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# Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound

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Chicago Booth and NBER

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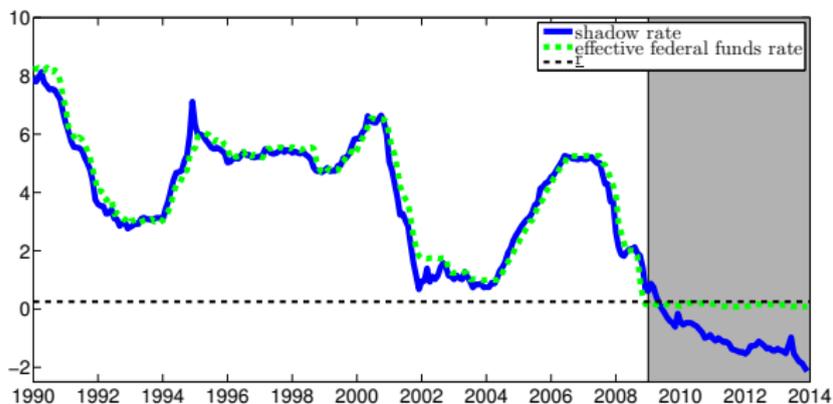
## Key question

What is the macroeconomic impact of monetary policy at the ZLB?

Conventional approach before ZLB

- ▶ VAR with the fed funds rate

But since December 2008, the fed funds rate has been near zero



# Challenges of zero lower bound

## Challenges

- ▶ What framework to study unconventional monetary policy?
  - ▶ The fed funds rate has been replaced by large-scale asset purchases and forward guidance as primary policy tools.
- ▶ How to describe the yield curve?
  - ▶ Gaussian ATSM allows negative interest rates.

## Shadow rate term structure model: Black (1995)

- ▶ Non-negative short rate:  $r_t = \max(\underline{r}, s_t)$
- ▶ Analytical solution does not exist in general

# Contributions

This paper

- ▶ an analytical approximation for SRTSM

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- ▶ an analytical approximation for SRTSM
- ▶ shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
- ▶ our shadow rate updated monthly by Atlanta Fed  
[www.frbatlanta.org/cqer/researchcq/shadow\\_rate.cfm](http://www.frbatlanta.org/cqer/researchcq/shadow_rate.cfm)

# Outline

- 1 Model
- 2 Shadow rate
- 3 Macroeconomic Implications
- 4 Conclusion

## Bond pricing

Risk-neutral factor dynamics:

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}, \quad \varepsilon_{t+1}^{\mathbb{Q}} \stackrel{\mathbb{Q}}{\sim} N(0, I).$$

▸ Pricing kernel

Pricing equation

$$P_t^n = \mathbb{E}_t^{\mathbb{Q}}[\exp(-r_t - r_{t+1} - \dots - r_{t+n-1})]$$

Yield

$$y_t^n = -\frac{1}{n} \log(P_t^n)$$

Forward rate

$$f_{n,n+1,t} = (n+1)y_{n+1,t} - ny_{nt}$$

# SRTSM and GATSM

## SRTSM

$$r_t = \max(\underline{r}, s_t)$$

$$s_t = \delta_0 + \delta'_1 X_t$$

## Forward rate

$$f_{n,n+1,t}^{SRTSM} = \underline{r} + \sigma_n^Q g\left(\frac{a_n + b'_n X_t - \underline{r}}{\sigma_n^Q}\right)$$

where  $g(z) = z\Phi(z) + \phi(z)$

▸  $a_n, b_n$

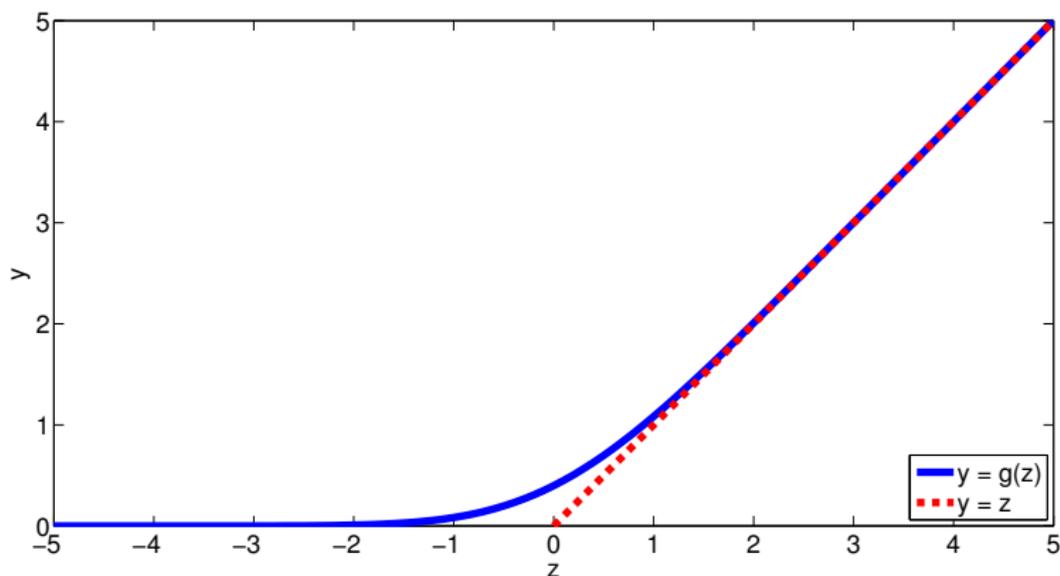
## GATSM

$$r_t = \delta_0 + \delta'_1 X_t$$

## Forward rate

$$f_{n,n+1,t}^{GATSM} = a_n + b'_n X_t.$$

# Property of $g(\cdot)$



$$f_{n,n+1,t}^{SRTSM} \quad \left\{ \begin{array}{l} \approx \underline{r}, \text{ at the ZLB} \\ \approx a_n + b'_n X_t = f_{n,n+1,t}^{GATSM}, \text{ when interest rates are high} \end{array} \right.$$

## Model fit

- ▶ GSW Data: monthly 1990-2013; maturities: 3m, 6m, 1y, 2y, 5y, 7y, 10y
- ▶ Estimation: Kalman filters [▶ details](#)
- ▶ Average absolute approximation error between 1990M1 and 2013M1 [▶ more](#)

