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# The Dynamic Macroeconomic Effects of Tax Policy in an Overlapping Generations Model

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## Abstract

The paper studies the dynamic allocation effects of tax policy in the context of an overlapping generations model of the Blanchard-Yaari type. The model is extended to allow for endogenous labor supply and three tax instruments: a capital income tax, labor income tax, and consumption tax. Analytical expressions and simple diagrams are used to discuss the impact, transition, and long-run effects of tax policy changes. It is shown that a part of the long-run incidence of capital and consumption taxes falls on capital when households' horizons are finite, whereas labor would fully bear the burden of these taxes in an infinite horizon model.

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**SUMMARY**

The paper employs a Blanchard-Yaari type of overlapping generations model—extended for endogenous labor supply—to study the dynamic macroeconomic effects of various tax instruments, viz., capital income taxes, labor taxes, and consumption taxes. The impact and long-run effects of tax policy changes are analyzed in a closed-economy setting. In addition, a simple graphical framework is developed that facilitates the analysis of the transition path toward the steady state.

The paper shows that overlapping generations do not affect the qualitative macroeconomic effects of capital and labor taxes found in the literature. An unanticipated and permanent increase in capital and labor taxes depresses long-run savings and consumption. In the short run, capital taxes boost consumption whereas labor taxes unambiguously depress consumption. However, the model illustrates that consumption taxes could have qualitatively different effects in a world of overlapping generations. Consumption taxes increase the steady-state capital stock if intertemporal labor supply is inelastic, whereas in a setting of finite horizons the capital stock would fall irrespective of the size of the labor supply effect.

The paper also shows that the long-run incidence of capital and consumption taxes partially falls on capital, whereas in a small open economy or with infinitely lived households capital can escape the entire long-run burden of taxation. Capital bears a larger burden of taxation in the short run than in the long run.

## I. INTRODUCTION

Taxing a particular activity can distort behavior of economic agents. This has macroeconomic implications, both on impact and in the long run, through its effect on savings and investment decisions. Moreover, tax policy not only affects the *intra*generational allocation of resources but also the distribution of resources between generations, the so-called *inter*generational allocation. The latter will be the focus of the present analysis. The paper studies the dynamic macroeconomic effects of tax policy in a world of overlapping generations. To this end a Blanchard (1985)-Yaari (1965) type of model for a closed economy is extended to incorporate endogenous labor supply and various tax instruments, viz. a capital income tax, labor income tax, and consumption tax. In particular, we analyze the dynamic effects of permanent and unanticipated increases in these taxes.

Overlapping generations models as developed by Blanchard and Yaari assume a simple demographic structure in which different generations coexist at any moment in time. Agents face an exogenous probability of death and are unconnected to previous generations due to the absence of a bequest motive. This multiperiod framework is very suitable to study intergenerational tax policy issues because it can fully characterize the transition path toward a new long-run equilibrium. The approach here derives analytical expressions for the short-run, transition, and long-run effects of small changes in tax policy. Knowledge of the entire adjustment path is of relevance to policy analysis because the short-run effects of a tax change may differ qualitatively from the long-run effects. The literature on the intergenerational effects of fiscal policy so far has primarily employed a Diamond (1965) type of overlapping generations model in which generations live for two periods (see Auerbach and Kotlikoff (1987, 1995), Laitner (1984), Keuschnigg (1994), and Ihori (1996)).[1] Recently, Bovenberg (1993, 1994), Nielsen and Sørensen (1991), and Bovenberg and Heijdra (1998) adopted the Blanchard specification of household behavior to explore analytically various fiscal policy issues. None of these studies, however, conducts a comparative analysis of tax instruments.

A second objective of the paper is to study the dynamic incidence of the three types of tax instruments. In a dynamic context, the burden of taxes is not only the reduction of current income but also the induced intertemporal general equilibrium effects on wages and rental rates. Little work has been done in this area until recently. Bovenberg (1993, 1994) and Nielsen and Sørensen (1991) have studied the incidence of capital income taxes in a Blanchard model for a small open economy with fixed labor supply. They show that in the long run capital can fully escape the burden of capital taxes because the after-tax rate of interest is fixed

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[1]In the Diamond (1965) model people work in the first period and retire in the second period, which implies that a period is roughly 30–40 years. In this framework, individuals from different generations have different marginal propensities to consume out of wealth because young agents save for retirement. In the Blanchard (1985) model, however, all generations have the same propensity to consume out of wealth. In both frameworks generations are different in the amount of wealth they have accumulated.

by world capital markets. Bovenberg and Heijdra (1998), however, employ a closed-economy Blanchard model—featuring an endogenous rate of interest—with inelastic labor supply, to argue that part of the long-run burden of capital taxes may fall on labor. Our model also assumes a closed economy, but features, in contrast to Bovenberg and Heijdra (1998), elastic labor supply.[2] This permits an assessment of the effects of changes in after-tax wages and household wealth on intertemporal labor supply, including its feedback effects on the equilibrium outcome. Moreover, analytical results beyond these in the literature on the dynamic incidence of labor and consumption taxes are reported. It is demonstrated that, for both capital and consumption taxes, labor can shift a part of the long-run incidence of taxation toward capital.

The dynamic macroeconomic model with finitely lived households in this paper has a general equilibrium structure. It features two factor markets—for labor and capital—and one market for final goods. Behavior at the micro level is grounded in intertemporal optimization of firms and households endowed with perfect foresight. The present analysis integrates public finance issues in a dynamic framework which facilitates the analysis of the key ways in which taxes impact on macroeconomic variables such as saving, consumption, and investment. The approach taken to provide an intuitive explanation of the comparative statics is similar to that set out by Jones (1965) who reduced a static two goods-two factors analysis to a simple supply and demand framework. Judd (1985, 1987) also deals with the dynamic effects of tax changes but his analysis is based on the assumption of a representative agent. Accordingly, there is no feedback effect from changes in the intergenerational distribution of income to aggregate savings. Furthermore, his analysis does not yield simple formula and pays less attention to interpreting the adjustment dynamics than the present analysis.

Another objective of the paper is to develop a simple diagrammatic apparatus to analyze the comparative static and transitional effects of tax policy changes. The graphical study of the general equilibrium effects of taxes on labor and capital markets can illustrate the changes that impact on the supply and demand side of the goods market. In addition, phase diagrams demonstrate the dynamic adjustment paths of consumption and the capital stock. The graphical framework is versatile since it can encompass various values of the intertemporal elasticity of labor supply. Accordingly, both the case of elastic and inelastic labor supply is addressed. In addition, the multi-period feature of the model makes it potentially suitable for the study of the effects of phased-in tax changes as well as anticipation effects.[3]

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[2]Heijdra (1994) also extends the Blanchard model to allow for variable labor supply, but assumes imperfect competition on the goods market and studies the output effects of a rise in public expenditure.

[3]See Judd (1985, 1987) for this type of analysis in a dynamic model featuring infinitely lived individuals and Auerbach and Kotlikoff (1987) for a 55-period numerical analysis in a Diamond (1965) type of overlapping generations framework. Anticipation effects and

(continued...)

The paper shows that the size of the labor supply effect crucially matters for the qualitative effects of consumption taxes on savings. Indeed, in the overlapping generations framework, the equivalence between a (permanent and unanticipated) increase in the taxes on labor income and consumption—which holds in models with infinitely lived households (see Atkinson and Stiglitz (1980))—breaks down. Both capital and labor taxes reduce savings over time and thus depress the long-run capital stock. Consumption taxes, however, may boost savings in the long run if the generational turnover effect is large enough to dominate the negative labor supply effect of consumption taxes. The generational turnover effect promotes savings because old agents pass away and are replaced by new agents who are born without any financial assets. Old generations are hit harder by the consumption tax since they predominantly consume out of the return on financial capital. Young generations, however, consume and save out of human capital which is increased by a rise in after-tax wages (assuming a given supply of labor). With exogenous labor supply, consumption taxes unambiguously increase savings, but for moderate and high values of the intertemporal labor supply elasticity the savings effect is negative.[4]

The rest of the paper is structured as follows. Section II sets out the Blanchard-Yaari overlapping generations model extended for endogenous labor supply and various tax instruments. The model consists of three sectors: a household sector which comprises a large number of cohorts which differ with respect to the level of their wealth, a perfectly competitive production sector, and a government sector. In Section III the steady state of the model is characterized and the dynamic behavior around the steady state is studied. Section IV discusses the dynamic allocation effects of a permanent unanticipated increase in capital income, labor income, and consumption taxes. This section will also address the issue of the dynamic incidence of taxes. Section V summarizes the main results of the paper.

