Financial Globalization, Home Equity Bias and International Risk-Sharing

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Abstract

Recent empirical evidence suggests that the massive wave of financial globalization has been accompanied by a decline in home equity bias and changes in international risk-sharing. The objective of this paper is to reconcile these facts in a theoretical framework. By increasing the number of assets that can be traded internationally we show that even if home equity bias is somehow reduced, international risk sharing is still above the one observed in the data. Explaining the linkages between financial globalization, home equity bias and international risk-sharing is still a challenge for current model of international portfolio allocation.

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1 Introduction

The last two decades have witnessed a dramatic increase in international capital flows. Lane and Milesi Ferretti (2001, 2006) have documented the increase in gross holdings of cross-country bond and equities for various countries. This unparalleled expansion in private international asset trade has largely been a developed-country phenomenon, but developing countries have also participated. Lane and Milesi-Ferretti (2006) shows indeed that gross external financial positions now exceed 100% of GDP for major industrialized countries.

This massive wave of financial globalization has been accompanied by changes in international portfolio in bonds and equities and changes in international risk sharing. Indeed, Sorensen et al (2007) document the implications of financial integration for international portfolio allocation and international risk sharing by examining OECD countries during the period 1993-2003. Their empirical analysis shows that financial globalization has been accompanied by a decline in home bias in debt and equity holdings with a corresponding increase in international risk-sharing. On average they find that from 1993 to 2003 and for the countries in their sample, the value of foreign equity holdings as a percentage of GDP increased from 0.09 to 0.31, the value of foreign debt holdings increased from 0.47 to 1.2 and a similar pattern arises for the value of equity and debt liabilities (from 0.1 to 0.37 for equities and 0.67 to 1.35 for debt). Their measures of international risk-sharing displays a related pattern: income risk sharing and consumption risk sharing increased quite steeply through the 1990s. Moreover their empirical analysis suggest that declining home bias has been associated with strongly increasing consumption risk sharing, where the effect is mostly due to an decline in equity home bias rather than debt home bias. In short the main conclusion from Sorensen et al (2007) is that home bias and international risk sharing are closely related phenomena. Obstfeld (2007) on the other hand measures the degree of risk-sharing by looking at averages of consumption growth and real exchange rates for various country as in the original Backus and Smith (1993) paper: using this metric there is a distinct negative relationship (i.e. faster consumption growth is associated with a real appreciation) in the data for the period going from 1991 to 2006 (the period of financial integration) suggesting a worsening rather than an improvement in international risk-sharing.

Is it possible to reconcile in a common framework the aforementioned stylized facts about international capital markets? That is, can we generate less home bias through financial globalization and what are the consequences of financial integration for international risk-sharing?

To address theoretically these questions, we construct a general equilibrium model with two countries (Home and Foreign) and two sectors (tradable and nontradable) where each country is specialized in its own tradable good: this model encompasses most of the current models that have been used in recent international portfolio allocation analysis[1]. In order to capture different stages of financial market integration we solve the model under various financial market configurations (see also Devereux and Sutherland, 2006b), different in the number of assets that are traded across countries going from the simple bonds economy to the case in which we allow trade in all equities and bonds. At most in our economy agents can trade equities in tradable, nontradable sectors and Home and Foreign nominal bonds. We allow for shocks to tradables and nontradables endowments and redistributive shocks to tradable and non-tradable income as to make the international asset structure incomplete.

Our model is driven mainly by supply shocks: in general following a positive supply shocks the real exchange rate depreciate and in order to share risk domestic agents will hold foreign assets to hedge against domestic income risks. In our model by introducing non-traded goods we allow for the possibility that, depending on the origin of the supply shock (i.e. traded versus non traded), the real exchange rate and relative consumption across countries can move in opposite directions (see Benigno and Thoenissen, 2007) so that home assets bias in our model could result as an implication of the lack of international risk sharing. This feature of the model though depends critically on the type of assets that are traded internationally: when only real bonds are traded internationally, risk sharing is limited and the comovements between real exchange rate and consumption induce agents to short home bonds (compatible with the evidence as in Coeurdacier and Gourinchas, 2008). The opposite result holds once we consider nominal bonds: risk sharing becomes perfect and agent go

[1] Our analysis is restricted, though, to a flexible price economy. Engel and Matsumoto (2007, 2008) and Devereux and Sutherland (2006b) consider models with sticky prices.

long on home bonds. As we allow for equities to be traded (either only tradable equities or both tradable and non-tradable equities) we find that home equity bias declines as risk sharing improves but quantitatively the correlation between consumption and real exchange rate is well above the one observed in the data. Moreover the possibility of trading nominal bonds along with equities generates once again perfect risk-sharing and a strong home equity bias.

In short the main result of our analysis is that explaining simultaneously the decline in home equity bias with the size of international risk-sharing as economies become more financially integrated constitutes still a serious challenge for international portfolio models.

In the next section we briefly review the related literature. Section 3 describes the model set-up. Section 4 describes the parametrization and Section 5 analyzes the implications of the different asset market structures.

2 Related Literature

Our work is related to several papers in different ways. On the role of non-tradeability in addressing the home equity bias, Stockman and Dellas (1989) made an earlier contribution to solve for the optimal portfolio with nontraded goods. They study an endowment economy with separable utility between nontraded and traded goods. Their optimal equity portfolio is a combination of a well diversified portfolio in traded good sector equities and a complete home bias portfolio in nontraded good sector equities. In another contribution Baxter, Jermann and King (1998) study portfolio allocation in an endowment economy and assume perfect substitutability in home and foreign traded goods: they find that the presence of nontraded goods cannot explain home bias because the optimal portfolio of traded good sector equities is well diversified. Moreover the optimal holdings of nontraded good sector equities can exhibit either home bias or anti-home bias depending on the elasticity of substitution between traded and nontraded goods. More recently Matsumoto(2007) and Collard, Dellas, Diba and Stockman (2008) have extended the previous work by allowing for differentiated home and foreign traded goods and non separable utility. Focusing on the role of non-traded factors, Engel and Matsumoto (2006) show, in a sticky price framework, that home bias may be optimal to hedge labor income risk. The closest paper to out analysis is the one by Hnatkovska (2005) who, differently from the ones just mentioned, considers a two-sector model with incomplete asset market structure: while we focus on the role of financial globalization she examines the dynamic of portfolio choice to reconcile the home bias in equity holdings and high turnover and volatility of international capital flows.

In terms of solution techniques our work use the methodology developed by Devereux and Sutherland (2006a) and Tille and van Wincoop (2007) who derive an approximation method for determining portfolio share in general equilibrium framework. Also, Evans and Hnatkovska (2007) examine a similar model with a related solution methodology.

Many recent papers have addressed the issue of home equity bias: Coeurdacier, Kollmann, and Martin (2007) present a 2-country, 2-good model with trade in equities and bonds. They focus on "redistributive shocks" – shocks that redistribute income between firm owners and workers – as a source of home bias in equity holdings. Coeurdacier, Kollmann, and Martin (2008) consider a similar model, but with investment specific technological change, which they argue can explain the home bias in equity holdings. Heathcote and Perri (2008) extend the analysis of Cole and Obstfeld (1991) who examines the role of relative prices in international risk-sharing by considering an economy with production. Engel and Matsumoto (2008) and Coeurdacier and Gourinchas (2008) examines the role of bonds in diversifying international risk. Finally P. Benigno (2008) studies the link between home equity bias and international risk-sharing in a framework with robust preferences.

3 A two-sector two-country model

We develop a basic two-country open economy model with tradable and non-tradedeable endowments. There is a home and a foreign country, with each country endowned with its own tradable good. Household maximize utility over infinite horizon and we consider different configurations of financial assets that can be traded. At most there are six assets traded consisting of home and foreign equity shares in tradable and non tradable sectors and home and foreign nominal bonds. The structure of the model is related to the production economies described in Benigno and Thoenissen

(2007), Chari et al. (2002) and Stockman and Tesar (1995). In what follows the general case with all assets traded is described in details.

3.1 Consumer Behavior

The representative agent in the home economy maximizes the expected present discounted value of the utility:

$$U_t = E \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_t^{1-\rho}}{1-\rho} + \chi \log \left(\frac{M_t}{P_t} \right) \right] \tag{1}$$

where β is the discount factor with $0 < \beta < 1$ M denotes money holdings and C represents a consumption index defined over tradable C_T and non tradable C_N consumption:

$$C_t = \left[\gamma_H^{\frac{1}{\kappa}} C_{T,t}^{\frac{\kappa-1}{\kappa}} + (1-\gamma_H)^{\frac{1}{\kappa}} C_{N,t}^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{2}$$

where κ is the elasticity of intratemporal substitution between C_N and C_T and γ_H is the weight that the households assign to tradable consumption. The tradable component of the consumption index is in turn a CES aggregate of home and foreign tradedeable consumption goods, C_H and C_F:

$$C_{T,t} = \left[\nu_H^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu_H)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where where θ is the elasticity of intratemporal substitution between C_H and C_F and ν_H is the weight that the households assigns to home tradable consumption. We adopt a similar preference specification for the foreign country except that variables are denoted with an asterisk and weights have the F sub-index. The consumption price index, defined as the minimum expenditure required to purchase one unit of aggregate consumption for the home agent is given by:

$$P_t = \left[\gamma_H P_{T,t}^{1-\kappa} + (1-\gamma_H) P_{N,t}^{1-\kappa} \right]^{\frac{1}{1-\kappa}} \tag{4}$$

Meanwhile, the traded goods price index, defined as the minimum expenditure required to purchase one unit of a traded good is given by:

$$P_{T,t} = \left[\nu_H P_{H,t}^{1-\theta} + (1-\nu_H) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{5}$$

In each country agents invest their nominal wealth W_t net of domestic holdings of nominal money, in two riskless bonds denominated in the home and foreign currency and four risky assets, which are claims on a fraction of home and foreign countries' tradable and non-tradable sector endowments. We denote the fraction of tradable and non-tradable endowments that accrue to shareholders as $k_{i,t}$ $(i = H, F, N, N^*)$. Thus, $(1 - k_{i,t})Y_{i,t}$ represents the non-financial income received by the agents. S_t denotes the nominal exchange rate. We have that the budget constraint of the home agent is :

$$\begin{aligned} W_t = {} & x_{1,t-1}P_{H,t}[V_{H,t} + k_{H,t}Y_{H,t}] + x_{2,t-1}S_tP_{F,t}^*[V_{F,t}^* + k_{F,t}Y_{F,t}^*] \qquad (6) \\ & + x_{3,t-1}P_{N,t}[V_{N,t} + k_{N,t}Y_{N,t}] + x_{4,t-1}S_tP_{N,t}^*[V_{N,t}^* + k_{N,t}^*Y_{N,t}^*] + R_{H,t}B_{H,t-1} \\ & + R_{F,t}^*S_tB_{F,t-1} + (1-k_{H,t})P_{H,t}Y_{H,t} + (1-k_{N,t})P_{N,t}Y_{N,t} + M_{t-1} - P_tC_t - M_t \end{aligned}$$

where W_t is net nominal wealth defined as:

$$W_t \equiv B_{H,t} + S_tB_{F,t} + x_{1,t}P_{H,t}V_{H,t} + x_{2,t}S_tP_{F,t}^*V_{F,t}^* + x_{3,t}P_{N,t}V_{N,t} + x_{4,t}S_tP_{N,t}^*V_{N,t}^* \tag{7}$$

$B_{H,t}$ is the home holdings of internationally traded home bond with , $B_{F,t}$ is the home holdings of internationally traded foreign bond, $x_{i,t}$ $(i = 1, 2, 3, 4)$ denote the shares of domestic and foreign tradable and nontradable sector equity held by the home agent. $V_{H,t}, V_{F,t}^*, V_{N,t}$ and $V_{N,t}^*$ are equity prices for holding claims on the tradable and nontradable sectors in each country while $Y_{H,t}, Y_{F,t}^*, Y_{N,t}$ and $Y_{N,t}^*$ represents the stochastic endowments (dividends). $R_{H,t}$ and $R_{F,t}^*$ represents the returns for holding home and foreign currency bonds. For the foreign country we have that the budget constraint is given by:

$$W_t^* = x_{1,t-1}^* \frac{P_{H,t}}{S_t}[V_{H,t} + k_{H,t}Y_{H,t}] + x_{2,t-1}^* P_{F,t}^*[V_{F,t}^* + k_{F,t}Y_{F,t}^*] \tag{8}$$
$$+x_{3,t-1}^* \frac{P_{N,t}}{S_t}[V_{N,t} + k_{N,t}Y_{N,t}] + x_{4,t-1}^* P_{N,t}^*[V_{N,t}^* + k_{N,t}^* Y_{N,t}^*] + \frac{R_{H,t}}{S_t} B_{H,t-1}^*$$
$$+R_{F,t}^* B_{F,t-1}^* + (1 - k_{F,t}) P_{F,t} Y_{F,t} + (1 - k_{N,t}^*) P_{N,t}^* Y_{N,t}^* + M_{t-1}^* - P_t^* C_t^* - M_t^*$$

where

$$W_t^* \equiv \frac{B_{H,t}^*}{S_t} + B_{F,t}^* + x_{1,t}^* \frac{P_{H,t}}{S_t} V_{H,t} + x_{2,t}^* P_{F,t}^* V_{F,t}^* + x_{3,t}^* \frac{P_{N,t}}{S_t} V_{N,t} + x_{4,t}^* P_{N,t}^* V_{N,t}^* \tag{9}$$