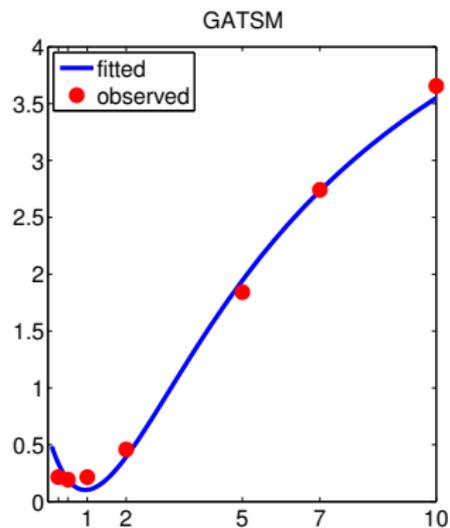
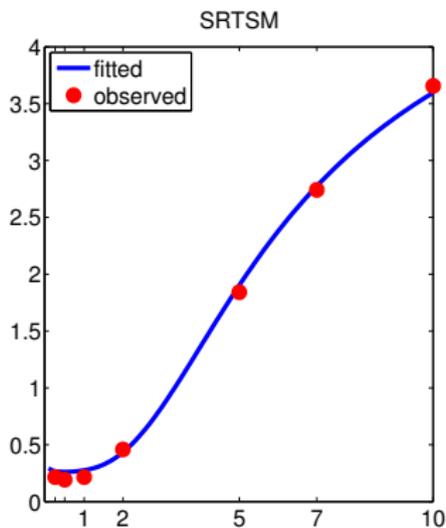
	3M	6M	1Y	2Y	5Y	7Y	10Y
forward rate error	0.01	0.02	0.04	0.13	0.69	1.14	2.29
forward rate level	346	357	384	435	551	600	636
yield error	0.00	0.01	0.01	0.04	0.24	0.42	0.78

