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Extreme Contagion in Equity Markets

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Contagion can be defined as the probability of observing large return realizations simultaneously across different financial markets (co-exceedances) rather than as increases in correlations. We introduce global extreme contagion measures constructed from bivariate extremal dependence measures. The measures are used to quantify both negative and positive equity returns contagion at the inter- and intraregional level for a number of mature and emerging equity markets during the past decade. Our results suggest that (a) contagion patterns differ significantly within regions and across regions, with Latin America showing a secular increase in contagion not matched by other regions or countries; (b) contagion is higher for negative returns than for positive returns; (c) only the 1998 Russian and Brazilian crises led to a global increase in contagion; and (d) extremal dependence measures of contagion and simple correlation measures are not highly correlated, with the exception of Latin America, suggesting that the use of correlations as a proxy for contagion may be misleading. [JEL F30, G10, G15, C10]

It has been widely documented that the comovement of prices in financial markets increases significantly during periods of stress, such as the Exchange Rate Mechanism crisis in 1992–93, the Mexican crisis in 1994, and the more recent

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financial crises in East Asia in 1997 and in Russia and Brazil in 1998.[1] Both policymakers and academics have interpreted upward changes in price comovements or correlations as evidence of a breakdown of current transmission mechanisms across financial markets, or contagion. Sudden correlation changes, however, are not necessary or sufficient to identify contagion or crisis periods, as they do not necessarily imply a structural change in the data generating process. For example, Boyer, Gibson, and Loretan (1997) and Forbes and Rigobon (2002) show that even if data are generated from a bivariate normal distribution with a constant correlation coefficient, the conditional correlation of the top 50 percent of the largest returns is different from the conditional correlation of the bottom 50 percent of the smallest returns. Furthermore, correlation analysis is unable to uncover nonlinear relationships.

Extreme Value Theory and Contagion

An alternative statistical approach to analyzing contagion in financial markets is extreme value theory (EVT). Multivariate EVT techniques quantify the joint behavior of extremal realizations (or co-exceedances) of financial prices or returns across different markets. Extremal realizations or exceedances are defined as those exceeding a large threshold value. The EVT approach to contagion, which has gained acceptance in recent years, captures well the belief of most observers in the private and policymaking communities that large shocks are transmitted across financial markets differently than small shocks.[2]

In EVT, there is contagion between two series, X and Y, with distribution functions F_X and F_Y, respectively, if

$$\lim_{u \to 1} \Pr\left(X > F_X^{-1}(u) \,\middle|\, Y > F_Y^{-1}(u)\right) \tag{1}$$

is statistically significant and different from zero, where F^{-1} is the inverse of the distribution function and u is the threshold value. Equation (1) states that as the threshold value approaches 1 from below, X exceeds the threshold value with positive probability, provided that Y has already exceeded the threshold. Another interpretation of equation (1) is that X and Y exhibit contagion if the probability of observing a large realization of X is higher after observing a large realization of Y; that is, the difference between the conditional and unconditional probability $\Pr(X > F_X^{-1}(u))$ is positive. The latter interpretation follows the definition of contagion given by Costinot, Roncalli, and Teiletche (2000). These authors suggest that there is contagion between X and Y if

$$\Pr\left(X > F_X^{-1}(u_1) \,\middle|\, Y > F_Y^{-1}(u_2)\right) \geq \Pr\left(X > F_X^{-1}(u_1)\right). \tag{2}$$

[1]Increased price synchronicity is also experienced during bull markets, though to a lesser extent. See Chan-Lau and Ivaschenko (2001) for a recent analysis of the 1990s' tech bubble.

[2]Bae, Karolyi, and Stulz (2003); Costinot, Roncalli, and Teiletche (2000); Hartmann, Straetmans, and de Vries (2001); Longin and Solnik (2001); Poon, Rockinger, and Tawn (2001); Quintos (2001); and Starica (1999), among others, are examples of recent empirical work that emphasizes this definition of contagion.

Equations (1) and (2) are equivalent in the asymptotic case of $u_1 = u_2 = u \rightarrow 1$, since

$$\lim_{u\to 1}\left[\Pr\left(X > F_X^{-1}(u) \mid Y > F_Y^{-1}(u)\right) - \Pr\left(X > F_X^{-1}(u)\right)\right]$$
$$= \lim_{u\to 1} \Pr\left(X > F_X^{-1}(u) \mid Y > F_Y^{-1}(u)\right).$$

Equation (1) suggests that evaluating contagion between two series is reduced to estimating the conditional probability of large realizations of one series conditional on large realizations of the other series. In the limit, as explained in more detail in Section II, the conditional probability can be approximated by the extremal dependence measure χ, defined as:

$$\chi = 2 - \lim_{u\to 1} \frac{\log C(u, u)}{\log u},$$

where the copula C represents the joint distribution of both series. It follows naturally that an increase (decline) in the value of χ implies an increase (decline) in contagion.

Coles, Heffernan, and Tawn (1999) note that the use of the χ measure is appropriate only when the series analyzed are asymptotically dependent; that is, in the limit, when the threshold $u \rightarrow 1$, extreme realizations of both series occur simultaneously. Estimates of χ overestimate the degree of dependence between two series if they are asymptotically independent; that is, in the limit, extreme realizations seldom occur simultaneously. For the case of asymptotically independent variables, Coles, Heffernan, and Tawn suggest using the extremal dependence measure $\bar{\chi}$:

$$\bar{\chi} = \lim_{u\to 1} \frac{2 \log(1 - u)}{\log \bar{C}(u, u)} - 1, \tag{3}$$

where $\bar{C}(u, u) = \Pr(X > F_X^{-1}(u), Y > F_Y^{-1}(u))$ is the joint survival probability. The measure $\bar{\chi}$ can be loosely interpreted as the correlation coefficient between both random variables.

Measuring contagion requires determining the asymptotic dependence class of the series analyzed and then using the appropriate extremal dependence measure χ or $\bar{\chi}$. It follows naturally to define extreme contagion as first proposed by Poon, Rockinger, and Tawn (2001): Given two random variables X and Y, with distribution functions F_X and F_Y, there is *extreme contagion* if (a) χ is statistically significant and different from zero, and X and Y are asymptotically dependent; or (b) $\bar{\chi}$ is statistically significant and different from zero, and X and Y are asymptotically independent. Therefore, for two random variables X and Y, it is possible to define the bivariate extreme contagion measure as

$$\text{Extreme Contagion Measures} = \begin{cases} \chi & \text{if } X \text{ and } Y \text{ are asymptotically independent} \\ \bar{\chi} & \text{if } X \text{ and } Y \text{ are asymptotically dependent} \end{cases}.$$

Changes in contagion can be linked to changes in the asymptotic dependence class and changes in the extreme contagion measure. Contagion increases if two series, previously asymptotically independent, become asymptotically dependent. Absent changes in the dependent class, contagion increases if the appropriate extreme contagion measure (χ for asymptotically dependent series, or $\bar{\chi}$ otherwise) increases.