Gross *nominal* equity returns and bonds returns expressed in terms of the home currency are as follows:

$$R_{1,t} \equiv \frac{P_{H,t}[V_{H,t} + k_{H,t}Y_{H,t}]}{P_{H,t-1}V_{H,t-1}}, \; R_{2,t} \equiv \frac{S_t P_{F,t}^*[V_{F,t}^* + k_{F,t}Y_{F,t}^*]}{S_{t-1} P_{F,t-1}^* V_{F,t-1}^*} \tag{10}$$
$$R_{3,t} \equiv \frac{P_{N,t}[V_{N,t} + k_{N,t}Y_{N,t}]}{P_{N,t-1}V_{N,t-1}}, \; R_{4,t} \equiv \frac{S_t P_{N,t}^*[V_{N,t}^* + k_{N,t}^* Y_{N,t}^*]}{S_{t-1} P_{N,t-1}^* V_{N,t-1}^*}$$
$$R_{5,t} \equiv R_{F,t}^* \frac{S_t}{S_{t-1}}, \; R_{6,t} \equiv R_{H,t}$$

V_i and Y_i are in units of the index good i, where $i = H, F, N, N^*$. We multiply them by the price of the respective index good, P_i, to find the nominal equity return expressed in home currency[2].

[2] The *real* returns will depend on the relative price of each index good with respect to the overall price level P :

We can express home agents' asset holdings as shares of wealth:[3]

$$\begin{gathered}
x_{1,t} P_{H,t} V_{H,t} \equiv \alpha_{1,t} W_t,\ x_{2,t} S_t P^*_{F,t} V^*_{F,t} \equiv \alpha_{2,t} W_t \\
x_{3,t} P_{N,t} V_{N,t} \equiv \alpha_{3,t} W_t,\ x_{4,t} S_t P^*_{N,t} V^*_{N,t} \equiv \alpha_{4,t} W_t \\
S_t B_{F,t} \equiv \alpha_{5,t} W_t,\ B_{H,t} \equiv \alpha_{6,t} W_t \\
\sum_{i=0}^{6} \alpha_{i,t} = 1.
\end{gathered} \tag{12}$$

Using the definition of gross nominal returns as in (10) and the shares of wealth invested in assets as defined in (12), we can rewrite the budget constraint in terms of excess returns over the

$$\begin{gathered}
r_{1,t} = R_{1,t} \frac{P_{t-1}}{P_t} \equiv \frac{\frac{P_{H,t}}{P_t}[V_{H,t} + Y_{H,t}]}{\frac{P_{H,t-1}}{P_{t-1}} V_{H,t-1}}, r_{2,t} \equiv R_{2,t} \frac{P_{t-1}}{P_t} = \frac{\frac{S_t P^*_{F,t}}{P_t}[V^*_{F,t} + Y^*_{F,t}]}{\frac{S_{t-1,f} P^*_{F,t-1}}{P_{t-1}} V^*_{F,t-1}} \\
r_{3,t} \equiv R_{3,t} \frac{P_{t-1}}{P_t} = \frac{\frac{P_{N,t}}{P_t}[V_{N,t} + Y_{N,t}]}{\frac{P_{N,t-1}}{P_t} V_{N,t-1}}, r_{4,t} = R_{4,t} \frac{P_{t-1}}{P_t} \equiv \frac{\frac{S_t P^*_{N,t}}{P_t}[V^*_{N,t} + Y^*_{N,t}]}{\frac{S_{t-1} P^*_{N,t-1}}{P_t} V^*_{N,t-1}} \\
r_{5,t} = R_{5,t} \frac{P_{t-1}}{P_t},\ r_{6,t} = R_{6,t} \frac{P_{t-1}}{P_t}
\end{gathered}$$

[3]Foreign agent's asset holdings as shares of wealth are defined as:

$$\begin{gathered}
x^*_{1,t} \frac{P_{H,t} V_{H,t}}{S_t} \equiv \alpha^*_{1,t} W^*_t,\ x^*_{2,t} P^*_{F,t} V^*_{F,t} \equiv \alpha^*_{2,t} W^*_t \\
x^*_{3,t} \frac{P_{N,t} V_{N,t}}{S_t} \equiv \alpha^*_{3,t} W^*_t,\ x^*_{4,t} P^*_{N,t} V^*_{N,t} \equiv \alpha^*_{4,t} W^*_t \\
B^*_{F,t} \equiv \alpha^*_{5,t} W^*_t,\ \frac{B^*_{H,t}}{S_t} \equiv \alpha^*_{6,t} W^*_t \\
\sum_{i=0}^{6} \alpha^*_{i,t} = 1
\end{gathered} \tag{11}$$

return on the home bond, $R_{xi} = R_{i,t} - R_{6,t}$ where $i = 1, ..., 5$[4]:

$$W_t = [R_{x1,t}\alpha_{1,t-1} + R_{x2,t}\alpha_{2,t-1} + R_{x3,t}\alpha_{3,t-1} + R_{x4,t}\alpha_{4,t-1} + R_{x5,t}\alpha_{5,t-1} + R_{6,t}]W_{t-1} \tag{14}$$
$$+(1 - k_{H,t})P_{H,t}Y_{H,t} + (1 - k_{N,t})P_{N,t}Y_{N,t} - P_tC_t - (M_t - M_{t-1})$$

3.2 Policy rules

We close the model by considering alternative policy rules. Although prices are fully flexible in our model, the way we specify policy rules matters as long as we have a nominal asset. This is because the return differential between home and foreign bonds is given by the rate of (unexpected) nominal exchange depreciation, which is affected by the policy rule in a flexible price setting. Consequently, equilibrium portfolio shares will be affected, which will then feed back into the model.

We focus on two cases: in the first one, policy authorities stabilize their own tradable prices ($P_{H,t} = 1$,and $P_{F^*,t} = 1$) and in the second one they stabilize domestic consumer prices ($P_t = 1$,and $P_t^* = 1$). Once we close the model the first order conditions that determine real money balances will determine M_t and M_t^* as a residual. Having a nominal bond with a CPI targeting rule is equivalent to having a real bond (or CPI indexed bond) with any policy rule in terms of equilibrium portfolio and model solution. Thus to facilitate discussion, we treat bonds under these two different policy specifications as different assets: we will refer to the nominal bond under CPI targeting rule as the real bond.

[4]Similarly, the budget constraint of the foreign agent can be expressed in terms of excess returns as:

$$W_t^* = \frac{S_{t-1}}{S_t}[R_{x1,t}\alpha^*_{1,t-1} + R_{x2,t}\alpha^*_{2,t-1} + R_{x3,t}\alpha^*_{3,t-1} + R_{x4,t}\alpha^*_{4,t-1} + R_{x5,t}\alpha^*_{5,t-1} + R_{6,t}]W^*_{t-1} \tag{13}$$
$$(1 - k_{F,t})P_{F,t}Y_{F,t} + (1 - k^*_{N,t})P^*_{N,t}Y^*_{N,t} - P^*_tC^*_t - (M^*_t - M^*_{t-1})$$

3.3 Equilibrium

Equilibrium in the asset markets requires home and foreign shares of stock to sum to one and home and foreign bonds to be in zero net supply:

$$x_{1,t} + x_{1,t}^* = 1,\ x_{2,t} + x_{2,t}^* = 1 \tag{15}$$
$$x_{3,t} + x_{3,t}^* = 1,\ x_{4,t} + x_{4,t}^* = 1$$
$$B_{F,t} + B_{F,t}^* = 0,\ B_{H,t} + B_{H,t}^* = 0$$

These together with equations (12) and its foreign counterpart imply the following:

$$\alpha_{1,t} W_t + \alpha_{1,t}^* S_t W_t^* = P_{H,t} V_{H,t} \tag{16}$$
$$\alpha_{2,t} \frac{W_t}{S_t} + \alpha_{2,t}^* W_t^* = P_{F,t}^* V_{F,t}^*$$
$$\alpha_{3,t} W_t + \alpha_{3,t}^* S_t W_t^* = P_{N,t} V_{N,t}$$
$$\alpha_{4,t} \frac{W_t}{S_t} + \alpha_{4,t}^* W_t^* = P_{N,t}^* V_{N,t}^*$$
$$\alpha_{5,t} \frac{W_t}{S_t} + \alpha_{5,t}^* W_t^* = 0$$
$$\alpha_{6,t} W_t + \alpha_{6,t}^* S_t W_t^* = 0$$
$$W_t + S_t W_t^* = P_{H,t} V_{H,t} + S_t P_{F,t}^* V_{F,t}^* + P_{N,t} V_{N,t} + S_t P_{N,t}^* V_{N,t}^*$$

The optimality conditions related to assets allocation for domestic and foreign households are given by the following set of equations

$$E_t \left[m_{t+1} R_{i,t+1} \right] = 1 \qquad i = 1, ..., 6. \tag{17}$$
$$E_t \left[m_{t+1}^* R_{i,t+1} \frac{S_t}{S_{t+1}} \right] = 1 \qquad i = 1, ..., 6.$$

where $m_{t+1} = \beta \frac{P_t}{P_{t+1}} \left(\frac{C_t}{C_{t+1}}\right)^{\rho}$ and $m^*_{t+1} = \beta \frac{P^*_t}{P^*_{t+1}} \left(\frac{C^*_t}{C^*_{t+1}}\right)^{\rho}$.

In what follows we also assume that there is complete pass-through, i.e. $P_{H^*,t} = P_{H,t}/S_t$,and $P_{F,t} = P_{F^*,t}S_t$. Equilibrium conditions in the good markets are obtained by equalizing the supply of each good with the demand obtained from the consumer intratemporal maximization problem.

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta} \left(\frac{P_{T,t}}{P_t}\right)^{-\kappa} \gamma_H \nu_H C_t + \left(\frac{P^*_{H,t}}{P^*_{T,t}}\right)^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\kappa} \gamma_F \nu_F C^*_t$$

$$Y^*_{F,t} = \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\theta} \left(\frac{P_{T,t}}{P_t}\right)^{-\kappa} \gamma_H (1-\nu_H) C_t + \left(\frac{P^*_{F,t}}{P^*_{T,t}}\right)^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\kappa} \gamma_F (1-\nu_F) C^*_t$$

$$Y_{N,t} = \left(\frac{P_{N,t}}{P_t}\right)^{-\kappa} (1-\gamma_H) C_t, \quad Y^*_{N,t} = \left(\frac{P^*_{N,t}}{P^*_t}\right)^{-\kappa} (1-\gamma_F) C^*_t$$

Finally asset prices are determined by combining the Euler equation with the definition of gross returns. Forward iteration together with no-bubble condition gives us the following:

$$V_{H,t} = \sum_{s=t+1}^{\infty} E_t \left\{ \beta^{s-t} \frac{u'(c_s)\frac{P_{H,s}}{P_s}}{u'(c_t)\frac{P_{H,t}}{P_t}} k_{H,s} Y_{H,s} \right\}, V^*_{F,t} = \sum_{s=t+1}^{\infty} E_t \left\{ \beta^{s-t} \frac{u'(c_s)\frac{S_s P^*_{F,s}}{P_s}}{u'(c_t)\frac{S_t P_{F,t}}{P_t}} k_{F,s} Y_{F,s} \right\} \quad (18)$$

$$V_{N,t} = \sum_{s=t+1}^{\infty} E_t \left\{ \beta^{s-t} \frac{u'(c_s)\frac{P_{N,s}}{P_s}}{u'(c_t)\frac{P_{N,t}}{P_t}} k_{N,s} Y_{N,s} \right\}, V^*_{N,t} = \sum_{s=t+1}^{\infty} E_t \left\{ \beta^{s-t} \frac{u'(c_s)\frac{S_s P^*_{N,s}}{P_s}}{u'(c_t)\frac{S_t P^*_{N,t}}{P_t}} k_{N^*,s} Y^*_{N,s} \right\}$$

3.4 Approximated solution

To solve the model we use the approximation techniques proposed in Devereux and Sutherland (2006a) and Tille and van Wincoop (2007). We approximate our model around the symmetric steady state in which steady-state inflation rates are assumed to be zero (see the appendix). Given our normalization home bias is the fraction invested by home agents in a country equity (both tradables $\bar{\alpha}_1$ and nontradables, $\bar{\alpha}_3$) minus the share of country's equity in the world equity supply (in steady state that would be $\bar{W}/(\bar{W}+\overline{SW}^*) = 1/2$. So home equity bias would arise if $\bar{\alpha}_1 + \bar{\alpha}_3 > 1/2$. If we want to measure equity bias in tradables only then the share of home country's traded sector equity in the world equity supply would be $\bar{P}_H\bar{V}_H/(\bar{P}_H\bar{V}_H + \bar{S}\bar{P}_F\bar{V}_F + \bar{P}_N\bar{V}_N + \bar{S}\bar{P}^*_N\bar{V}^*_N) = 1/4$ so that home bias in tradables would arise if $\bar{\alpha}_1 > 1/4$. If international asset trade in the non-tradable sector is not allowed then home equity bias would require $\bar{\alpha}_1 > 1/2$.

