $A_t = 1$, so that the only shock is the preference shock.

Note that in this case, the conditions for an efficient allocation (19) and (20) imply that the first best satisfies

$$-\frac{u_C\left(C_t, N_t\right)}{u_N\left(C_t, N_t\right)} = 1,$$

and

$$C_t + G = N_t.$$

Therefore the efficient allocation is constant, and is unaffected by the preference shock.

Let us consider a particular example, a deterministic version of the examples in Eggertsson (2009) and Christiano, Eichenbaum and Rebelo (2009). In their models, it is this shock - interacting with the zero bound - that generates a potentially big recession.

Assume that ξ_t evolves exogenously according to

$$\begin{aligned} \frac{\xi_t}{\xi_{t+1}} &< \beta \text{ for } t = 0, 1, 2, .., T-1, \\ \frac{\xi_t}{\xi_{t+1}} &= 1 \quad \text{for } t = T, T+1, T+2, .. \end{aligned}$$

The natural rate of interest is $\frac{1}{\beta}\frac{\xi_t}{\xi_{t+1}} < 1$ if $t < T$ and $\frac{1}{\beta} > 1$ for $t \geq T$. We set the nominal interest rate to $1 + i_t = 1$ for $t \leq T-1$ and $1 + i_t = \beta^{-1}$ for $t > T$. We set the path of consumption taxes according to

$$\frac{1+\tau_{t+1}^c}{1+\tau_t^c} = \beta \frac{\xi_{t+1}}{\xi_t}, \text{ for } t = 0, 1, 2, .., T-1.$$

And we set labor taxes as follows

$$\frac{\left(1+\tau_t^c\right)}{\left(1-\tau_t^n\right)} \frac{\theta}{\left(\theta-1\right)} = 1, \text{ for all } t.$$

[11] Eggertsson also considers tax changes, but only one at a time. As we show, it is key to be able to change the two taxes - consumption and labor income - jointly.

Note that, in this deterministic case, there is one degree of freedom in the choice of tax policy: the initial level of the consumption tax τ_0^c. Given an initial consumption tax, the equations above completely determine the paths of consumption and labor taxes. Consumption taxes increase over time for $t < T$ and then stabilize at some level τ^c for $t \geq T$. Labor taxes follow the opposite pattern: they decrease over time for $t < T$ and then stabilize at some level τ^n for $t \geq T$ with $\frac{1+\tau^c}{1-\tau^n}\frac{\theta}{\theta-1} = 1$.

The key is that the prices that matter for intertemporal decisions are consumer prices, which are gross of consumption taxes. The idea is to induce inflation in consumer prices, while keeping producer price inflation at zero. The result is negative real interest rates, and the distortions associated with producer price inflation are altogether avoided. This can be achieved by a simultaneous adjustment in consumption and labor taxes. A temporarily lower consumption tax ($\tau_t^c < \tau^c$) generates inflation in consumer prices. Why does the labor tax need to be temporarily raised ($\tau_t^n > \tau^n$)? The changes in consumption tax introduce undesirable variations in the marginal cost of firms: if the labor tax is kept unchanged at τ^n, the lower consumption tax ($\tau_t^c < \tau^c$) reduces the marginal cost of firms. This also creates incentives for producers to reduce their prices. This effect must therefore be counteracted by temporarily raising the labor tax ($\tau_t^n > \tau^n$).

This policy resembles the sales tax holiday proposal by Hall and Woodward at the end of 2008 and Feldstein in 2003 addressing the Japanese stagnation in the nineties. To implement the first best, however, it is important to note that labor taxes must be adjusted in the opposite direction of consumption taxes so as not to distort the intratemporal margin.

3.4 Time-consistency

Importantly, because our policy implements the efficient allocation, it is time-consistent. If a future planner were given an opportunity to revise this policy in the future, it would choose not to do so. This should be contrasted with the policy recommendations involving future commitments to low interest rates in Krugman (1998) and Eggertsson and Woodford (2003 and 2004a). These policies involve commitments to "being irresponsible" in the future by keeping the nominal interest rate below the natural rate of interest even when the latter turns back positive. When the future comes, a planner is tempted to renege on these commitments and raise interest rates as soon at the natural rate of interest turns positive.

This represents an additional advantage of flexible tax policy. Not only does it deliver a better allocation (the efficient one), it also has the benefit of not requiring costly commitments that might be difficult to make credible.

4 The linearized model

In order to relate our results more closely to the literature, we now analyze the log-linearized version of the model. As before, we assume $A_t = 1$, $G_t = G$, and $u(C_t, N_t, \xi_t) = u(C_t, N_t)\,\xi_t$.

Then, the following equations provide a log linear approximation[12] to the model above:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \sigma(\hat{\imath}_t - E_t \pi_{t+1} - r_t) + \sigma(E_t \widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t), \tag{24}$$

$$\pi_t = \kappa \widehat{y}_t + \kappa\psi\left(\widehat{\tau}^n_t + \widehat{\tau}^c_t\right) + \beta E_t \pi_{t+1}, \tag{25}$$

where $\pi_t = \ln \frac{P_t}{P_{t-1}}$, $\hat{\imath}_t = \ln\ (1+i_t)$, $\widehat{y}_t = \ln \frac{Y_t}{Y}$, $\widehat{\tau}^c_t = \ln \frac{(1+\tau^c_t)}{(1+\tau^c)}$, $\widehat{\tau}^n_t = \ln \frac{(1-\tau^n_t)}{(1-\tau^n)}$, and $r_t = \ln \beta^{-1} + \ln \xi_t - E_t \ln \xi_{t+1}$. Note that i_t and r_t are in levels, while the other variables are in deviations to the steady state. That is only for the convenience of defining the lower bound. The steady state has zero inflation, zero growth rate of taxes, and the nominal interest rate equal to the real, $\hat{\imath} = r = \ln \beta^{-1}$.

