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Recent Developments in Stress Testing Market Risk

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Paper presented at the Expert Forum on Advanced Techniques on Stress Testing: Applications for Supervisors
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Agenda

I. **Traditional stress tests for market risk**

II. **Maximum Loss as a risk measure uncovering harmful scenarios**

III. **Integration of market and credit risk stress testing**

I: Traditional Stress Tests

Ingredients for stress testing

- Portfolio: In our case the trading book (subject to market risk)
- Scenarios: possible market states $\boldsymbol{r}$
  $\boldsymbol{r} = (r_1,...,r_n)$ vector of risk factor values
  r_i are: interest rates, exchange rates, equity indices etc.
- Portfolio valuation function P as a function of $\boldsymbol{r}$: $P = P(\boldsymbol{r})$
- Current state of the market: $\boldsymbol{r}_{\mathrm{CM}}$
- Hence, current portfolio value: $P(\boldsymbol{r}_{\mathrm{CM}})$

Performing stress tests

1. Select scenarios $\boldsymbol{r}_{\mathrm{stress1}}$, $\boldsymbol{r}_{\mathrm{stress2}}$,... (according to some criterion)
2. Calculate portfolio values $P(\boldsymbol{r}_{\mathrm{stress1}})$, $P(\boldsymbol{r}_{\mathrm{stress2}})$,…
3. Derive some measure of riskiness of the scenarios

I: Traditional Stress Tests

How to select scenarios

- Standard scenarios
- Historical scenarios
- Subjective worst case scenarios

I: Dangers of Traditional Stress Tests

- A stress scenario for one portfolio might be a lucky strike for another portfolio
- Stress tests with standard and historical scenarios may nourish a false illusion of safety
- Subjective worst case scenarios are often too implausible to trigger management action

But: Stress Tests can be the basis of informed risk decisions ...

... if the scenarios are plausible

... if we are confident there are no worse scenarios

II: Maximum Loss

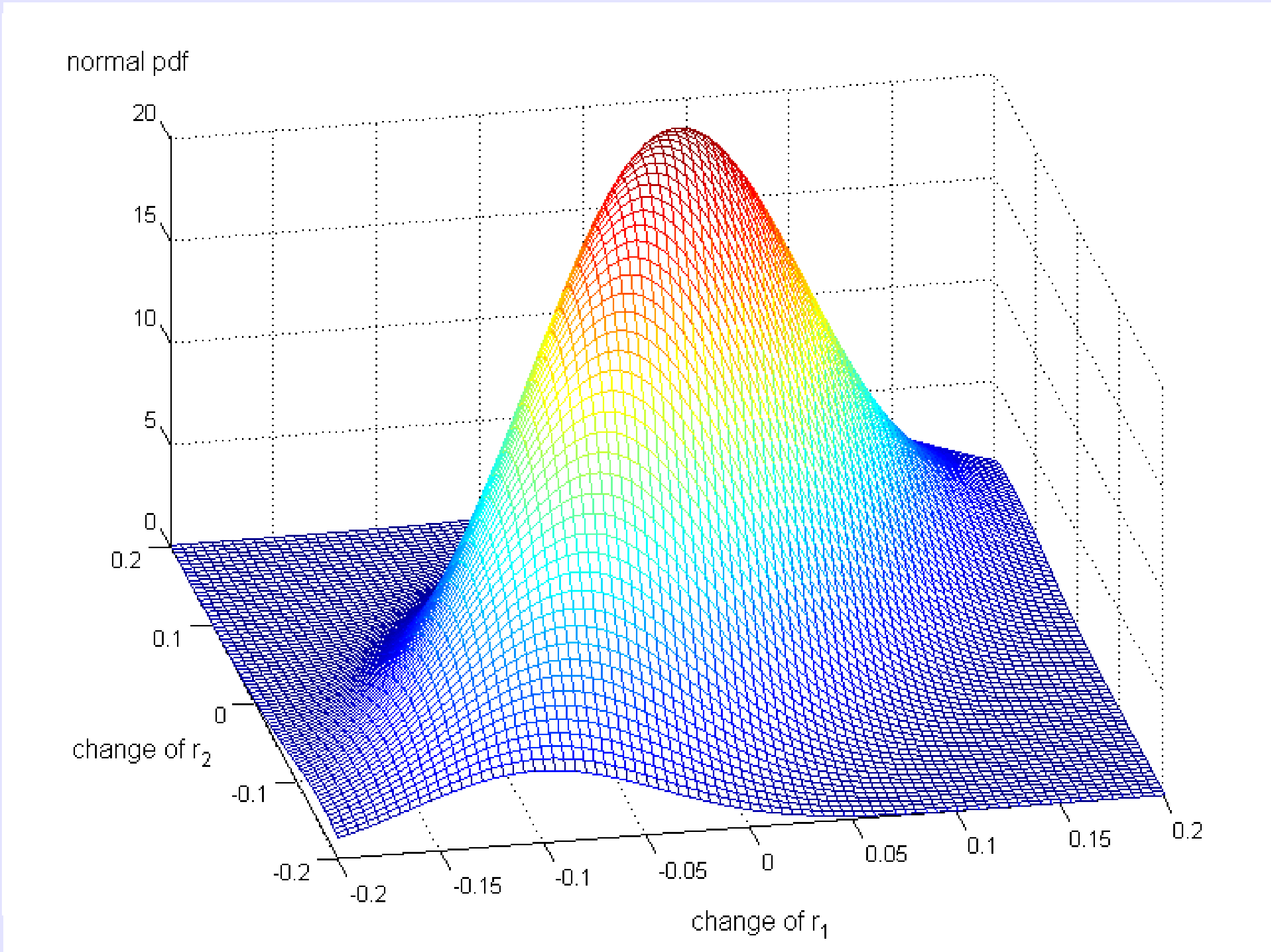
- Good overview on Maximum Loss in doctoral thesis by Studer (1997)
- Can be interpreted as a risk measure that avoids dangers of traditional stress tests
- Choose a **trust region** *TR*: A set of scenarios above a certain minimal plausibility threshold
- Maximum Loss defined as: $$\mathrm{MaxLoss}_{TR}(P) := \sup_{\mathbf{r} \in TR} \{P(\mathbf{r}_{\mathrm{CM}}) - P(\mathbf{r})\}$$
- "Above the plausibility threshold no loss worse than MaxLoss can happen"

