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The Macroeconomics of Debt Overhang

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Abstract

I analyze the interactions of debt overhang in multiple markets. When households' loans are underwater, households make inefficient consumption/saving decisions. When banks' bonds are underwater, banks refuse to finance new investments. The two problems can reinforce each other. From a macroeconomic perspective, the model suggests that financial bailouts can be efficient, that it might be optimal to favor the banks during mortgages renegotiations, and financial globalization may prevent government from undertaking efficient recapitalization programs unless countries agree to coordinate their efforts.

FIRST DRAFT.

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Myers (1977) shows that debt overhang can lead to under-investment. Firms in financial distress find it difficult to raise capital for new investments because the proceeds from these new investments mostly serve to increase the value of the existing debt instead of equity. Debt overhang can be alleviated if the various creditors and shareholders manage to renegotiate their contracts and restructure the balance sheets. This process is always challenging and costly, and a large body of empirical research has shown the economic importance of private renegotiation costs for firms in financial distress (see Gilson, John, and Lang (1990), Asquith, Gertner, and Scharfstein (1994), Hennessy (2004) among others). The social costs of renegotiation may be even larger than the private costs because renegotiations can trigger creditor runs among other firms. Moreover, from a theoretical perspective, one should expect renegotiation to be costly because otherwise debt would not discipline managers or reduce risk shifting (Jensen and Meckling (1976), Hart and Moore (1995)). Hence, debt holders are often dispersed which makes it difficult to renegotiate outside bankruptcy because of free-rider problems or contract incompleteness (see Bulow and Shoven (1978), Gertner and Scharfstein (1991), and Bhattacharya and Faure-Grimaud (2001)).

The paper is organized as follows. Section 1 sets up the model. Section 2 explains the debt overhang equilibrium. I explain the externalities and strategic complementarities in savings decisions, in investment decisions, and across savings and investment. I also discuss multiple equilibria.

Section 3 explains the role of renegotiations. I show that it is optimal for the government to subsidize renegotiations. I also show that it is optimal for the government to favor the banks in the renegotiations of household loans.

Section 4 discusses government interventions: assets buybacks, recapitalizations, debt guarantees.

Section 5 discusses the issues created by international banking operations. Section 6 concludes.

This paper relates to the literature on government bailouts. Most of the literature on government bailouts focuses on financial institutions. Gorton and Huang (2004) argue that there is a potential role for the government to bail out banks in distress because the government can provide liquidity more effectively than the private market. Diamond and Rajan

(2005) show that bank bailouts can increase excess demand for liquidity, which can cause further insolvency and lead to a meltdown of the financial system. Diamond (2001) emphasizes that governments should only bail out banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bank bailout policies can be designed such that they do not distort ex-ante lending incentives relative to strict bank closure policies. Kocherlakota (2009) analyzes resolutions to a banking crisis in a setup where insurance provided by the government generates debt overhang. He analyzes the optimal form of government intervention and finds an equivalence result similar to our symmetric information equivalence theorem. Debt overhang also plays a fundamental role in the model of Diamond and Rajan (2009). Lamont (1995) presents a simple macroeconomic model but does not analyze renegotiations or interventions. Philippon and Schnabl (2009) compare various forms of government bailouts in a partial equilibrium model of debt overhang. They find that, if firms and the government have the same information at the time that firms decide whether to participate in a government program, all interventions are equivalent. If, on the other hand, firms have better information than the government at the time that firms decide whether to participate in a government program, buying equity dominates asset buybacks and debt guarantees. Debt overhang has also been analyzed in the context of sovereign debt crisis (Krugman (1988), Aguiar, Amador, and Gopinath (forthcoming)).

1 Model

1.1 Technology and preferences

The economy is populated by a continuum of households $i \in [0,1]$, and a continuum of financial intermediaries (banks) $j \in [0,1]$. For simplicity, I do not introduce a separate non financial corporate sector. I assume that intermediaries own industrial projects. In doing so, I ignore contractual issues between the banks and the industrial sector.

The model has two dates, $t = 1, 2$. The utility of household i is:

$$U(i) = E\left[c_1(i) + \frac{c_2(i)}{\delta}\right] \tag{1}$$

Initial conditions.

Households and banks start the first period with outstanding financial claims that settle at the end of the second period. Figure 1 presents the balance sheets. The financial assets of households are the bonds and stocks issued by the intermediaries. The liabilities of households are the mortgages held by the banks. The assets of intermediaries are the loans (mortgages) made to households as well as investments in pre-existing physical assets that deliver exogenous payoffs z at time 2. The liabilities of intermediaries are the bonds they must pay back at date 2.

For simplicity, I assume that households have the same initial diversified portfolios of assets, and that they differ only by the face value of the loans they must repay at date 2. Let $F(m)$ be the cumulative distribution of the mortgages, with $m \in [0, \infty)$. Similarly, I assume that banks hold diversified portfolios of consumer credit, and that they differ only by the face value of the bonds they have issued. Let $G(b)$ be the distribution of the face values of banks' bonds, with $b \in \left[b^{\min}, b^{\max}\right]$ outstanding at date 0.

Time 1: Investment and saving.

At time 1, intermediaries receive an investment opportunity: they can spend $x \in \{0, X\}$ units of output at time 1 to create qx units of output at time 2, with $q > \delta$. Investment can only be made by intermediaries. Let n be the face value of the new claims issued by intermediaries to finance these investments.

Households receive an endowment which can be used for consumption at date 1, or for investment. For simplicity I assume that all individuals receive the same endowment $\bar{y}_1$. The endowments can be consumed or invested in financial claims issued by intermediaries. Let $\bar{x}$ be aggregate investment, and let $\bar{c}_1$ be aggregate consumption at time 1. Clearing the goods market requires

$$\bar{y}_1 = \bar{c}_1 + \bar{x}. \tag{2}$$

While intermediaries chose how much to invest, households chose how much to consume and how much to save in order to maximize their lifetime utility (1).

Time 2: Production and consumption.

At time 2, the exogenous assets produce z, and the new investments by intermediaries return qx. Aggregate output at time 2 is:

$$\bar{y}_2 = z + qx \tag{3}$$

It is also equal to aggregate consumption $\bar{c}_2$.