To determine the portfolio allocation we can rewrite the home and foreign portfolio choice equations given in (17) as follows:

$$E_t\left[m_{t+1}R_{x,t+1}\right] = 0$$
$$E_t\left[m^*_{t+1}R_{x,t+1}\frac{S_t}{S_{t+1}}\right] = 0$$

where $R'_{x,t+1} = [R_{1,t+1} - R_{6,t+1}, R_{2,t+1} - R_{6,t+1}, R_{3,t+1} - R_{6,t+1}, R_{4,t+1} - R_{6,t+1}, R_{5,t+1} - R_{6,t+1}]$ is the vector of excess returns using the home nominal bonds as a reference.

These two set of conditions imply the following equation that characterizes optimal portfolio choice up to a second order:

$$E_t\left[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1})\hat{R}_{x,t+1}\right] = 0$$

This is an orthogonality condition between excess returns in domestic currency and the difference in the nominal stochastic discount factors evaluated in the same currency. Since $E_t\left[\hat{R}_{x,t+1}\right] = 0$ up to a first order this condition can be expressed as:

$$Cov_t(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}, \hat{R}_{x,t+1}) = 0 \tag{19}$$

As shown by Devereux and Sutherland (2006a), to evaluate (19) and determine the portfolio shares, it is sufficient to take a first-order approximation of the remaining equilibrium conditions for which the only aspect of portfolio behavior that matter is $\bar{\alpha}_i$. The first order conditions of the rest of the model are summarized in the appendix.

4 Calibration

In this section, we outline our baseline calibration. We assume that the home and foreign economy are of equal size and are calibrated in a symmetric fashion. In choosing the parameters of utility function, we set β to match a 4% annual discount rate. As in Stockman and Tesar (1995) the coefficient of constant relative risk aversion, or the inverse of the intertemporal elasticity of substitution, ρ, is set to 2.

We calibrate the parameters pertaining to the consumption basket in the following way. The share of tradable goods in final consumption, γ, is 0.55, while the share of home goods in tradable consumption, ν, is 0.72. The calibration of this parameter is in line with other recent studies, such as Corsetti et al. (2008).

We assume an elasticity of substitution between home and foreign traded goods, θ, of 2.5 and an elasticity of substitution between traded and non-traded goods, κ, of 0.41, similar to what is suggested by Stockman and Tesar (1995)[5]. Given $\rho = 2$, this implies that utility is non-separable between traded and non-traded goods. Indeed, traded and non-traded goods are complements in our benchmark calibration since $\kappa\rho < 1$.

At first pass we proxy our process for tradable and non-tradable endowments in terms of the corresponding Solow residuals for each sector as in other studies. [6] To estimate these shocks Benigno and Thoenissen (2007) set US as the 'home' country and Japan plus the EU15 as the 'foreign' economy and use annual sectoral output and labour input from the Groningen Growth and Development Centre, 60-Industry Database which spans the years 1979 - 2002. They follow Backus, Kehoe and Kydland (1992) by imposing cross-country symmetry on the estimated shock process. Accordingly, endowments (dividends) are given by the following first order autoregressive process:

[5] Ostry and Reinhart (1992) estimate this parameter to be higher in the range of 0.66-1.44

[6] See Benigno and Thoenissen (2007), Corsetti et al (2008), Matsumoto (2007) and Stockman and Tesar (1995).

$$\begin{bmatrix} \log Y_{H,t} \\ \log Y^*_{F,t} \\ \log Y_{N,t} \\ \log Y^*_{N,t} \end{bmatrix} = \begin{bmatrix} \delta_T & 0 & 0 & 0 \\ 0 & \delta_T & 0 & 0 \\ 0 & 0 & \delta_N & 0 \\ 0 & 0 & 0 & \delta_N \end{bmatrix} \begin{bmatrix} \log Y_{H,t-1} \\ \log Y^*_{F,t-1} \\ \log Y_{N,t-1} \\ \log Y^*_{N,t-1} \end{bmatrix} + \begin{bmatrix} \upsilon_{H,t} \\ \upsilon^*_{F,t} \\ \upsilon_{N,t} \\ \upsilon^*_{N,t} \end{bmatrix} \tag{20}$$

where $\delta_T = 0.84$ and $\delta_N = 0.30$. For the variance-covariance matrix of endowment shocks, we set $V(\upsilon_H) = V(\upsilon_F) = 3.76$ and $V(\upsilon^*_N) = V(\upsilon^*_N) = 0.51$. We set the covariance between tradable and non-tradable endowments to zero at first pass to examine the role of the relative variance in determining the equilibrium portfolio and cross correlations.

In calibrating redistributive shocks we follow Coeurdacier, Kollman and Martin (2007). They compute the steady-state capital share to be 40% using data for G7 countries. Thus we set the mean capital share in tradable sector as $\bar{k}_H = 0.4$ and assume that the mean capital share in the non-tradable sector will be lower at $\bar{k}_n = 0.2$. Since under incomplete markets what matters for optimal portfolio is the *relative* size of shocks, we use the standard deviations of capital share and real GDP growth calculated by Coeurdacier et al. (2007) and set the relative size of redistributive versus endowment shocks to $V(\upsilon_{Ki})/V(\upsilon_{Yi}) = 1.2$ for $i = H, F, N, N^*$.The autoregressive process considered for redistributive shocks are as follows:

$$\begin{bmatrix} \log k_{H,t} \\ \log k^*_{F,t} \\ \log k_{N,t} \\ \log k^*_{N,t} \end{bmatrix} = \begin{bmatrix} \delta_{KT} & 0 & 0 & 0 \\ 0 & \delta_{KT} & 0 & 0 \\ 0 & 0 & \delta_{KN} & 0 \\ 0 & 0 & 0 & \delta_{KN} \end{bmatrix} \begin{bmatrix} \log k_{H,t-1} \\ \log k^*_{F,t-1} \\ \log k_{N,t-1} \\ \log k^*_{N,t-1} \end{bmatrix} + \begin{bmatrix} \upsilon_{k_H,t} \\ \upsilon^*_{k_F,t} \\ \upsilon_{k_N,t} \\ \upsilon^*_{k_N,t} \end{bmatrix} \tag{21}$$

Assuming redistributive shocks to be equally persistent as tradable endowment shocks we set $\delta_{KT} = \delta_{KN} = \delta_T = 0.84$.

After solving the model in terms of the state variables, we use these autoregressive processes to generate simulated time series of length T for the variables of interest and compute the moments of interest (standard deviation, covariances). We repeat this procedure J times and then compute the average of the moments. We compute the moments based on these Monte Carlo simulations because, under certain specifications, i.e. incomplete markets, our model is non-stationary.

5 Results

Before going into the details of each asset market structure we consider, we give an overview of how optimal portfolio shares and degree of international risk sharing vary across different degrees of financial market integration. The most basic financial market structure we consider is where there is international trade in only one non-contingent bond. Then we consider intermediate cases where only bonds or equities are internationally traded. The most complex asset market structure that we consider is the case in which all equities and bonds can be traded. Table 1 reports the results for the case where uncertainty is solely driven by shocks to tradable and non-tradable sector endowments while Table 2 considers redistributive shocks as additional sources of risk.

According to the spanning principle, markets will be potentially incomplete as long as the number of independent shocks hitting the economy is bigger than the number of assets available for trade. However, Table 1 shows that even with trade in only nominal bonds or trade in only tradable sector equities, full international risk sharing can be achieved when there are only endowment shocks. When redistributive shocks are added, full risk sharing can only be achieved with the most complex asset market configuration, where trade in all equities and bonds is possible.

Our results suggest that full international risk sharing can be consistent with a home or a foreign bias depending on the model specification. Full risk sharing-as reflected by a perfect correlation between relative consumption and real exchange rate- is consistent with both a foreign bias (the case with 2 tradable equity and 4 endowment shocks, Table 1), and a home bias (the case with 4 equities and 2 nominal bonds with redistributive shocks, Table 2).

Tables 1 and 2 also suggest that adding more sources of uncertainty while keeping the asset market structure unchanged, yields a switch from foreign equity bias to home equity bias, while decreasing the degree international risk sharing. To put it differently, moving towards a world with less uncertainty (a world with smaller number of shocks) would decrease the home bias and increase international risk sharing (Compare the results in Table 1 and 2 for the cases of 2 equities and 4 equities).

Table 1: 4 Endowment Shocks (Tradable and Non-tradable) - Different asset market structures under baseline calibration

	$\frac{\alpha_B}{\bar{C}}$	$\frac{\alpha_B^*}{\bar{C}}$	α_T	α_T^*	α_{NT}	α_{NT}^*	$Corr(\hat{C}_t - \hat{C}_t^*, \hat{Q}_t)$	
Asset Market Structure							(1)	(2)
Non-Contingent Bond	-	-	-	-	-	-	0.6870	0.3513
2 Real Bonds	-0.50	0.50	-	-	-	-	0.7212	0.3874
2 Nominal Bonds	1.85	-1.85	-	-	-	-	1	1
2 Equities (Tradable Sector)$^{(*)}$	-	-	-0.5214	1.5214	-	-	1	1
4 Equities (T & NT Sectors)$^{(**)}$	-	-	-0.3669	1.0766	0.2188	0.0775	1	1

Note: (1) Conditional cor.(2) Simulated Hp-filtered series.

All calculations under baseline calibration.

$^{(*)}\bar{k}_h = 0.4, \bar{k}_n = 0, ^{(**)}\bar{k}_h = 0.4, \bar{k}_n = 0.2$ and $\bar{k}_h = 0.4, \bar{k}_n = 0$ for bonds only cases.

Table 2: 4 Endowment Shocks and 4 redistributive Shocks (Tradable and Non-tradable) - Different asset market structures under baseline calibration

	$\frac{\alpha_B}{\bar{C}}$	$\frac{\alpha_B^*}{\bar{C}}$	α_T	α_T^*	α_{NT}	α_{NT}^*	$Corr(\hat{C}_t - \hat{C}_t^*, \hat{Q}_t)$	
Asset Market Structure							(1)	(2)
2 Equities (Tradable Sector)$^{(*)}$	-	-	0.6499	0.3501	-	-	0.7628	0.4853
4 Equities (T & NT Sectors)	-	-	0.4678	0.2419	-0.3620	0.6523	0.9168	0.7955
4 Equities& 2 Real Bonds	-0.1024	0.1024	0.4668	0.2428	-0.3636	0.6539	0.9171	0.7958
4 Equities& 2 Nominal Bonds	5.9563	-5.9563	0.7097	0	0.2905	-0.0002	1	1

Note: (1) Conditional cor.(2) Simulated Hp-filtered series.

All calculations under baseline calibration.