We now assume that monetary policy follows an interest rate rule that explicitly takes into account the lower bound on nominal interest rates

$$\hat{\imath}_t = \max\{0, r_t + \phi_\pi \pi_t + \phi_y \widehat{y}_t\}. \tag{26}$$

In this linear version of the model, if the parameters of the interest rate rule satisfy the Taylor principle, then given the tax policy, the interest rate rule implements a unique local solution to the linear system.

Consider the case where fiscal policy is not used, $\widehat{\tau}^c_t = 0$ and $\widehat{\tau}^n_t = 0$. As long as the lower bound does not bind, movements in the nominal interest rate can fully offset the preference shock affecting r_t. Indeed, the interest rate rule is defined so as to fully insulate output and inflation from this shock, so that in equilibrium, $\widehat{y}_t = 0$, and $\pi_t = 0$. The intuition is simple: shocks to the real interest rate should be absorbed one to one by changes in the nominal interest rate. In this way, the shock does not affect prices and therefore there is no change in output.

Note, on the other hand, that if the nominal interest rate is zero and there is a large enough negative shock to the real interest rate such that $r_t < 0$, this could result in deflation and, given the price frictions, output would drop. This is why the zero bound on interest rates can be a cost to policy.

Fiscal policy can also be used to respond to the shock, and fully stabilize the economy. Suppose the outcome of the interest rate rule is that the nominal interest rate is zero, $\hat{\imath}_t = 0$. From (24), it is clear that there will be a conditional growth rate of the consumption tax,

$$E_t \widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t = r_t,$$

that will satisfy the first equation for $\widehat{y}_t = E_t \widehat{y}_{t+1} = 0$ and $E_t \pi_{t+1} = 0$. From (25), there is an adjustment on the labor income tax,

$$\widehat{\tau}^n_t = -\widehat{\tau}^c_t,$$

that will satisfy the second equation for $\widehat{y}_t = 0$ and $\pi_t = E_t \pi_{t+1} = 0$. The interest rate rule (26) is satisfied.

[12] See the Appendix for the derivation of the linear approximation. The linear equations are similar to Eggertsson (2009).

5 A Model with Capital

The model can easily be extended to allow for capital accumulation. However, to achieve the first best, the tax policy must be enriched to include a tax on income from capital. To do so, assume that investment, I_t, is also an aggregate of the individual varieties

$$I_t = \left[\int_0^1 i_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{27}$$

Aggregate investment increases the capital stock according to

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{28}$$

Minimization of expenditure on the individual investment goods implies

$$\frac{i_{it}}{I_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}, \tag{29}$$

The budget constraints of the households now reads

$$\begin{aligned} & \frac{1}{1+i_t} \overline{B}_{t+1}^h + \sum_{s^{t+1}/s^t} Q_{t,t+1} B_{t,t+1} + P_t K_{t+1} \\ = \; & \overline{B}_t^h + B_{t-1,t}^h + U_t K_t + (1-\delta) P_t K_t - \\ & \tau_t^k \left(U_t K_t - \delta P_t K_t \right) + \left(1-\tau_t^n\right) W_t N_t - \left(1+\tau_t^c\right) P_t C_t - T_t \end{aligned} \tag{30}$$

U_t is the rental cost of capital. Note that the tax τ_t^k has an allowance for depreciation. We believe this is the most natural assumption. As we will show, it will have implications on the behavior of this tax rate when implementing the optimal allocation.

The marginal condition for capital is

$$P_t = \sum_{s^{t+1}/s^t} Q_{t,t+1} \left[P_{t+1} + \left(1-\tau_{t+1}^k\right) \left(U_{t+1} - \delta P_{t+1} \right) \right], \; t \geq 0 \tag{31}$$

The production function of each good i, y_{it}, uses labor, n_{it}, and capital and is given by

$$y_{it} = A_t F\left(k_{it}, n_{it}\right),$$

where A_t is an aggregate productivity shock and the production function is constant returns to scale.

The firm choices must satisfy

$$\frac{U_t}{W_t} = \frac{F_k \left(\frac{k_{it}}{n_{it}} \right)}{F_n \left(\frac{k_{it}}{n_{it}} \right)}$$

Let the corresponding cost function be $C_t = C\left(y_{it}; U_t, W_t\right)$. This is linear in y_{it}, so that marginal cost is a function of the aggregates only.

The optimal price set by these firms is

$$p_t = \frac{\theta}{(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} C_y \left(U_{t+j}, W_{t+j}\right),$$

where $C_y(.)$ is marginal cost, and $\eta_{t,j}$ are the same as in the model without capital.

Market clearing for each variety implies that

$$c_{it} + g_{it} + i_{it} = A_t F\left(n_{it}, k_{it}\right) \tag{32}$$

while market clearing for capital implies

$$K_t = \int_0^1 k_{it} di. \tag{33}$$

Using the demand functions (8), (7), it follows that[13]

$$C_t + G_t + I_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t}\right)^{-\theta} di\right]^{-1} A_t F\left(K_t, N_t\right). \tag{34}$$

An equilibrium for $\{C_t, N_t, K_t\}$, $\{p_t, P_t, W_t, U_t\}$, and $\left\{i_t, \tau_t^c, \tau_t^n, \tau_t^k\right\}$ is characterized by (14), (15), (17), and

$$\frac{U_t}{W_t} = \frac{F_k\left(\frac{K_t}{N_t}\right)}{F_n\left(\frac{K_t}{N_t}\right)}, \tag{35}$$

$$p_t = \frac{\theta}{(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} C_y \left(U_{t+j}, W_{t+j}\right), \tag{36}$$