Choice of trust region

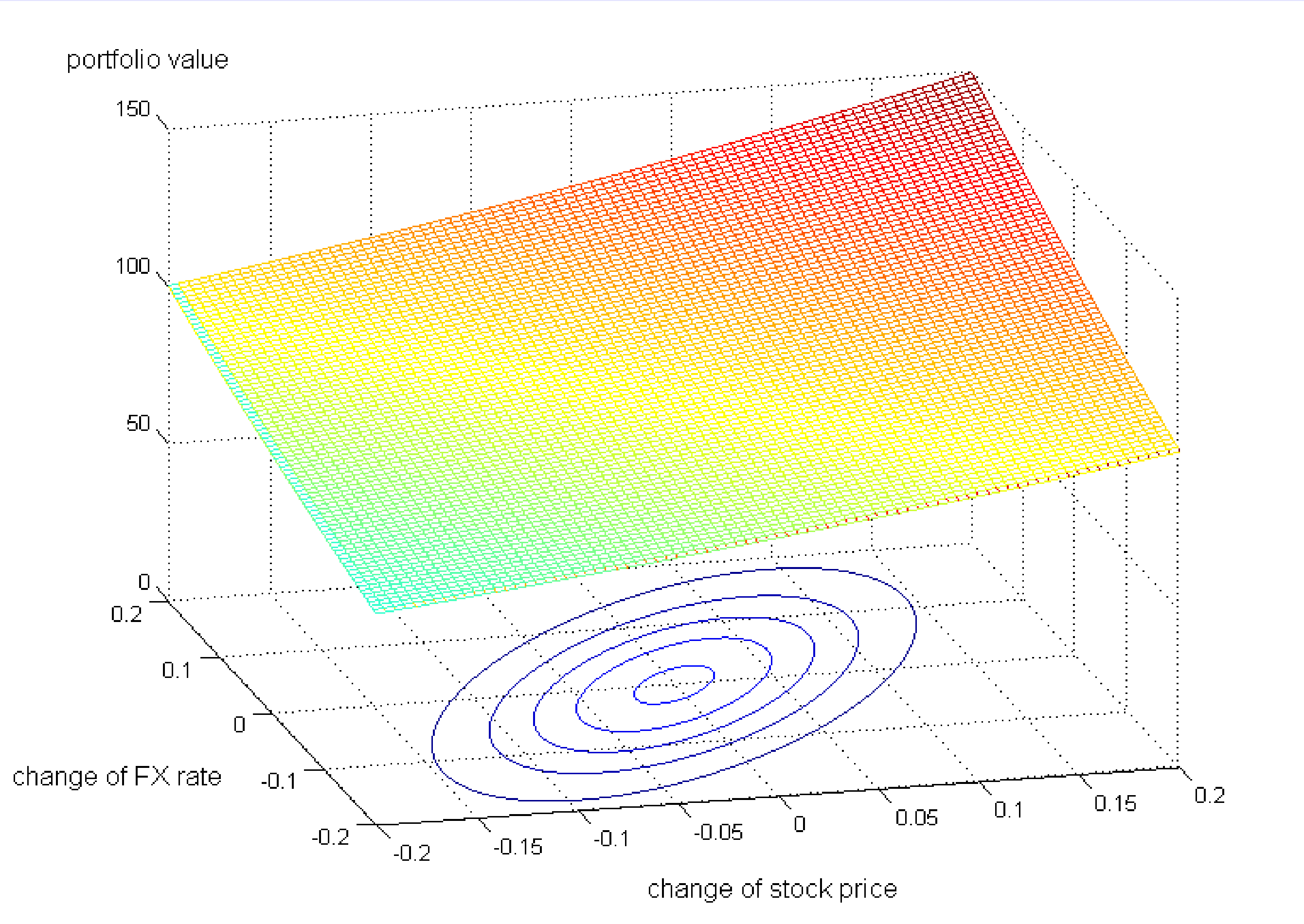
- By means of the multivariate risk factor distribution
- Trust region shall have some predefined probability (p) and contain only scenarios with "highest density"
- In case risk factors have an elliptic distribution (e.g. multivariate normal, Student-t): Trust region is an ellipsoid of scenarios with Mahalanobis distance to $\boldsymbol{r}_{\mathrm{CM}}$ below some threshold k_p: $$TR = \left\{\mathbf{r} : (\mathbf{r} - \mathbf{r}_{\mathrm{CM}})' \, \Sigma^{-1} \, (\mathbf{r} - \mathbf{r}_{\mathrm{CM}}) \leq k_p^{\,2}\right\}$$