# Model fit

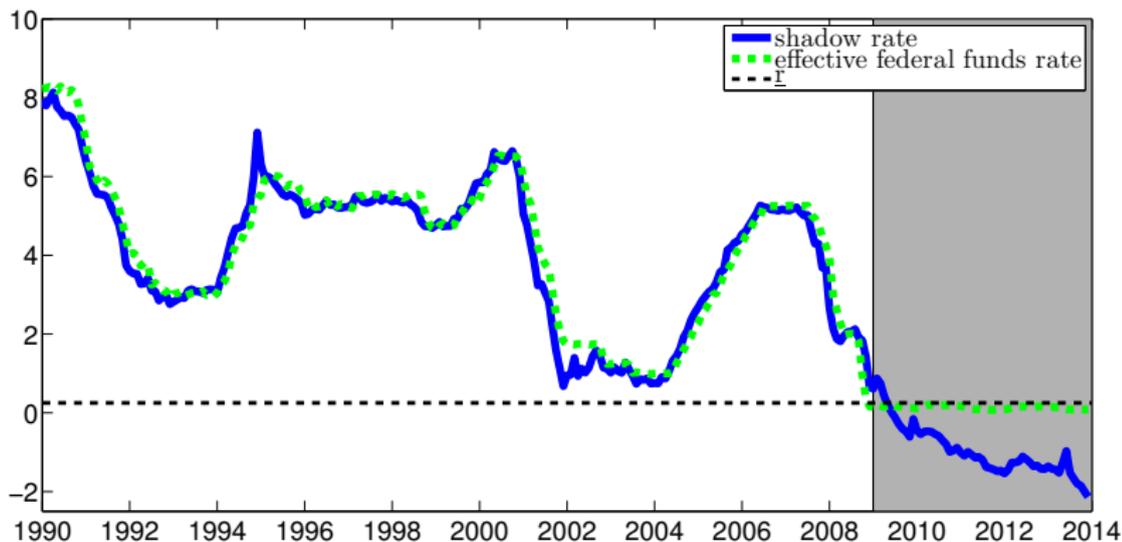
Log likelihood values ▶ specification

- ▶ SRTSM: 856; GATSM: 755

Figure: Average forward curve in 2012

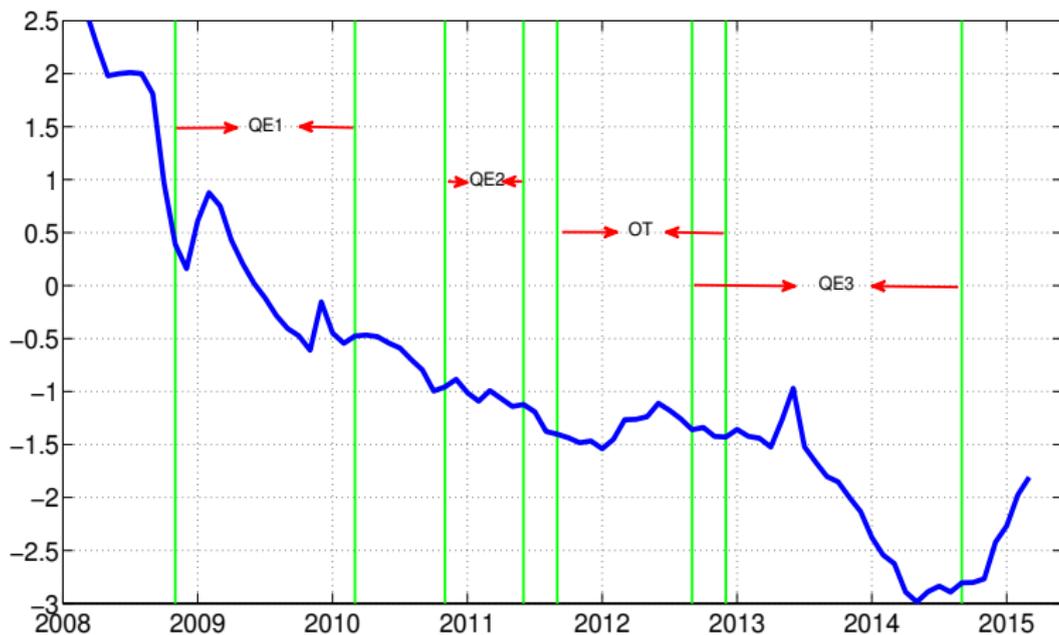


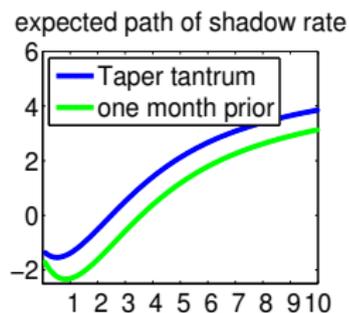
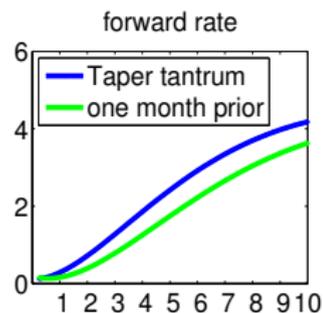
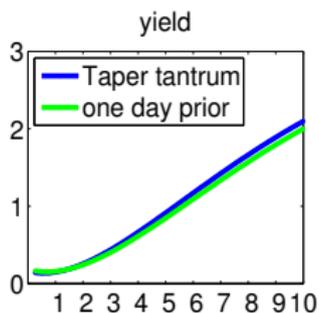
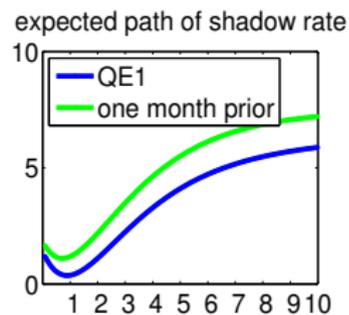
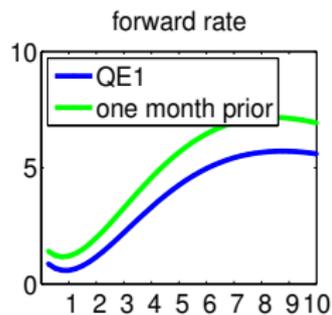
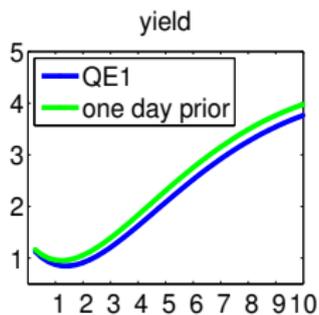
# Shadow rate



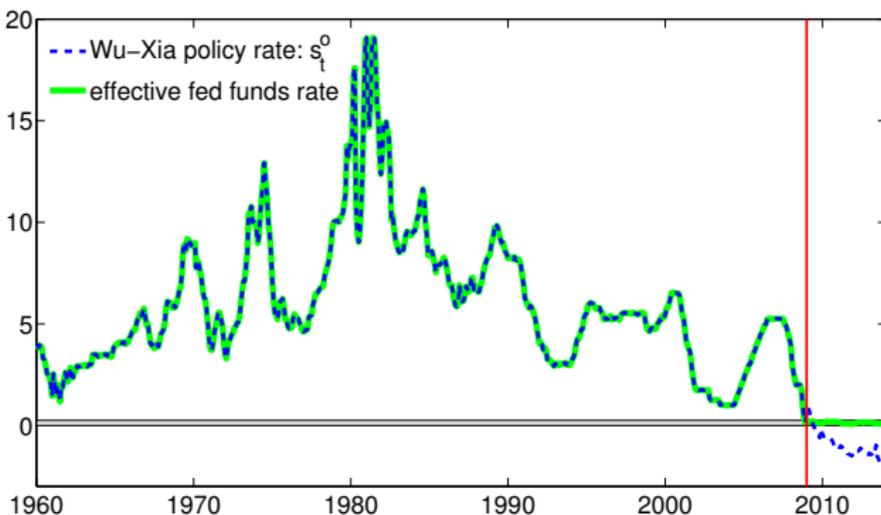
Summary for unconventional monetary policy?

# Shadow rate and LSAPs





# Monetary policy



$$s_t^o = \begin{cases} \text{effective federal funds rate} & \text{before 2009} \\ \text{shadow rate} & \text{since 2009} \end{cases}$$

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

## Factor augmented vector autoregression

Replace the fed funds rate with  $s_t^o$  in Bernanke, Boivin, and Eliasz (2005)

$$Y_t^m = a_m + b_x x_t^m + b_s s_t^o + \eta_t^m, \quad \eta_t^m \sim N(0, \Omega)$$

- ▶  $Y_t^m$ : 97 economic variables from 1960 to 2013
- ▶  $x_t^m$ : 3 underlying macro factors

Factor dynamics:

$$\begin{bmatrix} X_t^m \\ S_t^o \end{bmatrix} = \begin{bmatrix} \mu^x \\ \mu^s \end{bmatrix} + \begin{bmatrix} \rho^{xx} & \rho^{xs} \\ \rho^{sx} & \rho^{ss} \end{bmatrix} \begin{bmatrix} X_{t-1}^m \\ S_{t-1}^o \end{bmatrix} + \Sigma^m \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^{\text{MP}} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^{\text{MP}} \end{bmatrix} \sim N(0, I)$$

- ▶ monthly VAR(13)
- ▶  $\Sigma^m$ : Cholesky decomposition

# Measures of monetary policy

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## Hypothesis I

$$H_0 : \rho^{xs}(t < \text{Great Recession}) = \rho^{xs}(t > \text{Great Recession})$$

- ▶  $p = 0.29$  for  $s_t^o$

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## Hypothesis II

$$H_0 : \rho^{sx}(t < \text{Great Recession}) = \rho^{sx}(t > \text{Great Recession})$$

- ▶  $\rho = 1$  for  $s_t^o$
- ▶  $\rho = 1$  for EFFR

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## Hypothesis II

$$H_0 : \rho^{sx}(t < \text{Great Recession}) = \rho^{sx}(t > \text{Great Recession})$$

- ▶  $\rho = 1$  for  $s_t^o$
- ▶  $\rho = 1$  for EFR

Implication: researchers can use shadow rate to update earlier studies that had been based on the historical fed funds rate.

