IMF Working Paper

WP/98/153

INTERNATIONAL MONETARY FUND

Research Department

Monetary Policy in a Small Open Economy with Credit Goods Production

Prepared by Jorge A. Chan-Lau[1]

Authorized for distribution by Peter Isard

October 1998

Abstract

The paper analyzes the effects of monetary policy in a dynamic model of a small open economy with cash and credit goods production, where government consumption is financed by seignorage. It shows that the interrelationships between the growth rate of the monetary aggregate and the technological properties of the economy have an important bearing on the existence and uniqueness of equilibrium, the optimal inflation rate, and the occurrence of explosive hyperinflations. In consequence, the paper concludes that monetary policy does matter in the long run.

JEL Classification Numbers: D50, E42, F41

Keywords: Cash goods, credit goods, inflation, monetary policy, small open economy.

Author's E-Mail Address: jchanlau@imf.org

---

[1]I would like to thank Peter Clark and Peter Isard for their comments. I retain responsibility for any errors or omissions.

Contents Page

Summary

High inflation countries experience a dramatic expansion of their financial sector and declining growth rates. This provides indirect evidence that monetary policy does indeed affect the real sector of an economy in the long run, invalidating the classical dichotomy. In stark contrast to these observations, the results obtained from standard monetary models -- where the demand for money is based either on cash-in-advance constraints or money in the utility function-- usually indicate that monetary policy has a modest role influencing the long run behavior of the economy.

One simple way to explain this apparent conflict is by introducing credit goods in an otherwise standard cash-in-advance economy. The mechanism at work is as follows: Credit goods provide a hedge against the inflation tax. If inflation is higher, household demand for credit goods increases, and accordingly, so does the demand for credit services. Therefore, more productive resources are allocated to the production of credit services at the expense of the real goods production sector.

This paper introduces the credit goods mechanism in a cash-in-advance model of a small open economy and analyzes the effects of monetary policy when government consumption is financed by seigniorage. It shows that the interrelationships between the growth rate of the monetary aggregate and the technological properties of the economy have an important bearing on the existence and uniqueness of equilibrium , the optimal inflation rate, and the occurrence of explosive hyperinflations. In consequence, the paper concludes that monetary policy does matter in the long run.

I. INTRODUCTION

High inflation countries experience a dramatic expansion of their financial sector and declining growth rates. This provides indirect evidence that monetary policy does indeed affect the real sector of an economy in the long run, invalidating the classical dichotomy. In stark contrast to these observations, the results obtained from standard monetary models, where the demand for money is based either on cash-in-advance constraints or money in the utility function, usually find that monetary policy has a modest role influencing the long run behavior of the economy.

One simple way to explain the facts described above is by introducing credit goods in an otherwise standard cash-in-advance economy. The mechanism at work is as follows: Credit goods provide a hedge against the inflation tax. If inflation is higher, household demand for credit goods increases, and accordingly, so does the demand for credit services. Therefore, more productive resources are allocated to the production of credit services at the expense of the real goods production sector.

Costly credit has been incorporated into dynamic closed economy models by Gillman (1993), Aiyagari, Braun and Eckstein (1995) and Aiyagari and Eckstein (1996). Among the issues analyzed by the authors above are: the welfare costs of inflation; the long run relationship between inflation, growth and the size of the intermediation sector; and the comovement of the velocity of money and inflation. However, to our knowledge, costly credit have not been introduced in small open economy models. The objective of this paper is to fill this gap and analyze how monetary policy can affect output in the long-run using a dynamic model of a small open economy where credit is costly.

To accomplish this objective, we extend the closed economy model of Aiyagari and Eckstein (1996). The main differences between their setup and ours are: (1) the assumption of perfect capital mobility, (2) the inclusion of traded goods that are perfect substitutes of domestic produced goods, and (3) the assumption that labor is the only production input in both the credit and good production technologies. Under the additional assumptions of purchasing power parity and zero inflation in the rest of the world, it can be shown that the model is equivalent to a simpler one where there are no traded goods. Without loss of generality, we analyze the latter model for convenience.

The main results obtained here are:

- There exists a simple relationship between the nominal interest rate and output, that depends on the production technologies in the goods and credit services sector and the nominal interest rate set by the government. Under some simple assumptions, it is possible to establish a long-run relationship between inflation and output.

- The existence and uniqueness of an equilibrium depend on the nominal interest rate.
- Friedman's rule is a special case of the optimal inflation rate and is obtained under plausible conditions.
- Hyperinflations are not exclusively a monetary phenomena. Their existence depends crucially on the characteristics of the production technologies for the consumption goods and credit services.