The extreme contagion measures described above are useful to study extreme contagion for a reduced number of markets. As the number of markets analyzed increases, though, uncovering contagion patterns may become untractable on a pairwise basis. For example, there are 136 different pairwise combinations for the 17 equity markets analyzed here. Unfortunately, extending the bivariate analysis to the multivariate case is rather difficult. We propose, hence, two simple aggregate global extreme contagion measures to evaluate contagion at the intraregional and interregional levels. The first global extreme contagion measure is the relative share of asymptotic dependent bivariate combinations. This measure captures changes in contagion arising from changes in the number of markets exhibiting asymptotical dependence. The second global extreme contagion measure is the relative share of pairwise market combinations that are statistically significant. This measure complements the first one, as it indicates how many markets exhibit pairwise contagion regardless of their asymptotic dependence properties. For simplicity, when the bivariate and global extreme contagion measures are applied to the analysis of extreme negative (positive) returns, they will be referred to as bear (bull) market contagion.

One shortcoming of these two contagion measures is that they may fail to capture changes in contagion when pairwise country contagion is changing if the ratio of asymptotic dependent cases to total cases or the ratio of statistically significant extreme contagion cases remains constant. While it is tempting to use the arithmetic mean of statistically significant cases, the mean will include cases with different asymptotic dependence properties. Hence, we refrain from introducing this third global extreme contagion measure into our analysis.

Main Findings

Bear and bull market contagion is analyzed using the global extreme contagion measures discussed above during the period December 31, 1987, to October 25, 2001. The main findings are the following:

1. Bear market contagion during the second half of the past decade is stronger than bull market contagion within market classes, across markets, and within regions. This finding, which is consistent with those of earlier empirical studies, highlights the asymmetric nature of the financial transmission mechanism across stock markets.
2. The successive crises experienced during the second half of 1998—the debacle of Long Term Capital Management (LTCM), the Russian debt crisis, and the Brazilian currency crisis—constituted major shocks with a lasting impact on the global financial system, as they prompted sustained increases in bear market contagion. In contrast, the 1994 Mexican crisis appeared not to have a major impact on contagion, while the 1997 East Asian crisis had only a regional impact.

3. Under the assumption that increased financial integration increases a stock market's vulnerability to contagion, Latin American stock markets are more vulnerable to contagion from mature stock markets than are East Asian markets. At the regional level, Latin American stock markets have become more integrated during the past decade, as measured by the upward trend in both bear and bull market contagion. In contrast, the financial integration of East Asian stock markets has remained roughly the same.
4. Emerging stock markets are mostly affected by contagion with the United States and the United Kingdom. In particular, contagion with the United States is particularly strong in the Latin American region. Contagion between Japan and emerging stock markets is mostly negligible.
5. Latin America is the only region where the number of asymptotically dependent country-pairs has increased during periods of bear market contagion. This evidence suggests a stronger shift in the downside transmission mechanism in the region, as stronger contagion is reflected not only in higher values of the extremal dependence measures but also by changes in the asymptotic tail properties. There is nonetheless a decline in bear market contagion between Argentina and the rest of Latin America toward the end of the sample period.
6. Contagion measures and five-year rolling window equity market returns correlation are not highly correlated, with Latin American stock markets being the only exception. Hence, reliance on simple correlations as a measure of contagion could be misleading. The findings above point toward an overall average increase in contagion worldwide between 1987 and 2001, especially across mature markets, between mature and Latin American stock markets, and within Latin America. Furthermore, the results are similar regardless of whether stock returns are denominated in local currency or U.S. dollars and whether raw returns or filtered returns are used. The rest of this paper describes the results corresponding to U.S. dollar-denominated raw returns, but the complete set of results is available from the authors upon request.

Though this paper does not explore possible explanations for these results in depth, a number of potential hypotheses could explain the results. One hypothesis is that increased globalization has strengthened financial and trade linkages considerably. Therefore, changes to economic fundamental factors are transmitted more rapidly than in the past, leading to increased price comovements and contagion. In particular, increased use of complex financial instruments such as sovereign default swaps and arbitrage strategies by hedge funds may have increased the exposure and vulnerability of financial institutions in mature markets to emerging markets. One notable example was the systemic risk posed by LTCM, whose demise was linked to adverse price movements in emerging markets. Another possible hypothesis is that in the absence of changes in fundamental economic factors, herding behavior among investors may lead to bouts of irrational exuberance and unjustified panics, increasing the synchronization of financial prices. Anecdotal evidence may support the herd-behavior explanation, as the importance of dedicated investors relative to crossover investors in emerging markets has been declining in the last years of the study period. Therefore, the proportion of informed investors in emerging markets may have been shrinking steadily, increasing vulnerability to shifts in market sentiment.

I. Related Literature

The reliance on extreme value theory rather than correlation analysis is motivated by the limitations of the latter approach.[3] In fact, simple correlation analysis can be deceptive when studying financial market dependence, as shown by Boyer, Gibson, and Loretan (1997) and Embrechts, McNeil, and Straumann (1999).[4] Boyer, Gibson, and Loretan showed analytically and numerically that data with stationary distributions could generate spurious correlation breakdowns. Their results invalidate the common practice of splitting a sample according to the realized values of the data to identify different correlation regimes. Embrechts, McNeil, and Straumann showed that even in the static case, when the multivariate distribution characterizing financial time series does not change in time, the use of correlations as dependence measures is justified only for multivariate normal distributions. Otherwise, correlations fail to reveal the multivariate dependence structure: zero correlation does not indicate independence, and perfectly dependent variables do not exhibit absolute unity correlation. Moreover, correlations are not invariant under monotonic data transformations.

Extreme value theory has been increasingly used to study the tail behavior of univariate financial time series.[5] However, the application of multivariate extreme value theory to study the joint probability or dependence of extreme realizations (extremal dependence) in financial markets has been more limited. Starica (1999) proposed a spectral measure method to estimate the probabilities of joint extreme returns in series generated by models with constant conditional correlation models, a family that encompasses GARCH models. Applying this method to high-frequency data on European Union currencies uncovered a high level of dependence among extreme returns. Longin and Solnik (2001), using monthly data for the period January 1959 to December 1996, found that the pairwise correlation of extreme returns between the United States and France, Germany, Japan, and the United Kingdom (Group of Five, or G-5) increases in bear markets but not in bull markets.

Hartmann, Straetmans, and de Vries (2001) studied asset return linkages during periods of stress across stock markets, bond markets, and stock-bond contagion in the G-5 countries. These authors suggested that contagion was better understood from the perspective of the probability of joint crashes. They found that market crashes were rare occurrences, but once one market crashed, the conditional probability of a crash in a different market was about one in five. Similarly, Bae, Karolyi, and Stulz (2003) suggested that contagion is better measured by the joint probability of co-exceedances in more than one market. These authors used a multinomial logistic regression, an approach commonly used in epidemiology studies, to estimate contagion across countries within a region and across regions rather than extreme value theory. They found that contagion could be explained by interest

[3] Hilliard (1979), Eun and Shim (1989), Roll (1988, 1989), Bertero and Mayer (1990), and Baig and Goldfjan (1999) among others, are examples of the correlation analysis approach.

[4] See also Forbes and Rigobon (2002).