$\bar{k}_h = 0.4, \bar{k}_n = 0.2$ for all except (*) where $\bar{k}_h = 0.4, \bar{k}_n = 0$

6 i) Bonds

We start by focusing on the case in which the only assets traded are bonds denominated in domestic and foreign currency [7]. Agents receive the full endowment as a non-financial income. The portfolio

[7] This is equivalent to imposing $x_1 = x_3 = 1$, and $x_2 = x_4 = 0$ in home budget constaint (??) and $x_1^* = x_3^* = 1$, $x_2^* = x_4^* = 0$ in foreign budget constaint.

orthogonality condition given in terms of the relative pricing kernel in (19) can be written in terms of real exchange rate adjusted relative consumption and excess return on foreign bonds:

$$Cov_t(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1}}{\rho}, \hat{R}_{x,t+1}) = 0 \tag{22}$$

with $\hat{R}_{x,t} = \hat{R}_{F,t} - \hat{R}_{H,t}$. As discussed in Section 3, policy rules matter for portfolio choice since we are in the presence of nominal bonds. Thus in our analysis of bond portfolios we distinguish among different policy rules, focusing on special parametric cases for the analytical solution.

a) Domestic tradable price targeting, $\hat{P}_{H,t} = 0$ and $\hat{P}^*_{F,t} = 0$

When there is no consumption home bias ($\nu = 1 - \nu = 0.5$) the optimal bond portfolio position is:

$$\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = \frac{\gamma(\theta - 1)}{2(1 - \beta\delta_T)} \tag{23}$$

We note here that domestic households will be long in the domestic bond as long as $\theta > 1$. Moreover the bond position (in this special case) is not affected by shocks to the non-traded sector or volatility of both shocks. To explain this portfolio position it is useful to consider what would happen to the components of the portfolio orthogonality condition given in (22) in the case in which each country had a zero portfolio share ($\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = 0$). In particular we are interested in the zero portfolio solution of the relative consumption and the excess returns:

$$\hat{C}_t - \hat{C}^*_t - \frac{\hat{Q}_t}{\rho} = \frac{1}{\kappa\rho}\left(\frac{\theta-1}{\theta}\frac{1-\beta}{(1-\beta\delta_T)} + \frac{\theta-1}{\theta}\frac{\gamma(\kappa\rho-1)}{(1-\beta\delta_T)}\right)(\upsilon_{H,t} - \upsilon_{F,t}) + \frac{(1-\gamma)(\kappa\rho-1)}{(1-\beta\delta_N)}(\upsilon_{N,t} - \upsilon^*_{N,t}) \tag{24}$$

$$\hat{R}_{x,t} = \hat{R}_{F,t} - \hat{R}_{H,t} = \hat{S}_t - E_{t-1}\hat{S}_t = \frac{1}{\theta}(\upsilon_{H,t} - \upsilon_{F,t}) \tag{25}$$

Without any portfolio diversification, (24) shows that in response to a positive home country tradable shock, home relative consumption rises as long as $\theta > 1$ and $\kappa\rho > 1$ (tradable and non-tradable goods are gross substitutes). If $\kappa\rho < 1$ (i.e. the tradable and non-tradable are gross complements) agents would prefer to increase consumption of both tradable and non-tradable goods:

the fact that the supply of non-tradables has not changed prevent them for doing so, creating a possible negative effect on overall consumption. Still, for plausible parameters, relative consumption increases following a positive tradable endowment shock even when traded and non-traded goods are complements in consumption. A similar reasoning occurs when there is a shock to non-tradable endowment: in this case the substitutability ($\kappa\rho > 1$ or $\kappa\rho < 1$) matters for determining the sign of the relative consumption response.

To hedge against the consumption risk (captured by movements in (24)), agents would like to hold domestic currency bonds when their return falls following an increase in relative consumption. Indeed (25) shows that following a positive shock to home tradable endowment, excess return on home bonds falls, i.e. $\hat{R}_{x,t}$ rises, as the nominal exchange rate depreciates. Thus home bonds provide a good hedge against traded sector shocks for $\theta > 1$. We note here that excess return is not affected at all by the substitutability or complementarity of traded and non-traded goods or the presence of non-traded goods shocks: thus, agents cannot hedge against non-traded sector shocks when there is no consumption home bias and the only assets available are currency bonds.

When utility is separable ($\kappa\rho = 1$) the optimal bond portfolio position is:

$$\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = \frac{\gamma(1-\nu)(\kappa - 1 + 2\nu(\theta - \kappa))}{(1-\beta\delta_T)} \tag{26}$$

Since in this case $\kappa = \frac{1}{\rho}$, it is possible to rewrite the above expression as follows ($\rho = 1$ gives the log-investor under separable utility):

$$\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = \frac{\gamma(1-\nu)}{(1-\beta\delta_T)}\left(-\frac{\rho-1}{\rho} + 2\nu(\theta - \frac{1}{\rho})\right) \tag{27}$$

As before the bond position is not affected by shocks to the non-traded sector or volatility of the shocks. As long as there is consumption home bias ($\upsilon > 1/2$) home currency holdings will be positive. When each country has a zero portfolio share ($\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = 0$). the real exchange rate adjusted consumption differential and the excess returns are given by:

$$\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho} = \frac{1-\beta}{1-\beta\delta_T}\frac{(\kappa - 1 + 2\nu(\theta-\kappa))}{(1+2\nu(\theta-1))}(\upsilon_{H,t} - \upsilon_{F,t}) \tag{28}$$

$$\hat{R}_{x,t} = \hat{R}_{F,t} - \hat{R}_{H,t} = \hat{S}_t - E_{t-1}\hat{S}_t \tag{29}$$

$$= \{\frac{\kappa(1-4\nu(1-\nu)) + 4\theta\nu(1-\nu) + \beta(2\nu-1)(\kappa-1+2\nu(\theta-\kappa))}{(\kappa(2\nu-1)^2 + 4\theta(1-\nu)\nu)(1+2\nu(\theta-1))(1-\beta\delta_T)}$$
$$+\frac{\beta\delta_T}{(\kappa(2\nu-1)^2+4\theta(1-\nu)\nu)(1-\beta\delta_T)}\}(\upsilon_{H,t} - \upsilon_{F,t})$$

With separable utility relative consumption and excess returns are unaffected by shocks to non-traded sector endowment as shown by (28) and (29). When there is a positive endowment shock in the traded goods sector, home consumption of home traded goods will increase more than foreign consumption of home traded goods because of the home bias in consumption. Non-traded goods will become more expensive compared to traded goods in both countries. If $\theta > \kappa$, i.e. the elasticity of substitution between home and foreign traded goods is bigger than the elasticity of substitution between traded and non-traded goods; home will decrease its consumption of the foreign traded good more than it decreases its consumption of home non-traded good. The extent of this substitution away from foreign traded goods increases with the degree of consumption home bias ν.Thus, relative consumption increases more in the face of a traded sector shock when $\nu > \frac{1}{2}$ and $\theta > \kappa$. The low elasticity of substitution between traded and non-traded goods, i.e. $\kappa < 1$,limits the increase in relative consumption as consumers cannot easily substitute away from non-traded goods which are now relatively more expensive in both countries.

Excess returns become a complicated expression when $\nu \neq \frac{1}{2}$as shown in (29). Again the key parameters that determine excess returns on home bonds, which is negatively related to unexpected home currency depreciation, are the extent of consumption home bias captured by $(2\nu - 1)$ and the relative substitutability among home and foreign traded goods and among traded and non-traded goods as given by $(\theta - \kappa)$.

For plausible parameter values, excess returns on home bonds are negative following traded sector endowment shocks, while relative consumption is positive, which makes home bonds a good hedge against real exchange rate fluctuations.

In general, for our calibration, in this case ($\hat{P}_{H,t} = 0$ and $\hat{P}^*_{F,t} = 0$) domestic agents are long in home currency bonds (Table 3). Once agents hold the optimal amount of currency bonds, consumption and real exchange rate becomes perfectly correlated. Indeed, following a positive supply-side shock to the home economy's traded goods sector, home agents become wealthier and demand more goods of all types. Despite this, through their portfolio allocation they share risk with foreigners and transfer resources to them so that foreign consumption increases more than in the zero-portfolio case [SEE FIGURE 1].

For the special case in which there is no home bias and utility is separable it is possible to derive an analytical expression for the conditional correlation between consumption differential and the real exchange rate[8]:

$$Corr_{t-1}(\hat{C}_t - \hat{C}^*_t, \hat{Q}_t) = \frac{(1-\gamma)\frac{\sigma^2_N}{\sigma^2_T} - \gamma\Phi}{\sqrt{\left((1-\gamma)^2\frac{\sigma^2_N}{\sigma^2_T} + \gamma^2\Phi\right)\left(\frac{\sigma^2_N}{\sigma^2_T} + \Phi\right)}} \tag{30}$$

where $\Phi = \frac{(1-\beta)^2(2\tilde{\alpha}_B - \gamma(\theta-1))^2}{\gamma^2\theta^2(1-\beta\delta_T)^2}$. From (30) we can see that once we substitute the optimal portfolio allocation we obtain that the conditional correlation is always equal to 1 so that in this case there is perfect international risk sharing.

b) Consumer price targeting, $\hat{P}_t = 0$ and $\hat{P}^*_t = 0$ (Real Bond)

When utility is separable ($\kappa\rho = 1$) and there is no consumption home bias ($\nu = 1 - \nu = 0.5$)

[8] The conditional correlation helps us in understanding the determinants of international risk-sharing. In the quantitative analysis we report both the conditional cross correlation and the HP-filtered cross correlation which is the one comparable with the empirical evidence presented earlier.

the optimal bond portfolio position is:

$$\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = \frac{\gamma(\theta-1)}{2(1-\beta\delta_T)}\left(-\frac{(1-\beta)(1-\gamma)(\theta-1)}{(1-\gamma)^2\theta^2\rho(1-\beta\delta_T)\frac{\sigma_N^2}{\sigma_T^2}}\right) \tag{31}$$

The optimal portfolio position can be decomposed in two terms: the first term is the one that correspond to the portfolio under tradable price targeting while the second term is specific to consumer price targeting. Because of the second component, domestic households will be short in the domestic bond so that $\tilde{\alpha}_B < 0$[9]. Moreover, the optimal bond portfolio depends critically among other things on the relative variance between traded and non-traded sector shocks. When there are only non-tradable shocks $\frac{\sigma_N^2}{\sigma_T^2} \to \infty$, $\tilde{\alpha}_B \longrightarrow 0$. As before we consider what would happen in the case in which each country had a zero portfolio share ($\tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = 0$). In particular we are interested in the behavior of the two components of the portfolio orthogonality condition-relative consumption and the excess returns:

$$\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho} = \frac{\theta-1}{\theta}\frac{1-\beta}{(1-\beta\delta_T)}(\upsilon_{H,t} - \upsilon_{F,t}) \tag{33}$$

$$\begin{aligned} \hat{R}_{x,t} = \hat{R}_{F,t} - \hat{R}_{H,t} = \hat{S}_t - E_{t-1}\hat{S}_t = \hat{Q}_t - E_{t-1}\hat{Q}_t \\ = \frac{1}{\theta}\left(\frac{(1-\beta)(1-\gamma)(\theta-1)}{-\kappa(1-\beta\delta_T)}\right)(\upsilon_{H,t} - \upsilon_{F,t}) + \frac{1-\gamma}{\kappa}\left(\upsilon_{N,t} - \upsilon_{N,t}^*\right) \end{aligned} \tag{34}$$

From (34) we can see the link between optimal bond position and the excess return on foreign bonds. Following a positive tradable endowment shock, under consumer price targeting, the

[9]Under non-separability the expression that determines the optimal portfolio position is more complicated and the sign of the bond position depends on the degree of substitutability between tradable and non-tradable:

$$\begin{aligned} \tilde{\alpha}_B = -\tilde{\alpha}_{B^*} = -\left(\frac{\theta-1}{\theta}\right)^2\left(\frac{1-\beta}{2(1-\beta\delta_T)^2}\right)\left(\frac{\sigma_N^2}{\sigma_T^2}\right)^{-1}\frac{\gamma\left(1+\gamma(\kappa\rho-1)\right)^2(1-\beta\delta_N)}{\rho(1-\gamma)\Lambda} \\ +\frac{\gamma(\kappa\rho-1)}{2\rho(1-\beta\delta_N)} \end{aligned} \tag{32}$$

where $\Lambda = \kappa\rho(1-\beta\delta_N) - (1-\beta)(1-\gamma)(\kappa\rho-1)$.

exchange rate appreciate as long as $\theta > 1$. Home consumption on the other hand will increase compared to foreign consumption following the same positive tradable endowment shock. This implies that domestic bonds will be a poor hedge against consumption risk if tradable shocks are the main source of uncertainty. On the other hand, following a positive non-tradable endowment shock, excess return on home bonds will be higher while relative consumption will remain unchanged. This would also discourage holdings of home bonds [10]

In general, for our calibration, in this case ($\hat{P}_t = 0$ and $\hat{P}_t^* = 0$) domestic agents are short in home currency bonds (Table 3). Once agents hold the optimal amount of currency bonds, consumption and real exchange rate becomes imperfectly correlated. Indeed, following a positive supply-side shock to the home economy's traded goods sector, home agents become wealthier and demand more goods of all types. In this case there is no enough risk-sharing either through portfolio allocation or the terms of trade and because of this foreign consumption does not increase by as much as home consumption. Also the real exchange rate appreciates since the Balassa-Samuelson effect dominates the terms of trade effect [SEE FIGURE 2].

