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Probabilistic Sustainability of Public Debt: A Vector Autoregression Approach for Brazil, Mexico, and Turkey

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This paper examines the sustainability of fiscal policy under uncertainty in three emerging market countries—Brazil, Mexico, and Turkey. For each country, we estimate a vector autoregression that includes fiscal and macroeconomic variables. Retrospectively, a historical decomposition shows by how much debt accumulation reflects unsustainable policy, adverse shocks, or both. Prospectively, Monte Carlo techniques reveal the primary surplus required to keep the debt-GDP ratio from rising in all but the worst 50, 25, and 10 percent of circumstances. Such a value-at-risk approach presents a clearer menu of policy options than do frameworks currently in use. [JEL D61, E61, E62]
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A *sustainable* fiscal policy is often defined as one that can be continued into the future without modification—no adjustment of the primary surplus and no default (by inflation or otherwise). At a minimum, the intertemporal solvency criteria must be satisfied. More often, sustainability has come to mean that the debt stock (or its ratio to output) does not rise.

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In developing countries, it may be difficult to design fiscal policy in such a way that no further modification will ever be necessary. Despite the best intentions of policymakers to stabilize the debt, there is some chance that adverse movements of interest rates, exchange rates, output, and other key variables may cause persistent increases in government debt. In this sense, sustainability is *probabilistic*. Accordingly, a fiscal authority may seek to reduce the probability that such adjustments will be necessary, through further increases of the primary surplus. A primary surplus that reduces the probability of future adjustments to less than 50 percent will, on average, *reduce* the debt.

This paper examines the sustainability of fiscal policy under uncertainty in three emerging market economies, namely, Brazil, Mexico, and Turkey. Sustainability is assessed both *retrospectively* (If historical policies were to be continued into the future, would fiscal policy be sustainable—or will a modification of policies be required?) and *prospectively* (What policies should be undertaken today in order to prevent the need for further adjustments in the future?).

Other retrospective assessments of fiscal sustainability do not emphasize uncertainty. The *fiscal gap* calculations of Blanchard and others (1990) and Talvi and Végh (2000) (see also Croce and Juan-Ramón, 2003) show how high a primary surplus must be in order to ensure sustainability. Such accounting-based (not econometric) calculations typically assume full knowledge about certain key variables, namely, long-run GDP growth and interest rates.

Econometric *solvency tests* introduced by Hamilton and Flavin (1986) (later extended by Wilcox, 1989; Trehan and Walsh, 1990 and 1991; Hakkio and Rush, 1991; and others) are also retrospective. They tell us whether the historical trajectory of fiscal data can be sustained into the future—but not how shocks to key variables will affect debt accumulation.

Prospective sustainability assessments feature mainly accounting (not econometric) frameworks wherein uncertainty appears in a rudimentary way. For example, the International Monetary Fund (2003b) template includes stress tests that examine fiscal outcomes if adverse shocks to certain key variables (interest rates or economic growth) occur in isolation.

However, such prospective assessments may require econometric (rather than accounting) approaches in order to better capture interactions between key variables and hence their *joint* impact on debt accumulation. For this reason, several studies (for example, Hoffmaister and others, 2001; Garcia and Rigobon, 2004; Hostland and Karam, 2005; Koeva, 2005; Celasun, Debrun, and Ostry, 2006; Guerson, 2006; and Penalver and Thwaites, 2006) propose using multivariate stochastic simulations of future debt behavior (consistent with an econometric approach).

Our approach to sustainability, both retrospective and prospective, differs from previous work. Retrospectively, beginning in some base period, the evolution of debt is attributed to either a baseline policy or accumulated shocks—movements of certain key explanatory variables that were not anticipated at the base period. A simple vector autoregression (VAR) model yields such *historical decompositions*. Hence, if under the no-shock forecast, the debt-GDP ratio does not rise, fiscal policy is sustainable in the way discussed by Blanchard and others (1990) and Talvi and Végh (2000). Innovations to nonpolicy variables that affect debt accumulation may be thought of as luck. Innovations to the primary balance contain both discretionary departures from an implicit fiscal policy rule and luck.

Prospectively, Monte Carlo simulations yield forecasts of debt-GDP ratio whose variance increases with horizon length. Several previous authors (for example, Garcia and Rigobon, 2004) present such simulations as familiar fan chart forecasts. However, as an additional step, we illustrate how such simulations are related to policy goals. Fan charts show precisely the adverse outcomes that policymakers typically wish to avoid. In our specific policy objective function—one also discussed in Tanner and Carey (2005)—the optimal primary balance is linked to both the level and variability of interest payments. Consistent with such an objective function, we present a menu of policy possibilities: alternative values of the primary surplus that would be required to stabilize debt at alternative probability levels (in our case, 75 and 90 percent) and horizons (in our case, from one to five years).[1]

Of course, when policies change, so might the behavior of market participants, but in ways not captured by the econometric model. This is the "Lucas critique." For example, the level of the primary surplus may itself affect interest rates (both level and volatility) in a way not captured in the data. This is an inherent shortcoming of both our work and other extant debt sustainability frameworks. It would be better to address the Lucas critique in a full-fledged general equilibrium model of debt sustainability, but work along these lines is still in its infancy.

I. Probabilistic Fiscal Sustainability: An Overview

Basic Identities

Any discussion of fiscal sustainability should begin with the public sector's budget constraint. In any period, we have

$$b_{t-1}(1+r) + \gamma_t - \tau_t = b_t, \tag{1}$$

where b is the real government debt, γ the noninterest expenditures, and τ is the tax revenues.

[1]The choice of probability and horizon reflects specific parameter values in the objective function. This is essentially a value-at-risk approach to fiscal policy (see also Barnhill and Kopits, 2003; and Adrogué, 2005).

Forward substitution of budget constraint over an infinite horizon Equation (1) yields the intertemporal constraint (from a period zero perspective):

$$b_{-1}(1+r) - E\left\{\sum_{t=0}^{\infty} ps_t/(1+r)^t\right\} = E\{\lim_{t\to\infty}[b_t/(1+r)^t]\}, \tag{2}$$

where $ps_t \equiv \tau_t - \gamma_t$ is the expected primary surplus. The well-known intertemporal solvency or "no-Ponzi game" condition is

$$E\{\lim_{t\to\infty}[b_t/(1+r)^t]\} = 0. \tag{3}$$

The goal is to satisfy conditions (2) and (3) without default—inflationary or otherwise. (State contingent government debt is not considered in this paper.) Thus, combining Equations (2) and (3), and manipulating, we obtain the familiar intertemporal solvency condition (from a period zero perspective):

$$b_{-1} = E\left\{\sum_{t=0}^{\infty} ps_t/(1+r)^{t-1}\right\}. \tag{3'}$$

Sustainability Under Uncertainty: Some Simple Examples

In this paper, the terms "constant policy," "constant primary surplus," and "constant primary balance" are used interchangeably. A constant policy that is sustainable is one in which the government can run the same primary surplus indefinitely into the future without resorting to default (by inflation or otherwise).

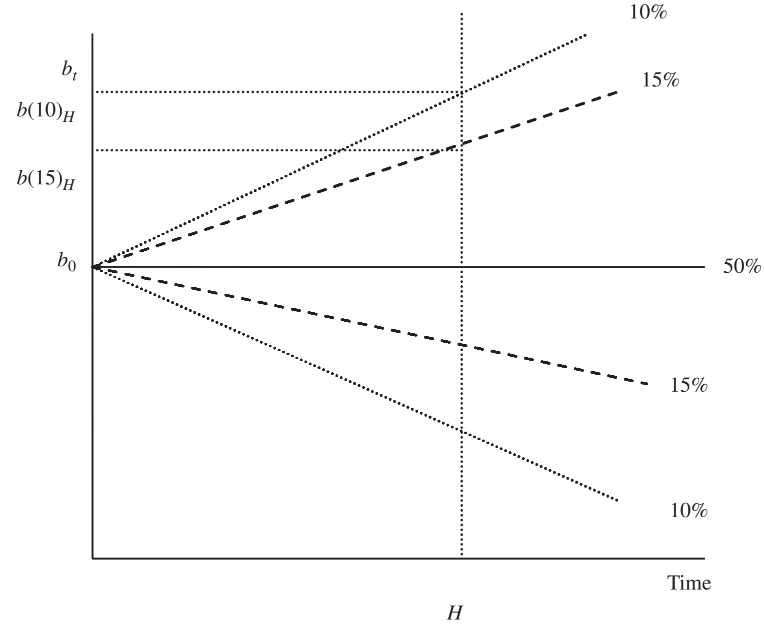
The message of Equation (3′) is clear: if $r_t > 0$ and if debt rises, to avoid default, the primary surplus *must* also rise. Under certainty, the sustainable *ps*, the constant value that guarantees satisfaction of Equations (2) and (3) without default is $ps = rb$. This policy, essentially Barro's (1979) tax smoothing idea, permits a government to stabilize the level of debt that it has inherited from a previous period. Note that if all variables of the model are written as ratios to GDP, and real GDP grows, that condition will be rewritten using a growth-adjusted discount factor, namely: $ps = [(r-g)/(1+g)]b$, where g = real GDP growth.

We now expand the discussion of fiscal sustainability to include a simple form of uncertainty. The economy is assumed to be open. Assume that the real interest rate r follows a *stationary* process, namely,

$$r_t = \bar{r} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma),$$

where $\bar{r}$ is the average interest rate. Recall the one-period budget constraint: $b_t = b_{t-1}(1+r_t) - ps_t$. Assume that, over a given horizon H $(0 < t < H)$, the government sets the primary surplus according to $ps_t = \bar{r}b_{t-1}$. It can be easily shown that, even if the real interest rate is stationary, debt will follow a random walk, $b_t = b_{t-1} + \omega_t$, where the term $\omega_t = (r_t - \bar{r})b_{t-1}$ is mean zero.

Figure 1. Debt Stabilization: 50 Percent Probability That $b_t \leq b_0$



This example is similar to ones discussed by Barro (1979) and Sargent (1987, p. 385).