[5] See Jansen and de Vries (1991), Longin (1996), Jondeau and Rockinger (1999), Phoa (1999), and Tsay (1999), among many others.

rates, changes in exchange rates, and conditional stock return volatility. Quintos (2001) constructed a measure of extremal correlation that does not depend on the specification of a dependence (copula) function and used it to analyze contagion, defined as significant extremal correlation changes in exchange rates, between Thailand and a number of Asian countries.

Costinot, Roncalli, and Teiletche (2000) suggested that dependence among financial markets was better modeled using copulas rather than correlation analysis, even if the latter is corrected using techniques such as those proposed by Forbes and Rigobon (2002). In the case of extreme returns, they found that the probability of joint exceedance for the Dow Jones and the French CAC40 stock market indexes increased dramatically when copulas were used rather than the bivariate normal distribution. They also analyzed the dependence between exchange rates and stock market returns during the 1997 Asian crisis. Their results provide stronger evidence of contagion than those obtained using correlation analysis.

Finally, Poon, Rockinger, and Tawn (2001) noted that the studies described above assumed asymptotic dependence. Intuitively, two series exhibit asymptotic dependence (independence) when in the limit, tail realizations of both series always (never) occur together. If the series analyzed are asymptotically independent, the use of conventional extreme value theory methods would overestimate the degree of extremal dependence. Poon, Rockinger, and Tawn described simple methods, first developed by Coles, Heffernan, and Tawn (1999), to test whether two series are asymptotically independent and to estimate their extremal dependence. The methods are applied to daily stock market returns in France, Germany, Japan, the United Kingdom, and the United States, with the sample divided into three different periods so that extremal dependence in bear and bull markets can be analyzed. They found that asymptotic independence characterized most of the pairwise combinations of stock returns, and that extremal dependence was much stronger in bear markets than in bull markets.

This paper, which also follows the methodology proposed by Coles, Heffernan, and Tawn, is closely related to the work of Poon, Rockinger, and Tawn referred to above. The main differences are as follows. First, a wider set of equity markets is considered in this study, and special attention is paid to the transmission of shocks across different market classes, for example, mature and emerging. Second, the use of five-year rolling window estimates of contagion permits tracking their historical evolution without assuming a priori that structural breaks took place in certain periods. Finally, the behavior of contagion measures is evaluated vis-à-vis the behavior of simple equity returns correlation, a measure widely favored in the policymaking community, to assess whether they convey the same amount of information. The next section describes the theory and empirical methodology used in this study.

II. Theory and Empirical Methodology

Although the rapidly expanding statistical literature on bivariate extreme value theory offers a number of alternative EVT measures, as described in Malevergne and Sornette (2002), the preferred measure of extremal contagion adopted in this paper is the one first proposed by Coles, Heffernan, and Tawn (1999), since it accounts for the asymptotic properties of the tail distributions. Coles, Heffernan, and Tawn suggest a two-step approach to estimate extremal dependence. The first step is to determine whether two series are asymptotically independent; in other words, the type of asymptotic joint tail distribution that governs the behavior of the series in the limit. Intuitively, two series are asymptotically independent (dependent) when infinitely large realizations of each series never (always) occur simultaneously. Notice, though, that asymptotically independence does not preclude the simultaneous realization of large but finite realizations. After the asymptotic properties are determined, Coles, Heffernan, and Tawn propose two different dependent measures, $\bar{\chi}$ and χ, for the case of asymptotically independent and dependent series, respectively.

This methodological framework implies that equity return contagion between two countries can be measured along two dimensions. The first dimension of contagion is linked to changes in the asymptotic properties of the joint tail distribution of equity returns. In fact, contagion is stronger between two countries if the equity return series are asymptotically dependent, since it implies that very large equity return realizations in both countries occur simultaneously. In contrast, asymptotic independence implies that very large equity return realizations never occur together. Hence, for two countries with a previously asymptotically independent joint tail distribution, contagion increases if the joint tail distribution becomes asymptotically dependent. Moreover, an increase in the number of country-pair returns exhibiting asymptotically dependence suggests increased contagion worldwide, as structural changes in the transmission mechanism lead to changes in the asymptotic properties of the joint tail distribution. The second dimension is associated with changes in the dependent measure when the type of asymptotic tail property remains unchanged. In this case, an increase in contagion between two countries is reflected in an increase in the dependence measure rather than changes in the joint tail distribution. Keeping these observations in mind, we explain the methodological framework below.

Given two positive random variables, X and Y, it is natural to relate their extremal dependence to either their conditional probability limit

$$\lim_{x,y\to\infty} \Pr(X > x \mid Y > y), \tag{4}$$

which measures the probability of an extreme realization of X conditional on the realization of an extreme realization of Y, or the joint survivor function defined as

$$\lim_{x,y\to\infty} \Pr(X > x, Y > y), \tag{5}$$

which measures the joint probability of large realizations of X and Y. In most empirical applications, the joint probability distribution function, $F(x, y) = \Pr(X < x, Y < y)$, which is required to estimate the probabilities above, is not known. However, it is possible to estimate their joint probability distribution if their univariate margin distributions are known using a copula.

The copula C is the unique function that relates the univariate marginal distributions of two random variables X and Y to their joint distribution (Nelsen, 1999):

$$F(x, y) = C\{F_X(x), F_Y(y)\}, \tag{6}$$

where $F_X(x) = F(x, \infty)$ and $F_Y(y) = F(\infty, y)$ are the cumulative distributions of X and Y, respectively. Therefore, knowledge of the copula function C and the univariate distributions of X and Y is sufficient to have complete information on the joint distribution of X and Y. In addition, copulas are invariant to nondecreasing transformations of the variables, so it is common practice to transform (X,Y) to uniform margins $(W,Z) = (F_X(X), F_Y(Y))$.[6] The copula function C, or joint distribution function, can be estimated from the sample by counting the number of occurrences belonging to a given event; that is, the number of realizations of X and Y that exceeded the threshold level u. Alternatively, a parametric functional form can be specified and estimation methods such as maximum likelihood can be used to obtain the parameters.

The information about the extremal dependence of two random variables in the copula C can be summarized by two dependence measures, χ and $\bar{\chi}$. The dependence measure χ is defined as

$$\begin{aligned} \chi &= \lim_{u\to 1} \Pr(W > u \mid Z > u) \\ &\approx \lim_{u\to 1} 2 - \frac{\log C(u, u)}{\log u} \\ &= \lim_{u\to 1} \chi(u), \end{aligned} \tag{7}$$

where $(U,V) = (F_X(X), F_Y(Y))$, and C is the copula that describes the joint probability of U and V. For independent variables, $\chi(u) = 0$; for perfect dependence, $\chi(u) = 1$. Notice that in the limit $\Pr(W > u \mid Z > u) = \Pr(Z > u \mid W > u)$, so that extremal dependence between two random variables is symmetric. One of the shortcomings of using χ as the only dependence measure is that for large but finite values of u, estimates of $\chi(u)$ may be constant and positive even if the variables are independent in the limit; that is, $\lim_{u\to 1} \chi(u) \to 0$.

