WP/16/45

IMF Working Paper

A Closer Look at Sectoral Financial Linkages in Brazil
I: Corporations' Financial Statements

by Izabela Karpowicz, Fabian Lipinsky, and Jongho Park

INTERNATIONAL MONETARY FUND

 WP/16/45

IMF Working Paper

Western Hemisphere Department

A Closer Look at Sectoral Financial Linkages in Brazil
I: Corporations' Financial Statements

Prepared by Izabela Karpowicz, Fabian Lipinsky and Jongho Park[1]

Authorized for distribution by Alfredo Cuevas

March 2016

Abstract

Understanding the interplay between firms' balance sheets and the macro-economic environment is important for understanding of the Brazilian economy. A close examination of developments in the nonfinancial corporate sector up to the early 2015 reveals weak equity growth, declining profitability, and rising leverage. The empirical work suggests that adverse shocks to financial variables lead to weaker real GDP growth in Brazil through their effect on corporate leverage, borrowing costs, and default frequencies. An estimation based on a DSGE model with financial frictions indicates that the recent economic downturn in Brazil is largely driven by a decrease in total factor productivity and by negative financial shocks.

JEL Classification Numbers: F34, F44, G15, G30, D58, E44.

Keywords: balance sheet analysis, DSGE Bayesian estimation, emerging economies

Author's E-Mail Address: ikarpowicz@imf.org; flipinsky@imf.org; Park@econ.umd.edu

[1] This paper was written when Jongho Park was a summer intern at the IMF. We are grateful for helpful comments from Paolo Cavallino, Sean Craig, Alfredo Cuevas, Andrea Gamba, Gavin Gray, Ivo Krznar, Luis Brandao Marques, Troy Matheson and Martin Saldinas. For help with S&P Capital IQ database we are grateful to Thierry Kpehor.

CONTENTS PAGE

I. Overview

After more than a decade of relatively strong economic growth, Brazil has entered a period of macroeconomic adjustment. In 2015, economic activity contracted sharply, inflation rose above 10 percent, and the government registered its second consecutive primary fiscal deficit—the first deficits in more than a decade. Rising unemployment and declining real wages are putting pressure on households' debt service capacity, while corporate leverage and debt service burden are increasing. To complicate the picture, the ongoing corruption probe on the oil producing company—Petrobras—is spreading to other large firms and its ramifications are paralyzing activity for suppliers and companies in other sectors. External factors have been unfavorable too, with risks of a further decline in commodity prices, and excessive financial volatility triggered by the anticipated U.S. monetary policy normalization. Financial conditions have tightened and policy space is limited, amid persistently high inflation, rising government debt, political tensions, and the prospect of a prolonged period of adjustment.

Consistent with the weakening economic environment, more recent data show a sharp increase in leverage, and deteriorating profitability and liquidity in the Nonfinancial corporate (NFC) sector. Financial linkages have broadened over recent years, as revealed by the balance sheet matrix analysis (BSA) and financial network analysis (FNA); the NFC sector has become the largest issuer of gross liabilities, the most exposed to the rest of the world, and the most vulnerable to exchange rate fluctuations.[2] This sector has also witnessed a secular decline in profitability, and more recently, also a decline in equity, shrinking cash buffers, and increasing leverage.

In this paper we examine NFC sector balance sheet vulnerabilities and find a link between balance sheets and the real business cycle in an empirical setting. Looking at disaggregated micro-level data, we show that the transmission channels of financial shocks to the real economy are active, and that a rise in NFC leverage can propagate financial shocks to the real economy through an increase in borrowing costs and default frequencies.

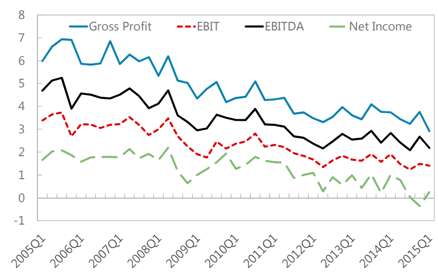
The paper is structured as follows. Section II presents a micro data study of NFC and households balance sheets focusing on a recent period, up to 2015Q1; section III presents the empirical results that examine the relationship between real GDP growth and selected financial variables and develops a DSGE model with financial shocks that is applied to Brazil; section IV concludes.

[2] The evolution of sectoral balance sheets in Brazil from 2007 to 2014 is presented in the forthcoming, part II of this paper—Intersectoral Balance Sheet Matrices.

II. A Closer Look at NFCs' Balance Sheets

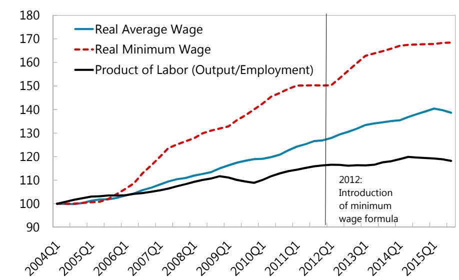
Corporate sector profitability has been on a declining trend.[3] All standard indicators of corporate sector profitability (measured relative to total assets) have been on a downward path since 2005. Net income has also been trending down and turned negative in 2015 (Figure 1).[4] Increases in the average wage, led by the minimum wage growth, have outstripped labor productivity growth, weighing on corporate profitability (Figure 2). The share of profit in income has been cyclical (Tosoni, 2014) and has fallen for about a decade. In addition, in the last few years, the end of the commodity super cycle and the appreciation of the currency have also played a role.

Figure 1. Brazil: Profitability Indicators
(In percent share of total assets)



Source: Capital IQ.

Figure 2. Brazil: Wage and Labor Productivity Growth
(Index; 2004=100)



Sources: IBGE and authors calculations.

The decline in profitability has affected equity growth and contributed to rising leverage. Assets have grown in real terms recently, but at a relatively slow pace. Meanwhile, non-equity liabilities have increased faster than assets, contributing to a pick-up in leverage (Figure 3).[5] This trend accelerated in the first half of 2015, and appears to be associated with declining equity (in real and absolute terms, and as a share of total assets). Equity began contracting in late 2012, and remained weak through the following quarters, with the exception of a small recovery in 2014Q2.[6] At the same time, the share of debt-to-assets (Figure 4) has increased since 2011, gathering pace in 2014, and surpassing the share of equity-to-assets in 2015 for the first time in the past decade.[7] In light of declining profitability, NFCs leverage is becoming a concern.

