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The Insurance Role of Social Security: Theory and Lessons for Policy Reform

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Abstract

This paper examines the impact of social security on welfare. The provision of social security reduces precautionary savings and encourages early retirement. Consequently, it lowers aggregate capital, employment, output, and consumption. On the other hand, it also provides old age insurance. This trade-off is examined using a life-cycle general equilibrium model. The paper finds that the current U.S. Social Security system can improve welfare even though the levels of aggregate output, employment, capital, and consumption fall relative to their levels without such a system. The welfare gains arise from insurance against living much longer than expected.

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SUMMARY

Pay-as-you-go public pension systems have been blamed for distorting saving and labor supply behavior. The current U.S. system, in addition, has a long-term financial imbalance, offers a low rate of return, and has low credibility—many young workers have little confidence that they will receive the benefits promised on retirement. Several proposals to reform the social security system have therefore been advanced.

Previous studies on social security and its reforms have neglected the welfare gains from old-age insurance that it provides. This paper overcomes this limitation and uses a life-cycle general equilibrium model with a large number of heterogeneous, risk-averse, rational agents to examine its impact. The agents in the model face life-span uncertainty, so they do not know how long retirement will last. They can save to provide for their own retirement, but there are no private annuity markets (this market is in reality thin). Hence, although all agents save rationally for retirement ex ante, those who ex post live much longer than expected can end up with little savings and will consume only minimal amounts. Social security insures against this outcome and can therefore raise welfare. On the other hand, social security eliminates the precautionary motive associated with uncertainty about longevity, and encourages early retirement. Individuals therefore save and work less, and aggregate savings, employment, and output fall. The model examines this trade-off, and shows that the welfare gains from old-age insurance can outweigh the welfare losses from lower output and savings.

The basic model is extended to examine various problems with the current social security system, and the results are then used to assess how proposed reforms might enhance or undermine the net value of social security.

I. Introduction

The U.S. Social Security system is primarily an unfunded pay-as-you-go system, in which payroll taxes of current workers are used to finance the benefits of current retirees.[2] As a result of the long-term decline in fertility rates and improvements in life expectancy, the number of retirees per worker, or dependency ratio, is expected to rise in the next decades (Figure 1).[3] This aging of the population compromises the financial balance of the pay-as-you-go system. Current projections indicate that Social Security will not be able to pay promised benefits on time starting in 2030. This long-term financial imbalance could be corrected by a 2.17 percent payroll tax increase. However, there is currently little support for such a tax raise, and other changes to the Social Security system are being considered.[4]

To date, the proposed reforms have been compared using the ratio of the present value of expected benefits to the present value of tax payments—the money worth ratio; see, for example, Gramlich (1996). Alternatively, they are evaluated by considering their impact on the *level* of output or aggregate savings. Kotlikoff et al. (1996), for example, use this approach to show that the privatization of Social Security could bring welfare gains. However, the authors admit that their paper "leaves as an important and open question the effect on risk sharing of privatizing Social Security."

The goal of this paper is to correct this deficiency and evaluate Social Security using a welfare measure which includes this insurance role.[5] The analysis in this paper provides a framework for evaluating the effectiveness of Social Security in providing insurance against

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[2]In this paper, Social Security refers only to retirement insurance, Old Age and Survivor Insurance (OASI). Disability and hospital insurance are not considered.

[3]The sharp anticipated rise in the dependency ratio starting in 2010 reflects the baby bust (from the Great Depression to World War II) followed by the post-World War II baby boom.

[4]One of the main tasks of the Social Security Administration Advisory Council (1997) was to make recommendations to eliminate the long-term deficit without the 2.17 percent tax increase. The council could not agree on one single approach and instead presented three different proposals. These plans are discussed in section IV.

[5]Previous studies of Social Security which address this insurance element are: Abel (1987), who finds that fully funded Social Security can improve welfare even when there is an annuity market provided agents are sufficiently risk averse; Hubbard and Judd (1987) who find that Social Security does not raise welfare when young workers face borrowing constraints; and more recently, Diamond (1996), who discusses this insurance aspect and suggests results similar to those presented in this paper, but makes no formal analysis of Social Security insurance.

old age, and is useful in gauging how proposed reforms might enhance or undermine this insurance role.

The fact that agents cannot insure completely against the uncertain time of death because of the lack of annuity markets implies that markets are incomplete. Hence, the representative-agent approach cannot be used. Instead micro-economic behavior is modeled directly and the behavior of many agents is simulated. Aggregate variables and social welfare functions can be then computed from these simulations. A further advantage of using such a rich model is that it can also be used to examine the effect of Social Security on the *distribution* of wealth and consumption.

The outline of this paper is as follows. In the next section, I present the basic model, describe how it is solved, and calibrate the model parameters. In section III, I first discuss the solution to the basic model, paying particular attention to the distortion of saving behavior and the net welfare impact of Social Security, and then consider four extensions of the basic model. In section IV, the reforms proposed by the Social Security Administration Advisory Council (1997) are briefly presented and evaluated using the results from the basic model and its extensions. Section V concludes.

II. A Model of Social Security with Insurance

A life-cycle general equilibrium model with a large number of heterogeneous, risk-averse, rational agents is presented in this section. The agents in the model face life-span uncertainty, so they do not know how long retirement will last. They can save to provide for their own retirement but there are no private annuity markets (in reality this market is thin[6]). Hence, although all agents save appropriately (rationally) for retirement *ex-ante*, those who *ex-post* live much longer than expected can end up with little savings and will consume only minimal amounts. Social Security provides insurance against this outcome and can therefore raise welfare. On the other hand, Social Security distorts saving behavior because it reduces the need to provide for one's own retirement. Therefore, individuals save less and aggregate savings and output fall when Social Security is available. The model examines this tradeoff.[7]

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[6]Adverse selection, which drives premiums up, is often cited as the main reason for this. Kotlikoff et. Al. (1996) also mention that inflation risk may be responsible for why these "annuities are a rarity in today's market."

[7]An extension of the model in section III considers the additional distortion of Social Security on the supply of labor.

A. The Model Economy

The model economy is populated by a large number of identical firms, by many households of mass one,[8] and by a government whose sole function is to run a Social Security program. The households are infinitely lived dynasties or families. However, these households go through life-cycles from age one up to at most M.[9] Let N_j be the fraction of households of age j, and s_j be the probability of a household surviving to age $j+1$ conditional on surviving up to age j. These survival probabilities are exogenously determined and imply a unique age distribution since,

$$N_{j+1} = s_j N_j \tag{1}$$

and,

$$\sum_{j=1}^{M} N_j = 1 \tag{2}$$

Figure 2 shows the survival probabilities for the United States for 1992. These are computed from the fraction of people who die at a given age. The implied and actual age distributions are shown in Figure 3. The shape of the implied distribution differs mostly from the actual one for ages under 50. This reflects the effect of time-varying birthrates which is absent from the model.

The Production Sector

Since the firms are all identical, it suffices to consider the behavior of a single representative firm. This representative firm produces the only good of this economy, which can be used for consumption or investment. The good is produced according to the Cobb-Douglas production function:

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[8]In this paper I neglect population growth. Hugget (1996) shows how a constant rate of population growth can be incorporated into the analysis. However, the policy tradeoffs investigated in this paper are independent of such population growth.

[9]The household's age can be thought of as the age of the head of the family. A person becomes the head of the family at the beginning of the period in which he has age one and remains the head until he dies (at most M years later). The next head of the dynasty, who has age one, assumes control of the family at the beginning of the following period. This sequence of events continues indefinitely.

$$f(K,N) = K^{\alpha} N^{1-\alpha} \quad with \ \alpha \in (0,1) \tag{3}$$

Here K and N are capital and labor inputs, respectively, and α is the capital's share of output. The firm rents capital from the households and hires labor. Assuming competitive behavior, the firm's profit maximization problem is:

$$\max_{K,N} f(K,N) - wN - rK - \delta K \tag{4}$$

where r is the rental rate of capital, w is the wage, and δ is the depreciation rate of capital. The necessary first order conditions for profit maximization yield the following equations for the factor prices:

$$\begin{aligned} r &= \alpha \left(\frac{K}{N}\right)^{\alpha-1} - \delta \\ w &= (1-\alpha)\left(\frac{K}{N}\right)^{\alpha} \end{aligned} \tag{5}$$

Social Security

The government administers a Social Security program which provides income for retired households. In this model, households work from ages 1 up to age ρ (unless they die before reaching age ρ). That is, $\rho + 1$ is the age of retirement. Although workers can accumulate wealth to provide income for retirement, there is no market for annuities or private pensions. The only pension available is that provided by the government's Social Security system. This program is financed by payroll taxes with a uniform tax rate τ. Therefore, working households receive after-tax earnings from labor of $w(1-\tau)$. Also, the gross replacement rate is η, so a retired household receives a Social Security benefit of ηw. The condition for balance of the Social Security budget is therefore:

$$w\tau \sum_{j=1}^{\rho} N_j = \eta w \sum_{j=p+1}^{M} N_j \tag{6}$$

The left-hand side is total tax revenue from working households and the right-hand side is the total amount of retirement benefits paid out. From equation (6), it is clear that the tax rate is:

$$\tau = \eta \cdot \; (\textit{dependency ratio}) \tag{7}$$

where the *dependency ratio* is the ratio of the number of retired households to the number of working households.

The Households

Households can only work up to age ρ. Once they are retired, their only earnings are from their accumulated wealth and from Social Security payments. Therefore, the problem of a household with age j can be expressed as:

$$\begin{aligned} V_j(k) &= \max_{k'>0}\{U(c) + \beta[s_j V_{j+1}(k') + (1-s_j)V_1(k')]\} \\ & s.t. \; c = (1 + r)k + w_j - k' \end{aligned} \tag{8}$$

for $j = 1,...,M$. Note that since the maximum age is M then $s_M = 0$. $\beta \in (0, 1)$ is the discount factor. The factor prices, r and w, and the lifetime labor earning profile, w_j, are taken as given by the household. The value functions, $V_1(\cdot),... V_2(\cdot),... V_M(\cdot)$, are indexed according to age and are functions of current period wealth, k. Households solve their problems by choosing the following period's wealth, k', which determines current period consumption, c. Note that k' is restricted to be strictly positive so no borrowing is allowed. Also, since labor earnings depend on age, j, the lifetime labor earnings profile, w_j, is:

$$w_j = \begin{cases} (1 - \tau)w & \textit{for } j \leq \rho \\ \eta w & \textit{for } \rho < j \leq M \end{cases} \tag{9}$$

It is important to note the strong inter-generational link from equation (8), as all households, regardless of age, care about the well-being of the next generation. This is what the right hand side term with $V_1(\cdot)$ represents. This aspect of the model is very different from the assumption used in the Auerbach-Kotlikoff type life-cycle model, in which generations are not linked in this way. The strong inter-generational link in the model presented here magnifies the impact of life-span uncertainty because outliving one's resources means not only lowers consumption in old age, but also results in smaller bequests, which affects the well-being of the children.

One would therefore anticipate that the gains from social security in this model, would be larger than the gains found using an Auerbach-Kotlikoff type model.[10]

To completely specify the model, the utility function, $U(\cdot)$, is assumed to be of the constant relative risk aversion type:

$$U(c) = \begin{cases} \dfrac{c^{1-\sigma}}{1-\sigma} & when\ \sigma \neq 1 \\ \log(c) & when\ \sigma = 1 \end{cases} \tag{10}$$

The solution to the functional equations in (8) yields the following M saving and consumption rules:

$$\begin{aligned} k' &= g_j(k) \\ c &= h_j(k) \end{aligned} \tag{11}$$

for $j = 1,\ldots,M$.

B. Model Solution

In this sub-section, I first define a steady state equilibrium and then describe how to solve for it.

Definition: A *steady state equilibrium* is a set of prices, r and w, saving functions, $g_1(.),\ldots,g_M$, consumption functions, $h_1(.),\ldots,h_M(.)$, tax rate, τ, replacement rate, η, and aggregate quantities, Y, I, C, K, and N, such that:

(i) r and w solve the representative firm's maximization problem, (4), and are given by (5).

(ii) $g_1(.),\ldots,g_M(.)$ are the saving functions that solve the functional equations in (8) taking r and w as given.

(iii) $h_1(.),\ldots,h_M(.)$ are the consumption functions that solve the functional equations in (8) taking r and w as given.

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[10]It would be interesting to replace the term $(1-s_j)V_1(k')$ in (8) by $\xi(1-s_j)V_1(k')$ where $\xi \in (0,1)$. Here, ξ measures the strength of the inter-generational linkage. $\xi=1$ is assumed in this paper, and $\xi=0$ is assumed in Auerbach-Kotlikoff models.

(iv) K is aggregate capital and is consistent with the households' saving functions:

$$K = \int_{\Omega_1} g_1(k^i) + \int_{\Omega_2} g_2(k^i)di + \ldots + \int_{\Omega_M} g_M(k^i)di \tag{12}$$

where k^i is the wealth of the i^{th} household, and Ω_j is the set of all households of age j.

(v) C is aggregate consumption and is also consistent with the households' consumption functions:

$$C = \int_{\Omega_1} h_1(k^i)di + \int_{\Omega_2} h_2(k^i)di + \ldots + \int_{\Omega M} h_M(k^i)di \tag{13}$$

(vi) N is aggregate employment and is given by $N = \sum_{j=1}^{\rho} N_j$, where N_j is the fraction of the population of age j, and ρ is the age of the oldest working households.

(vii) I is aggregate investment and is given by: $I = \delta K$.

(viii) Y is aggregate output and is given by: $Y = K^{\alpha}N^{1-\alpha}$.

(ix) τ is the tax rate given by (6) or (7), and it is the rate which balances the Social Security budget constraint for a given replacement ratio, η.

(x) The aggregate resource constraint is satisfied: $Y = C+I$.

Numerical methods are used to solve for this equilibrium using the following steps:

1. Obtain the age distribution implied by the survival probabilities $(s_1,\ldots,s_M)$. The maximum working age, ρ, then implies a unique value of aggregate labor supply, N.

2. Assume a given wealth distribution. This distribution implies a particular value for aggregate capital, K^*. This value of K^* and N can now be used to obtain a set of factor prices and lifetime labor earning profile using (5) and (9).