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = E_t \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)} \left[1 + \left(1-\tau_{t+1}^k\right)\left(\frac{U_{t+1}}{P_{t+1}} - \delta\right)\right], \tag{37}$$

$$C_t + G_t + K_{t+1} - (1-\delta) K_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t}\right)^{-\theta}\right]^{-1} A_t F\left(K_t, N_t\right). \tag{38}$$

As before, we do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

Efficient allocations As before, at the efficient allocation, the marginal rate of technical substitution between any two varieties must be equal to one, so

$$c_{it} = C_t\,;\; g_{it} = G_t\,;\; i_{it} = I_t.$$

The efficiency conditions for the aggregates are:

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{1}{A_t F_n\left(K_t, N_t\right)}, \tag{39}$$

[13] Since the production function is constant returns to scale, $F\left(k_{it}, n_{it}\right) = F_k\left(\frac{k_{it}}{n_{it}}\right) k_{it} + F_k\left(\frac{k_{it}}{n_{it}}\right) n_{it}$ and $\frac{k_{it}}{n_{it}}$ is the same across firms, $\frac{k_{it}}{n_{it}} = \frac{K_t}{N_t}$.

$$u_C\left(C_t,N_t,\xi_t\right)=\sum_{s^{t+1}/s^t}\beta u_C\left(C_{t+1},N_{t+1},\xi_{t+1}\right)\left[A_{t+1}F_k\left(K_{t+1},N_{t+1}\right)+1-\delta\right] \tag{40}$$

and

$$C_t+G_t+K_{t+1}-(1-\delta)\,K_t=A_tF\left(K_t,N_t\right). \tag{41}$$

Policy variables and prices with variable interest rates We first set $\tau_t^c=0$. As before, so as to achieve production efficiency, the price level must be constant across time and states. The aggregate resource constraint (38) becomes (41). When $P_t=P$, (36) becomes

$$P=\frac{\theta}{(\theta-1)}C_y\left(U_t,W_t\right).$$

so that nominal marginal cost must be constant. Since $C_y\left(U_t,W_t\right)=\frac{U_t}{A_tF_k}=\frac{W_t}{A_tF_n}$, from (14), it must be that

$$-\frac{u_C\left(C_t,N_t,\xi_t\right)}{u_N\left(C_t,N_t,\xi_t\right)}=\frac{\left(1+\tau_t^c\right)\frac{\theta}{(\theta-1)}}{\left(1-\tau_t^n\right)A_tF_n\left(K_t,N_t\right)}, \tag{42}$$

implying that $\frac{1+\tau_t^c}{1-\tau_t^n}=\frac{\theta-1}{\theta}$ so the labor income tax will have to be $1-\tau_t^n=\frac{\theta}{\theta-1}$. The nominal wage will be such that (14) is satisfied.

$$-\frac{u_C\left(C_t,N_t,\xi_t\right)}{u_N\left(C_t,N_t,\xi_t\right)}=\frac{P}{\left(1-\tau_t^n\right)W_t},$$

and the nominal interest rate must move with the real rate to satisfy

$$u_C\left(C_t,N_t,\xi_t\right)=\left(1+i_t\right)E_t\left[\beta u_C\left(C_{t+1},N_{t+1},\xi_{t+1}\right)\right].$$

The rental cost of capital satisfies (35). Finally, the tax rate on capital income must be chosen to satisfy the marginal condition for capital (37).

$$u_C\left(C_t,N_t,\xi_t\right)=E_t\left\{\beta u_C\left(C_{t+1},N_{t+1},\xi_{t+1}\right)\left[1+\left(1-\tau_{t+1}^k\right)\left(\frac{\theta-1}{\theta}A_{t+1}F_k\left(K_{t+1},N_{t+1}\right)-\delta\right)\right]\right\}.$$

Clearly the capital income tax must be moving with shocks in order to implement the efficient allocation. It is no longer the case that the efficient allocation can be implemented with constant taxes.[14]

It is interesting to note, though, that this is the case because we assume, as is standard, that firms can deduct depreciation expenses from the capital income tax, i.e., the tax is paid on $(U_t-\delta P_t)K_t$. If, instead, we had assumed that the tax was paid on the gross return U_tK_t, the marginal condition for capital would be

$$\begin{aligned}&u_C\left(C_t,N_t,\xi_t\right)\\=\;&E_t\left\{\beta u_C\left(C_{t+1},N_{t+1},\xi_{t+1}\right)\left[1-\delta+\left(1-\tau_{t+1}^k\right)\frac{\theta-1}{\theta}A_{t+1}F_k\left(K_{t+1},N_{t+1}\right)\right]\right\},\end{aligned}$$

and, setting a constant tax, $\left(1-\tau_{t+1}^k\right)\frac{\theta-1}{\theta}=1$, would be consistent with the optimal allocation.

[14] Standard New Keynesian models usually have labor only and assume taxes are not flexible. If instead they considered capital, the nonflexiblity of taxes would be costly.

Policy variables and prices at the zero bound When the natural rate of interest is negative, the efficient allocation can no longer be implemented with constant consumption and labor taxes. But it can still be implemented with flexible taxes.

As before, we set the nominal interest equal to the natural rate of interest whenever the latter is positive, and to zero otherwise. The intertemporal condition, with a constant price level, is as before

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = \left(1+i_t\right) E_t\left[\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)}\right]$$

which imposes restrictions on the path of consumption taxes. There are multiple paths that satisfy these constraints. The labor income tax will have to move to compensate for the movements in the consumption tax, satisfying condition (42) above.