For the special case in which there is no home bias and utility is separable it is possible to derive an analytical expression for the conditional correlation between consumption differential and the real exchange rate:

[10] Under non-separability we have more complicated expression for the consumption differential and the excess return:

$$\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho} = \frac{\theta-1}{\theta}\frac{1-\beta}{(1-\beta\delta_T)}\frac{1+\gamma(\kappa\rho-1)}{\kappa\rho}(\upsilon_{H,t}-\upsilon_{F,t}) + \frac{1-\beta}{(1-\beta\delta_N)}\frac{(1-\gamma)(\kappa\rho-1)}{\kappa\rho}\left(\upsilon_{N,t}-\upsilon_{N,t}^*\right) \tag{35}$$

$$\begin{aligned}\hat{R}_{x,t} = \hat{R}_{F,t} - \hat{R}_{H,t} &= \hat{S}_t - E_{t-1}\hat{S}_t \\ &= -\frac{\theta-1}{\theta}\frac{1-\beta}{(1-\beta\delta_T)}\frac{1-\gamma}{\kappa}(\upsilon_{H,t}-\upsilon_{F,t}) \\ &+\frac{1-\gamma}{\kappa}\frac{\Lambda}{(1+\gamma(\kappa\rho-1))(1-\beta\delta_N)}\left(\upsilon_{N,t}-\upsilon_{N,t}^*\right)\end{aligned} \tag{36}$$

$$Corr_{t-1}(\hat{C}_t - \hat{C}_t^*, \hat{Q}_t) = \frac{-(1-\gamma)\Upsilon_1\Upsilon_2\frac{\sigma_N^2}{\sigma_T^2} + \kappa\gamma^2\Theta}{\sqrt{\left((1-\gamma)^2\Upsilon_1^2\frac{\sigma_N^2}{\sigma_T^2} + (\kappa\gamma^2)^2\Theta\right)\left(\Upsilon_2^2\frac{\sigma_N^2}{\sigma_T^2} + \Theta\right)}} \tag{37}$$

where

$$\begin{aligned}
\Upsilon_1 &= \frac{2\tilde{\alpha}_B(1-\beta) - \gamma\kappa(1-\beta\delta_N)}{2\tilde{\alpha}_B(1-\beta)(1-\gamma) - \gamma\kappa(1-\beta\delta_N)} \\
\Upsilon_2 &= \frac{(1-\beta\delta_N)}{2\tilde{\alpha}_B(1-\beta)(1-\gamma) - \gamma\kappa(1-\beta\delta_N)} \\
\Theta &= \left[\frac{\theta-1}{\theta}\frac{1-\beta}{2\tilde{\alpha}_B(1-\beta)(1-\gamma) - \gamma\kappa(1-\beta\delta_T)}\right]^2
\end{aligned}$$

Even if (37) is still a complicated expression, it is possible to show that when the relative variance of tradable shocks increases ($\frac{\sigma_N^2}{\sigma_T^2}$ becomes lower), domestic agents short more domestic currency bonds and international risk sharing worsen. So that in this case imperfect risk sharing is driven by the Balassa-Samuelson effect combined with CPI targeting (see also Benigno and Thoenissen (2007)) and it is the outcome of an optimal portfolio allocation.

To have a better idea on how the different factors affect the cross-correlation and the home currency position, in Figures 3-5 we do sensitivity analysis by varying the $\frac{\sigma_N^2}{\sigma_T^2}$, θ and κ. The fact that more negative home currency position is associated with lower consumption-real exchange rate cross-correlation is a robust feature of this case as we consider lower volatility of non-tradable shocks, higher θ (as long as $\theta > 1$) and lower κ. While in general we find that while this case is compatible with lack of imperfect risk sharing, quantitatively the cross-correlation between consumption differential and real exchange rates is still above the one measured empirically. For example a negative cross correlation would be consistent with a relative variace ratio, $\frac{\sigma_N^2}{\sigma_T^2}$, extremely low. So even if international asset trade is restricted to nominal currency bonds, it is difficult for this class of models to replicate the empirical evidence.

Table 3 summarizes optimal home bond positions and implied relative consumption real exchange rate correlations under the two policy rules we consider for our benchmark calibration and for the special cases of symmetric prefences and separable utility.

Table 3: 2 Nominal Bonds, 4 Endowment Shocks - Baseline model and sensitivity analysis

	Home bonds	$\frac{Cov_t(\hat{R}_{x,t},\hat{Q}_t)}{Var(\hat{R}_{x,t})}$		$Corr(C_t - C_t^*, Q_t)$	
	$\alpha/\bar{C}$	(1)	(2)	(1)	(2)
$\hat{P}_{H,t}= 0$ and $\hat{P}^*_{F,t}= 0$					
Baseline calibration	1.85	0.2088	0.2095	1	1
$\nu = 0.5$	2.06	0.0000	-0.0021	1	1
$\kappa = 0.5$	1.83	0.2420	0.2420	1	1
$\hat{P}_t= 0$ and $\hat{P}^*_{F,t}= 0$					
Baseline calibration	-0.50	1	0.9830	0.7212	0.3874
$\nu = 0.5$	-0.83	1	0.9853	0.8142	0.5265
$\kappa = 0.5$	-0.23	1	0.9832	0.7298	0.4511

Note: (1) Conditional (2) Simulated Hp-filtered series. $R_{x,t}=R_{F,t}-R_{H,t}$,i.e. excess return on foreign bonds.

6.1 ii) Equities

6.1.1 Tradable sector equities

We now focus on another incomplete asset market structure in which we only allow for international trade in tradable sector equities. This case is obtained by letting $B_H = B_F = 0$, $x_3 = 1$ and $x_4 = 0$ in home budget constraint given by (6) and $B^*_H = B^*_F = 0$, $x^*_3 = 0$ and $x^*_4 = 1$ in foreign budget constraint given by (8). Recall that in this case home equities bias in tradable would require $\bar{\alpha}_T > 1/2$ since world equities supply consists only of tradable equities.

We distinguish different cases depending on the number of shocks and the fraction of the endowment which is distributed to household (i.e "labor income") (See Table 4). In order to generate home bias for our baseline calibration we need to have redistributive shocks as in the analysis of Coeurdacier, Kollman and Martin (2007). Without redistributive shocks, agents go long on foreign equity the bigger is the share of labor income. Intuitively when the share of labor income increases, foreign equities becomes a better hedge against domestic shock so that agents tend to reduce their home equity position. In the presence of redistributive shocks because of uncertainty related to the fraction of labor income, households will tend to have more domestic equities. In general the presence of redistributive shocks does not affect the consumption differential at the zero portfolio solution (i.e. $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ depends only on tradable and non-tradable endowment shocks) while it does affect the excess return since, for example, a positive redistributive shock that increase labor income reduces the return on home equities. This is why foreign equities are not a good hedge against redistributive shocks and agents increase holdings of domestic tradable equities. This effect is bigger the bigger the size of the redistributive shock relative to tradable endowment shock [SEE FIGURE 7].

In the case with endowment shocks only, international risk-sharing is perfect despite the presence of non-tradable shocks: by choosing optimally their portfolio of tradable equities agents perfectly insure themselves against fluctuations in relative consumption.On the other hand, the presence of redistributive shocks tend to lower the cross correlation between consumption and real exchange rate. Relative consumption and real exchange rate cam move in opposite directions in the face of a tradable endowment shock [SEE FIGURE 6]. As in the only bonds case, decreasing the relative volatility of non-tradable shocks, $(\frac{\sigma_N^2}{\sigma_T^2})$ reduces the degree of risk-sharing but it doesn't have any effect on the share of tradable equities; the higher is the elasticity of intratemporal substitution (for $\theta > 1$) the lower is the consumption real exchange rate cross-correlation but the lower is the degree of home bias [SEE FIGURES 8 AND 9]. In the presence of redistributive shocks, since the equities also hedge against fluctuations in non-financial income in addition to fluctuations in real exchange rate, the covariance-variance ratio of excess returns and real exchange rate decreases and becomes more in line with data reported in van Wincoop and Warnock (2007).

In terms of the empirical evidence discussed at the beginning, this case suggests that a model with international asset trade in tradable sector equities would be consistent with imperfect risk-sharing and home bias in equities as long as there are redistributive shocks. From a quantitative point of view, though, the cross-correlation between consumption differential and the real exchange rate is still above the one observed in the data. The presence of non-tradable shocks contributes to lower it but not sufficiently at least for plausible calibration of $\frac{\sigma_N^2}{\sigma_T^2}$.

Table 4: 2 Tradable Sector Equities, Different Combinations of Endowment and Redistributive Shocks - Baseline model and sensitivity analysis

International Trade in Tradable Sector Equities			$\frac{Cov(\hat{R}_{x,t},\hat{Q}_t)}{Var(\hat{R}_{x,t})}$		$Corr(C_t - C_t^*, Q_t)$	
	α_T	α_T^*	(1)	(2)	(1)	(2)
2 Tradable Endowment Shocks						
$\bar{k}_h = 1, \gamma = 1, \nu = 0.5$	0.5000	0.5000	0	0	1	1
$\bar{k}_h = 1, \gamma = 0.55, \nu = 0.72$	0.3914	0.6086	-0.9973	-1.0026	1	1
$\bar{k}_h = 0.4, \gamma = 0.55, \nu = 0.72$ (Baseline)	-0.5214	1.5214	-0.9973	-1.0026	1	1
4 Endowment Shocks (Tradable&NT)						
$\bar{k}_h = 1, \gamma = 0.55, \nu = 0.72$	0.3914	0.6086	-0.9973	-0.9960	1	1
$\bar{k}_h = 0.4, \gamma = 0.55, \nu = 0.72$ (Baseline)	-0.5214	1.5214	-0.9973	-0.9960	1	1
4 Endowment+2 Redistributive Shocks						
$\bar{k}_h = 1, \gamma = 0.55, \nu = 0.72$	0.8600	0.1400	-0.2294	-0.2246	0.7628	0.4853
$\bar{k}_h = 0.4, \gamma = 0.55, \nu = 0.72$ (Baseline)	0.6499	0.3501	-0.2295	-0.2247	0.7628	0.4853

Note: (1) *Conditional* (2) *Simulated Hp−filtered series.* $R_{x,t} = R_{2,t} - R_{1,t}$,*i.e. excess return on foreign equity.*

6.1.2 Tradable and Non-tradable Sector Equities

We now allow also for equity trade in the non-tradable sector. In this case home equity bias in tradable would require $\bar{\alpha}_T > 1/4$ since world equities supply consists only of tradable and non tradables equities while home equity bias would require $\bar{\alpha}_T + \bar{\alpha}_{NT} > 1/2$. In Table 5 we report the detailed results for this case. As in the previous case the results depend critically on the presence of redistributive shocks and the fraction of endowment which is distributed to households. When there are only endowment shocks, international asset markets are complete: if the weight on nontradable, γ, is 0.5, utility is separable between tradable and non-tradable goods ($\kappa\rho = 1$) and there is no home bias in preferences ($\nu = 0.5$), with both tradable and non-tradable sector endowments fully capitalisable ($k_h = 1, k_n = 1$), we replicate the Stockman and Dellas (1989) outcome of full-diversification in tradable sector equities and full home bias in non-tradable sector equities. This specific case implies $\bar{\alpha}_T + \bar{\alpha}_{NT} = 3/4$. Assuming tradable and non-tradable goods are complements ($\kappa\rho < 1$)$-$keeping everything else constant- leaves the share of tradable equities unchanged while increases the portfolio share of foreign non-tradable sector equity as discussed in Collard, Dellas, Diba, Stockman (2007). For the separable case but allowing for consumption home bias ($\nu = 0.72$) we get foreign bias in tradable sector equity (consistent with Table 4). But there is still home equity bias in overall portfolio as the home agents hold the total home non-tradable equity stock (again this is consistent with Collard et. al. (2007)). Once we reduce the steady-state capital share of tradable and non-tradable endowment to 0.4 and 0.2, respectively, agents hedge against domestic shock by increasing their holding of foreign equities: home agents short home tradable sector equity and sell some of the home non-tradable equity stock to buy more of the foreign tradable sector equity, which creates a foreign bias in overall portfolio.

Once we allow for redistributive shocks asset markets become incomplete. As in the previous case, the presence of redistributive shocks determines a sizable home bias in tradable sector equities. In the presence of redistributive shocks agents hedge against movements in the real exchange rate coming from the relative prices of non-tradable by increasing also their share of non-tradable equities. But once we allow for an increase in labor income share ($\bar{k}_n < 1$) agents hedge against domestic shock to non-tradable by increasing their holding of foreign non-tradable equities.reverting the previous result and reducing significantly the degree of home bias.