3. The household's problem, (8), is solved by numerical value function iteration to obtain the saving and consumption decision rules.

4. The behavior of a large number of households (more than 10,000) is simulated for many periods. The actual value of K and the wealth distribution can be obtained from these simulations.

5. If the actual value of K is close to K^* and the actual wealth distribution is close to the assumed one, then an equilibrium has been found; otherwise repeat steps 2 to 4.

C. Calibration

To compute the steady state variable values, one needs to first choose parameter values for α, β, δ, and σ. These preference and technology parameters are drawn from the real business cycle literature; see, for example, Prescott (1986). The time period is one year. Therefore, the discount factor is set to $\beta = 1.04^{-1}$, the annual capital depreciation rate is 10 percent ($\delta = 0.1$), the capital's share of output is $\alpha = 0.36$, and the coefficient of relative risk aversion is set to one ($\sigma = 1$).

Agents go through life cycles from ages 1 (real-life age of 20), up to a maximum of M=75 (real-life age of 94). Households work up to age ρ=45, so the real-life retirement age is 65. The survival probabilities are obtained, as mentioned above, from expected death data for 1992 and are shown in Figure 2.

The replacement rate is set equal to the replacement rate of average earning individuals, η = 43 percent.[11] This benefit level requires a tax rate of τ = 13.25 percent to balance the Social Security budget. This tax rate is higher than the current tax rate of 10.52 percent for OASI, but is closer to the 13 percent rate which would bring the current system into long-term balance (Social Security Administration Advisory Council (1997)).

III. The Insurance Role of Social Security

A. The Basic Model

In this sub-section I use the basic model to examine how Social Security affects individual household saving behavior. Since aggregate variables can be obtained by summing over all household-level variables, the model also has implications for the effect of Social Security on aggregates. In addition, the net welfare impact of Social Security can be computed from individual-level variables using a social welfare function.

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[11]In practice, the replacement rate varies with retirement age and life-time earnings, rising with the former and falling with the latter. Myers (1993), Table 2.10, reports replacement rates ranging from 24 percent to 58 percent for 1990.

Savings

I start by discussing the effect of Social Security on the saving behavior of individual households, and then look at the quantitative impact on aggregate savings.

First, consider the case of no Social Security ($\eta = \tau = 0$). Sample time paths for wealth and consumption for a single household are shown in Figure 4. Households accumulate wealth during most of their years of employment. They dissave largely during retirement, but also dissave a small amount in their early working years, particularly if they inherited substantial wealth. It is evident that the swings in wealth are larger than those of consumption, since wealth is used to keep consumption as smooth as possible. As households approach retirement, the saving rate increases and consumption drops. This occurs because households discount future consumption and would rather consume more early on. Consequently, they delay accumulating wealth until near retirement. During retirement, consumption drops at a slightly faster pace due to the labor income loss.

Sample time paths of an individual household's wealth and consumption are shown in Figure 5 when Social Security is provided with a 43 percent replacement rate. In this case, the lifetime earnings profile of the household is more even; see equation (9). Hence, the need to use wealth to provide for retirement and smooth the stream of consumption is reduced. This has two effects. First, the fluctuations in household's wealth are smaller and, second, the average wealth holdings are also smaller because of the reduced precautionary saving motive.

The reduced need for precautionary savings leads to lower wealth accumulation in the economy as a whole and reduces the capital stock. Also, since capital is the only variable input to production (aggregate employment is exogenous and fixed), then output and aggregate consumption are also reduced. By comparing the first two columns of Table 1 one observes the quantitative effect of Social Security on capital and other aggregate variables.[12]

If only aggregate variables were used to evaluate Social Security, then one would conclude that Social Security lowers welfare. However, these variables are unsatisfactory measures of welfare because level comparisons neglect the *insurance* value of Social Security. One advantage of the approach used in this paper, it that measures of welfare which incorporate this insurance role can be used. I perform this welfare evaluation of Social Security in the following sub-section.

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[12]These results agree, at least qualitatively, with the evidence and arguments presented in Feldstein (1974), (1996a). He argues that the distortions due to Social Security discourage savings substantially. Feldstein (1996b), for example, calculates that Social Security reduced national savings by 62 percent. The empirical evidence is, however, inconclusive on this point. The results of Leimer and Lesnoy (1982) suggest, for example, that Social Security may not have any significant effect on aggregate savings.

Welfare

In the model, agents of the same age are identical *ex ante* since they have identical preferences. However, they are heterogenous *ex post* because they experience different histories. Some dynasties will experience life-cycles with long lives, whereas others will experience life-cycles with short lives. The latter households accumulate more wealth than the former ones.[13] Hence households of the same age differ by wealth holdings, and the model steady state is described by a *distribution* of wealth. Since Social Security affects the saving decision of the households, it also affects this wealth distribution.

The impact of Social Security on the distributions of wealth, consumption and utility are shown in Figure 6, 7, and 8, respectively.[14] In contrast, the utility distribution in Figure 8 is the steady state distribution of the discounted lifetime utilities of all the agents in the economy and therefore gives a measure of *ex ante* welfare for this economy.[15]

As can be observed from comparing the plots in Figure 6, there are fewer households with low levels of wealth when Social Security is provided. This reflects the transfer that Social Security provides to retired households and is particularly important to the households that have lived many years and have run their wealth down. That is, it is particularly important for the *poor and elderly*.

The assistance provided to the old with little wealth is also noticeable in Figures 7 and 8. When there is no Social Security, there is a long lower tail in the consumption distribution due to the few households who live for many years and have used up most of their wealth and consume little. On the other hand, when Social Security is available this lower tail disappears

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[13]This is a problem with this basic model since households who experience shorter life-cycles and who experience deaths more frequently will tend to be better off. A model extension in Section III overcomes this problem by recognizing the value of leisure and that of longevity.

[14]Although the goal of this model is not to explain the wealth distribution, Hugget (1996) shows that a similar model can match features of the observed U.S. wealth distribution.

[15]Define the utility for the i^{th} household as:

$$\underline{U}^i = \sum_{t=0}^{\infty} \beta^t U(c_t^{\,i})$$

The distribution of utility is obtained from the simulations of a larger number of households who experience different histories. Enough periods are simulated so that $\underline{U}^i$ converges for all agents. Figure 8 shows the distribution of the $\underline{U}^i$'s.

since the very old are being protected. The same argument carries over to the effect of Social Security on utility. The clipping of the lower tail of the utility distribution is associated with an increase in the average utility and therefore with an increase in welfare. Note that average utility is the measure of welfare when a utilitarian social welfare function is used (every agent receives an equal weight).[16]

Changes in the first two moments of the wealth, consumption, and utility distributions due to Social Security are given in Table 2. These figures show how much average wealth and consumption fall while average utility rises. They also indicate the degree to which inequalities are reduced. In addition, the net value of Social Security is reported in the last three rows of the table. These calculations indicate that households would willingly give up 4.5 percent of aggregate consumption in order to have Social Security. Using 1995 data, this means that Social Security is worth $221.7 billion.

The main result of this sub-section is that Social Security *can improve welfare even though the levels of aggregate output, capital, investment, and consumption fall.* This occurs because the benefits from insurance against old-age poverty are higher than the losses due to lower consumption levels (which in turn is due to lower aggregate savings). This illustrates how the use of the money-worth ratio or of any other measure which ignores the value of insurance can be misleading in evaluating Social Security. The model also indicates that the source of these welfare gains is the protection provided to the very old.

Optimal Insurance

The welfare gains arise because Social Security completes the missing annuity market. Further welfare gains are possible if Social Security completes this market 100 percent, and provides full insurance against the loss of income during retirement. That is, if Social Security benefits are exactly equal to the after tax labor earnings of a working household,

$$(1 - \tau)w = \eta w \tag{14}$$

For the economy at hand, a tax rate of $\tau = 23.56$ percent and a replacement rate of $\eta = 76$ percent, satisfy this condition. This Social Security program results in a constant lifetime earnings profile, and yields the highest possible welfare gains.

Figure 9 shows the value of Social Security for different replacement rates, and confirms that the highest gains are obtained when $\eta = 76$ percent. Moreover, in this case the steady state level of capital is that given by the modified golden rule, and the rate of return on capital equals the rate of time preference. In this full-insurance case, individual uncertainty has

[16]The methodology used in this paper is not restricted to this welfare function, and other social welfare functions could easily be used.

been eliminated, all agents are identical (the wealth distribution is degenerate), and the model economy could be described by a complete market, representative agent model.[17]

The results of this sub-section should be interpreted with care because many distortions are absent from the model. In practice, it is likely that the distortions introduced by such a generous program would be extremely high. Particularly problematic are the disincentives on the labor supply that this program would create. Therefore, the results of this sub-section should not be interpreted as an argument for increased generosity of Social Security. Rather, the discussion is included here to help understand the sources of welfare gains in the model.

B. The Failure of Private Insurance

In this sub-section I examine whether private retirement insurance could deliver similar welfare gains to those computed above.[18] If private insurance were effective, then one could argue that there is no need for the current public program, and that perhaps the latter may be limiting the availability of private insurance. To explore this issue, the basic model is modified by replacing Social Security with a model of private retirement insurance.

The goal of this sub-section is, however, not to examine thoroughly how private retirement insurance schemes fail (we know they do), but rather to *illustrate* why this might be so. Therefore, the model of private insurance used here is admittedly simple and differs in several aspects from actual private insurance schemes. One of the main differences is that private insurance in the model is financed on a pay-as-you-go basis rather than being fully funded. Another difference, is that only young households are allowed to purchase insurance. That is, households must decide at a young age whether to participate in the insurance program, and if they choose not to purchase insurance then, they will not have another chance to buy insurance later on in life. Specifically, assuming that only agents of age 1 can choose whether to purchase insurance, the problem of a household of age 1 becomes,

$$V_1(k) = \max_{I\in\{0,1\}}\{Vinsured(k),\ Vuninsured(k)\} \tag{15}$$

where,

[17]This result is consistent with that in Aiyagari (1995) who shows that in an economy with uninsurable idiosyncratic risk, capital taxation could increase welfare because it reduces the capital stock to its modified golden rule level.

[18]Private insurance is characterized by being actuarially fair and voluntary.

$$\begin{aligned} Vinsured(k) &= \max_{k'>0}\{U(c) + \beta[s_j V_2(k') + (1-s_j)V_1(k')]\} \\ & s.t.\ c = (1+r)k + w_j - k' \end{aligned} \tag{16}$$

is the value of being insured, and

$$\begin{aligned} Vuninsured(k) &= \max_{k'>0}\{U(c) + \beta[s_j \tilde{V}_2(k') + (1-s_j)V_2(k')]\} \\ &= s.t.\ c = (1+r)k + \tilde{w}_j - k' \end{aligned} \tag{17}$$

is the value of being uninsured. The household chooses whether to buy insurance *($I = 1$)* or not *($I = 0$)*, and how much wealth to hold next period, k'. Hence, the solution to (15) is a rule on the insurance selection, $I = i(k)$, a saving function, $k' = g(k)$, and a consumption function, $c = h(k)$.

The problem of an insured household with age $j > 1$ can be expressed as:

$$\begin{aligned} V_j(k) &= \max_{k'>0}\{U(c) + \beta[s_j V_{j+1}(k') + (1-s_j)V_1(k')]\} \\ & s.t\ c = (1+r)k + w_j - k' \end{aligned} \tag{18}$$

for $j = 2,\ldots,M$. Similarly, the problem of an uninsured household with age j can be expressed as:

$$\begin{aligned} \tilde{V}_j(k) &= \max_{k'>0}\{U(c)+\beta[s_j \tilde{V}_{j+1}(k')+(1-s_j)V_1(k')]\} \\ & s.t.\ \ c=(1+r)k+w_j-k' \end{aligned} \tag{19}$$

for $j = 2,\ldots,M$. Here $V_j(k)$ and $\tilde{V}_j(k)$ are the value functions of insured and uninsured households of age j, respectively. Note that the model consists now of $2M - 1$ functional equations, instead of M equations for the basic model.

The lifetime earning profiles of insured households is:

$$w_j = \begin{cases} (1 - \tilde{\tau})w & \text{for } j \le \rho \\ \tilde{\eta}w & \text{for } \rho < j \le M \end{cases} \tag{20}$$

where $\tilde{\tau}$ is the premium $\tilde{\eta}w$ is the insurance payout. The profile for uninsured households is simply:

$$w_j = \begin{cases} w & \text{for } j \le \rho \\ 0 & \text{for } \rho < j \le M \end{cases} \tag{21}$$

Assuming a competitive insurance industry, the zero profit condition for the private insurance sector is:

$$\tau \sum_{j=1}^{\rho} N_j = \tilde{\eta} \sum_{j=\rho+1}^{M} N_j \tag{22}$$

where N_j is now interpreted as the proportion of insured households of age j. This condition implies that this private insurance scheme is actuarially fair, since total payments equal total benefits paid out at the aggregate level. [19]

An equilibrium can be defined in an analogous way to that in the previous section. Before attempting to solve for this equilibrium, it is useful to examine the insurance decision function, *i(k),* when all equilibrium conditions are satisfied except for (22). This function is shown in Table 3. For very low premiums, all households would purchase insurance. As the premium is raised, the wealthier households choose not to buy insurance. The fraction of households who do not get insured increases as the premium rises. Hence, the model exhibits *adverse selection* since those who are less likely to need insurance do not purchase insurance.[20] When the premium reaches 4 percent no household buys insurance. In fact, no households purchase insurance when the premium is raised to the level needed to satisfy (22). Therefore, *actuarially fair insurance would not be willingly purchased. In other words, the only equilibrium level of private insurance is no insurance at all.*

Figure 10 illustrates why agents in the model choose not to buy insurance. The value of insurance is compared to the value of being uninsured at various ages. At real-life age 20 (top panel in Figure 10) all households value no insurance over insurance and therefore prefer

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[19]A more realistic model would make payments and benefits dependant on individual variables as well.