Now the capital income tax will also have to move to account for the changes in the consumption tax:[15]

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = E_t\left\{\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)}\left[1+\left(1-\tau_{t+1}^k\right)\left(\frac{\theta-1}{\theta} A_{t+1} F_k\left(K_{t+1}, N_{t+1}\right)-\delta\right)\right]\right\}.$$

Going back to the experiment of Section 3.3, when the zero bound is temporarily binding, we must now supplement consumption and labor taxes with capital taxes. The reason is simple. When capital is introduced in the model, the increasing path of consumption taxes, that is necessary to circumvent the zero bound constraint, acts as an undesirable tax on capital. Its effects on capital accumulation must therefore be counteracted with an offsetting capital subsidy. This subsidy must remain in place as long as the natural rate of interest is negative (until period T).

6 The irrelevance of the zero bound in more general environments

We have shown that tax policy can be used to achieve full efficiency, when nominal interest rates are at the zero bound. In order for this to be the case, it must be that there are no idiosyncratic shocks, that the initial distribution of prices across firms is degenerate, that profit taxes are used to finance the subsidies to production, and that lump sum taxes are used to finance government spending. We find the extreme case to be particularly illustrative of the point we want to make, but the result is more general. In these cashless economies with sticky prices, whatever policy can do with the nominal rate, can also be done with tax policy. But tax policy can do more: The zero bound constraint can be made irrelevant. This is the case, regardless of whether full efficiency can be attained. We now make this explicit.

We modify the model in Section 2 and allow for productivity shocks to be idiosyncratic. The production function of each good i, now, uses labor, n_{it}, according to

$$c_{it} + g_{it} = y_{it+j} = A_t A_{it} n_{it}, \tag{43}$$

[15] Note that, contrary to the case with a flexible interest rate and no consumption taxes, in this case a flexible capital income tax rate is necessary even if the tax base is the gross capital income.

where A_t is an aggregate shock and A_{it} is an uncorrelated firm specific productivity shock.

Let $\xi_{it} \in \{0, 1\}$ be the random variable, such that, if $\xi_{it} = 1$, the firm can change the price. The draws are *i.i.d.* over time and across firms with $E_{t-1}[A_{it}] = 1$. The firms that are able to change prices choose the price p^*_{it} to maximize profits

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left[p^*_{it} y_{it+j} - W_{t+j} n_{it+j} \right]$$

where output $y_{it+j} = c_{it+j} + g_{it+j}$ must satisfy the technology constraint and the demand function

$$y_{it+j} = \left(\frac{p^*_{it}}{P_{t+j}} \right)^{-\theta} Y_{t+j},$$

obtained from (8) and (7), where $Y_{t+j} = C_{t+j} + G_{t+j}$.

The optimal price set by these firms is

$$p^*_{it} = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_t A_{it+j}}, \tag{44}$$

where

$$\eta_{t,j} = \frac{(\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau^C_{t+j}\right)} \left(P_{t+j}\right)^{\theta-1} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau^C_{t+j}\right)} \left(P_{t+j}\right)^{\theta-1} Y_{t+j}}.$$

The price of firm i is $p_{it} = p^*_{it}$ if $\xi_{it} = 1$, and $p_{it} = p_{it-1}$, otherwise.

6.1 Equilibria

Using the demand functions (8), (7), it follows that

$$C_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} A_t^{-1} A_{it}^{-1} di \right]^{-1} N_t. \tag{45}$$

An equilibrium for $\{C_t, N_t\}$, $\{p_{it}, p^*_{it}, P_t, W_t\}$, and $\{i_t, \tau^c_t, \tau^n_t\}$ is characterized by households marginal conditions (14), (15) with $R_t \geq 1$, the price setting constraint (44), above, the condition for the price level (6), where $p_{it} = p^*_{it}$ if $\xi_{it} = 1$, and $p_{it} = p_{it-1}$, otherwise, and the resource constraints (45).

If, at time zero, firm i cannot optimally choose the price, because $\xi_{i0} = 0$, then $p_{i0} = p_{i,-1}$, and there is a distribution of these initial prices which is not necessarily degenerate.

6.2 The efficient flexible price allocation

If prices were flexible, then firms would set prices according to

$$p_{it} = \frac{\theta}{\theta - 1} \frac{W_t}{A_t A_{it}}.$$

The aggregate price level would be

$$P_t = \frac{\theta}{\theta - 1} \frac{W_t}{A_t} \left[\int_0^1 \left(\frac{1}{A_{it}} \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

and the resource constraints would be

$$C_t + G_t = A_t \left[\int_0^1 (A_{it})^{\theta-1} di \right]^{\frac{1}{\theta-1}} N_t. \tag{46}$$

Substituting the nominal wage from the households intratemporal condition (14), we have

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{1+\tau_t^c}{1-\tau_t^n} \frac{\theta}{\theta-1} \frac{1}{A_t} \frac{1}{\left[\int_0^1 (A_{it})^{\theta-1} di \right]^{\frac{1}{\theta-1}}}.$$

This condition and the resource constraints (46) are the only implementability conditions. The efficient allocation can be achieved by setting $\frac{1+\tau_t^c}{1-\tau_t^n} \frac{\theta}{\theta-1} = 1$ (taxes are required to counteract the monopoly distortion). One possibility is to set $\tau_t^c = 0$ and $\frac{1}{1-\tau^n} = \frac{\theta-1}{\theta}$.

