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Valuation Effects and the Dynamics of Net External Assets

Michael B. Devereux and Alan Sutherland

Presented by Mike Devereux and Alan Sutherland

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Michael B Devereux[†]and Alan Sutherland[‡] April 2008 (Preliminary and Incomplete)

Abstract

Recent data show that the traditional current account can be a highly inaccurate measure of the change in the net foreign assets (NFA). A number of 'valuation effects' have been identified which contribute to changes in NFA but which are not properly captured by the conventionally measured current account. This paper makes use of recent developments in the analysis of portfolio allocation in general equilibrium to investigate valuation effects in a simple model. We identify a number of valuation effects which correspond to those emphasised by other authors. Broadly speaking, the valuation effects in the model correspond to those measured in the data, and have the effect of enhancing cross country risk sharing. But there is a key distinction between 'unanticipated' and 'anticipated' valuation effects. We find that unanticipated valuation effects arise only at higher orders of approximation and are small for a reasonable parameterisation of the model.

Keywords: Valuation effects.

JEL: F41, F32, F37

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[†]CEPR and Department of Economics, University of British Columbia, 997-1873 East Mall, Vancouver, B.C. Canada V6T 1Z1. Email: devm@interchange.ubc.ca Web: www.econ.ubc.ca/devereux/mdevereux.htm

[‡]CEPR and School of Economics and Finance, University of St Andrews, St Andrews, Fife, KY16 9AL, UK. Email: ajs10@st-and.ac.uk Web: www.st-and.ac.uk/~ajs10/home.html

1 Introduction

Open economy macroeconomic models typically pay close attention to the current account as a measure of the evolution of an economy's net external assets. The growth of current account imbalances, and in particular the US current account deficit, has brought this linkage to the forefront of economic policy discussion. Since countries must satisfy intertemporal budget constraints, large and growing current account deficits will reduce net external assets and require that future trade surpluses be generated.

This traditional view of the current account has been put into question more recently, however. A series of detailed and careful data construction studies that have taken place over the past decade suggest that traditional measures of the current account may give a highly inaccurate measure of the movement of an economy's net external wealth (Lane and Milesi Ferretti 2001, 2006). These studies show that corrected measures of net external assets must incorporate changes in asset prices, returns, and foreign exchange rates that impact on the value of an economy's net external wealth through separate 'valuation effects' on gross assets and liabilities. Moreover, given the explosive growth in cross-border capital flows since the mid 1990's, leading to huge increases in the scale of gross external assets and liabilities, these previously unmeasured valuation effects on net external assets have risen dramatically relative to the traditional measures of the current account. A number of recent studies have emphasized the empirical relevance of these valuation effects (Tille 2003, Higgens et al. 2005, Lane and Milesi Ferretti, 2005, Gourinchas, 2007).

Despite the by now well recognized importance of correctly measuring the impact of valuation changes, until recently, there has been little impact of these new empirical findings on the traditional modeling of the current account and net external asset movements in open economy macro models. One of the key reasons for this is that it has proven difficult to incorporate classic principles of portfolio choice into the conventional dynamic general equilibrium open economy model. But recent developments in the literature has developed techniques for making progress in combining portfolio choice with general equilibrium open macro models¹. This paper makes use of these new techniques to provide a qualitative and quantitative analysis of the ability of theoretical models to account for valuation effects in the evolution of net external assets, and to explore the interaction

¹See, for instance Couerdacier (2005), Evans and Hnatkovska, (2005), Kollman (2006), Engel and Matsumoto (2006), Devereux and Sutherland (2006), (2007), Tille and Van Wincoop, (2007).

between valuation effects and traditional measures of the current account.

We begin by developing a highly simplified two-country endowment economy model in which each country faces two sources of risk - one from capital income, which is assumed to be internationally diversifiable through equity sales, and the other from labor income, which cannot be directly diversified. Although this model is extremely simple, it allows us to illustrate in an analytical example the main elements of the dichotomy between the traditionally measured current account and the valuation channel in determining the movement of net external assets. Defining the valuation channel of as the gap between the movement of net external assets and the standard measure of the current account, we show that the valuation effect may be broken into anticipated and unanticipated components. The anticipated component of the valuation effect captures expected excess returns on a country's portfolio due to differences in the covariance risk associated with each country's traded equity. Such country risk premia allow, in principle, for permanent imbalances in national current accounts. In addition, there may be time-varying anticipated excess return that are associated with current account adjustment. The unanticipated component of the valuation effect captures the way in which national portfolios are structured so as to hedge against consumption risk. A basic property of the unanticipated valuation component is that it should co-vary negatively with the traditional current account. The model also allows for a decomposition of unanticipated valuation effects into those coming from movements in rates of return on assets, and those coming from movements in the portfolio holdings.

Having defined these different components of valuation effects, we go on to provide a quantitative account of the importance of each component in the evolution of net assets. We show that the model indicates that anticipated valuation effects are extremely small, except for counterfactually high values of risk aversion and differences in country endowment volatilities. But unanticipated valuation components may represent a large fraction of the volatility of net external assets, even when the model is calibrated to realistic sizes of gross national portfolios. Moreover, unanticipated valuation effects in the model behave in quite a similar fashion to those imputed from the net foreign assets (NFA) data - in particular, they dominate the movements in GDP, they tend to be negatively correlated with the current account, and they are approximately i.i.d.

One aspect of the recent portfolio discussion emphasizes the difference between the effects of shocks to returns for a given portfolio, and the effects of adjustment in the portfolio itself, sometimes called 'portfolio rebalancing' (see Hau and Rey, 2007). In our model, both effects form part of the dynamics of NFA. Unanticipated valuation channels involve both shocks to returns, and movements in portfolio holdings. But in our quantitative decomposition of the volatility of net external assets, the latter channel plays at best a small role. By far the biggest driver of the volatility of net external assets is the unanticipated movement in returns, holding the portfolio constant. Portfolio adjustment and movements in expected returns can also create anticipated valuation effects. But our analysis suggests that these effects arise only at higher orders of approximation and are quantitively very small.

The paper's contribution is also pedagogical. We document how valuation effects enter in the evolution of net foreign assets, and at what order of approximation each effect is important. To this extent, the paper can be seen as a theoretical underpinning for some traditional 'portfolio balance' modeling, which combined goods and asset market modeling in one framework, but based on assumed rules of thumb behaviour with respect to portfolio composition. At the same time, our analysis naturally places a limit on the potential importance of each component of valuation effects. In one sense, our results suggest that in order to support the importance of some key elements of portfolio balance models, it would be necessary to develop models of risk that differ fundamentally from those of the standard intertemporal stochastic model that underlies the traditional open economy macro framework, used in this paper.

There is a large and growing literature on valuation effects and current account dynamics in general equilibrium models. Notable recent papers are Cavallo and Tille (2006), Ghironi, Lee and Rebucci (2006), and Pavlova and Rigibon (2007). Cavallo and Tille (2006) and Ghironi, Lee and Rebucci (2006) provide a careful quantitative accounting of the impact of valuation effects in models in which the portfolio structure is calibrated to match the data. Pavlova and Rigibon (2007) present a rich continuous time dynamic model in which the portfolio rules can be obtained in closed form, but follow a different line of inquiry from that considered here.

The paper is structured as follows. The next section discusses some properties of the data on the current account and net external assets. We then set out a simple example model of the current account in the face of capital and labour income risk. Following this, we discuss the properties of the solution method for portfolio choice. We then explore some analytical results on valuation effects. After this, we present quantitative results on

the importance of anticipated and unanticipated valuation effects.

2 The Current Account and Valuation Effects in the Evolution of Net External Assets

In this section, we provide a brief description of the evolution of net external assets and their decomposition into factors driven by the conventional measure of the current account, and those driven by valuation effects². We begin by reviewing the importance of valuation effects in the dynamics of net external assets for a subset of OECD countries. Start with a simple decomposition of net external assets into the conventional current account as measured in balance of payments data, and valuation terms. Thus, for country *i* at time *t*, we have:

$$NFA_{it} - NFA_{it-1} = CA_{it} + VAL_{it}$$

We can obtain these objects from the data by using the IMF/Lane-Milesi-Ferretti External Wealth of Nations (EWN) data set on international investment positions, and separately, from reported balance of payments data on the current account. As discussed in Lane and Milesi-Ferretti (2006), Tille (2003), and Gourinchas (2007), movements in VAL_{it} are driven among other effects, by asset price and exchange rate changes which cause revisions in the value of gross external assets and liabilities, but are not incorporated in the income account as returns paid or received on gross external liabilities or assets.