[20]For a discussion of adverse selection see Schulz (1995).

to remain uninsured. Households of age 41 (middle panel in Figure 10)differ in their preference for insurance depending on their wealth. Only the least wealthy households would prefer to be insured. The bottom panel of Figure 10 shows that at age 49 all households prefer to be insured regardless of their wealth. Therefore, private insurance is not chosen in the model because only the young are allowed to purchase insurance.[21]

There are three reasons why younger households value no insurance over insurance. First, because households discount future consumption, they prefer to consume more early on, and can do so when they are uninsured since they do not pay the premium. Second, as households age, time-of-death uncertainty is resolved. Younger workers may die before ever reaching retirement, so they have a risk of paying for insurance that they might not end up needing. This risk is reduced as households near retirement. Third, the value of retirement benefits relative to the total contributions increases as households get older. This occurs in the model because total expected benefits are independent of age, but total contributions decrease as households age since older households have fewer years left to retirement (benefits and contributions do not depend on age in the model). In practice, total expected benefits would be closely tied to contributions according to the individual's age under a private insurance scheme. However, this is not likely to overturn the results of this section, because the model makes insurance a better buy than is likely to be in practice. Hence, private insurance might be even less appealing in reality than in the model. [22]

The main result of this sub-section was to show that private insurance may face problems so serious as to make it unfeasible in practice. I examined only two problems, adverse selection and restriction in the timing of insurance purchase, and found that the problems were so severe that no private insurance would be purchased.[23] In reality there are additional informational problems with private insurance because the purchasers of insurance are likely to have more information about their expected life span that insurance companies. *The model suggests that even when there are no such informational asymmetries, adverse selection can be so severe as to prevent the existence of widely available private retirement insurance.* Therefore, the fact that private insurance markets are thin is probably not due to the existence of Social Security. This suggests that Social Security is probably not crowding

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[21]Note that this is still an incomplete market model since insurance sales are severely restricted in the model.

[22]This result is dependant on the way that private insurance is modeled here, as well as on the parametrization of the model. A full study of the failure of private insurance schemes is beyond the scope of this paper.

[23]Incidentally, Social Security has the additional advantages over private pensions in that it is fully portable from one job to another and it protects workers who change jobs before they are vested in private pension plans.

out private insurance, and the welfare gains from the Social Security program computed in the last section are probably not overstated.

C. Old-Age Insurance Instead of Retirement Insurance

In this subsection I consider the effect of raising the age at which benefits are collected. This is important since the welfare gains arise because of insurance against old age, not against retirement.

To examine this question, I modify the basic model as follows. Suppose that agents still retire at age $\rho = 65$, but can only collect benefits when they reach age λ $(\lambda > \rho)$. Therefore, households have to take care of themselves from age 65 to λ. Hence this insurance scheme has a *deductible*, and Social Security provides protection only against a catastrophe (living *much longer* than expected).

In Table 4, I present the welfare calculations for different values of λ. Note that raising λ from 65 to 75 lowers the value of Social Security by only 13 percent. That is, *most of the welfare gains can be obtained with less than half the current Social Security taxes if benefits are paid only to those above age 75.* Given current life-span expectancies (see Figure 11), approximately 65 percent of all households would expect to receive some benefits.

In Figure 8, I showed the distribution of utility with and without Social Security. The effect of retirement insurance was to clip the lower tail of the distribution. Old-age insurance, on the other hand, does not eliminate this lower tail, it only makes it thinner. This is because there are still a few households who run their wealth down in the ten years from retirement until the time when they collect benefits at age 75. Evidently, the welfare gains from the complete elimination of this lower utility tail are not very large.

D. Low Confidence in Social Security

This extension of the basic model is relevant for today's policy debate. The Social Security Advisory Council (1997) has identified the unprecedented low level of confidence in the system as one of the main problems with the program. Many believe that they will receive substantially reduced benefits and will have a large negative return to their tax contributions. There is, therefore, a perceived risk that the government will default on promised benefits. In practice this default can occur if the government fails to adjust benefits correctly for price increases, or if it fails to adjust tax brackets for inflation (so real taxes of benefits increase over time), or if it decides to means-test benefits in the future.[24]

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[24]This last option has been discussed by The Bipartisan Commission on Entitlements and Tax

(continued...)

A simple way of incorporating this risk of benefit default into the basic model is by assuming that a fraction, ψ, of the households will not receive any retirement benefits in spite of having paid taxes during their working years. In reality, workers may fear only a partial default and the risk is likely to vary across agents (e.g., wealthier households may face a higher risk if benefits become means-tested in the future). Nevertheless, this simple model modification is useful to examine how default risk affects the value of Social Security.

Suppose that households find out at age ρ whether they will receive benefits or not. If benefits are received at age ρ then they will be received until death; otherwise no benefits will be received at all until death. Then, letting $V(\cdot)$ Stand for the value of receiving the promised benefits, and $\tilde{V}(\cdot)$ stand for the value when suffering a default, the problem of a household of age $j = \rho$ can be expressed as,

$$V_j(k) = \max_{k'>0}\{U(c) + \beta s_j[1 - \psi)V_{j+1}(k') + \psi\tilde{V}_{j+1}(k')] + \beta(1 - s_j)V_1(k)\} \quad s.t.\ c = (1 + r)k + w_j - k' \tag{23}$$

The problem of households who do not suffer a default is:

$$V_j(k) = \max_{k'>0}\{U(c) + \beta[s_j V_{j+1}(k') + (1 - s_j)V_1(k)]\} \quad s.t.\ c = (1 + r)k + w_j - k' \tag{24}$$

where the lifetime earning profile, w_j, is,

$$w_j = \begin{cases} (1 - \tau)w & for\ j \le \rho \\ \eta w & for\ \rho < j \le M \end{cases} \tag{25}$$

Similarly, the problem for households who suffer the default is:

$$\tilde{V}_j(k) = \max_{k'>0}\{U(c) + \beta[s_j\tilde{V}_{j+1}(k') + (1 - s_j)V_1(k)]\} \quad s.t.\ c = (1 + r)k + \tilde{w}_j - k' \tag{26}$$

[24](...continued)
Reform.

with a lifetime earnings profile of $\tilde{w}_j$,

$$w_j = \begin{cases} (1 - \tau)w & \text{for } j \leq \rho \\ 0 & \text{for } \rho < j \leq M \end{cases} \tag{27}$$

The problem of households of ages 1 through ρ - 1 is also given by (24).

Since the government defaults on a fraction ψ of its obligations, the budget constraint of the Social Security program is now:

$$\tau \sum_{j=1}^{\rho} N_j = (1 - \psi) \cdot \eta \sum_{j=\rho+1}^{M} N_j \tag{28}$$

Equation (28) shows that the possibility of default lowers the dependency ratio by a factor of $(1 - \psi)$, and that defaults can be used to restore balance to the Social Security budget. This is precisely why defaults might be expected when there are financial imbalances, as is the case today.

The results for different default probabilities are given in Table 5. As expected, the risk of default undermines the insurance value of Social Security. There are two main reasons for this. First, given this default risk, households save more and consume less; consequently welfare is lowered. Second, some households who would benefit from Social Security (i.e., those households who have lived much longer than expected) suffer defaults.

It is interesting to note that lowering the probability of default has a more than proportional impact on raising welfare. Therefore, there are sizable welfare gains from raising the credibility of the system, and *there is scope for raising welfare while cutting benefits if the system becomes more credible in the process.*

E. Early Retirement and the Value of Leisure

The analysis so far has only considered the distortion of Social Security on savings. However, the provision of Social Security also distorts the supply of labor by encouraging early retirement;[25] see, for example, Feldstein (1974). Here, I modify the basic model to allow for early retirement and make the decision to retire early endogenous. I find that since Social

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[25]The social security tax also distorts the labor decision of workers, particularly low-wage earners. However, I ignore this effect in this discussion, and concentrate only on the decision to retire early.

Security subsidizes retirement, the labor supply decreases and the welfare losses also rise. However, there are still net welfare gains from having Social Security.[26]

To illustrate the potential impact of Social Security on labor supply, I present changes in labor force participation rates for males and females in the top panel of Figure 12. The decline in the labor force participation rate of males is more pronounced for those between ages 55 and 64 (lower panel in Figure 12). In fact, the labor force participation rate for males in this age group fell 17.5 percent from 1970 to 1994.[27] Arguably, part of this drop has been due to the availability of Social Security.

I now extend the basic model by making the decision to retire early endogenous. Specifically, suppose that agents have the option of retiring at age $\rho - \phi$, where $\phi > 0$. If a household chooses to retire, then it remains out of the labor force until death. If instead it chooses not to retire, then it remains employed until age ρ. As before, all households aged $\rho + 1$ or older are retired. In order to make the retirement decision meaningful leisure must now have some value. This is incorporated by assuming the following form of the utility function:

$$\tilde{U}(c,l) = U(c) + v \cdot (1 - l) \quad \textit{where } v > 0$$

where $U(\cdot)$ is given by (10), l is 1 when the household works and 0 otherwise. The parameter v is a measure of the value of leisure and it determines the number of households who choose to retire early when Social Security is provided.

The problem of households aged $j = 1,\ldots,\rho - \phi - 1$ is:

$$\begin{aligned} V_j(k) &= \max_{k'>0}\{\tilde{U}(c,1) + \beta[s_j V_{j+1}(k') + (1 - s_j)V_1(k)]\} \\ & s.t.\ c = (1 + r)k + w_j - k' \end{aligned} \qquad (30)$$

with the same notation as before. The interesting problem is that faced by households of age $j = \rho - \phi$,

[26]The measure of welfare used in this case reflects the value of leisure.

[27]The Statistical Abstract of the U.S. (1995), Table 627.

$$V_j(k) = \max_{l \in \{0,1\}}\{Vretire(k), Vwork(k)\} \tag{31}$$

where *Vretire(k)* is the value of retiring early $(l = 0)$, and *Vwork(k)* is the value of working $(l = 1)$. These expressions are given by,

$$\begin{aligned} Vwork(k) &= \max_{k'>0}\{\tilde{U}(c,1) + \beta[s_j V_{j+1}(k') + (1 - s_j)V_1(k)]\} \\ & s.t.\ c = (1 + r)k + w_j - k' \end{aligned} \tag{32}$$

and,

$$\begin{aligned} Vretire(k) &= \max_{k'>0}\{\tilde{U}(c,0) + \beta[s_j V_{j+1}(k') + (1 - s_j)V_1(k)]\} \\ & s.t.\ c = (1 + r)k + \tilde{w}_j - k' \end{aligned} \tag{33}$$

Note that $\tilde{V}(\cdot)$ is the value function when the household retires early. The solution to (31) gives not only saving and consumption but also a decision for early retirement:

$$l = R(k) \tag{34}$$

Households who choose to remain in the labor force solve the problem,

$$\begin{aligned} V_j(k) &= \max_{k'>0}\{\tilde{U}(c,1) + \beta[s_j V_{j+1}(k') + (1 - s_j)V_1(k')]\} \\ & s.t.\ c = (1 + r)k + w_j - k' \end{aligned} \tag{35}$$

and those that retire early solve, .

$$\begin{aligned} \tilde{V}_j(k) &= \max_{k'>0}\{\tilde{U}(c,0) + \beta[s_j \tilde{V}_{j+1}(k') + (1 - s_j)V_1(k')]\} \\ & s.t.\ c = (1 + r)k + \tilde{w}_j - k' \end{aligned} \tag{36}$$

Equations (35) and (36) apply for households with age $j = \rho - \phi + 1, \ldots, \rho - 1$.

A household of age $j = \rho$ who chooses early retirement solves,

$$
\begin{aligned}
\tilde{V}_j(k) &= \max_{k' > 0} \{ \tilde{U}(c,1) + \beta [s_j V_{j+1}(k') + (1 - s_j) V_1(k')] \} \\
& s.t. \ c = (1 + r)k + \tilde{w}_j - k'
\end{aligned}
\tag{37}
$$

and a household of the same age who stayed in the labor force solves,

$$
\begin{aligned}
V_j(k) &= \max_{k' > 0} \{ \tilde{U}(c,1) + \beta [s_j V_{j+1}(k') + (1 - s_j) V_1(k')] \} \\
& s.t. \ c = (1 + r)k + w_j - k'
\end{aligned}
\tag{38}
$$

All households who are $\rho + 1$ years old or older solve the same problem as in (30). Finally, the lifetime labor income profile for the households who do not choose to retire early, w_j, is,

$$
w_j = \begin{cases} (1 - \tau)w & \text{for } j \le \rho \\ \eta w & \text{for } \rho < j \le M \end{cases}
\tag{39}
$$

and the profile for those who chose to retire early, $\tilde{w}_j$, is,

$$
w_j = \begin{cases} (1 - \tau)w & \text{for } j < \rho - \phi \\ 0 & \text{for } \rho - \phi \le j \le \rho \\ \eta w & \text{for } \rho < j \le M \end{cases}
\tag{40}
$$

The first step in solving the model is to specify a value for the leisure parameter, v. As a benchmark I will choose a value of v which has the maximum possible effect that is compatible with observed changes in the labor participation rate of older males. Specifically, suppose that all of the decline in the labor force participation rate from 1970 to 1994 of males between ages 55 and 64 is due to the provision of Social Security. As noted above, this drop was of 17.5 percent. Since there were 9.98 million men in this group in 1994, this decline in the participation rate would have caused 1.75 million workers to take early retirement. Given that the total labor force in 1994 was 131.3 million, this corresponds to a 1.33 percent decline in the labor force. Hence, I will choose the value of v such that aggregate labor supply drops by 1.33 percent when Social Security is provided.

An equilibrium can be defined in the same way as in section II. Solving for an equilibrium is now more complicated because the aggregate labor supply is now endogenous.

The solution procedure is as follows: (1) Assume an endogenous labor supply of N_0 when *no* Social Security is provided. (2) Choose an initial value for ν. (3) Guess an initial wealth distribution and the implied aggregate capital, K^*. (4) Assume that aggregate labor supply when Social Security is provided is 1.33 percent less than N_0. (5) Use K^* and aggregate labor supply to calculate prices τ and w using equation (5). (6) Solve the household problem to obtain saving functions, consumption functions as well as the decision to retire early (equation (34)), when Social Security *is provided*. (7) Simulate the behavior of a large number of households for many periods. From these simulations obtain the wealth distribution, aggregate capital, and endogenous aggregate employment. If these quantities and wealth distribution coincide with the assumed ones in steps (3) and (4) then proceed to the next step. Otherwise, repeat steps (2)–(6) with a new initial guess for ν and a new initial wealth distribution. (8) Finally, re-solve the model *without* Social Security with the value of ν just obtained and solve for the endogenous labor supply. If this is the same as N_0, then we are done. Otherwise repeat steps (1)–(7) using another initial value for N_0. Clearly these steps are computationally very involved because one needs to solve for the labor supply, and many iterations are needed to back out the desired value of ν. For simplification, I solve only for the equilibrium where N_0 equals aggregate employment in the basic model of section II.