6.3 Implementability with interest rate policy only

We now turn to the sticky price economy. In this section, we restrict the consumption tax and the labor tax to be constant $\tau_t^c = \tau^c$, $\tau_t^n = \tau^n$ with $\frac{1+\tau_t^c}{1-\tau_t^n} \frac{\theta}{\theta-1} = 1$. Then the set of equilibria for $\{p_{it}, p_{it}^*, P_t, \tau^n, C_t, N_t, Y_t\}$ is restricted by

$$p_{it}^* = \frac{\theta}{(1-\tau^n)(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{-\frac{P_{t+j} u_N\left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}{u_C\left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}}{A_t A_{it+j}}, \tag{47}$$

where

$$\eta_{t,j} = \frac{(\alpha\beta)^j U_C(t+j) (P_{t+j})^{\theta-1} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j U_C(t+j) (P_{t+j})^{\theta-1} Y_{t+j}},$$

obtained by replacing the nominal wage from (14) into (44);

$$P_t = \left[\int_0^1 {p_{it}}^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \tag{48}$$

where $p_{it} = p_{it}^*$ if $\xi_{it} = 1$, and $p_{it} = p_{it-1}$, if $\xi_{it} = 0$;

$$\frac{u_C(C_t, N_t, \xi_t)}{P_t} = \beta (1+i_t) E_t \frac{u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{P_{t+1}}; \tag{49}$$

the resource constraints

$$C_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} A_{it}^{-1} di \right]^{-1} A_t N_t; \tag{50}$$

and the zero bound constraint $i_t \geq 0$.

There are two reasons why the flexible price allocation might not be implemented: the zero bound on nominal interest rates and the presence of idiosyncratic shocks. The first reason is by now familiar. The second reason is new. With idiosyncratic shocks, at the efficient allocation, the relative price for any two given firms responds to the relative idiosyncratic shocks that these firms face so that $\frac{p_{it}}{p_{jt}} = \frac{A_{jt}}{A_{it}}$. With sticky prices, it is impossible to replicate this volatile pattern of relative prices.

6.4 Implementability with both interest rates and tax policy

With flexible tax rates, an equilibrium for $\{p_{it}, p_{it}^*, P_t, \tau_t^c, \tau_t^n, i_t, C_t, N_t, Y_t,\}$ is restricted by

$$p_{it}^* = \frac{\theta}{\theta - 1} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{\frac{-\left(1+\tau_{t+j}^c\right) P_{t+j} u_N\left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}{\left(1-\tau_{t+j}^n\right) u_C\left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}}{A_t A_{it+j}} \tag{51}$$

where

$$\eta_{t,j} = \frac{(\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} \left(P_{t+j}\right)^{\theta-1} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} \left(P_{t+j}\right)^{\theta-1} Y_{t+j}},$$

together with (48), where $p_{it} = p_{it}^*$ if $\xi_{it} = 1$, and $p_{it} = p_{it-1}$, if $\xi_{it} = 0$;

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right) P_t} = \left(1 + i_t\right) E_t \left[\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right) P_{t+1}}\right]; \tag{52}$$

(50); and finally the restriction that the zero bound constraint be verified $i_t \geq 0$.

Condition (51) can be rewritten recursively as

$$p_{it}^* = \eta_{t,0} \frac{\theta}{(\theta-1)} \frac{\frac{-(1+\tau_t^c) P_t u_N(C_t, N_t, \xi_t)}{(1-\tau_t^n) u_C(C_t, N_t, \xi_t)}}{A_t A_{it}} + \left(1 - \eta_{t,0}\right) E_t p_{it+1}^*. \tag{53}$$

where

$$\eta_{t,0} = \frac{\frac{U_C(t)}{\left(1+\tau_t^C\right)} \left(P_t\right)^{\theta-1} Y_t}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} \left(P_{t+j}\right)^{\theta-1} Y_{t+j}}. \tag{54}$$

Note that the weight $\eta_{t,0}$ depends on the path for the consumption taxes.

When flexible taxes can be used, the zero bound constraint does not restrict the set of implementable allocations and prices. To see this, consider a sequence for prices and allocations $\{p_{it}, p_{it}^*, P_t, C_t, N_t, Y_t\}$ that satisfies (51), (48), (52), and (50), but does not necessarily satisfy the zero bound constraint. We denote by $\{\tau_t^c, \tau_t^n, i_t\}$ the corresponding sequence of taxes and nominal interest rates, and we denote by $\eta_{t,0}$ the quantity defined in equation (54) for this allocation.

The same allocation and process for prices can be implemented with another sequence $\{\tilde{\tau}_t^c, \tilde{\tau}_t^n, \tilde{\imath}_t\}$ for taxes, in such a way that the zero bound constraint is satisfied. We now explain how to construct consumption and labor taxes that implement the original allocation with the new interest rate $\tilde{\imath}_t = \max\{i_t, 0\}$. The key is to construct consumption taxes in such a way that (52) holds and the weights are unchanged, $\tilde{\eta}_{t,0} = \eta_{t,0}$.

In order to perform this construction recursively, it is useful to represent the realization of uncertainty as a tree. Consider a history (a node in the tree) and assume that $\tilde{\tau}_t^c$ has been chosen. We construct $\tilde{\tau}_{t+1}^c$ across all the possible continuation histories (the descendent nodes) simultaneously in such a way that

$$\frac{1}{1+\tilde{\imath}_t} = E_t \left[\frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{u_C\left(C_t, N_t, \xi_t\right)} \frac{P_t}{P_{t+1}} \frac{1+\tilde{\tau}_t^c}{1+\tilde{\tau}_{t+1}^c}\right]$$

and

$$\frac{1}{\eta_{t,0}} = 1 + \alpha E_t \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{u_C\left(C_t, N_t, \xi_t\right)} \frac{P_t}{P_{t+1}} \frac{1+\tilde{\tau}_t^c}{1+\tilde{\tau}_{t+1}^c} \left(\frac{P_{t+1}}{P_t}\right)^{\theta} \frac{Y_{t+1}}{Y_t} \frac{1}{\eta_{t+1,0}}. \tag{55}$$