## II. THE MODEL OF OVERLAPPING GENERATIONS

### A. Households

The household section is modeled by an overlapping generations framework as developed by Yaari (1965) and Blanchard (1985). At each instant of time a large cohort of agents is born

(...continued)
temporary policy changes are not addressed in the current paper but will be left for further extensions.

[4]Auerbach and Kotlikoff (1987, p. 68) also show, using their dynamic computable general equilibrium model for the U.S. economy, that a rise in consumption taxes increases savings if the intertemporal labor supply response is small. However, they do not provide a further analysis of this result.

which face a constant probability of death, $\beta \geq 0$, per unit of time.[5] The size of the total population at any moment in time is constant and normalized to unity. New generations are not connected to old generations since households do not have a bequest motive. Hence, agents are born without financial assets. In the absence of bequests, households contract actuarially fair (reverse) life insurance with annuities companies. Consumers receive a rate of return on their financial wealth (i.e., $\beta$) at each instant of time during their lives, but when they die their entire stock of assets goes to the insurance company.

A representative agent of the generation born at time $v \leq t$ derives utility at time $\tau$ from private consumption, $C(v,\tau)$, and leisure, $1\text{-}L(v,\tau)$:[6]

$$U(v,t) \equiv \int_t^{\infty} \left[ \epsilon_C \log C(v,\tau) + (1-\epsilon_C) \log[1-L(v,\tau)] \right] \exp[(\alpha+\beta)(t-\tau)] d\tau, \tag{2.1}$$

where $\alpha$ and $\epsilon_C$ denote, respectively, the constant pure rate of time preference ($\alpha \geq 0$) and the consumption share in utility. The logarithmic specification of utility imposes unit *inter*temporal and *intra*temporal elasticities of substitution. The agent's dynamic budget identity in real terms is given by:

$$\dot{A}(v,t) = [r(t)+\beta] A(v,t) + W(t)[1-t_L(t)] + T(t) - X(v,t), \tag{2.2}$$

where $A(v,t)$ is financial wealth, $r(t)$ is the real rate of interest, $W(t)$ is the age-independent real wage rate, $t_L(t)$ is the tax rate on labor income, and $T(t)$ are net lump-sum transfers which are uniform across all generations. The agent receives $r(t)A(v,t)$ as interest payments on its financial wealth at time t and $\beta A(v,t)$ is the premium received from the insurance company. A dot above a variable denotes a time derivative, e.g., $\dot{A}(v,t) \equiv dA(v,t)/dt$. *Full* consumption, $X(v,t)$, is defined as the sum of the value of goods consumption and the opportunity cost of leisure consumption:

$$X(v,t) \equiv [1+t_C(t)] C(v,t) + [1-t_L(t)] W(t)[1-L(v,t)], \tag{2.3}$$

where $t_C(t)$ is a proportional tax rate on private consumption. A No-Ponzi-Game (NPG) solvency condition needs to be imposed to prevent agents from accumulating debt indefinitely:

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[5]The property of a constant probability of death implies that a young person has the same expected lifetime (given by $1/\beta$) as a very old person. Accordingly, expected lifetimes increase for smaller values of $\beta$.

[6]Leisure is defined as the household's time endowment (which is normalized to unity) less labor supply, $L(v,t)$.

$$\lim_{\tau \to \infty} A(v,\tau) \exp\left[ -\int_t^\tau [r(\mu) + \beta] \mathrm{d}\mu \right] = 0, \tag{2.4}$$

where the effective rate of interest (i.e., the sum of the real rate of interest and the probability of death) is used as the discount factor.

The representative consumer of vintage $v$ is endowed with perfect foresight and maximizes life-time utility (2.1) subject to the household's dynamic budget restriction (2.2) and the NPG-condition (2.4). The optimization problem can be solved in two stages which yield the entire time path of consumption and labor supply. In the first stage the consumer chooses the optimal time profile of full consumption. The first-order conditions yield the full consumption Euler equation:

$$\dot{X}(v,\tau) = \left[ r(\tau) - \alpha \right] X(v,\tau). \tag{2.5}$$

By integrating (2.5) forward and substituting the resulting expression in the household's budget constraint (which is (2.2) integrated with the NPG condition (2.4) imposed) one derives full consumption for period $t$ as a linear function of total household wealth:

$$X(v,t) = (\alpha + \beta)\left[ A(v,t) + H(t) \right], \tag{2.6}$$

where $H(t)$ is expected lifetime human wealth:

$$H(t) \equiv \int_t^\infty \left[ W(\tau)\left[ 1 - t_L(\tau) \right] + T(\tau) \right] \exp\left[ -\int_t^\tau \left[ r(\mu) + \beta \right] \mathrm{d}\mu \right] \mathrm{d}\tau. \tag{2.7}$$

Intuitively, $H(t)$ represents the after-tax market value of the household's time endowment, i.e., the present discounted value of maximum future after-tax wage income. The constant marginal propensity to consume out of total wealth, $\alpha + \beta$, follows from assuming an intertemporal substitution elasticity of unity. Under this assumption the substitution and income effect of a change in the real rate of interest exactly cancel.

In the second stage the representative consumer of cohort $v$ decides about the consumption-leisure choice in each time period. This yields simple expressions relating individual consumption and leisure demand to full consumption:

$$C(v,t) = \frac{\epsilon_C X(v,t)}{1 + t_C(t)}, \qquad 1 - L(v,t) = \frac{(1 - \epsilon_C) X(v,t)}{W(t)\left[ 1 - t_L(t) \right]}. \tag{2.8}$$

Due to the constant probability of death, the size of a cohort declines deterministically through time (i.e., in an exponential fashion), although individuals are uncertain about their time of death. Following Blanchard (1985) the household sector can be aggregated by integrating over all currently alive generations. For example, aggregate private consumption, is defined as:

$$C(t) \equiv \beta \int_{-\infty}^{t} C(v,t)\, e^{\beta(v-t)} \mathrm{d}v, \tag{2.9}$$

where $\beta e^{\beta(v-t)}$ is the size of a cohort of vintage $v$ as of time $t$. Similarly, we can obtain the aggregate values for $L(t)$, $A(t)$, $T(t)$, $H(t)$, and $X(t)$. This yields the following equations describing the behavior of the aggregate household sector:

$$X(t) = (\alpha + \beta)\left[A(t) + H(t)\right], \tag{2.10}$$

$$\dot{A}(t) = r(t)A(t) + W(t)\left[1 - t_L(t)\right] + T(t) - X(t), \tag{2.11}$$

$$\dot{H}(t) = \left[r(t) + \beta\right]H(t) - W(t)\left[1 - t_L(t)\right] - T(t). \tag{2.12}$$

Aggregate financial wealth accrues at the rate $r(t)$ while individual financial wealth accumulates at a faster rate $r(t)+\beta$. Since $\beta A(t)$ is a transfer from those who die to those who remain alive it therefore does not enter the economy-wide budget constraint. By differentiating household consumption (2.8) with respect to time, using (2.5), the Euler-equation for the growth in individual consumption is obtained:

$$\frac{\dot{C}(v,t)}{C(v,t)} = r(t) - \alpha - \frac{\dot{t}_C(t)}{1 + t_C(t)}. \tag{2.13}$$

Equation (2.13) is the familiar Keynes-Ramsey rule which says that the rate of change of individual consumption is positively related to the wedge between the real rate of interest and the pure rate of time preference. An immediate rise in the consumption tax rate (i.e., $\dot{t}_C(t)=0$) does not affect the growth rate of private consumption, while a gradually introduced increase in the consumption tax rate (i.e., $\dot{t}_C(t)>0$) works like a capital income tax and flattens the tilt of the consumption profile (see Auerbach and Kotlikoff, 1987). Aggregating (2.13) yields the Modified Keynes-Ramsey (MKR) rule which describes aggregate consumption growth modified for the existence of overlapping generations:

$$\frac{\dot{C}(t)}{C(t)} = \left[r(t) - \alpha - \frac{\dot{t}_C(t)}{1+t_C(t)}\right] - \epsilon_C\beta(\alpha+\beta)\left[\frac{A(t)}{[1+t_C(t)]C(t)}\right]$$
$$= \frac{\dot{C}(v,t)}{C(v,t)} - \beta\left(\frac{C(t)-C(t,t)}{C(t)}\right). \tag{2.14}$$

Aggregate consumption growth differs from consumption growth for individual cohorts because of the turnover of generations. At each instant a cross-section of the existing population dies and is replaced by a new generation. Since average consumption exceeds consumption by newly born agents, the generational turnover effect drags down aggregate consumption growth. See also Section III for a further exposition.