---

[3] See Appendix I for data sources and coverage. A brief discussion of households' sector balance sheets is included in Appendix II.

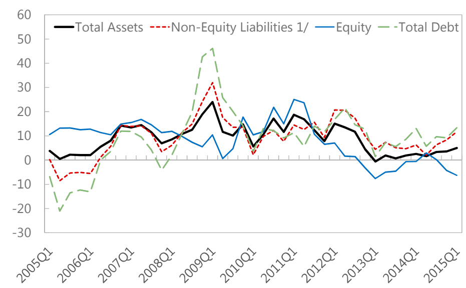
[4] Gross profit (GP) = sales-cost of sales; EBITDA = GP-other operating expenses; EBIT = EBITDA-depreciation and amortization; Net income = EBIT-interest, tax and non-operating income.

[5] Non-equity liabilities and equity sum to total assets by the accounting identity.

[6] A decrease in net income is associated with lower retained earnings and lower equity. Therefore, decreasing profitability results in declining equity. In Figure 1, profitability measures mostly remain positive while equity growth rate contracts after 2012. Comparing average shares of firms with negative profit over two different 10-quarter sample periods, 2010Q2−2012Q3 and 2012Q4−2015Q1, we find that the share increases from 27 to 31 percent. This suggests that the equity contraction started in late 2012 and was driven by firms shifting from positive to negative net profit at an extensive margin.

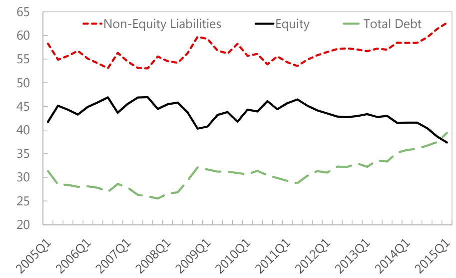
[7] The increase in leverage ratio is driven both by the increase in non-equity liabilities and the decrease in equity. However, we cannot identify based on available data if the increase in non-equity liabilities is driven by exchange rate movements or by active debt issuance.

Figure 3. Brazil: Annual Change in Balance Sheet Positions
(In real annual percentage change)



Source: Capital IQ.

Figure 4. Brazil: Balance Sheet Positions
(In percent share of total assets)

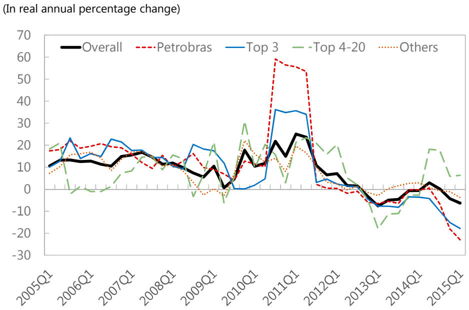


Source: Capital IQ.

1/ Includes accounts payable, income taxes payable, accrued expense, deferred credit, deferred taxes, negative goodwill, and reserves.

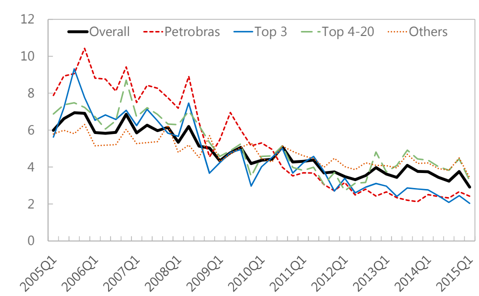
While most firms show declining equity and profitability, these patterns are most pronounced among the three largest firms in the sample—Petrobras, Eletrobras, and Vale. These three firms alone account for approximately 26 percent of the total assets, with the next 16 largest firms (ranked 4 to 20) accounting for 24 percent of assets. For Vale and Petrobras, the ongoing decline in commodity prices portends continuous stress on profitability. With the exception of the firms ranked 4 to 20 by assets, all other firms began exhibiting negative equity growth in 2014Q2, with the three largest firms suffering the most rapid declines (Figure 5).[8] These trends are consistent with the pattern of declining profitability displayed in Figure 6. Starting from above average in 2010, profitability in the largest three firms has also declined faster than in other firms after 2012.

Figure 5. Brazil: Real Equity Growth Rate
(In real annual percentage change)



Source: Capital IQ.

Figure 6. Brazil: Gross Profit
(In percentage share of total assets)



Source: Capital IQ.

The asset side of the balance sheet points to growing liquidity concerns, along with a slowdown of investment. Here, the focus is on three items on the asset side of the balance sheet: “Cash and Liquidity”, “Plant and Properties” or “Capital”, and “Long-term Investments”.[9] Annual growth of liquid assets has contracted in recent quarters (Figure 7). However, this “cash” is not used to accumulate physical assets, as suggested by the evolution of “Plants and Properties” over the same period; the share of “Capital” in total assets is decreasing, albeit at a slow pace (Figure 8). While fully in line with recent economic

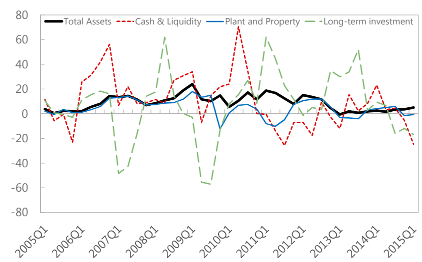
---

[8] The sharp increase in equity of Petrobras during 2010Q3−2011Q2 reflects the company’s recapitalization and the acquisition of the Libra oil field.

[9] “Cash and Liquidity” refers to instantly marketable assets, and mostly corresponds to “cash” in theoretical models. This item is not used in production. However, one important purpose of holding cash is for precautionary purposes, in case of financial distress (Bates et al., 2009). “Plant and Properties” corresponds to “capital” in theoretical models and carries information on firms’ production capacity. “Long-term Investments” mostly includes longer term financial instruments.