▶ Robustness



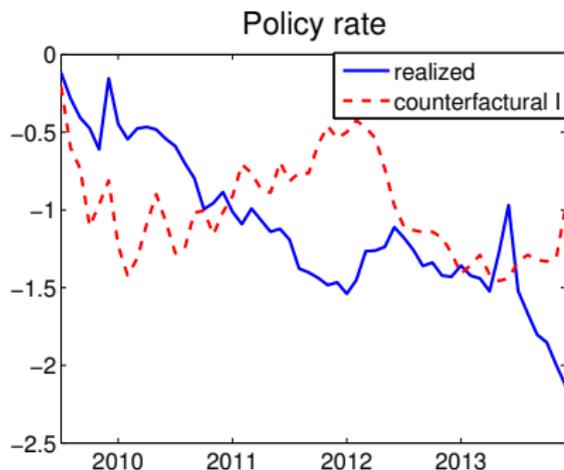
# Historical decomposition

What if there had been no monetary policy shocks?

- ▶ realized:  $\varepsilon_t^{\text{MP}} = \hat{\varepsilon}_t^{\text{MP}}$
- ▶ counterfactual:  $\varepsilon_t^{\text{MP}} = 0$  for ZLB

Unconventional monetary policy

- ▶ reduced the shadow rate by 0.4% between 2011 and 2013.



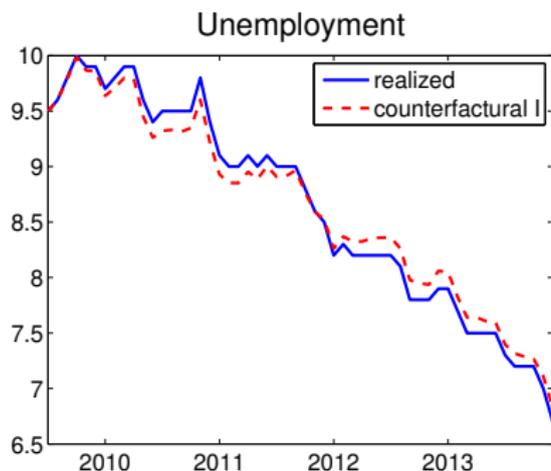
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Unconventional monetary policy

- ▶ reduced unemployment by 0.13% in Dec 2013. [▶ More](#)



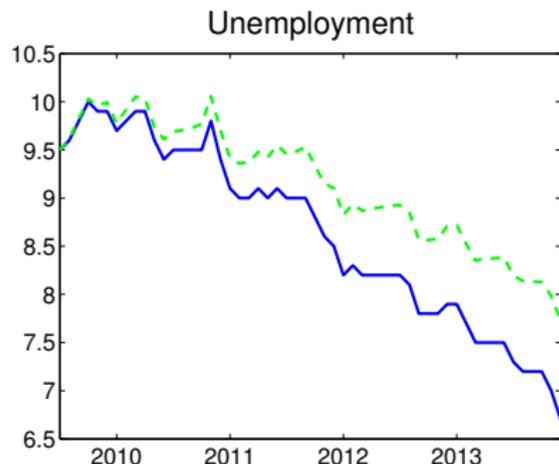
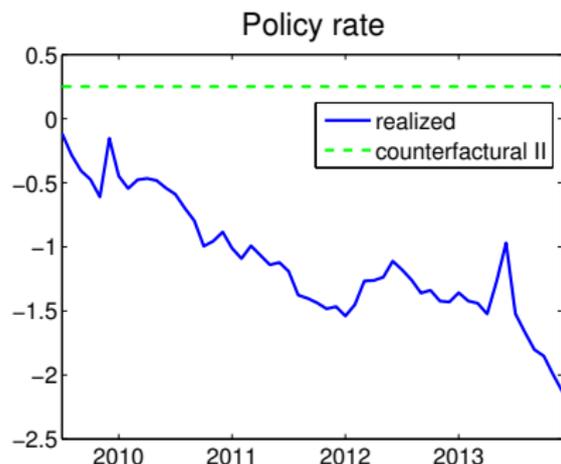
# Counterfactual II

What if the shadow rate had been kept at  $\underline{r}$ ?

- ▶ counterfactual:  $\varepsilon_t^{\text{MP}}$  is such that  $s_t^o = \underline{r}$  at ZLB

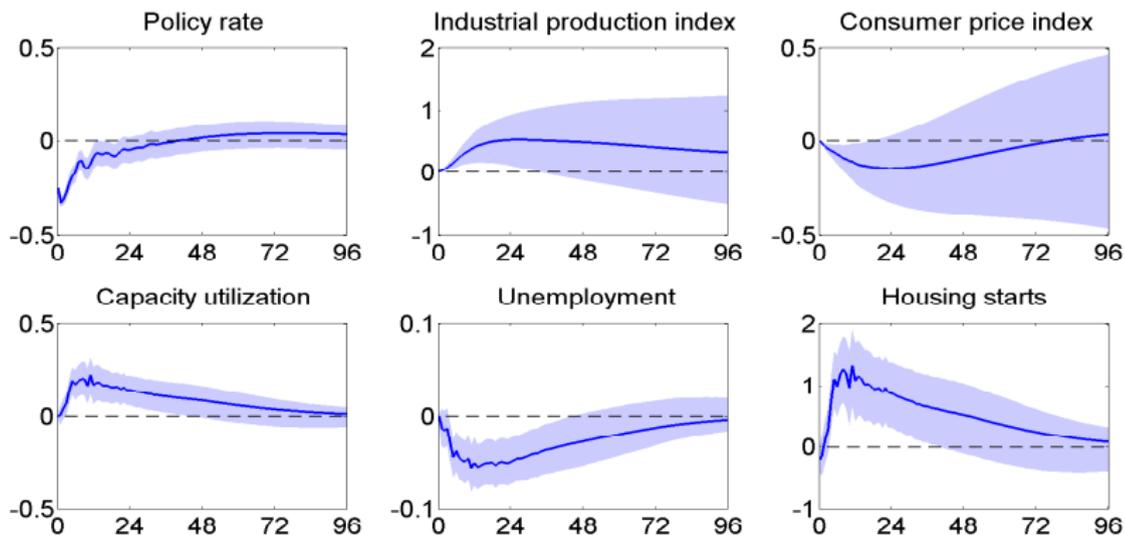
Unconventional monetary policy

- ▶ reduced unemployment by 1% in December 2013 [▶ More](#)