1.2 First best equilibrium

Since $q > \delta$, it is optimal to invest as much as possible. The first best saving curve is $S^*(r) = y_1 1_{r \geq \delta}$, while the first best investment curve is $I^*(r) = X 1_{r \leq q}$. If $\bar{y}_1 > X$, there is excess savings, and the equilibrium is such that $\bar{x} = X$, $\bar{c}_1 = \bar{y}_1 - X$ and the interest rate is $r = \delta$. Alternatively, if $\bar{y}_1 < X$, investment is constrained by available savings, so that $\bar{x} = \bar{y}_1$, $\bar{c}_1 = 0$ and the interest rate is q.[1] In the remaining of the paper, I assume an interior solution for consumption at time 1:

Assumption 1: Excess savings. *The aggregate endowment exceeds the investment capacity at time 1:*

$$\bar{y}_1 \geq X.$$

The required return in the first best equilibrium is therefore $r^{FB} = \delta$, and the first best quantities are $c_1^{FB} = \bar{y}_1 - X$, and $c_2^{FB} = \bar{y}_2^{FB} = z + qX$.

2 Debt overhang equilibrium

The key assumption of the debt overhang model is that outstanding claims are senior.

Assumption 2: **Debt overhang**. *The initial banks' bonds b (resp. households' loans m) are senior to the claims that can issued at date 1. Renegotiating the claims cost κ_b (resp. κ_m).*

I assume for now that $\kappa_b = \kappa_m = \infty$ so renegotiation is impossible at date 1. I analyze renegotiations in Section 3.

2.1 Consumption and saving

Financial contracts are settled at time 2. I have specified the face values of the contracts: m owed by individuals to banks, b and n owed by banks to individuals. Let ρ_ω be the actual repayment on claim $\omega = \{m, b, n\}$, and let $\bar{\rho}_\omega$ denote aggregate repayments.

[1] Note that in this case banks are indifferent between investing and not investing. Since projects are indivisible at the micro level, a fraction of banks invest $x = X$, while the remaining banks invest nothing. The fraction is such that the equilibrium condition $\bar{x} = \bar{y}_1$ is satisfied.

Individuals hold diversified portfolios of equity and debt, so their financial income is only a function of aggregate payments from intermediaries. Let r be the gross rate of return between dates 1 and 2. The wealth at time 2 of an individual who saved s at time 1 is:

$$w(s) = \bar{\rho}_b + \bar{e} + rs, \tag{4}$$

Individual with mortgage loan m defaults if $m \geq w(s)$. In case of default at date 2, individuals consume nothing and their financial wealth goes to the banks holding the loans. As we shall see, underwater households want to dis-save. They can sell their liquid stocks and bonds holdings for a value $(\bar{\rho}_b + \bar{e})/r$. The program of the individual is therefore

$$\mathcal{H} : \max y_1 - s + E\left[\frac{c_2}{\delta}\right],$$

subject to $s \in [-(\bar{\rho}_b + \bar{e})/r, y_1]$, and $c_2(s, m) = \max(0, w(s) - m)$. The next lemma characterizes the solution.

Lemma 1 *The savings at date 1 of an individual with debt m is*

$$s(m) = y_1 \cdot 1_{m \in [0, \hat{m}]} - \frac{\bar{\rho}_b + \bar{e}}{r} \cdot 1_{m \in (\hat{m}, \infty)}, \tag{5}$$

where

$$\hat{m} \equiv (r - \delta)\left(y_1 + \frac{\bar{\rho}_b + \bar{e}}{r}\right). \tag{6}$$

Proof. Since the benchmark model is deterministic, we can write the household's program as:

$$\mathcal{H} : \max_{s \in [-(\bar{\rho}_b + \bar{e})/r, y_1]} y_1 - s + \frac{1}{\delta}\max(0, w(s) - m).$$

If $r < \delta$ it is clear that $s = -(\bar{\rho}_b + \bar{e})/r$ for all households. In this case $\hat{m}$ is negative and $m > \hat{m}$ for all households. If $r \geq \delta$, we have to consider the incentives of the household given its outstanding loan m. If the household decides to consume all it can at time 1, and default at time 2, it gets

$$c_1^{\max} = y_1 + (\bar{\rho}_b + \bar{e})/r$$

On the other hand, the household could decide to save everything. If $m > \bar{\rho}_b + \bar{e} + ry_1$, the household would still be insolvent and it would obviously be optimal to chose $c_1 = c_1^{\max}$. If on the other hand, the household is solvent conditional on saving all it can, then it consumes

$$c_2^{\max}(m) = ry_1 + \bar{\rho}_b + \bar{e} - m$$

The marginal household is therefore characterized by the debt level $\hat{m}$ such that $c_2^{\max}(\hat{m}) = \delta c_1^{\max}$. ∎

Without debt overhang, the program would be linear. Debt overhand makes it convex so the household chooses corner solutions: either $s = -(\bar{\rho}_b + \bar{e})/r$ or $s = y_1$. Households with loans above $\hat{m}$ consume as much as possible at date 1 and default at date 2. Households with loans below $\hat{m}$ save all their income ($s = y_1$) as long as $\delta \geq r$ and they do not default at date 2. The intuition is simple. If individuals have too much debt, they have no incentives to save in conventional ways because their savings are given to banks. Equation (6) is intuitive. The second term in parenthesis is simply NPV of households' assets, and $r - \delta$ is the spread of the market expected return over the rate of time preference. The spread is required to induce saving under debt overhang.

Debt overhang induces households to make inefficient decisions. In the simple stylized model presented here, they consume more at date 1. In reality, they could also decrease investment in maintenance of their homes, decrease job searching efforts, increase time spent avoiding repayments, etc. The robust effect is that they decrease investment in assets and activities that can be appropriated by their creditors, and they increase consumptions and activities that cannot be taken away from them in case of personal bankruptcy.

2.2 Borrowing and investment

We solve the investment problem by backward induction. At time 2, banks hold diversified portfolios of consumer credit, so they receive $\bar{\rho}_m$. Banks repay according to seniority rules. Their exogenous assets yield z. Therefore, a bank with investment x, initial bonds b and new liabilities n defaults at time 2 if and only if:

$$z + qx + \bar{\rho}_m < b + n. \tag{7}$$

The repayments from banks to their claim holders follow strict priority rules. Senior debt holders (holding the initial long term bonds) are paid first:

$$\rho_b = \min(b, z + qx + \bar{\rho}_m);$$

then junior debt holders (holding the notes issued at time 1):

$$\rho_n = \min\left(n, z + qx + \bar{\rho}_m - \rho_b\right);$$

and finally the shareholders:

$$e = z + qx + \bar{\rho}_m - \rho_b - \rho_n.$$

At time 1, the investment decision is made to maximize shareholder value, taking as given the initial outstanding liabilities b. For any bank, we can use the participation constraint of new investors $E[\rho_n] = rx$ to write shareholder value at time 1 as:

$$E_1[e|x] = z + \bar{\rho}_m + (q - r)x - \min\left(b, z + qx + \bar{\rho}_m\right). \tag{8}$$