The structure of the paper is as follows. Section II presents the model. Section III defines the concept of a competitive equilibrium in this economy. Section IV analyzes government spending financed by seigniorage when money grows at a constant rate. Section V characterizes the optimal inflation tax and determines the conditions that rule out hyperinflations. Section VI concludes.

II. THE MODEL

It is assumed that there is perfect capital mobility and that the domestic country is too small to influence the world real interest rate, r^w. Therefore, the Fisher relationship between nominal and real interest rate is satisfied:

$$1 + R_t = (1 + r^w)(1 + \pi_t)\,, \tag{1}$$

where R is the nominal domestic interest rate and π is the domestic inflation rate.

The domestic country is inhabited by a representative household who is infinitely lived and endowed with a unit of labor in every period. The household supplies labor to the domestic firms that produce goods and credit services at the competitive wage. The household's utility, V, is given by a time additive separable utility representation

$$V \equiv \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{2}$$

where β is the subjective discount rate, U is the current utility function, and c_t is the amount of a consumption basket consumed by the household. The utility function satisfies the following properties: $U' > 0$ and $U'' < 0$.

The consumption basket, c_t, must be less or equal to the sum of the amount of cash goods, c_{1t}, and credit goods, c_{2t}, purchased by the household plus government transfers, τ_t :

$$c_t \leq c_{1t} + c_{2t} + \tau_t. \tag{3}$$

Equation (3) implies that cash and credit goods are perfect substitutes.[2] Existence of a steady state with constant consumption requires that the subjective discount rate satisfies the following assumption:

Assumption 1 $\beta = 1/(1 + r^w)$

At first glance, it seems unnatural to assume that there is no trade and that economic decisions are affected only by the equalization of the domestic real rate with the world interest rate. However, this model is equivalent to one with traded goods under the following assumptions: (1) the traded goods is a perfect substitute of both cash and credit goods, (2) purchasing power parity holds, and (3) there is no inflation in the rest of the world. These three assumptions are relatively standard, so the results will remain unchanged either in this model or its traded goods equivalent. Since working with this model is simpler, we refer the reader to the appendix, where we prove the equivalence between the two economies.

The timing of transactions is the same one as in Lucas and Stokey (1987). The assets market opens first and the household purchases bonds using the money available at the beginning of the period, which is given by the sum of previous money holdings, M_t, carried over from period $t-1$, an injection of money, X_t, by the government, and the payoff from holding a nominal amount of bonds issued by the government, $B_t/(1 + R_{t-1})$, where R_{t-1} is the nominal interest rate paid from period $t-1$ to period t. Because of perfect capital mobility, the nominal interest rate has to satisfy the Fisher equation (1). After transactions take place in the assets market, it closes and the goods market opens. The household buys the cash good subject to the cash-in-advance constraint using the balances that remain after purchasing government bonds, so its budget constraint is given by:

$$p_{1t}c_{1t} + \frac{B_{t+1}}{(1 + R_t)} \leq M_t + X_t + B_t. \tag{4}$$

Finally, after the closing of the goods market, the household receives its salary, W_t, and allocates the remaining balances between next period money holdings, M_{t+1}, and credit goods, c_{2t}:

$$p_{1t}c_{1t} + p_{2t}c_{2t} + M_{t+1} + \frac{B_{t+1}}{(1 + R_t)} \leq M_t + X_t + B_t + W_t. \tag{5}$$

There are two different types of goods in the economy: simple goods, to which we refer hereafter plainly as goods, that can be used as cash goods or inputs to produce credit

[2] If both goods are not perfect substitutes, then it is possible to determine endogenously what fractions of total consumption are bought using cash and credit. However, this results do not change the qualitative results delivered by the model. A detailed analysis can be found in Gillman (1993) and Aiyagari, Braun and Eckstein (1995).

services, and credit goods. The goods sector uses the technology F to produce output Y_t using labor services n_{1t}:

$$Y_t = F(n_{1t}). \tag{6}$$

Output Y_t can either be consumed as a cash good or used as an input to produce the credit good. Therefore, the price of one unit of output must be equal to p_{1t}, to rule out arbitrage. The amount of goods used as an input to produce credit services is given by:

$$F\left(n_{1t}\right) - c_{1t}^s - g_t - \tau_t,$$

where g_t is government spending.