The impact of Social Security on aggregates is given in the last column of Table 1. Aggregate capital, output, and consumption fall because of the drop in labor supply due to the workers who retire early. Not surprisingly, adding the labor supply distortion to the distortion of savings increases the losses due to Social Security.

The distributional and welfare impact of Social Security is shown in Table 6 and Figures 13, 14, and 15. The definitions of the distributions in these figures are the same as those used in Figures 6, 7, and 7; however, the utility distribution is now based on the utility function, $\tilde{U}(c,l)$, so the value of leisure is reflected in these calculations. The insurance aspect is noticeable in Figure 15. As before, Social Security clips the lower tail of the utility distribution by protecting the elderly. The value of Social Security is reduced slightly from that computed in section II, from 4.5 percent of aggregate consumption to 4.43 percent.

As in section II, Social Security reduces disparities in wealth holdings (Figure 13) and consumption (Figure 14). But now the spread of the utility distribution is widened and an upper tail appears (Figure 15). This is because now the wealthy are allowed to retire early and enjoy more leisure. To the extent that Social Security promotes and subsidizes early retirement, then it is not only protecting the poor and elderly, but is also favoring the very wealthy (who are the most likely to take early retirement).

The main result from this section is that the results from section II still hold when labor supply is endogenous and leisure has value. Specifically, welfare can rise when Social Security is provided even though aggregate capital, employment, output, and consumption fall. The net welfare gains of the current U.S. Social Security program are on the order of 4.4–4.5 percent of aggregate consumption or around $220 billion (in 1995 dollars).

IV. Options for Reform

Although the analysis of this paper is general and does not examine explicitly the impact of the specific policy reforms proposed by the Social Security Administration Advisory Council (1997), the results so far provide valuable lessons for the current debate on policy reform.

A. The Proposed Reforms

One of the main tasks of the Social Security Administration Advisory Council (1997) was to make recommendations to eliminate the long-term deficit without raising taxes by 2.17 percent. The council presented three different proposals: the Maintenance Benefit (MB) plan, the Individual Accounts (IA) plan, and the Personal Security Account (PSA) plan.

The MB plan proposes to continue the current system, subject to some modifications. For example, the taxes on benefits would be increased, coverage would be extended to state and local government employees, and payroll taxes would be raised by 1.6 percent in the year 2045. The plan also recommends a study to invest a portion of the trust fund assets in a stock index fund in order to avoid raising taxes in 2045.

Under the IA plan, individual accounts would be created alongside the current Social Security system. A 1.6 percent tax increase would fund these accounts. Workers could select a number of investment options, but the accounts would be administered by the government. At retirement, the funds in the individual accounts would be converted to an annuity to supplement the Social Security benefits. In addition, the Social Security system would be modified by raising the taxes on benefits, increasing coverage of state and local government employees, speeding the already scheduled increase in the age of eligibility for full benefits, and reducing benefit growth of middle- and high-wage workers.

The PSA plan proposes to establish an individual account for each worker. Five percent of taxes would be allocated to these accounts, and these funds could be invested according to the workers' choices but would be privately managed. The rest of the taxes would finance benefits to survivors and the disabled. At retirement, a worker would receive the return from the PSA as well as a flat benefit about $416 a month in 1996 dollars. Under the plan, coverage of state and local government employees would be increased, the age of eligibility would be raised faster than under the current system, and there would be changes to benefits and their taxation. The transition to this system would be costly because the 5 percent of taxes diverted to the accounts would not be available to pay for current retirees. To pay for this transition, a 1.5 percent tax increase would be required.

B. Lessons for Reform

Several aspects of these plans, such as their rate of returns, their administrative costs, or the investment of the trust fund in equities, have already been discussed (Gramlich (1996), Diamond (1996)). However, to date the plans have not been compared according to the old-age insurance that they provide. In this sub-section, I take a step in filling this gap using results from the model of the previous sections.

The key result from the analysis of this paper is that the pooling of risk through a pay-as-you-go Social Security system can deliver higher welfare than rational individual saving. This suggests that the MB plan can provide the highest welfare and the PSA plan might provide the least. It also suggests that, if mandatory savings are used (as in the IA or PSA plans), it is preferable to convert the savings to an annuity upon retirement. This is because those individuals who live for many more years than expected could withdraw "too fast" from their saving accounts and could end up in poverty.

Another result from the model is that the welfare gains from Social Security stem from protecting the very old. Hence, the suggestion to raise the retirement age at which benefits can be collected is a good way to restore financial balance while minimizing the welfare losses due to lower benefits. All plans involve raising this age, but perhaps a speedier increase, as proposed under the IA and PSA plans, might be a better way to restore balance and avoid raising taxes in the future.

V. Conclusions

The goal of this paper was to examine the old-age insurance value of Social Security and to compare proposed reforms according to their insurance value. The results are subject to several qualifications. First, many distortions of Social Security are left out of the model, and incorporating them is likely to lower the net welfare gains found here. Feldstein and Samwick (1996), for example, argue that Social Security introduces significant additional distortions on the labor supply and on compensation. Second, the sensitivity of the results to parameter values and functional forms remains to be explored. Particularly important are the choice of the utility function, the degree of risk aversion, the social welfare function, as well as the degree of intergenerational linkage assumed in the model. Third, the insurance value of the proposed reform plans remains to be examined explicitly. Details of the plans could alter the conclusions drawn from the general analysis. For example, the insurance value of the two tiers in the PSA plan has not been considered in this paper.

With these caveats in mind, the main insights from the analysis are:

- Risk pooling from Social Security can provide higher welfare gains than individual savings. Even if Social Security lowers aggregate savings, output and employment, there can still be net welfare gains from having such a program because of the insurance it provides.

- These gains arise from the protection it gives to those who live longer than expected.

- Adverse selection problems can be so severe as to prevent the wide use of private retirement insurance. Hence, the need for a mandated system.

- Today's low confidence in the system undermines the insurance value of Social Security.

In turn, these insights have the following implications for the proposed Social Security reforms:

- The MB plan is likely to provide better old-age insurance than the IA or PSA plans since individual savings provide less insurance than the current Social Security program.

- If private accounts are set up, converting them to annuities upon retirement is preferred to leaving them as saving accounts since even rational agents could withdraw savings "too fast."

- Raising the age at which benefits are collected is a way to restore financial balance to the Social Security system while maintaining most of the welfare benefits.

The main message of this paper -- that the pooling of risk through a pay-as-you-go Social Security system can provide higher welfare gains than individual savings -- should not be surprising if one thinks of Social Security as an insurance scheme. Particularly, once one recalls how other forms of insurance work. After all, most of us do not accumulate enough savings to replace our homes in the event they are destroyed. Rather we take advantage of the fact that home damage is an unlikely event, and buy home insurance instead of *each of us* having to accumulate enough savings to replace our homes. Yet, this is what plans to privatize Social Security through the creation of retirement saving accounts would amount to.

Table 1. Impact of Social Security on Aggregates

	No Social Security	Social Security 1/ (No Value of Leisure)	Social Security 2/ (Value of Leisure)
Replacement rate, η	--	43%	43%
Tax rate, τ	--	13.25%	14.01%
Retirement age	65	65	60 or 65
Dependency ratio	30.83%	30.83%	32.58%
Output, Y	1.379	1.312	1.295
Consumption, C	0.985	0.969	0.953
Investment, I	0.394	0.343	0.338
Capital, K	3.93	3.42	3.38
Employment, N	0.764	0.764	0.7543
Interest rate, r	2.61%	3.8%	3.78%
Wage, w	1.154	1.098	1.0985

1/ There is no value of leisure and labor supply is exogenous.
2/ Leisure has value and labor supply is endogenous.

Table 2. Distributional and Welfare Impact of Social Security

(Exogenous Labor Supply)

	No Social Security	Social Security
Replacement, η	--	43%
Wealth distribution		
Mean	3.93	3.42
Standard deviation	2.87	2.17
Consumption distribution		
Mean	0.985	0.969
Standard Deviation	0.318	0.121
Utility distribution		
Mean	-2.176	-1.031
Standard deviation	4.902	2.510
Value of Social Security 1/		
As a percentage of aggregate consumption	--	4.5%
In 1995 dollars	--	$221.7 billion
As a percentage of 1995 GDP	--	3.06%

1/ This value is based on the amount of aggregate consumption that households would give up to have this Social Security program.

Table 3. Decision to Buy Private Insurance

Premium, τ	$i(k)$ 1/
0%	*1* for all *k*
1.0%	*1* for all *k*
1.5%	*1* for $k \leq 35.0$, *0* for $k > 35.0$
2.0%	*1* for $k \leq 33.7$, *0* for $k > 33.7$
2.5%	*1* for $k \leq 31.9$, *0* for *31.9*
3.0%	*1* for $k \leq 31.1$, *0* for $k > 31.1$
4.0%	*0* for all *k*

1/ $i(k)=1$ when insurance is purchased; $i(k)=0$ otherwise.

Table 4. Old-Age Insurance

(Replacement rate η=43% in all cases)

Age at Which Agents Start Receiving Benefits, λ	Taxes	Mean Utility	Value of Social Security 1/ (Percent of aggregate consumption)
No social security	0%	-2.176	--
65	13.25%	-1.031	4.5%
75	6.11%	-1.1776	3.9%
80	3.38%	-1.3246	3.3%

1/ This value is based on the amount of aggregate consumption that households would give up to have this Social Security program.

Table 5. Risk of Default

Probability of Default, ψ	Mean Utility	Value of Social Security 1/ (Percent of Aggregate Consumption)
0%	-1.031	4.5%
5%	-1.100	4.23%
10%	-1.165	3.97%

1/ This value is based on the amount of aggregate consumption that households would give up to have this Social Security program.

Table 6. Distributional and Welfare Impact of Social Security

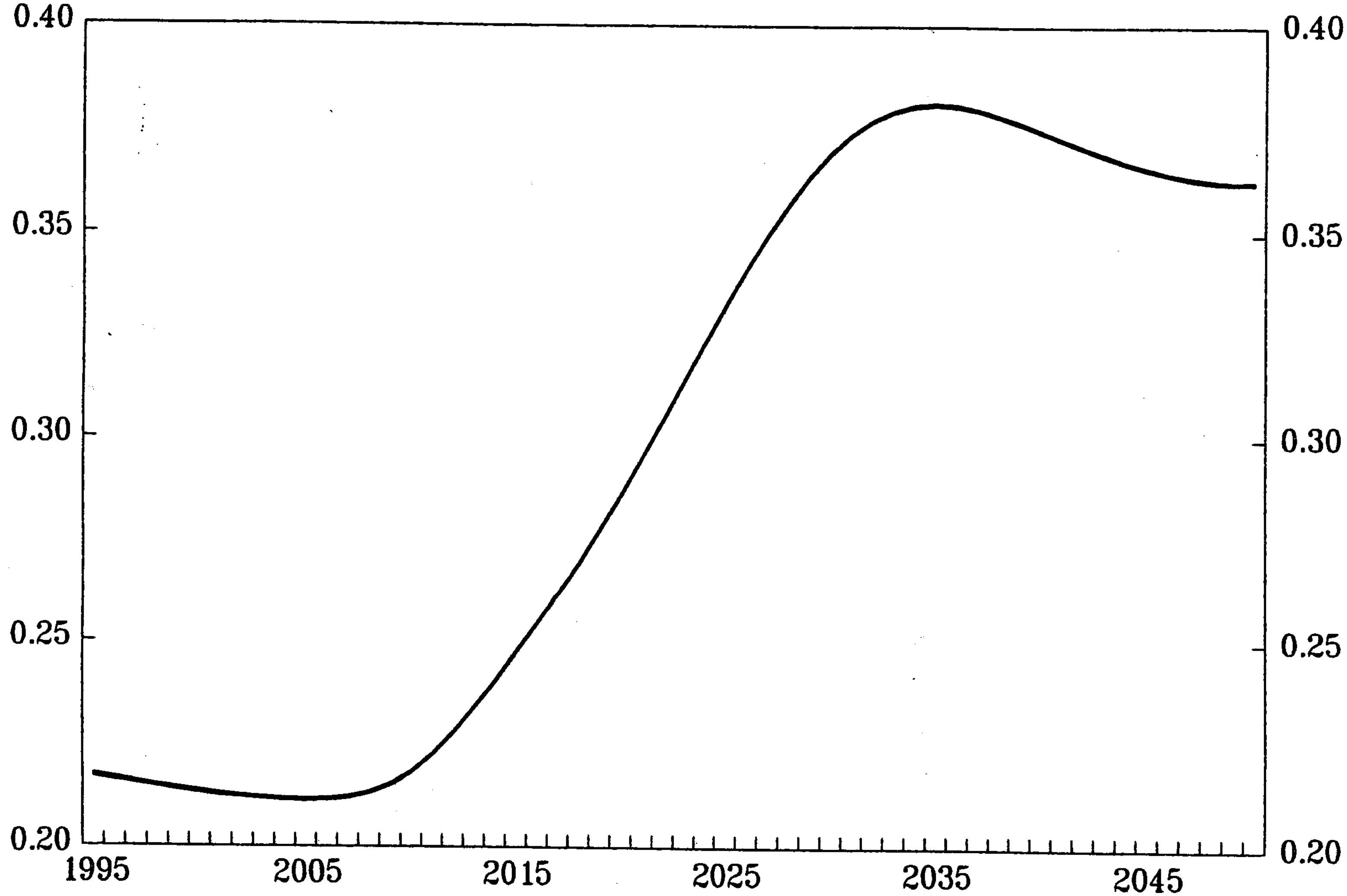
(Endogenous Labor Supply)

	No Social Security	Social Security
Replacement rate, η	--	43%
Wealth distribution		
Mean	3.93	3.38
Standard deviation	2.893	2.540
Consumption distribution		
Mean	0.985	0.953
Standard deviation	0.315	0.127
Utility distribution		
Mean	3.521	4.4615
Standard deviation	2.515	3.642
Value of Social Security 1/		
As a percentage of aggregate consumption	--	4.43%
In 1995 dollars	--	$218 billion
As a percentage of 1995 GDP	--	3.01%

1/ This value is based on the amount of aggregate consumption that households would give up to have this Social Security program.