But as in the case above, changes in the structure of international portfolio do not have any significant impact on the degree of international risk-sharing: what determines a lower cross correlation between consumption and real exchange rate (for our calibration it ranges from 0.75 to 0.8) is the presence of redistributive shocks.

While quantitatively the home equity position is highly sensitive to parameter specification, we note that as we increase the number of assets (from 2 to 4) we observe a combined reduction of both home equity bias and an increase in international risk-sharing.

Table 5: Tradable and Non-tradable Sector Equities, Different Combinations of Endowment and Redistributive Shocks - Baseline model and sensitivity analysis

Tradable and Non-Tradable Sector Equities	α_T	α_T^*	α_{NT}	α_{NT}^*	$Corr(\hat{C}_t - \hat{C}_t^*, \hat{Q}_t)$	
4 Endowment Shocks (Tradable&NT)					(1)	(2)
Cases with $\bar{k}_h = 1, \bar{k}_n = 1$	0.25	0.25	0.50	0	1	1
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.25	0.25	0.50	0	1	1
$\gamma = 0.5, \nu = 0.5, \kappa = 0.41$	0.25	0.25	0.4794	0.0206	1	1
$\gamma = 0.5, \nu = 0.72, \kappa = 0.5$	0.2006	0.2994	0.50	0	1	1
Cases with $\bar{k}_h = 0.4, \bar{k}_n = 0.2$	-0.1666	0.8333	0.3333	0	1	1
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	-0.1666	0.8333	0.3333	0	1	1
$\gamma = 0.55, \nu = 0.72, \kappa = 0.41$	-0.3669	1.0766	0.2188	0.0715	1	1
4 Endowment & 4 Redistributive Shocks						
Cases with $\bar{k}_h = 1, \bar{k}_n = 1$						
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.4423	0.0577	0.3856	0.1144	0.9185	0.7751
$\gamma = 0.5, \nu = 0.5, \kappa = 0.41$	0.4423	0.0577	0.3610	0.1390	0.9319	0.8097
$\gamma = 0.5, \nu = 0.72, \kappa = 0.5$	0.4309	0.0691	0.3489	0.1511	0.9124	0.7861
Cases with $\bar{k}_h = 0.4, \bar{k}_n = 0.2$						
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.4744	0.1923	-0.0482	0.3814	0.9185	0.7751
$\gamma = 0.55, \nu = 0.72, \kappa = 0.41$	0.4678	0.2419	-0.3620	0.6523	0.9168	0.7955

Note: (1) *Conditional* (2) *Simulated Hp−filtered series.*

6.1.3 Bonds, tradable and Non-tradable equities

We now allow also for bonds trade along with equities trade (full financial integration). As before home equity bias in tradable would require $\bar{\alpha}_T > 1/4$ while home equity bias would require $\bar{\alpha}_T + \bar{\alpha}_{NT} > 1/2$.Here bond portfolios are expressed as a share of total wealth. [11] In Tables 6 and 7, we report the results for the cases in which we allow for international trade in real and nominal bonds, respectively.[12]

[11] To facilitate comparison with 2 bonds only case given in Table 2, where we express bond portfolio as a share of steady state consumption, i.e. $\alpha/\bar{C}$, we can multiply the bond shares given in Table 4 by steady state wealth-consumption ratio $\frac{\bar{W}}{\bar{C}} = \frac{\beta}{1-\beta}$.

[12] Having a nominal bond with a CPI targeting rule is equivalent to having a real bond (or CPI indexed bond) with any policy rule in terms of equilibrium portfolio and model solution. Thus to facilitate discussion, we treat bonds under these two different policy specifications as different assets: we will refer to the nominal bond under CPI targeting rule as the real bond.

Compared with the only equities case, the presence of real bonds doesn't add much: the equity position is basically the same as in the previous case and the bond position is negative as expected. As before we note the instability of the non-tradable equity position once we reduce the steady state capital share of tradable and non-tradable endowment. Also there are no substantial differences in terms of cross-correlation between consumption and real exchange rate.

With nominal bonds on the other hand there is no portfolio diversification: agents optimally hold home equities both in tradable and non-tradable endowment. As in the bonds only case, agents are long in domestic currency bonds and there is perfect international risk-sharing.

These results are consistent with recent theoretical framework that have addressed the home equity bias in various setting with market completeness or incompleteness like Coeurdacier, Kollmann and Martin (2007, 2008), Coeurdacier and Gourinchas (2008) and Collard et al. (2007).

Despite higher financial integration, home equity bias is still a robust feature of the model contradicting the empirical evidence that suggests a decline in home bias in the last decade.

Table 6: Real Bonds, Tradable and Non-Tradable Sector Equities with Endowment and Redistributive Shocks - Baseline model and sensitivity analysis

Real Bonds, T and NT Equities	α_T	α_T^*	α_{NT}	α_{NT}^*	α_B	α_B^*	$Corr(\hat{C}_t - \hat{C}_t^*, \hat{Q}_t)$ (1)	(2)
Cases with $\bar{k}_h = 1, \bar{k}_n = 1$	0.4423	0.0577	0.3877	0.1123	-0.0143	0.0143	0.9228	0.7844
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.4423	0.0577	0.3877	0.1123	-0.0143	0.0143	0.9228	0.7844
$\gamma = 0.5, \nu = 0.5, \kappa = 0.41$	0.4423	0.0577	0.3642	0.1358	-0.0137	0.0137	0.9347	0.8164
$\gamma = 0.5, \nu = 0.72, \kappa = 0.5$	0.4306	0.0694	0.3485	0.1515	-0.0014	0.0014	0.9128	0.7866
Cases with $\bar{k}_h = 0.4, \bar{k}_n = 0.2$								
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.4743	0.1923	-0.0409	0.3742	-0.0478	0.0478	0.9228	0.7844
$\gamma = 0.55, \nu = 0.72, \kappa = 0.41$	0.4668	0.2428	-0.3636	0.6539	-0.0041	0.0041	0.9171	0.7958

Note: (1) *Conditional* (2) *Simulated Hp−filtered series.*

Table 7: Real Bonds, Tradable and Non-Tradable Sector Equities with Endowment and Redistributive Shocks - Baseline model and sensitivity analysis

Nominal Bonds, T and NT Equities	α_T	α_T^*	α_{NT}	α_{NT}^*	α_B	α_B^*	$Corr(\hat{C}_t - \hat{C}_t^*, \hat{Q}_t)$ (1)	(2)
Cases with $\bar{k}_h = 1, \bar{k}_n = 1$	0.5000	0.0000	0.5000	0	0.0750	-0.0750	1	1
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.5000	0.0000	0.5000	0	0.0750	-0.0750	1	1
$\gamma = 0.5, \nu = 0.5, \kappa = 0.41$	0.5000	0.0000	0.4997	0.0003	0.0750	-0.0750	1	1
$\gamma = 0.5, \nu = 0.72, \kappa = 0.5$	0.5000	0	0.5000	0	0.0666	-0.0666	1	1
Cases with $\bar{k}_h = 0.4, \bar{k}_n = 0.2$								
$\gamma = 0.5, \nu = 0.5, \kappa = 0.5$	0.6666	0	0.3334	0	0.2500	-0.2500	1	1
$\gamma = 0.55, \nu = 0.72, \kappa = 0.41$	0.7097	0	0.2905	-0.0002	0.2385	-0.2385	1	1

Note: (1) *Conditional* (2) *Simulated Hp−filtered series.*

7 Conclusion

This paper shows that current portfolio models are unable to address the current trend in international financial markets. The main challenge is to replicate within the same model the observed pattern in the international portfolio allocation with the current (low) degree of international risk-sharing. A promising avenue in this respect might be to depart from standard preference assumption (see for example P. Benigno, 2008) or to consider alternative information structure (as in Tille and van Wincoop, 2008). In a current work in progress we are investigating the extent to which Epstein-Zin preferences could help in addressing the aforementioned regularities.

8 Appendix

Steady state

From the Euler equations (17), $\bar{m} = \bar{m}^* = \beta$ and $\overline{R}_1 = \overline{R}_2 = \overline{R}_3 = \overline{R}_4 = \overline{R}_5 = \bar{R}_6 = \beta^{-1}$.

From the equations that define the nominal returns of the assets (10) we obtain that $\frac{\bar{V}_h}{\bar{k}_h \bar{Y}_h} = \frac{\bar{V}_f}{\bar{k}_f \bar{Y}_f} = \frac{\bar{V}_n}{\bar{k}_n \bar{Y}_n} = \frac{\bar{V}_n^*}{\bar{k}_n^* \bar{Y}_n^*} = \frac{\beta}{1-\beta}$. The asset market equilibrium given in 16 imply the following in the steady-state:

$$\bar{\alpha}_1\bar{W} + \bar{\alpha}_1^*\bar{S}\bar{W}^* = \bar{P}_h\bar{V}_h$$

$$\bar{\alpha}_2\frac{\bar{W}}{\bar{S}} + \bar{\alpha}_2^*\bar{W}^* = \bar{P}_f^*\bar{V}_f^*$$

$$\bar{\alpha}_3\bar{W} + \bar{\alpha}_3^*\bar{S}\bar{W}^* = \bar{P}_n\bar{V}_n$$

$$\bar{\alpha}_4\frac{\bar{W}}{\bar{S}} + \bar{\alpha}_4^*\bar{W}^* = \bar{P}_n^*\bar{V}_n^*$$

$$\bar{\alpha}_5\frac{\bar{W}}{\bar{S}} + \bar{\alpha}_5^*\bar{W}^* = 0$$

$$\bar{\alpha}_6\bar{W} + \bar{\alpha}_6^*\bar{S}\bar{W}^* = 0$$

$$\bar{W} + \bar{S}\bar{W}^* = \bar{P}_h\bar{V}_h + \bar{S}\bar{P}_f^*\bar{V}_f^* + \bar{P}_n\bar{V}_n + \bar{S}\bar{P}_n\bar{V}_n^*$$

As the initial wealth distribution is not determined we choose steady state net wealth as in P. Benigno (2007) and assume $\bar{W} = \overline{SW}^*$. We normalize $\bar{k}_i$ and $\bar{Y}_i$ for $i = h, f^*, n, n^*$ such that $\bar{P}_h\bar{V}_h = \bar{S}\bar{P}_f\bar{V}_f = \bar{P}_n\bar{V}_n = \bar{S}\bar{P}_n^*\bar{V}_h^* = 0.5\bar{W}$. With this normalization, steady-state asset shares satisfy:

$$\bar{\alpha}_1 + \bar{\alpha}_1^* = 0.5$$

$$\bar{\alpha}_2 + \bar{\alpha}_2^* = 0.5$$

$$\bar{\alpha}_3 + \bar{\alpha}_3^* = 0.5$$

$$\bar{\alpha}_4 + \bar{\alpha}_4^* = 0.5$$

$$\bar{\alpha}_5 + \bar{\alpha}_5^* = 0$$

$$\bar{\alpha}_6 + \bar{\alpha}_6^* = 0$$

This normalization also implies that $\bar{\alpha}_i = 0.5\bar{x}_i$ for $i = 1, ..., 4$.So in our framework home bias is the fraction invested by home agents in a country equity (both tradables $\bar{\alpha}_1$ and nontradables, $\bar{\alpha}_3$) minus the share of country's equity in the world equity supply (in steady state that would be $\bar{W}/(\bar{W} + \overline{SW}^*) = 1/2$. So home bias would arise if $\bar{\alpha}_1 + \bar{\alpha}_3 > 1/2$.