When variables are asymptotically independent, the dependence measure χ is equal to zero by definition, so it does not contain information about the relative strength of dependence for large but finite realizations of the variables. Coles, Heffernan, and Tawn (1999) suggest using the dependence measure $\bar{\chi}$ defined as

$$\bar{\chi} = \lim_{u\to 1} \bar{\chi}(u), \tag{8}$$

where the function $\bar{\chi}(u)$ is defined as

$$\bar{\chi}(u) = \frac{2\log(1-u)}{\log \bar{C}(u, u)} - 1, \tag{9}$$

[6]In recent work, Hu (2002) uses copulas to estimate dependent patterns in major financial markets. In contrast to the work presented here, as well as in the related literature section, Hu models the complete joint distribution of realizations rather than the tail realizations only.

with $\bar{C}(u,v) = 1 - u - v + C(u,v)$, and $\bar{\chi} \in [-1,1]$. Note that $\bar{C}(u,v)$ is simply the joint probability $\Pr(U > u, V > v)$. Therefore, if the random variables are asymptotically independent, it is possible to determine whether the extremal dependence is either positive or negative.

Estimating χ and $\bar{\chi}$ requires estimating $C(u,u) = \Pr(W < u, Z < u)$ and $\bar{C}(u,u) = \Pr(W > u, Z > u)$ and replacing their values in equations (7) and (9). This can be accomplished using the following nonparametric procedure.[7] First, select the threshold value that determines the tail region. In this paper, the threshold value is 0.95, which means that only returns in the bottom (top) 5 percent quantile are considered in the analysis. It should be noted, though, that there are other ways to select the threshold value, as suggested in Danielsson and de Vries (1997); Embrechts, Kluppelberg, and Mikosch (1997); and Longin and Solnik (2001). Second, estimate empirically $C(u, u)$ and $\bar{C}(u, u)$ from the data and calculate χ and $\bar{\chi}$. For example, estimate $C(u, u)$ for $u = 0.95$ by counting the joint realizations of W and Z below 0.95, and divide by the number of total observations. Third, if the null hypothesis $\bar{\chi} = 1$ can be rejected, then contagion is measured by $\bar{\chi}$. Otherwise, contagion is measured by χ.

Once the bivariate contagion measures have been estimated, constructing the global extreme contagion measures is straightforward, as they consist of simple ratios of the total number of asymptotically dependent cases and statistically significant bivariate contagion cases to total cases analyzed.

III. Results

The methodology described in the previous section is used to analyze weekly stock market returns in mature and emerging stock markets. Mature markets analyzed in this study include those of France, Germany, Japan, the United Kingdom, and the United States. Emerging markets include those of Argentina, Brazil, Chile, and Mexico in Latin America, and Hong Kong SAR, Indonesia, Malaysia, the Philippines, the Republic of South Korea, Singapore, Taiwan Province of China, and Thailand. For each country, weekly stock market returns are calculated from the corresponding U.S. dollar-denominated and local currency-denominated Morgan Stanley Capital Index for the period December 31, 1987, to October 25, 2001. The sample encompasses the Mexican crisis in 1994, the East Asia crisis in 1997, the Russian crisis and the collapse of LTCM in the second half of 1998, the Brazilian crisis in early 1999, and the early stages of the Argentinian debt crisis in 2001.[8]

[7] See Poon, Rockinger, and Tawn (2001) for an alternative parametric approach.

[8] The assumption of a fixed rolling window implies that the financial transmission mechanism, and hence contagion, changes over time and supposedly renders the results obtained here vulnerable to the criticism put forward by Forbes and Rigobon (2002); namely, using a fixed rolling window is equivalent to sorting out the data by the level of volatility. We offer two answers to this critique. First, on a practical level, the results presented here did not change substantially when the dependence measures were estimated recursively by increasing the sample size one observation at a time. Second, on an intuitive level, studies such as Chakrabarti and Roll (2000) suggest that crises are mainly characterized by dramatic declines in asset values rather than increased volatility, and, hence, adjustments for volatility should not be applied.

Global extreme contagion measures for the bottom 5 percent negative returns, bear market contagion, and the top 5 percent positive returns, bull market contagion, were estimated using a fixed five-year rolling window. The contagion measures were estimated both for raw equity returns and filtered equity returns obtained after fitting a TARCH (1,1).[9] The use of filtered equity returns is justified by the fact that stock returns exhibit serial dependence, while bivariate extreme value methods assume that the data are i.i.d. Diebold, Schuermann, and Stroughair (1998) pointed out that in some cases, not correcting for serial dependence could result in poor tail estimators. They suggest first modeling the conditional mean and variance of the returns and analyzing the properties of the standardized residuals, an approach followed by other authors, including McNeil and Frey (2000); Poon, Rockinger, and Tawn (2001); Patton (2001); and Hu (2002). The results by Poon, Rockinger, and Tawn suggest that extremal dependence measures obtained using raw stock return series would in general be higher than those obtained using filtered series, especially for those associated with positive extreme realizations.

Overall, our results indicate that discriminating between local currency or U.S. dollar-denominated indices and filtering the data do not change the results significantly. Hence, this section describes only the results obtained using raw U.S. dollar-denominated equity returns. The evolution of contagion is assessed using the two global extreme contagion measures proposed here by (a) describing changes in the asymptotic properties of the tail distribution as measured by changes in the number of country-pairs displaying asymptotic dependence and (b) analyzing changes in the number of significant contagion cases; that is, those statistically different than zero for a 95 percent confidence level. Finally, the section concludes with a detailed examination of extreme contagion within and across markets.[10]

Asymptotic Properties of the Tail Distribution

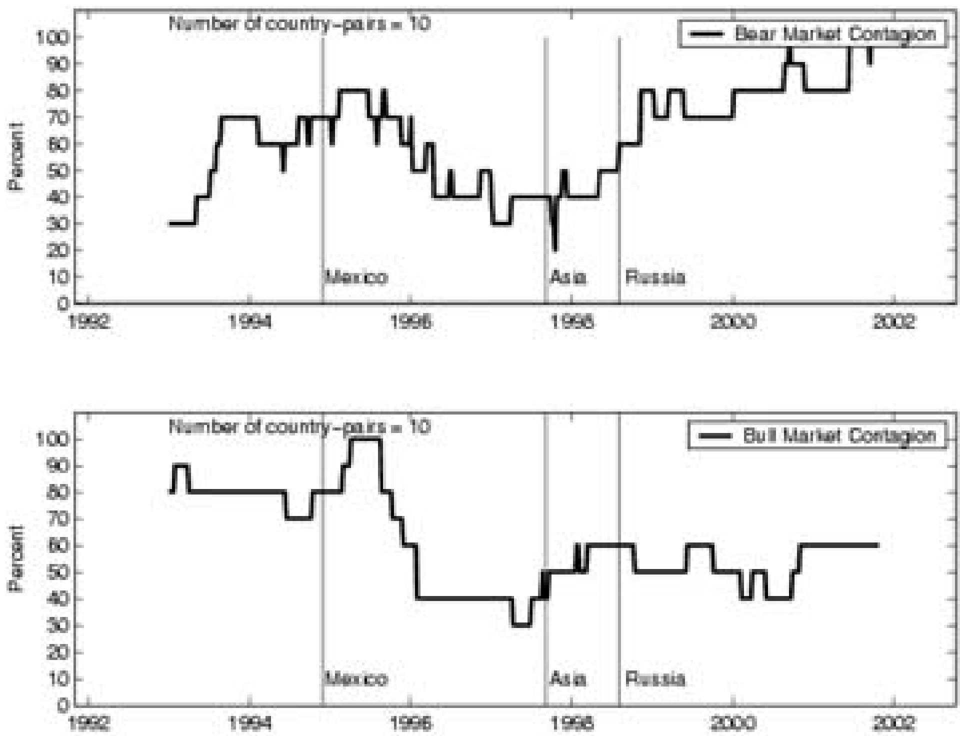
As explained above, contagion is stronger between two countries if their equity return series are asymptotically dependent rather than independent, as the former tail property implies the simultaneous realization of very large equity return realizations in both countries. Therefore, an increase in the number of country-pair returns exhibiting asymptotic dependence suggests increased contagion, as structural changes in the transmission mechanism lead to changes in the asymptotic properties of the joint tail distribution.