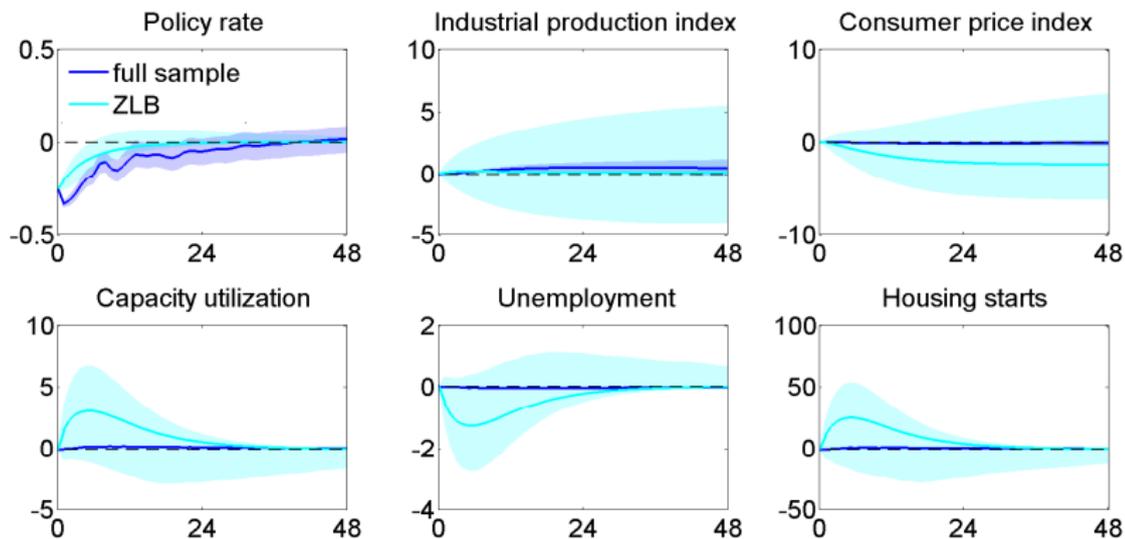
# Impulse response: full sample

A -25bps monetary policy shock



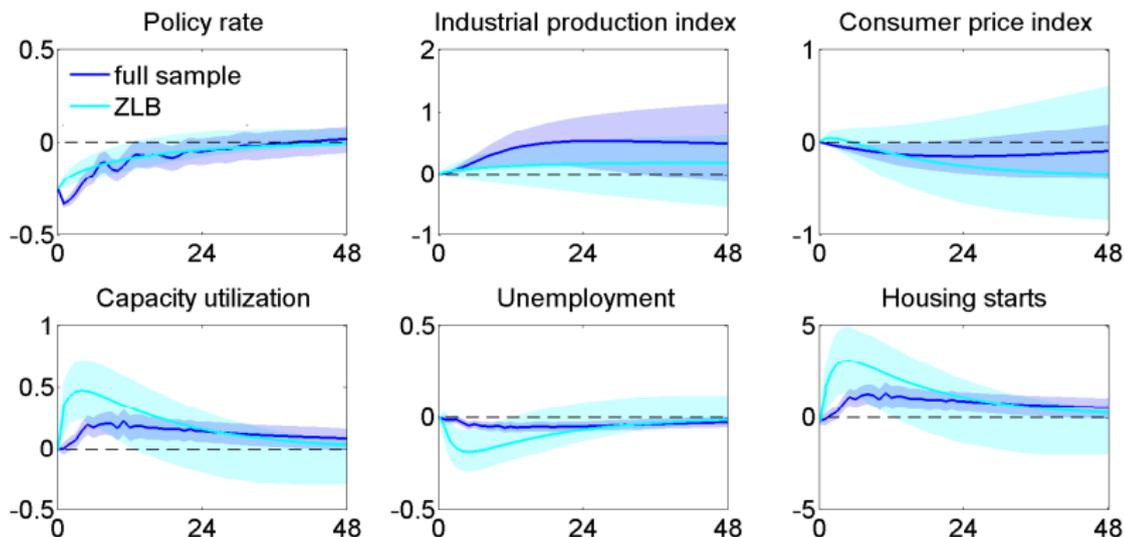
# Full sample FAVAR(13) vs. ZLB FAVAR(1)