Equation (8) says that the returns to investing are the NPV of the project $(q - r)x$ – as it would be in the first best economy – minus the transfers to existing bond holders. We therefore have the following lemma:

Lemma 2 *The investment function of a bank with debt level b is*

$$x(b) = X \cdot 1_{b \leq \hat{b}} \cdot 1_{r \leq q}, \tag{9}$$

where

$$\hat{b} \equiv z + \bar{\rho}_m + (q - r)X. \tag{10}$$

Proof. Investment takes place if and only if

$$E_1[e|x = X] > E_1[e|x = 0]$$

or

$$(q - r)X > \min\left(b, z + qX + \bar{\rho}_m\right) - \min\left(b, z + \bar{\rho}_m\right)$$

A bank that is always solvent ($b \leq z + \bar{\rho}_m$) will invest if and only if $q > r$. A bank that is never solvent ($z + \bar{\rho}_m + qX < b$) will never invest. The marginal bank is indifferent between investing and not investing when $(q - r)X = b - z - \bar{\rho}_m$. This defines the debt threshold for investment $\hat{b}$. ■

We saw earlier how debt overhang led households to make inefficient savings decisions. In the case of banks, we see that debt overhang leads to under-investment in new projects at time 1. Banks with debt above $\hat{b}$ do not invest even though the projects have positive net present value.

2.3 Equilibrium conditions

To understand the macroeconomic equilibrium conditions, we need to bring together the balance sheets of banks and individuals. An important relation comes from the aggregate balance sheets of banks. We know that $\bar{\rho}_b + \bar{e} = z + q\bar{x} + \bar{\rho}_m - \bar{\rho}_n$ and $\bar{\rho}_n = r\bar{x}$, therefore:

$$\bar{\rho}_b + \bar{e} = z + (q - r)\,\bar{x} + \bar{\rho}_m. \tag{11}$$

We can now present the equilibrium conditions.

Proposition 1 *The equilibrium conditions of the debt overhang model are the flow of payments from households to banks:*

$$\bar{\rho}_m(\hat{m}) = \int_0^{\hat{m}} m dF(m), \tag{12}$$

the individual default threshold:

$$\hat{m} = (r - \delta)\left(y_1 + \frac{z + (q - r)\,\bar{x} + \bar{\rho}_m(\hat{m})}{r}\right), \tag{13}$$

the aggregate savings condition (for $r > \delta$):

$$\bar{x} = y_1 F(\hat{m}) - \frac{z + (q - r)\,\bar{x} + \bar{\rho}_m}{r}\left(1 - F(\hat{m})\right), \tag{14}$$

and the aggregate investment condition (for $r < q$):

$$\bar{x} = XG\left(z + \bar{\rho}_m(\hat{m}) + (q - r)\,X\right). \tag{15}$$

Proof. The repayments of a household with debt level m to its bank is m if $m < \hat{m}$ and 0 otherwise. In the aggregate, we therefore get (12). Using (11) we can rewrite (6) as (13). If we aggregate individual savings (5), we get

$$S = y_1 F(\hat{m}) - \frac{\bar{\rho}_b + \bar{e}}{r}\left(1 - F(\hat{m})\right).$$

Using (11) and $S = \bar{x}$, we then obtain (14). Finally, if we aggregate investment decisions (9), we get (15). ■

We have 4 unknowns $\bar{x}$, r, $\hat{m}$, ρ and 4 equations. Consider the first two equilibrium conditions, (12) and (13), holding $\bar{x}$ and r constant. Equation (12) says that the flow of payments from individuals to banks, $\bar{\rho}_m$, increases with the default threshold $\hat{m}$. Equation (13) characterizes the optimal savings behavior of an individual, given her expectations about future income from intermediaries, $q\bar{x} + \bar{\rho}_m$, and given excess returns on savings, $(r - \delta)\, y_1$. The higher these are, the more the individual is willing to save. In this schedule, $\hat{m}$ is increasing in $\bar{x}$ and in r, and in $\bar{\rho}_m$.

Lemma 3 *Debt overhang creates strategic complementarities in savings decisions*

Proof. From equation (12), we know that repayments $\bar{\rho}_m$ are increasing in $\hat{m}$

$$\frac{\partial \bar{\rho}_m}{\partial \hat{m}} = \hat{m} f\left(\hat{m}\right)$$

From equation (13) we know that $\hat{m}$ is increasing in $\bar{\rho}_m$

$$\frac{\partial \hat{m}}{\partial \bar{\rho}_m} = 1 - \delta / r$$

■

Individuals eventually receive the income of intermediaries. So if $\bar{\rho}_m$ increases, banks receive more, and so do individuals, because they hold the bonds and stocks of intermediaries. These are the first complementarities in the model. Lemma 3 suggests that the savings system is not stable when $(1 - \delta/r)\, \hat{m} f\left(\hat{m}\right) > 1$, even without the consequences on investment.

The aggregate savings curve (14) says, in equilibrium, the savings of solvent households must finance investment plus the net dis-savings of insolvent agents. Along the savings curve, $\hat{m}$ must increase in $\bar{x}$: more capital spending requires more agents to save. A higher interest rate, on the other hand, decreases the NPV of the insolvent's agent portfolios of liquid claims, so fewer agents need to save to finance the same $\bar{x}$, and we would expect $\hat{m}$ to decrease with r.

Consider now equation (15).

$$XG\left(z + \bar{\rho}_m + (q - r)\, X\right) = \bar{x}$$

If, as is plausible, we have $\partial \hat{m}/\partial \bar{x} > 0$, then both the LHS and the RHS are increasing in $\bar{x}$. The intuition is the following. When more banks finance new projects, economic value is created. This value eventually trickles down to households, who are thus less likely to default on their loans. This makes banks more solvent, and therefore more willing to finance new investment.

Let us define:

$$\hat{m}_q \equiv (q-\delta)\left(y_1 + \frac{z+\bar{\rho}_m(\hat{m}_q)}{q}\right) \tag{16}$$

We can state a general property about the equilibrium interest rate.