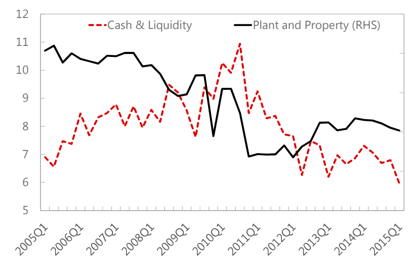
developments in Brazil, these balance sheet pressures raise concerns for activity in the corporate sector, as a decline in fixed investment affects future productive capacity.

Figure 7. Brazil: Annual Growth in Assets
(In real annual percentage change)



Source: Capital IQ.

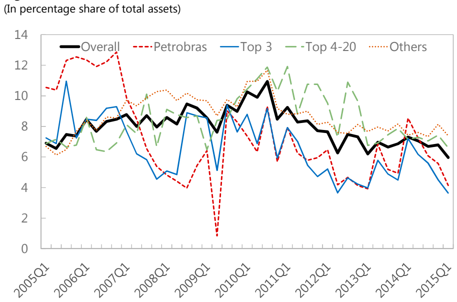
Figure 8. Brazil: Assets Compositions
(In percent share of total assets)



Source: Capital IQ.

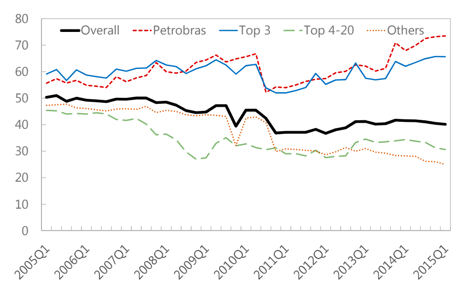
Across firms, cash and liquid assets appear broadly stable for small- and medium-sized corporates, but are falling rapidly in large companies since the beginning of 2014. While the share of liquid assets in total assets has declined after their post-crisis peak, this indicator masks important divergences across firms. The three largest firms, especially Petrobras, are driving the overall downward trend. The liquidity of firms ranked 4 to 20 by total assets has been broadly stable for around 2 years, while smaller firms have increased their liquid assets slightly (Figure 9). The fraction of "Plant and Property" to total assets of the three largest firms has increased steadily since 2010, while the fraction of "Plant and Property" of other firms has declined (Figure 10). Investment has thus continued favoring large firms that were most profitable a decade ago but whose profitability has declined since then (Figure 6). This may be due to the fact that large firms have easier access to financing at more favorable terms, and have less incentive to hold cash buffers for precautionary purposes. Going forward, the new low level of commodity prices will depress the accumulation of physical capital in firms such as Petrobras and Vale.

Figure 9. Brazil: Cash to Total Assets Ratios
(In percentage share of total assets)



Source: Capital IQ.

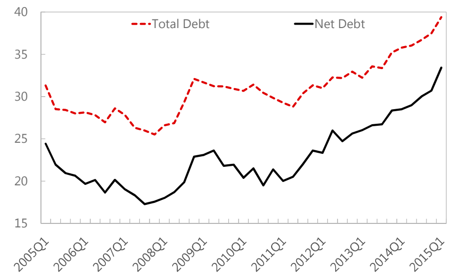
Figure 10. Brazil: P&P to Total Assets Ratios
(In percentage share of total assets)



Source: Capital IQ.

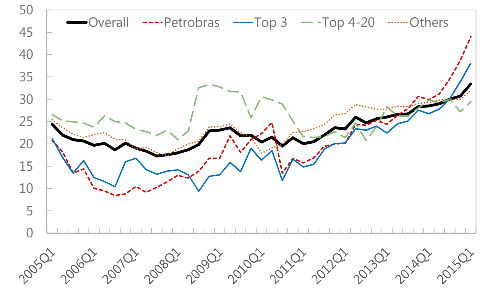
Corporate sector net debt has increased over the past several years, with larger firms showing more aggressive borrowing. Recent studies focusing on Brazil have already pointed to a substantial increase in debt of listed Brazilian firms over the past decade, with a rise in U.S. dollar denominated debt and a substantial increase of bonds issued in local currency (Kang and Saborowski, 2015). The leverage ratio has steadily increased with the trend accelerating in 2015. In light of decreasing cash buffers, the increase in leverage is mostly driven by a pick-up in net debt (total debt minus cash and liquidity). By the beginning of 2015, leverage had reached its highest level in more than a decade, with the fraction of total debt exceeding that of equity for all firms except those ranked 4 to 20 by assets. The pattern is universal across firms but especially pronounced among the three largest firms which are also experiencing declining profitability and shrinking cash buffers (Figures 11 and 12).

Figure 11. Brazil: Leverage Ratios
(In percent share of total assets)



Source: Capital IQ.

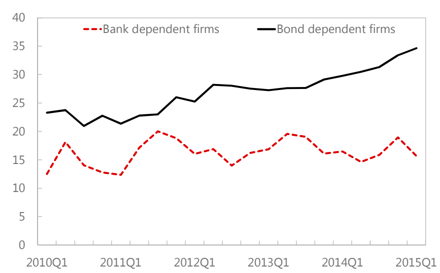
Figure 12. Brazil: Net Debt to Total Assets
(In percentage share of total assets)



Source: Capital IQ.

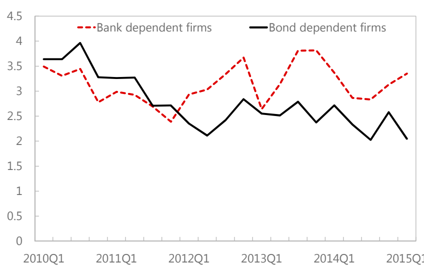
Rising leverage is largely driven by bond financing and bond-dependent firms, most notably by Petrobras. Consistent with previous findings related to EM bond markets (Shin, 2013, Turner, 2014, Caballero et al. 2015, and IMF 2015b) and recent literature on Brazil (IMF, 2015), the bond market in Brazil has been growing at a rapid pace. While bank-dependent firms have shown stable leverage, leverage among bond-dependent firms has been steadily increasing (Figure 13)[10]; bond-dependent firms have also begun to be less profitable (Figure 14). Indeed, large Brazilian firms have financed operations increasingly through bonds issuance since 2012, most notably Petrobras.

Figure 13. Brazil: Net Debt to Total Assets
(In percent share of total assets)



Source: Capital IQ.