  (Σ is the co-variance-matrix)

II: Trust Region: Area of Highest Density



II: Within Trust Region: Find Scenario with Smallest Portfolio Value (= Maximum Loss)



II: Benefits of Maximum Loss

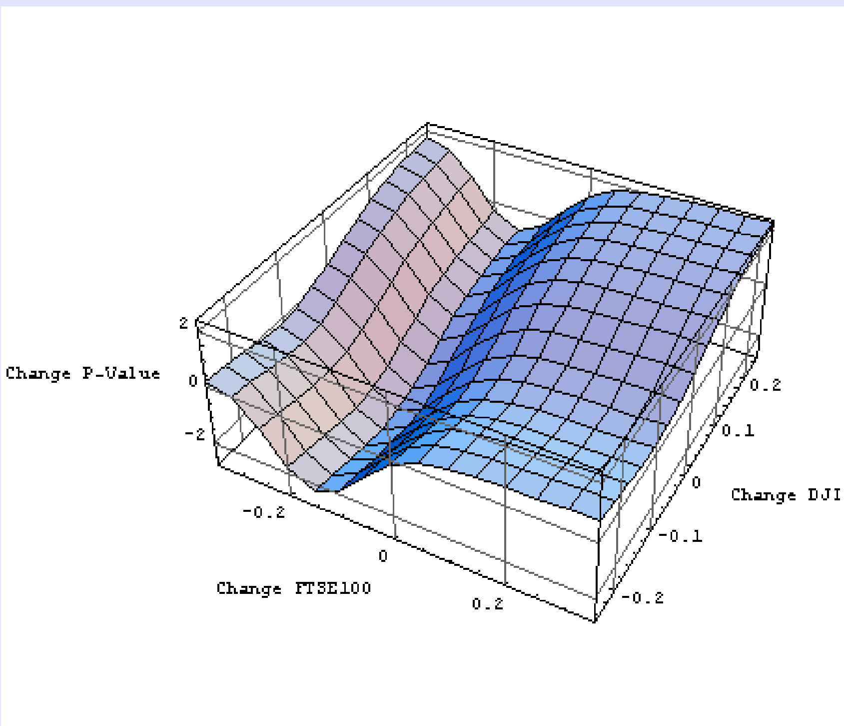
- Maximum Loss not only quantifies risks but also identifies a worst case scenario (among all scenarios in the trust region)
- Searching for worst case scenarios yields more harmful and more plausible scenarios than other ways to identify stress scenarios
- Sample portfolio consisting of options on different international stock indices
  - Stress scenarios are identified in different ways
    - Worst case according to the recommendations of the DPG (Derivatives Policy Group)
    - Recurrence of Black Friday in October 1987
    - Worst case scenario implied by Maximum Loss