Lemma 4 *All equilibria must have $r > \delta$. If $qy_1F(\hat{m}_q) \leq (z+\bar{\rho}_m(\hat{m}_q))(1-F(\hat{m}_q))$, then there is a unique equilibrium with $r \geq q$ and no investment ($\bar{x}=0$). If $qy_1F(\hat{m}_q) > (z+\bar{\rho}_m(\hat{m}_q))(1-F(\hat{m}_q))$, then all equilibria have strictly positive investment and $r \in (\delta, q)$.*

Proof. Suppose $r \leq \delta$. Then $\hat{m} \leq 0$ and $F(\hat{m}) = 0$ and savings would be negative, which is not feasible. Hence $r > \delta$. At the other extreme, suppose that $r \geq q$. Then $\bar{x} = 0$ and the savings equilibrium requires

$$y_1F(\hat{m}) = \frac{z+\bar{\rho}_m}{r}(1-F(\hat{m}))$$

Is this an equilibrium? Suppose $r = q$. Then we can solve for $\hat{m}_q$ defined above. If at $r = q$ we have $y_1F(\hat{m}_q) > \frac{z+\bar{\rho}_m}{q}(1-F(\hat{m}_q))$ then there is no equilibrium with $r \geq q$. Because an increase in r from $r = q$ would only increase $\hat{m}$ and therefore $y_1F(\hat{m}) >> \frac{z+\bar{\rho}_m}{r}(1-F(\hat{m}))$. On the other hand, if $y_1F(\hat{m}_q) \leq \frac{z+\bar{\rho}_m}{q}(1-F(\hat{m}_q))$, then there is a unique equilibrium with $r \geq q$. This equilibrium has no investment. ■

2.4 Open Economy

The open economy with exogenous interest rate is easier to analyze. The savings curve (14) becomes irrelevant, and the equilibrium conditions are simply:

$$\begin{aligned}
\bar{\rho}_m(\hat{m}) &= \int_0^{\hat{m}} m dF(m) \\
\frac{\hat{m}}{r-\delta} &= y_1 + \frac{z+(q-r)\bar{x}+\bar{\rho}_m(\hat{m})}{r} \\
\bar{x} &= XG(z+\bar{\rho}_m(\hat{m})+(q-r)X)
\end{aligned}$$

Response to shocks.

The exogenous variables in the model are z and y_1. We can use them to understand how the economy reacts to shocks. If we take the total differential of the system, we get

$$\begin{aligned}\left(\frac{r}{r-\delta}-\hat{m}f\left(\hat{m}\right)\right)d\hat{m} &= \left(q-r\right)d\bar{x}+rdy_1+dz \\ d\bar{x} &= Xg\left(\hat{b}\right)\left(\hat{m}f\left(\hat{m}\right)d\hat{m}+dz\right)\end{aligned}$$

And therefore

$$\left(\frac{r}{r-\delta}-\left(1+\hat{\gamma}\right)\hat{m}f\left(\hat{m}\right)\right)d\hat{m}=rdy_1+\hat{\theta}dz$$

where I have defined

$$\hat{\gamma}\equiv\left(q-r\right)Xg\left(\hat{b}\right) \tag{17}$$

The system is locally stable if

$$\frac{r}{r-\delta}>\left(1+\hat{\gamma}\right)\hat{m}f\left(\hat{m}\right). \tag{18}$$

In the first best case, neither z nor y_1 has any impact on investment. Exogenous shocks to y_1 or to z affect investment in the debt overhang economy. Negative shocks increase the fraction of non performing loans, decrease the quality of banks balance sheets, and therefore decrease investment.

Multiple equilibria

The complementarities explained above can also give rise to multiple equilibria. Here is a simple example to illustrate the potential for multiple equilibria. There are $1-\pi$ agents with no debt, $m=0$, and π agents with debt $m=M$. We consider an equilibrium where a share η of the indebted agents default. In other words, $F\left(\hat{m}\right)=1-\pi+\left(1-\eta\right)\pi=1-\eta\pi$. Then

$$\bar{\rho}_m=\left(1-\eta\right)\pi M$$

Individuals must be indifferent, so $\hat{m}=M$:

$$M=\left(r-\delta\right)\left(y_1+\frac{z+\left(q-r\right)\bar{x}+\bar{\rho}_m}{r}\right)$$

Finally

$$\bar{x}=XG\left(z+\bar{\rho}_m+\left(q-r\right)X\right)$$

Consider good equilibrium where $\eta = 0$, $\bar{\rho}_m = \pi M$. Of course this equilibrium can only happen in the open economy because it has excess savings since $y > X$. So this equilibrium requires that the economy runs a surplus. The rate is pinned down by the world market and the investment curve is $\bar{x} = XG\left(z + \pi M + (q - r) X\right)$. This can be an equilibrium if and only if it is indeed the case that:

$$M \leq \hat{m} = (r - \delta) \left(y_1 + \frac{z + (q - r) XG\left(z + \pi M + (q - r) X\right) + \pi M}{r} \right)$$

Note that $(r - \delta) \pi / r < 1$ so the condition is not trivially satisfied.

Now consider the polar opposite, where all indebted agents default: $\eta = 1$ and $\bar{\rho}_m = 0$. The investment curve is $\bar{x} = XG\left(z + (q - r) X\right)$. Indeed $\eta = 1$ if

$$M > (r - \delta) \left(y_1 + \frac{z + (q - r) XG\left(z + (q - r) X\right)}{r} \right)$$

So the condition for multiple equilibria is

$$G\left(z + (q - r) X\right) < \frac{r}{(r - \delta)(q - r)} \frac{M}{X} - \frac{ry_1 + z}{(q - r) X} \leq G\left(z + \pi M + (q - r) X\right) + \frac{\pi}{(q - r)} \frac{M}{X}$$

Multiple equilibria are possible if the mass of investing banks increases enough from $G\left(z + (q - r) X\right)$ to $G\left(z + \pi M + (q - r) X\right)$. This is similar to a global violation of the local stability constraint (18).

2.5 Summary

The debt overhang economy has the following features.

1. Banks have outstanding mortgages on their books, and the performance of these loans is affected by macroeconomic conditions

2. Banks can finance new investment in the corporate sector, but their willingness to do so decreases if their legacy loans are non-performing.

3. Households' balance sheets improve when banks finance new investments because households are residuals claimants.

4. When households balance sheets improve, they are less likely to default on their existing mortgage loans, and this improves the balance sheet of the banks. The banks are then more willing to finance new investments.

5. Finally, in the closed-economy context, there would be an additional effect coming from the crowding out of investment by the current consumption of households with under-water loans. This effect is not present in the open-economy because the supply of savings is fully elastic.

The first externality in the model is that one bank does not take into account the impact of increased activity on the default rate of other banks' loans when deciding to make new loans. The second externality is that one household does not take into account the impact of its default on the balance sheets of the banks.