A constant policy primary surplus, $\bar{r}b_0$, is a sustainable one if the debt does not rise above b_0; if the debt does rise, the authorities will then have to raise the primary surplus. Critically, the probability that the constant policy is sustainable is only 50 percent. That is, there is a 50 percent probability that the debt will rise; in this case, the primary surplus will have to be adjusted upward. Such a policy is shown in Figure 1.

This figure shows *constant policy* (constant *ps*). In this policy, the debt $b_t \geq b_0$ with a probability of 50 percent at any time t. At horizon H, the debt $b_H \geq b(15)_H$ with a probability of 15 percent; $b_H \geq b(10)_H$ with 10 percent probability.

Because debt follows a random walk, deviations of b from b_0 are permanent—hence an adjustment of the primary surplus is required. Tanner and Carey (2005) note that such an environment might generate the debt intolerance suggested by Reinhart, Rogoff, and Savastano (2003). That is, the long-run debt level is inherently unpredictable. Markets may be intolerant to even low levels of debt precisely because they doubt that the adjustment required to keep the government (ex ante) solvent will be politically feasible.

Policy Possibilities and the Objective Function

One may ask, "Why choose a policy whose probability of success is 50 percent?" There is nothing special about 50 percent. Rather, such a policy might be thought of as merely one option from a broader menu of *policy possibilities*: a higher primary surplus will make success more likely (decrease the probability that another adjustment will be required).

Thus, consider a general policy reaction function that includes such a broad menu of possibilities, namely, $ps_t = (\bar{r} + \bar{\kappa})b_{t-1}$. Note that $\bar{\kappa}$ is chosen by the policymaker; it should be the result of some optimal policy decision. For example, Tanner and Carey (2005) propose a constant absolute risk aversion/exponential objective (loss) function:

$$\Phi(\tau_t) = -1/\phi\, exp(-\phi\tau_t), \tag{4a}$$

where τ_t is the tax rate ($ps \equiv \tau - \gamma$, $\gamma \equiv$ primary expenditures) and $\phi \leq 0$ is a "prudence" parameter. It can be shown that minimizing this function subject to the intertemporal budget constraint yields a fiscal rule

$$ps_t^* \equiv [\tau_t^* - \gamma_t] = \kappa_t^* + rb_{t-1}, \tag{4b}$$

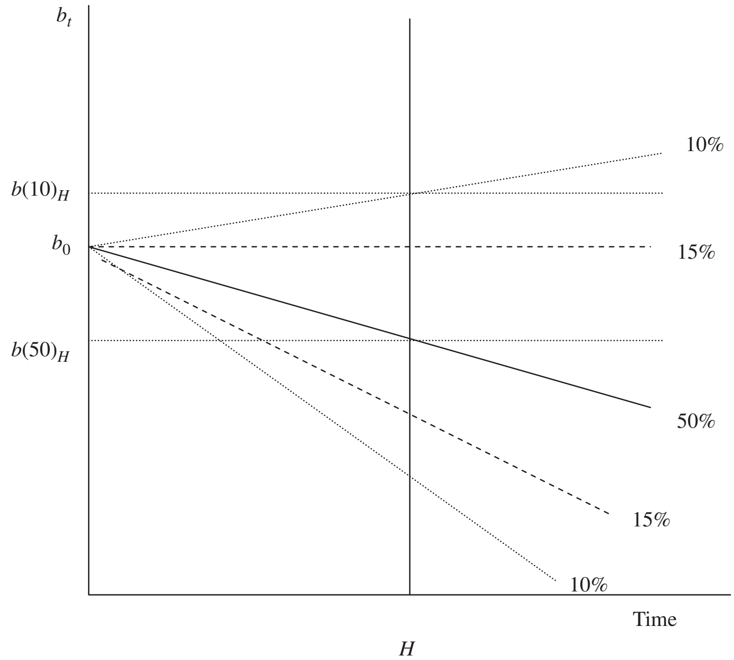
where $\kappa_t^* \equiv -\phi\sigma_t^2/2 \geq 0$ and σ_t^2 is the variance of the tax burden $(\gamma + rb_t)$. Now consider an authority whose (precautionary) objective is to cushion the country's residents from future adjustments. For such a government, $\phi < 0$ and therefore $\kappa_t^* > 0$. The country's primary surplus at any time is positively related to the variance of the tax burden $(\gamma + rb_t)$, and the debt ratio falls over time. By contrast, if the authority has no such precautionary objective, $\phi = 0$ and $\kappa_t^* = 0$. For this special case only, the optimal policy is $ps_t^* = rb_{t-1}$— debt stabilization/tax smoothing as discussed by Barro (1979), Sargent (1987), Blanchard and others (1990), and Burnside (2005).

Figure 2 shows one hypothetical policy for which $\kappa^* > 0$. Over an H-period horizon, the debt drops on average (that is, probability 50 percent) to $b(50)_H$; $b_H \geq b_0$ with 15 percent probability; $b_H \geq b(10)_H$ with 10 percent probability.

II. Methodology: VAR and Monte Carlo Simulation

The above discussion of debt accumulation and sustainability easily generalizes to a multivariate framework. To do so, we develop a VAR system. In this section, we describe the VAR system, including how it is ordered. Then we discuss the idea of *retrospective sustainability*: whether or not the debt would have risen if certain shocks had not occurred (as revealed by a *historical decomposition*). Finally, our concept of *prospective sustainability* asks, "What is the probability that debt (and hence primary surpluses) will rise above their current levels over a horizon of H years into the future?" We perform a Monte Carlo simulation of the VAR to compute such probabilities. In this sense, our paper is similar to Garcia and Rigobon (2004) and Celasun, Debrun, and Ostry (2006). However, in a more normative vein, we extend our analysis to illustrate several policy possibilities (such as those discussed above).

Figure 2. Debt Reduction: 15 Percent Probability That $b_t > b_0$



Description and Ordering of the VAR System

A VAR framework is well suited to examine debt accumulation and fiscal sustainability. The endogenous variables contained in the $(K \times 1)$ vector Y will include both fiscal and nonfiscal variables. One example is

$$Y = [ip_t, ps_t, \varepsilon_t, r_t]', \tag{5a}$$

where ip_t is the percent change of industrial production, ps the primary surplus, ε_t the growth of the real bilateral exchange rate vis-à-vis the U.S. dollar, and r_t is the real interest rate. Likewise, the vector of block exogenous variables X_t is also country specific, typically including discrete (dummy) variables for exchange rate regime or crisis dum_t, but also includes lagged oil price growth op_t.

Thus, the reduced form VAR system of order p is written[2]

$$\begin{aligned} Y_t =& [A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p}] \\ &+ [B_0 + C_0 X_t + C_1 X_{t-1} + C_2 X_{t-2}] + u_t, \end{aligned} \tag{5b}$$

[2]Oil prices are exogenous: they are determined in world markets and not affected by country-specific variables. Dummy variables may be thought of as a way to control for structural breaks in the series.

where the A_i's ($i = 1, \ldots, p$) are the matrices of autoregressive coefficients of order ($K \times K$; $K = 4$ in this example), B_0 is the column matrix of constant terms of order ($K \times 1$), and the C_i's ($i = 0$, 1, and 2) are the matrices of parameters associated with the other exogenous variables. C_i is a matrix of order ($K \times 2$) and u_t is a vector of error terms that are assumed to be a zero-mean independent white noise process with a time-invariant positive definite covariance matrix $E(u_t, u_t') = \Sigma_u$.[3] Data for all countries are discussed in Table A1 in Appendix I. Unit root tests presented in Table B1 in Appendix II reveal that all elements of Y_t and X_t are stationary ($I(0)$). Standard procedures conducted prior to estimation (Akaike, Schwarz) are used to choose lag length.

From the reduced form system of Equation (5b), one may derive a corresponding structural model that is obtained by assuming that the structural shocks are orthogonal; we impose certain plausible restrictions on elements of the system (5b). To do so, we use the well-known Choleski factorization, which implies a recursive relationship between the variables. We justify this recursive structure below.[4]

The industrial production, or GDP, equation (the first equation in system (5b)) represents the reduced form for aggregate output (which is determined by solving the aggregate supply and demand curves for output by eliminating changes in the price level—that is, inflation).

The primary surplus, or fiscal, equation (the second equation in system (5b)) reflects the government's fiscal policy behavior, which we allow to respond contemporaneously to output shocks (for example, automatic stabilizers), but not to other variables. This is logical: fiscal policy changes are made less frequently than other policy changes.

The third equation in system (5b) summarizes the central bank's behavior vis-à-vis the exchange rate (real, bilateral against the U.S. dollar). Shocks to this equation may be interpreted as innovations from international capital markets. We thus assume that the central bank allows the exchange rate to adjust immediately to output shocks, fiscal policy shocks, and shocks from international capital markets.

The real interest rate or monetary equation (the fourth equation in system (5b)) may be thought of as the central bank's interest rate rule. Potentially, the real interest rate could adjust immediately to output shocks, fiscal policy shocks, and shocks from international capital markets. Shocks to this equation may be interpreted as departures from the central bank's interest rate rule.

The Choleski decomposition is discussed further in Appendix II. Note also that debt level itself (b_t) is not among the endogenous variables but

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[3]The process defined in Equation (5) is assumed stable, insofar as $det(I_4 - A_1 x - A_2 x^2 - \ldots - A_p x^p) \neq 0$ *for* $|x| \leq 1$. Thus, the polynomial defined by the determinant of the autoregressive operator has no roots in or on the complex unit circle. Thus, the system's stability is not inconsistent with the stochastic nature of debt.

[4]We are indebted to an anonymous editor for critical input in this section.

instead is inferred from elements of Y_t by using the budget identity $b_t = b_{t-1}(1+r_t) - ps_t$. Note also that the random walk example from the previous section generalizes to the VAR setting: $\bar{r}_t$ can be reinterpreted as the conditional mean of the real interest rate—the forecast of r_t obtained from the fourth equation of system (5b).