ZLB with effective federal funds rate



# Full sample FAVAR(13) vs. ZLB FAVAR(1)

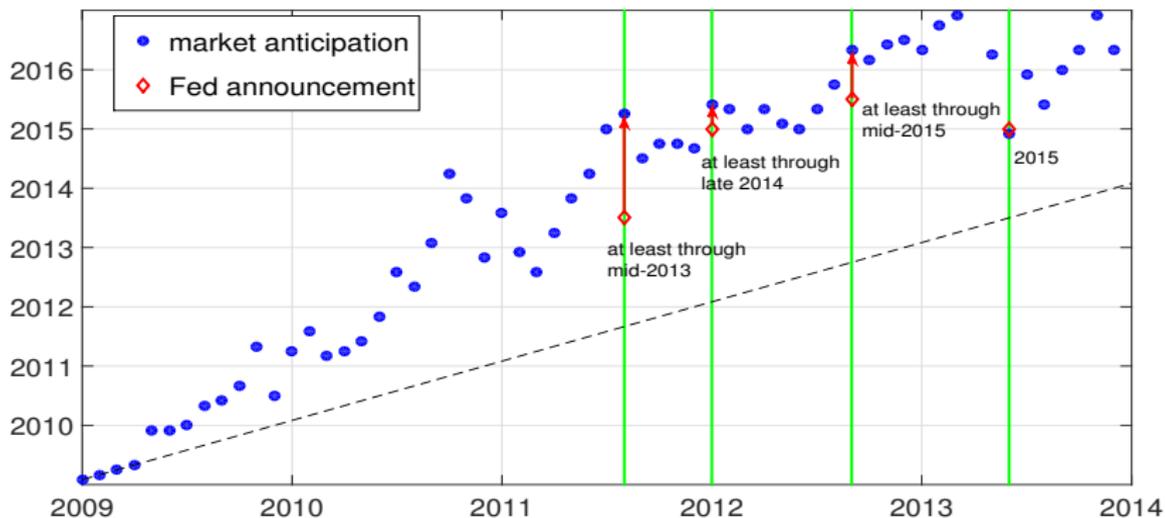
## ZLB with shadow rate



# Forward guidance

## ZLB duration

$$\tau_t = \inf\{\tau_t \geq 0 | s_{t+\tau} \geq \underline{r}\}.$$



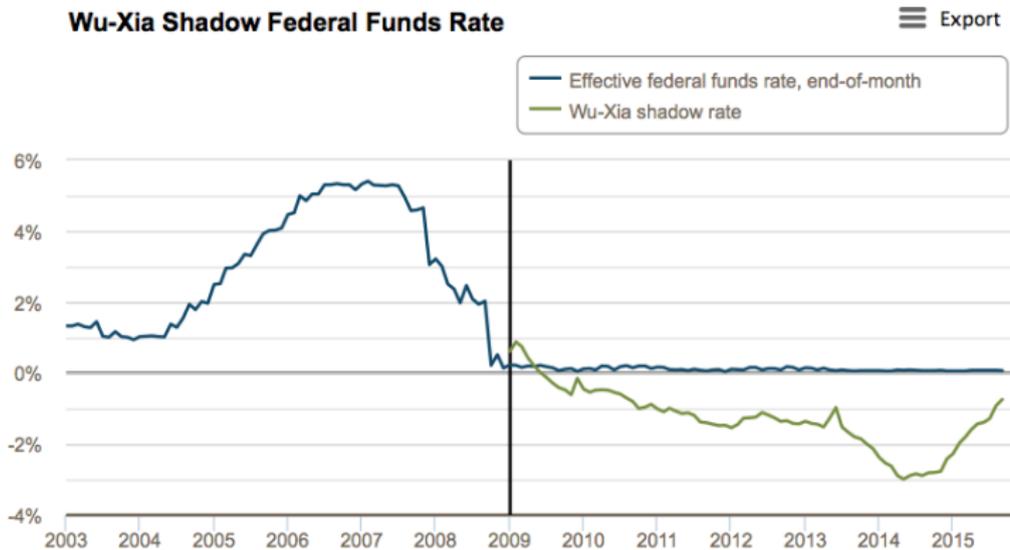
# Conclusion

## Method

- ▶ Develop an approximation for bond prices in the SRTSM

## Economics

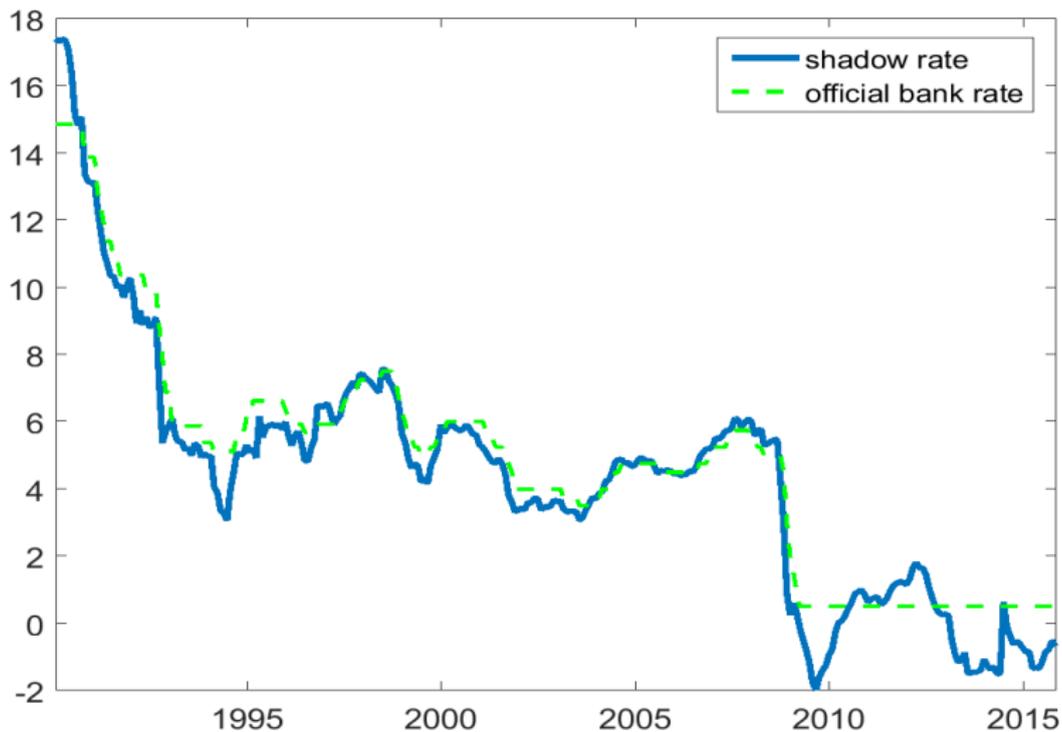
- ▶ The shadow rate exhibits similar dynamic correlations with economic variables after the Great Recession as the fed funds rate did earlier in data.
- ▶ Unconventional monetary policy lowered the unemployment rate by 0.13% in December 2013.



Sources: Board of Governors of the Federal Reserve System and Wu and Xia (2014)

Source: [www.frbatlanta.org/cqer/researchcq/shadow\\_rate.cfm](http://www.frbatlanta.org/cqer/researchcq/shadow_rate.cfm)

# ECB shadow rate



# Pricing kernel

Factor dynamics:

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I).$$

Pricing kernel

$$\begin{aligned} m_{t+1} &= r_t + \frac{1}{2} \lambda_t' \lambda_t + \lambda_t' \varepsilon_{t+1} \\ \lambda_t &= \lambda_0 + \lambda_1 X_t \end{aligned}$$

where  $\mu^Q = \mu - \Sigma \lambda_0$ , and  $\rho^Q = \rho - \Sigma \lambda_1$

Pricing equation

$$P_t^n = \mathbb{E}_t[\exp(-m_{t+1}) P_{t+1}^{n-1}]$$

## Bond recursions

$$a_n = \delta_0 + \delta'_1 \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \mu^Q - \frac{1}{2} \delta'_1 \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \Sigma \Sigma' \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right)' \delta_1,$$
$$b'_n = \delta'_1 (\rho^Q)^n.$$

▶ Back

## Model specification

$\underline{r} = 0.25$ , interest rate on reserves

three factors

Normalization: restrict  $\mathbb{Q}$  parameters

Repeated eigenvalues

$$\rho^{\mathbb{Q}} = \begin{bmatrix} \rho_1^{\mathbb{Q}} & 0 & 0 \\ 0 & \rho_2^{\mathbb{Q}} & 1 \\ 0 & 0 & \rho_2^{\mathbb{Q}} \end{bmatrix}.$$