Credit services, which are sold at price p_{st}, are produced with the technology G using labor services n_{2t}:

$$S_t = G(n_{2t}). \tag{7}$$

Once produced, credit services are used as inputs to produce the credit good according to the following Leontieff fixed proportions technology:

$$c_{2t}^s = \min(Y_t - c_{1t}^s - g_t, S_t), \tag{8}$$

where c_1^s and c_2^s are the supply of cash goods and credit goods respectively. No arbitrage implies the following relationship among prices

$$p_{2t} = p_{1t} + p_{st}. \tag{9}$$

The following standard assumption asserts that both technologies are increasing in the amount of labor input:

Assumption 2 *The production technologies F and G satisfy $F' > 0$, $G' > 0$.*

The remaining agent in our model is the government, whose only role in the economy is to finance government spending via seigniorage. Given an initial government spending, g_0, an initial money supply, M_0, and an outstanding amount of nominal debt, B_0, the government chooses a sequence of spending, $\{g_t\}_{t=1}^{\infty}$, monetary injections, $\{X_t\}_{t=1}^{\infty}$, and lump-sum transfers, $\{\tau_t\}_{t=1}^{\infty}$, paths such that it balances its budget period after period:

$$p_{1t}g_t + p_{1t}\tau_t = X_t + \frac{B_{t+1}^g}{1+R_t} - B_t^g. \tag{10}$$

This implies the following law of motion of the money supply:

$$M_{t+1}^s = M_t^s + X_t. \tag{11}$$

III. EQUILIBRIUM

A competitive equilibrium must satisfy market clearing conditions, the household's optimality conditions, profit maximization by the firms and the Fisher equation. We proceed to present these conditions. Market clearing conditions are summarized by the equations below:

$$n_{1t} + n_{2t} = 1, \tag{12}$$

$$c_{1t} = c_{1t}^s, \tag{13}$$

$$c_{2t} = c_{2t}^s = G(n_{2t}), \tag{14}$$

$$c_t = c_{1t} + c_{2t} + \tau_t = Y_t - g_t = F(n_{1t}) - g_t, \tag{15}$$

$$B_t = B_t^g, \tag{16}$$

$$M_t = M_t^s. \tag{17}$$

The interpretation of the market clearing equations is straightforward. The first states that the total demand for labor services should be equal to labor supply. The second and third conditions state that consumption of the cash and credit good must be equal to their supply. The fourth condition state that total consumption of the household is equal to total production net of government consumption. Finally, the last two conditions are necessary for market clearing in the assets market.

Utility maximization by the household yield the optimality conditions:

$$U'(c_t) = \beta \frac{p_{1t}}{p_{1t+1}} U'(c_{t+1})(1 + R_t), \tag{18}$$

$$\frac{p_{2t}}{p_{1t}} = 1 + R_t. \tag{19}$$

The first order condition is the standard Euler equation and determines the optimal intertemporal substitution. The second equation is a no arbitrage relationship derived from the assumption of perfect substitution between the cash and credit goods. As Aiyagari and Eckstein (1996) noted, this is the channel that breaks the dichotomy between the nominal and real sectors, and hence, it is crucial to the results of the paper.

Profit maximization by the firms result in wage equalization across sectors, yielding the following relationship between employment in both sectors:

$$F'(n_{1t}) = \frac{p_{st}}{p_{1t}} G'(n_{2t}). \tag{20}$$

From equations (9) and (19):

$$\frac{p_{st}}{p_{1t}} = R_t,$$

which implies

$$F'(n_{1t}) = R_t G'(n_{2t}). \tag{21}$$

Definition 1 *A **competitive equilibrium** is the sequence of allocations* $\{c_t, c_{1t}, c_{2t}, c^s_{1t}, c^s_{2t}, n_{1t}, n_{2t}, B_t, M_t, B^s_t, M^s_t, g_t, \tau_t\}_{t=1}^{\infty}$, *and prices* $\{p_{1t}, p_{2t}, R_t\}_{t=0}^{\infty}$ *such that they satisfy the market clearing equations (12)-(19), the optimality equations (18)-(19), the profit maximization equations (20)-(21) and the Fisher equation.*

IV. Seigniorage Financing with a Constant Money Growth Rate

We characterize the steady states of this model for the particular case in which bonds are issued in zero net supply and zero lump-sum transfers. Thus, government spending is financed only by seigniorage. In this case, it is simple to show that inflation is equal to:

$$\pi_t = \frac{p_{1t+1}}{p_{1t}} - 1 = \frac{p_{2t+1}}{p_{2t}} - 1.$$

The Euler equation reduces to:

$$1 = \beta(1 + \overline{R})/(1 + \overline{\pi}) = \beta(1 + r^w),$$

where $\overline{\pi}$ and $\overline{R}$ denote steady state inflation and nominal interest rate respectively. Because of Assumption 1, the steady state Euler equation is always satisfied. This implies that the model is compatible with the existence of multiple equilibria characterized by constant consumption, though this is not necessarily the case. We show below under what conditions multiple equilibria exist.