This can be seen as a system of two equations in the unknowns $\tilde{\tau}^c_{t+1}$. This system always has a solution as long as the two equations are not colinear. A necessary and sufficient condition is that $\left(\frac{P_{t+1}}{P_t}\right)^\theta \frac{Y_{t+1}}{Y_t} \frac{1}{\eta_{t+1,0}}$ is not constant across the possible continuation histories, or in other words that this date-$t+1$ random variable is not predictable at time t. We then set labor taxes as follows

$$\frac{1-\tilde{\tau}^n_t}{1+\tilde{\tau}^c_t} = \frac{1-\tau^n_t}{1+\tau^c_t}.$$

We have proved the following result: modulo a technical condition, every allocation that can be implemented with a combination of taxes and monetary policy that does not necessarily respect the zero lower bound constraint can also be implemented with a different combination of taxes and monetary policy that does respect the zero lower bound constraint. Our proof can easily be adapted to show the stronger results that the interest rate is a redundant instrument when flexible taxes can be used. While the nominal interest rate is a redundant policy instrument when taxes are also used for stabilization, taxes are not redundant instruments. For example, if taxes are not used, then the set of implementable allocations will be restricted by the zero bound on nominal interest rates.

It is important to emphasize that even with flexible taxes, the efficient allocation cannot be implemented. This would require a richer set of instruments, i. e. consumption and labor taxes specific to each firm in the economy.

7 Conclusions

The main conclusion of this paper is that in the standard New-Keynesian model, the zero bound constraint on nominal interest rates is not a relevant restriction on policy when both fiscal and monetary policy are flexible. In response to a recent literature on using inefficient monetary or government spending policies to circumvent the zero bound constraint in the New-Keynesian model, we show that tax policy can do that at zero cost.

The argument that fiscal policy can neutralize the effects of the zero bound is very simple. Suppose the objective of policy was to lower real rates. If nominal rates cannot be lowered, real rates can still be low if expected inflation is high. Getting all prices to move together in response to aggregate conditions—so expected inflation is high—may come at a cost. Note that the relevant inflation to consider is producer price inflation. Indeed, it may be costly to get all producers in the economy to raise all future prices uniformly. But inflation arising from a reduction in current consumption taxes (or increases in future consumption taxes) is easy to achieve, can be announced and implemented at zero cost, and brings down real interest rates.

Movements in consumption taxes would in general distort other margins. For this reason we have to use a model where those decisions are explicitly modelled, and allow for other taxes as well. In a standard new-Keynesian model, we show that, if consumption and labor income taxes are both used, it is possible to compensate for the distortions and achieve the first best. We then analyze the same economy but with capital accumulation. The main results extend to this case, as long as flexible capital income taxes are also used. Importantly, because our policy implements the efficient allocation, it is time-consistent: if a future planner were given an opportunity to revise this policy in the future, it would choose not to do so.

We first consider an environment where the first best can be implemented, even at the zero bound. This assumption makes the results particularly stark, but the irrelevance of the zero bound constraint holds more generally. We consider an extension of the model where the full efficient allocation cannot be achieved, because of idiosyncratic shocks or because the initial distribution of prices of the different firms is not degenerate. Productive efficiency can no longer be achieved, but tax policy can undo the zero bound restriction on nominal interest rates.

In order for the zero bound to be ineffective, taxes must be flexible. But, are taxes flexible enough? After witnessing the policy response to the recent crisis in the US and elsewhere, it is hard to argue for lack of flexibility of any fiscal policy. In any case, our point is also normative, meaning that if taxes were not flexible, they should be made flexible. There are also many examples of movements in sectorial or state level taxes with the purpose of stimulating spending. Interesting examples are the tax holidays on sales taxes in many states in the US,[16] and programs such as the Consumer Assistance to Recycle and Save (CARS) set up in June 2009.[17]

We have analyzed these questions in a model with sticky prices but flexible wages. It should be clear that our policies can be adapted to an economy with sticky wages, provided that the employer and employee components of the payroll tax can be adjusted separately.

We have analyzed the implications of a particular restriction on the nominal interest rate, that it cannot be negative. But for the economy of a small state in a federation or a small economy in a monetary union, the nominal interest rate is always beyond control. The implications for stabilization policy are similar to the ones we have seen in this paper, applied to an apparently very different issue. If interest rate policy cannot be adjusted, tax policy can still be, and the constraints on the nominal rate can be made irrelevant. Common nominal interest rates do not have to be too low or too high.

In the economy we have analyzed, we do not consider good specific taxes. And concluded that fiscal policy at the zero bound can do as well as monetary policy away from the zero bound. In an environment where different sectors are hit by different shocks, or affected differently by common shocks, fiscal policy that treats different sectors differently can do better than monetary policy, whether at the zero bound or away from it.

[16] It is customary for many states in the US to announce yearly sales tax holidays for specific sets of goods. They typically last for only a few days.

[17] Commonly known as Cash for Clunkers, this was a temporary subsidy for the trading in and purchase of a new, more fuel efficient, vehicle. The initial budget was set to one billion dollars and planned to last for five months. Due to the high number of applications, it was terminated after the second month, and the final budget was close to three billion.