▶ Back

# Kalman filters

State equation

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I)$$

observation equation for SRTSM  $\Rightarrow$  extended Kalman filter

$$f_{n,n+1,t}^o = \underbrace{\underline{r} + \sigma_n^Q g \left( \frac{a_n + b'_n X_t - \underline{r}}{\sigma_n^Q} \right)}_{f_{n,n+1,t}^{SRTSM}} + \eta_{nt}, \eta_{nt} \sim N(0, \omega)$$

observation equation for GATSM  $\Rightarrow$  Kalman filter

$$f_{n,n+1,t}^o = \underbrace{a_n + b'_n X_t}_{f_{n,n+1,t}^{GATSM}} + \eta_{nt}, \eta_{nt} \sim N(0, \omega)$$

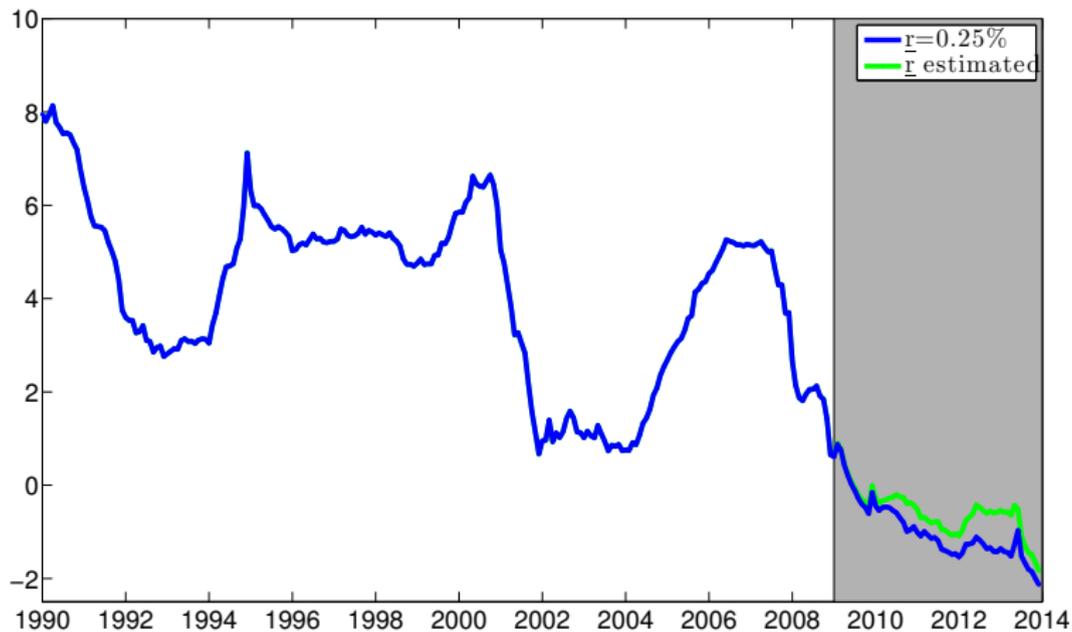
## Approximation error for ZLB

Average absolute approximation error between 2009M1 and 2013M1

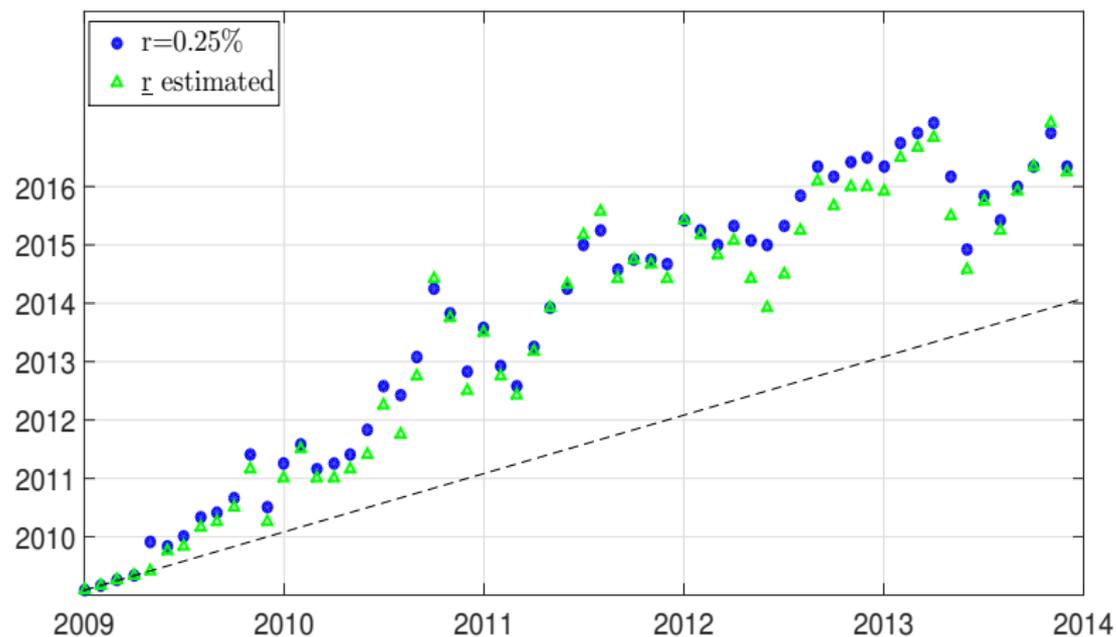
	3M	6M	1Y	2Y	5Y	7Y	10Y
forward rate error	0.00	0.01	0.06	0.43	2.50	3.51	5.41
forward rate level	23	26	46	111	326	418	481
yield error	0.00	0.00	0.01	0.10	0.91	1.50	2.37