Figure 14. Brazil: EBITDA to Total Assets
(In percent share of total assets)

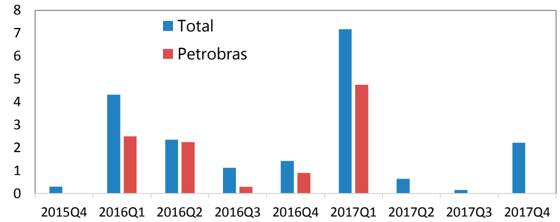


Source: Capital IQ.

Petrobras's total gross debt reached US$130 billion in 2015Q2, and return on equity and assets turned negative in the last quarter of 2014. The company's debt maturity profile provides a buffer in the near term, with only 10 percent of debt due in 2016 and 85 percent of principal payments due after 2017. Petrobras has a sizable portion of external bonds, with a significant amount maturing in the first quarter of 2017 (Figure 15). The company has made significant cuts to capital expenditure plans to alleviate financing needs and to deleverage.

[10] Debt instruments are classified into two categories: bank loans and bonds. Data on the former is directly available from Capital IQ, while the latter is derived as total debt net of bank loans. Firms are categorized as being "bank-dependent" if bank loans account for more than 80 percent of total debt; all other firms are "bond-dependent".

Figure 15. Brazil: External Debt Maturing[1]
(Billions of US dollars)



A vector-autoregression (VAR) estimation suggests that shocks to the NFC leverage ratio affects growth. To assess the effect of the recent increase in NFCs' leverage on growth, we estimate a bivariate VAR with total debt-to-equity-ratio and year-on-year real GDP growth rate for the sample period 2005Q1−2015Q1.[11] The intuition behind this approach is that an increase in leverage beyond a certain level pushes up default probabilities and increases borrowing costs, effectively contributing to defaults and/or leading to de-leveraging, a slowdown in lending/borrowing, and a generalized decline in growth.[12] We identify the leverage shock by imposing a recursive ordering assumption. In particular, we assume that the leverage ratio responds to GDP growth rate both contemporaneously and with a lag, while GDP growth rate responds to the leverage ratio only with a lag. This identifying assumption is in line with existing literature according to which real aggregate variables respond to financial variables only with a lag (as in, for instance, Caballero et al., 2015).

The response of the GDP growth rate to a one standard deviation unanticipated increase in leverage ratio (about 15 percent) is "u-shaped". The real GDP growth rate reaches its trough around four quarters after the shock, and gradually reverts back to its steady state growth rate. The real GDP growth rate is approximately one percentage point lower than in the absence of a shock to the leverage ratio (Figure 16). The VAR analysis supports the view that the current increase in NFCs' leverage ratio may further slow the economy. The result is essentially identical if we replace total debt-to-equity-ratio with the net debt-to-equity ratio. The Granger-causality tests with two lags confirm (at 1 percent significance level) that net debt-to-equity and total debt-to-equity help predict GDP while the GDP does not have predictive power on the other two variables.[13]

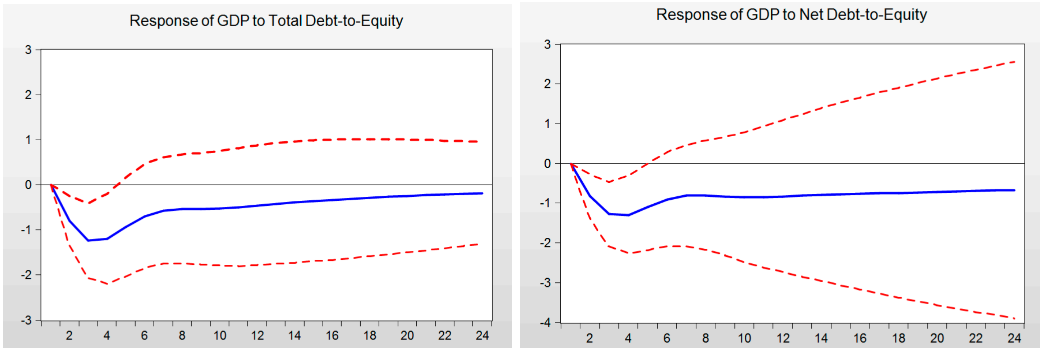
[11] We limit our analysis to a bivariate VAR due to the short sample period.

[12] An example from Brazilian reality is a sharp turnaround in the corporate strategy of Petrobras. Up until 2015Q2, Petrobras strategy focused on meeting specific investment and production targets. When leverage and debt repayment capacity became a concern, Petrobras' strategy was realigned to concentrate on de-leveraging. This change in strategy was not triggered by an unexpected financial shock; it was rather consequence of a perceived long lasting change in prospects for oil.

[13] As a robustness test, we change the ordering of variables and results remain essentially identical. Our results are robust to the order of VAR.

Figure 16. Impulse Response Function of a Shock to Firm Leverage

(Percent)



Source: Authors' estimates.

III. Macroeconomic Consequences of Financial Shocks

A. Risk Pricing and GDP

Financial variables play an important role in characterizing business cycles across emerging market economies (EMs). The empirical literature on bond spreads and business cycles shows that bond spreads contain useful information for forecasting real variables, and shocks to bond spreads generate real economic fluctuations. However, empirical testing has been for the most limited to advanced economies, in particular the United States and the Euro Area (Gilchrist et al., 2009; Gilchrist and Zakrajšek, 2012; and Gilchrist and Mojon, 2014). In EM setting, empirical analysis has mostly focused on the sovereign bond market, also pointing to the link between spreads and the real economy (Akinci, 2013, and Uribe and Yue, 2006). The recent pickup in the EMs' corporate bond issuances has also brought attention to the effects of negative financial shocks in corporate bond markets, which are found to cause recessions (Caballero et al., 2015).[14]

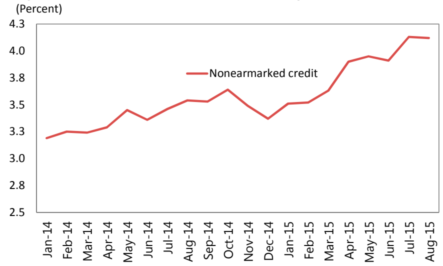
Financial variables associated with the performance of Brazil's NFCs point to a recent worsening trend since 2014. The increase in leverage generally exposes firms to higher financing costs and pushes up default probabilities. Access to external credit by corporates has suffered since early 2015 and credit supply has been tightening also domestically. As a result of these pressures, companies in various sectors have signaled their intention to resort to asset sales and other operations in order to generate cash and reduce dependence on new borrowing, and some are in discussions with creditors.[15] Corporate NPLs have started

[14] These papers do not, however, directly account for quantity measures in their estimates, such as sectoral leverage ratios or bond holdings, for instance. In their more recent work, however, Fernandez and Gulan (2015) document some empirical facts on corporate leverage and EMs business cycles.