### B. Firms

The production sector is characterized by a large number of firms that produce an identical good under perfect competition. Net output,[7] $Y(t)$, is produced according to a Cobb-Douglas technology with labor, $L(t)$, and physical capital, $K(t)$, as homogeneous factor inputs which are rented from households:

$$Y(t) = \gamma_0 L(t)^{\epsilon_L} K(t)^{1-\epsilon_L}, \qquad 0 < \epsilon_L < 1,\ \gamma_0 > 0, \tag{2.15}$$

where $\epsilon_L$ denotes the share of before-tax wage income in net output. The representative firm maximizes its net present (or equity) value:

$$V(t) = \int_t^\infty \left[[1-t_K(\tau)][Y(\tau) - W(\tau)L(\tau)] + \delta K(\tau) - I(\tau)\right]\exp\left[-\int_t^\tau r(\mu)\,d\mu\right]d\tau, \tag{2.16}$$

with respect to labor and capital subject to the production function (2.15) and the capital accumulation constraint, $\dot{K}(t) = I(t) - \delta K(t)$, where $I(t)$ denotes gross investment, $\delta$ is the constant rate of depreciation of capital, and $t_K(t)$ represents a tax on capital income. There are no adjustment costs associated with investment. The first-order conditions imply that the marginal productivity of labor and capital equal the producer costs of these factors:

$$\epsilon_L\left(\frac{Y(t)}{L(t)}\right) = W(t), \tag{2.17a}$$

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[7]This means that production is measured after allowing for depreciation of the capital stock.

$$(1-\epsilon_L)\left(\frac{Y(t)}{K(t)}\right) = \frac{r(t)}{1-t_K(t)}. \qquad (2.17b)$$

Combining the expressions in equations (2.17a-b) yields the wage-rental cost ratio as a function of the capital-labor ratio. By substituting (2.17a-b) in (2.16) and using the property of linear homogeneity of the production function, we can derive that the market value of the firm is equal to the replacement value of its capital stock, i.e., $V(t)=K(t)$.

### C. The Government and Market Equilibrium

Government expenditure consists of lump-sum transfers to households. The government finances its spending by employing one (or a mix) of the following tax instruments: (i) a capital income tax, (ii) a labor income tax, or (iii) a tax on consumption. The model abstracts from debt policy implying that the government needs to balance its budget in each time period. The dynamic government budget identity is given by:

$$T(t) = t_K(t)\left[Y(t) - W(t)L(t)\right] + t_L(t)\,W(t)L(t) + t_C(t)C(t). \qquad (2.18)$$

In a dynamic macroeconomic model with perfect foresight, behavior of households, firms, and the government today depends on future economic conditions. Equilibrium therefore requires that agents' behavior be consistent not only with current 'prices' (i.e., wages, interest rates, and tax rates), but also with the entire path of future prices. Due to the time-separable nature of the utility function (see equation (2.1)), past household decisions only affect current behavior through the household's accumulated stock of capital.

At each instant of time, factor and goods markets clear instantaneously. In this closed economy households can only accumulate domestic assets. As a result, financial market equilibrium requires that $A(t)=K(t)$. Wage flexibility ensures that the supply of labor by households matches labor demand by firms. Goods market equilibrium is obtained when the supply of goods equals aggregate demand, which consists of private consumption plus net investment: $Y(t)=C(t)+\dot{K}(t)$.

## III. MODEL PROPERTIES

This section investigates the stability of the model and sets out the graphical apparatus that is employed to provide an intuitive account of the general equilibrium effects.

## A. Stability

Local stability of the model can be studied by linearizing the nonlinear system of equations around an initial steady state (see Judd (1982)). The main expressions are given in Table 1.

The dynamic part of the model consists of the aggregate capital stock (which is a predetermined variable) and aggregate consumption (which is a forward-looking or jump variable). By using labor demand (T.4), labor supply (T.6), and the aggregate production function (T.7), a useful 'quasi-reduced form' expression for aggregate output is obtained:

$$\tilde{Y}(t) = \phi(1-\epsilon_L)\tilde{K}(t) - (\phi-1)\left[\tilde{C}(t) + \tilde{t}_L(t) + \tilde{t}_C(t)\right], \tag{3.1}$$

where $\tilde{Y}(t) \equiv dY(t)/Y$, $\tilde{K}(t) \equiv dK(t)/K$, $\tilde{C}(t) \equiv dC(t)/C$, $\tilde{t}_L(t) \equiv dt_L(t)/(1\text{-}t_L)$, $\tilde{t}_C(t) \equiv dt_C(t)/(1\text{+}t_C)$, and $\phi$ is a crucial parameter representing the intertemporal labor supply effect:

$$1 \le \phi \equiv \frac{1+\omega_{LL}}{1+\omega_{LL}(1-\epsilon_L)} \le \frac{1}{1-\epsilon_L}, \tag{3.2}$$

where $\omega_{LL}(\equiv(1\text{-}L)/L\geq 0)$ is the leisure to labor ratio, which also represents the intertemporal substitution elasticity of labor supply. Note that $\phi=1$ if labor supply is exogenous (since $L=1$, which implies that $\omega_{LL}=0$). Higher values of $\omega_{LL}$ correspond to a larger intertemporal labor supply effect. By using (3.1) and (T.5) in (T.1)-(T.2), the dynamical system characterizing the evolution of the economy can be written in terms of $\tilde{K}$ and $\tilde{C}$:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{C}}(t) \end{bmatrix} = \begin{bmatrix} \dfrac{r\phi}{1-t_K} & -\dfrac{r\phi}{(1-t_K)(1-\epsilon_L)} \\ -(r-\alpha) - r\left[1-\phi(1-\epsilon_L)\right] & (r-\alpha) - r(\phi-1) \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{C}(t) \end{bmatrix} - \begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix}, \tag{3.3}$$

where $\dot{\tilde{K}}(t) \equiv d\dot{K}(t)/K$, $\dot{\tilde{C}}(t) \equiv d\dot{C}(t)/C$ and the square matrix on the right-hand side of (3.3) is the Jacobian, $J$, evaluated at the steady state. The (potentially time-varying) forcing terms $\gamma_K(t)$ and $\gamma_C(t)$ are given by:

$$\gamma_K(t) \equiv \frac{r(\phi-1)}{(1-t_K)(1-\epsilon_L)}\left[\tilde{t}_L(t) + \tilde{t}_C(t)\right],$$
$$\gamma_C(t) \equiv r(\phi-1)\left[\tilde{t}_L(t) + \tilde{t}_C(t)\right] - (r-\alpha)\tilde{t}_C(t) + r\tilde{t}_K(t) + \dot{\tilde{t}}_C(t). \tag{3.4}$$