Retrospective Sustainability

This paper has highlighted the role of macroeconomic shocks (both internal and external) on the evolution of public debt. We may thus ask about retrospective sustainability, "What would have been the evolution of public debt if certain shocks had not occurred?" The *historical decomposition* representation of a VAR, although less widely used than the (equivalent) variance decomposition, is better suited to answer such questions. For a precise derivation from the moving average representation of system (5b), see Appendix II. Below we present a simplified version thereof.

In any VAR, each element y_i of the vector Y can be expressed as the sum of two elements: (1) a baseline projection of that variable $y_i(base)_t$, conditional on all information available in the base period M; and (2) the (orthogonal) impacts of shocks from all variables thereon, including "own" shocks, accumulated from $M+1$ forward. That is, in any period $M+j$ $0 \le M \le M+j \le T$, the historical decomposition of y_i is

$$y_{i,M+j} = y_i(base)_{M+j} + z^*_{i1j} + z^*_{i2j} + \cdots + z^*_{iIj}, \tag{6}$$

where $y_i(base)_{M+j}$ incorporates all information about a variable that is available before time $M+1$, and z^*_{ikj} represents the impacts of the kth variable ($k = 1, 2, 3, \ldots I$) on the ith variable, accumulated from $M+1$ through $M+j$. Thus, Equation (6) yields a result that is counterfactual: what the value of variable y_i would have been if there had been no shocks to the kth variable y_k: $y_{i,M+j}(omit\ k) \equiv y_{iM+j} - z^*_{ikj}$.

As an example, consider the effects of industrial production (ip) shocks on debt accumulation. We note that ip has effects on both the interest rate and the primary surplus. Thus, counterfactually, the interest rate and the primary surplus, when purged of the effects of ip, would be written $r_{M+j}(omit\ ip) \equiv r_{M+j} - z^*_{r,ip,j}$ and $ps_{M+j}(omit\ ip) \equiv ps_{M+j} - z^*_{ps,ip,j}$. And combining these terms into a one-period budget constraint yields a measure of debt b when purged of the effects of ip. In any period, that measure is $b_{M+j}(omit\ ip) = b_{M+j-1}(omit\ ip) * (1 + r_{M+j}(omit\ ip)) - ps_{M+j}(omit\ ip)$.

Prospective Sustainability

To prospectively assess fiscal sustainability, simulations of the VAR system with randomly generated shocks are presented. Specifically, the simulated value of the debt b(sim) for any period $t > J$ is

$$b(sim)_t = b(sim)_{t-1^*}(1 + r(sim)_t) + pd(sim)_t, \tag{7a}$$

where simulated values of the interest rate and primary deficit, r(sim) and pd(sim), respectively, are

$$r(sim)_t = \zeta_{r0} + \zeta^*_{r1t} + \zeta^*_{r2t} + \cdots + \zeta^*_{rIt}, \tag{7b}$$

and

$$pd(sim)_t = \zeta_{p0} + \zeta^*_{p1t} + \zeta^*_{p2t} + \cdots + \zeta^*_{pIt}, \tag{7c}$$

where ζ_{r0} and ζ_{p0} are the assumed mean levels of the real interest rate and primary surplus, and the terms ζ^*_{rit} and ζ^*_{pit} are simulated impacts of shocks to variable i on the real interest rate and primary deficit, respectively.

Several other papers, including Garcia and Rigobon (2004); Celasun, Debrun, and Ostry (2006); and Penalver and Thwaites (2006), perform similar analyses. They present forecasts of debt—the mean value along with upper- and lower-confidence intervals that increase with time (fan charts). However, to expand on these previous papers, we use the simulations to learn about the policy possibilities discussed above. That is, we calculate the average primary surplus required to keep debt-GDP from growing with probabilities of w percent (for example, 90, 75, 50 percent) over horizons of one to five years.

Previous Econometric Work on Fiscal Sustainability

In our econometric treatment of fiscal sustainability, we depart from some previous research; see below for a brief discussion of why we do so. In a preliminary version of this paper (Tanner and Samake, 2006), we provide a more detailed literature review.

One approach would be to empirically estimate a (retrospective) fiscal rule such as $ps_t = \kappa + \alpha b_{t-1} + error_t$. It can be shown that ex ante solvency (equation (3)) is satisfied if there is a positive relationship between the primary surplus and debt ($\alpha > 0$). Papers that present variants of this test include Bohn (1998 and 2005), IMF (2003a), and Tanner and Ramos (2003). Unfortunately, estimates of the coefficient α may be inherently biased. The price level, and hence real debt b_{t-1}, will be endogenous because the price level itself adjusts to changes in expected future discounted primary surpluses (consistent with the fiscal theory of the price level). Moreover, mere solvency may be of little interest to policymakers. For example, suppose that $\kappa < 0$ and/or $\alpha < r$. In such a case, even while the government formally satisfies Equation (3), the debt (or its ratio to GDP) will still grow over time. Therefore, ever-increasing primary surpluses will be required to offset ever-increasing interest payments. Finally, such a test may indicate that a government is insolvent when it is not. To see this, suppose that α is statistically not different from zero, but $\kappa > rb_{t-1}$. Here, the government will be reducing debt and/or accumulating assets; hence, primary surpluses may be reduced in the future.

As an alternative, some authors have examined the present value of government liabilities by testing for the stationarity of the interest-inclusive deficit ($rb_{t-1}-ps_t$) or for the cointegration of tax revenues τ and interest-inclusive expenditures $\gamma+rb$. A precursor to this research was Hamilton and Flavin (1986); see also Kremers (1989), Trehan and Walsh (1990 and 1991), Hakkio and Rush (1991), Bohn (1991), Corsetti and Roubini (1991), Tanner and Liu (1994), Ahmed and Rogers (1995), Tanner (1995), and Uctum and Wickens (2000).

Without other coefficient restrictions, such tests may indicate that the government formally satisfies Equation (3) even if debt grows. Of course, such tests may be made more stringent. For example, we might impose a one-to-one cointegrating linkage between taxes τ and interest-inclusive expenditures $\gamma+rb$ with a zero intercept ($\kappa=0$). Such a restriction implies that debt is stabilized—something we might also learn from mere casual observation.

III. Brazil, 2000–05

In Brazil, public debt has grown from about 30 percent of GDP during the 1990s to about 51 percent in 2005 (all debt figures are adjusted for recognition of contingent liabilities as in Tanner and Ramos, 2003). This happened in the context of several dramatic changes in economic policies, including the Real Plan of 1994 (when inflation fell dramatically), increased exposure to global market shocks (spillovers from the Mexican, Asian, and Russian crises of 1995, 1997, and 1998, respectively), and the pegged exchange rate regime (within a narrow band) that ended in 1999 in crisis and devaluation.

The period 2000–05 is well suited for sustainability analyses such as those proposed in this paper. In 2000, in the aftermath of the crisis and while both an IMF program and an inflation targeting regime were being inaugurated, the primary surplus was substantially increased, from about zero to just over 3 percent of GDP. At that time the debt-GDP ratio was about 49 percent, but it was envisaged (see, for example, Ramos, 2001) that the fiscal adjustment would be large enough to gradually reduce the debt ratio to about 47 percent of GDP by 2005.

However, this hope was dashed. Instead, between 2000 and 2004, the debt ratio rose. Notwithstanding additional primary adjustments during this period, electoral uncertainties helped boost the real interest rate, real depreciation, and hence debt (whether denominated in domestic or foreign currency). In 2002, the debt ratio peaked at about 60 percent of GDP. Then, by 2004, both output and Brazil's currency, the real, recuperated, helping the debt ratio to fall—but not enough to meet the initial projections.

Econometric Preliminaries

The analysis of Brazil focuses on a vector Y_t of endogenous variables:

$$Y(Brazil)_t = [ip_t, ps_t, \varepsilon_t, r_t]', \tag{8}$$

where ip_t is the industrial production index, ps_t is the primary surplus, ε_t is real depreciation (bilaterally, against the U.S. dollar), and r_t is the (implicit) average real interest factor calculated directly from the budget constraint, namely, $r_t = ([b_t - ps_t]/b_{t-1}) - 1$, a measure of the real cost of government borrowing. Exogenous variables include the change in oil prices and discrete dummy intercept variables, as discussed below.

The frequent, dramatic regime changes in Brazil make it difficult to choose the sample period for estimation. Our principal estimates (see Table B2 in Appendix II) use the period from mid-1995 to mid-2005. To account for regime shifts, we include two dummy intercept variables: one to isolate the exchange rate crisis ($D = 1$ for $t = 1999{:}1$–$1999{:}4$ and $D = 0$ otherwise), and one to isolate the more recent floating rate period ($D = 0$ for $t = 1995{:}5$–$1999{:}4$, $D = 1$ thereafter). We also include a time trend in the estimates (thus capturing increases in the primary surplus during 2000–05). Standard tests (Aikaike, Schwarz) suggest that four lags should be included (see Appendix II).

We also discuss alternative estimates in Appendix II. These include results using only post-1999 data. Although these options do not yield identical results, they do confirm that the upsurge in debt from 2001 to 2003 was largely the result of innovations to exchange rates and interest rates, rather than baseline policy.

Retrospective Analysis, 2000–05

Table 1a presents results of a retrospective analysis whose baseline begins in mid-2000 ($M = 2000{:}5$) and ends in mid-2005. At the beginning of this period, Brazil's debt ratio was about 50 percent of GDP. As mentioned above, the debt ratio first rose and then fell dramatically; it returned to its initial ratio to GDP by 2005.