3 Renegotiations

A theoretical solution to debt overhang is renegotiation between equity and debt holders. If renegotiation is costless, efficiency is restored. In practice, however, renegotiation often requires bankruptcy, which is a costly process. Indeed, a large body of empirical research has shown the economic importance of private renegotiation costs for firms in financial distress (see Gilson, John, and Lang (1990), Asquith, Gertner, and Scharfstein (1994), Hennessy (2004) among others). The social costs of renegotiation may be even larger than the private costs because renegotiations can trigger creditor runs among other firms. Moreover, from a theoretical perspective, one should expect renegotiation to be costly because otherwise debt would not discipline managers or reduce risk shifting (Hart and Moore (1995), Jensen and Meckling (1976)). Hence, debt holders are often dispersed which makes it difficult to renegotiate outside bankruptcy because of free-rider problems or contract incompleteness (see Bulow and Shoven (1978), Gertner and Scharfstein (1991), and Bhattacharya and Faure-Grimaud (2001)).

In our framework, there are two separate debt overhand frictions, one with savers, one with intermediaries.

3.1 Renegotiating mortgages

The efficiency gains depend on the market interest rate. When r is above δ, households should save. Some do not because they would transfer too much to the banks. At cost κ_m, the household and the banks can engage in efficient renegotiations. Without renegotiations, the banks gets 0 at date 2, and the household gets utility from c_1 only, which is equivalent to consumption at date 2 equal to

$$\delta c_1^{\max} = \delta \left(y_1 + \left(\bar{\rho}_b + \bar{e} \right) / r \right)$$

With efficient bargaining the household would save since $r > \delta$ and the income available at date 2 would be $ry_1 + \bar{\rho}_b + \bar{e}$. The net surplus at time 1 from efficient renegotiation is

$$\begin{aligned} \Sigma_m &= \frac{ry_1 + \bar{\rho}_b + \bar{e} - \delta c_1^{\max}}{r} - \kappa_m \\ \Sigma_m &= \left(1 - \frac{\delta}{r}\right)\left(y_1 + \frac{\bar{\rho}_b + \bar{e}}{r}\right) - \kappa_m \end{aligned}$$

If $\Sigma_m > 0$ renegotiations take place. Let θ be the share of surplus that banks obtain. This means that the bank now receives at time 2

$$\tilde{\rho}_m = \theta r \Sigma_m$$

while households consume

$$\tilde{c}_2 = \delta c_1^{\max} + (1 - \theta) r \Sigma_m$$

Note that using (11), we also obtain

$$\Sigma_m \equiv \left(\frac{r - \delta}{r}\right)\left(y_1 + \frac{z + (q - r)\bar{x} + \bar{\rho}_m}{r}\right) - \kappa_m$$

It seems plausible that the costs of renegotiations depend on the amount by which the loan is under water. I therefore assume that κ_m depends on $m - \hat{m}$

$$\kappa_m = \kappa_m (m - \hat{m}),$$

where the function $\kappa_m(.)$ is increasing, convex, and such that $\kappa_m(0) = 0$.

If we still define $\hat{m}$ as the point of efficient behavior without renegotiations, we can now define a new cutoff $\tilde{m}$ by $\Sigma_m(\tilde{m}) = 0$:

$$\kappa_m (\tilde{m} - \hat{m}) = \left(\frac{r - \delta}{r}\right)\left(y_1 + \frac{z + (q - r)\bar{x} + \bar{\rho}_m}{r}\right)$$

We can write

$$\begin{aligned} r\Sigma_m(m) &= \hat{m} - r\kappa_m(m - \hat{m}) \\ \bar{\rho}_m &= \int_0^{\hat{m}} m dF(m) + \theta \int_{\hat{m}}^{\tilde{m}} r\Sigma_m(m) dF(m) \end{aligned}$$

The unknowns are $r, \hat{m}, \bar{x}$. Under loans renegotiations, we therefore have the following equilibrium conditions:

$$\begin{aligned}
\hat{m} &= (r-\delta)\left(y_1 + \frac{z + (q-r)\bar{x} + \bar{\rho}_m}{r}\right) \\
\bar{x} &= y_1 - (1 - F(\tilde{m}))\frac{\hat{m}}{r-\delta} - \int_{\hat{m}}^{\tilde{m}} \kappa_m (m - \hat{m})\, dF(m) \\
\bar{x} &= XG(z + \bar{\rho}_m + (q-r)X)
\end{aligned}$$

and the definitions of $\tilde{m}$ and $\bar{\rho}_m$ as functions of $r, \hat{m}, \bar{x}$ are

$$\begin{aligned}
r\kappa_m(\tilde{m} - \hat{m}) &= \hat{m} \\
\bar{\rho}_m &= \int_0^{\hat{m}} m dF(m) + \theta \int_{\hat{m}}^{\tilde{m}} (\hat{m} - r\kappa_m(m - \hat{m}))\, dF(m)
\end{aligned}$$

Proposition 2 *If the government can influence renegotiations of household mortgages, it is optimal to favor the banks.*

Proof. The bargaining power of the banks only enters through the repayments flows $\bar{\rho}_m$. A higher θ increases the locus of the $\bar{\rho}_m$ function. But from the investment equation (15), we know that a higher $\bar{\rho}_m$ increases aggregate investment. Since $\bar{x}$ is too low, it is optimal to favor the banks. ■

The reason for this result is that households own the banks. So in the aggregate, what they pay as debtors they receive as shareholders. But increasing $\bar{\rho}_m$ decreases bank debt overhang, so it is beneficial.