[15] Petrobras and CSN (steel company) have announced plans to sell assets and the latter renegotiated its debt with Caixa and BNDES in August. Trinufo Participaçoes has improved its financial profile by raising cash from an asset sale to a company in China in August. General Shopping has made a restricted offer of shares whose proceeds will be used to repurchase a perpetual bond. Some relatively big names have also filed for bankruptcy protection, including several firms in the construction and energy sectors (e.g., Galvao, OAS, Shahin) since late 2014.

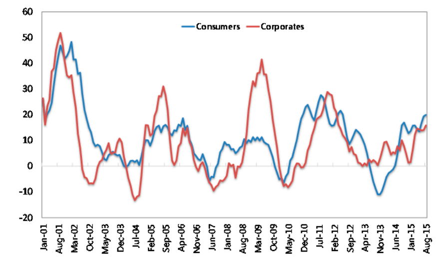
pointing up, albeit slowly (Figure 17) while delinquencies on bills and loans registered by Serasa to August of 2015 reached historical heights, mostly due to unpaid bills to nonbanks (electricity and telecom).[16] This indicator has an estimated lagged effect on banks NPLs of around 1 year, suggesting further increases in NPLs in the short term (Figure 18). Corporate default probabilities are also on the upward path (Figure 19).

Figure 17. Brazil: Corporate NPLs on Outstanding Credit
(Percent)



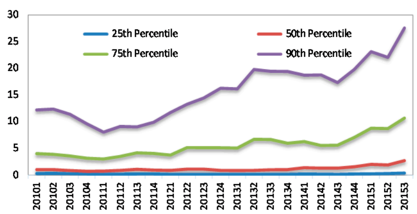
Source: Central Bank of Brazil.

Figure 18. Brazil: Default Index
(3-months moving average of y/y monthly growth, percent)



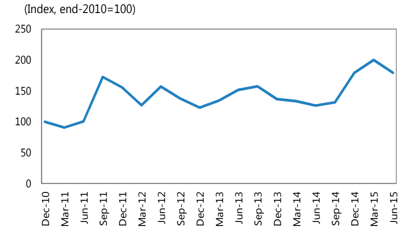
Sources: Serasa Experian.

Figure 19. Brazil: Corporate Default Probabilities
(1-Yr EDF9, Percent)



Source: SERASA Experian.

Figure 20. Brazil: CEMBI
(Index, end-2010=100)



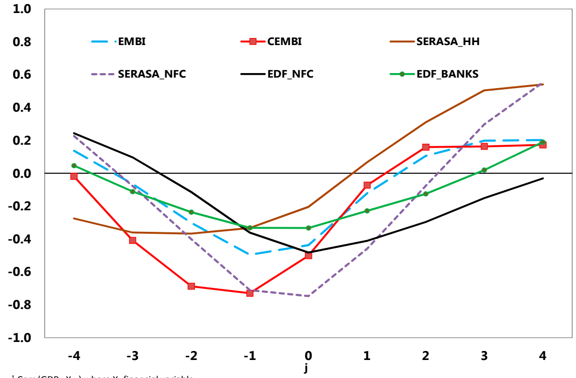
Source: Bloomberg.

A number of financial indicators are correlated with real GDP growth—and appear to lead real GDP growth by a few quarters. The correlation between year-on-year real GDP growth rates and various contemporaneous and lagged financial indicators (X_{t+j}) is displayed in Figure 21. The 6 indicators chosen represent the financial conditions of the sovereign, corporate, household and financial sectors of the economy.[17] All the financial indicators are negatively correlated with the GDP growth rate and most of the indicators lead growth (for example, the correlation of the one-period-ahead CEMBI with the current GDP growth rate is -0.8, and the contemporaneous correlation is approximately -0.5), which implies that unfavorable financial conditions in the current period tend to be associated with slower growth in the future. In general, the corporate indicators of financial pressure exhibit the strongest correlation with the GDP growth rate. These findings are consistent with previous empirical findings which suggest that corporate and sovereign spreads behave in a countercyclical fashion in EMs (Neumeyer and Perri, 2005; Caballero et al., 2015).

[16] According to Serasa Experian, in August 2015, the number of delinquent companies reached 4 million, and their overall debt amounted to R$91 billion.

[17] See Appendix I for a description of variables.

Figure 21. Brazil: Correlations of Growth and Financial Variables [1]
(Percent)



[1] Corr (GDP_t, X_{t+j}) where X=financial variable.
Sources: Serasa Experian, Moody's Credit Edge, Bloomberg; and authors' estimates.