We can derive VAL_{it} indirectly, since NFA_{it} is reported in the EWN data-base (and updated using the IMF IIP), and CA_{it} is observable from Balance of Payments data. To make the VAL_{it} variable comparable with our model, we scale by GDP. Thus, we define

$$val_{it} = \frac{(NFA_{it} - NFA_{it-1})}{GDP_{it}} - \frac{CA_{it}}{GDP_{it}} \equiv \Delta nx_{it} - ca_{it},$$

Since NFA_{it} and CA_{it} are reported in US dollars, we use US dollar GDP_{it} from the OECD database. The variable val_{it} is constructed for a sample of 23 OECD countries for the period 1980-2006. Table 1 describes the characteristics of val_{it} . The first column of the Table reports the fraction of the total variation in Δnx_{it} accounted for by valuation

²Similar discussion is provided in Kollman (2006), Gourinchas (2007), Lane and Milesi Ferretti (2006) among others.

effects; $var_i = var(val_i)/var(\Delta nx_i)$ over the sample. For most countries, this is well above 50 percent. The average value is 0.90, and the US is highest at 1.39. Thus, in terms of accounting for the variation in net external assets, we see that valuation effects represent a very large component. In particular, for most countries, this completely dominates the share attributable to the trade balance or the current account in the variation of net external assets.

The valuation term is of course not independent of the current account itself. The second column of Table 1 reports the correlation coefficient between val_{it} and ca_{it} for each country. As we discuss below, a property of an optimal risk sharing portfolio across a range of general equilibrium models is that a country should receive an excess return on its portfolio that is negatively correlated with the trade balance. Depending on the specification of the asset market structure, this may translate into a negative correlation between ca_i and val_i . The evidence on this in Table 1 is mixed. For 14 of the 23 countries in the sample, $corr(ca_i, val_i)$ is negative, with the highest negative correlation being for the US.

Kollman (2006) has previously noted that Δnx_{it} is approximately *i.i.d.* for most countries, while the current account displays substantial persistence. Here, when we impute the valuation effect as the difference between the two, we find that val_{it} inherits the persistence properties of the Δnx_{it} series. The measured val_{it} has no serial correlation for almost all countries. Table 1 reports the results from the AR(1) regression $val_{it} = c_1 + c_2 val_{it-1}$ for each country. The AR(1) coefficient is insignificant for almost all countries. In the model below, we show that val_{it} should be very close to an *i.i.d.* process.

Since the model's predictions for risk-sharing are mainly concerned with the trade account, we also report the following decomposition:

$$vai_{it} = \Delta nx_{it} - ta_{it}$$

where ta_{it} is the trade account to GDP ratio. Thus vai_{it} is the sum of the valuation term to GDP ratio, plus the income account to GDP ratio. In practice, vai_{it} and ta_{it} behave very similarly. Figures 1 and 2 illustrate the time series for val_t and vai_t respectively, for a number of countries. The two measures of valuation are highly correlated for these countries, since the dynamics of the trade balance and the current account are almost the same, in most countries. Table 2 reports the identical results to Table 1 for this decomposition. As before, the variance of the valuation term is very high relative to the variance of net external assets - the average value is again about .9. Thus, a large component of nx is driven by portfolio effects, rather than trade balance effects. In addition, we find that the correlation between *vai* and *ta* is negative now for most countries. Finally, constructed in this way, *vai* is transitory - the AR(1) coefficient is again insignificant for most countries.

Most of these stylized facts are 'first-order' in nature. That is, they just involve the features of the second moments of the data. Interpreted this way, as we discuss below, val and vai can thus be thought of as the result of an optimal risk-sharing portfolio. Gourinchas (2007) refers to these as 'unpredictable' valuation terms, because they can interpreted as implicit insurance against business cycle shocks. But other recent discussion of valuation effects in international financial data stress the presence of 'predictable' valuation effects at the national level, meaning that there are predictable excess returns on some component of a country's gross assets relative to the same component in its gross liabilities. As a rough measure of this, Tables 1 and 2 compute the average valuation effect over the sample for each country. In principle, if valuation changes were just attributable to first order risk-sharing, then this should be a very small number. In fact, it is negative, and a relatively large share of GDP for many countries. For the US, it is positive and 1.4 percent of GDP. Gourinchas and Rey (2005) estimate a substantial excess return on US assets relative to liabilities, for all components of its international portfolio. For portfolio equity and debt securities, Curcuru, Dvorak, and Warnock (2008) argue that the actual excess return to the US is quite small. But for FDI, Higgins, Klitgaard and Tille (2006) find a 2-3 percent persistently higher return on US assets abroad than foreign assets held in the US.

Lane and Milesi Ferretti (2007) provide an overview of some of the measurement problems inherent in these estimates. Lane and Milesi Ferretti (2005) take a larger sample of countries, and find that average rates of return on assets and liabilities have had significant differences over substantial periods of time for many countries.

Gourinchas and Rey (2007) highlight a somewhat different predictable valuation effect. They find that, conditional on an increase in the US trade balance deficit, the US experiences a predictable persistent increase in the excess return on its international investment portfolio, thereby reducing the required increase in the future trade balance surplus required to achieve overall intertemporal budget balance.

While unpredictable valuation gains or losses are relatively easy to model, in terms

of an optimal insurance arrangement, it has proven much more difficult to integrate the findings of predictable excess returns into general equilibrium modeling. This is because these effects are of a 'higher order' nature. Some writers have employed old style 'Portfolio Balance' models in order to conduct theoretical analysis of predictable and time-varying excess returns. But these models remain unsatisfactory because of the *ad hoc* nature of the modeling of risk. In our analysis below, we examine higher order approximations of portfolio choice within a standard general equilibrium framework, and explore the degree to which they give rise to predictable valuation effects on the evolution of net external assets.

3 A Simple Example Model

We first illustrate how the decomposition of the measured current account and valuation effects interact in a simple two-country endowment model with only two traded assets, and a single world consumption good.

Agents in the home country have a utility function of the form

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(C_{\tau}) \tag{1}$$

where C is consumption and $u(C_{\tau}) = (C_{\tau}^{1-\rho})/(1-\rho)$.

The budget constraint for home agents is given by

$$\alpha_{1,t} + \alpha_{2,t} = \alpha_{1,t-1}r_{1,t} + \alpha_{2,t-1}r_{2,t} + Y_t - C_t \tag{2}$$

where Y is the endowment received by home agents, $\alpha_{1,t-1}$ and $\alpha_{2,t-1}$ are the real holdings of the two assets (purchased at the end of period t-1 for holding into period t) and $r_{1,t}$ and $r_{2,t}$ are gross real returns. The stochastic process determining endowments and the nature of the assets and the properties of their returns are specified below.

We call $W_t = \alpha_{1,t} + \alpha_{2,t}$ the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents). Defining $r_{x,t} = r_{1,t} - r_{2,t}$ as the "excess return" on asset 1, the budget constraint can then be re-written as

$$W_t = \alpha_{1,t-1}r_{x,t} + r_{2,t}W_{t-1} + Y_t - C_t.$$
(3)

At the end of each period agents select the portfolio of assets to hold into the following period. The first-order condition for the choice of $\alpha_{1,t}$ can be written in the following form

$$E_t \left[u'(C_{t+1})r_{1,t+1} \right] = E_t \left[u'(C_{t+1})r_{2,t+1} \right]$$
(4)

Foreign agents face a similar portfolio allocation problem with an analogous budget constraint.

Assets are assumed to be in zero net supply, so market clearing in asset markets implies $\alpha_{1,t-1} + \alpha_{1,t-1}^* = 0$ and $\alpha_{2,t-1} + \alpha_{2,t-1}^* = 0$. To simplify notation in this example, we can drop the subscript from $\alpha_{1,t}$ and simply refer to α_t . Note that $\alpha_{1,t} = -\alpha_{1,t-1}^* = \alpha_t$, $\alpha_{2,t} = W_t - \alpha_t$ and $\alpha_{2,t}^* = W_t^* + \alpha_t$, where W^* is foreign net external assets, and $W_t + W_t^* = 0$.

Assume that endowments are the sum of 'capital income' components, $Y_{K,t}$ and 'labour income' components $Y_{L,t}$, so that

$$Y_t = Y_{K,t} + Y_{L,t}.$$
 (5)

We assume that the two countries may trade assets represent claims on capital income, but labour income is non-diversifiable. The endowments are determined by the following simple stochastic processes

$$\log Y_{K,t} = \mu \log Y_{K,t-1} + \varepsilon_{K,t}, \qquad \log Y_{L,t} = \mu \log Y_{L,t-1} + \varepsilon_{L,t}$$

where $\varepsilon_{K,t}$, $\varepsilon_{L,t}$, are zero-mean i.i.d. shocks which are symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_K] = \sigma_K^2$, $Var[\varepsilon_L] = \sigma_L^2$. We assume that $Cov[\varepsilon_K, \varepsilon_L] = \sigma_{KL}$. Foreign income processes are defined analogously, and we assume zero covariance between home and foreign income shocks.