Figure 1

UNITED STATES
PROJECTED DEPENDENCY RATIO 1/

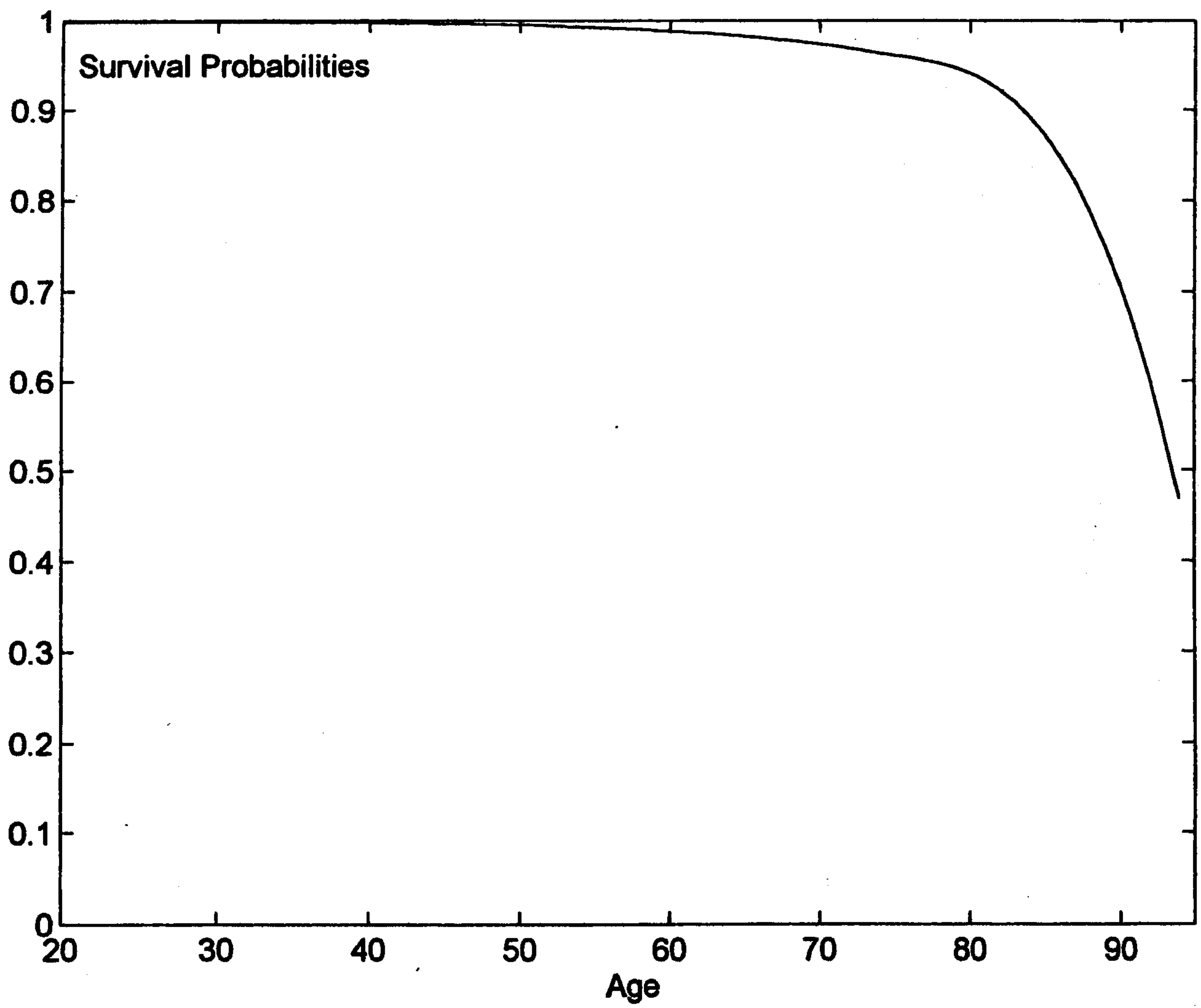


Source: Day (1996).

1/ The number of retirees per worker.

Figure 2

UNITED STATES
SURVIVAL PROBABILITIES 1/

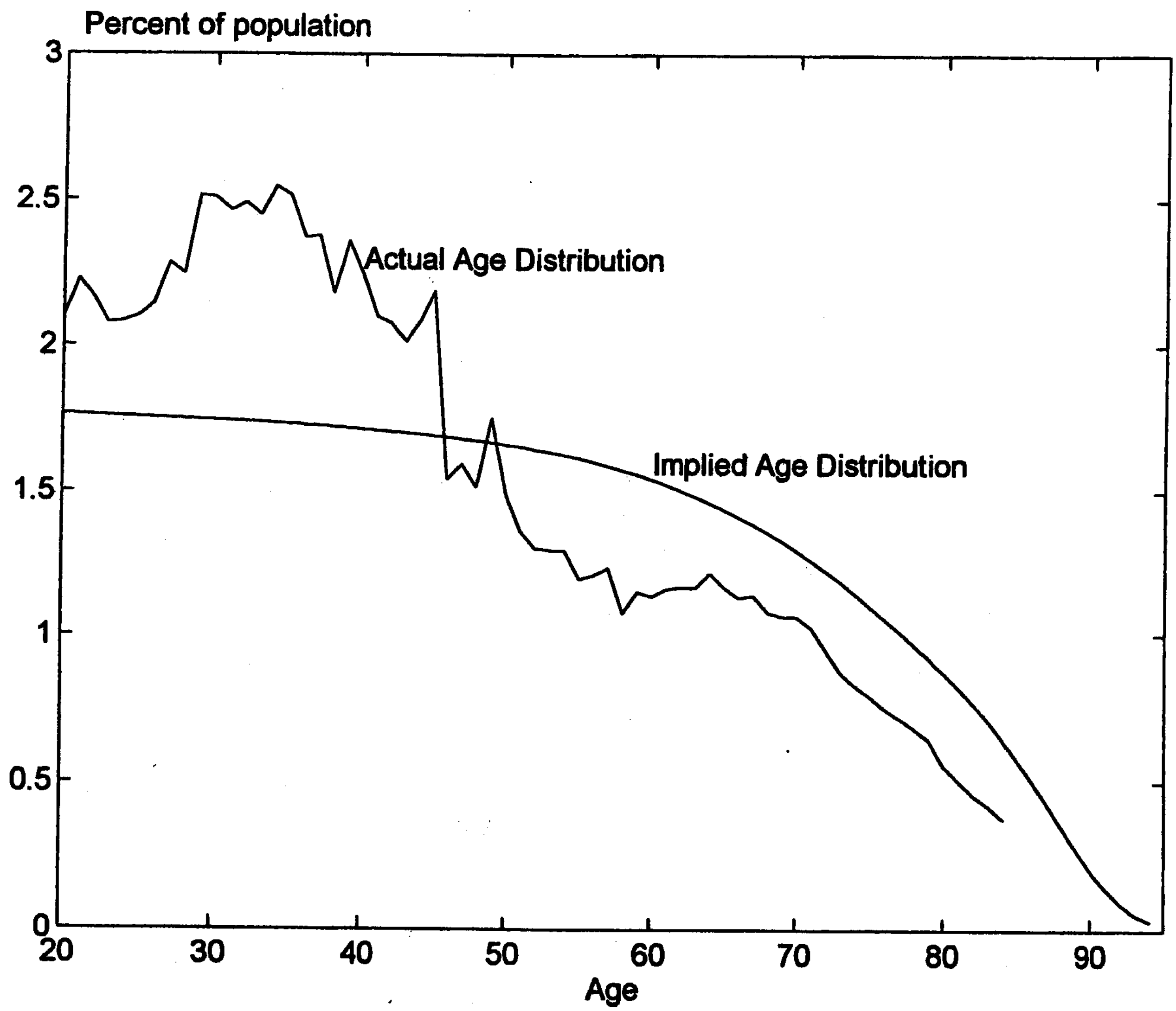


Source: Statistical Abstract of the U.S. (1995), table 116. Data for 1992.

1/ Probability of surviving from one year to the next for a given age.

Figure 3

UNITED STATES
ACTUAL AND IMPLIED AGE DISTRIBUTIONS 1/



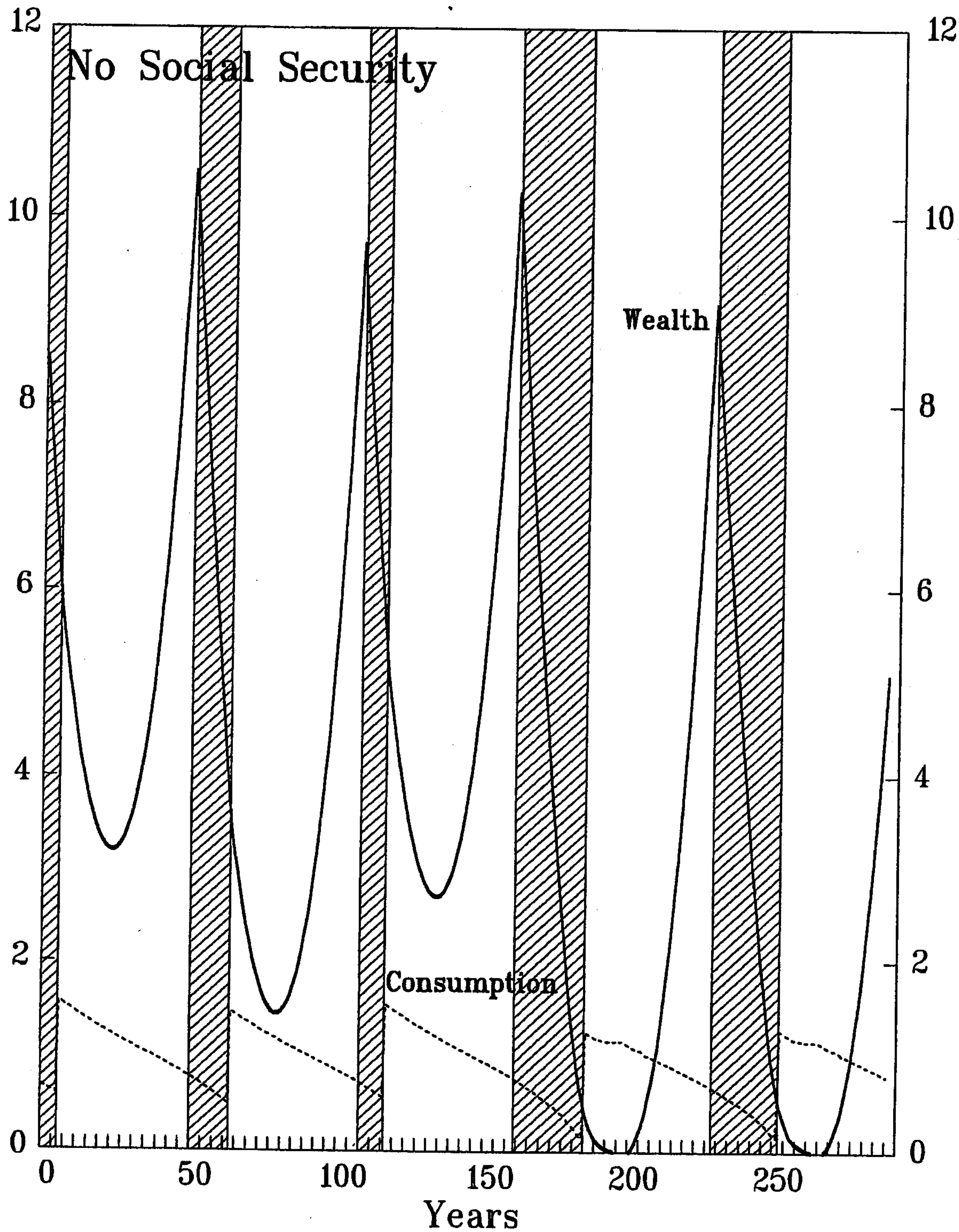
Source: Actual age distribution from the Statistical Abstract of the U.S. (1995).

1/ The implied age distribution is computed from the survival probabilities in figure 2.