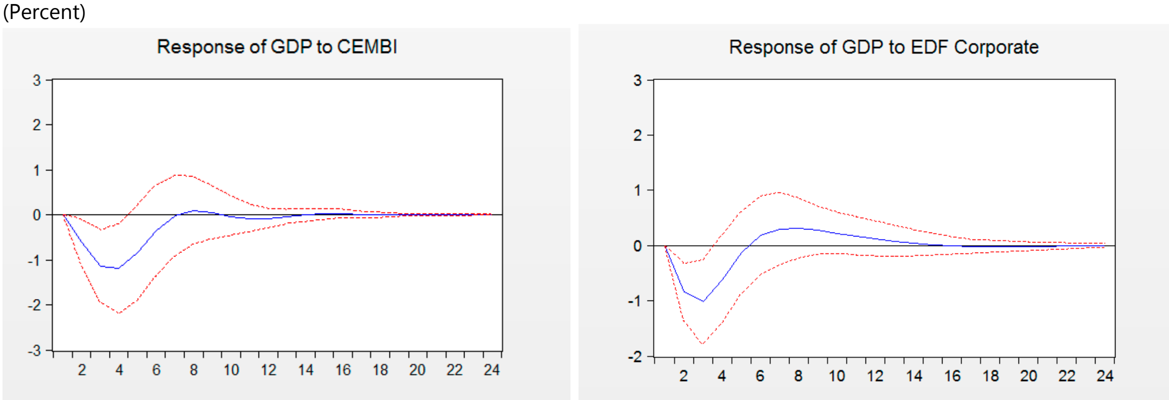
Bivariate VARs are used again to examine causal relationships between financial variables and growth. Because the aforementioned correlations do not necessarily establish a causal relationship between financial market conditions and real economic activity, we estimate a simple bivariate VAR model with corporate financial indicators and the year-on-year real GDP growth rate.[18] As we did earlier, we identify the shock to leverage by imposing a recursive assumption. In particular, we assume that the real GDP growth rate affects financial indicators both contemporaneously and with a lag, while financial indicators affects GDP growth rate only with a lag.

A worsening in financial variables is found to be associated with slower growth in Brazil. The impulse-response function of the real GDP growth rate to a one standard deviation unanticipated increase in the CEMBI (about 60 basis points) is “u-shaped”. The GDP growth rate reaches its trough four quarters after the increase in the CEMBI, and gradually reverts back to the steady state. At its trough, the real GDP growth rate is approximately one percentage point lower than in the absence of the financial shock. The deterioration of GDP growth in response to an unanticipated increase in the corporate default index is most felt in the third quarter, but its trough effect on GDP growth is similar (Figure 22). The Granger causality tests suggest that CEMBI and corporate EDFs help to predict real GDP growth, but real GDP growth does not have predictive power on these financial variables.[19]

[18] There are other relevant macroeconomic variables that affects business cycles, such as U.S. and domestic monetary policy rates. Considering the relatively short sample period, the dimension of VAR is limited. Relying on previous empirical literature that finds a significant effect of financial variables on GDP growth rate after controlling for these variables (Akinci, 2013, Uribe and Yue, 2006, and Caballero et al., 2015), analysis of a larger VAR is left for future research.

[19] The result for CEMBI is robust to the order of VAR.

Figure 22. Brazil: Impulse Response Function of a Financial Shock
(Percent)



B. DSGE Estimation

In this section we estimate a DSGE model with financial frictions using a Bayesian method. Our main objective is to quantify the role of financial shocks over the business cycle in Brazil. A DSGE model estimation allows for straightforward identification of the structural shocks of the economy. All estimable equations are derived from the optimizing behavior of the agents, and all structural shocks are directly modeled and, thus, identified. In particular, we consider five aggregate shocks in the economy: preference shocks, aggregate labor productivity shocks, investment efficiency shocks, government spending shocks, and financial shocks.

The EM literature has developed some open economy DSGE models with financial frictions. Neumeyer and Perri (2005), Uribe and Yue (2006), and Chang and Fernandez (2013) show that financial sectors play non-negligible roles in shaping EM business cycles, suggesting that a close investigation of balance sheets of each economic sector (both at a macro and micro level) may reveal important information for assessing current and future conditions of the economy. Fernandez and Gulan (2015) have added to this line of thought by explicitly incorporating the corporate balance sheet channel into the model. A DSGE estimation exercise turned particularly useful in quantifying the role of various shocks over the business cycles. Similar to Smets and Wouters (2007), who have estimated a New Keynesian model in a U.S. setting, Christiano et al. (2014) have estimated a financial accelerator model with risk shocks, and found that risk shocks account for a large fraction of fluctuations of the U.S. economy. In an EM setting, Chang and Fernandez (2013) conducted a similar exercise using data from Mexico and their estimation result, too, supports the presence of financial frictions.

Our model is adapted from Bernanke et al. (1999) financial accelerator, augmented for financial shocks following Christiano et al. (2014). In contrast to standard real business cycle models, in the financial accelerator model the financial sector is explicitly considered. Firms engage in financial contracts to finance their investment and, once contracts are closed, production takes a place, idiosyncratic shocks materialize, and firms may default depending on the realization of shocks. Since state verification is costly, lenders request a risk premium from borrowers. If the variance of idiosyncratic productivity shock is high, lenders request a higher risk premium.[20] Christiano et al. (2014) introduce risk shocks into the financial accelerator model by assuming that the variance of a firm's idiosyncratic productivity is stochastic. Aggregate shocks are realized at the beginning of the production period, while idiosyncratic shocks are realized after input decisions are taken and financial contracts are finalized. The model is a simplified "real" version of Christiano et al. (2014), which we briefly discuss in Box 1 and, in more detail, in Appendix III.

Box 1. The Financial Accelerator Model

The characterization of the financial contract plays a key role in the model, thus, we first describe how default productivity threshold is determined; we then define the expected return of firm, and, finally, we discuss an optimal financial contract under financial frictions in relation to financial shocks.

Default threshold

The firm purchases capital $q_t k_t$, and finances the purchase by raising external debt b_t for a given level of equity (net worth) e_t. The accounting identity imposes $q_t k_t = b_t + e_t$. At the end of the period, the firm receives a return on capital $\epsilon_{i,t+1} R_{k,t+1}$, where $\epsilon_{i,t+1}$ represents idiosyncratic productivity, and $R_{k,t} = \left(\frac{\alpha y_t}{q_{t-1} k_{t-1}} + (1-\delta)\frac{q_t}{q_{t-1}}\right)$ is the aggregate return on capital determined by the aggregate state of the economy. Earnings is defined as $\epsilon_{i,t+1} R_{k,t+1} q_t k_t - R_{b,t+1} b_t$, where the first term represents revenue from the operation, and the second term is a sum of principal and interest. If the realization of the idiosyncratic productivity shock decreases revenue to the point that it is insufficient to service the external debt, firms default. The default threshold is defined as:

$$\epsilon_{i,t+1}^* = \frac{R_{b,t+1} b_t}{R_{k,t+1} q_t k_t}$$

and implies that firms endogenously determine a default threshold for given levels of $R_{k,t+1}$, $R_{b,t+1}$, b_t, and $q_t k_t$.