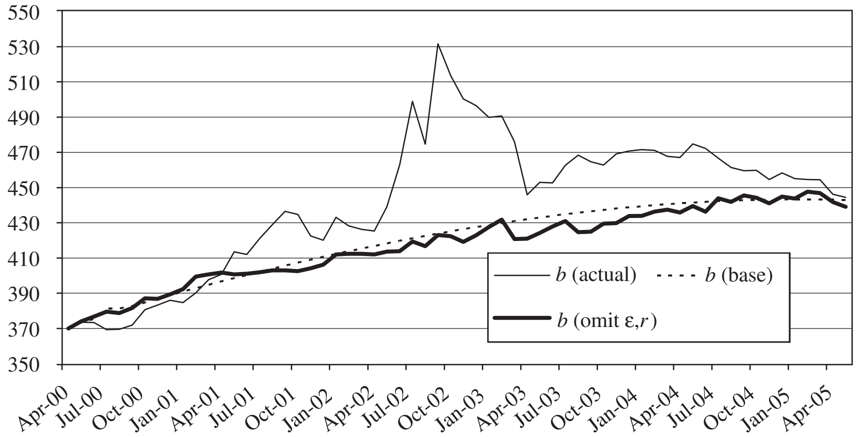
Movements in exchange rates and interest rates (reflecting shifts in investor sentiment) jointly explained more than 97 percent of variation in the debt ratio over this period. Figure 3 illustrates this: for most of the period, the debt level purged of these two shocks ($b(omit\ \varepsilon + r)$) is close to the baseline level ($b(base)$). When the debt peaks in September 2002, the cumulative (adverse) impact of these shocks ($b_t - b(omit\ \varepsilon + r)_t > 0$) was about 13 percent of GDP.

Thereafter, pressures subsided and the real recuperated. By mid-2005 (end sample) the gap ($b_t - b(omit\ \varepsilon, r)_t < 0$) fell to about 0.6 percent of GDP.[5]

As Figure 4 shows, innovations in output (industrial production) also affected the debt accumulation process—but to a much smaller extent than exchange rates and interest rates (less than 2 percent of total variation). In 2003, debt levels purged of the output shocks are higher than observed levels

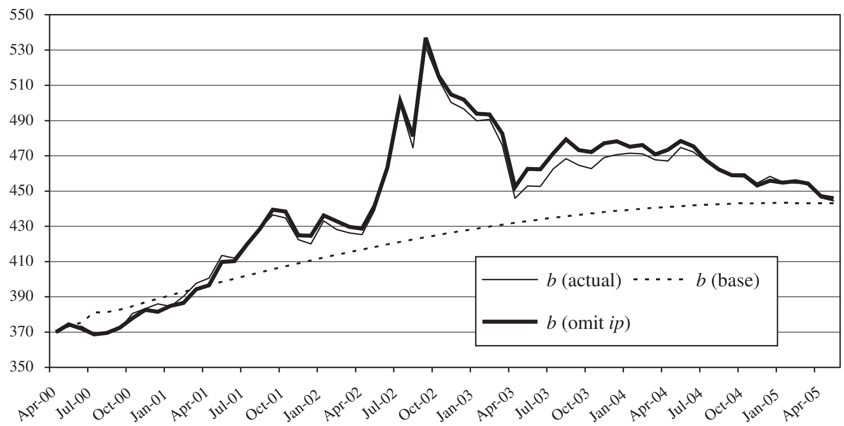
[5]That is, end-period values for b_t and $b(\text{base})_t$ are very close. In Appendix II, we show that this result depends on the precise specification.

Figure 3. Brazil: Real Public Debt Purged of Exchange Rate and Interest Rate Shocks $b(omit\ \varepsilon, r)$ in billions of constant reais (1995=100)



Source: Central Bank of Brazil.

Figure 4. Brazil: Real Public Debt Purged of Industrial Production Shocks $b(omit\ ip)$ in billions of constant reais (1995=100)



Source: Central Bank of Brazil.

$(b(omit\ ip)_t - b_t > 0)$—that is, during this period, industrial production shocks helped reduce the debt. In August 2003, the gap between $b(omit\ ip)_t$ and b_t is about 1.3 percent of GDP.

As mentioned above, there were several fiscal adjustments between 2000 and 2005: the primary surplus ratio rose from about 3.5 percent of GDP at end-2000 to more than 6 percent by mid-2005 (4.8 percent end-year). We

Table 1a. Brazil: Retrospective Analysis, 2000:5–2005:6

Historical Decomposition of Debt (b)	Percent of GDP
Initial debt (b_M)	50.5
End-period debt (b_{M+J})	50.8
Baseline projection ($b(base)_{M+J}$)	50.7
Shock component (b_{M+J}–($b(base)_{M+J}$)	0.2

Explaining Variation in Debt (b)	Percent of Total Variation, (b)	$b(omit)_{M+J}$ Percent of GDP
Oil price (*poil*)	0.8	51.2
Exchange rate, interest rate (ε, r)	97.2	50.2
Industrial production (*ip*)	1.9	51.0
Primary surplus (*ps*)	0.1	50.9
Initial primary surplus ratio ($ps(base))/GDP$		3.5
Debt-stabilizing constant ps/GDP		4.0

Source: Central Bank of Brazil.

capture such movements with a time trend.[6] From a tax-smoothing perspective, it might have been preferable if the initial adjustment in 2000 had been somewhat larger, thus avoiding the need for subsequent adjustments. As an illustrative calculation (Table 1a, bottom), if the primary surplus had initially been adjusted to about 4 percent of GDP (rather than 3.5 percent), the debt ratio would have been stabilized—without the need for further increases in the primary surplus (relative to GDP).

Prospective Analysis from 2005 Onward

Prospective analyses are presented in panels 1 and 2 of Table 1b. The upper portion of each panel shows a baseline scenario: the primary surplus for the initial year of the simulation (2005:6) is designed to roughly correspond to the actual policy: a primary surplus of about 4½ percent of GDP. The second portion of both tables presents the menu alternatives for probabilistic debt sustainability.

Panel 1 of Table 1b presents a scenario in which the real interest factor is assumed to be 12.8, in line with recent history. Mean GDP growth is assumed

[6]However, the counterfactual is derived from estimates that do not include a time trend. Note that the counterfactual mean primary surplus is calculated in constant-price reais; accordingly, the *ratio* of the primary surplus to GDP will fall somewhat over time.

Table 1b. Brazil: Prospective Sustainability from 2005:6

	Time Horizon (Years)				
	1	2	3	4	5
Panel 1. Average real interest rate $r \approx 12.8$ percent (historical value); GDP growth ≈ 4 percent; first-year primary surplus ps/GDP ≈ 4.5 percent					
Statistics					
Mean	52.19	52.51	53.05	53.65	54.72
Standard deviation	6.64	9.95	13.39	16.08	19.35
Median	51.82	51.93	51.41	51.28	51.73
75th percentile	56.63	58.55	61.29	62.30	64.18
90th percentile	60.79	65.98	70.38	74.87	79.31
Stabilizing debt (b) with probability					
50 percent; requires initial ps/GDP of:	5.44	5.17	5.14	5.14	5.20
Average debt ratio, end of horizon	51.40	51.40	51.40	51.40	51.40
75 percent; requires initial ps/GDP of:	9.31	7.78	7.41	6.85	6.66
Average debt ratio, end of horizon	47.30	45.75	43.85	43.57	42.79
90 percent; requires initial ps/GDP of:	12.92	10.55	8.75	8.92	8.60
Average debt ratio, end of horizon	43.52	39.79	39.41	34.13	31.38
Panel 2. Average real interest rate $r \approx 8.5$ percent (2005 survey data); GDP growth ≈ 3.5 percent (IMF estimate); first-year primary surplus ps/GDP ≈ 4.5 percent					
Statistics					
Mean	48.5	46.3	44.1	41.7	39.5
Standard deviation	3.8	5.3	6.7	7.5	8.4
Median	48.4	45.9	43.6	41.3	38.9
75th percentile	51.0	49.6	48.6	46.1	44.5
90th percentile	53.5	53.7	53.1	51.7	50.2
Stabilizing debt (b) with probability					
90 percent; requires initial ps/GDP of:	6.6	5.5	5.0	4.6	…
Average debt ratio, end of horizon	46.4	44.2	42.5	41.6	…

to be just under 4 percent; this reflects the optimistic side of current market assessments (for example, Jaeger, 2006).

Simulations (1,000 draws) reveal a modest increase in the mean debt-GDP ratio, from 51.4 percent in 2005 to 54.7 percent in 2010. They also show the probability of less desirable outcomes: by 2010, the debt ratio exceeds 64.2 and 79.3 percent with probabilities of 25 and 10 percent, respectively.

Panel 2 of Table 1b presents a menu of policy alternatives. The first line shows that government must run a primary surplus of at least 5 percent of GDP if it wishes to stabilize the debt on average, over any horizon. The third line shows the surplus that would be required to keep the debt from rising with 75 percent probability. Note that as the horizon increases, the required primary surplus falls. For a one-year horizon, such an objective would require a 9.3 percent primary surplus; on average, the debt ratio would fall to 47.3 percent of GDP (line 4). By contrast, applying this objective over a five-year horizon requires a primary surplus of only 6.7 percent of GDP; on average, the debt-GDP ratio falls to 42.8 percent in 2010. This confirms that the benefits of debt reduction accumulate over time. The fifth line shows the surplus that would be required to keep the debt from rising with 90 percent probability. Applying this objective over a five-year horizon implies that the primary surplus must be 8.6 percent of GDP; on average, the debt-GDP ratio falls to 31.4 percent in 2010.

Alternatively, Panel 2 of Table 1b presents a more moderate scenario. Interest rates are lower, about 8½ percent (average) over the period, reflecting survey data from 2005. Also, in this scenario, GDP growth is also lower—about 3½ percent (consistent with recent IMF estimates). Under these assumptions, by 2010, average debt falls to 39½ percent. By end-of-horizon, the probability that debt does not rise is greater than 90 percent. The table also shows the adjustment required to stabilize debt with a 90 percent probability for shorter horizons. For a three-year horizon, for example, the required initial primary surplus is 5 percent of GDP.

A caveat should be placed on these results. Under Brazil's recent debt management strategy, the fraction of debt that is denominated or indexed to the U.S. dollar has dropped substantially. This may reduce exchange rate risk, but only to the extent that exchange rate shocks are not transmitted to interest rates (that is, deviations from uncovered parity). In fact, some (unreported) simulations wherein exchange rate shocks were omitted yielded results very close to those presented in Panel 2 of Table 1b.

Counterfactual Prospective, 2000–05

Table 1c provides another "rearview mirror": it summarizes what the model might have shown if it had been used in 2000—using only the data then available.[7] The upper part of the table indicates that the assumed primary surplus (3.5 percent of GDP for the initial year and slightly less thereafter) might not have been perceived to stabilize the debt beyond the initial year.