	Relative Loss	Plausibility
Worst DPG	- 183%	once in 10 yrs
Black Friday	- 154%	once in 19 yrs
Worst Case (ML)	**- 279%**	once in **8** yrs

II: Benefits of Maximum Loss

Identifying key risk factors of the worst case scenario = Locating the vulnerable spots of a portfolio

Example: Again option portfolio



	Risk Factors	Rel. Changes	Loss	Explanatory Power
Report 1	**FTSE100**	**-13%**	**206** %	**74%**
Report 2	**FTSE100** **DJI**	**-13%** **-8%**	**264** %	**94%**
Report 3	**FTSE100** **DJI** **NIK225**	**-13%** **-8%** **-5%**	**271** %	**97%**

$$\text{Explanatory Power} = \frac{\text{Loss}\,(\mathbf{r}_{\text{report}})}{\text{Loss}\,(\mathbf{r}_{\text{worst case}})}$$

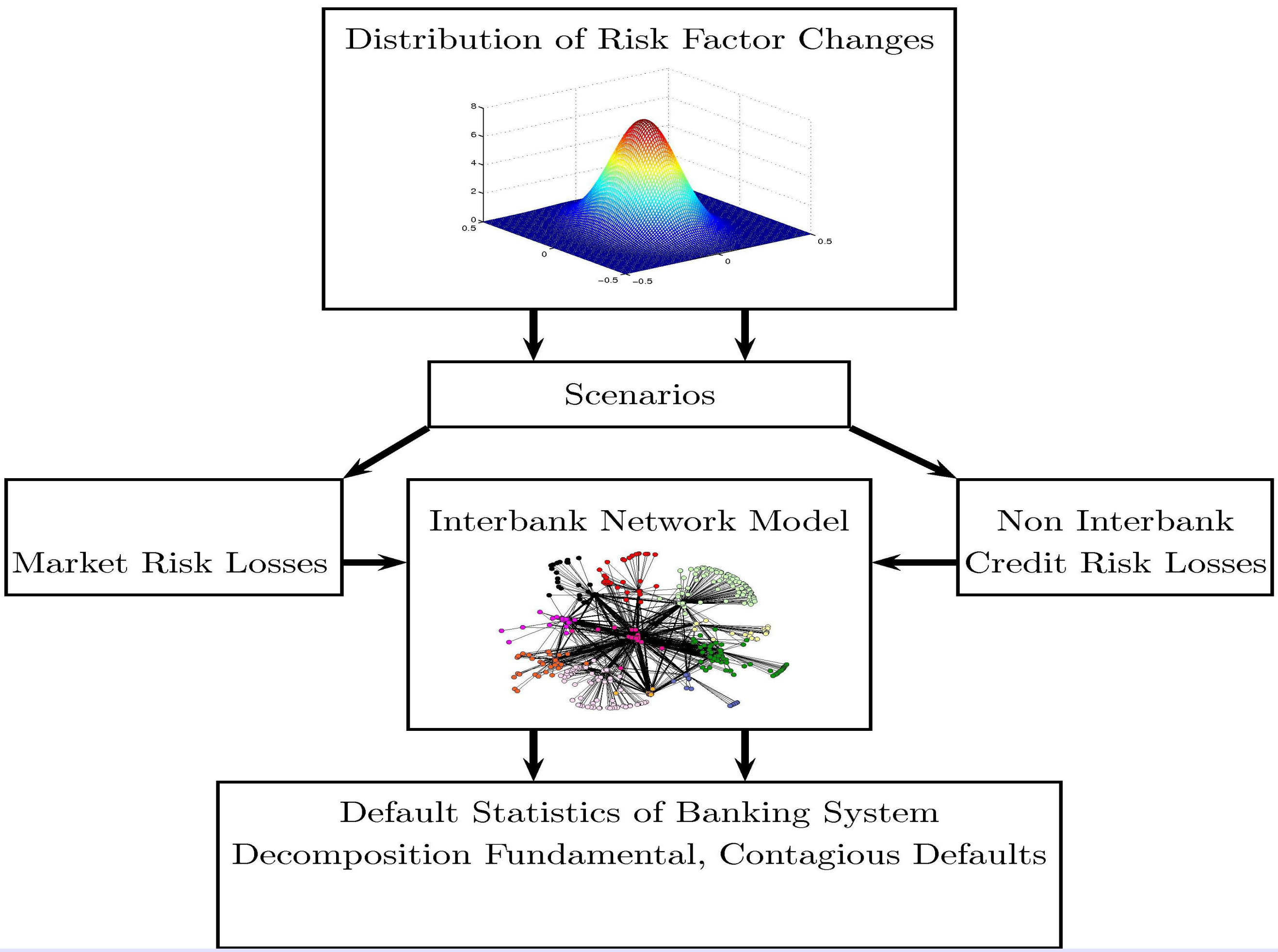
II: The Problem of Dimensional Dependence