▶ back

# Robustness



# Robustness



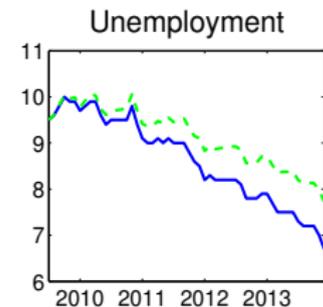
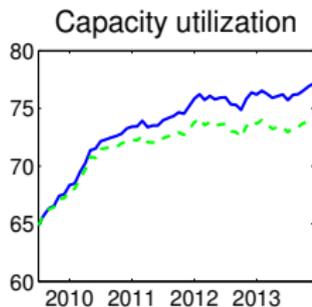
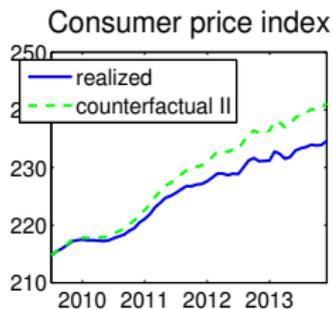
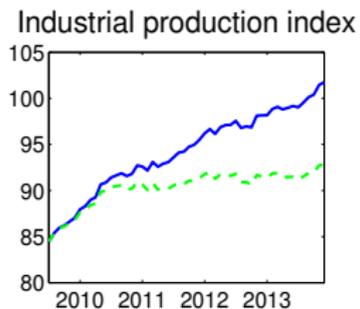
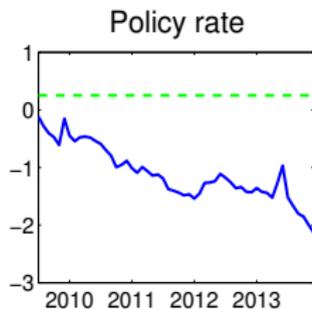
# Robustness

		$p$ -value for $\rho_1^{XS} = \rho_3^{XS}$	$p$ -value for $\rho_1^{SX} = \rho_3^{SX}$
	Baseline	0.29	1.00
A1	estimate $\underline{r}$	0.18	1.00
A2	2-factor SRTSM	0.13	0.97
A3	Fama-Bliss	0.38	1.00
A4	5-factor FAVAR	0.70	1.00
A5	6-lag FAVAR	0.09	0.98
	7-lag FAVAR	0.19	0.97
	12-lag FAVAR	0.22	1.00

▶ Back

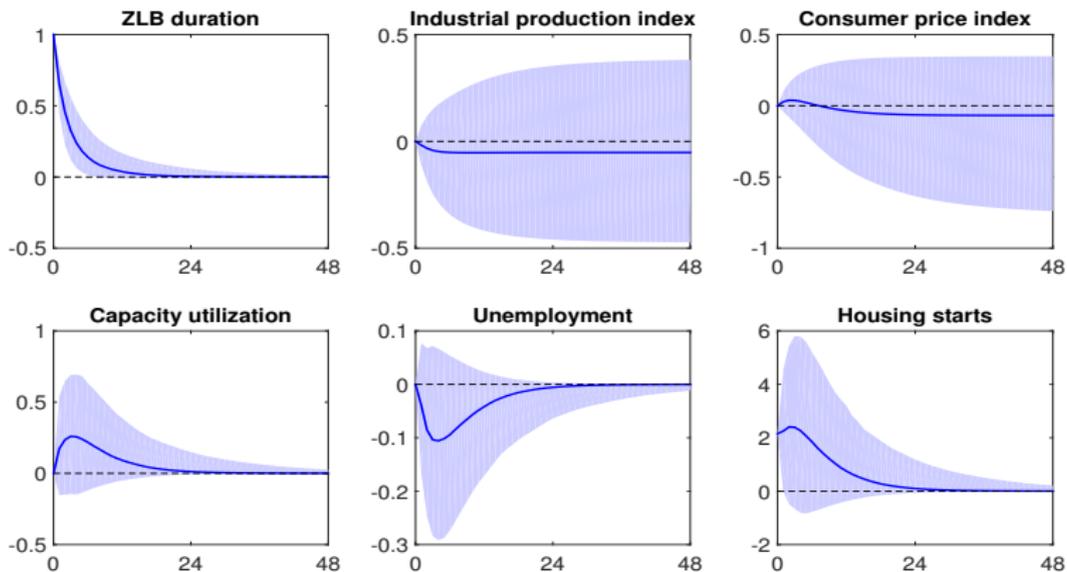


# Counterfactual II

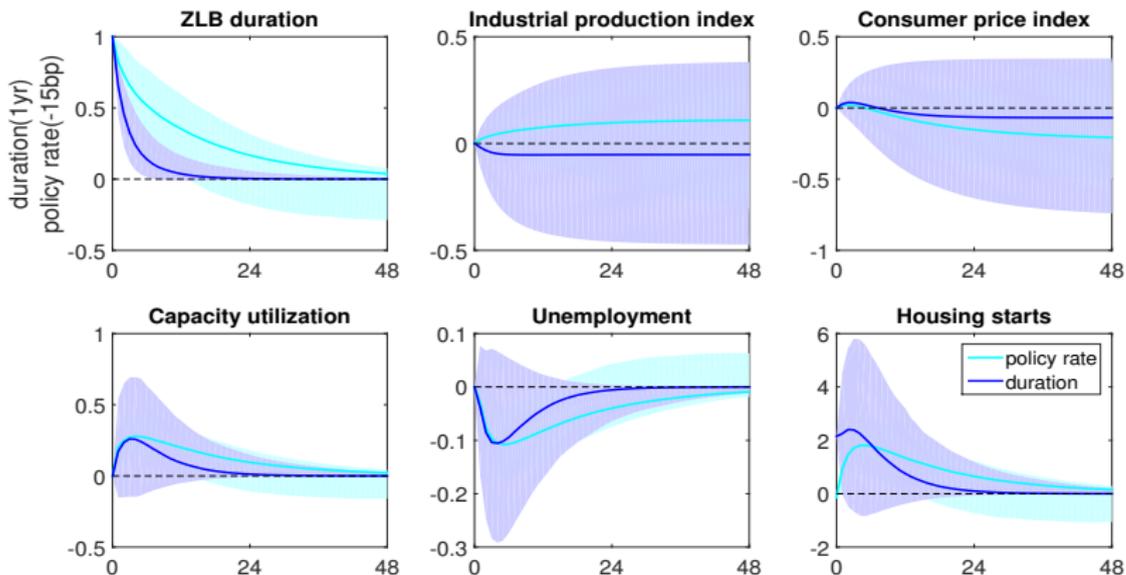


# Impulse responses: forward guidance

A monetary policy shock to increase the ZLB by 1 year



# Forward guidance vs. shadow rate



Unemployment rate decreases by 0.1% with

- ▶ a one year increase in the expected ZLB duration
- ▶ 15 basis-point decrease in the policy rate

▶ Back