Figure 4

UNITED STATES

HOUSEHOLD WEALTH AND CONSUMPTION 1/



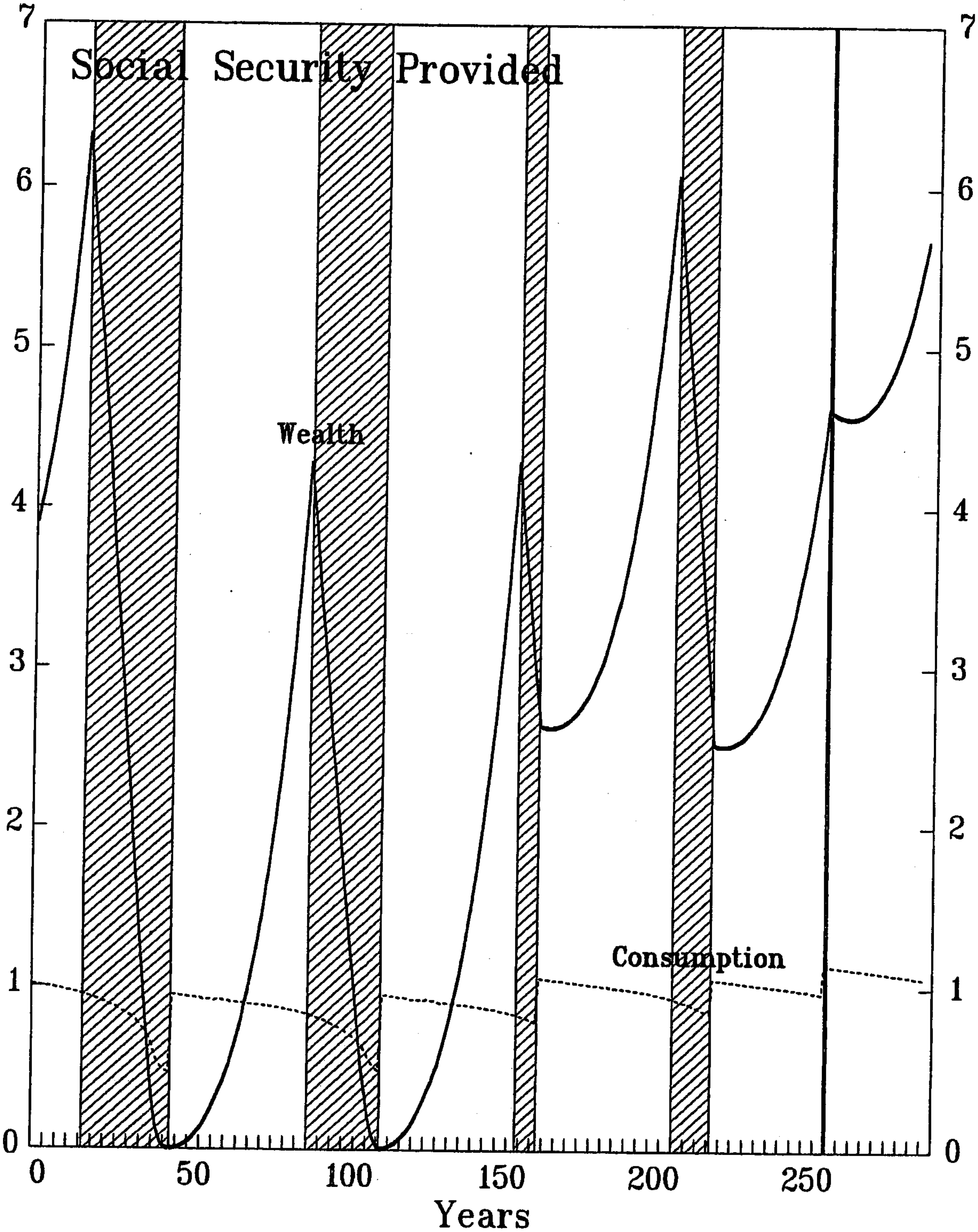
Source: Model Simulations.

1/ Shaded area indicates number of years spent in retirement.

Figure 5

UNITED STATES

HOUSEHOLD WEALTH AND CONSUMPTION 1/

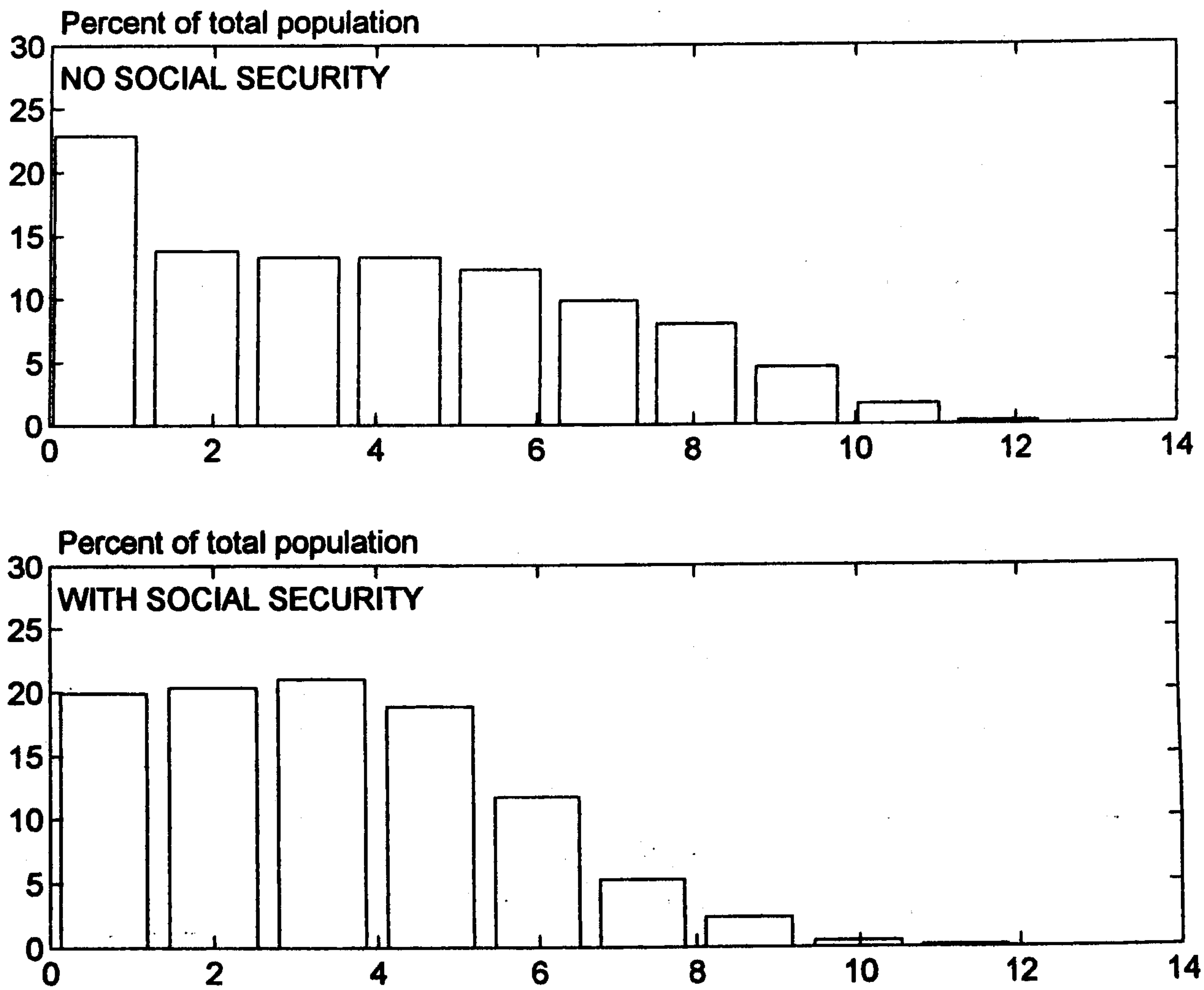


Source: Model Simulations.

1/ Shaded area indicates number of years spent in retirement.

Figure 6

UNITED STATES
WEALTH DISTRIBUTION (BASIC MODEL) 1/

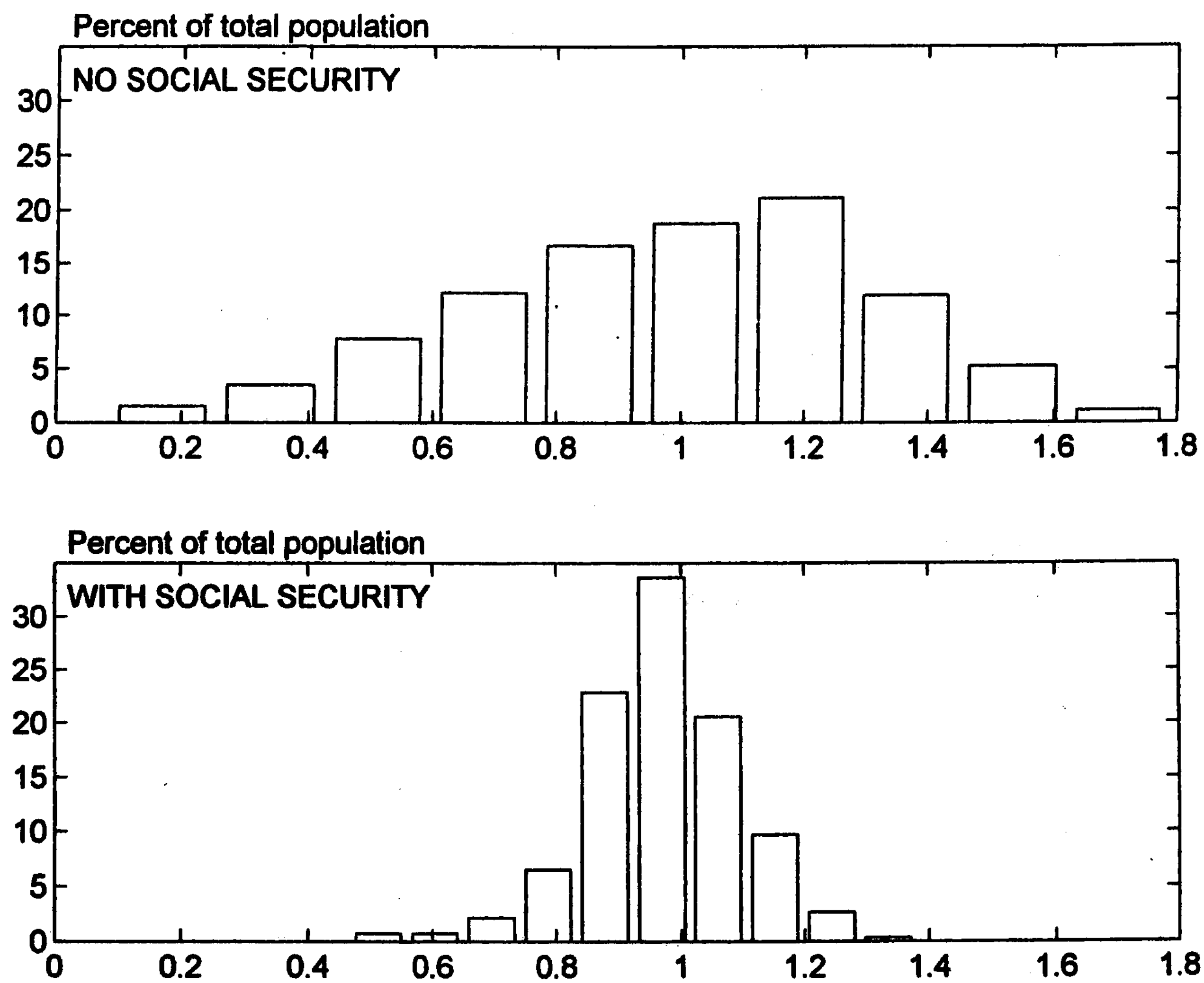


Source: Model simulations.

1/ The x-axis represents values for wealth.

Figure 7

UNITED STATES
CONSUMPTION DISTRIBUTION (BASIC MODEL) 1/

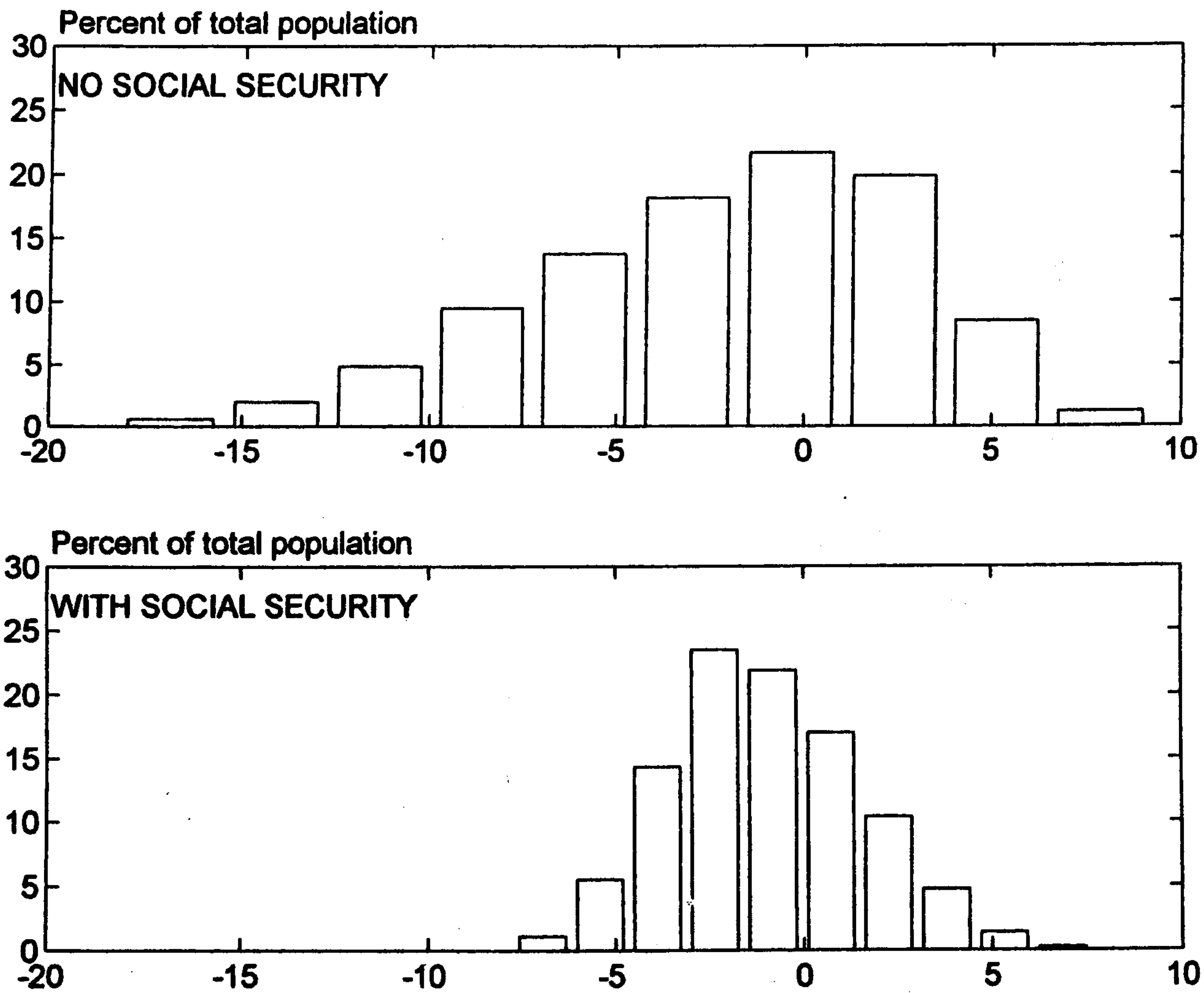


Source: Model simulations.