- n … number of risk factors on which the portfolio depends
- Let's consider an elliptic risk factor distribution; trust regions are then ellipsoids
- The trust region shall have probability p
- When k (the "radius" of the ellipsoid) is fixed, p depends on k (and on n): e.g. for multivariate normal distribution:

$$p(k,n) = 1 - F_{\chi_n^2}(k^2) = 1 - \frac{1}{2^{n/2}\Gamma(n/2)} \int_0^{k^2} s^{\frac{n}{2}-1} e^{-\frac{s}{2}} ds$$

- To get trust regions with some predefined probability p, k has to increase as n increases
- If we add an "empty risk factor" (i.e. a factor on which the portfolio value does not depend), the radius k has to increase in order to hold p fixed
- We therefore are searching for MaxLoss within a larger trust region when we add an empty risk factor
- Also MaxLoss is likely to be larger once having added an empty risk factor

III: Systemic Risk Monitor (SRM) – Basic Structure



III: Risk Factors in SRM

- SRM analysis market risk and credit risk simultaneously
- As risk factors we have market risk factors as well as credit risk drivers
- Time horizon in SRM is 3 months
  - Implies that length of risk factor time series will be limited; e.g. with quarterly data starting in 1980 we get about 100 data points
  - For numerical stability (estimation of covariance matrix for the grouped t-copula) number of observations should clearly exceed number of risk factors
- Therefore parsimonious selection of risk factors (trade-off with accuracy of valuation)
  - Interest rates:
    - 5 currencies EUR, USD, CHF, JPY, GBP
    - Maturities: 3 months, 1 year, 5 years, 10 years
  - 2 equity indices (national, international)
  - 4 exchange rates: EUR vis-à-vis USD, CHF, JPY, GBP
- Credit risk drivers should be able to explain PDs in different industrial sectors

$$\mu_{i,t} = \frac{e^{X_t \beta_i}}{1 + e^{X_t \beta_i}} + \varepsilon_{i,t}$$

  - 8 credit risk factors were selected (e.g. GDP, Consumer Price Index, Unemployment rate, international stock index)

III: Stress Testing in SRM

- 26 market risk factors + 8 credit risk factors = 34 risk factors
- These factors we wish to model statistically
  - Allows for a Monte Carlo-simulation for analyzing the actual situation (sampling from the un-conditional distribution)
  - Allows for a Monte Carlo-simulation for **stress testing** (sampling from the conditional distribution)
- For stress testing, a set of risk factors is set to some predefined values
- Remaining factors are sampled from the conditional distribution

III: Statistical Modeling of Risk Factors

- Multivariate distribution of risk factors is estimated in a 2-step procedure:
  - Step 1: Modeling of **marginal distribution** of each risk factor by models which are optimized with respect to their out-of sample density forecast
  - Step 2: Modeling of **dependencies** between individual risk factors by a grouped t-copula
- Our goal is to have enough flexibility in order to capture
  - Marginal distributions of the various risk factors
  - Patterns of dependence between risk factors
- Market risk factors and Credit risk factors are treated in a common statistical model

III: Selecting Marginal Distributions

Goal: Find a statistical model for risk factor changes over the horizon of one quarter

Aspects to be considered when selecting a model

- Maybe modeling risk factors at higher frequencies (basic periods: daily, weekly,…) and aggregating these models to a quarter can exploit information contained in higher frequency data
- Maybe there are GARCH effects even for quarterly data
- Different alternatives for the distribution of residuals:
  - Normal
  - Student t
  - Extreme value

III: Marginal Distributions: Tests and Results

- The resulting 36 models were applied to 19 (sufficiently long) time series
- 2 statistical tests were applied to each model:
  - **Test 1**: According to de Raaij und Raunig (2002)
  - **Test 2**: Kolmogorov-Smirnov-Test for N(0,1)
- Table shows number of accepted time series per model (according to test 1 and 2)