Rather, over a five-year horizon (2000 through 2005) the mean debt-GDP ratio rises from 50.4 to 55.3 percent. More important, the simulations suggest more dramatic increases in the debt ratio might have been foreseen: over a five-year horizon, simulated debt exceeds 62.1 and 70.8 percent with probabilities of 25 and 10 percent, respectively.

The bottom part of Table 1c suggests that somewhat more stringent fiscal adjustment would have been necessary to insure against sharp increases in the

[7]The dummy variable in the equation here applies to 1999:1–1999:3—the exchange rate crisis and its immediate aftermath only. For these simulations, mean output growth is adjusted to conform roughly to expectations held at that time.

Table 1c. Brazil: Counterfactual Prospective Sustainability from 2000:5
(First-year primary surplus ps/GDP ≈ 3.5 percent)

	Time Horizon (Years)				
	1	2	3	4	5
Statistics					
Mean	50.43	51.23	52.27	53.54	55.26
Standard deviation	4.37	6.41	8.48	10.03	12.12
Median	50.22	51.15	52.15	53.11	54.33
75th percentile	53.47	55.23	57.73	59.44	62.08
90th percentile	56.29	59.66	63.44	66.76	70.82
Stabilizing debt *(b)* with probability					
50 percent; requires initial ps/GDP of:	3.21	3.81	4.01	4.10	4.18
Average debt ratio, end of horizon	50.76	50.13	50.66	50.93	51.42
75 percent; requires initial ps/GDP of:	6.21	5.62	5.54	5.38	5.36
Average debt ratio, end of horizon	47.62	46.74	45.60	45.12	44.49
90 percent; requires initial ps/GDP of:	8.65	7.43	7.00	6.62	6.50
Average debt ratio, end of horizon	45.08	42.84	40.76	39.47	37.77

debt-GDP ratio that did occur. For a five-year horizon (by 2005), primary surpluses of about 4.2, 5.4, and 6.5 percent would have stabilized the debt with probabilities of 50, 75, and 90 percent, respectively.

IV. Mexico

Mexican public debt contains both a traditional component and a sizable augmentation. Over the past two decades, the traditional component fell from almost 100 percent of GDP in the late 1980s to around 18 percent of GDP in 2005. The augmentation, which includes liabilities associated with bank bailouts and development borrowing, became important after the crisis of 1994–95. When summed, the two elements reached about 45.3 percent of GDP in 2005. Because only the traditional component of public debt is available on a monthly basis, the retrospective analysis covers only this measure. However, the prospective analysis highlights the augmented measure.[8]

Econometric Preliminaries

Estimates reported in Table B3 in Appendix II use monthly data from mid-1997 to mid-2005. (This period thus omits both the 1994 crisis and its aftermath.) For Mexico, the vector $Y(Mexico)_t$ is defined as

$$Y(Mexico)_t = [ip_t, ps_t, \varepsilon_t, \Delta b, r_t]', \tag{9}$$

[8]We do not consider implicit liabilities of public social security systems (see Bauer, 2002).

where Δb_t is the real operational deficit (that is, the change in real debt), and r_t is the real interest rate as traditionally defined $(1+i_\tau)/(1+\pi_\tau)-1$. As detailed in Appendix II, six lags are included. This specification incorporates the familiar discrepancy between overall balance (above the line) and the change in government debt (below the line). (This discrepancy is also present in data from Turkey but not from Brazil.) Of course Δb_t contains information about other variables included in the VAR, namely, the primary deficit and real interest rates. But the VAR filters out precisely these effects. Because shocks to Δb_t are orthogonal to both exchange rates and real interest rates, they are treated as an additional (error) component of the primary balance. Note also that, in this ordering, all other shocks except those to the real interest rate r are assumed to affect Δb contemporaneously. Thus, central bank decisions (shocks to r) are assumed to affect the fiscal balance with a lag. However, although estimated separately, the impacts of orthogonal innovations to primary and operational deficits are presented as a single "fiscal shock" to conserve space (Table 2a and Figure 6).

Retrospective Analysis, 1998–2005

In the base period ($M = 1999{:}5$), the debt-GDP ratio was about 19 percent. By the end of the sample, the baseline forecast debt ratio was 17.7 percent (Table 2a). Thus, in the absence of shocks, the debt ratio in Mexico would have fallen slightly over the 1999–2005 period. Hence, looking at the traditional measure in isolation, fiscal policy was *sustainable*.

According to Table 2a, over the entire period, the impacts of shocks to interest rates and real exchange rates (combined) and industrial production helped to slightly reduce the debt ratio, while oil price shocks had a small but positive impact on the debt, suggesting that oil windfalls were spent, not saved. Furthermore, Figure 5 suggests that the beneficial impacts of interest rate reductions and a stronger peso were most important in 2001–02.

However, the discretionary shocks represented a slightly positive impact on the debt. As Table 2a shows, omitting these shocks would have further reduced the debt by about 0.5 percent of GDP. As Figure 6 shows, such shocks were especially important during 2000–01 and again in 2004–05; there were discretionary expansions of fiscal policy during these periods. Table 2a reveals that, even though revenues from the state-run oil company (PEMEX) are a substantial source of public sector revenue, oil price changes played an only minor role in debt accumulation. This suggests that oil price changes were transmitted both to revenues and to spending in a way that left the primary balance unchanged.[9]

[9]If entered in *levels*, either the oil prices or real PEMEX revenues did display larger impacts on the debt than those presented here. However, because both variables are level nonstationary (unlike the other variables), such a finding is potentially spurious and hence not emphasized.

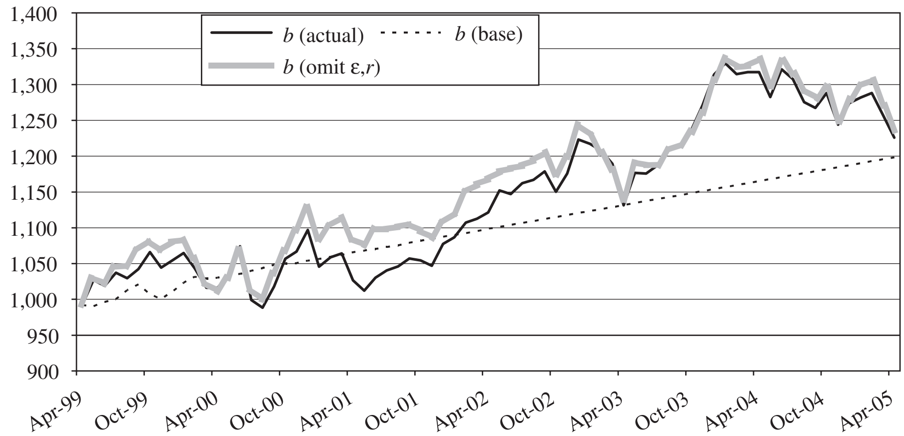
Table 2a. Mexico: Retrospective Sustainability, 1999:5–2005:4

Historical Decomposition of Debt (*b*)	**Percent of GDP**
Initial debt (b_M)	19.70
End-period debt (b_{M+J})	18.10
Baseline projection ($b(base)_{M+J}$)	17.69
Shock component (b_{M+J}–($b(base)_{M+J}$)	0.41

Explaining Variation in Debt (*b*)	**Percent of Total Variation, *b***	**$b(omit)_{M+J}$ Percent of GDP**
Exchange rate, interest rate (ε, *r*)	15.5	18.23
Deficit (Δ*b*, −*ps*)	79.6	17.40
Industrial production (*ip*)	2.2	18.29
Oil price (*poil*)	2.8	18.08

Source: Central Bank of Mexico.

Figure 5. Mexico: Real Public Debt Purged of Exchange Rate and Interest Rate Shocks *b*(*omit* ε, *r*) in billions of real pesos (2000=100)

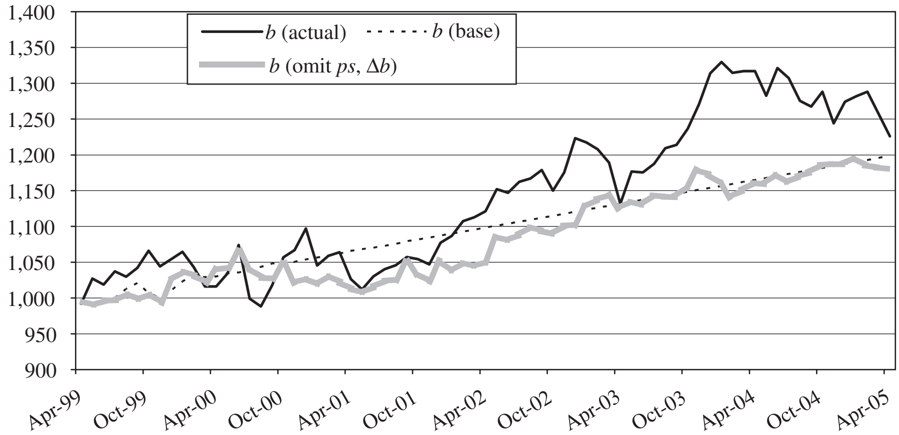


Source: Central Bank of Mexico.

Prospective Analysis from 2005 Onward

For a prospective analysis, the upper part of Table 2b presents a scenario that is comparable to one presented in a recent IMF Staff Report (IMF, 2005) insofar as similar interest rates, GDP growth rates, and average overall surpluses are assumed in both. The key feature of both scenarios is that a

Figure 6. Mexico: Real Public Debt Purged of Deficit Shocks *b*(*omit ps*, Δ*b*) in billions of real pesos (2000 = 100)



Source: Central Bank of Mexico.