If we want to measure equity bias in tradables only then the share of home country's traded sector equity in the world equity supply would be $\bar{P}_h\bar{V}_h/(\bar{P}_h\bar{V}_h + \bar{S}\bar{P}_f\bar{V}_f + \bar{P}_n\bar{V}_n + \bar{S}\bar{P}_n^*\bar{V}_h^*) = 1/4$ so that home bias in tradables would arise if $\bar{\alpha}_1 > 1/4$. If international asset trade in the non-tradable sector is not allowed then $\bar{\alpha}_1 > 1/2$.All relative prices are equal to 1 in the symmetric steady-state and $\bar{S} = 1$. Using the goods market equilibrium conditions given in (**??**), we can pin down steady-state consumption relative to tradable and nontradable sector output as follows:[13]

$$\frac{\bar{C}}{\bar{Y}_H} = \frac{\bar{C}^*}{\bar{Y}_H} = \frac{1}{\gamma_H\nu_H + \gamma_F\nu_F}$$
$$\frac{\bar{C}}{\bar{Y}_F} = \frac{\bar{C}^*}{\bar{Y}_F} = \frac{1}{\gamma_H(1-\nu_H) + \gamma_F(1-\nu_F)}$$
$$\frac{\bar{C}}{\bar{Y}_N} = \frac{\bar{C}^*}{\bar{Y}_N} = \frac{1}{1-\gamma_H}$$
$$\frac{\bar{C}}{\bar{Y}_N^*} = \frac{\bar{C}^*}{\bar{Y}_N^*} = \frac{1}{1-\gamma_F}$$

with

$$\bar{C}_H = \gamma_H\nu_H\bar{C}, \qquad \bar{C}_F = \gamma_H(1-\nu_H)\bar{C}^*$$
$$\bar{C}_H^* = \gamma_F\nu_F\bar{C}, \qquad \bar{C}_F^* = \gamma_F(1-\nu_F)\bar{C}^*$$

First Order Approximation to the Rest of the Model:

First Order Approximation to the Rest of the Model:

For any variable y ,except R_x, we define log-deviation from the steady-state, $\widehat{y}$ as $\widehat{y} = \log(\frac{y_t}{\bar{y}})$. The log-deviation of excess return $\widehat{R}_{xi,t}$ can be characterized as $\widehat{R}_{xi,t} = \frac{R_{xi,t} - \overline{R}}{\overline{R}}$.

Combining home and foreign Euler equations:

[13] Using the budget constraints, (14, 13) and normalizing resdistributive parameters so that $\bar{k}_h = \bar{k}_f$, $\bar{k}_n = \bar{k}_n^*$, we can show that steady-state consumption in home and foreign countries are equalized, i.e. $\bar{C} = \bar{C}^*$.

$$E_t\left[\widehat{m}_{t+1}-\widehat{m}^*_{t+1}+\widehat{S}_{t+1}-\widehat{S}_t\right]=0 \tag{38}$$

Home stochastic discount factor:

$$\widehat{m}_t=\widehat{P}_{t-1}-\widehat{P}_t+\rho\widehat{C}_{t-1}-\rho\widehat{C}_t \tag{39}$$

Foreign stochastic discount factor:

$$\widehat{m}^*_t=\widehat{P}^*_{t-1}-\widehat{P}^*_t+\rho\widehat{C}^*_{t-1}-\rho\widehat{C}^*_t \tag{40}$$

Home budget constraint:

$$\begin{aligned}\widehat{W}_t &= \frac{1}{\beta}\widehat{W}_{t-1}+\frac{1}{\beta}\widehat{R}_{6,t}+\widehat{R}_{x1,t}\widetilde{\alpha_1}+\widehat{R}_{x2,t}\widetilde{\alpha_2}+\widehat{R}_{x3,t}\widetilde{\alpha_3}+\widehat{R}_{x4,t}\widetilde{\alpha_4}+\widehat{R}_{x5,t}\widetilde{\alpha_5}\\ &+(1-\bar{k}_H)\frac{\bar{P}_H\bar{Y}_H}{\bar{W}}(\hat{P}_{H,t}+\hat{Y}_{H,t})+(1-\bar{k}_N)\frac{\bar{P}_N\bar{Y}_N}{\bar{W}}(\hat{P}_{N,t}+\hat{Y}_{N,t})-\bar{k}_H\frac{\bar{P}_H\bar{Y}_H}{\bar{W}}\hat{k}_{H,t}-\bar{k}_N\frac{\bar{P}_N\bar{Y}_N}{\bar{W}}\hat{k}_{N,t}\\ &-\frac{\overline{PC}}{\overline{W}}(\widehat{P}_t+\widehat{C}_t)\end{aligned} \tag{41}$$

where

$$\begin{aligned}\widetilde{\alpha_i} &= \frac{\overline{\alpha_i}}{\beta}\text{ for } i=1,...,6.\\ \frac{\bar{P}\bar{C}}{\bar{W}} &= \left(\frac{1-\beta}{\beta}\right)\frac{1}{1-(1-\bar{k}_H)(\gamma_H\nu_H+\gamma_F\nu_F)-(1-\bar{k}_N)(1-\gamma_H)}\\ \frac{\bar{P}_H\bar{Y}_H}{\bar{W}} &= \frac{\bar{P}_H\bar{Y}_H}{\bar{P}\bar{C}}\frac{\bar{P}\bar{C}}{\bar{W}}=\left(\frac{1-\beta}{\beta}\right)\frac{\gamma_H\nu_H+\gamma_F\upsilon_F}{1-(1-\bar{k}_H)(\gamma_H\nu_H+\gamma_F\nu_F)-(1-\bar{k}_N)(1-\gamma_H)}\\ \frac{\bar{P}_N\bar{Y}_N}{\bar{W}} &= \frac{\bar{P}_N\bar{Y}_N}{\bar{P}\bar{C}}\frac{\bar{P}\bar{C}}{\bar{W}}=\left(\frac{1-\beta}{\beta}\right)\frac{1-\gamma_H}{1-(1-\bar{k}_H)(\gamma_H\nu_H+\gamma_F\nu_F)-(1-\bar{k}_N)(1-\gamma_H)}\end{aligned}$$

From Euler equations we can show that

$$\hat{R}_{6,t} = -E_{t-1}\hat{m}_t$$

Home Non-Traded Goods Market Clearing:

$$\widehat{Y}_{N,t} = \frac{\bar{C}}{\bar{Y}_N}(1-\gamma_H)\hat{C}_t - \frac{\bar{C}}{\bar{Y}_N}\kappa(1-\gamma_H)(\widehat{P}_{N,t} - \widehat{P}_t) \tag{42}$$

Foreign Non-Traded Goods Market Clearing:

$$\widehat{Y}^*_{N,t} = \frac{\bar{C}^*}{\bar{Y}^*_N}(1-\gamma_F)\hat{C}^*_t - \frac{\bar{C}^*}{\bar{Y}^*_N}\kappa(1-\gamma_F)(\widehat{P}^*_{N,t} - \widehat{P}^*_t) \tag{43}$$

Home Traded Goods Market Clearing:

$$\begin{aligned} \widehat{Y}_{H,t} &= \frac{\bar{C}_H}{\bar{Y}_H}\hat{C}_t + \frac{\bar{C}^*_H}{\bar{Y}_H}\hat{C}^*_t + \kappa\frac{\bar{C}^*_H}{\bar{Y}_H}\hat{Q}_t \\ &\quad - \left(\theta(1-\nu_H) - (\kappa-\theta)(\nu_H - \upsilon_F)\frac{\bar{C}^*_H}{\bar{Y}_H}\right)\left(\hat{P}_{H,t} - \hat{P}_{F,t}\right) \\ &\quad -\kappa(1-\gamma_H)\left(\hat{P}_{T,t} - \hat{P}_{N,t}\right) \end{aligned} \tag{44}$$

Foreign Traded Good Market Clearing

$$\begin{aligned} \widehat{Y}^*_{F,t} &= \frac{\bar{C}_F}{\bar{Y}_F}\hat{C}_t + \frac{\bar{C}^*_F}{\bar{Y}_F}\hat{C}^*_t + \kappa\frac{\bar{C}^*_F}{\bar{Y}_F}\hat{Q}_t \\ &\quad - \left(\theta\nu_H - (\kappa-\theta)(\nu_F - \upsilon_H)\frac{\bar{C}^*_F}{\bar{Y}_F}\right)\left(\hat{P}_{F,t} - \hat{P}_{H,t}\right) \\ &\quad -\kappa(1-\gamma_H)\left(\hat{P}_{T,t} - \hat{P}_{N,t}\right) \end{aligned} \tag{45}$$

where $\hat{Q}_t$ is the real exchange rate defined as:

$$\hat{Q}_t = \hat{P}_t^* + \hat{S}_t - \hat{P}_t. \tag{46}$$

It is possible to decompose the real exchange rate as follows:

$$\hat{Q}_t = (v - v^*)\hat{T}_t + (1 - \gamma^*)(\hat{P}_{N,t}^* - \hat{P}_{T,t}^*) - (1 - \gamma)(\hat{P}_{N,t} - \hat{P}_{T,t})$$

Home CPI:

$$\hat{P}_t = \gamma_H \hat{P}_{T,t} + (1 - \gamma_H)\hat{P}_{N,t} \tag{47}$$

Home Traded Price Index:

$$\hat{P}_{T,t} = \nu_H \hat{P}_{H,t} + (1 - \nu_H)\hat{P}_{F,t} \tag{48}$$

Foreign CPI:

$$\hat{P}_t^* = \gamma_F \hat{P}_{T,t}^* + (1 - \gamma_F)\hat{P}_{N,t}^* \tag{49}$$

Foreign Traded Price Index:

$$\hat{P}_{T,t}^* = \nu_F \hat{P}_{H,t}^* + (1 - \nu_F)\hat{P}_{F,t}^* \tag{50}$$

Law of one price holds for imports:

$$\hat{P}_{F,t} = \hat{P}_{F,t}^* + \hat{S}_t \tag{51}$$

$$\hat{P}_{H,t}^* = \hat{P}_{H,t} - \hat{S}_t \tag{52}$$

Equity Prices:

$$\hat{V}_{H,t} - \beta E_t \hat{V}_{H,t+1} = \rho \hat{C}_t - (\hat{P}_{H,t} - \hat{P}_t) - \rho E_t \hat{C}_{t+1} + E_t(\hat{P}_{H,t+1} - \hat{P}_{t+1}) + (1-\beta) E_t(\hat{k}_{H,t+1} + \hat{Y}_{H,t+1}) \tag{53}$$

$$\hat{V}^*_{F,t} - \beta E_t \hat{V}^*_{F,t+1} = \rho \hat{C}_t - (\hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_t) - \rho E_t \hat{C}_{t+1} + E_t(\hat{P}^*_{F,t+1} + \hat{S}_{t+1} - \hat{P}_{t+1}) + (1-\beta) E_t(\hat{k}_{F,t+1} + \hat{Y}_{F,t+1}) \tag{54}$$

$$\hat{V}_{N,t} - \beta E_t \hat{V}_{N,t+1} = \rho \hat{C}_t - (\hat{P}_{N,t} - \hat{P}_t) - \rho E_t \hat{C}_{t+1} + E_t(\hat{P}_{N,t+1} - \hat{P}_{t+1}) + (1-\beta) E_t(\hat{k}_{N,t+1} + \hat{Y}_{N,t+1}) \tag{55}$$

$$\hat{V}^*_{N,t} - \beta E_t \hat{V}^*_{N,t+1} = \rho \hat{C}_t - (\hat{P}^*_{N,t} + \hat{S}_t - \hat{P}_t) - \rho E_t \hat{C}_{t+1} + E_t(\hat{P}^*_{N,t+1} + \hat{S}_{t+1} - \hat{P}_{t+1}) + (1-\beta) E_t(\hat{k}^*_{N,t+1} + \hat{Y}^*_{N,t+1}) \tag{56}$$

Realized Excess Returns:

$$\hat{R}_{x1,t} = \hat{R}_{1,t} - \hat{R}_{6,t} = \beta(\hat{V}_{H,t} - E_{t-1}\hat{V}_{H,t}) + (1-\beta)(\hat{k}_{H,t} + \hat{Y}_{H,t} - E_{t-1}\hat{k}_{H,t} - E_{t-1}\hat{Y}_{H,t}) + \hat{P}_{H,t} - E_{t-1}\hat{P}_{H,t} \tag{57}$$

$$\hat{R}_{x2,t} = \hat{R}_{2,t} - \hat{R}_{6,t} = \beta(\hat{V}^*_{F,t} - E_{t-1}\hat{V}^*_{F,t}) + (1-\beta)(\hat{k}_{F,t} + \hat{Y}^*_{F,t} - E_{t-1}\hat{k}_{F,t} - E_{t-1}\hat{Y}^*_{F,t}) + \hat{P}^*_{F,t} - E_{t-1}\hat{P}^*_{F,t} + \hat{S}_t - E_{t-1}\hat{S}_t \tag{58}$$

$$\hat{R}_{x3,t} = \hat{R}_{3,t} - \hat{R}_{6,t} = \beta(\hat{V}_{N,t} - E_{t-1}\hat{V}_{N,t}) + (1-\beta)(\hat{k}_{N,t} + \hat{Y}_{N,t} - E_{t-1}\hat{k}_{N,t} - E_{t-1}\hat{Y}_{N,t}) + \hat{P}_{N,t} - E_{t-1}\hat{P}_{N,t} \tag{59}$$

$$\hat{R}_{x4,t} = \hat{R}_{4,t} - \hat{R}_{6,t} = \beta(\hat{V}^*_{N,t} - E_{t-1}\hat{V}^*_{N,t}) + (1-\beta)(\hat{k}^*_{N,t} + \hat{Y}^*_{N,t} - E_{t-1}\hat{k}^*_{N,t} - E_{t-1}\hat{Y}^*_{N,t}) + \hat{P}^*_{N,t} - E_{t-1}\hat{P}^*_{N,t} + \hat{S}_t - E_{t-1}\hat{S}_t \tag{60}$$

$$\hat{R}_{x5,t} = \hat{R}_{5,t} - \hat{R}_{6,t} = \hat{S}_t - E_{t-1}\hat{S}_t \tag{61}$$