Expected earnings

For every given level of return on capital, and interest and debt level, firms survive as long as total assets are greater or equal to total liabilities. Accordingly, the expected earnings (prior to the realization of the idiosyncratic productivity shock) is equal to:

$$\int_{\epsilon_{i,t+1}^*}^{\infty} \left(\epsilon_{i,t+1} R_{k,t+1} q_t k_t - R_{b,t+1} b_t\right) f\left(\epsilon_{i,t+1}\right) d\epsilon_{i,t+1}$$

where $f(\,)$ is a PDF of ϵ. We can further simplify this to

$$\left(1 - \Delta(\epsilon_{t+1}^*)\right) R_{k,t+1} k_t - R_{b,t+1} b_t \left(1 - F(\epsilon_{i,t+1}^*)\right)$$

where $F(\,)$ is CDF of ϵ, and $\Delta\left(\epsilon_{i,t+1}^*\right) \equiv \int_0^{\epsilon_{i,t+1}^*} \epsilon_{i,t+1} f\left(\epsilon_{i,t+1}\right) d\epsilon_{i,t+1}$.

Optimal debt contract

Firms maximize expected earnings, choosing debt b_t, and the interest rate $R_{b,t+1}$ taking e_t as given, subject to the lenders' individual rationality condition. More formally, the optimal debt contract solves the following maximization problem:

[20] The characterization of the financial contract in this study mostly follows Townsend (1979).

$$Max\ \left(1-\Delta(\epsilon^*_{t+1})\right)R_{k,t+1}k_t - R_{b,t+1}b_t\left(1-F\left(\epsilon^*_{i,t+1}\right)\right) \tag{1}$$

$$subject\ to$$

$$R_{b,t+1}b_t\left(1-F\left(\epsilon^*_{i,t+1}\right)\right)+\Delta(\epsilon^*_{t+1})R_{k,t+1}q_tk_t(1-\mu)=R_{d,t}b_t \tag{2}$$

$$e_t = q_tk_t - b_t.$$

where $R_{d,t}$ is the return on risk-free assets. Equation (2) characterizes the lenders' individual rationality condition, which guarantees lenders' participation in the contract. The first term on the left-hand side represents the repayment of debt in case of firms' survival. The second term in (2) represents the liquidation value of the firm in case of default. Default is costly, since a fraction μ of the net worth is lost in the liquidation process, and it results in additional risk premium for lenders who participate in the contract. The right-hand side of the equation represents lenders' "outside option" constituted by a risk free bond. Because lenders lend to a large number of firms, they are able to diversify risk and they pay their own creditors in all states of the world whatever they invested plus the risk-free interest. The first order conditions with respect to $R_{b,t+1}$ and b_t are:

$$\frac{Prob\left(R_{k,t+1}\right)}{R_{d,t}}\left(-b_t\left(1-F(\epsilon^*_{t+1})\right)\right)+\frac{Prob\left(R_{k,t+1}\right)}{R_{d,t}}\lambda^F_{(t+1)}\left(b_t(1-F(\epsilon^*_{t+1})-f(\epsilon^*_{t+1})\epsilon^*_{t+1}\mu)\right)=0$$

$$E_t\left(\frac{R_{k,t+1}}{R_{d,t}}\left(1-\Delta(\epsilon^*_{t+1})-\epsilon^*_{t+1}\left(1-F(\epsilon^*_{t+1})\right)+\lambda^F_{t+1}\left(\epsilon^*_{t+1}\left(1-F(\epsilon^*_{t+1})\right)+\Delta(\epsilon^*_{t+1})(1-\mu)\right)\right)\right)$$
$$= E_t(\lambda^F_{t+1})$$

Financial shocks

The first order conditions of the financial contract imply that $f(\epsilon^*_{t+1})$, $\Delta(\epsilon^*_{t+1})$, and the default probability $F(\epsilon^*_{t+1})$, are key determinants of the contract outcome. Assuming that $\epsilon_{i,t+1}$ is log-normally distributed $LN(\mu,\sigma_t^2)$, and normalizing the mean of $\epsilon_{i,t+1}$ to unity, it can be shown that $\epsilon_{i,t+1}\sim LN\left(-\frac{\sigma_t^2}{2},\sigma_t^2\right)$. Under this assumption,

$$F(\epsilon^*_t)=\Phi\left(\left(ln\epsilon^*_t+\frac{\sigma_t^2}{2}\right)/\sigma_t\right)$$

$$f(\epsilon^*_t)=\frac{1}{\epsilon^*_t\sigma_t}\phi\left(\left(ln\epsilon^*_t+\frac{\sigma_t^2}{2}\right)/\sigma_t\right)$$

$$\Delta(\epsilon^*_t)=1-\Phi\left(\left(-ln\epsilon^*_t+\frac{\sigma_t^2}{2}\right)/\sigma_t\right)$$

where Φ and ϕ are the CDF and the PDF of the standard normal distribution respectively. It is straightforward from the expression that the variance of idiosyncratic productivity σ_t is a key determinant of default probability. In the standard financial accelerator model, the variance of idiosyncratic shocks is constant. As a result, the risk premium is constant for a given level of aggregate shocks. We introduce financial shocks by further assuming that the variance of the idiosyncratic productivity shock is stochastic, as in Christiano et al. (2014):

$$\sigma_t=(1-\rho)\sigma_{ss}+\rho\sigma_{t-1}+\epsilon_{s,t}$$

Under such assumption, the risk premium varies not only due to other aggregate shocks, but also due to exogenous changes in the variance of idiosyncratic productivity. For any given level of other aggregate shocks, shocks to the variance directly affect the default threshold, and in turn, affect also the outcome of the financial contract. *Since the exogenous changes in the variance of idiosyncratic shocks independently affect the risk premium, we interpret exogenous changes in the variance as financial shocks.*

Household and government behavior

The representative agent maximizes utility, subject to his budget constraint $c_t + i_t + T_t = y_t$, and a capital accumulation equation (investment is subject to adjustment costs). The agent chooses consumption c_t, capital k_t, and labor n_t. Output is equal to $y_t = k_{t-1}^{\alpha}\left(z_{y,t}n_t\right)^{1-\alpha}$ where $z_{y,t}$ represents the aggregate productivity shock, which is AR(1). The government finances its stochastic spending through lump-sum taxation $g_{ss}z_{g,t} = T_t$. The government spending shock $z_{g,t}$ is also AR(1). We assume that the preference on consumption is stochastic, represented by $z_{c,t}$, also AR(1). Shocks to the preference directly affect households' intertemporal optimality conditions, and thus, the consumption-saving decisions. We introduce also investment efficiency shocks, $z_{i,t}$, and assume that the fraction of investment converted into capital follows a stochastic process. Preference, investment efficiency, and government spending shocks all together constitute aggregate demand shocks.