Table 2b. Mexico: Prospective Sustainability from 2005:6
(Initial period primary surplus ps/GDP ≈ 2.1 percent)

	Time Horizon (Years)				
	1	2	3	4	5
Statistics					
Mean	45.0	43.5	41.9	40.2	38.7
Standard deviation	2.1	3.4	4.2	4.9	5.6
Median	44.9	43.3	41.5	40.0	38.1
75th percentile	46.5	45.7	44.6	43.1	42.0
90th percentile	47.7	47.9	47.6	46.6	46.0
Stabilizing debt (*b*) with probability					
75 percent requires initial ps/GDP of:	3.2	2.3	1.9	1.6	1.5
Average debt ratio, end of horizon	42.7	43.1	42.6	42.3	41.9
90 percent requires initial ps/GDP of:	4.4	3.2	2.8	2.4	2.3
Average debt ratio, end of horizon	42.7	41.2	39.9	39.1	38.0

modest primary surplus—about 2.1 percent of GDP—will reduce the debt over a five-year horizon from 40 percent of GDP in 2005 to just under 38 percent in 2010. However, the table presents a somewhat different picture of the risks associated with such a policy. By 2010, there is a 10 percent chance that debt will exceed 46 percent.

The bottom part of Table 2b presents a menu of policy options for Mexico. For example, to stabilize the debt at the current value of 45.3 percent of GDP with a 75 percent probability, over a five-year horizon, an initial primary surplus of about 1.5 percent of GDP would be required; to stabilize the debt with a 90 percent probability requires a primary surplus of about 2.3 percent of GDP. As before, a longer horizon implies a less stringent adjustment.

Some caveats should be made regarding debt sustainability for governments that obtain revenues from nonrenewable resources or temporary bonanzas (in Mexico, petroleum). Typically, spending from a windfall should be spread out over time. Our estimations suggest, however, that this has not happened in Mexico. Rather, recent high oil revenues have helped the authorities to boost spending—not the surplus.

Therefore, if oil revenues fall, in order to maintain the same primary surplus, the Mexican authorities will have to (symmetrically) cut *spending*. Conversely, if the Mexican authorities wished to retain both current levels of expenditures and the primary surplus, it would have to raise additional (nonoil) revenues.

V. Turkey

Until recently, Turkey has suffered from chronic high inflation and fiscal imbalances. In the currency crises of 1994 and especially in 2000–01, the recognition of unrecoverable assets in the banking sector, along with defensive interest rate hikes, substantially increased debt. However, more recently, under an IMF program that featured a primary surplus exceeding 6½ percent of GDP, this adjustment, along with higher GDP growth and a real appreciation of the Turkish lira, helped reduce the debt ratio from more than 90 percent of GDP in 2000 to about 55½ percent in 2005–06.

Econometric Preliminaries and Retrospective Analysis

Estimations (Table B4 in Appendix II) use monthly data from mid-1994 to mid-2005. For Turkey, vector Y_t is defined exactly as for Mexico (see equation (9)); eight lags are included. To account for the extraordinary nature of the currency crisis in 2001, the estimates include a crisis dummy that equals unity for the crisis period 2001:2–2001:6 and zero otherwise.

The retrospective analysis suggests that, prior to the 2001 currency crisis, fiscal policy was unsustainable. As Table 3a shows, in mid-1996, the initial debt-GDP ratio was about 43.2 percent. Note that shocks to fiscal policy itself account for 60 percent of the total variation in debt—see also Figure 7. In turn, most of this variability—about three-fourths—is a result of shocks from below the line, including the assumption of financial sector obligations. (In the prospective analysis below, such shocks are omitted.) Between 1996 and 1998, expansionary deficit shocks helped boost the debt. Thereafter, policy tightened somewhat. From mid-1996 through late 2000, if the deficit shocks are omitted, the debt is higher than baseline by about 1.9 percent of GDP. However, this effect was largely offset by shocks to industrial production; omitting such shocks reduces the debt by about 1.9 percent of

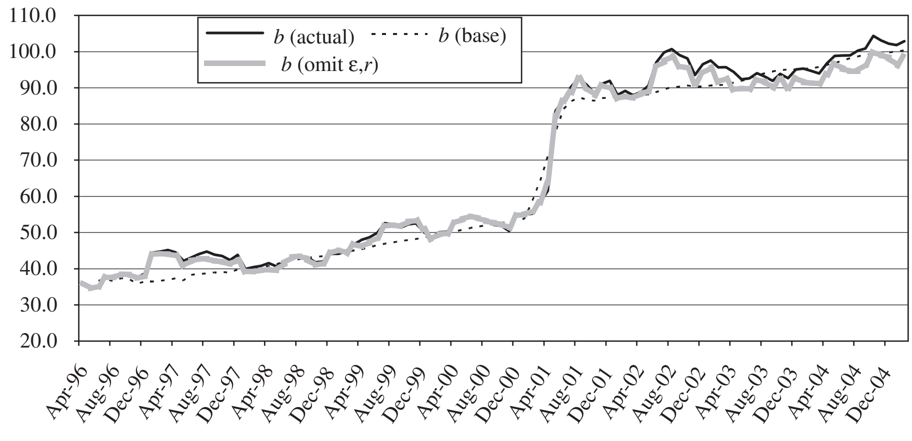
Table 3a. Turkey: Retrospective Analysis, 1997:5–2000:9

Historical Decomposition of Debt (b)	**Percent of GDP**
Initial debt (b_M)	43.2
End-period debt (b_{M+J})	48.8
Baseline projection ($b(base)_{M+J}$)	49.1
Shock component (b_{M+J}–($b(base)_{M+J}$)	–0.2

Explaining Variation in Debt (b)	**Percent of Total Variation, b**	**$b(omit)_{M+J}$ Percent of GDP**
Oil price ($poil$)	8.3	48.1
Exchange rate, interest rate (ε, r)	20.4	49.3
Deficit (Δb, $-ps$)	60.0	50.9
Industrial production (ip)	11.3	47.1

Source: Central Bank of Turkey.

Figure 7. Turkey: Real Public Debt Purged of Exchange Rate and Interest Rate Shocks $b(omit\ \varepsilon, r)$ in billions of new Turkish lira (2000=100)



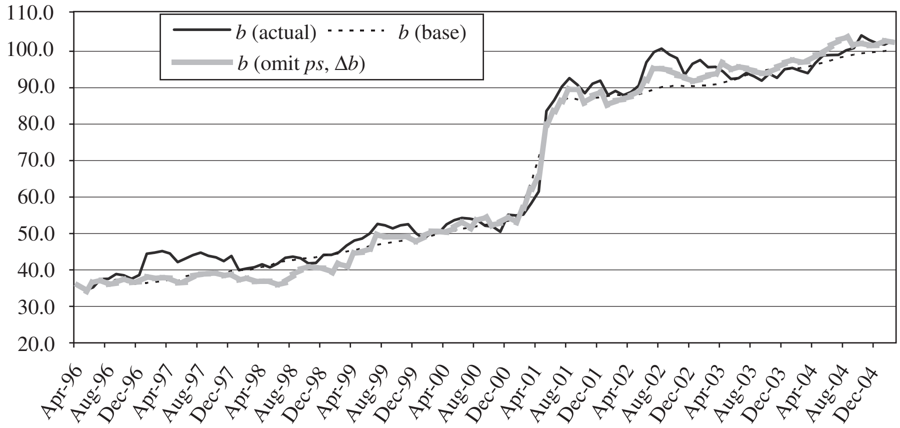
Source: Central Bank of Turkey.

GDP. After the 2001 crisis, volatility in exchange rates and interest rates helped increase debt levels—see also Figure 8; omitting such shocks leaves debt lower than baseline by about 1 percent of GDP.

Prospective Analysis from 2005 Onward

Prospective simulations give reason for cautious optimism. The upper portion of Table 3b presents a scenario that broadly reflects recent assumptions about

Figure 8. Turkey: Real Public Debt Purged of Deficit Shocks *b*(*omit ps*, Δ*b*) in Billions of New Turkish Lira (2000=100)



Source: Central Bank of Turkey.

Table 3b. Turkey: Prospective Sustainability from 2005:5
(Average primary surplus ps/GDP ≈ 6.5 percent, years 1 through 5)

	Time Horizon (Years)				
	1	**2**	**3**	**4**	**5**
Statistics					
No-shock scenario	49.6	45.5	40.8	36.2	31.7
Mean	51.3	49.2	46.0	42.8	39.4
Standard deviation	6.3	9.7	11.7	13.5	15.1
Median	50.9	48.0	44.8	40.4	37.3
75th percentile	55.3	55.0	52.9	50.9	47.2
90th percentile	59.9	61.9	61.3	60.8	58.9
Stabilizing debt with probability					
90 percent; requires initial ps/GDP of:	10.8	9.5	8.5	8.1	7.5
Average *ps/GDP* years 1–5	10.3	9.1	8.0	7.6	7.1
Average debt ratio, end of horizon	47.4	43.8	41.2	38.2	36.5

Turkey: the mean primary surplus is assumed to be maintained at about 6.5 percent of GDP in 2006 and afterward; average economic growth is just under 5 percent and the average real interest rate is about 8 percent a year. Importantly, the prospective scenario, like the retrospective, includes shocks to the primary balance (*ps*) and the real interest payments. However, as mentioned above, below-the-line shocks—such as public assumption of financial sector obligations—are prospectively assumed to be zero.

The Turkish case illustrates how macroeconomic shocks do not always cancel out for debt accumulation. In the absence of any shocks (no-shock scenario, first line of Table 3b), the debt-GDP ratio falls from 55.5 percent of GDP to about 32 percent of GDP by 2010. However, when random shocks are included, the mean debt ratio falls less dramatically, to about 39 percent of GDP. Regarding risks, the table reveals that over the five-year horizon, there is a 10 percent probability that the debt-GDP ratio will rise to at least 59 percent of GDP.

The bottom part of Table 3b shows the primary surplus that is required to stabilize the debt with a 90 percent probability—for certain horizons. As with Brazil, a longer horizon implies a smaller adjustment. For a one-year horizon, such an objective would require a primary surplus of just under 11 percent of GDP in the first year (declining thereafter, about 10 percent over the entire horizon). By contrast, for a five-year horizon, the required primary surplus must average about 7 percent of GDP over the entire horizon.