Figure 1 summarizes the main findings concerning changes in the asymptotic properties of the tail distribution. For all country-pairs analyzed, the number of asymptotically dependent cases has never exceeded 30 percent of all cases during the period under study, for both bear and bull market contagion. In the case of bear market contagion, the number of asymptotically dependent cases increased sharply in the second half of 1997 following the East Asia crisis as a result of increases in

[9]See Zakoian (1994) and Glosten, Jagannathan, and Runkle (1993) for a description of the TARCH model.

[10]Because of space limitations, only selected figures have been included. A set of complete figures is available from the authors upon request.

Figure 1. Number of Asymptotically Dependent Cases—Mature Markets
(In percent)



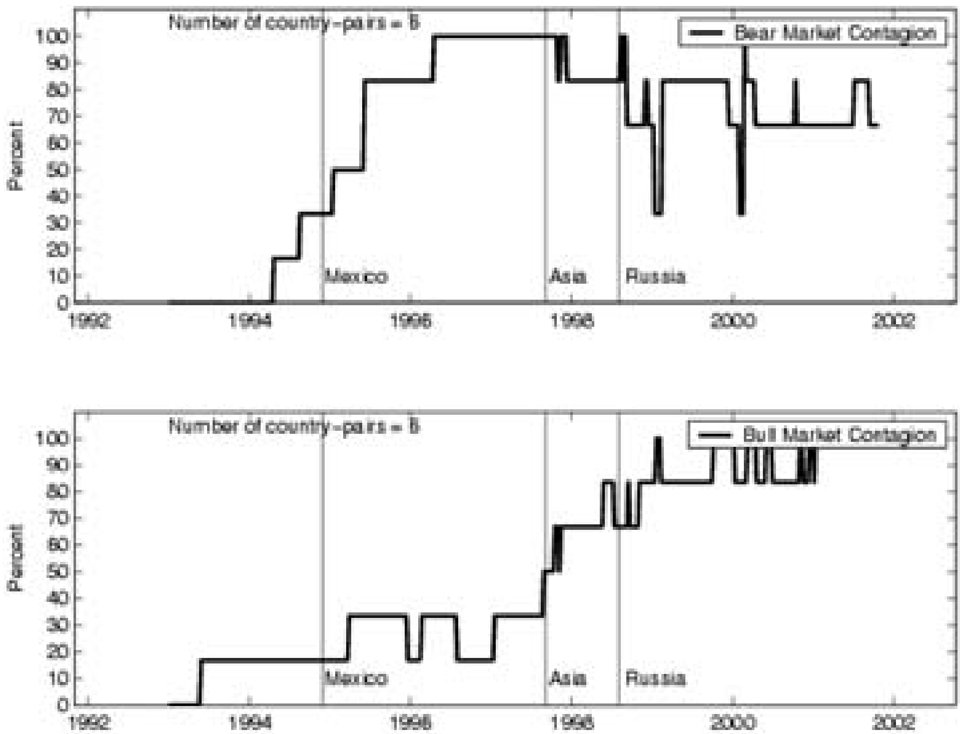
the number of asymptotically dependent cases between Latin America and both mature markets and East Asia. However, by end-1998, the number of asymptotically dependent cases was almost negligible. It should be noted, though, that for Latin America, and to a lesser extent in mature markets, the number of asymptotically dependent cases has been increasing since end-1998 (Figure 2).

In the case of bull market contagion, the most salient feature is the increase in the number of asymptotically dependent cases in early 1996 because of increased dependence between mature markets and emerging markets and within emerging markets, with dependence measured by the greater number of cases classified as asymptotically dependent. However, soon after the 1996 peak, the number of asymptotically dependent cases began declining steadily and now stands at the same level as in the case of bear market contagion. Interestingly, for any given period except during the East Asia crisis, asymptotic dependence is more prevalent during bull market contagion than during bear market contagion.[11]

[11]Results showing how dependence has changed for each individual country-pair during the past decade, for example, whether the countries' stock return series are asymptotically dependent or independent, were also obtained. However, no clear pattern worth reporting emerged.

Figure 2. Number of Asymptotically Dependent Cases—Latin America

(In percent)



Is Contagion Significant?

A second way to assess contagion trends is to evaluate changes in the number of contagion cases that are statistically different than zero. Figure 3 shows the number of significant contagion cases as the percentage of all country-pairs analyzed.

In the case of bear market contagion, the data show a sharp decline in the ratio of significant contagion cases to total cases from 40 percent in late 1995 to roughly 20 percent by mid-1997. The decline was reversed dramatically in the wake of the East Asia crisis, as significant contagion cases jumped from 30 percent before the crisis to 50 percent by end-1998. The Russian default on rouble-denominated debt in August 1998 and the LTCM debacle in October 1998 coincided with another increase of significant contagion cases from 10 percent to 60 percent. Finally, by the time of the Brazilian devaluation in early 1999, significant contagion cases reached 70 percent. This evidence suggests that, overall, bear market contagion increased following the late 1990s crises.

However, the empirical evidence varies widely within regions and across regions. For mature markets, only the 1998–99 crises led to an increasing number of significant contagion cases. For Latin America, the Mexican crisis appears to be the event that marked the beginning of a gradual increase in significant contagion