Table 1. Linearized Version of the Model

$$\dot{\tilde{K}}(t) = \left(\frac{r}{(1-t_K)(1-\epsilon_L)}\right)\left[\tilde{Y}(t) - \tilde{C}(t)\right] \tag{T.1}$$

$$\dot{\tilde{C}}(t) = (r-\alpha)\left[\tilde{C}(t) + \tilde{t}_C(t) - \tilde{K}(t)\right] + r\tilde{r}(t) - \dot{\tilde{t}}_C(t) \tag{T.2}$$

$$\begin{aligned}\tilde{T}(t) &= (1+t_C)\left[\tilde{t}_C(t) + \left(\frac{t_C}{1+t_C}\right)\tilde{C}(t)\right] \\ &+ \epsilon_L(1-t_L)\left[\tilde{t}_L(t) + \left(\frac{t_L}{1-t_L}\right)\tilde{Y}(t)\right] + (1-\epsilon_L)(1-t_K)\left[\tilde{t}_K(t) + \left(\frac{t_K}{1-t_K}\right)\tilde{Y}(t)\right]\end{aligned} \tag{T.3}$$

$$\tilde{Y}(t) - \tilde{L}(t) = \tilde{W}(t) \tag{T.4}$$

$$\tilde{Y}(t) - \tilde{K}(t) = \tilde{t}_K(t) + \tilde{r}(t) \tag{T.5}$$

$$\tilde{L}(t) = \omega_{LL}\left[\tilde{W}(t) - \tilde{t}_L(t) - \tilde{C}(t) - \tilde{t}_C(t)\right] \tag{T.6}$$

$$\tilde{Y}(t) = \epsilon_L\tilde{L}(t) + (1-\epsilon_L)\tilde{K}(t) \tag{T.7}$$

*Shares and parameters:*

| | |
|---|---|
| $\epsilon_L \equiv WL/Y$ | Share of before-tax wage income in real net output |
| $\omega_{LL} \equiv (1-L)/L$ | Ratio of leisure to labor |
| $\alpha$ | Pure rate of time preference |
| $\beta$ | Probability of death |

The following notational conventions are adopted. A deviation of a variable relative to the initial steady state is denoted by a tilde (~), e.g., $\tilde{K}(t) \equiv dK(t)/K$. For the tax tildes are defined as follows: $\tilde{t}_i(t) \equiv dt_i(t)/(1-t_i)$, for $i=K, L$, $\tilde{t}_C(t) \equiv dt_C(t)/(1+t_C)$ and $\tilde{T}(t) \equiv dT(t)/Y$. For the time rate of change of a variable we use a tilde and a dot: $\dot{\tilde{K}}(t) \equiv d\dot{K}(t)/K = \dot{K}(t)/K$, since $\dot{K}(0)=0$ in the initial steady state, except $\dot{\tilde{t}}_C(t) \equiv d\dot{t}_C(t)/(1+t_C)$.

The equilibrium is a saddle point. Saddle-point stability holds because the determinant of the Jacobian matrix is negative:

$$\det(J) \equiv -\lambda_1\lambda_2 = -\frac{r\phi\epsilon_L[2r-\alpha]}{(1-t_K)(1-\epsilon_L)} < 0, \tag{3.5}$$

where the characteristic roots are denoted by $-\lambda_1 < 0$ and $\lambda_2 > 0$. The fact is used that in the general case of finite horizons ($\beta > 0$) the steady-state rate of interest exceeds the pure rate of time preference, i.e., $r > \alpha$. Proposition 1 summarizes some stability results for the model that will prove useful in the discussion of tax policy changes.

PROPOSITION 1: *The model satisfies the following properties: (a) the model is locally saddle-point stable, (b) the characteristic roots are* $-\lambda_1<0$ *and* $\lambda_2>0$. *The unstable root satisfies the inequality* $\lambda_2>2r-\alpha$.

## B. Graphical Apparatus

In order to facilitate the discussion, the model is first summarized graphically by means of two schedules used in Figures 2 to 4 below. The CSE curve represents all points for which the goods market is in equilibrium with a constant capital stock ($\dot{\tilde{K}}(t)=0$). The MKR curve is the MKR rule, which represents the steady-state aggregate Euler equation augmented for the turnover of generations ($\dot{\tilde{C}}(t)=0$). The CSE curve is obtained by rewriting the first equation in (3.3) in steady-state form:

$$\tilde{C}(t) = (1-\epsilon_L)\tilde{K}(t) - \left(\frac{(1-\epsilon_L)(1-t_K)}{r\phi}\right)\gamma_K(t), \tag{3.6}$$

which is unambiguously upward sloping in $(\tilde{C}(t), \tilde{K}(t))$ space. The dynamic forces operating on the economy off the CSE curve are obtained from the first equation of (3.3). Points above the CSE curve are associated with a falling capital stock over time because both goods consumption is too high and labor supply (and hence production) is too low. The opposite is the case for points below the CSE curve.

The MKR curve is obtained by using the steady-state version of the second equation of (3.3):

$$\left[(r-\alpha)-r(\phi-1)\right]\tilde{C}(t) = \left[r-\alpha+r\left[1-\phi(1-\epsilon_L)\right]\right]\tilde{K}(t) + \gamma_C(t). \tag{3.7}$$

The term in square brackets on the right-hand side of (3.7) is positive (see (3.2)) but the sign of the term in square brackets on the left-hand side is ambiguous. The slope of the MKR curve is thus ambiguous because it is determined by two effects which work in opposite directions, viz. the *generational turnover* (GT) effect and the *labor supply* (LS) effect. The intuition behind these two effects can best be explained by looking at the two polar cases separately.

**Labor supply effect with infinite lives**

The pure LS effect is isolated by studying the model with endogenous labor supply and infinitely lived representative agents, i.e., $\phi > 1$ and $\beta = 0$. This version of the model could be viewed as a limit of an overlapping generations model as lifetimes grow large. In that case the MKR curve represents points for which the real after-tax rate of interest equals the rate of time preference, $r[C,K,t_L,t_C,t_K] = \alpha$, so that the slope of MKR depends on the partial derivatives $r_C \equiv \partial r/\partial C$ and $r_K \equiv \partial r/\partial K$. In order to explain the intuition behind these partial derivatives, Figure 1 depicts the situation on the markets for production factors. A useful expression for the (inverse) demand for capital ($K^D$) is obtained by combining (T.5) and (T.7):

$$\tilde{r}(t) = -\epsilon_L\left[\tilde{K}(t) - \tilde{L}(t)\right] - \tilde{t}_K(t). \tag{3.8}$$

In terms of panel (a) of Figure 1, the $K^D$ schedule is downward sloping and an increase in employment or a decrease in the capital tax shifts $K^D$ up. For a given stock of capital (which is represented by the vertical schedule $K_0$ in Figure 1(a)), the real (after-tax) interest rate clears the rental market for capital. In the infinite horizon model the long-run supply curve of capital is horizontal and coincides with the dashed line in Figure 1(a).

By using (T.4) and (T.7), the (inverse) demand for labor ($L^D$) can be written as:

$$\tilde{W}(t) = -(1 - \epsilon_L)\left[\tilde{L}(t) - \tilde{K}(t)\right]. \tag{3.9}$$

In terms of panel (b) of Figure 1, $L^D$ is a downward sloping schedule. An increase in the stock of capital enhances labor productivity and thus shifts the $L^D$ schedule up. Labor supply, $L^S$, is a standard upward sloping schedule and shifts to the left if consumption,[8] the labor tax, or the consumption tax rises (see (T.6)).

An increase in consumption (from $C_0$ to $C_1$) shifts the labor supply curve to the left, say from $L^S(W,C_0)$ to $L^S(W,C_1)$ in Figure 1(b), and for a given capital stock, employment falls from $L_0$ to $L_1$. This reduces the marginal product of capital, shifts the demand for capital to the left, say from $K^D(r,L_0)$ to $K^D(r,L_1)$ in Figure 1(a), and causes a fall in the rate of interest. This explains why $r_C < 0$.