The log-linearized model characterized by 23 equations from (38) to (61) involves the following sequence of 25 variables:

$\{\hat{m}, \hat{m}^*, \hat{C}, \hat{C}^*, \hat{P}, \hat{P}^*, \hat{P}_H, \hat{P}_F, \hat{P}_H^*, \hat{P}_F^*, \hat{P}_T, \hat{P}_T^*, \hat{P}_N, \hat{P}_N^*, \hat{S}_t, \hat{Q}_t,$

$\hat{R}_{x1}, \hat{R}_{x2}, \hat{R}_{x3}, \hat{R}_{x4}, \hat{R}_{x5,t}, \hat{V}_H, \hat{V}_F^*, \hat{V}_N, \hat{V}_N^*, \hat{W}\}$

We close the model by either a domestic tradable price targeting rule, i.e. $\hat{P}_{F,t}^* = 0$ and $\hat{P}_{H,t} = 0$, or by a CPI targeting rule, i.e. $\hat{P}_t^* = 0$ and $\hat{P}_t = 0$

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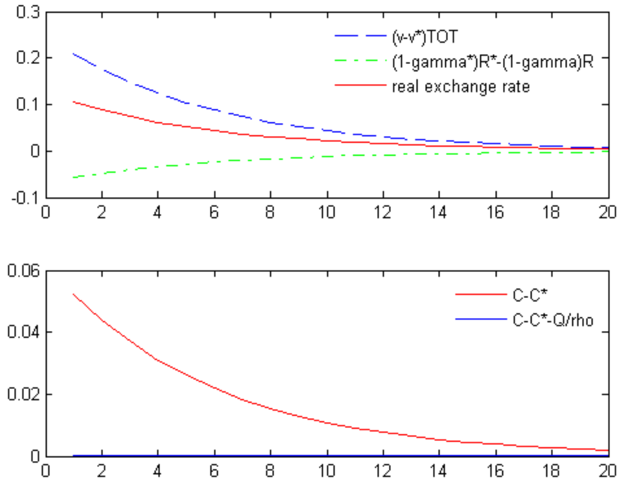

Figure 1: Impulse response to domestic tradable endowment shock for the case with 2 nominal bonds & tradable PPI targeting under benchmark calibration

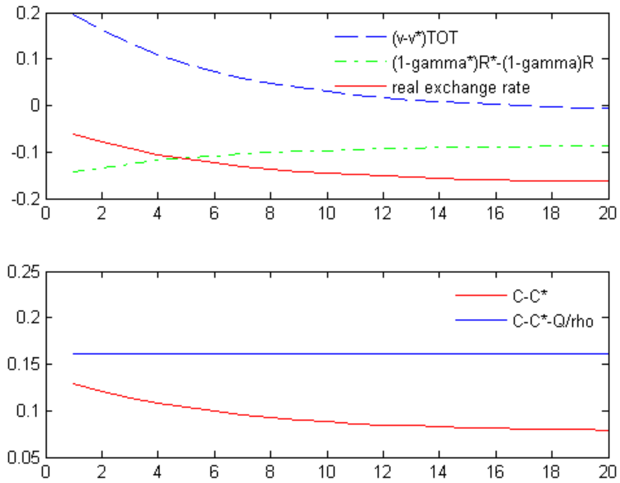

Figure 2: Impulse response to domestic tradable endowment shock for the case with 2 nominal bonds & CPI targeting under benchmark calibration

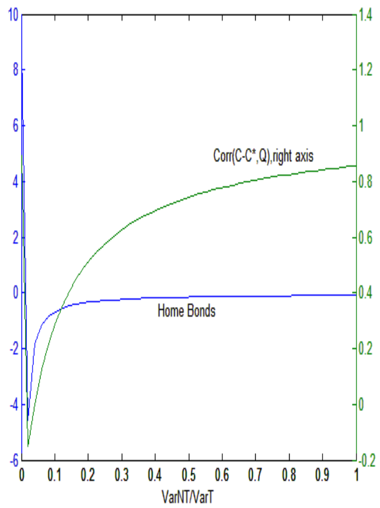

Figure 3: Sensitivity Analysis. Home bond position and cross-correlation between the real exchange rate and relative consumption for the case of 2 nominal bonds with CPI targeting for different values of relative variance of non-tradable to tradable endowment shocks

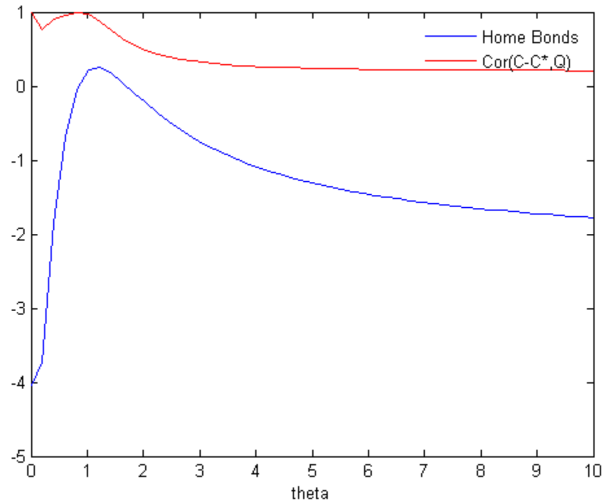

Figure 4: Sensitivity Analysis. Home bond position and cross-correlation between the real exchange rate and relative consumption for the case of 2 nominal bonds with CPI targeting for different values of θ.

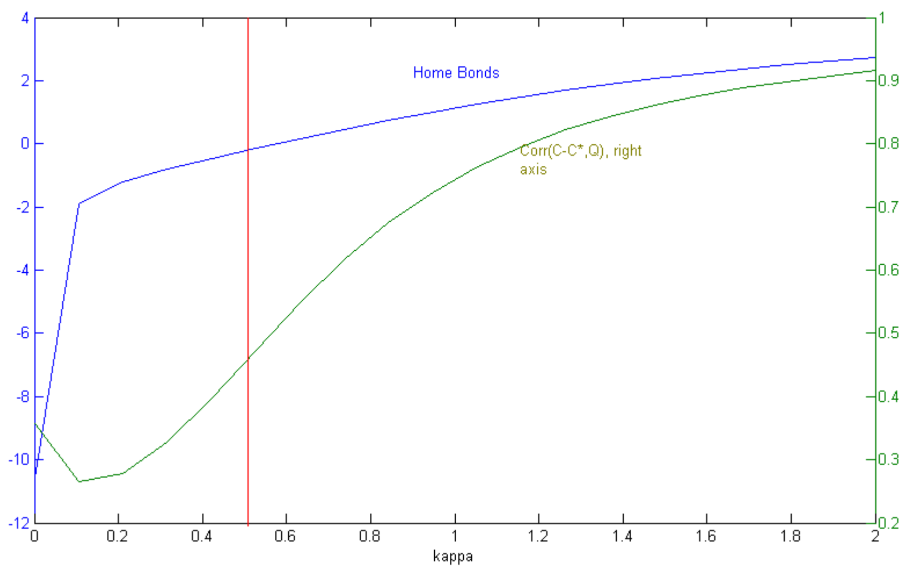

Figure 5: Sensitivity Analysis. Home bond position and cross-correlation between the real exchange rate and relative consumption for the case of 2 nominal bonds with CPI targeting for different values of κ.

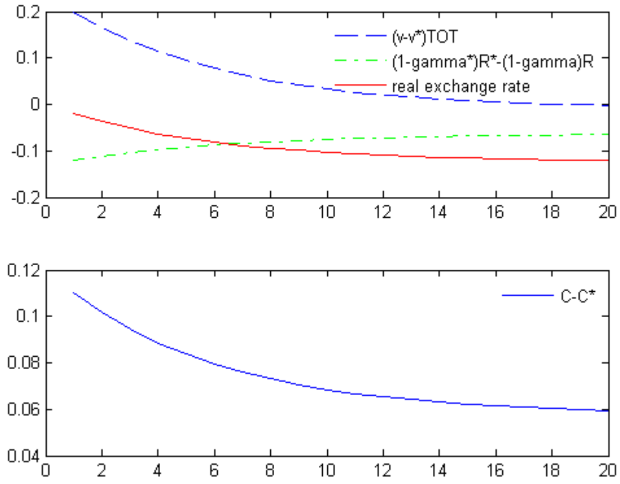

Figure 6: Impulse response to domestic tradable endowment shock for the case with 2 tradable sector equities under benchmark calibration

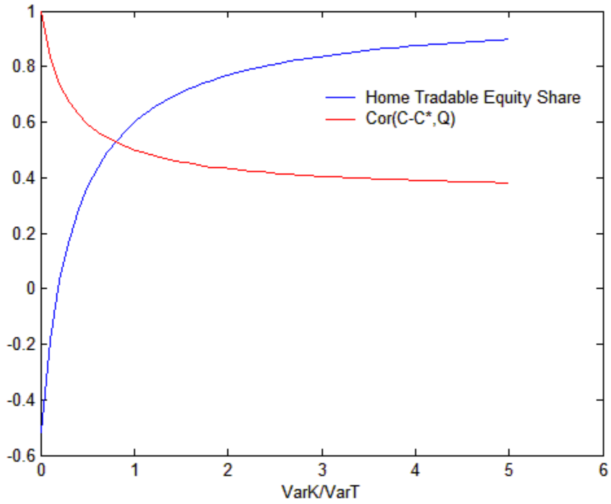

Figure 7: Sensitivity Analysis. Home equity share in home portfolio and cross-correlation between the real exchange rate and relative consumption for the case of 2 tradable sector equities for different values of relative variance of redistributive shocks to tradable endowment shocks.

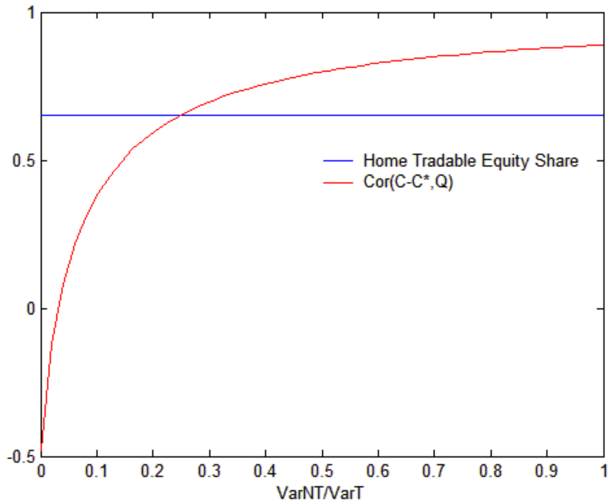

Figure 8: Sensitivity Analysis. Home equity share in home portfolio and cross-correlation between the real exchange rate and relative consumption for the case of 2 tradable sector equities for different values of relative variance of non-tradable to tradable endowment shocks

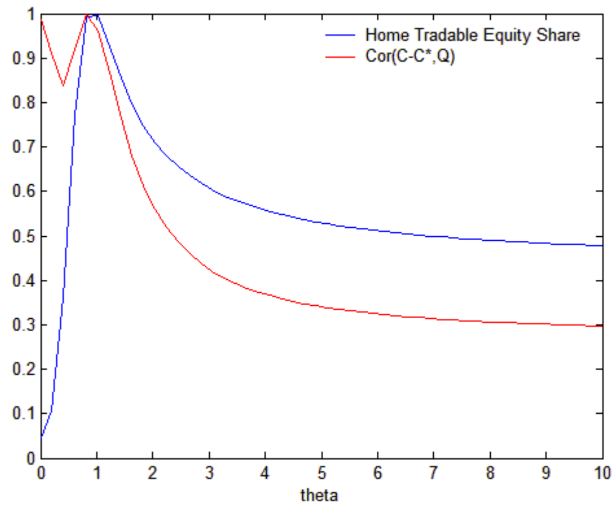

Figure 9: Sensitivity Analysis. Home equity share in home portfolio and cross-correlation between the real exchange rate and relative consumption for the case of 2 tradable sector equities for different values of θ.