The two traded assets are equity claims on the home and foreign capital income. The real payoff on a unit of home equity is $Y_{K,t}$ and the real price is $Z_{E,t-1}$. Thus the gross real rate of return on home equity is

$$r_{1,t} = \frac{Y_{K,t} + Z_{E,t}}{Z_{E,t-1}} \tag{6}$$

The real return on foreign equity is defined analogously, where $Z_{E,t-1}^*$ is the price of the foreign equity.

The first-order conditions for home consumption is

$$C_t^{-\rho} = \beta E_t \left[C_{t+1}^{-\rho} r_{2,t+1} \right]$$
(7)

Finally, equilibrium consumption plans must satisfy the resource constraint

$$C_t + C_t^* = Y_t + Y_t^*$$
 (8)

A competitive equilibrium in this example is defined by (3), (4) and its foreign counterpart, (6) and the analogous equation for r_{2t} , (7) and its foreign counterpart, and (8). These implicitly give the solutions for the equilibrium values of C, C^* , $r_1, r_2, Z_{E,t}, Z_{E,t}^*$, W_t and α_t .

4 Approximation of portfolio holdings and excess returns

Before we analyze the nature of portfolio valuation effects on the evolution of net external assets, we first discuss the nature of portfolio solutions within this model. In particular, we define some terms relating to the true and approximated solutions for gross portfolio holdings and equilibrium asset returns.

Consider an approximation of α up to order N

$$\alpha_t = \bar{\alpha} + \hat{\alpha}_t^{(1)} + \hat{\alpha}_t^{(2)} + \dots + \hat{\alpha}_t^{(N)} + O\left(\epsilon^{N+1}\right)$$
(9)

where $\bar{\alpha}$ is the zero-order component (i.e. α at the point of approximation) and $\hat{\alpha}^{(i)}$ is the order-*i* component of the deviation of α from $\bar{\alpha}$. In what follows we will confine attention to the first two terms in this approximation, $\bar{\alpha}$ and $\hat{\alpha}_t^{(1)}$. Notice that, by definition, $\bar{\alpha}$ is constant and therefore captures the average or steady-state element of portfolio holdings, while the (first-order) time varying element in portfolio holdings is captured by $\hat{\alpha}_t^{(1)}$.

Notice also that agents make their portfolio decisions at the end of each period and are free to re-arrange their portfolios each period. In a recursive equilibrium, therefore, the equilibrium asset allocation will be some function of the state of the system in each period - which is summarised by the state variables. We therefore postulate that the true portfolio (i.e. the equilibrium portfolio in the non-approximated model) is a function of state variables, $\alpha_t = \alpha(Z_t)$ where Z is the vector of state variables. We can therefore deduce that $\hat{\alpha}_t^{(1)}$ is a linear function of the first-order deviation of Z from \overline{Z} , i.e.

$$\hat{\alpha}_t^{(1)} = \gamma \hat{Z}_t^{(1)}$$

where γ is a vector of coefficients (to be determined).

When analysing a DSGE model up to first-order accuracy, the standard solution approach is to use the non-stochastic steady-state of the model as the approximation point,

(i.e. the zero-order component of each variable) and to use a first-order approximation of the model's equations to solve for the first-order component of each variable. Neither of these steps can be used in the above model. It is very simple to see why. In the non-stochastic equilibrium the portfolio optimality conditions imply

$$r_{1,t+1} = r_{2,t+1} \tag{10}$$

i.e. both assets pay the same rate of return. This implies that, for given W, all portfolio allocations pay the same return, so any value for α is consistent with equilibrium. Thus the non-stochastic steady state does not tie down a unique portfolio allocation. A similar problem arises in a first-order approximation of the model. First-order approximation of equations the portfolio optimality conditions imply

$$E_t[\hat{r}_{1,t+1}^{(1)}] = E_t[\hat{r}_{2,t+1}^{(1)}] + O\left(\epsilon^2\right)$$
(11)

i.e. both assets have the same expected rate of return. Again, any value of α is consistent with equilibrium.

So neither the non-stochastic steady state nor a first-order approximation of the model provide enough equations to tie down the zero or first-order components of α . The basic problem is easy to understand in economic terms. Assets in this model are only distinguishable in terms of their risk characteristics and neither the non-stochastic steady state, nor a first-order approximation, capture the different risk characteristics of assets. In the case of the non-stochastic steady state there is, by definition, no risk, while in a first-order approximation there is certainty equivalence.

This statement of the problem immediately suggests a solution. It is clear that the risk characteristics of assets only show up in the second-moments of model variables, and it is only by considering higher-order approximations of the model that the effects of second-moments can be captured. This fundamental insight has existed in the literature for many years. It was first formalised by Samuelson (1970), who established that, in order to derive the zero-order component of the portfolio, it is necessary to approximate the portfolio problem up to the second order. Our solution approach follows this principle. In Devereux and Sutherland (2006) we show that a second-order approximation of the portfolio optimality conditions provides a condition which makes it possible to tie down the zero-order component of α . Having established this starting point, it is relatively straightforward to extend the procedure to higher-order components on α . Samuelson

(1970) in fact states a general principle that, in order to derive the Nth-order component of the portfolio, it is necessary to approximate the portfolio problem up to order N + 2. By following this principle, in Devereux and Sutherland (2007) we show that the solution for the first-order component of α can be derived from third-order approximations of the portfolio optimality conditions.

Now consider equilibrium expected returns, $E_t[r_{1,t+1}]$ and $E_t[r_{2,t+1}]$ or, more specifically, the expected *excess* return, defined as $x_t = E_t[r_{1,t+1} - r_{2,t+1}]$. Consider an approximation of x_t up to order N

$$x_t = \bar{x} + \hat{x}_t^{(1)} + \hat{x}_t^{(2)} + \dots + \hat{x}_t^{(N)} + O\left(\epsilon^{N+1}\right)$$
(12)

What can our solution approach tell us about equilibrium excess returns at different orders of approximation? First, notice that equations (10) and (11) tell us immediately that $\bar{x} = \hat{x}_t^{(1)} = 0$. It follows therefore that expected excess returns only deviate from zero at orders 2 and higher. In Devereux and Sutherland (2006) we show that $\hat{x}_t^{(2)}$ can solved in conjunction with $\bar{\alpha}$. Furthermore, we show that $\hat{x}_t^{(2)}$ can be written as a function of one-period-ahead conditional second moments of first-order realised asset returns and consumption. Because these one-period-ahead conditional second moments are non-timevarying $\hat{x}_t^{(2)}$ will also be non-time varying. In fact $\hat{x}_t^{(2)}$ can naturally be thought of as the steady-state equilibrium expected excess return which corresponds to steady-state equilibrium asset holdings, $\bar{\alpha}$.

In a similar way, in Devereux and Sutherland (2007b we show that the third-order component of excess returns, $\hat{x}_t^{(3)}$, can be solved in conjunction with the first-order component of asset holdings, $\hat{\alpha}_t^{(1)}$. We show there that $\hat{x}_t^{(3)}$ can be written in terms of expected products of first and second-order realised asset returns and consumption. Furthermore, just as $\hat{\alpha}_t^{(1)}$ is time varying, it follows that $\hat{x}_t^{(3)}$ is also time varying and it is possible to show that $\hat{x}_t^{(3)}$ is a linear function of the first-order component of state variables, i.e.

$$\hat{x}_t^{(3)} = \delta \hat{Z}_t^{(1)}$$

where δ is a vector of coefficients which are functions of one-period-ahead conditional second moments (i.e. δ is order-2 and thus the right-hand side is order-3). $\hat{x}_t^{(3)}$ can naturally be thought of as the time varying element of excess returns that corresponds to the first-order time varying element of portfolio holdings.