---

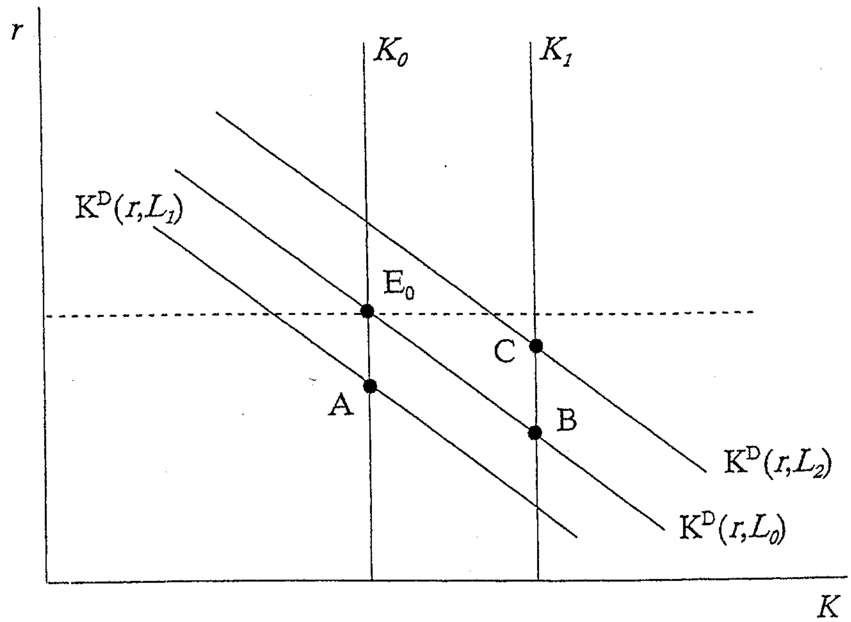
[8]The negative wealth (or intertemporal substitution) effect in labor supply vanishes if the following utility function is selected (see Greenwood and others (1988)):

$$U(v,t) = \int_t^\infty \log\left[X(v,\tau)\right]\exp\left[(\alpha+\beta)(t-\tau)\right]d\tau, \quad X(v,\tau) = C(v,\tau) - \frac{L(v,\tau)^{1+1/\sigma}}{1 + 1/\sigma},$$

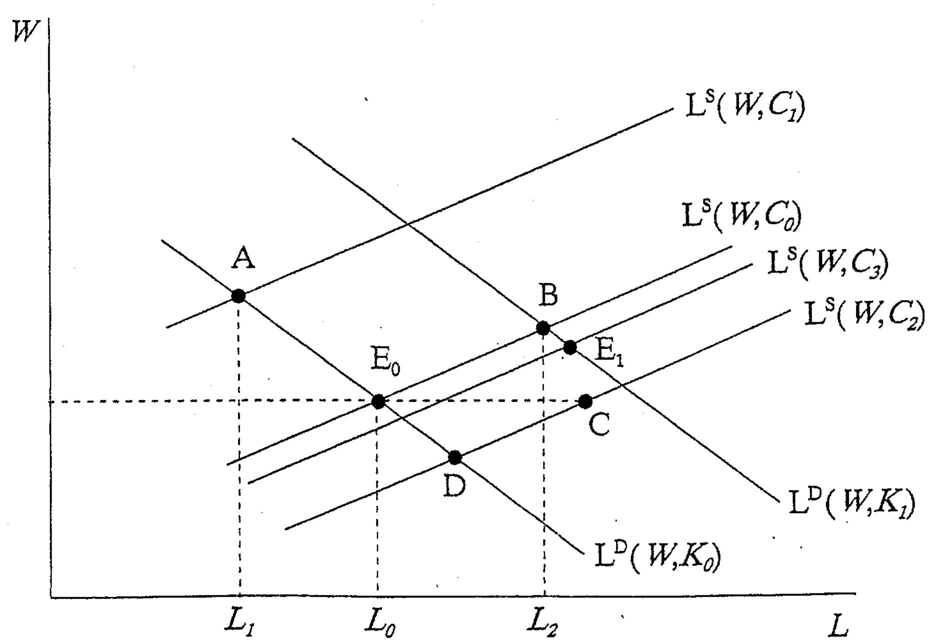
where $\sigma$ represents the intratemporal elasticity of labor supply. Labor supply is then given by $L(v,t) = [(1-t_L(t))W(t)/(1+t_C(t))]^\sigma$.

Figure 1. The Markets for Capital and Labor

*Panel (a) The Rental Market for Capital*

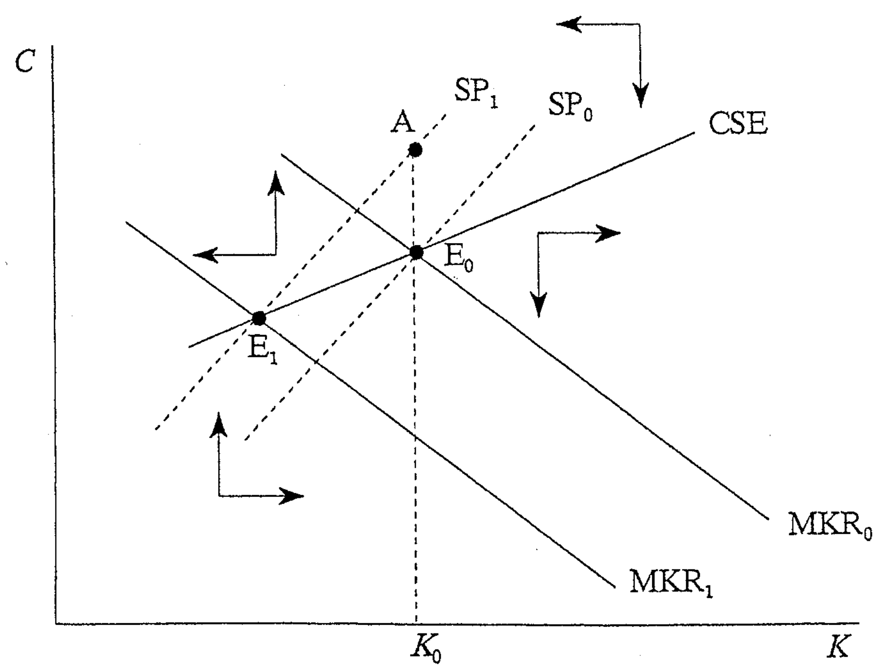



*Panel (b) The Labor Market*



An increase in the capital stock (from $K_0$ to $K_1$) has two effects. First, the *direct* effect leads to a rightward shift of the capital supply curve which, for a given level of employment, leads to a reduction in the rental price of capital. In terms of Figure 1(a) the direct effect is represented by the move from $E_0$ to B. There is also an *indirect* effect, because the increase in the capital stock boosts labor demand, say from $L^D(W,K_0)$ to $L^D(W,K_1)$ in Figure 1(b). For a given level of consumption, this leads to an expansion of employment from $L_0$ to $L_2$, represented by the move from $E_0$ to B. This increase in employment, in turn, boosts the demand for capital, say from $K^D(r,L_0)$ to $K^D(r,L_2)$ in Figure 1(a). The indirect effect thus represents the move from point B to point C directly above it. It is not difficult to show, however, that the direct effect of the capital stock always dominates the employment-induced effect, so that the rate of interest depends negatively on the capital stock, $r_K < 0$, and the MKR curve in Figure 2 is downward sloping (as $dC/dK \equiv -r_K/r_C < 0$ in that case). Points to the left of the curve are associated with a low capital stock, a high rate of interest, and a rising full consumption profile.

Figure 2. The Effects of the Capital Income Tax




## Generational turnover effect with exogenous labor supply

The pure GT effect is isolated by studying the model with exogenous labor supply and finitely lived agents, for which $\omega_{LL}$=0 (and thus $\epsilon_C$=1 and $\phi$=1) and $\beta$>0. In that case the MKR curve represents points for which the tilt to the consumption profile of individual households is precisely sufficient to ensure the turnover of financial assets across generations, $r[K,t_K]-\alpha=\beta(\alpha+\beta)K/[C(1+t_C)]$,[9] where the rate of interest now does not depend on consumption, the labor tax, and the consumption tax because labor supply is exogenous. From Figure 1 it is clear that with a fixed supply of labor, only the direct effect of a change in the capital stock remains.

The MKR curve is upward sloping because of the turnover of generations. Its slope can be explained by appealing directly to equation (2.14) (with $\epsilon_C=1$ and $A=K$) and Figure 3. Suppose that the economy is initially on the MKR curve, say at point $E_0$. Now consider a higher level of consumption, say at point B. With the same capital stock, both points feature the same rate of interest. Accordingly, individual consumption growth, $\dot{C}(v,t)/C(v,t)=r-\alpha$, coincides in the two points. Expression (2.14) indicates, however, that aggregate consumption growth depends not only on individual growth but also the *proportional* difference between average consumption and consumption by a newly born generation, i.e., $[C(t)-C(t,t)]/C(t)$. Since newly born generations start without any financial capital,[10] the *absolute* difference between average consumption and consumption of a newly born household depends on the average capital stock and is thus the same in the two points. Since the level of aggregate consumption is higher in B, this point features a smaller proportional difference between average and newly born consumption, thereby raising aggregate consumption growth.