Steady state government consumption is given by:

$$\overline{g} = \overline{\pi} m,$$

where m denotes steady state real balances. Given $\overline{c}_1$, the amount of consumption of the cash good in the steady state, real balances demanded by the household are given by:

$$m = \frac{\overline{c}_1}{1 + \overline{\pi}},$$

and this determines the amount of government expenditures as a function of steady state consumption and inflation:

$$\overline{g} = \frac{\overline{\pi}\overline{c}_1}{1 + \overline{\pi}}.$$

The relationship between consumption and the nominal interest rate, where the latter can be controlled by the government through the choice of the monetary growth rate, is determined by equation (21) together with the labor market clearing condition:

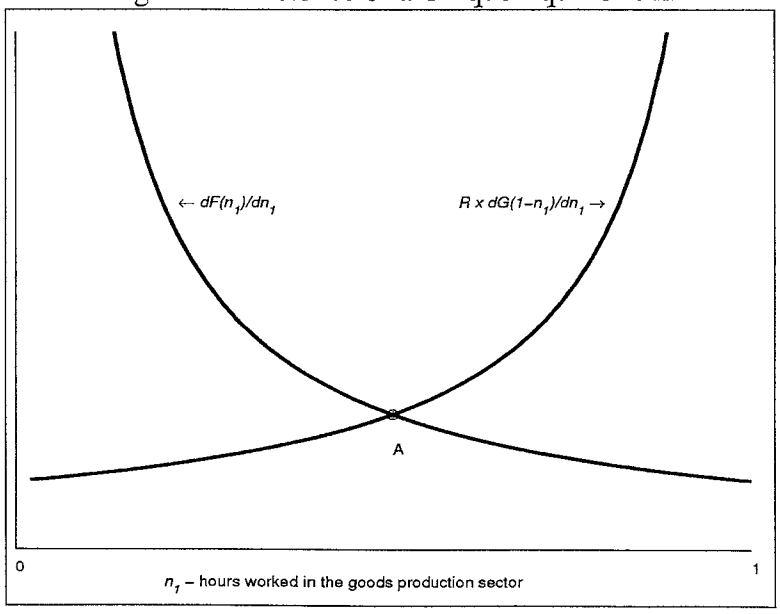
$$F'(\overline{n}_1) = R_t G'(1 - \overline{n}_1),$$

where the overlined variables denote steady state values. A more transparent way to express the relationship above is to rewrite it such that the nominal interest rate becomes a function of steady state hours supplied to the goods producing sector:

$$\overline{R} = H(\overline{n}_1) = \frac{F'(\overline{n}_1)}{G'(1 - \overline{n}_1)}. \tag{22}$$

Recalling that total output and consumption are an increasing function of hours worked in the good producing sector, the last equation gives us a relationship between inflation and output, or equivalently, between inflation and employment in the goods production sector, as measured by n_1 . However, the existence of this relationship depends on the characteristics of the production technologies available to the goods and credit services sectors, that is F and G respectively. For example, Figure 1 shows the case of a unique equilibrium given the nominal interest rate $\overline{R}$.

Figure 1. Existence of a Unique Equilibrium



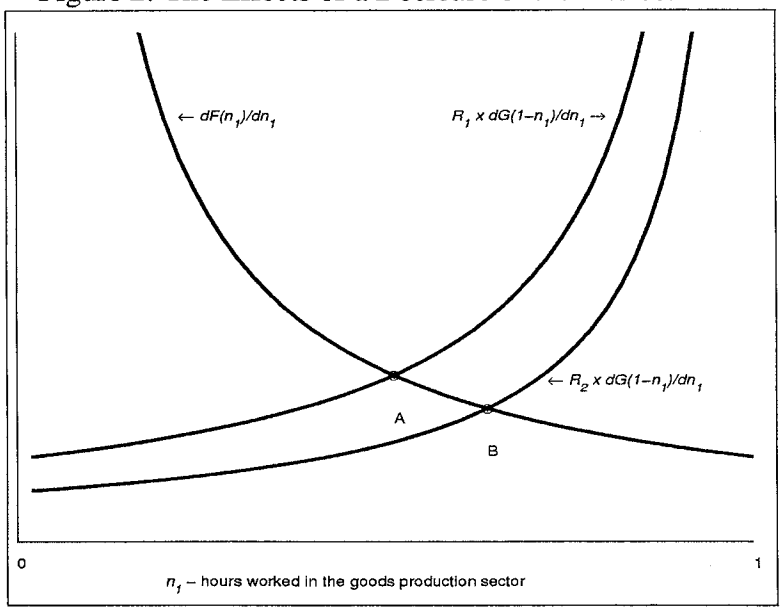
This case assumes that the marginal productivity of both technologies becomes infinite when labor supply in that sector becomes very small. The following lemma establish when a unique equilibrium exists.