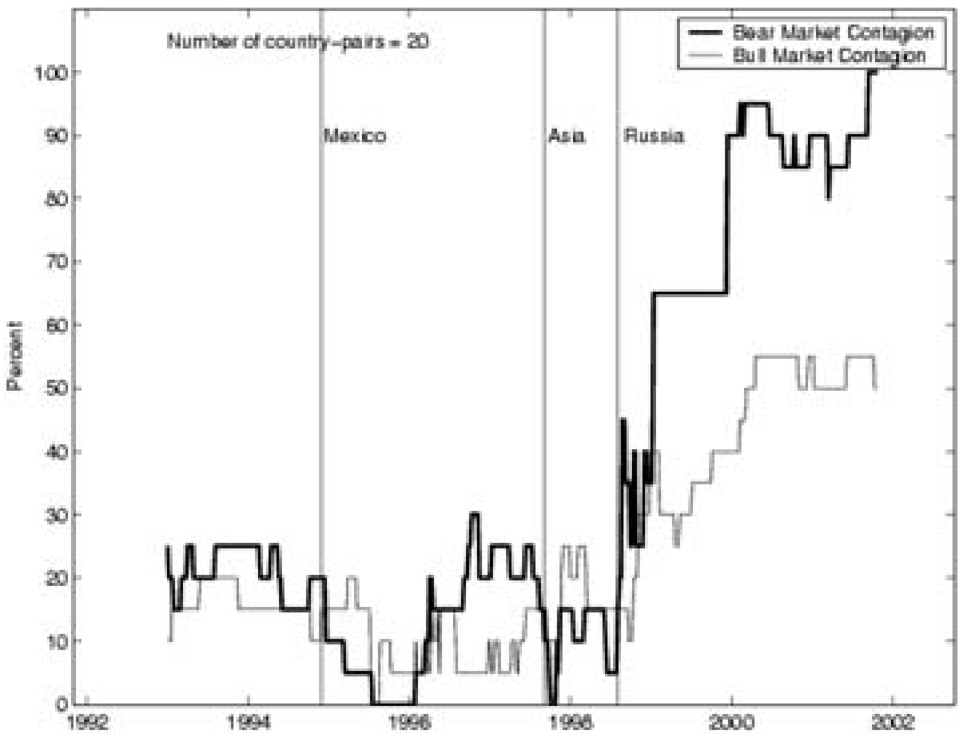
Figure 3. Significant Contagion Cases—All Markets
(In percent)

cases, from 35 percent before the crisis to 100 percent by early 1996. For East Asia, significant contagion cases declined from 1995 to mid-1997 and increased sharply from 55 percent before the 1997 crisis to 95 percent by mid-1998.

Across regions, significant contagion cases between mature markets and Latin America increased substantially only in the second half of 1998, from 10 percent before the Russian GKO crisis to 50 percent by late 1998. They jumped again to 70 percent in early 1999 and increased gradually to 100 percent by the second half of 2001 (Figure 4). The pattern is quite different for contagion between mature markets and East Asia. The number of significant contagion cases experienced a short-lived increase in the aftermath of the 1997 East Asia crisis. Subsequently, the number of significant cases declined until the 1998–99 crises (Figure 5). With respect to contagion between Latin America and East Asia, the dramatic data show that the East Asia crisis signaled a watershed event (Figure 6).

Figure 3 also shows that in the case of bull market contagion, the number of significant contagion cases increased worldwide in early 1999, mostly because of rising trends in emerging markets and between mature markets and emerging markets. In contrast, the number of significant contagion cases between Latin America and East Asia (Figure 6) did not increase significantly, while in the case of mature markets, they have declined since late 1995.

Figure 4. Significant Contagion Cases—Mature Markets and Latin America
(In percent)



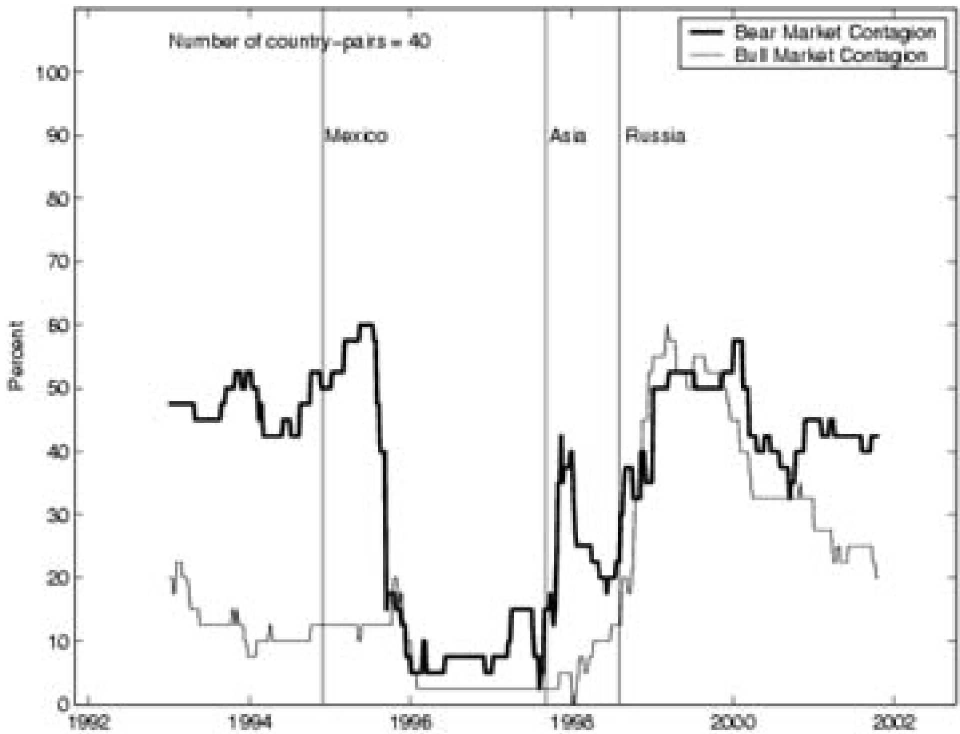
Contagion Across Mature Stock Markets

There are a number of distinctive bear market contagion patterns across mature stock markets. First, the extent of contagion of other mature markets with the United States was mostly flat and statistically insignificant before 1998, with the exception of some spikes in the 1995–96 period. It should be noted that contagion increased following the Mexican and Asian crises in 1994 and 1997, respectively, but remained statistically insignificant.

Contagion between the U.S. and European stock markets changed dramatically following the rapid succession of financial crises in the second half of 1998. Indeed, from late 1998 to mid-1999, there was a steep increase in contagion, which leveled off around late 1999 to early 2000. There was a similar buildup in bear market contagion between the United States and Japan, though contagion became statistically significant only in early 2001. By end-2001, contagion turned statistically insignificant once more.

Second, contagion between European stock markets exhibits a V-shaped pattern: contagion declined gradually from 1995 to 1997, leveled off during 1997–98, and, from late 1998 on, it followed a similar pattern as contagion with the United States. It should be noted that contrary to contagion with the United States, contagion across European stock markets was statistically significant in the 1995–97 period and

Figure 5. Significant Contagion Cases—Mature Markets and East Asia
(In percent)