5 Valuation Effects in the Example Model

Now let us focus on the definition of the current account and valuation effects within the simple model outlined above. Equation (3) can be rearranged to give a representation for the change in net external wealth as

$$W_t - W_{t-1} = \alpha_{1,t-1}r_{x,t} + (r_{2,t} - 1)W_{t-1} + Y_t - C_t$$
(13)

We wish to explore how the change in net foreign assets is broken down into the conventional measured current account, and valuation effects associated with movements in asset prices, asset returns, and portfolio composition. Because in general, there is no exact solution for α_{1t} , we need to take approximations in order to identify these valuation effects. Following Devereux and Sutherland, (2007), we may approximate (13) around an initial symmetric steady state with $\bar{W} = 0$, and $\bar{C} = \bar{Y}$ in the following manner:

$$\hat{W}_{t} - \hat{W}_{t-1} = \hat{Y}_{t} + \frac{1}{2}\hat{Y}_{t}^{2} - \hat{C}_{t} - \frac{1}{2}\hat{C}_{t}^{2} \\
+ \frac{1 - \beta}{\beta}\hat{W}_{t-1} + \frac{1}{\beta}\hat{W}_{t}\hat{r}_{2,t} \\
+ \tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t}^{2} - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t}^{2})) + \hat{\alpha}_{t-1}\hat{r}_{x,t} + O\left(\epsilon^{3}\right)$$
(14)

where $\hat{C}_t = \frac{C_t - \bar{C}}{\bar{C}}$, and similarly for \hat{Y}_t and \hat{r}_{1t} , and \hat{r}_{2t} , $\hat{r}_{x,t} = \hat{r}_{1t} - \hat{r}_{2t}$, and $\hat{W}_t = (W_t - \bar{W})/\bar{C}$. The portfolio expressions $\tilde{\alpha}$ and $\hat{\alpha}_t$ are defined as $\tilde{\alpha} = \bar{\alpha}/(\beta \bar{Y})$ and $\hat{\alpha}_t = \frac{1}{\beta \bar{Y}}(\alpha_t - \bar{\alpha}) = \frac{\alpha_t}{\beta \bar{Y}} - \tilde{\alpha}$. As discussed above, we define $\bar{\alpha}$ as the 'steady state' portfolio, while $\hat{\alpha}_t$ is defined as the first order deviation of α_t from the steady state portfolio. Because of the nature of the approximation, both measures are re-scaled by steady state output.

Expression (14) decomposes the change in net external wealth into conventional current account effects and portfolio valuation effects. The first line represents the trade balance, evaluated up to second order. The second and third lines represent the sum of the conventional income account and the valuation components of the net external assets. The value of $\tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t+1}^2 - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t+1}^2))$ gives the impact on net external wealth of an excess return on asset 1 relative to asset 2. The size and sign of this effect obviously depends on the portfolio position $\tilde{\alpha}$. When the home country takes a negative position in asset 1, $\tilde{\alpha} < 0$, and a positive excess return on asset 1 reduces its net external wealth. The term $\tilde{\alpha}\hat{r}_{x,t} = \tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t+1}^2 - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t+1}^2))$ can be further decomposed into an *anticipated* valuation effect, and an *unanticipated* valuation effect. Up to a first order approximation, expected excess returns are zero, so all ex-post movements in $r_{x,t}$ are unanticipated. But when evaluated to a second order, there may be anticipated excess returns, due to differences in the degree to which the two assets contribute to consumption variability. The final term represents the impact of portfolio re-adjustment. To the extent that home consumers shift their portfolio towards assets that have a higher ex-post return, their net external wealth will rise, for a given current account, and for given valuation effects on the pre-existing portfolio. Since we are evaluating only up to second order accuracy, the term $\hat{\alpha}_{t-1}\hat{r}_{x,t}$ is a mean-zero, i.i.d. process. This is because, given that $\hat{\alpha}_{t-1}$ is a first order term, $\hat{r}_{x,t}$ is accurate only up to first order, to make the whole expression $\hat{\alpha}_{t-1}\hat{r}_{x,t}$ second order. Hence, the impact of shifts in the portfolio can influence the evolution of net external assets only by generating unanticipated valuation effects. Nevertheless, they still contribute to fluctuations in net external assets.

We wish to provide an investigation of the importance of each of these valuation components of the dynamics of net external assets, and to compare this to the behaviour of the conventional definition of the current account. We first do this within the context of the simple analytical model described above, and then in terms of a more elaborate model developed in succeeding sections.

Following Devereux and Sutherland (2006), it is easy to compute the first order solutions for consumption, asset prices, and asset returns. Given this, we may then compute the steady state optimal portfolio as follows

$$\tilde{\alpha} = -\frac{1}{2(1-\beta)} \frac{\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\sigma_{KL} + \sigma_{KL}^*)}{\sigma_K^2 + \sigma_K^{2*}}$$
(15)

The optimal steady state portfolio takes a negative weight on home equity, and a positive weight on foreign equity, so long as $\phi(\sigma_K^2 + \sigma_K^{2*}) + (1 - \phi)(\sigma_{KL} + \sigma_{KL}^*) > 0$. A fully diversified portfolio would have each country holding half of the other's equity to GDP ratio, or $\tilde{\alpha}_F = -\frac{\phi}{2(1-\beta)}$. The degree of home bias in equity holdings will depend on the correlation between capital and labour income in each country. Thus, if σ_{KL} and σ_{KL}^* are less than zero, we have $|\tilde{\alpha}| < |\tilde{\alpha}_F|$, and there is home bias in equity holdings. Note that even with home bias in equity holdings, there can be significant gross positions. For instance, when the home country holds only 10 percent of foreign equity, $\tilde{\alpha} = -\frac{0.1\phi}{2(1-\beta)}$. With a discount factor of 0.96 and a capital income share of $\phi = 0.36$, this is equivalent to a gross asset and liability position of 0.45 times GDP.

5.1 First Order Valuation Effects

To provide an illustration, we may compute the importance of valuation effects in the variation of net external assets up to the *first order*. This is the standard approach taken in much of the earlier literature on valuation effects. Taking the first order approximation of (13), we may write:

$$\hat{W}_{t} - \hat{W}_{t-1} = \frac{1-\beta}{\beta} \hat{W}_{t-1} + \hat{Y}_{t} - \hat{C}_{t} + \tilde{\alpha}(\hat{r}_{1,t} - \hat{r}_{2,t}) + O\left(\epsilon^{2}\right)$$
(16)

Expression (16) is simpler than (14) due to the absence of the second order terms. Now the only valuation effect is due to the excess return on asset 1 relative to asset 2.

There are two ways to perform a decomposition on (16). The first approach, and the clearest way within this model, is to take the first term $\frac{1-\beta}{\beta}\hat{W}_{t-1} + \hat{Y}_t - \hat{C}_t$, as a measure of the conventional current account, and the second term $\tilde{\alpha}(\hat{r}_{1,t} - \hat{r}_{2,t})$, as a measure of valuation effects which impact on the net external assets, but do not directly enter into the current account. To the extent that the current account does not directly incorporate equity returns, this is consistent with standard balance of payments accounting.

An alternative approach is to decompose r_{1t} and r_{2t} into dividend and capital gains terms, and to assume that the dividend payments are incorporated in the measurement of the current account. In this model, both decompositions have very similar implications, so we focus only on the first one. The appendix (to be added) reports the results based on the second decomposition of valuation terms.

Following the decomposition discussed in section 2, we may state that

$$\hat{W}_t - \hat{W}_{t-1} = CA_t + VAL_t$$

where CA_t is the current account to GDP ratio, and VAL_t corresponds to the valuation term. In the example model, these definitions correspond to:

$$CA_{t} = \frac{\beta}{2} \frac{(1-\mu)}{(1-\beta\mu)} \left[\phi(\hat{Y}_{k,t} - \hat{Y}_{k,t}^{*}) + (1-\phi)(\hat{Y}_{l,t} - \hat{Y}_{l,t}^{*}) \right] - \tilde{\alpha} \frac{(1-\beta)^{2}}{(1-\beta\mu)} (\varepsilon_{k,t} - \varepsilon_{k,t}^{*})$$
(17)

$$VAL_{1,t} = \tilde{\alpha} \frac{(1-\beta)}{(1-\beta\mu)} (\varepsilon_{k,t} - \varepsilon_{k,t}^*)$$
(18)

The measured current account contains two terms. The first term is the familiar textbook definition of the current account. When there is a rise in home relative to foreign income, whether capital or labour income, the current account will improve, so long as shocks are not fully persistent. The second term captures the impact of portfolio reallocation effects on consumption, and therefore the current account. The valuation term represents the income gain or less due to unanticipated changes in the excess return on assets. The scale of this will depend on the portfolio position $\tilde{\alpha}$, given in (15).

At the first order level, we note that the valuation term $\tilde{\alpha}(\hat{r}_{1,t} - \hat{r}_{2,t})$ is i.i.d. This seems to accord with the property of the implied valuation effects in the data.

How large are valuation effects, relative to the measured current account, in the evolution of net external assets? We can compute this in this example model. Take the special case where $\sigma_K^2 = \sigma_L^2 = \sigma_K^{*2} = \sigma_L^{*2}$, and $\sigma_{KL} = \sigma_{KL}^*$. Define $VR = \frac{Var_{t-1}(VAL_{1t})}{Var_{t-1}(\hat{W}_t - \hat{W}_{t-1})}$. Then, using (15), (17), and (18), we can establish that:

$$VR = \frac{(\phi\sigma_K^2 + (1-\phi)\sigma_{KL})^2}{(1-\mu)^2(1-\phi)^2(\sigma_K^2 - \sigma_{KL})^2 + \mu^2(\phi\sigma_K^2 + (1-\phi)\sigma_{KL})^2}.$$
(19)

Theoretically, this can take any range between zero and infinity. When $\phi = 1$ or $\sigma_{KL} = \sigma_K^2$, there are effectively complete markets, and the right hand side of (19) is $1/\mu^2$, which always exceeds unity. If shocks are quite transitory, then the optimal portfolio keeps net external assets very stable, and the valuation ratio is very high. On the other hand, for low or negative σ_{KL} , the optimal portfolio stance is small, due to home bias, and the valuation ratio may be very small.