3.2 Renegotiating bank debt

The investment rule is to invest when $b < \hat{b} = z + (q-r)X + \bar{\rho}_m$. When this constraint binds, banks shareholders and bondholders would like to renegotiate the face values of senior liabilities b. The NPV of foregone investment is $(q/r - 1)X$ so renegotiation takes place if and only if $\Sigma_b > 0$ where

$$\Sigma_b \equiv (q/r - 1)X - \kappa_b$$

Without renegotiations, the shareholders get 0 and the bond holders get $z + \bar{\rho}_m$. Let θ be the bargaining power of the shareholders. After renegotiations, the shareholders get $\theta\Sigma_b$

while the debt holders get $\bar{\rho}_m + z + (1-\theta)\,\Sigma_b$. For the banks that do not renegotiate, we have still have $\rho_b + e = z + (q-r)\,x + \bar{\rho}_m$ and

$$\hat{b} = z + \bar{\rho}_m + (q-r)\,X$$

For the banks that renegotiate, we have

$$\rho_b + e = z + (q-r)\,X + \bar{\rho}_m - r\kappa_b\left(b - \hat{b}\right)$$

The marginal bank is such that $\Sigma_b = 0$, which defines the cutoff $\tilde{b}$

$$\kappa_b\left(\tilde{b} - \hat{b}\right) \equiv (q/r - 1)\,X$$

In the aggregate, the flow payments equation (11) becomes

$$\bar{\rho}_b + \bar{e} = z + (q-r)\,\bar{x} + \bar{\rho}_m - r\int_{\hat{b}}^{\tilde{b}} \kappa_b\left(b - \hat{b}\right) dG\,(b) \tag{19}$$

Equation (12) is unchanged: $\bar{\rho}_m\,(\hat{m}) = \int_0^{\hat{m}} m dF\,(m)$. Equation (13) is now

$$\hat{m} = (r-\delta)\left(y_1 + \frac{\bar{\rho}_b + \bar{e}}{r}\right)$$

with the new flows payments from equation (19). The aggregate savings curve (14) becomes

$$S = y_1 - (1 - F\,(\hat{m}))\,\frac{\hat{m}}{r-\delta} - \int_{\hat{b}}^{\tilde{b}} \kappa_b\left(b - \hat{b}\right) dG\,(b)$$

The aggregate investment curve (15) now becomes

$$\bar{x} = XG\left(\tilde{b}\right)$$

We can therefore state the following proposition:

Proposition 3 *The government is indifferent to the sharing of the surplus in the renegotiations of bank debt.*

This is in sharp contrast with the case of loans. The reason is that households own the banks. Therefore they do not care (in the aggregate) whether the bondholders gain or lose at the expense of shareholders.

4 Government interventions

If the deadweight losses from debt overhang and bankruptcy is high, there might be room for an intervention by the government. The government can alleviate the debt overhang problem by providing capital to the banks, or by helping insolvent households.

4.1 Financial bailouts

The government has several ways to bail out the financial system. In an equity injection, the government provides cash to the banks, and asks for equity in exchange. In a debt guarantee program, the government offers to guarantee the new debt issued by banks. The banks can then borrow at the risk free rate. In an asset buyback program, the government offers to buy back the toxic assets from the banks above market price. Philippon and Schnabl (2009) show that, absent private information on the side of banks, all these programs are equivalent. For simplicity, I focus here on a pure cash transfer and I abstract from deadweight losses from taxation.

The government taxes the endowment at time 1 and raises an amount τ. It can then transfer this amount to the banks. After the transfer, each bank must borrow only $X - \tau$ from investors. These investors require expected returns $r(X - \tau)$. If the banks is always solvent with the new cash τ, i.e. if $z + \bar{\rho}_m + \tau > b$, then the bank invests. The marginal bank is such that it is insolvent if it does not invest, and solvent if it does, in which case it gets $e = z + \bar{\rho}_m + qX - r(X - \tau) - b$. The investment condition becomes

$$b < \hat{b} = z + \bar{\rho}_m + (q - r)X + r\tau \tag{20}$$

Banks that do not invest can lend the cash τ, and for these banks we have $\rho_b + e = z + \bar{\rho}_m + r\tau$. If we aggregate across all banks, the flow payments equation (11) becomes

$$\bar{\rho}_b + \bar{e} = z + \bar{\rho}_m + r\tau + (q - r)\bar{x} \tag{21}$$

Equation (12) is unchanged: $\bar{\rho}_m(\hat{m}) = \int_0^{\hat{m}} m dF(m)$. Equation (6) becomes

$$\hat{m} = (r - \delta)\left(y_1 - \tau + \frac{\bar{\rho}_b + \bar{e}}{r}\right) \tag{22}$$

Using the new flow equation (21), we see that equation (13) does not change

$$\hat{m} = (r - \delta)\left(y_1 - \tau + \frac{z + \bar{\rho}_m + r\tau + (q - r)\bar{x}}{r}\right) = (r - \delta)\left(y_1 + \frac{z + \bar{\rho}_m + (q - r)\bar{x}}{r}\right)$$

with the new flows payments from equation (19). The aggregate savings curve (14) is also unchanged because household savings decrease by τ to $y_1 - \tau - (1 - F(\hat{m})) \frac{\hat{m}}{r-\delta}$ while bank savings increase by τ, and the demand for funds is now only $\bar{x} - \tau$. Hence (14) is unchanged. Finally, the aggregate investment curve (15) now becomes

$$\bar{x} = XG\left(z + \bar{\rho}_m + (q - r) X + r\tau\right)$$

So equations (12), (13), and (14) are unchanged, while the schedule of aggregate investment curve (15) moves up. Hence, investment must increase.

In the open economy case, we get

$$\left(\frac{r}{r-\delta} - (1 + \hat{\gamma})\, \hat{m} f(\hat{m})\right) d\hat{m} = \hat{\gamma} r d\tau$$

with $\hat{\gamma}$ defined in (17). This equation shows that the fraction of insolvent households decrease with the financial bailout. Similarly, aggregate investment increases by

$$d\bar{x} = Xg\left(\hat{b}\right)\left(\hat{m} f(\hat{m})\, d\hat{m} + r d\tau\right)$$

So investment goes up for two reasons: the direct impact on banks' liquidity, and the indirect impact on the performance of their mortgages.

Proposition 4 *Financial bailouts increase welfare by increasing investment and increasing the fraction of solvent households.*

Of course, in reality, these efficiency gains must be compared the deadweight losses from taxation. But we will see in Section 5 how financial globalization changes this outcome even without deadweight losses from taxes.