VI. Comparisons and Conclusions

This paper has examined the sustainability of fiscal policy under uncertainty in three emerging market economies—namely, Brazil, Mexico, and Turkey—both retrospectively and prospectively. Our retrospective assessment differs from previous work in the way that it decomposes the effects of a baseline policy, policy shocks, and other shocks. Our prospective approach to debt sustainability has at least two advantages over those currently in use. First, an econometric framework like ours uses data to inform the policy process in a richer and more sophisticated way than accounting frameworks. Second, we believe that our framework communicates a clearer statement of policymaker preferences and a clearer menu of their options than other current frameworks. In addition to illustrating adverse outcomes, our framework reveals how we might avoid such outcomes—and with what probability.

The three countries chosen—Brazil, Turkey, and Mexico—share some similar features. All three countries are emerging markets with substantial public debt-GDP ratios (50 percent or greater). In recent years, all three countries have suffered fiscal imbalances and have struggled to reduce inflation. But, among these three countries, there are important differences in the debt accumulation process. Shocks to fiscal policy itself have a much greater impact on debt accumulation in Mexico and Turkey than in Brazil. Interest rate and exchange rate shocks, although important in all three countries, were especially important in Brazil.[10] In that context, considerable attention was paid to the shocks that each country individually faced.

[10]Such results may reflect the fact that the standard deviation of the primary surplus relative to debt (s.d.$(ps)/b$) is lower in Brazil (s.d.$(ps)/b = 0.54$) than in either Mexico (s.d.$(ps)/b = 1.97$) or Turkey (s.d.$(ps)/b = 1.30$). Note that "deficit" shocks are not strictly comparable across countries. Because Brazil has better data, above and below the line deficit measures are equal. Hence "deficit" shocks in Brazil are limited to the primary balance. By contrast, in Mexico and Turkey, there were large "below-the-line" elements—for example, the bank recapitalizations in Turkey—that were also included in a "deficit" shock. Also, as discussed in the text, a time trend for the primary surplus was included only for Brazil.

Dummy variables, especially for Brazil and Turkey, reflected judgment that certain events (for example, exchange rate crises) were sui generis.

In terms of policy, the analysis for debt sustainability in these three countries can be termed optimistic—but only conditionally so. In the case of Brazil, the current primary surplus (about 4¼ percent of GDP) will continue to yield debt reduction only if real interest rates remain stable or fall. In the case of Mexico, although the country appears to be stabilizing debt, it may be doing so by drawing down its petroleum-based wealth. In the case of Turkey, the optimistic scenario is conditioned on maintenance of substantial primary surpluses—about 6½ percent of GDP.

As a next research step, the analysis in this paper might be placed into a general equilibrium model (see Mendoza and Oviedo, 2004). Doing so would permit a true welfare analysis of the costs and benefits of preempting further fiscal adjustment. The optimal primary surplus and debt reduction path would thus be tailored to a country's specific circumstances.

APPENDIX I

Table A1. Sources and Definitions of Variables Used

Variable	Brazil	Mexico	Turkey
Industrial production (*ip*)	CB—11064[1]	CB—SR953[1]	CB—TP.TSY01
Primary balance (*ps*)	CB—4649; Consolidated	CB—SG17	CB- TP.KB.K19
Debt (*b*)	CB—4478; Consolidated[2]	CB—SG199; IMF Staff Report	CB-TP.KB.A09; IMF Staff Report[3]
Inflation (π)	CB—188; National CPI	CB—SP1 CPI	IMF-IFS S64 CPI
Exchange rate (ε)	IMF-IFS RF (period average.)	IMF-IFS RF (period average.)	IMF-IFS RF (period average)
Interest rate (*r*)	CB 11; Selic[4]	IMF-IFS 60C.ZF (Treasury bill rate)	IMF-IFS 60B.ZF (Interbank rate)

Sources: Central bank websites: Brazil: http://www.bcb.gov.br/?ECONOMY; Mexico: http://www.banxico.org.mx/tipo/estadisticas/index.html; Turkey: http://tcmbf40.tcmb.gov.tr/cbt-uk.html.

[1]Seasonally adjusted.

[2]Historical "traditional" fiscal numbers from Central Bank of Mexico; prospective calculations based on "augmented" fiscal figures from recent IMF Staff Reports (see Staff Report 06/352, Table 2; http://www.imf.org/external/pubs/cat/longres.cfm?sk = 19981.0).

[3]Foreign debt: yearly, from IMF Staff Reports; monthly *foreign* debt numbers are interpolated (see Staff Report 06/268, Table 2; http://www.imf.org/external/pubs/cat/longres.cfm?sk = 19469.0).

[4]Used for supplementary calculations.

APPENDIX II

Econometric Methodology and Estimates

Model Setup and Preliminary Tests

Without loss of generality, VAR system (5b) can be rewritten as

$$Y_t = \sum_{i=1}^{p} A_i Y_{t-i} + DX_t + u_t,$$

$$E(u_t u_t') = \Sigma_u, \quad u_t \sim N(0, \Sigma_u), \tag{A.1}$$

where Y_t is the vector of endogenous variables; X_t is the vector of deterministic factors, including the constant terms, trend, dummy variables, and oil price; and u_t is a vector of error terms. As discussed in the text, elements of (Y_t) differ across countries. For Brazil, we have

$$Y_t'(Brazil) = (ip_t, ps_t, \varepsilon_t, r_t).$$

For Mexico and Turkey, to capture below-the-line adjustments, the real operational deficit (that is, the change in real debt Δb_t) is also included:

$$Y_t'(Mexico\ or\ Turkey) = (ip_t, ps_t, \varepsilon_t, \Delta b_t, r_t)$$

Equation (B.1) can be expressed in a *moving-average* form:

$$Y_t = C(L)(DX_t + u_t) = \sum_{i=0}^{+\infty} ((D(L)L^i(X_t)) + C(L)u_t), \tag{A.2}$$

where L is the lag operator. Assume both sides of Equation (A.2) are stationary. In this case, Equation (A.2) can be written as

$$Y_t = \sum_{i=0}^{+\infty} D_i X_{t-i} + \sum_{i=0}^{+\infty} \phi_i u_{t-i} = Z_t + \sum_{i=0}^{+\infty} \phi_i u_{t-i,} \tag{A.3}$$

where $D_i = D(L)$ and $Z_t = \sum_{i=0}^{+\infty} D(L)L^i X_t$.

This representation assumes that all endogenous elements of the vector Y_t are stationary (that is, an $I(0)$ process). Thus, prior to the estimations, unit root tests (augmented Dickey-Fuller, 1979, and Phillips-Perron, 1988) are performed on the individual variables. Results are presented in Table B1. These tests confirm that all elements of Y_t, and oil price growth (an element of Z_t) are stationary. Thus, estimates of the (raw) VAR are summarized for Brazil in Table B2, Mexico in Table B3, and Turkey in Table B4.

Choleski Factorization

Note that Equation (A.1) can be written in matrix form as

$$B(L) = \sum_{i=0}^{\infty} B_i L^i,$$

$$B(L)Y_t = DX_t + u_t(\text{matrix lag operator}, B_0 = I). \tag{A.4}$$

Assuming $B(L)$ to be invertible, Equation (A.4) can also be written as a moving average, namely, $Y_t = C(L)(DX_t + u_t)$, where $C(L) = B(L)^{-1}$. Structural, orthogonal disturbances

Table B1. Brazil, Mexico, and Turkey: Model Specifications and Diagnostic Tests

Univariate Stationarity Tests, Endogenous Variables

	Brazil		**Mexico**		**Turkey**	
Dates	1995:5–2005:5		1997:8–2005:5		1994:5–2005:3	
Lags	4		4		4	
	ADF	**PP**	**ADF**	**PP**	**ADF**	**PP**
ip	−5.61**	−16.46**	−2.99*	−10.97**	−8.05**	−17.87**
ps	−1.51	−7.89**	−6.71**	−11.89**	−5.19**	−11.40**
ε	−4.21**	−7.21**	−4.59**	−8.42**	−5.55**	−7.48**
R	−4.20**	−10.88**	−2.84	−5.53**	−4.80**	−9.47**
Δ*b*	…	…	−4.57**	−10.31**	−4.86**	−9.72**

Note: ADF = Augmented Dickey-Fuller (1979)*T*-test; PP = Phillips-Perron (1988)tests. ** and * indicate rejection of the null hypothesis of nonstationarity at the 99 and 95 percent levels, respectively, according to MacKinnon's (1991) critical values (performed in Regression Analysis of Time Series or RATS package). Variables: ip_t = monthly percent change, industrial production; pd_t = real primary surplus; ε_t = monthly percentage in bilateral real exchange rate (against US$); r_t = real interest rate; and Δb_t = real operational deficit (monthly change in real debt).

Table B2. Brazil: Summary of Estimation, VAR 1995:5–2005:5
(99 degrees of freedom; four lags)

	Dependent Variable			
	ip	***ps***	**ε**	***r***
***F*-statistic**				
ip (lagged)	2.33	0.67	1.54	0.77
ps (lagged)	2.01	1.82	1.41	1.54
ε (lagged)	0.84	0.41	2.92	2.54
r (lagged)	0.61	0.82	18.99	1.38
poil (lagged)	0.37	1.61	0.83	1.57
***T*-statistic**				
DUM99	0.10	−6.52	1.98	2.06
$R(\text{bar})^2$	0.03	0.40	0.50	0.03

Note: VAR = vector autoregression.