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8 Appendix: The log-linearized model

As productivity shocks play no particular role, we assume that $A_t = 1$ for all t, so (16) becomes

$$\frac{p_t}{P_t} = \frac{\theta}{(\theta-1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{P_t} \tag{56}$$

The steady state has

$$\begin{aligned} C_t &= C,\, N_t = N,\, \xi_t = 1, \tau_t^c = \tau^c, \tau_t^n = \tau^n \\ P_t &= p_t = P, 1+i = \beta^{-1} \end{aligned}$$

so that

$$\eta_{t,j} = (1-\alpha\beta)(\alpha\beta)^j \text{ , and } \frac{\theta}{(\theta-1)} W = P.$$

If we log-linearize equation (15) , using (18) to replace labor, we obtain

$$\lambda \widehat{C}_t + \Gamma \widehat{\xi}_t - \widehat{\tau}_t^c \simeq \hat{\imath}_t - \ln \beta^{-1} - E_t \pi_{t+1} + \lambda E_t \widehat{C}_{t+1} + \Gamma E_t \widehat{\xi}_{t+1} - E_t \widehat{\tau}_{t+1}^c \tag{57}$$

where

$$\begin{aligned} \lambda &= \frac{C}{uc}(u_{CC} + u_{CN}) \\ \Gamma &= \frac{\xi}{u_C} u_{C\xi} = \frac{u_{C\xi}}{u_C} = 1 \text{ if } \xi_t \text{ is multiplicative} \\ \widehat{C}_t &= \ln \frac{C_t}{C} \\ \widehat{\xi}_t &= \ln \xi_t \\ \widehat{\tau}_t^c &= \ln \frac{(1+\tau_t^c)}{(1+\tau^c)} \\ \pi_{t+1} &= \ln \frac{P_{t+1}}{P_t} \\ \hat{\imath}_t &= \ln (1+i_t) \end{aligned}$$

Linearization of the aggregate resource constraint yields

$$C_t + G_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t} \right)^{-\theta} \right]^{-1} A_t N_t$$

assuming that government consumption is constant, delivers

$$\frac{C}{C+G} \widehat{C}_t = \widehat{y}_t$$

So, if we let $g^{-1} = \frac{C}{C+G}$, then

$$\widehat{C}_t = g \widehat{y}_t,$$

If we also assume that the shock ξ_t is multiplicative, so $\Gamma = 1$, we can write equation (57) as

$$\lambda g \widehat{y}_t + \widehat{\xi}_t - \widehat{\tau}_t^c \simeq \hat{\imath}_t - \ln \beta^{-1} - E_t \pi_{t+1} + \lambda g E_t \widehat{y}_{t+1} + E_t \widehat{\xi}_{t+1} - E_t \widehat{\tau}_{t+1}^c$$

or, letting $\sigma = 1/\lambda g$,

$$\widehat{y}_t \simeq E_t \widehat{y}_{t+1} + \sigma \left[\hat{\imath}_t - E_t \pi_{t+1} - \left(\ln \beta^{-1} + \widehat{\xi}_t - E_t \widehat{\xi}_{t+1} \right) \right] - \sigma \left(E_t \widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t \right)$$

On the other hand, linearization of (56) , delivers

$$\ln p_t \simeq \ln \frac{\theta}{(\theta - 1)} + \ln E_t \sum_{j=0}^{\infty} \eta_{t,j} W_{t+j}$$

But

$$W_t = \frac{(1 + \tau^c_t) P_t}{(1 - \tau^n_t)} \left[-\frac{u_C (C_t, N_t, \xi_t)}{u_N (C_t, N_t, \xi_t)} \right]^{-1}$$

so

$$\ln p_t \simeq \ln \frac{\theta}{(\theta - 1)} + \ln E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{\left(1 + \tau^c_{t+j}\right) P_{t+j}}{\left(1 - \tau^n_{t+j}\right)} \left[-\frac{u_C \left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}{u_N \left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)} \right]^{-1}$$

or

$$\ln p_t - \ln P_t \simeq \ln \frac{\theta}{(\theta - 1)} + \ln E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{\left(1 + \tau^c_{t+j}\right) \frac{P_{t+j}}{P_t}}{\left(1 - \tau^n_{t+j}\right)} \left[-\frac{u_C \left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}{u_N \left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)} \right]^{-1}$$

The log-linearization of the second term in the right hand side is given by

$$\ln E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{\left(1 + \tau^c_{t+j}\right) \frac{P_{t+j}}{P_t}}{\left(1 - \tau^n_{t+j}\right)} \left[-\frac{u_C \left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)}{u_N \left(C_{t+j}, N_{t+j}, \xi_{t+j}\right)} \right]^{-1} \simeq (1 - \alpha\beta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\Omega_{t+j} \right]$$

where

$$\Omega_{t+j} = \widehat{\tau}^c_{t+j} + \widehat{\tau}^n_{t+j} + \pi_t(j) + \phi \widehat{C}_{t+j} - \gamma \widehat{\xi}_{t+j}$$

where

$$\begin{aligned} \pi_t(j) &= \ln \frac{P_{t+j}}{P_t} \\ \widehat{\tau}^n_t &= \ln \frac{(1 - \tau^n_t)}{(1 - \tau^n)} \end{aligned}$$

and

$$\begin{aligned} \phi &= (-1) \frac{C}{U_C (-U_N)} \left[(U_{CC} + U_{NC}) (-U_N) - U_C (U_{NC} + U_{NN}) \right] \\ \gamma &= \frac{-1}{U_N^2} \left[U_{C\xi} U_N - U_C U_{N\xi} \right] \end{aligned}$$

Note that if, as we will assume, $u\left(C_{t+j}, N_{t+j}, \xi_{t+j}\right) = u\left(C_{t+j}, N_{t+j}\right) \xi_{t+j}$, then $\gamma = 0$. Note also that $\phi > 0$.