4.2 Households bailouts

Consider the simple model with two groups of households. There are $1 - \pi$ agents with no debt, $m = 0$, and π agents with debt $m = M$. We consider an equilibrium where a share η of the indebted agents default. In other words, $F(\hat{m}) = 1 - \pi + (1 - \eta)\pi = 1 - \eta\pi$, and

$$\bar{\rho}_m = (1 - \eta)\, \pi M.$$

The government levies a per-capita tax τ on the households with no debt and transfers τ' it to the households with debt M. The budget constraint requires

$$(1-\pi)\,\tau = \pi\tau'$$

The default threshold is now

$$\hat{m} = (r-\delta)\left(y_1 + \tau' + \frac{z + (q-r)\,\bar{x} + \bar{\rho}_m}{r}\right) \tag{23}$$

The savings curve is

$$\bar{x} = (1-\eta\pi)\,y_1 - \eta\pi\left(\frac{z + (q-r)\,\bar{x} + \bar{\rho}_m}{r} + \tau'\right)$$

Finally the investment curve is still

$$\bar{x} = XG\,(z + \bar{\rho}_m + (q-r)\,X)\,.$$

Proposition 5 *In a closed economy, households' bailouts can backfire.*

Proof. Consider a candidate equilibrium with $r = q$ and no investment. What is the condition for a no investment outcome with $\eta = 1$? The critical value is

$$\hat{m}_q = (q-\delta)\left(y_1 + \tau' + \frac{z}{q}\right)$$

If $\hat{m}_q < M$, then all indebted households indeed default. Without investment the savings condition is

$$(1-\pi)\,y_1 \leq \pi\left(\frac{z}{q} + \tau'\right)$$

The condition for the bad outcome is therefore

$$\frac{y_1}{\pi} \leq y_1 + \tau' + \frac{z}{q} \leq \frac{M}{q-\delta}$$

Whether τ' helps or hurt depends on which side is tighter. If M is large so that the second condition is satisfied (this means households with high debt are underwater even with the transfer), then the bad outcome might be avoided by the fact that $y_1/\pi > y_1 + z/q$. But with the transfer τ', the savings curve goes down because the government transfers money to agents who consume too much. If the transfer is such that $y_1/\pi \leq y_1 + \tau' + z/q$ then the bad outcome becomes an equilibrium, while it was not an equilibrium before. ■

In an open economy, however, this type of transfer is always good because the default threshold in (23) improves while the savings curve is irrelevant. Then an increase in τ' simply improve the performance of mortgage loans, and this has a positive indirect impact on investment.

5 International coordination

Now imagine a world of small open economies. Households own domestic and foreign stocks and bonds. Let α be the share of foreign assets in domestic households' portfolios. Similarly, let β be the share of foreign mortgages in domestic banks' portfolios. Without interventions, equation (12) is unchanged, $\bar{\rho}_m(\hat{m}) = \int_0^{\hat{m}} m dF(m)$, and the equilibrium conditions are:

$$\begin{aligned} \hat{m} &= (r-\delta)\left(y_1 + (1-\alpha)\frac{\bar{\rho}_b + \bar{e}}{r} + \alpha\frac{\bar{\rho}_b^* + \bar{e}^*}{r}\right) \\ \bar{\rho}_b + \bar{e} &= z + (1-\beta)\,\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)\,\bar{x} \\ \bar{x} &= XG\left(z + (1-\beta)\,\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)\,X\right) \end{aligned}$$

Consider now the financial bailout of Section 4. If we aggregate all the economies, it is clear that we go back to the previous analysis. In other words, global bailouts improve efficiency. But this does not mean that an individual country would find it in its own interest to bail out its financial system. This is the issue that we now analyze.

Consider a domestic bailout. The government taxes the endowment at time 1 and raises an amount τ. It can then transfer this amount to the domestic banks. With the transfer, the cutoff becomes:

$$\hat{b} = z + (1-\beta)\,\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)\,X + r\tau$$

In the aggregate, the flow payments equation (11) becomes

$$\bar{\rho}_b + \bar{e} = z + (1-\beta)\,\bar{\rho}_m + \beta\bar{\rho}_m^* + r\tau + (q-r)\,\bar{x}$$

Equation (6) becomes

$$\hat{m} = (r-\delta)\left(y_1 - \tau + (1-\alpha)\frac{\bar{\rho}_b + \bar{e}}{r} + \alpha\frac{\bar{\rho}_b^* + \bar{e}^*}{r}\right)$$

If the country is small, it does not take into account the impact of its decision on the foreign payouts $\bar{\rho}_b^* + \bar{e}^*$. Putting the pieces together, we get

$$\begin{aligned} \frac{r}{r-\delta}\hat{m} &= ry_1 - \alpha r\tau + (1-\alpha)\left(z + (1-\beta)\,\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)\,\bar{x}\right) + \alpha\left(\bar{\rho}_b^* + \bar{e}^*\right) \\ \bar{x} &= XG\left(z + (1-\beta)\,\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)\,X + r\tau\right) \end{aligned}$$

If we differentiate the system, we get

$$\begin{aligned} \left(r/\left(r-\delta\right) - (1-\alpha)\,(1-\beta)\,\hat{m}f\left(\hat{m}\right)\right)d\hat{m} &= (1-\alpha)\,(q-r)\,d\bar{x} - \alpha r d\tau \\ d\bar{x} &= Xg\left(\hat{b}\right)\left((1-\beta)\,\hat{m}f\left(\hat{m}\right)d\hat{m} + rd\tau\right) \end{aligned}$$

And therefore

$$\left(\frac{r}{r-\delta} - (1-\alpha)\,(1-\beta)\,(1+\hat{\gamma})\,\hat{m}f\left(\hat{m}\right)\right)d\hat{m} = \left((1-\alpha)\,\hat{\gamma} - \alpha\right)rd\tau$$

with $\hat{\gamma}$ defined in (17).

Proposition 6 *Domestic financial bailouts are less efficient when banks operate internationally, and when households diversify their financial portfolios. When* $(1-\alpha)\,\hat{\gamma} < \alpha$*, financial bailouts lead to more mortgage defaults.*

Corollary 1 *Financial globalization creates the need for coordination in financial bailouts.*

It is clear that countries have an incentive to free-ride on foreign recapitalization programs.

6 Conclusion

I have presented a simple model where debt overhang occurs simultaneously in two markets. Households' mortgages can be under-water, and this can lead them to take inefficient actions to avoid transferring wealth to their creditors. Banks's bonds can be under-water, and this can lead them to pass on positive NPV projects. The two problems interact in equilibrium in a way that can amplify shocks or even lead to multiple equilibria.

When I study government interventions, I find that governments might want to favor the banks during the renegotiations of mortgages, and that financial bailouts can improve economic efficiency.

In open economies, I find that bailouts might require coordination among countries, since it is possible that no country would chose to bail out its financial system, even though coordinated bailouts could improve or restore efficiency.

References

Aghion, P., P. Bolton, and S. Fries (1999): "Optimal Design of Bank Bailouts: The Case of Transition Economies," *Journal of Institutional and Theoretical Economics*, 155, 51–70.

Aguiar, M., M. Amador, and G. Gopinath (forthcoming): "Investment Cycles and Sovereign Debt Overhang," *Review of Economic Studies*.