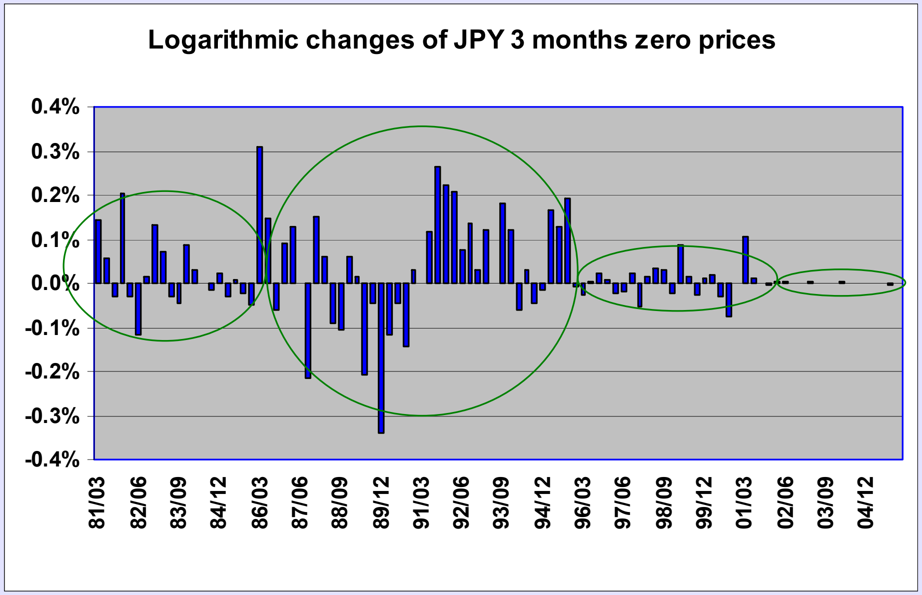
		No GARCH						GARCH					
		Normal		t		EVT		Normal		t		EVT	
		T1	T2	T1	T2	T1	T2	T1	T2	T1	T2	T1	T2
Basic period	1d	0	0	0	0	0	0	0	0	0	2	0	0
	5d	0	0	0	1	0	0	0	0	0	1	0	0
	10d	0	0	0	2	0	1	0	0	0	3	0	2
	20d	0	2	1	6	0	1	0	2	0	7	0	4
	30d	0	5	8	16	1	8	0	5	11	14	2	10
	60d	9	16	9	13	13	15	10	17	11	15	15	15

Results

- Aggregation does not yield good results: Quarterly data already contain all the information
- GARCH models are slightly superior to constant volatility
- Extreme value distribution for residuals appears best

III: Marginal Distributions: Final Model Selection

- No aggregation of higher frequency data, i.e. use quarterly data directly



- GARCH
  - Testing procedure favors consideration of GARCH effects
  - Makes sense for analysis of current situation
  - Should be used with care for stress tests
- Distribution of Residuals
  - Extreme value distribution performs best in the test procedures
  - Simulations show that extreme value distribution leads to too extreme risk factor movements
  - SRM now uses t-distribution as marginals
    - Also leads to extreme risk factor movements in some cases
    - Hence restriction: degrees of freedom > 4,1

III: Match between Historical and Generated Data

- Many time series show more extreme movements than a normal distribution (small degree of freedom)
- Sample of 10,000 scenarios (no GARCH) is compared with the historical input data
  - Standard deviations match pretty well
  - Kolmogorov-Smirnov test for the null hypothesis, that historical data and generated sample have the same distribution
  - Is rejected only in two cases at alpha = 0.05

Risk factor	# degrees of freedom	Standard deviation Hist. Data	Standard deviation Generated Sample	p-Value KS-Test
Usd	3336.3	6.0%	6.0%	0.975
Chf	4.1	2.8%	2.9%	0.927
Jpy	4.8	5.5%	5.7%	0.670
Gbp	6.3	4.6%	4.6%	0.543
EquityAt	4.1	10.9%	9.9%	0.971
EquityNonAt	4.1	8.3%	8.0%	0.813
Eur 03M	4.1	0.2%	0.1%	0.884
Eur 01Y	4.2	0.6%	0.6%	0.980
Eur 05Y	14.6	2.5%	2.5%	0.935
Eur 10Y	7.3	4.4%	4.3%	0.874
Usd 03M	4.1	0.3%	0.2%	0.297
Usd 01Y	4.1	1.2%	1.0%	0.981
Usd 05Y	4.7	4.0%	4.0%	0.668
Usd 10Y	4.8	6.7%	6.7%	0.382
Chf 03M	4.1	0.2%	0.2%	0.846
Chf 01Y	4.6	0.8%	0.8%	0.926
Chf 05Y	9.0	1.9%	1.9%	0.922
Chf 10Y	9.6	3.2%	3.2%	0.821
Jpy 03M	4.1	0.3%	0.2%	0.000
Jpy 01Y	4.1	0.8%	0.5%	0.044
Jpy 05Y	4.1	2.6%	2.5%	0.677
Jpy 10Y	4.1	4.6%	4.5%	0.964
Gbp 03M	4.1	0.3%	0.2%	0.710
Gbp 01Y	4.1	1.0%	1.0%	0.853
Gbp 05Y	4.1	3.0%	3.0%	0.825
Gbp 10Y	4.1	5.3%	5.0%	0.919