Lemma 1 *If $F'(.)$ and $G'(.)$ are continous in $[0,1]$ and satisfy $F'(0) > G'(1)$, $F'(1) < G'(0)$, then for any given nominal interest rate, there exists a unique labor allocation $n_1(\overline{R})$ that satisfies equation (1).*

Proof. Define the function $A : [0,1] \to \mathcal{R}$ as $A(n) = F(n) - G(1-n)$. A is continous, strictly increasing and satisfies $A(0) < 0$,$A(1) > 0$. Therefore, by the intermediate value theorem, there exists a unique n^* in $[0,1]$ such that $F(n^*) = G(n^*)$. ■

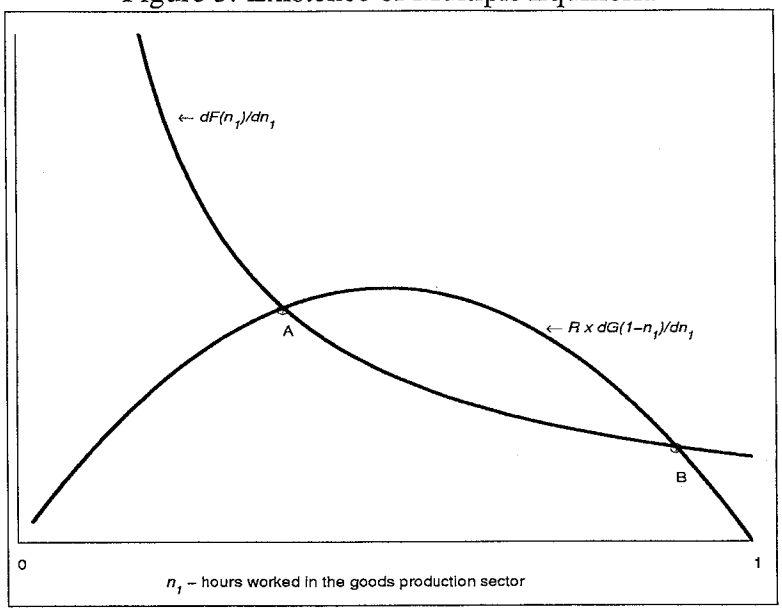
Clearly, monetary policy can be welfare enhancing: by reducing the nominal interest rate, that is equivalent to reducing the inflation rate or the growth rate of the monetary aggregate, the government can increase the number of hours spent working in the goods production sector. Because the household's welfare is an increasing function of goods output, the household is better off. Figure 2 shows the positive effect of a decrease in the nominal interest rate, which results in higher levels of consumption and output and is associated with a decrease in inflation, as the equilibrium changes from point A to B.

Figure 2. The Effects of a Decrease of the Interest Rate



However, uniqueness of equilibrium is not the only possible situation. If the conditions stated in Lemma 1 are not satisfied, there could exist multiple equilibria or no equilibrium at all, given a fixed nominal interest rate. Figure 3 shows the existence of multiple equilibria when the marginal productivity of one of the technologies does not exhibit a monotonic behavior with respect to labor input, in this case, the credit services sector. This equilibrium corresponds to the case in which the production technology G exhibits nonmonotonic marginal productivity.

Figure 3. Existence of Multiple Equilibria



In contrast to other models that exhibit multiple equilibria, monetary policy can eliminate the indeterminacy by imposing a different nominal interest rate. Figure 4 depicts the situation in which an increase of the nominal interest rate from R_1 to R_2 eliminates equilibrium indeterminacy at the expense of reducing welfare, since both equilibria A and B Pareto dominate equilibrium C.