1/ The x-axis represents values for consumption.

Figure 8

UNITED STATES
UTILITY DISTRIBUTION (BASIC MODEL) 1/

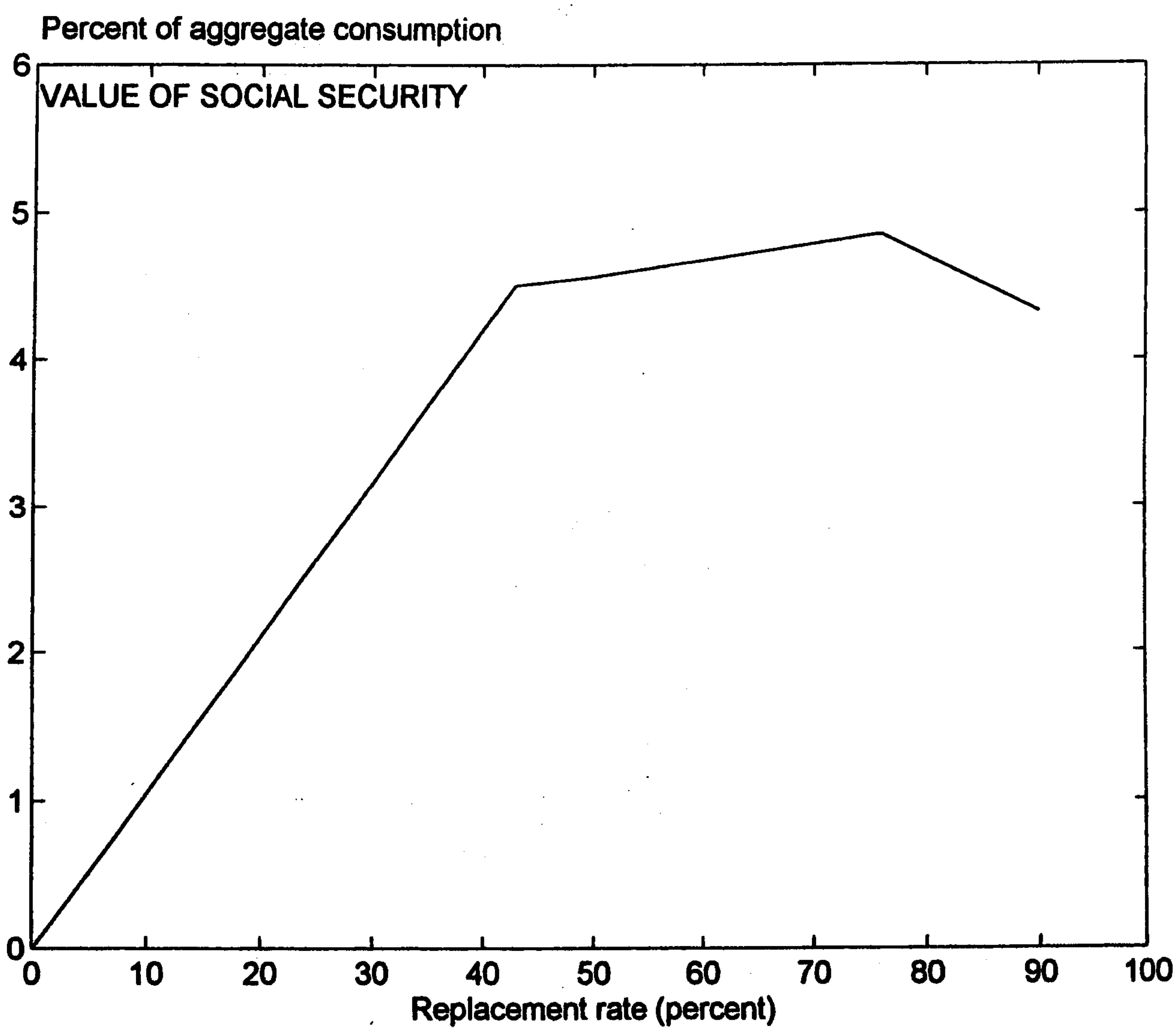


Source: Model simulations.

1/ The x-axis represents values for utility.

Figure 9

UNITED STATES
VALUE OF SOCIAL SECURITY (BASIC MODEL) 1/

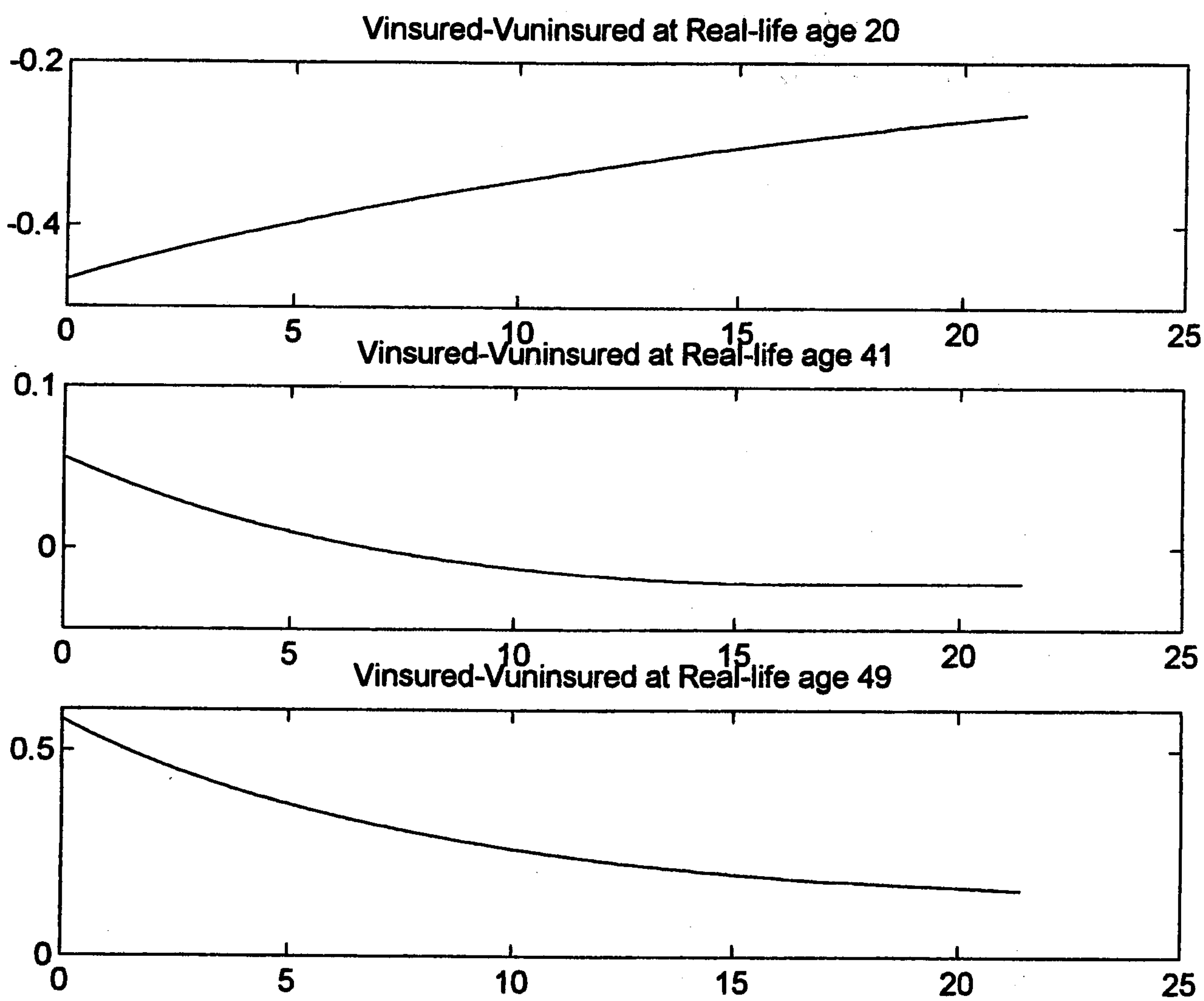


Source: Model simulations.

1/ This is the percentage of aggregate consumption that households would give up in order to have a Social Security program with a given replacement rate.

Figure 10

UNITED STATES
VALUE OF INSURANCE MINUS THE VALUE OF NO-INSURANCE 1/

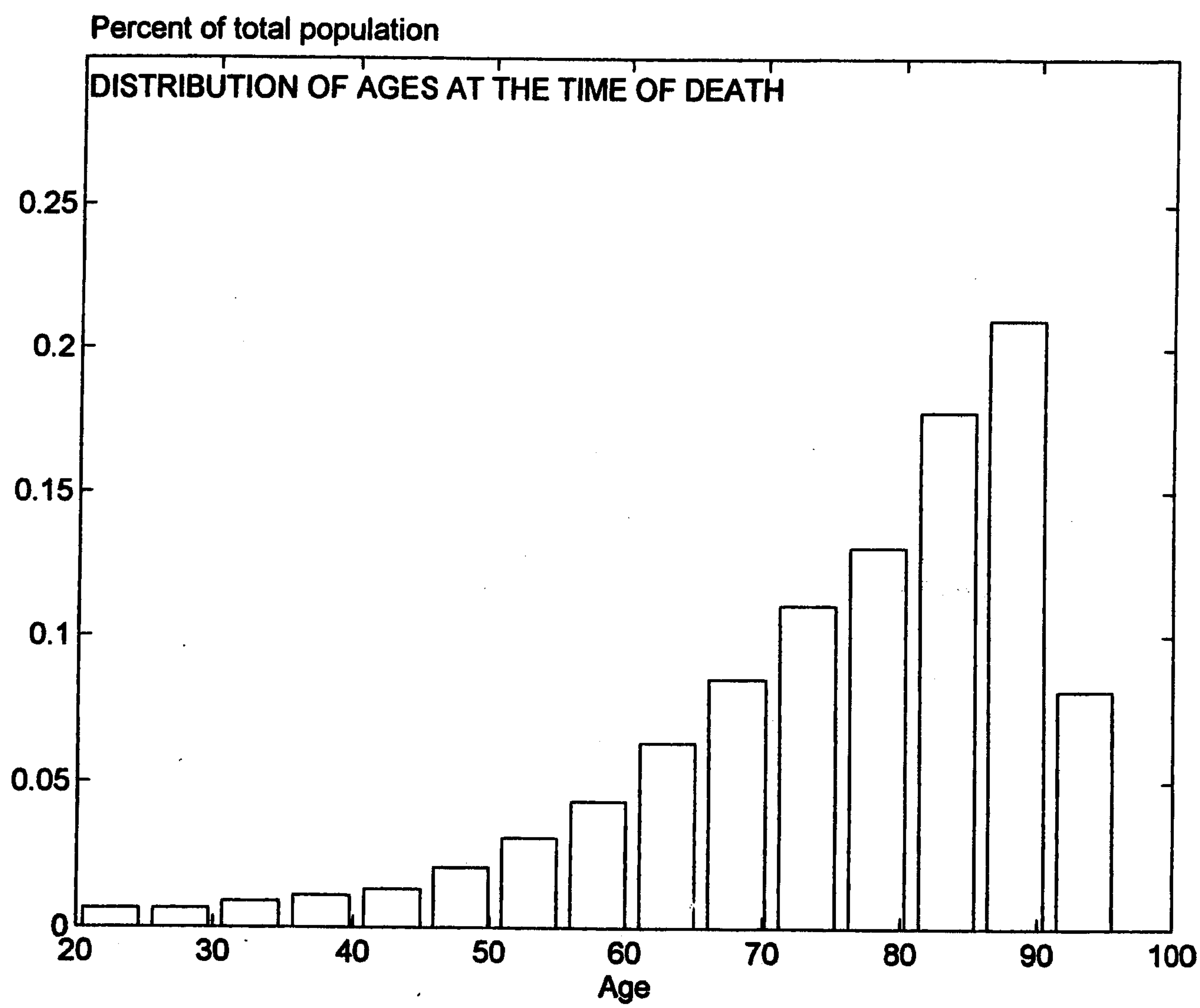


Source: Model simulations.

1/ This reveals the preference of households for private retirement insurance at different ages. The x-axis represents values for household wealth.

Figure 11

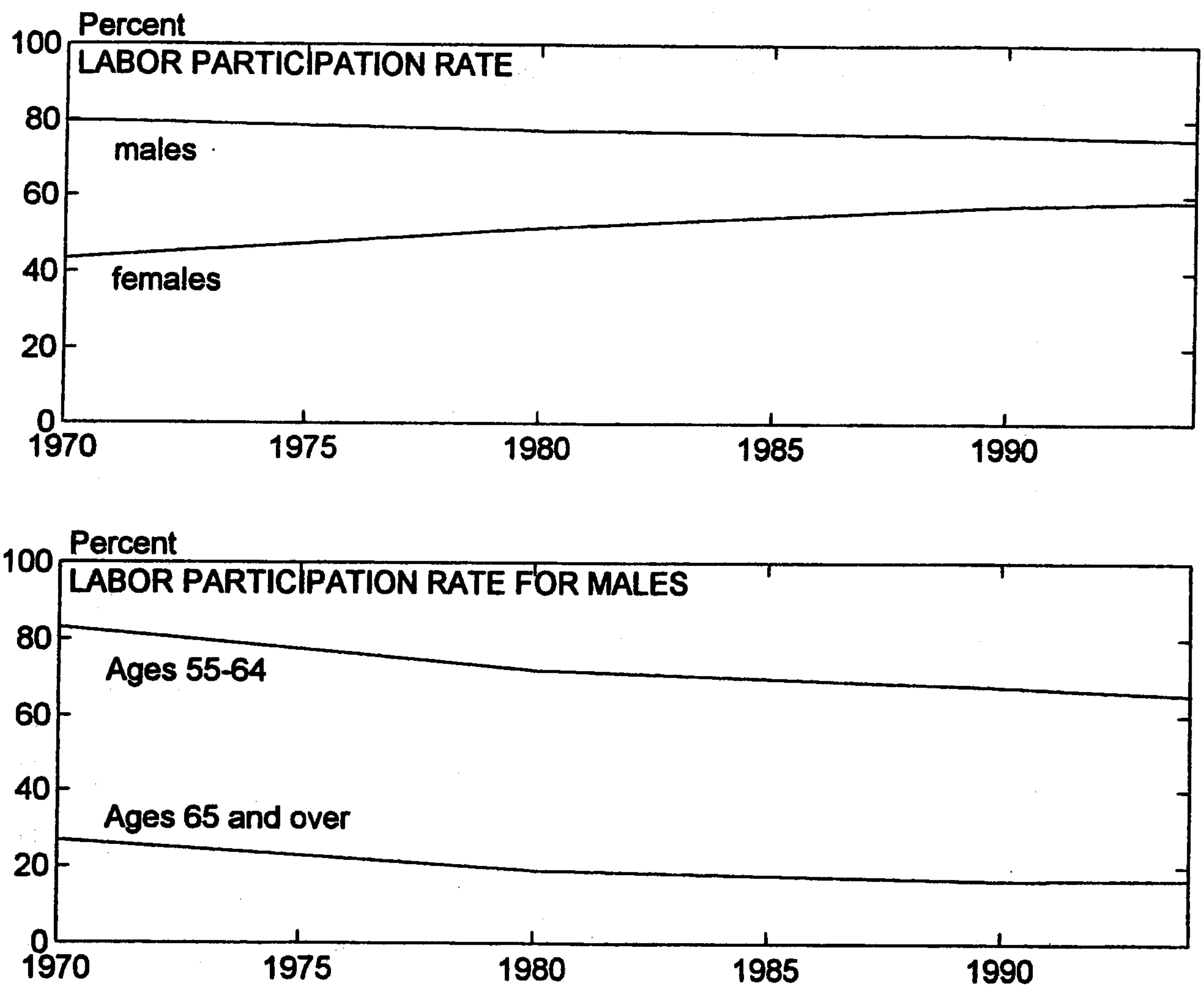
UNITED STATES
DISTRIBUTION OF AGES AT THE TIME OF DEATH



Source: Computed from the survival probabilities from figure 2.