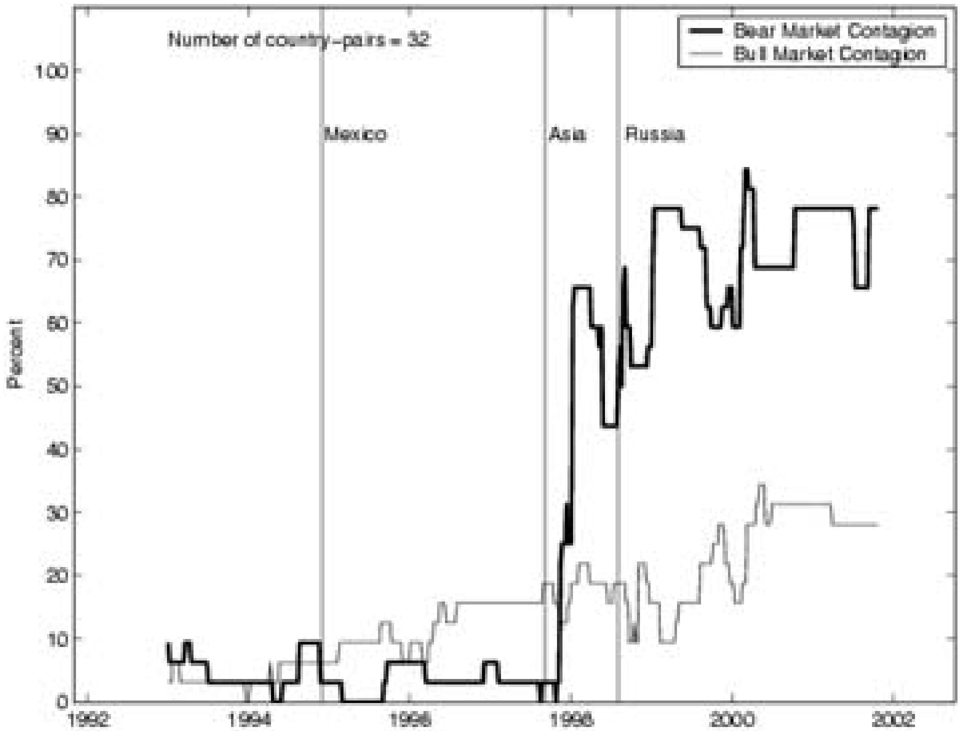
became less significant in the post-1998 period. Finally, contagion between Japanese and European stock markets exhibit the same V-shaped pattern that characterizes contagion across Europe. However, contagion has been mostly insignificant.

Some observations are worth noting regarding bull market contagion patterns in mature stock markets. First, bull market contagion between the U.S. and European stock markets was roughly constant, stronger than the corresponding bear market contagion, and statistically significant for substantial periods before 1997. Contagion declined gradually in 1997 and increased again in 1998. During the post-1998 period, bull market contagion has been weaker than bear market contagion, and only contagion between the United States and France has failed to show statistical significance. Second, contagion across European stock markets has been roughly constant and statistically significant for most of the period under study. Finally, bull market contagion with Japan, which used to be statistically significant and stronger than bear market contagion before 1996, has been declining gradually since end-1992.

Contagion Between Mature Markets and Latin America

In Latin America, contagion with the United States became statistically significant only after the second half of 1998, a finding that reinforces the notion that the 1998

Figure 6. Significant Contagion Cases—Latin America and East Asia
(In percent)



crises were major global shocks to the international financial system. Indeed, in 1999 contagion with the United States climbed up to levels never seen before and remained roughly constant until the end of the sample period. Contagions with European stock markets have been increasing steadily and became statistically significant around mid-2000. Contagion with Japan is nonexistent. Because of Latin America's strong economic and financial linkages with the United States, it is not surprising that contagion with the former is stronger than contagion with other mature stock markets.

Bull market contagion with the United States and European stock markets experienced increases in 1997 and 1998 for most Latin American stock markets. However, with the exception of contagion with the United States and Germany in the case of Argentina and Brazil, contagion has remained statistically insignificant. Contagion with Japan is not statistically significant, though estimates for Brazil, which increased significantly in 1998, are only marginally insignificant. On average, estimates of bull market contagion are lower than the corresponding estimates of bear market contagion.

Contagion Between Mature Markets and East Asia

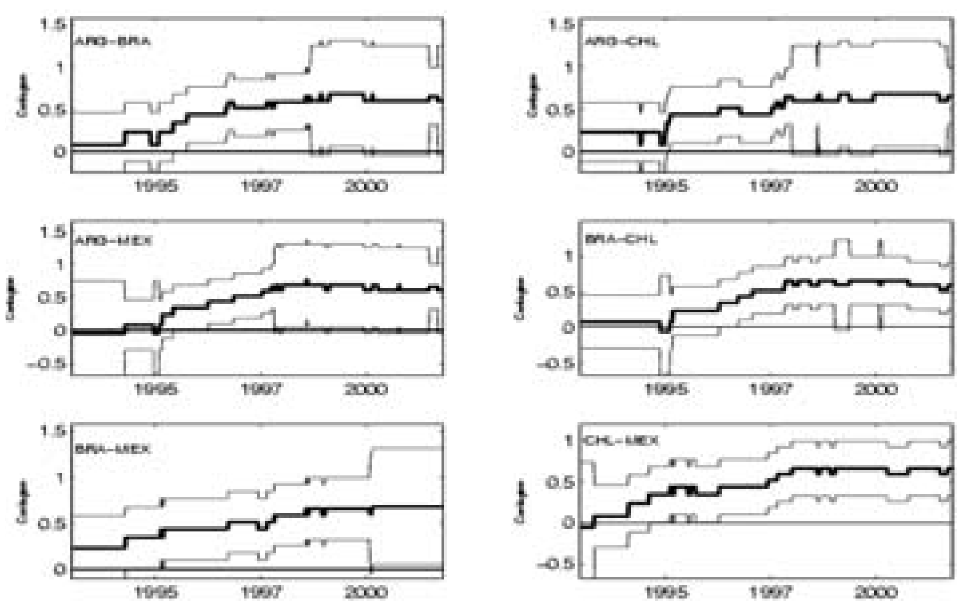
In general, bear market contagion with mature markets has not been statistically significant for East Asian stock markets. By the end of 2001, the region's stock markets appeared to be linked most closely to the United States and the United Kingdom. Overall, though, stock market linkages between East Asia and the developed world are weaker than those corresponding to Latin America, suggesting that financially the latter region is better integrated to the rest of the world.

Similarly, bull market contagion with mature markets has not been statistically significant during most of the period under study. By end-2001, there was significant contagion only between the United Kingdom and Hong Kong SAR. As in the case of Latin America, estimates of bull market contagion are lower than the corresponding estimates of bear market contagion. Interestingly, financial linkages between East Asia and Japan are not as strong as those between Latin America and the United States, as bear and bull market contagion is statistically insignificant.

Contagion Within Latin America

Within Latin America, bear market contagion increased steadily from 1992 to 1998 and then began leveling off in 1999. Figure 7 shows the appropriate bivariate extreme contagion measure and the 95 percent confidence levels. During the period 1996–98, bear market contagion was statistically significant for most country-pairs and has remained significant with the exception of country-pairs including Argentina. Nevertheless, contagion with Argentina has been marginally insignificant only from 1999 on and, in the case of Chile, became significant by end-2001. It is interesting to note that for the pairs Argentina-Brazil and Argentina-Mexico, contagion became significant in early 2001 but turned insignificant again by end-2001—

Figure 7. Bear Market Contagion in Latin America



Note: Please refer to the Appendix for country name abbreviations.