In order to restore zero growth of aggregate consumption, the capital stock must rise (to point C). The larger capital stock not only reduces individual consumption growth by decreasing the rate of interest but also raises the drag on aggregate consumption growth due to the turnover of generations because a larger capital stock widens the gap between average wealth (i.e., the wealth of the generations that pass away) and wealth of the newly born.

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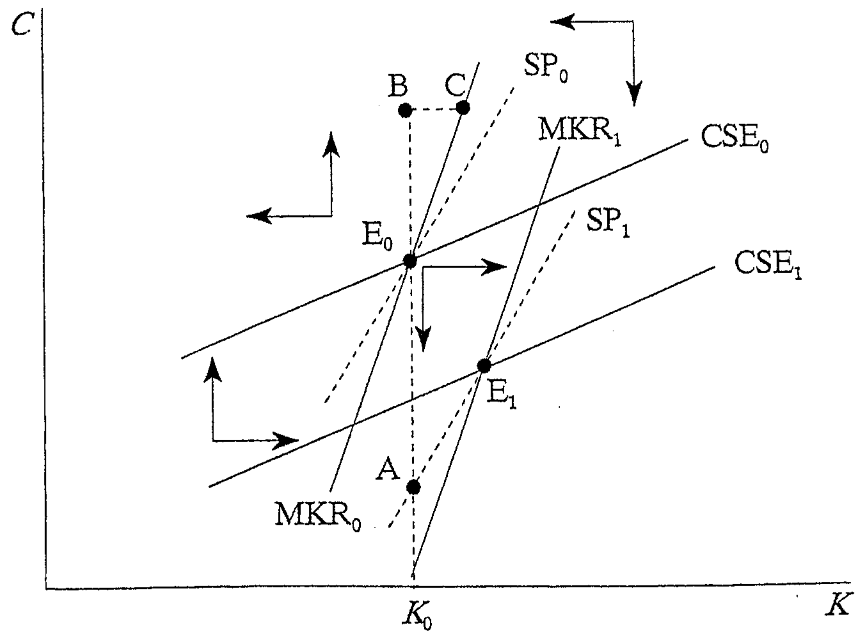
[9]The higher *effective* rate of time preference in the finite horizon case leads to a higher value of the marginal product of capital and correspondingly a lower capital stock than in the case of an infinite-horizon model.

[10]The share of human wealth in total wealth of an agent belonging to generation $v$ is given by:

$$\alpha_H(v) \equiv \frac{H(0)}{K(v,0) + H(0)} = \exp[(r-\alpha)v], \qquad 0 \leq \alpha_H(v) \leq 1.$$

where $\alpha_H(v)$ is decreasing in the age, $-v$, of the generation. Very old generations (with $v\to-\infty$) have accumulated over their life a large stock of financial assets which implies a negligible share of human capital in total wealth ($\alpha_H(v)\to 0$). Young agents (with $v\to 0$), however, own no financial wealth so that $\alpha_H(v)\to 1$ and thus fully consume out of human wealth.

Figure 3. Exogenous Labor Supply and Finite Lives




For points to the left of the MKR curve, the capital stock is low and, consequently, the interest rate is higher than the rate of time preference so that the consumption profile is rising. The opposite holds for points to the right of the MKR curve. In terms of Figure 3, steady-state equilibrium is attained at the intersection of CSE and MKR in point $E_0$. Given the configuration of arrows, it is clear that this equilibrium is saddle-point stable, and that the saddle path, $SP_0$, is upward sloping and steeper than the CSE curve.

## IV. THE DYNAMIC ALLOCATION EFFECTS OF TAX POLICY

In this section the dynamic allocation effects of the three tax instruments on the macro-economy are considered. In addition, the dynamic incidence of tax increases is analyzed. Analytical expressions for the impact, transition, and long-run effects are computed and the economic intuition is explained with the aid of simple diagrams. To keep matters simple, only unanticipated permanent shocks are studied, and the time at which the shock occurs is normalized to zero. It is assumed that the revenues of the taxes are rebated to households in a lump-sum fashion so that the government maintains a balanced budget in each time period.

As was demonstrated in Section III, the slope of the MKR curve depends on the balance of the LS and GT effects. If the LS effect dominates the GT effect the MKR curve slopes downward. As it turns out, the *qualitative* effects of the capital and labor taxes do not depend on the slope of the MKR curve. For that reason the intuitive discussion of the effects for those two taxes is based on the premise that the LS effect dominates the GT effect. The analytical expressions that are given, however, cover the most general version of the model.[11]

### A. Capital Income Tax

The effects of the capital tax are illustrated with the aid of Figure 2 (and with Figure 1). An increase in the capital tax shifts the $K^D$ schedule in Figure 1(a) down so that the rate of interest falls in the impact period because the amount of capital is predetermined. This makes current consumption more attractive compared to future consumption. As a result, consumption rises on impact:

$$\tilde{C}(0) = \frac{r\tilde{t}_K}{\lambda_2} > 0, \tag{4.1}$$

which is represented in Figure 2 by the vertical jump from point $E_0$ to A on the new saddle path associated with the new equilibrium. The increase in consumption shifts the labor supply curve $L^S$ in Figure 1 to the left so that, for a given labor demand $L^D$, employment and output fall and the gross wage rate rises:

$$-\left(\frac{\epsilon_L}{1-\epsilon_L}\right)\tilde{W}(0) = \epsilon_L\tilde{L}(0) = \tilde{Y}(0) = -\left(\frac{r(\phi-1)}{\lambda_2}\right)\tilde{t}_K < 0. \tag{4.2}$$

The reduction in employment prompts a further downward shift in the $K^D$ schedule which has a negative impact effect on the (after-tax) rate of interest:

$$\tilde{r}(0) = -\left(\frac{\lambda_2 + r(\phi-1)}{\lambda_2}\right)\tilde{t}_K < 0. \tag{4.3}$$

The simultaneous increase in consumption and decrease in production crowds out net investment so that over time the capital stock falls. The fall in the interest rate renders the optimal time profile for consumption downward sloping so that consumption also starts to decline, after the initial increase, and the economy moves from point A to the ultimate equilibrium at point $E_1$ according to:

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[11]In doing so the need to discuss the GT effect for these two taxes is obviated since it does not affect the qualitative allocation effects. For the consumption tax, the GT effect does, however, play a crucial role. For that case, the intuition is explained with the version of the model for which labor supply is exogenous so that the GT effect dominates the LS effect.

$$\tilde{K}(t) = \mathrm{A}(\lambda_1,t)\tilde{K}(\infty), \qquad \tilde{C}(t) = \tilde{C}(0)\left[1-\mathrm{A}(\lambda_1,t)\right] + \tilde{C}(\infty)\mathrm{A}(\lambda_1,t), \tag{4.4}$$

where $\mathrm{A}(\lambda_1,t)\equiv 1-\exp(-\lambda_1 t)$ is an adjustment term, $\lambda_1$ is the adjustment speed of the economy,[12] and $\tilde{K}(\infty)$ and $\tilde{C}(\infty)$ are, respectively, the long-run effects on the capital stock and consumption:

$$\tilde{K}(\infty) = -\left(\frac{r}{\epsilon_L[2r-\alpha]}\right)\tilde{t}_K < 0, \qquad \tilde{C}(\infty) = -\left(\frac{r(1-\epsilon_L)}{\epsilon_L[2r-\alpha]}\right)\tilde{t}_K < 0. \tag{4.5}$$

The rise in the capital tax rate decreases the capital stock as well as consumption in the long run, particularly if the pure rate of time preference, the share of before-tax wage income, and the after-tax rate of interest are low. Without overlapping generations, the steady-state capital stock and steady-state consumption fall according to $\tilde{K}(\infty)=-\tilde{t}_K/\epsilon_L$ and $\tilde{C}(\infty)=(1-\epsilon_L)\tilde{K}(\infty)$.