Table B3. Mexico: Summary of Estimation, VAR 1997:5–2005:5
(59 degrees of freedom; six lags)

	Dependent Variable				
	ip	***ps***	**ε**	**Δ*b***	***r***
***F*-statistic**					
ip (lagged)	1.65	0.82	2.47	1.85	0.28
ps (lagged)	1.69	3.84	2.40	2.58	3.01
ε (lagged)	0.37	0.74	1.08	0.38	0.45
Δ*b* (lagged)	0.47	0.78	4.56	0.64	0.37
r (lagged)	1.26	2.98	1.74	1.25	5.10
poil (lagged)	1.21	0.60	0.28	0.65	1.01
$R(\text{bar})^2$	0.05	0.26	0.30	0.04	0.36

Table B4. Turkey: Summary of Estimation, VAR 1994:5–2005:3
(81 degrees of freedom; eight lags)

	Dependent Variable				
	ip	***ps***	**ε**	**Δ*b***	***R***
***F*-statistic**					
poil (lagged)	1.26	0.99	0.53	1.88	0.47
ip (lagged)	5.97	0.65	1.26	1.65	1.00
ps (lagged)	3.42	1.08	1.43	0.90	2.34
ε (lagged)	1.48	0.48	4.02	1.13	1.27
Δ*b* (lagged)	0.85	0.98	2.20	0.65	0.63
r (lagged)	2.55	1.74	2.32	2.61	1.71
***T*-statistic**					
Crisis dummy	−2.19	−0.34	4.94	0.38	−0.76
$R(\text{bar})^2$	0.52	0.08	0.47	0.38	0.27

(v_t), namely, $u_t = \Omega v_t$, are uncovered by imposing certain restrictions on $C(L)$. Note that

$$\Sigma_u = \Omega E(v_t v_t')\Omega' = \Omega\Omega_t', \tag{A.5}$$

where $E(v_t v_t')$ implies a normalization of the orthogonal disturbances. In addition, the recursive Choleski structure discussed in the text guarantees (at least just) identification of Ω.

The Historical Decomposition

We now formally develop the idea of a historical decomposition. In the text, we claimed that any of the elements of endogenous vector Y (indexed by k) may be represented as the sum of a baseline forecast plus the sum of accumulated shocks from period M forward.

To more formally derive this notion, we expand Equation (B.3):

$$Y_{M+j} = \left[Z_{M+j} + \sum_{i=j}^{+\infty} \phi_i u_{M+j-i}\right] + \left[\sum_{i=0}^{j-1} \phi_i u_{M+j-i}\right], \tag{A.6}$$

where M and j are such that $0 \leq M \leq M+j \leq T$, and where T is the total number of observations. The first term on the right-hand side of Equation (A.6) is the forecast of Y_{t+j} based on information available at time M. This vector may be thought of as containing baseline forecasts: $Y_{k,\,M+j}(base)$. The second term is the part of Y_{t+j} resulting from innovation in periods $M+1$ to $M+j$—shocks that occurred *after* the baseline period M. Expanding further, we show the individual moving-average representation for each of the K endogenous variables:

$$Y_{M+j} = \begin{bmatrix} Y_{1,\,M+j} \\ \vdots \\ Y_{k,\,M+j} \\ \vdots \\ Y_{K,\,M+j} \end{bmatrix} = \begin{bmatrix} Z_{1,\,M+j} \\ \vdots \\ Z_{k,\,M+j} \\ \vdots \\ Z_{K,\,M+j} \end{bmatrix} + \begin{bmatrix} \sum_{h=1}^{K}\left[\sum_{i=j}^{\infty} \phi_{1h,i} u_{h,M+j-i}\right] \\ \vdots \\ \sum_{h=1}^{K}\left[\sum_{i=j}^{\infty} \phi_{kh,i} u_{h,M+j-i}\right] \\ \vdots \\ \sum_{h=1}^{K}\left[\sum_{i=j}^{\infty} \phi_{Kh,i} u_{h,M+j-i}\right] \end{bmatrix}$$

$$(\phi_{kh,i}\text{'s are elements of } \phi_i) \tag{A.7}$$

Thus, we may express any of the endogenous elements $Y_{k,\,M+j}$, $k = (1, 2, \ldots, K)$ at time $M+j$, in the following scalar form:

$$Y_{k,\,M+j}(base) = Z_{k,\,M+j} + \left[\sum_{h=1}^{K}\left[\sum_{i=j}^{\infty} \phi_{kh,i} u_{h,M+j-i}\right]\right]. \tag{A.8}$$

Now, we may formally derive the z^* terms:

$$z^*_{kh,j} = \sum_{i=0}^{j-1} \phi_{kh,\,i} u_{h,\,M+j-i}. \tag{A.9}$$

Next, we may remove from any element of Y the effects of shocks to element $h = \eta$. We define this as $Y_{k,\,M+j}(omit\ \eta)$.:

$$Y_{k,\,M+j}(omit\,\eta) = Y_{k,M+j} - z^*_{k\eta,j} \equiv Y_{k,M+j} - \sum_{i=0}^{j-1} \phi_{k\eta,i} u_{h,M+j-i}. \tag{A.10}$$

Thus, $Y_{k,\,M+j}(omit\ \eta)$ is similar to Equation (6) in the text. It is a counterfactual: the value of $Y_{k,\,M+j}$ that would have been observed if there had not been any innovations to element η between period M and period $M+j$.

Brazil: Alternative Estimates

As mentioned in the text, frequent and dramatic regime changes in Brazil present challenges in choosing the sample period for estimation. As reported above, the principal estimates use the period from mid-1995 to mid-2005; these estimates include two dummy intercept variables: exchange rate crisis ($D=1$ for $t=$1999:1–1999:4 and $D=0$ otherwise), floating rate period ($D=0$ for $t=$1995:5–1999:4, $D=1$ thereafter), and a time trend (to capture increases in the primary surplus during 2000–05). However, as summarized in Table B5, in addition to the reported estimates (version (i)) the four alternatives are versions (ii) through (v).

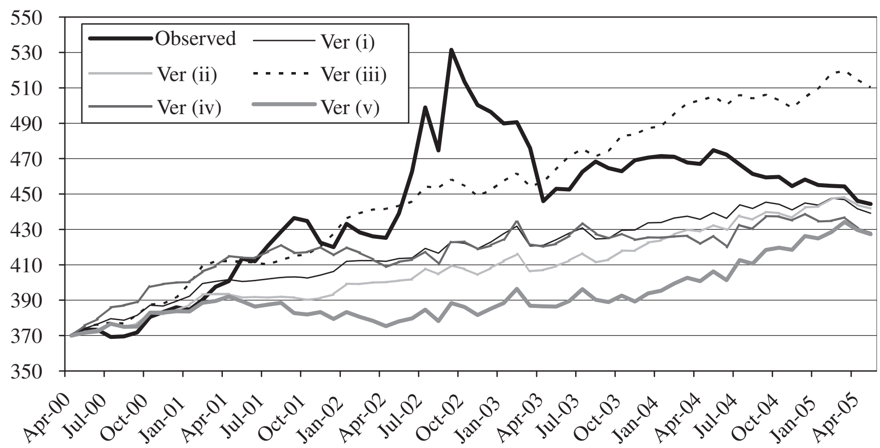
Figure B1 suggests that version (iii) is substantially different from the other versions, insofar as the baseline debt level as of end-2005 is substantially higher than in the other versions. This probably reflects the fact that, for version (iii)—unlike for versions (i) and (ii)—the dummy variable for the 1999 exchange rate crisis was omitted. (Versions (iv) and (v) use only post-crisis data.) Thus, version (iii) yields a policy conclusion that differs

Table B5. Brazil: Summary of Alternative Estimates (Versions (i)–(v))

Version	Sample	Crisis Dummy 1998:11–1999:3	Flex Regime Dummy 1999:4–2005:6	Time Trend
(i)*	1995:5–2005:6	Yes	Yes	Yes
(ii)	1995:5–2005:6	Yes	Yes	No
(iii)	1995:5–2005:6	No	Yes	No
(iv)	1999:4–2005:6	n.a.	n.a.	Yes
(v)	1999:4–2005:6	n.a.	n.a.	No

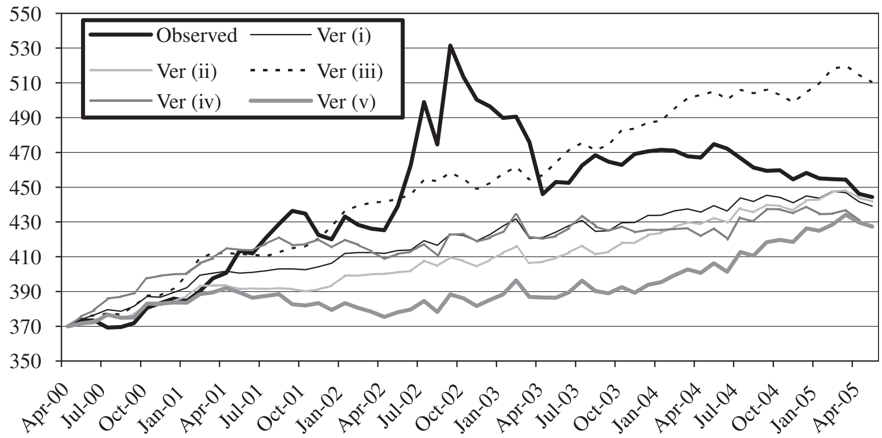
Note: * = main version reported in text; n.a. = not available.

Figure B1. Brazil: Observed and Baseline Debt, Alternative Estimates
(Versions (i)–(v) in billions of constant reais (1995 = 100))



Source: Central Bank of Brazil. (See Appendix I for details.)

Figure B2. Brazil: Observed Debt and "Purged" Debt *b*(*omit* ε, *r*) Alternative Estimates *(Versions (i)–(v) in billions of constant reais (1995=100))*



Source: Central Bank of Brazil. (See Appendix I for details.)

dramatically from the other versions: in the absence of appreciation of the real exchange rate that occurred after 2003, Brazil's fiscal policy would have been unsustainable even if there had been no shocks; that is, over j periods $(b(base)_{M+j}/GDP_{M+j} > b(base)_M/GDP_M$.

Figure B2 shows debt levels purged of exchange rate and interest rate effects (*b* (*omit* ε, *r*)) for versions (i)–(v). This figure highlights the fact that, in all versions, upsurge in debt from 2001–03 was largely the result of innovations to exchange rates and interest rates—not baseline policy. However, as in Figure B1, version (iii) is substantially different from the other versions. Consistent with the version (iii) baseline in Figure B1, this figure suggests that, by end-2005, the debt level would have been substantially higher *had pressures on the real not eased up*. However, such a conclusion must be considered with caution: in version (iii), the 1999 crisis is not treated as a sui generis event.

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