In the data, we saw that the average variance of the valuation channel to the variance of net external exports was approximately 0.9 for both NFA decompositions. Figure 1 reports the value for VR for various ranges of $\tilde{\alpha}$. We set $\sigma_K^2 = \sigma_L^2 = \sigma_K^{*2} = \sigma_L^{*2} = .02^2$, $\beta = 0.96$, $\phi = 0.36$, and $\mu = 0.9$, using conventional parameterizations, and assuming persistent shocks. The size of the valuation component is obviously critically related to the gross external portfolio (15). This in turn depends on the degree to which labor and capital income co-vary within countries. For illustrative purposes, we report the valuation term as σ_{KL} and σ_{KL}^* are varied. For $\sigma_{KL} = -\phi/(1-\phi)\sigma_K^2$ and $\sigma_{KL}^* = -\phi/(1-\phi)\sigma_K^{*2}$ we have $\tilde{\alpha} = 0$, and both valuation terms are zero. As $\sigma_{KL} \to \sigma_K^2$ and $\sigma_{KL}^* \to \sigma_K^{*2}$ (assuming that $\sigma_L^2 = \sigma_K^2$ and $\sigma_L^{*2} = \sigma_K^{*2}$), $\tilde{\alpha}$ goes to $-\frac{0.5}{1-\beta}$.

We set $\sigma_{KL} = \sigma_{KL}^*$, and vary σ_{KL} between $-(.0135)^2$ and 0.02^2 . For $\sigma_{KL} = -(0.0135)^2$, the home country holds a positive gross external position in foreign equity equal to 80 percent of GDP matched by a negative position in home equity of an equal size (approximately the case of the US). But as σ_{KL} rises to 0.02^2 , there is a perfect correlation between capital and labor income, and each country take a very large positive (negative) position in foreign (home) equity. The horizontal axis in Figure 3 illustrates the value of $\tilde{\alpha}$ induced by changes in σ_{KL} . The figure illustrates that the share of the volatility of net external assets attributable to valuation terms is increasing sharply in $-\tilde{\alpha}$. For $-\tilde{\alpha} = 0.8$, for instance, VR = 0.68. As σ_{KL} rises, however, $-\tilde{\alpha}$ rises, and the size of the valuation component increases, with VR exceeding 1 as $-\tilde{\alpha}$ rises above 1.4. The case of lower persistence is quite different. When $\mu = 0.75$, for instance, then initially, VR begins quite low. But as $-\tilde{\alpha}$ rises, the VR increases much faster.

This example suggests that in principle, a model of efficient risk-sharing can explain the large size of valuation effects in the data. But a second property of the unanticipated valuation term is its covariance with the measured current account. As we saw in our description of the data, for the majority of countries, the covariance between the conventional current account measure and the valuation effect is negative. This also extends to the trade balance measure. In this example, using the solution for $\tilde{\alpha}$, we may establish that:

$$\operatorname{cov}_{t-1}(VAL_{1t}, CA_t) = -0.25 \frac{\left(\phi(\sigma_K^2 + \sigma_K^2) + (1 - \phi)(\sigma_{KL} + \sigma_{KL}^*)\right)^2}{(1 - \beta\mu)(\sigma_K^2 + \sigma_K^2)} < 0$$

There is a negative covariance between the trade balance and the unanticipated valuation term. This is intuitive. A positive capital income shock increases the return to capital, but also increases income. If agents are hedged by taking a negative (positive) position in home (foreign) stocks, the current account improves while at the same time there is a negative return on the external portfolio. Likewise, in the model there is a negative covariance between GDP and VAL_{1t} , as seen in most countries data

It is apparent that the endogenous portfolio is critical to explaining the size of valuation effects. By this we mean that increases in the volatility of shocks do not translate into higher valuation effects, absent the endogenous response of the optimal portfolio. Figure 4 illustrates this for our example model. Starting again at the initial calibration where $-\tilde{\alpha} = 0.8$, we illustrate the impact of a rise in σ_K^2 and σ_K^{*2} on VR under two scenarios. In the first scenario, $\tilde{\alpha}$ adjusts according to (15). In the second, $-\tilde{\alpha}$ is held constant at 0.8. We see that all of the increase in the importance of valuation is coming from the response of the optimal portfolio. In the case where $\tilde{\alpha}$ is held constant, the valuation ratio actually falls. Increased volatility in an of itself does not necessarily increase the valuation component of net external assets. In this example, increased volatility for a given $\tilde{\alpha}$ raises the volatility of the current account, relative to that of the valuation term. It is only when the portfolio is endogenous that the valuation term grows in importance.

5.2 Second Order Valuation Effects: Anticipated Excess Returns

The second type of valuation effect discussed above is the *anticipated* excess return on the external portfolio. As mentioned, this is zero up to a first order. But evaluated up to the second order, we obtain the following expression

$$E_{t-1}\left[\hat{r}_{x,t} + \frac{1}{2}\hat{r}_{x,t}^{2}\right] = \frac{\rho}{2}\frac{(1-\beta)}{(1-\beta\mu)}\left(\phi(\sigma_{K}^{2} - \sigma_{K}^{2*}) + (1-\phi)(\sigma_{KL} - \sigma_{KL}^{*})\right) + O\left(\epsilon^{3}\right)$$
(20)

The expected excess return on the home country asset is positive if the volatility of the home capital income shock exceeds that of the foreign shock, and the covariance of capital and labour income shocks at home exceed those in the foreign country. Intuitively, if $\sigma_K^2 < \sigma_K^{2*}$, then the foreign capital income shock is more responsible for world consumption volatility than the home shock. Investors in both countries then must receive a higher expected return on the foreign asset. Even if $\sigma_K^2 = \sigma_K^{2*}$, however, if $\sigma_{KL} < \sigma_{KL}^*$, then again world consumption volatility is more correlated with the home asset return, and there is a risk premium on the foreign asset.

A risk premium on the foreign asset translates into a expected long run current account imbalance in the following way. Take expectations of (14). Since this example model has a unit root in net assets, we have $E_{t-1}(\hat{W}_t - \hat{W}_{t-1}) = 0$. Then we must get

$$E_{t-1}\left[\frac{1-\beta}{\beta}\hat{W}_{t-1} + \frac{1}{\beta}\hat{W}_{t}\hat{r}_{2,t} + \hat{Y}_{t} + \frac{1}{2}\hat{Y}_{t}^{2} - \hat{C}_{t} - \frac{1}{2}\hat{C}_{t}^{2}\right] + \tilde{\alpha}E_{t-1}\left[\hat{r}_{x,t} + \frac{1}{2}\hat{r}_{x,t}^{2}\right] = 0$$

The first term is the expected current account surplus, evaluated up to second order, while the second term is the expected excess return on the external portfolio. If a country holds an external portfolio which commands a positive risk premium, so that $\tilde{\alpha}E_{t-1}\left[\hat{r}_{x,t} + \frac{1}{2}\hat{r}_{x,t}^2\right] > 0$, then it can exhibit a permanent average current account deficit. For instance, if $\phi(\sigma_K^2 - \sigma_K^{2*}) + (1 - \phi)(\sigma_{KL} - \sigma_{KL}^*) < 0$, then country 1's asset is less correlated with world consumption risk. Since $\tilde{\alpha} < 0$, we then have $\tilde{\alpha}E_{t-1}\left[\hat{r}_{x,t} + \frac{1}{2}\hat{r}_{x,t}^2\right] > 0$, and country 1 can have a permanent current account deficit equal to this. By acting as a 'safe haven', a country with a low volatility of output can gain on average, if it is willing to hold more risky foreign assets. How big can this safe haven effect on the current account be within our simple example? To estimate this, we must combine the solution for $\tilde{\alpha}$ with the expected excess return within the model to obtain:

$$-\frac{\rho}{4} \frac{\left[\phi(\sigma_K^2 + \sigma_K^{2*}) + (1 - \phi)(\sigma_{KL} + \sigma_{KL}^*)\right] \left[\phi(\sigma_K^2 - \sigma_K^{2*}) + (1 - \phi)(\sigma_{KL} - \sigma_{KL}^*)\right]}{(1 - \beta\mu)(\sigma_K^2 + \sigma_K^{2*})}$$

The two key parameters determining the size of this expression are the coefficient of relative risk aversion, and the degree of persistence in endowment shocks. Figure 2 illustrates the excess return and the current account effect. The figure assumes that $\sigma_K^2 = 0.01^2$, and $\sigma_K^{2*} = 0.04^2$, indicating that the foreign country has a much more volatile endowment process. The covariance between capital and labor income in each country is varied in order to allow variation in the value of $\tilde{\alpha}$. We again assume that $\beta = 0.96$, $\phi = 0.36$, $\mu = 0.9$, and assume that $\rho = 5$, indicating a high rate of risk aversion, but still well within the range used in asset pricing studies. Again, $-\tilde{\alpha}$ is on the horizontal axis.