In the long run, the output effect equals the consumption effect, i.e., $\tilde{Y}(\infty)=\tilde{C}(\infty)$ (see (T.1)). By combining this result with (4.5) and (T.7) we find that employment is unchanged in the long run: $\tilde{L}(\infty)=0$. In terms of Figure 1(b), the reduction in the capital stock shifts $L^D$ down but the fall in consumption shifts $L^S$ to the right. There is no effect on employment but the before-tax wage unambiguously falls in the long run:

$$\tilde{W}(\infty) = -\left(\frac{r(1-\epsilon_L)}{\epsilon_L[2r-\alpha]}\right)\tilde{t}_K < 0. \tag{4.6}$$

After-tax wages (defined as $W^N(t)=W(t)[1-t_L(t)]+T(t)$) are given in the long run by:

$$(1-\epsilon_L)^{-1}\tilde{W}^N(\infty) = \epsilon_L(1-t_L)(1+\omega_{LL})\tilde{K}(\infty) + (1-t_K)\left[\tilde{t}_K + \left(\frac{t_K}{1-t_K}\right)\tilde{K}(\infty)\right], \tag{4.7}$$

where $\tilde{W}^N(t)\equiv dW^N(t)/Y$. The first term on the right-hand side of (4.7) represents the effect of the long-run capital stock on before-tax wages in the new steady state. Before-tax wages decline since workers have less capital to work with. The term between brackets on the right-hand side of (4.7) depicts the positive effect on the long-run after-tax wage of a rise in the lump-sum transfers provided by the government. The first term between brackets represents the positive tax rate effect of a higher capital tax. However, raising capital taxes erodes the capital tax base (if $t_K>0$) which depresses lump-sum transfers and the after-tax wage. The net effect on the long-run after-tax wage is positive if the tax rate effect is large enough to

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[12]The properties of the adjustment term are as follows: $\mathrm{A}(\lambda_1,0)=1-\lim_{t\to\infty}\mathrm{A}(\lambda_1,t)=0$ and $d\mathrm{A}(\lambda_1,t)/dt\geq 0$.

dominate the combined negative effect of the fall in the long-run gross wage and the erosion of the tax base. By substituting (4.5) in (4.7) we obtain:

$$(1-\epsilon_L)^{-1}\tilde{W}^N(\infty) = \frac{\epsilon_L(r-\alpha)(1-t_K) - r\left[t_K + (1-t_L)\epsilon_L\omega_{LL} - \epsilon_L t_L\right]}{\epsilon_L\left[2r-\alpha\right]}\tilde{t}_K. \tag{4.7'}$$

For exogenous labor supply (i.e., $\omega_{LL}$=0) and without preexisting capital and labor taxes (i.e., $t_K$=$t_L$=0) long-run after-tax wages remain above their initial equilibrium value. However, when intertemporal labor supply is sufficiently elastic, after-tax wages are likely to decline in the new steady state.

The long-run effect on the after-tax rate of interest is obtained by using (4.5) and (T.5) and noting that $\tilde{Y}(\infty)=\tilde{C}(\infty)$:

$$\tilde{r}(\infty) = -\left(\frac{r-\alpha}{2r-\alpha}\right)\tilde{t}_K < 0. \tag{4.8}$$

With overlapping generations ($r>\alpha$), the after-tax return remains below its initial steady-state equilibrium value. Thus, capital continues to bear part of the incidence of the capital tax in the long run, particularly if the rate of time preference is low and the initial rate of interest is large. This result is also obtained by Bovenberg and Heijdra (1998) for the case of exogenous labor supply. In the absence of overlapping generations (i.e., $\beta$=0 and $r$=$\alpha$, see Judd (1985) and Chamley (1985)) or in a small open economy (see Bovenberg (1993)), in contrast, capital can escape the entire long-run burden of capital taxation. In a small open economy, the world capital market fixes the after-tax rate of return on assets since physical capital is perfectly mobile internationally. With infinite lives, the long-run after-tax interest rate is fixed by the rate of time preference. We thus generalize the incidence results of Bovenberg and Heijdra (1998) to the case of endogenous labor supply. Since there is no long-run employment effect, our incidence results coincide with their results in the long run. However, in the short run, the elastic labor supply response allows labor to more than fully escape the burden of capital taxes even when households are infinitely lived.

PROPOSITION 2: *With overlapping generations ($r>\alpha$) a part of the long-run incidence of the capital tax is borne by the owners of capital. In the short run, workers bear less of the capital tax burden than in the long run, particularly if labor supply is very elastic (since $\tilde{W}^N(0)>0$ and $\tilde{W}^N(\infty)<0$). Without overlapping generations ($r=\alpha$) or in a small open economy the full long-run burden of capital taxes is shifted to labor.*

### B. Labor Income Tax

The impact, transition, and long-run effects of the labor tax can be studied with the aid of Figures 1 and 4. In Figure 1, an increase in the labor tax shifts $L^S$ to the left and $K^D$ down.

This causes a reduction in the interest rate and a fall in consumption which shifts $L^S$ to the right. The net effect on the labor market is a shift in $L^S$ to the left. The impact effect on consumption is represented by the downward shift in Figure 4 from $E_0$ to A:

$$\tilde{C}(0) = -\left(\frac{(\phi-1)\left[\lambda_2-(2r-\alpha)\right]}{\lambda_2\phi}\right)\tilde{t}_L < 0, \tag{4.9}$$

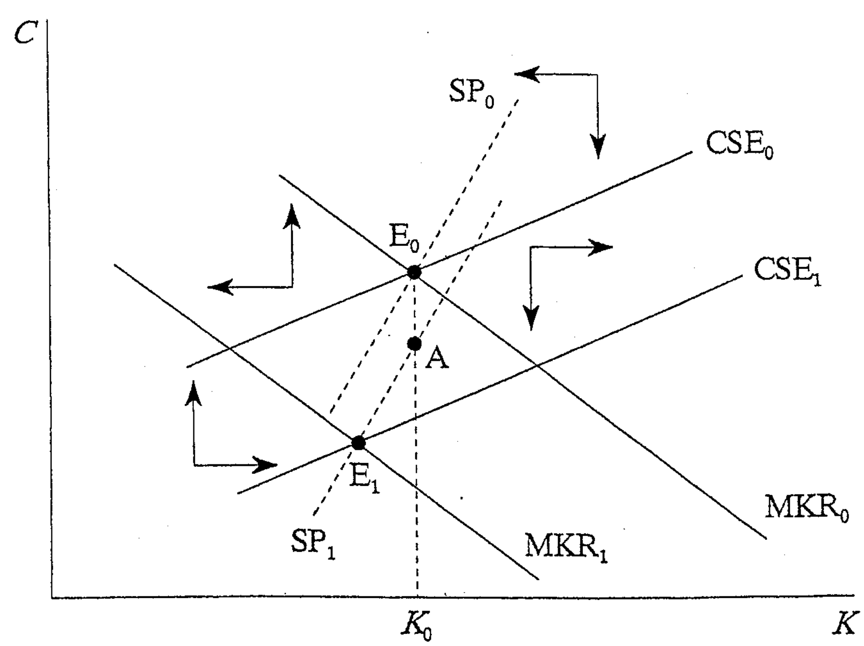
where the term in square brackets on the right-hand side is positive by Proposition 1(b). The impact effects on the other variables are:

$$-\left(\frac{\epsilon_L}{1-\epsilon_L}\right)\tilde{W}(0) = \epsilon_L\tilde{L}(0) = \tilde{Y}(0) = \tilde{r}(0) = -\left(\frac{(\phi-1)\left[\lambda_2+(\phi-1)(2r-\alpha)\right]}{\lambda_2\phi}\right)\tilde{t}_L < 0. \tag{4.10}$$

Gross wages as well as after-tax wages rise in the short run while the short-run rate of interest declines. Thus, capital bears a part of the labor tax burden in the short run. The transition path from A to $E_1$ is again described by (4.4), where the long-run effects are now given by:

$$\tilde{K}(\infty) = \tilde{C}(\infty) = \tilde{Y}(\infty) = \tilde{L}(\infty) = -\left(\frac{\phi-1}{\phi\epsilon_L}\right)\tilde{t}_L < 0. \tag{4.11}$$

Figure 4. The Effects of the Labor Income and Consumption Tax



The labor tax leads to a reduction in labor supply and the capital stock but it does not affect the optimal capital-labor ratio. This explains why the gross wage and the rate of interest are unchanged in the long run as well: $\tilde{W}(\infty)=\tilde{r}(\infty)=0$ (see (3.8) and (3.9)). However, the long-run after-tax wage unambiguously declines if labor supply is elastic:[13]

$$\tilde{W}^N(\infty) = -\frac{\left[1+\omega_{LL}(1-t_L)\right]\epsilon_L\omega_{LL}}{1+\omega_{LL}}\tilde{t}_L < 0. \tag{4.12}$$

PROPOSITION 3: *Under both finite and infinite horizons workers fully bear the long-run burden of labor taxes,* $\tilde{W}^N(\infty)<0$. *In the short run, however, capital bears a part of the incidence of the labor tax,* $\tilde{W}^N(0)>0$.