Figure 12

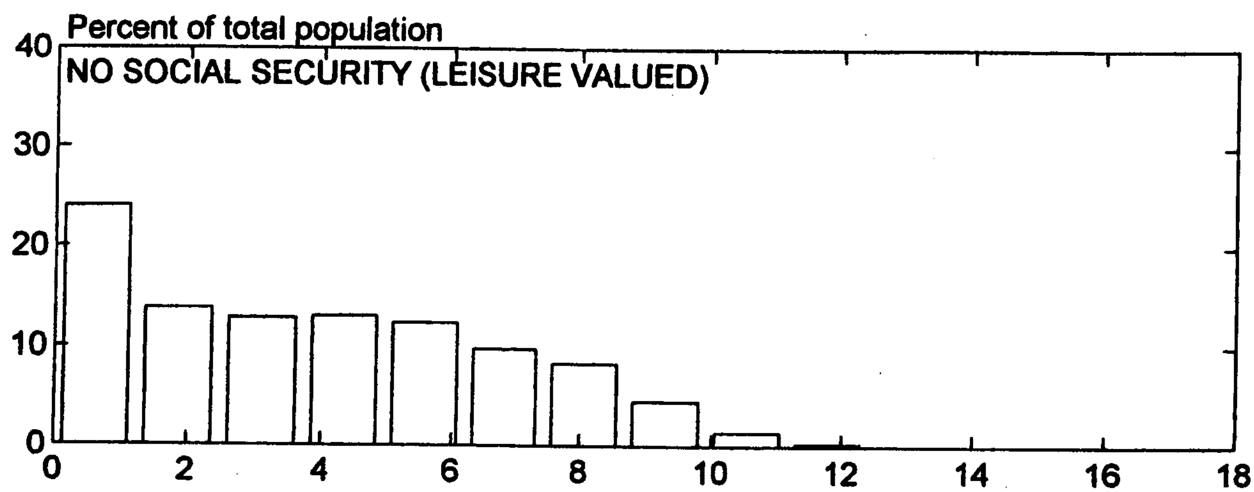
UNITED STATES
LABOR PARTICIPATION RATES

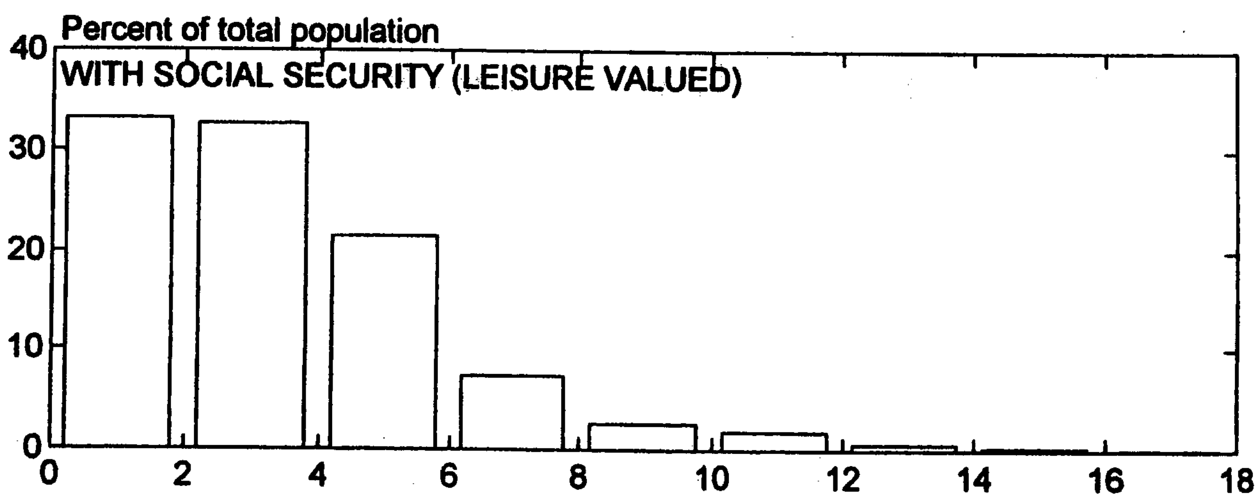


Source: Statistical Abstract of the U.S. (1995).

Figure 13

UNITED STATES
WEALTH DISTRIBUTION (EXTENDED MODEL: LEISURE HAS VALUE) 1/



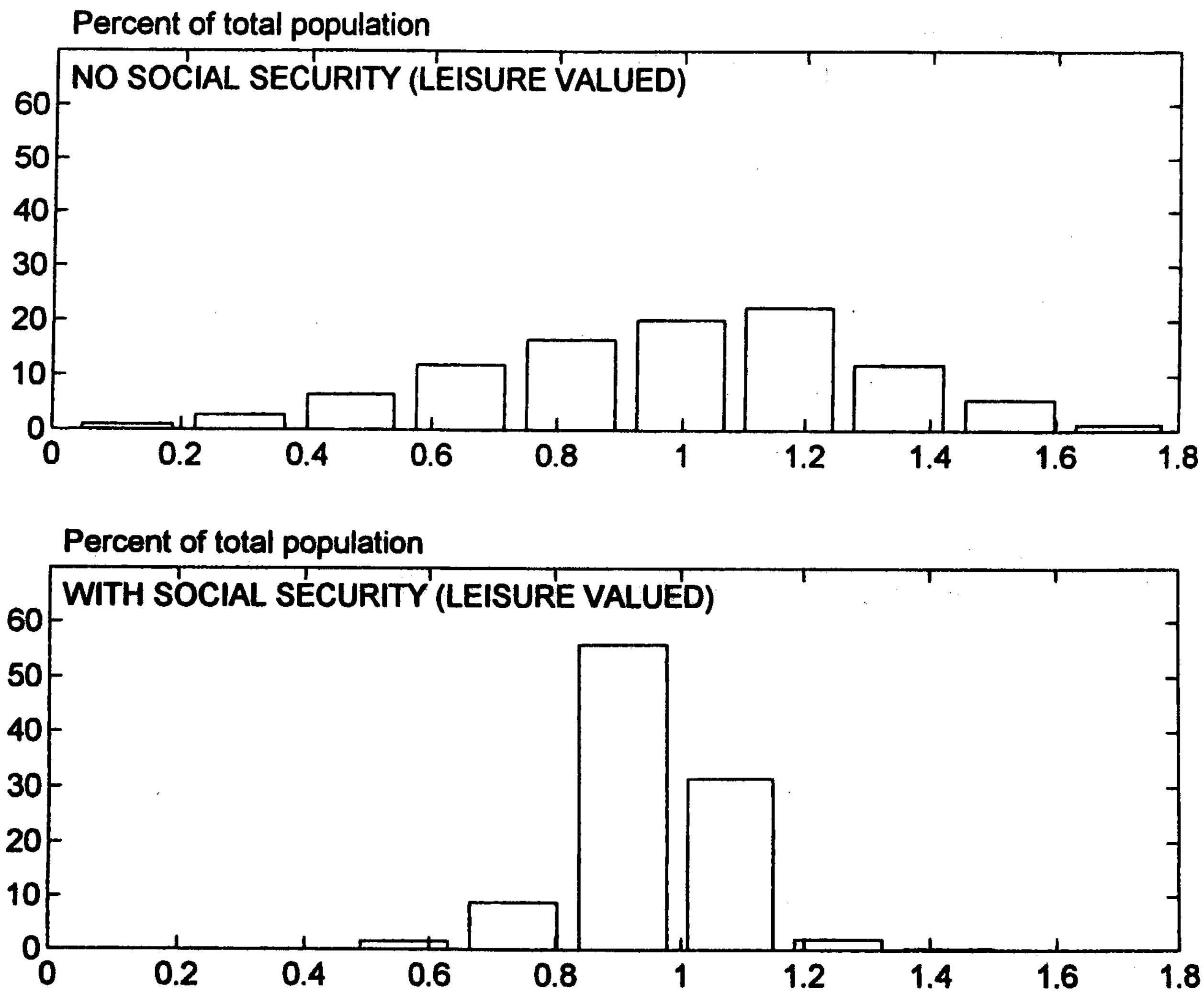


Source: Model simulations.

1/ The x-axis represents values for wealth.

Figure 14

UNITED STATES
CONSUMPTION DISTRIBUTION (EXTENDED MODEL: LEISURE HAS VALUE) 1/

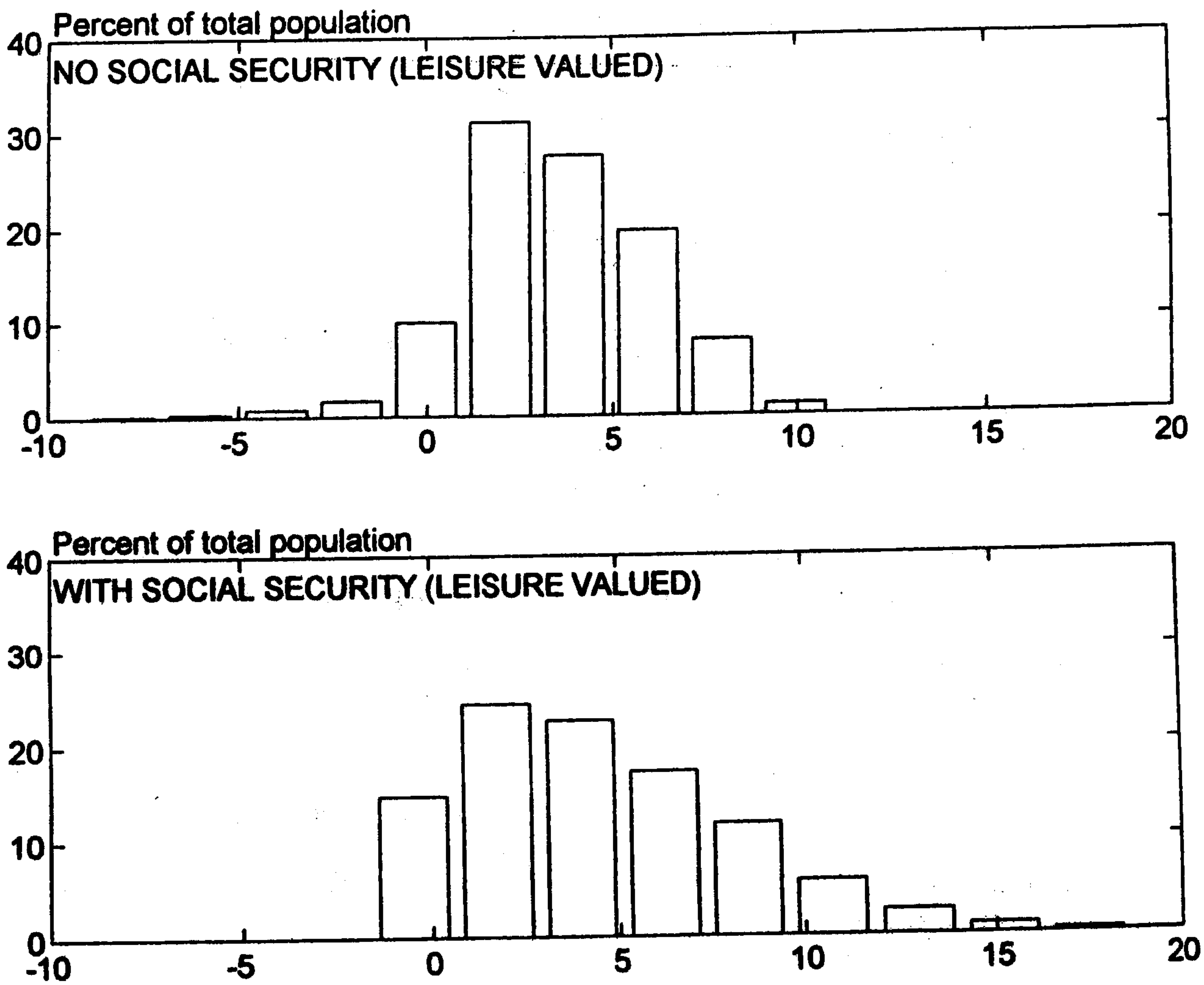


Source: Model simulations.

1/ The x-axis represents values for consumption.

Figure 15

UNITED STATES
UTILITY DISTRIBUTION (EXTENDED MODEL: LEISURE HAS VALUE) 1/



Source: Model simulations.

1/ The x-axis represents values for utility.

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