Asquith, P., R. Gertner, and D. Scharfstein (1994): "Anatomy of Financial Distress: An Examination of Junk-Bond Issuers," *The Quarterly Journal of Economics*, 109, 625–658.

Bhattacharya, S., and A. Faure-Grimaud (2001): "The Debt Hangover: Renegotiation with Non-contractible Investment," *Economics Letters*, 70, 413–419.

Bulow, J., and J. Shoven (1978): "The bankruptcy decision," *Bell Journal of Economics*, 9, 437–456.

Diamond, D. (2001): "Should Japanese Banks Be Recapitalized?," *Monetary and Economic Studies*, pp. 1–20.

Diamond, D., and R. Rajan (2005): "Liquidity Shortages and Banking Crises," *Journal of Finance*, 60(2), 615–647.

Diamond, D. W., and R. G. Rajan (2009): "Fear of fire sales and the credit freeze," NBER WP 14925.

Gertner, R., and D. Scharfstein (1991): "A Theory of Workouts and the Effects of Reorganization Law," *the journal of finance*, 46, 1189–1222.

Gilson, S., K. John, and L. Lang (1990): "Troubled debt restructuring: an empirical study of private reorganization of firms in default," *Journal of Financial Economics*, 27, 315–353.

Gorton, G., and L. Huang (2004): "Liquidity, Efficiency and Bank Bailouts," *American Economic Review*, 94, 455–483, NBER WP9158.

Hart, O., and J. Moore (1995): "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management," *American Economic Review*, 85(3), 567–585.

Hennessy, C. A. (2004): "Tobin's Q, Debt overhang, and Investment," *the journal of finance*, 59, 1717–1742.

Jensen, M., and W. Meckling (1976): "Theory of the firm: managerial behavior, agency costs and ownership structure," *Journal of Financial Economics*, 3, 305–360.

Kocherlakota, N. (2009): "Assessing Resolutions of the Banking Crisis," Working Paper.

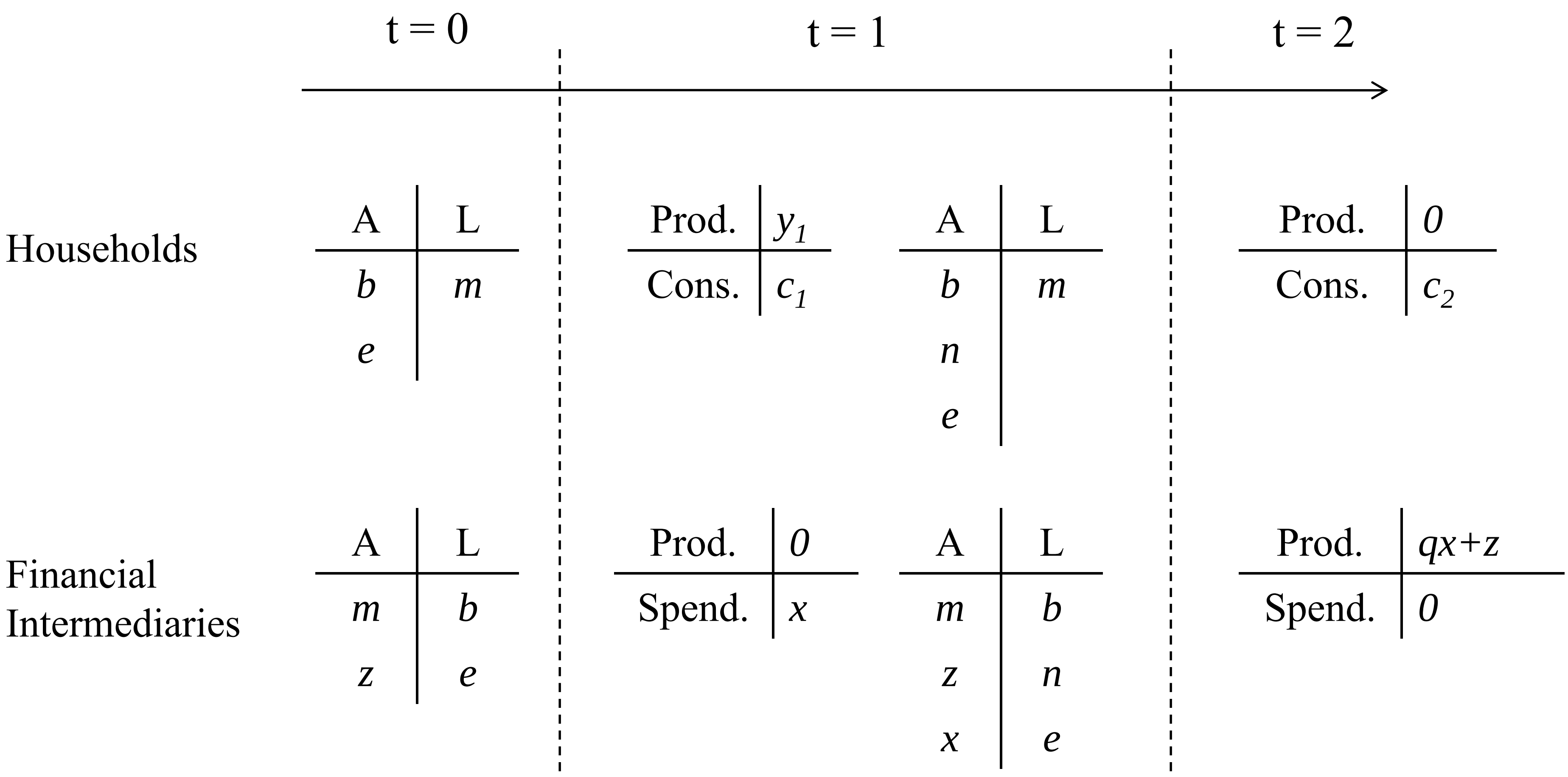
Krugman, P. (1988): "Financing vs. forgiving a debt overhang," NBER WP 2486.

Lamont, O. (1995): "Corporate-Debt Overhang and Macroeconomic Expectations," *American Economic Review*, 85(5), 1106–1117.

Myers, S. C. (1977): "Determinants of Corporate Borrowing," *Journal of Financial Economics*, 5, 147–175.

Philippon, T., and P. Schnabl (2009): "Efficient Recapitalization," NBER WP 14929.

Fig 1: Timing, Technology and Balance Sheets



Notes: b=bonds, e=equity, m=loans, n=new bonds, x=physical investment. All assets (A) and liabilities (L) measured at book values at the end of the period. The corporate non financial sector is integrated with the financial intermediaries.

Figure 2: First Best Equilibrium