### C. Consumption Tax

It is clear from the definition of the policy shock terms in equation (3.4) that the consumption tax is equivalent to a labor tax in the case where there are no overlapping generations.[14] Indeed, in the infinitely lived representative agent model $r=\alpha$ holds in steady state so that $\gamma_C(t)$ and $\gamma_K(t)$ are affected equally by the two tax rates. In this case, the consumption tax (or equivalent labor income tax) does not affect the household's intertemporal tradeoff between consumption today and tomorrow. Savings are only affected through the income (or wealth) effect of the consumption tax which shifts the present value budget constraint of the household inward. All results discussed in the previous section therefore also apply for the consumption tax when the birth rate is zero.

In the presence of overlapping generations this familiar equivalence result no longer holds because the GT effect now works in the opposite direction to the LS effect (see (3.4)). In order to explain the intuition behind the GT effect in the presence of a consumption tax, the case is considered where the GT effect dominates the LS effect ($r-\alpha>r(\phi-1)$) so that the MKR curve is upward sloping. This is shown in Figure 3 where labor supply is assumed to be exogenous. An increase in the consumption tax shifts both the MKR and CSE schedules to the right so that consumption jumps down on impact from $E_0$ to A. Figure 1 can be used to explain this effect. An increase in the consumption tax shifts $L^S$ to the left and $K^D$ down in

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[13]If households do not receive any transfers from the government, the elasticity of labor supply has no effect on the long-run incidence of the labor tax change. This is in line with incidence analyses embedded in neoclassical growth models with variable labor supply (i.e., Feldstein (1974)). In the next section it is shown that the intertemporal labor supply response does matter for the incidence analysis of consumption taxes.

[14]The change in the consumption tax must also be unanticipated and permanent for the equivalence result to hold.

Figure 1. This causes a reduction in the interest rate and a fall in consumption which shifts $L^S$ to the right. The net effect on the labor market is a shift of $L^S$ to the left. The impact effect on consumption is given by:

$$\tilde{C}(0) = -\left(\frac{(\phi - 1)(\lambda_2 - r) + (r - \alpha)}{\lambda_2 \phi}\right)\tilde{t}_C < 0, \tag{4.13}$$

where it should be noted that Proposition 1(b) implies that $\lambda_2 > r$. The impact effects on the other variables are:

$$-\left(\frac{\epsilon_L}{1-\epsilon_L}\right)\tilde{W}(0) = \epsilon_L \tilde{L}(0) = \tilde{Y}(0) = \tilde{r}(0) = -\left(\frac{(\phi - 1)\left[\lambda_2 + (\phi - 1)r + (r - \alpha)\right]}{\lambda_2 \phi}\right)\tilde{t}_C < 0. \tag{4.14}$$

In the representative agent model (or with a dominant LS effect) a fall in the interest rate implies a downward-sloping optimal consumption profile (see above) but this is no longer so if the GT effect dominates the LS effect. In that case, though the individual consumption profiles are still downward sloping (see (2.13)), the aggregate consumption profile is upward sloping (see Figure 3). In terms of equation (2.14), the first term on the right-hand side falls but the second term more than offsets this effect. Because old agents have more wealth and consume more than recently born agents, the former are hit harder by the consumption tax than the latter so that the proportional difference between the old and young agents falls.

The reduction in aggregate consumption outweighs the reduction in production so that net investment takes place and the economy starts to move from A to the ultimate equilibrium in point $E_1$. The transition path is still described by the expressions in (4.4) and the long-run effects are:

$$\tilde{K}(\infty) = \left(\frac{r - \alpha - r(\phi - 1)}{\phi\epsilon_L[2r - \alpha]}\right)\tilde{t}_C, \quad \tilde{C}(\infty) = \left(\frac{(r-\alpha)(1-\phi\epsilon_L) - r(\phi - 1)}{\phi\epsilon_L[2r-\alpha]}\right)\tilde{t}_C. \tag{4.15}$$

With a dominant GT effect the capital stock rises in the long run, whereas it would fall if the GT effect is fairly weak. Consumption falls in the long run unless labor supply is not very elastic. Indeed, if labor supply is exogenous then $\phi = 1$ and consumption rises in the long run. Of course, in the new steady state the long-run effects on output and consumption are the same, i.e., $\tilde{Y}(\infty) = \tilde{C}(\infty)$ (see (T.1)). By using the long-run results for consumption, output, and the capital stock in (T.4)-(T.6), we obtain:

$$\tilde{L}(\infty) = -\left(\frac{\phi - 1}{\phi\epsilon_L}\right)\tilde{t}_C < 0, \quad \tilde{r}(\infty) = -\left(\frac{r-\alpha}{2r-\alpha}\right)\tilde{t}_C \le 0,$$
$$\tilde{W}(\infty) = \left(\frac{(r-\alpha)(1-\epsilon_L)}{\epsilon_L[2r-\alpha]}\right)\tilde{t}_C \ge 0. \tag{4.16}$$

With finite horizons, the consumption tax leads to a fall in the rate of interest and an increase in the gross wage rate. After-tax wages rise more than gross wages in the long run since households receive the transfers financed by the consumption tax. Part of the burden of the consumption tax is thus borne by owners of capital, a result which is impossible with infinitely lived agents or in a small open economy (see the similar discussion surrounding the capital income tax in Section IV.A above).

PROPOSITION 4: *With overlapping generations, a part of the long-run incidence of the consumption tax is shifted to capital. In the absence of intergenerational disconnectedness or in a small open economy the entire burden of consumption taxes is borne by labor.*

### D. Discussion

The main findings in this section can be summarized as follows. First, both capital and labor taxes depress economic activity in the long run. An increase in the proportional capital tax reduces savings and discourages capital formation (see equation (4.5)) whereas the labor tax reduces capital formation provided labor supply is endogenous (see (4.11)). These findings can also be derived in a framework with an infinitely lived agent implying that the generational structure does not affect the qualitative effects on capital formation and aggregate consumption.

The results are less definite with a consumption tax because the labor supply and generational turnover effects of a policy change work in opposite directions. If the former effect is dominated by the latter, an increase in the consumption tax leads to an increase in the capital stock in the long run. The equivalence between proportional consumption and labor taxes, which is observed in an infinite horizon model, thus fails to hold in an overlapping-generations world. If the generational turnover effect is dominated by the labor supply effect, the labor and consumption tax have qualitatively very similar effects on the economy.

## V. CONCLUSIONS

In this paper the dynamic macroeconomic effects of various tax policy instruments within a framework of finitely lived households have been considered. An overlapping-generations model of the Blanchard-Yaari type was extended to incorporate endogenous labor supply and three tax policy instruments, viz. a capital income tax, labor income tax, and consumption tax. This framework allowed for a study of the entire transition path toward a new long-run equilibrium. Analytically, expressions for the short-run, transition, and long-run effects of permanent and unanticipated changes in the tax measures were derived. In addition, a simple graphical apparatus was developed to facilitate the understanding of the dynamic effect of tax policy changes on labor, capital, and goods markets.

The paper yields a number of results. First, both capital and labor income taxes depress savings and thus have qualitatively similar effects on the steady-state capital stock.

Consumption taxes, however, may boost the capital stock in the long run if the *generational turnover* effect dominates the *labor supply* effect. This is only likely to happen if labor supply is inelastic. The generational turnover effect is positive because the consumption tax hits harder the consumption of old generations who mainly consume out of accumulated financial wealth. This alleviates the drag on consumption growth caused by the passing away of old wealthy generations that are replaced by newly born generations who do not own any financial wealth. If labor supply is elastic, the reduction in aggregate consumption depresses labor supply and capital demand which causes a fall in the rate of interest and therefore in individual savings. Second, the long-run incidence of capital and consumption taxes partially falls on capital whereas in a small open economy or with infinitely lived households capital is able to escape the entire long-run burden of taxation. Indeed, with overlapping generations, capital bears a larger burden of taxation in the short run than in the long run.

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