For values of $-\tilde{\alpha}$ in the range of 0.5 to 1, the effect of differential risk on the current account is extremely small. A 'safe haven' country in this range could expect to have a current account deficit of between 0.008 and 0.012 percent of GDP. The excess return on the external portfolio is only about .01 of a percent. As total leverage rises, the impact on the current account rises, but remains a negligible share of GDP. Thus, in this simple model, the potential for anticipated excess returns to differences in country riskiness is very small.

5.3 Second order valuation effects: Portfolio Rebalancing

In section 5.1, we showed how an optimal portfolio position gives rise to returns on the net external portfolio that reduces consumption risk, and may account for much of the volatility of net external assets, evaluated up to the first order. But there are aspects of optimal portfolio choice not fully captured by the analysis of section 5.1. The approximation of (14) is taken up to a second order. This means that the relevant valuation term that corresponds to the empirical measures is given by

$$VAL_{2t} = \tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t}^2 - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t}^2)) + \hat{\alpha}_{t-1}\hat{r}_{x,t}$$

Relative to the measure VAL_{1t} , this contains the impact of portfolio adjustment on returns, and hence on the evolution of net external assets. Even though the term $\hat{\alpha}_{t-1}\hat{r}_{x,t}$ is i.i.d., it still allows for risk sharing at the second order. How important is this in the determination of the variance of net external assets? That is, how much additional risk-sharing is offered by this portfolio adjustment term? In order to answer this question, we have to go beyond the solution (15), which gives the steady state (or zero order) component of the portfolio. The expression $\hat{\alpha}_t$ represents the first order component of the portfolio solution. It captures the extent to which the real portfolio is itself adjusted in response to movements in the underlying state variables of the economy. In Devereux and Sutherland (2007), it is shown that there is an analytical solution for this, which (for this model) can be written as:

$$\hat{\alpha}_{t}^{(1)} = \gamma_{1} \hat{Y}_{K,t} + \gamma_{2} \hat{Y}_{K,t}^{*} + \gamma_{3} \hat{Y}_{L,t} + \gamma_{4} \hat{Y}_{L,t}^{*} + \gamma_{5} \hat{W}_{t}$$
(21)

where the γ_i coefficients are complicated functions of parameters and the moments of shocks, and are described in the appendix.

What is the intuition for the dependence of α on the shocks and net wealth, as captured in (21)? To see this, go back to the portfolio selection condition (4), which indicates that the individual wishes to keep the expected product of marginal utility times excess returns equal to zero. When (4) is evaluated up to a second order, this is equivalent to keeping the conditional one-step ahead covariance of log consumption and excess returns equal to zero. A constant portfolio $\tilde{\alpha}$ is sufficient to achieve this. But when we take a third order approximation in order to obtain $\hat{\alpha}$, the conditional means of consumption and asset returns will affect overall portfolio risk, and agents will have to adjust their portfolio to hedge against this. These adjustments in turn affect the correct measure of valuation, evaluated up to a second order. With some slight abuse of terminology, we call this a 'portfolio rebalancing' effect. In response to movements in the conditional means of consumption and asset returns, agents desire to adjust their portfolio holdings.

How important are these higher order effects? In Table 3, we report the results of the

$\tilde{\alpha}$	VR_1	VR_2	$corr_1$	$corr_2$	$corr_3$
30	.18	.035	077	064	-0.46
43	.32	.028	098	089	045
55	.45	.023	120	113	044
68	.57	.018	142	137	042
8	.68	.014	164	160	041
93	.77	.011	186	182	039
-1.05	.85	.009	207	204	038
-1.17	.91	.007	228	226	037
-1.30	.96	.006	248	246	036
-1.42	1.01	.005	268	264	034

Table 3: 2^{nd} -order Approx. to Current Account

valuation terms when we solve the model up the second order. In Table 3, we define the valuation ratios VR_1 and VR_2 respectively as

$$VR_1 = var(\tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t}^2 - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t}^2))/var(W_t - W_{t-1}),$$

and

$$VR_2 = var(\hat{\alpha}_{t-1}\hat{r}_{x,t})/var(W_t - W_{t-1}).$$

Thus, VR_1 is a measure of the importance of the direct movement in excess returns on the portfolio on the volatility of net foreign assets. VR_2 is a measure of the volatility in 'portfolio rebalancing' as a share of the volatility of net foreign assets. VR_1 is almost the same as the first order solution VR from Figure 3. As gross asset positions rise, the importance of movements in excess returns on the portfolio grows larger. VR_2 was not measured before. But in fact, it is small. The adjustment of portfolios contributes is at most 3.5 percent of the variation in net external assets. Moreover, as the size of the gross asset positions rise, this falls. In the baseline calibration of the previous section, where $\tilde{\alpha} = -0.8$, movements in the portfolio contribute only 1.4 percent of the volatility of net foreign assets.

Despite the small size of the portfolio adjustment term in accounting for the movement in net external assets, it still exhibits the risk-sharing properties of the first order solution. In Table 3,

$$corr_1 = corr(\tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t}^2 - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t}^2)) + \hat{\alpha}_t \hat{r}_{x,t}, CA_t),$$

$$corr_2 = corr(\tilde{\alpha}(\hat{r}_{1,t} + \frac{1}{2}\hat{r}_{1,t}^2 - (\hat{r}_{2,t} + \frac{1}{2}\hat{r}_{2,t}^2)), CA_t),$$

and

$$corr_3 = corr(\hat{\alpha}_t \hat{r}_{x,t}, CA_t)$$

Thus, the overall valuation term, and the two subcomponents of the valuation term covary negatively with the current account. Portfolio rebalancing does play a role as part of the optimal portfolio. But relative to the first order effect of having an optimally chosen fixed portfolio, this role has only a minor impact on the evolution of net external assets.

5.4 Third Order Valuation Effects: Portfolio Adjustment

In the previous sections we have considered the valuation terms arising in the first and second-order approximations of $\hat{W}_t - \hat{W}_{t-1}$. Notice that these valuation terms depend on the zero-order and first-order component of gross asset holdings, $\tilde{\alpha}$, and the first and second-order components of excess returns. In fact the only anticipated valuation term that arises is the product of the zero-order component of α and the second-order component of expected excess returns. As explained, both the zero-order component of α and the second-order component of x are non-time-varying. So the *anticipated* valuation term in the second-order approximation of $\hat{W}_t - \hat{W}_{t-1}$ is also non-time-varying.

Recent literature (in particular Gourinchas and Rey, 2007) has emphasised the possibility that portfolio adjustment may generate additional *predictable time-varying valuation* effects which affect $\hat{W}_t - \hat{W}_{t-1}$ during the transitional phase that follows a shock. For instance, if home households respond to a shock by increasing α , an additional predictable valuation term will arise while α is above its steady-state value. Likewise, predictable valuation effects can be generated by predictable movements in expected excess returns following a shock. Gourinchas and Rey's evidence suggests that a negative shock to the US trade balance is followed by a predictable increase in the excess return on the US portfolio. If the expected excess return on the portfolio rises following a shock, an additional valuation effect will exist for as long as the expected excess return is above its steady state value. We now wish to examine whether these valuation terms can arise in our example model. If so, how large are they? Moreover do they increase or decrease $\hat{W}_t - \hat{W}_{t-1}$ during the transitional phase following a shock?

Clearly, in order to answer these questions it is necessary to analyse the time varying behaviour of α and x. As discussed in section 4 this requires solving for higher order

components of α and x. More specifically it is necessary (at least) to solve for the firstorder component of α and the *third-order* component of x. In the previous section we have already introduced the time varying solution for $\hat{\alpha}_t^{(1)}$. In conjunction with this first order solution for $\hat{\alpha}_t^{(1)}$ we may also derive the third-order behaviour of x as a linear function of the state variables as follows

$$x_t^{(3)} = \delta_1 \hat{Y}_{K,t} + \delta_2 \hat{Y}_{K,t}^* + \delta_3 \hat{Y}_{L,t} + \delta_4 \hat{Y}_{L,t}^* + \delta_5 \hat{W}_t$$
(22)

where again the δ_i coefficients are complicated functions of parameters and the moments of shocks.