Table 1. Correlation Between Extremal Dependence Measures of Contagion and Conditional Correlations for Equity Markets

	USA	GBR	FRA	DEU	JPN	ARG	BRA	CHL	MEX	SGP	HKSAR	IDN	KOR	MYS	PHL	TPC	THA
USA		0.91	0.85	0.83	0.64	0.75	0.93	0.91	0.82	0.36	0.11	0.02	0.94	0.60	0.60	0.80	0.40
GBR	0.51		0.50	0.64	0.20	0.81	0.76	0.92	0.94	0.14	0.76	–0.05	0.77	0.22	0.90	0.13	0.66
FRA	0.03	0.57		0.89	0.53	0.78	0.59	0.74	0.79	0.89	0.00	–0.61	0.92	0.70	0.87	0.60	0.66
DEU	0.71	–0.16	0.28		0.50	0.91	0.75	0.90	0.94	0.77	0.38	–0.17	0.83	0.63	0.63	0.27	0.62
JPN	0.93	0.93	0.39	0.79		0.64	0.67	0.53	0.86	0.82	0.86	–0.53	–0.22	0.87	0.56	0.15	0.61
ARG	0.90	0.62	0.51	0.95	0.52		0.93	0.94	0.96	0.60	0.80	0.91	0.97	0.94	0.89	0.93	0.91
BRA	0.88	0.44	0.28	0.83	0.87	0.97		0.96	0.98	0.58	0.14	0.57	0.96	0.62	0.79	0.52	0.84
CHL	0.67	0.21	–0.12	0.89	–0.26	0.50	0.98		0.94	0.85	0.47	0.84	0.83	0.80	0.79	0.86	0.62
MEX	0.72	0.81	–0.10	0.68	0.58	0.95	0.93	0.85		0.22	0.38	0.69	0.73	–0.11	0.74	0.13	0.83
SGP	0.82	0.63	0.28	–0.07	0.63	0.96	0.91	0.14	0.63		0.15	0.45	0.56	0.70	0.72	0.17	0.29
HKSAR	0.23	0.85	0.20	0.45	0.80	0.32	0.92	–0.22	0.42	0.53		0.48	0.35	0.61	0.22	0.62	0.59
IDN	0.46	0.78	0.65	0.50	0.97	0.72	0.62	–0.02	0.63	0.89	0.81		0.81	0.24	0.70	0.41	0.58
KOR	0.86	0.58	0.58	0.80	–0.27	0.93	0.29	0.80	0.60	–0.11	–0.19	0.97		0.45	0.64	0.55	0.83
MYS	0.67	–0.08	0.77	0.17	0.46	0.75	0.09	–0.50	0.60	0.77	–0.42	0.64	0.26		0.72	0.76	0.59
PHL	0.24	0.82	0.78	0.14	0.75	0.17	0.86	0.34	–0.20	0.73	0.81	0.88	0.90	0.52		0.76	0.82
TPC	0.56	–0.05	0.29	0.55	0.63	0.42	0.29	0.24	0.25	0.58	0.87	0.86	0.32	0.43	0.46		0.43
THA	0.42	0.85	0.71	0.04	0.85	0.84	–0.46	–0.08	–0.37	0.25	–0.08	0.91	0.17	0.17	0.37	0.59	

Upper triangular matrix: Correlation between extremal dependence measures of contagion for large negative returns and conditional stock market return correlations.
Lower triangular matrix: Correlation between extremal dependence measures of contagion for large positive returns and conditional stock market return correlations.
Note: Please refer to the Appendix for country name abbreviations.

clear evidence of decoupling. Bull market contagion across Latin American stock markets has been also trending upward since 1998, with the only exception being the pair Argentina-Chile. Moreover, since 1998, contagion for almost all country-pairs has been statistically significant. In contrast to bear market contagion, there is no decoupling from Argentina for large positive market returns. As in the previous cases analyzed, bull market contagion is weaker than bear market contagion.

Contagion Within East Asia

Contagion has been roughly constant and significant for most country-pairs in East Asia during the entire period under study. The 1997 East Asia crisis prompted an increase in contagion for a number of country-pairs for which contagion was not significant before the crisis, such as Hong Kong SAR-Korea, Hong Kong SAR-Taiwan Province of China, and Korea-Malaysia. Contagion with Singapore and Hong Kong SAR, the two most important regional financial centers, has been significant for most country-pairs since end-1992 and for all country-pairs since end-1997. In contrast with Latin America, there is no secular increasing trend in East Asia. Bull market contagion patterns are similar to those of bear market contagion, although bull market contagion is weaker. This evidence suggests that financial transmission mechanisms across East Asian stock markets may have remained unchanged during the past decade.

Contagion Between Latin America and East Asia

Contagion between Latin America and East Asia was statistically insignificant before 1998. However, by late 1998 estimates of contagion increased sharply, and for some country-pairs including Argentina-Korea, Argentina-Taiwan Province of China, Chile-Hong Kong SAR, Chile-Thailand, and Mexico-Thailand, contagion became statistically significant. Contagion with Brazil, the major economy in Latin America, was significant for Malaysia and Taiwan Province of China only in 1999 and 2000, respectively. It is of interest to consider the degree of contagion between Argentina and Hong Kong SAR since both countries had currency board arrangements. In early 2000, contagion was significant, but by mid-2001, decoupling took place and contagion turned insignificant. With few exceptions, bull market contagion across Latin American and East Asian stock markets has been statistically insignificant for the past decade. The exceptions are Argentina and Singapore, and Brazil and Hong Kong SAR, with contagion significant since 1998 and 2000, respectively.

Differences Between Contagion Measures and Correlation Measures

Correlations between the contagion measures used in this paper and five-year rolling window equity market returns conditional correlations suggest at times quite different patterns of contagion (Table 1). For example, the correlation between bear contagion measures and conditional correlations among mature markets range from 0.20 (for Japan and the United Kingdom) to 0.91 (for the United States and the

United Kingdom). Therefore, reliance on conditional correlations could lead to false alarms, by overstating contagion risk not reflected in increases in extremal dependence measures, or create a false sense of security. The only exception is Latin America, where the correlation of contagion measures and simple correlations exceeds 0.93 for every pairwise combination of countries. However, even in Latin America the use of simple correlations is not always validated since contagion measures and correlations sometimes move in different directions during short periods.

IV. Conclusions

This paper has studied how contagion across equity markets has evolved during the past decade. The authors used dependence measures based on the joint behavior of co-exceedances of equity returns for pairwise combinations of equity market returns. These measures, which are based on extreme value theory, avoid the problems associated with the use of simple Pearson correlations.

The results indicate that (a) contagion patterns differ significantly within and across regions, with Latin America showing a secular increase in contagion not matched by other regions or countries; (b) contagion is higher for negative returns than for positive returns; (c) only the 1998 Russian and Brazilian crises led to a global increase in contagion; and (d) extremal dependence measures of contagion and simple correlation measures are not highly correlated, with the exception of Latin America, suggesting that the use of correlations as a proxy for contagion can at times be misleading.

APPENDIX

Country Abbreviations Used in This Paper

Mature markets

FRA = France
DEU = Germany
JPN = Japan
GBR = United Kingdom
USA = United States

Latin America

ARG = Argentina
BRA = Brazil
CHL = Chile
MEX = Mexico

East Asia

HKSAR = Hong Kong SAR
IDN = Indonesia
KOR = Republic of Korea

MYS	= Malaysia
PHL	= Philippines
SGP	= Singapore
TPC	= Taiwan Province of China
THA	= Thailand

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