Figure 4. Elimination of Equilibrium Indeterminacy

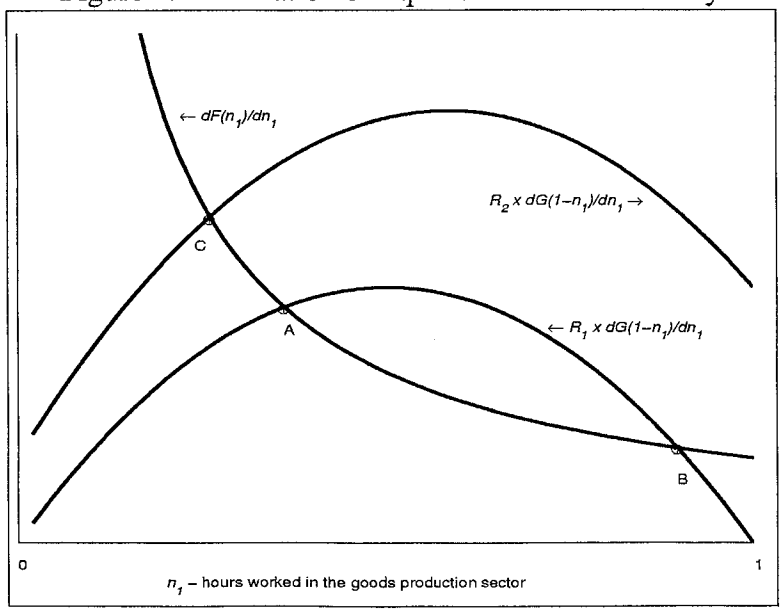


Figure 5 shows a case in which there is no equilibrium compatible with the given nominal interest rate, as shown in Figure 5, though a decrease in the nominal interest rate from R_1 to R_2 determines a unique equilibrium at a higher consumption level. This situation is shown in Figure 6.

Figure 5. A Case where Equilibrium does not Exist

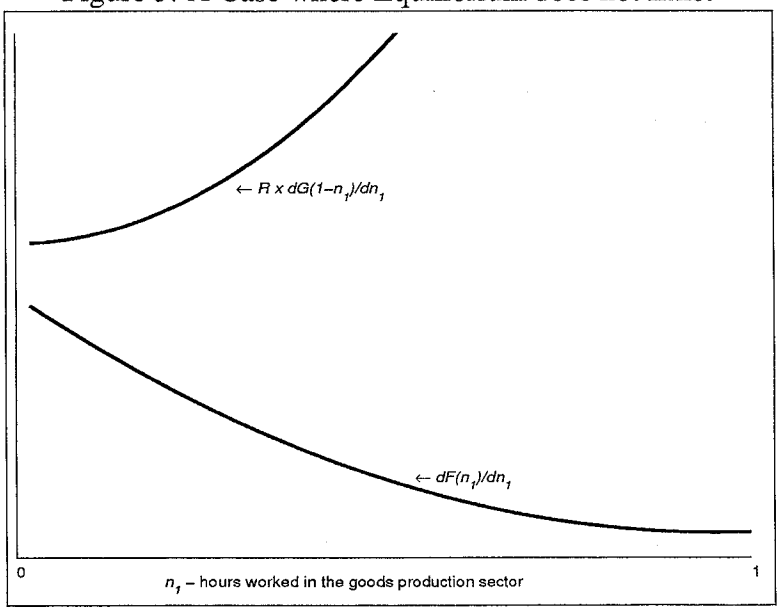
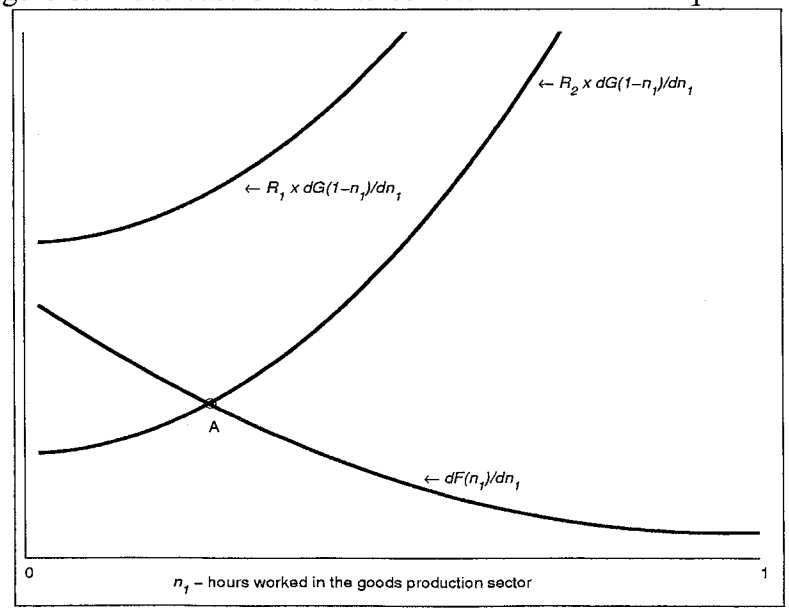


Figure 6. A decrease of the interest rate determines an equilibrium



Summarizing, the existence and uniqueness of equilibrium depends both on the technological characteristics of the economy and the rate of growth of the monetary aggregate determined by the monetary authorities.