How do these terms enter the approximate expected evolution of NFA? The answer is that, at the level of a second-order approximation of $\hat{W}_t - \hat{W}_{t-1}$, as given in equation (14), neither of these terms has any (predictable) effect. While the term $\hat{\alpha}_{t-1}\hat{r}_{x,t}$ in equation (14) does depend on $\hat{\alpha}^{(1)}$, this does not give rise to a *predictable* valuation effect, because $\hat{\alpha}_{t-1}\hat{r}_{x,t}$ is i.i.d. up to the second order. As shown in the previous section, such adjustments will contribute to the variance of net external assets, but by definition, any *predictable* time-variation in $\alpha_{t-1}r_{xt}$ will be zero at the second-order and will only emerge at the third-order or higher. These effects are likely to contribute a very small fraction of the overall variation of net external assets.

To see this more clearly, note that a third-order expansion of expected $\hat{W}_t - \hat{W}_{t-1}$ yields the following

$$E\left[\hat{W}_{t} - \hat{W}_{t-1}\right] = E\left[\hat{Y}_{t} + \frac{1}{2}\hat{Y}_{t}^{2} + \frac{1}{6}\hat{Y}_{t}^{3} - \hat{C}_{t} - \frac{1}{2}\hat{C}_{t}^{2} - \frac{1}{6}\hat{C}_{t}^{3}\right]$$
(23)

$$+E\left[\frac{1-\beta}{\beta}\hat{W}_{t-1}+\frac{1}{\beta}\hat{W}_{t}\hat{r}_{2,t}\right]+\tilde{\alpha}\hat{x}_{t}+\hat{\alpha}_{t-1}\hat{x}_{t}+O\left(\epsilon^{4}\right) \qquad(24)$$

An important feature of this approximation is that it does not contain any higher approximations of α than those we have already obtained.

To evaluate (23) it is necessary to evaluate the two final terms up to third order. Substituting for $\hat{\alpha}_{t-1}$ and \hat{x}_t , expanding and deleting terms of order higher than 3 yields

$$\tilde{\alpha}x_t + \hat{\alpha}_{t-1}x_t = \underbrace{\tilde{\alpha}\hat{x}^{(2)}}_{\text{constant}} + \underbrace{\tilde{\alpha}\hat{x}^{(3)}_t}_{\text{time variation in }x} + \underbrace{\hat{\alpha}^{(1)}_{t-1}\hat{x}^{(2)}}_{\text{time variation in }\alpha}$$
(25)

The first term in this expression is just the second-order expected valuation effect identified in previous sections. The second and third terms are the additional expected valuation effects which arise in the third-order approximation. The second term, $\tilde{\alpha}\hat{x}_{t}^{(3)}$, is time varying because $\hat{x}_{t}^{(3)}$ is time-varying. And the third term, $\hat{\alpha}_{t-1}^{(1)}\hat{x}^{(2)}$, is time varying because $\hat{\alpha}_{t-1}^{(1)}$ is time varying. Hence $\tilde{\alpha}\hat{x}_{t}^{(3)}$ captures the effect of time varying expected returns on $E\left[\hat{W}_{t} - \hat{W}_{t-1}\right]$ while $\hat{\alpha}_{t-1}^{(1)}\hat{x}^{(2)}$ captures the effect of time varying portfolio holdings on $E\left[\hat{W}_{t} - \hat{W}_{t-1}\right]$. These two terms thus capture exactly the transitional, time-varying expected return valuation effects identified at the start of this section.

While these terms only enter a third-order approximation of the $E\left[\hat{W}_t - \hat{W}_{t-1}\right]$ equation, it is not necessary to solve the model up to this order to be able to evaluate these terms. In fact once we have obtained the solutions of the form (21) and (22), these valuation effects can be evaluated directly from first-order impulse response of the state variables. In the next section, a numerical calculation of these terms will be presented in the context of numerical impulse responses.

6 A Quantitative Examination of Valuation Effects in the Example Model

To illustrate the role and potential magnitude of the different valuation effects, we consider some example impulse responses following an innovation to capital income. These are shown in Figure 5. Again we set $\sigma_K^2 = \sigma_L^2 = \sigma_K^{*2} = \sigma_L^{*2} = .02^2$, $\beta = 0.96$, $\phi = 0.36$, and $\mu = 0.9$ and we choose $\sigma_{KL} = \sigma_{KL}^* = -0.0136^2$ so that $\tilde{\alpha} = -0.8$, i.e. home households hold a gross position in foreign equity equal to about 80% of GDP. As noted above, this corresponds to a case where home households hold approximately 91% of home equity and foreign households hold approximately 91% of foreign equity (i.e. there is a substantial degree of home equity bias). Figure 5 shows the impact of a -1% shock to capital income in the home country (Y_K) in period 1. The impact on total income is shown in panel (a). Home country income falls by 0.36% on impact. Panel (b) shows that consumption in the home economy falls by approximately 0.22% in period 1. The impact effect of the shock is therefore to push the home economy into a trade deficit of approximately 0.14% of GDP. This deficit declines to zero as the effects of the shock fade.

While the home economy runs a trade and current account deficit following the shock, net foreign assets rise sharply in period 1 and then decline. The sharp rise in NFA in period 1 reflects the first-order unanticipated valuation effect that arises from the effects of the shock on realised equity returns. The shock to home country capital income implies a sharp unanticipated fall in the price of home equity so there is an unanticipated negative excess return on home equity (i.e. \hat{r}_x is negative).³ Home households have a negative external position in home equity (i.e. $\tilde{\alpha}$ is negative) so the negative excess return in home equity represents a positive valuation effect. The shock to \hat{r}_x is approximately -0.3% so this first-order valuation effect is approximately 0.24% of GDP (i.e. -0.3×-0.8). This is illustrated in panel (k).

By evaluating the γ_i coefficients in (21) we are also able to trace out the dynamic effect of the shock on gross portfolio holdings. These are shown in panel (d) and panel (e). Here $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are home households' holdings of, respectively, home and foreign equity. Panels (d) and (e) show that the movements in gross equity holdings are, in the case of this parameterisation of the model, significantly larger than the movement in NFA. The shock induces home households to increase their gross holdings of home equity by over 1% of GDP while their holdings of foreign equity are reduced by an almost equivalent amount. As discussed in Devereux and Sutherland (2007), the lower conditional mean of consumptions and asset returns leads to a temporarily lower gross portfolio requirement. As a result, the negative position in home equity is reduced (so $\hat{\alpha}_1$ is positive), and the positive position in foreign equity is also reduced ($\hat{\alpha}_2$ is negative).

Evaluation of the δ_i coefficients in (22) allows us also to plot the effects of the shock on the (third-order) expected path of the excess return (i.e. $E[\hat{r}_x]$). This is illustrated in panel (h). The shock leads to a persistent reduction in the expected excess return. The magnitude of this effect is, however, very small (which is not surprising given that this is a third-order effect). $E[\hat{r}_x]$ falls by 0.000015% following the shock and gradually returns to zero as the effects of the shock fade.

The dynamic responses of $\hat{\alpha}$ and $E[\hat{r}_x]$ provide us with the information necessary to calculate the two third-order valuation effects in (25). These are illustrated in panel (l). The plot labelled val(α) represents the value of the third term in (25) while the second term in (25) is labelled val(rx). It can be seen from panel (l) that $\hat{\alpha}_{t-1}^{(1)}\hat{x}^{(2)}$ is zero in this parameterisation of the model. This reflects the symmetric nature of the parameterisation,

³Notice from panel (g) that the prices of both home and foreign equity fall following the shock. The price of foreign equity falls because the expected future rate of return on all equity has to be above its steady state value to be consistent with the rising path of consumption. The price of home equity obviously falls more than the price of foreign equity because the persistent shock to home capital income reduces the income stream to holders of home equity.

which implies that $\hat{x}^{(2)} = 0$. Dynamic adjustment of $\hat{\alpha}_t^{(1)}$ therefore does not generate any predictable valuation effect. Panel (1) shows however that $\tilde{\alpha}\hat{x}_t^{(3)}$ is positive following the shock. This reflects the fact that $E[\hat{r}_x]$ is negative (see panel (h)) while $\tilde{\alpha}$ is also negative. The persistent negative value of $E[\hat{r}_x]$ therefore represents a positive valuation effect for home households. This effect is, however, minute. At its largest it is only 0.000012 % of GDP! This should be compared to the trade deficit, which is 0.14% of GDP in the period of the shock.