Thus, we can write

$$\begin{aligned} \widehat{p}_t &\simeq (1 - \alpha\beta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}^c_{t+j} + \widehat{\tau}^n_{t+j} + \pi_t(j) + \phi \widehat{C}_{t+j} - \gamma \widehat{\xi}_{t+j} \right] \\ &\simeq (1 - \alpha\beta) \left[\begin{array}{c} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi \widehat{C}_t - \gamma \widehat{\xi}_t \right] \\ + (\alpha\beta) E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}^c_{t+j+1} + \widehat{\tau}^n_{t+j+1} + \pi_t(j) + \phi \widehat{C}_{t+j+1} - \gamma \widehat{\xi}_{t+j+1} \right] \end{array} \right] \end{aligned}$$

where $\widehat{p}_t = \ln \frac{p_t}{P}$. But note that

$$\pi_t(j) = \ln \frac{P_{t+j}}{P_t} = \ln \frac{P_{t+1}}{P_t} \frac{P_{t+j}}{P_{t+1}} = \ln \frac{P_{t+1}}{P_t} + \ln \frac{P_{t+j}}{P_{t+1}} = \pi_{t+1} + \pi_{t+1}(j-1)$$

so we can write the equation as

$$
\begin{aligned}
\widehat{p}_t &\simeq (1-\alpha\beta)\left[\begin{array}{c} \left[\widehat{\tau}_t^c+\widehat{\tau}_t^n-\phi\widehat{C}_t-\gamma\widehat{\xi}_t\right]+ \\ (\alpha\beta)\,E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left[\widehat{\tau}_{t+j+1}^c+\widehat{\tau}_{t+j+1}^n+\pi_{t+1}+\pi_{t+1}(j-1)+\phi\widehat{C}_{t+j+1}-\gamma\widehat{\xi}_{t+j+1}\right]\end{array}\right] \\
&= (1-\alpha\beta)\left[\begin{array}{c} \left[\widehat{\tau}_t^c+\widehat{\tau}_t^n+\phi\widehat{C}_t-\gamma\widehat{\xi}_t\right]+(\alpha\beta)\,E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left[\pi_{t+1}\right]+ \\ (\alpha\beta)\,E_t\sum_{j=0}^{\infty}(\alpha\beta)^j\left[\widehat{\tau}_{t+j+1}^c+\widehat{\tau}_{t+j+1}^n+\pi_{t+1}(j-1)+\phi\widehat{C}_{t+j+1}-\gamma\widehat{\xi}_{t+j+1}\right]\end{array}\right] \\
&= (1-\alpha\beta)\left[\widehat{\tau}_t^c+\widehat{\tau}_t^n+\phi\widehat{C}_t-\gamma\widehat{\xi}_t\right]+(\alpha\beta)\,E_t\left[\pi_{t+1}\right]+(\alpha\beta)\,E_t\widehat{p}_{t+1}
\end{aligned}
$$

But the log linearization of (17) delivers

$$\ln P_t \simeq \alpha\ln P_{t-1}+(1-\alpha)\ln p_t^*$$

so

$$\ln P_t-\alpha\ln P_t \simeq \alpha\ln P_{t-1}+(1-\alpha)\ln p_t^*-\alpha\ln P_t$$

or

$$\widehat{p}_t \simeq \frac{\alpha}{1-\alpha}\pi_t$$

Replacing above

$$\frac{\alpha}{1-\alpha}\pi_t \simeq (1-\alpha\beta)\left[\widehat{\tau}_t^c+\widehat{\tau}_t^n+\phi\widehat{C}_t-\gamma\widehat{\xi}_t\right]+(\alpha\beta)\,E_t\left[\pi_{t+1}\right]+(\alpha\beta)\,E_t\frac{\alpha}{1-\alpha}\pi_{t+1}$$

or

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c+\widehat{\tau}_t^n+\phi\widehat{C}_t-\gamma\widehat{\xi}_t\right]+\frac{1-\alpha}{\alpha}(\alpha\beta)\,E_t\left[\pi_{t+1}\right]+(\alpha\beta)\,E_t\pi_{t+1}$$

so

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c+\widehat{\tau}_t^n+\phi\widehat{C}_t-\gamma\widehat{\xi}_t\right]+\beta E_t\pi_{t+1}$$

Finally, recall that

$$\widehat{C}_t = g\widehat{y}_t,$$

so

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c+\widehat{\tau}_t^n+\phi g\widehat{y}_t-\gamma\widehat{\xi}_t\right]+\beta E_t\pi_{t+1}$$

Letting

$$
\begin{aligned}
\kappa &= (1-\alpha\beta)\frac{1-\alpha}{\alpha}\phi g \\
\psi &= (\phi g)^{-1}
\end{aligned}
$$

we obtain

$$\pi_t \simeq \kappa\psi\left(\widehat{\tau}_t^c+\widehat{\tau}_t^n\right)+\kappa\widehat{y}_t-\kappa\psi\gamma\widehat{\xi}_t+\beta E_t\pi_{t+1}$$

We assume that the shock ξ_t is multiplicative, so $\gamma=0$. If we let $r_t=\left(\ln\beta^{-1}+\widehat{\xi}_t-E_t\widehat{\xi}_{t+1}\right)$, the system can be written as

$$\pi_t \simeq \kappa\widehat{y}_t+\kappa\psi\left(\widehat{\tau}_t^n+\widehat{\tau}_t^c\right)+\beta E_t\pi_{t+1}$$

$$\widehat{y}_t \simeq E_t\widehat{y}_{t+1}-\sigma(\hat{\imath}_t-E_t\pi_{t+1}-r_t)+\sigma(E_t\widehat{\tau}_{t+1}^c-\widehat{\tau}_t^c)$$

with the constraint that $\hat{\imath}_t \geq 0$.