V. The Optimal Inflation Tax and Hyperinflation

The rule proposed by Friedman (1969), that money should grow at a constant rate, and if possible, this rate should be negative and equal to the inverse of the real interest rate, obtains under some special conditions of the production technologies in the goods and credit services sectors.

Lemma 2 *If $F'(1)$ is bounded above and $\lim_{n_1 \uparrow 1} G'(1-n_1) \to \infty$, then the optimal nominal rate is $R = 0$ and the Friedman rule is optimal.*

Proof. Taking the limit in equation (22) we obtain

$$\lim_{n_1 \to 1} \frac{F'(n_1)}{G'(1-n_1)} = 0 = R,$$

that is, the nominal interest rate must be equal to zero. Because of perfect capital mobility, it implies a negative inflation rate of $1/(1+r^w) - 1$. Clearly, in the case that the world interest rate is zero, the optimal inflation tax is zero. ■

Because of the previous lemma, Friedman's rule optimize household's utility since they allocate their total endowment of labor to the production of consumable goods. In general, it is not necessarily true that $\lim_{n_1 \uparrow 1} G'(1-n_1) \to \infty$. However, if $G'' < 0$, then it is possible to determine the optimal inflation rate.

Lemma 3 *Let F and G satisfy the following: $F'(1)$ is bounded above, $G'' < 0$ and $\lim_{n_1 \uparrow 1} G'(1-n_1) < \infty$. Then the optimal inflation rate is given by*

$$\pi^* = \frac{1+R^*}{1+r^w}$$

where R^* satisfies

$$R^* = \lim_{n_1 \to 1} \frac{F'(n_1)}{G'(1-n_1)}$$

Proof. Under the conditions stated in the lemma, the limit

$$\lim_{n_1 \to 1} \frac{F'(n_1)}{G'(1-n_1)}$$

is well defined and finite. Satisfaction of the Fisher relationship yields the optimal inflation rate. ■

It is straightforward to find the necessary conditions that rule out explosive hyperinflations, i.e. when the inflation rate, and hence the nominal interest rate, becomes unbounded.

Lemma 4 *If* $G' \neq 0$ *and* $F' < \infty$, *explosive hyperinflation cannot occur.*

Proof. The conditions of the lemma imply that the nominal interest rate cannot be infinite, therefore, we can rule out equilibria with explosive hyperinflation. ■

An implication of the lemma is that countries experiencing explosive hyperinflation do not necessarily satisfy one of the conditions. Again, though monetary policy determines the steady state of the economy, the constraints imposed by the real sector of the economy cannot be ignored. Therefore, explosive hyperinflation, is both a monetary problem and a structural problem.

Finally, the following lemma establishes the necessary assumptions for the existence of a well determined relationship between inflation and output:

Lemma 5 *If for any given nominal interest rate there is a unique steady state, then there is a well determined long run relationship between inflation and output. In particular, this is true if the conditions of Lemma 4.1. are satisfied.*

VI. CONCLUDING REMARKS

We have developed a cash-in-advance constraint model of a small open economy with cash and credit goods. As in its closed economy counterparts, the calssical dichotomy breaks down. Accordingly, the interrelationships between monetary policy and the technological characteristics of the economy have an important bearing on: (1) the existence and uniqueness of equilibrium, (2) the optimal inflation rate, (3) the long run relationship between inflation and growth and (4) the conditions that rule out explosive hyperinflations.

We proceed to discuss some possible extensions of the model presented here. The analysis has been conducted under the assumption of perfect substitution among credit goods

and cash goods, with no trade. The analysis remains valid in the case of a economy with trade, where the imported good is a perfect substitute, purchasing power parity holds and the world inflation is zero. Nevertheless, it is worth exploring how the model's behavior changes when goods are imprefect substitutes.

The effect of inflation in domestic investment can be analyzed if capital stock accumulation is introduced in the model. In addition, high inflation is also associated with uncertainty. A full analysis requires extending the model to a stochastic setup and to relax the assumption of time-additive utility, since it forces the subjective discount rate to be equal to the inverse of the world interest rate.[3]

Finally, implicit in the analysis is the assumption that the inflation rate is identical to the devaluation rate of the domestic currency. Extensions of the model disentangling the tight link among these two quantities would shed more light about the effects of monetary policy in open economies.