As a further illustration of the size of the third-order valuation effects consider an asymmetric case where $\sigma_K^2 = 0.01^2$ and $\sigma_K^{*2} = .04^2$, i.e. a case where foreign capital income is more volatile than home capital income. This implies a steady-state risk premium in foreign equity of 0.0079% (i.e. $\hat{x}^{(2)} = -0.0079\%$). In this case time variation in $\hat{\alpha}_t^{(1)}$ generates a non-zero valuation effect via the term $\hat{\alpha}_t^{(1)} \hat{x}^{(2)}$. Impulse responses (not reported) show that this valuation effect is negative (because $\hat{x}^{(2)}$ is negative and $\hat{\alpha}_t^{(1)}$ is positive) and it has a maximum absolute value of 0.0001 following a -1% shock to home capital income. Again this is minute in comparison to the trade deficit created by the shock.

We can conclude this section therefore by saying that the stabilizing impact of a fall in the trade balance on the path of expected returns, as identified empirically by Gourinchas and Rey (2007), exists in our model in theory. But in practice, it can play essentially no role at all in the adjustment process. To obtain an economically meaningful pattern of time varying expected valuation effects through movements in excess returns, it is obvious that one would need to develop a model in which time-varying risk played a much bigger role than it does here.

7 Extending the Model: Habit Persistence

One clear failure of the basic model is its inability to explain substantial excess returns on country portfolios. Yet the data suggest these exist for a number of countries. Here we extend the basic model to exhibit habit persistence in consumption. We show that this extension allows for much higher average excess returns.

Now instead of (1), assume that agents in the home and foreign countries have utility

given by:

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(C_{\tau} - \omega \overline{C}_{\tau-1}), \qquad (26)$$

where \overline{C} represents aggregate consumption, and $u(x) = \frac{1}{1-\rho}x^{1-\rho}$. Thus, habits are 'external' to the individual. In equilibrium, we must have $C = \overline{C}$. Apart from this single change, the model is as before.

Using the analogous version of () from above, we have the condition

$$E_t (C_{t+1} - \omega \overline{C}_t)^{-\rho} r_{xt+1} = 0,$$

Taking a linear approximation of this condition, and combining it with the analogous foreign condition, we can obtain the identical condition for steady state portfolio determination that we obtained in the model without habit persistence. The key reason is that, with external habits, \overline{C}_t is taken as given by individuals, and, given that the steady state (or zero order) portfolios are evaluated up to 2nd order approximation only, \overline{C}_t drops out of the condition determining the optimal portfolio.

Figure 6 illustrates the relationship between $\hat{\alpha}$ and $\tilde{\alpha}E_{t-1}\left[\hat{r}_{x,t} + \frac{1}{2}\hat{r}_{x,t}^2\right]$ with habit persistence, under the same set of parameter values as section 5.2, assuming the $\omega = 0.9$. As the size of gross assets rises, then the expected valuation effect can rise substantially above the level that held under the baseline specification. When $\tilde{\alpha} = -3.4$, representing a large degree of gross leverage (but still in line with observations for some countries) we have an expected valuation effect above 1 percent of GDP.

8 Extending the Model: Exchange Rate Valuation Effects

An obvious drawback in the analysis so far is the absence of relative price changes in current account adjustment, and further, the absence of exchange rate movements in the composition of valuation effects. We correct for this now by extending the model to a two-commodity structure, and allowing for trading in nominal bonds as well as equity.

The model is quite standard. All agents in the home country have utility functions of the form:

$$U = E_0 \sum_{t=0}^{\infty} \left[\frac{C_t^{1-\rho}}{1-\rho} + \frac{1}{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} \right]$$
(27)

where $\rho > 0$, C is a consumption index defined across all home and foreign goods, $\frac{M}{P}$ are real money balances, and E is the expectations operator. The consumption index C for home agents is given by:

$$C_t = \left[\mu^{\frac{1}{\theta}} C_{Ht}^{\frac{\theta-1}{\theta}} + (1-\mu)^{\frac{1}{\theta}} C_{Ft}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$
(28)

where C_H and C_F are aggregators over individual home and foreign produced goods. The parameter θ in (28) is the Armington elasticity of substitution between home and foreign goods. The parameter μ measures the importance of consumption of the home good in preferences. With $\mu > 0.5$, there is home bias in preferences.

The aggregate consumer price index for home agents is therefore:

$$P_t = \left[\mu P_{Ht}^{1-\theta} + (1-\mu) P_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$
(29)

where P_H and P_F are the aggregate price indices for home and foreign goods.

The budget constraint of the home country agent is then:

$$P_t C_t + W_{t+1} = P_{Ht} Y_t + P_t \sum_{k=1}^N \alpha_{k,t-1} r_{kt}$$
(30)

where W_t denotes the net value of nominal wealth for the home agent. The final term represents the total return on the home country portfolio. We allow now for trade in Nassets, where in our case $N \leq 4$. In addition to trading in equity, we assume that agents can trade in the nominal bonds of either country's currency.

The conditions for consumers' utility maximization are standard. The home consumer's demand for home and foreign goods may be written as:

$$C_H = \mu \left(\frac{P_H}{P}\right)^{-\theta} C, \qquad C_F = (1-\mu) \left(\frac{P_F}{P}\right)^{-\theta} C.$$

Optimal consumption and portfolio choices are characterised by the conditions:

$$C_t^{-\rho} = \beta E_t C_{t+1}^{-\rho} r_{N,t+1}, \tag{31}$$

$$E_t C_{t+1}^{-\rho}(r_{k,t+1} - r_{N,t+1}) = 0, \qquad k = 1..N - 1.$$
(32)

TO BE COMPLETED

9 Conclusion

TO BE WRITTEN

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Table 1					
	var(val)/var(dnx)	average(val)	correl(val,ca)	correl(val,gdp)	ar coefficient
australia	0.88	0.003	0.3	-0.031	0.128
austria	1.12	-0.004	-0.342	0.05	0.202
canada	1.12	0.006	-0.386	0.045	0.614**
denmark	0.62	-0.008	0.217	-0.03	-0.078
finland	0.94	-0.028	0.053	-0.165	0.316
france	0.94	-0.004	-0.058	-0.002	-0.19
germany	0.45	0.003	0.244	0.078	-0.126
iceland	0.44	-0.023	0.086	-0.179	-0.17
ireland	0.92	-0.002	0.053	-0.227	-0.083
italy	1.05	-0.003	-0.26	-0.155	-0.015
japan	0.98	-0.006	-0.125	0.06	-0.387
korea	0.88	-0.022	-0.312	0.301	0.07
Mexico	1.28	-0.009	-0.504	0.218	-0.164
netherlands	0.99	-0.045	-0.077	0.042	-0.212
new zlnd	0.99	-0.008	-0.082	-0.078	0.389*
norway	0.6	-0.009	-0.353	-0.273	0.15
portugal	0.42	0.001	0.088	0.317	-0.009
Spain	0.7	-0.011	0.108	-0.016	0.006
Sweden	1.14	-0.024	-0.349	-0.068	0.038
Switzerland	1.14	-0.011	-0.355	-0.063	0.093
Turkey	0.71	-0.016	-0.007	-0.19	0.104
UK	0.89	0.004	0.01	-0.101	-0.243
US	1.4	0.014	-0.537	0.11	0.313

Table 2					
	var(val)/var(dnx)	average(val)	correl(val,tb)	correl(val,gdp)	ar coefficient
australia	0.88	-0.033	0.299	-0.019	0.146
austria	1.49	0.017	-0.577	0.132	0.383
canada	1.07	-0.038	-0.292	0.057	0.539**
denmark	0.78	-0.03	0.124	-0.031	0.014
finland	0.99	-0.062	-0.043	-0.111	0.34
france	0.93	0.002	0.006	0.035	-0.199
germany	0.57	-0.024	0.327	0.209	0.117
iceland	0.55	-0.058	0.351	-0.257	-0.05
ireland	1.22	-0.152	-0.452	-0.16	0.135
italy	1.05	-0.014	-0.272	-0.083	-0.029
japan	1	-0.007	-0.126	-0.019	0.447*
korea	1.02	-0.028	-0.367	0.359	0.192
Mexico	1.37	-0.034	-0.578	0.339	-0.033
netherlands	1.02	-0.053	-0.138	0.065	-0.268
new zlnd	0.99	-0.076	-0.057	-0.072	0.408*
norway	0.57	-0.03	-0.188	-0.299	0.118
portugal	0.67	0.056	0.089	0.371	0.520*
Spain	0.81	0.012	0.191	0.02	0.147
Sweden	0.96	-0.053	-0.012	0.005	-0.121
Switzerland	1	0.067	-0.095	0.089	0.04
Turkey	0.85	0.023	-0.115	-0.277	0.154
UK	0.86	0.017	0.052	-0.052	-0.255
US	1.29	0.019	-0.476	0.07	0.26



Figure 1



Figure 2









Figure 5 continued