III: Match between Historical and Generated Data

2 selected time series

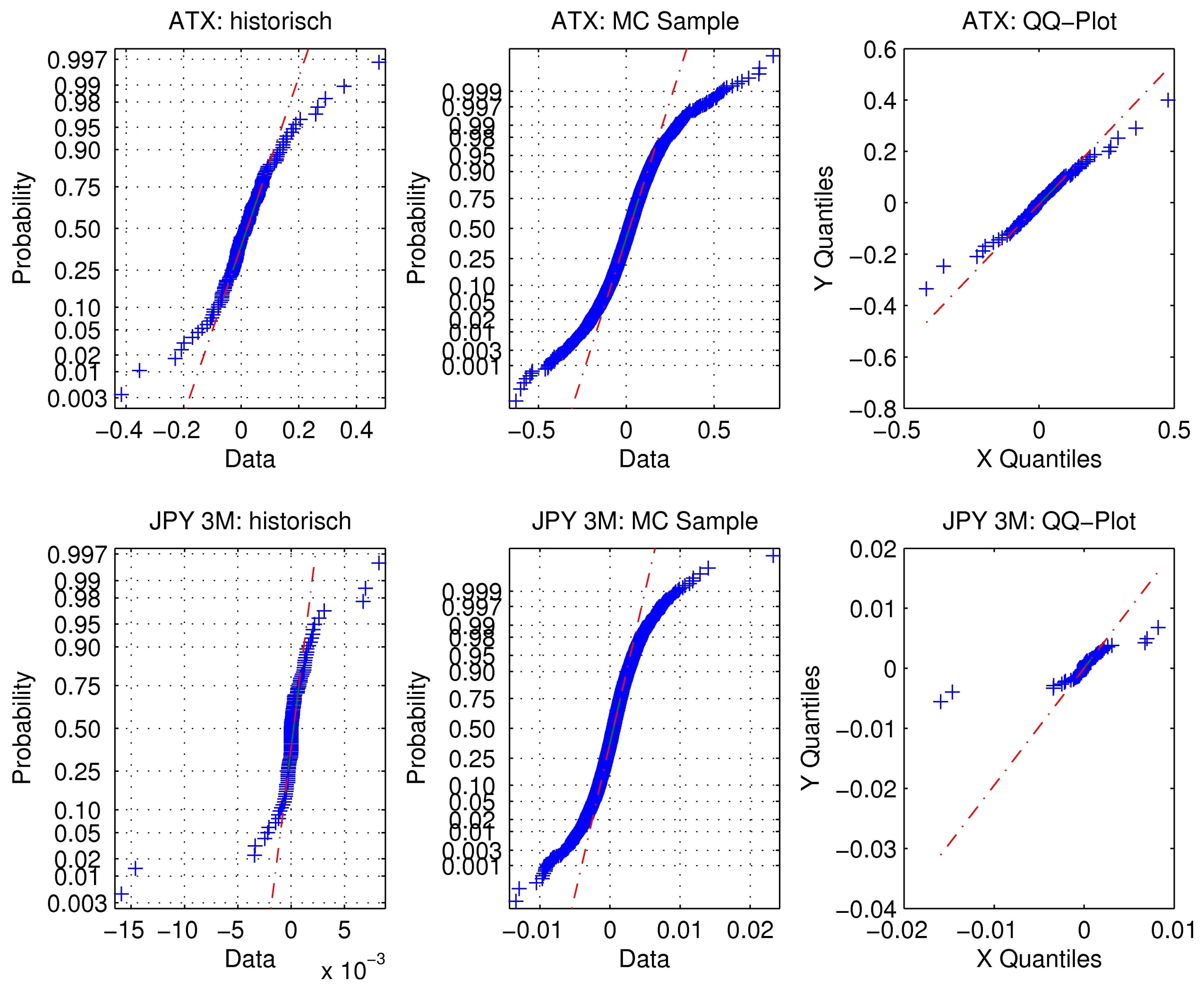
- Austrian traded index (top)
- JPY 3 months (bottom)

Left: Historical data in a normal probability plot

Middle: Generated sample (size=10,000; no GARCH) in a normal probability plot

Right: QQ-Plot for assessing the match between historical and generated data

- ATX fits well
- JPY 3M: historical extremes are not captured (a result of the restriction dof > 4,1)