[3] Mendoza(1991) discusses in detail the problems associated to time-additive utility functions in stochastic small open economy models.

A. Derivation of the Household's Optimality Conditions

The optimality conditions (18) and (19) are derived from the Lagrangian corresponding to the household maximization problem:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\Bigg\{U(c_t)-\lambda_t(c_t-c_{1t}-c_{2t})-\mu_t\left(c_{1t}-\frac{M_t+X_t}{p_{1t}}-\frac{B_t}{p_{1t}}+\frac{B_{t+1}}{p_{1t}(1+R_t)}\right)\\ -\gamma_t\left(c_{2t}\frac{p_{2t}}{p_{1t}}+\frac{M_{t+1}}{p_{1t}}-\frac{M_t+X_t}{p_{1t}}-\frac{B_t}{p_{1t}}+\frac{B_{t+1}}{p_{1t}(1+R_t)}-w_t+c_{1t}\right),\end{aligned}$$

where λ, μ and γ are the Lagrange multipliers associated with the constraints (3), (4) and (5). The first order conditions of this problem are given by:

$$U'(c_t)=\lambda_t, \tag{23}$$

$$\lambda_t=\mu_t+\gamma_t, \tag{24}$$

$$\lambda_t=\gamma_t\frac{p_{2t}}{p_{1t}}, \tag{25}$$

$$\frac{\gamma_t}{p_{1t}}=\beta\frac{\mu_{t+1}+\gamma_{t+1}}{p_{1t+1}}, \tag{26}$$

$$\mu_t+\gamma_t=\beta(1+R_t)(\mu_{t+1}+\gamma_{t+1})\frac{p_{1t}}{p_{1t+1}}, \tag{27}$$

and transversality conditions

$$\lim_{t\to\infty}\beta^t\lambda_t=0. \tag{28}$$

Equation (18) follows from replacing equations (23) and (24) in equation (25), while replacing (24), (25)) and (26) in (27) gives equation (19)

B. Equivalence of the Model with a Model with Traded Goods and Perfect Substitutability among Goods

The model described in the main body of the paper is equivalent to an economy with an additional traded good that is a perfect substitute of both the cash and credit goods. In this case, the constraints faced by the representative household are given by

$$c_t \leq c_{1t} + c_{2t} + c_{3t} + \tau_t, \tag{29}$$

$$p_{1t}c_{1t} + p^*_{3t}e_t c_{3t} + \frac{B_{t+1}}{(1+R_t)} \leq M_t + X_t + B_t, \tag{30}$$

$$p_{1t}c_{1t} + p_{2t}c_{2t} + p^*_{3t}e_t c_{3t} + M_{t+1} + \frac{B_{t+1}}{(1+R_t)} \leq M_t + X_t + B_t + W_t, \tag{31}$$

where c_3 is the amount of traded goods consumed, p^*_3 the foreign price of the good and e the exchange rate, measured in units of domestic currency per unit of foreign currency. Let λ, μ and γ be the Lagrange multipliers associated to equations (29), (30) and (31). The first order conditions of this problem are identical to (23)-(27), together with

$$\lambda_t = (\mu_t + \gamma_t)e_t \frac{p^*_{3t}}{p_{1t}},$$

is equivalent to the purchasing power parity relationship

$$e_t p^*_{3t} = p_{1t}$$

Assuming that the foreign price is fixed and equal to one, the behavior of this economy is the same as the one described in the main text, with an additional equation that determines the devaluation rate as a function of the price level p_1.

REFERENCES

Aiyagari, S.R., T. Braun and Z. Eckstein, 1995, "Transaction services, inflation, and welfare," Federal Reserve Bank of Minneapolis, Research Department, Working paper 551.

Aiyagari, S.R. and Z. Eckstein, 1996, "Interpreting monetary stabilization in a growth model with credit goods production," in *Financial Factors in Stabilization and Growth*, ed. by M. Blejer, Z. Eckstein, Z. Herkowitz and L. Leiderman (Cambridge, MA: Cambridge University Press)

Friedman, M, 1969, *The optimum quantity of money and other essays*(Chicago: Aldine)

Gillman, M, 1993, "The welfare cost of inflation in a cash-in-advance economy with costly credit," *Journal of Monetary Economics* 31, pp. 97-115.

Lucas, R.E., Jr., and N. Stokey, 1987, "Money and interest in a cash-in-advance economy," *Econometrica* 55, pp. 491-513.

Mendoza, E., 1991, "Real business cycles in a small open economy," *American Economic Review* 81, pp.797-818