III: Modeling Dependencies: Grouped t-Copula

- **Copula** models dependencies between risk factors
  - Copula is the part of the multivariate distribution which is not contained in the marginal distributions
- Concept of t**ail-dependence** for assessing dependencies
  - The coefficient of tail-dependence λ between two variables is defined as:

$$\lambda := \lim_{v \to 1^-} \mathbb{P}(X_1 > G_1^{-1}(v) \mid X_2 > G_2^{-1}(v)) > 0;$$

  - Is roughly speaking the probability that one variable is very large (small) given the other variable is very large (small)
  - In case $\lambda > 0$, “one variable can pull up (down) the other variable”
- For the multivariate normal distribution we have $\lambda = 0$ (no tail-dependence)
  - Real data show tail-dependence
- An alternative is given by the t-copula
  - There is tail-dependence between risk factors ($\lambda > 0$)
  - Scenarios can be generated easily in a Monte Carlo-simulation
  - Drawback: between all risk factors there is the same tail-dependence

III: Modeling Dependencies: Grouped t-Copula

- As an alternative to the t-copula the **grouped t-copula** was introduced by Daul et al. (2003)
  - Risk factors are arranged into groups
  - Within each group risk factors have the same tail-dependence
  - Each group is characterized by a parameter (degrees of freedom)
- Grouped t-copula was adopted for SRM
  - Is suited equally well for MC-simulations as the plain t-copula
  - In SRM risk factors were arranged into 4 groups (in parentheses: estimated degrees of freedom)
    - Credit risk factors (20)
    - FX (14)
    - Equity (5)
    - Interest rates (11)

III: Simulation

- In SRM we need the multivariate distribution in order to generate scenarios for the Monte Carlo-simulation
- For the grouped t-copula efficient algorithms exist for sampling from the
  - un-conditional distribution
  - conditional distribution (a set of risk factors is set to predefined values)
- E.g. algorithm for un-conditional distribution (Daul et al. (2003))

1. *Generate a random vector* $Z \sim N(0,\rho)$*, where* ρ *is a linear correlation matrix, and generate an independent random variable* $U \sim U(0,1)$
2. *Denote by* G_ν *the distribution function of* χ^2_ν *and let* $R_k := G^{-1}_{\nu_k}(U)$ *for* $k = 1, ..., m$.
3. *Then the vector*

$$Y = \left(\frac{Z_1}{\sqrt{R_1/\nu_1}}, \ldots, \frac{Z_{s_1}}{\sqrt{R_1/\nu_1}}, \frac{Z_{s_1+1}}{\sqrt{R_2/\nu_2}}, \ldots, \frac{Z_{s_1+s_2}}{\sqrt{R_2/\nu_2}}, \ldots \frac{Z_n}{\sqrt{R_m/\nu_m}} \right)$$

   has by equation (A.18) a grouped t-copula with Student marginals.

4. *Denote by* t_ν *the distribution function of the one-dimensional Student distribution. Then*

$$X := \left(H_1^{-1}(t_{\nu_1}(Y_1)), \ldots, H_{s_1}^{-1}(t_{\nu_1}(Y_{s_1})), H_{s_1+1}^{-1}(t_{\nu_2}(Y_{s_1+1})), \ldots, H_n^{-1}(t_{\nu_m}(Y_{s_n}))\right),$$

   have grouped t-copula and marginal distribution functions H_i, $i = 1, \ldots, n$.

Literature

Breuer T., M. Jandacka, G. Krenn (2005): “Towards an Integrated Measurement of Credit and Market Risk“, presentation at a workshop held by the Research Task Force of the Basel Committee for Banking Supervision http://www.bis.org/bcbs/events/rtf05breuer.pdf

Daul S., E. DeGeorgi, F. Lindskog, A. McNeil (2003): “The grouped t copula with an application to credit risk”, *RISK* Vol. 16, pp 73-76

de Raaij G., B. Raunig (2002): “Evaluating Density Forecasts from Models of Stock Market Returns“, Working Paper 59, Oesterreichische Nationalbank

McNeil A., R. Frey (2000): “Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach”, *Journal of Empirical Finance* 7

Oesterreichische Nationalbank (1999): Guidelines on Market Risk, Volume 5: Stress Testing http://www.oenb.at/en/img/band5ev40_tcm16-20475.pdf

Studer G. (1997): “Maximum Loss for Measurement of Market Risk”, Doctoral Thesis, Swiss Federal Institute of Technology, Zürich