The model provides a useful understanding of how financial shocks are transmitted through firms' balance sheets to the real economy. The empirical analysis in previous sections established the link between the leverage ratio and real GDP growth, and the link between financial variables, represented by corporate spreads, and real GDP growth. The model links these findings by providing an interpretation of how deteriorating financial conditions affect firms' balance sheets health, and in turn, macroeconomic performance. In the model, corporate bond spreads increase immediately in response to financial shocks. As the financial environment deteriorates, the net worth (or equity) of firms decreases, leading to an increase in the leverage ratio. Due to higher spreads and higher leverage, firms face higher financial distress and consequently the default rate increases. In response, in following periods, firms start de-leveraging and, over the course of this process, hire and invest less, which leads to a contraction of the economy.

As a first step, to quantify the role of financial shocks, we estimate aggregate TFP, financial, and investment efficiency shocks along with all structural parameters. We parameterize the variance of investment and preference shocks at a substantially low level to focus on the role of financial shocks.[21] We use cyclical components of HP filtered GDP, consumption and investment along with the CEMBI for the sample period 2005Q1−2015Q1 to estimate the financial shocks. Next, we calculate the historical contribution of each shock to HP filtered GDP, consumption and investment as in Smets and Wouters (2007): (i) we derive equilibrium conditions of the DSGE model, and estimate parameters that best fit the data for the sample period; (ii) we parameterize the model with the estimation results, feed in the actual data series into the equilibrium conditions, and calculate the realization of each shock for the sample period. This allows us to calculate the contribution of each shock to the realization of the historical data series.

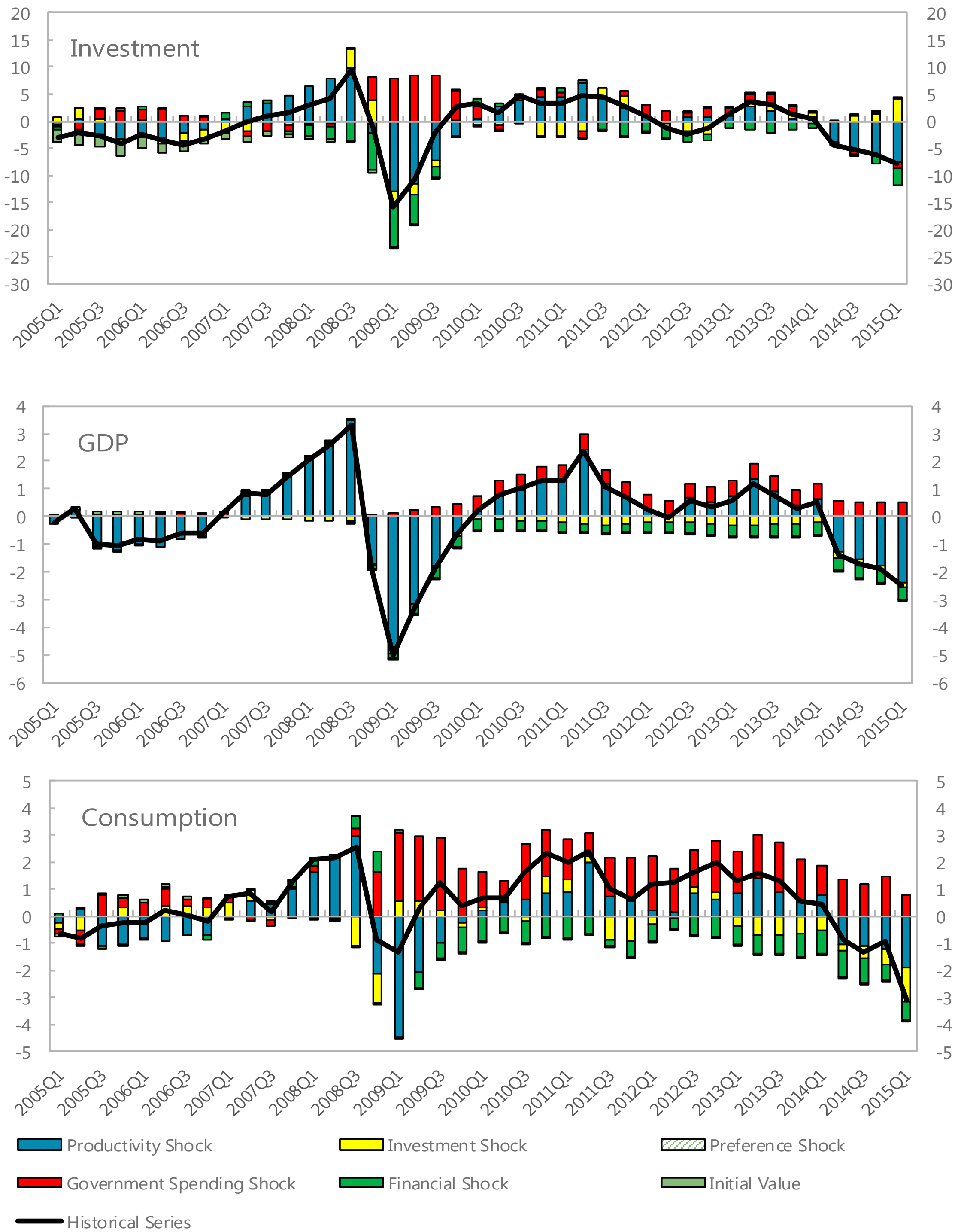
We find that the effect of financial shocks on real variables is non-negligible. While aggregate productivity shocks account for a large fraction of GDP and investment

[21] See Tables 1 and 2 in Appendix III for detail on estimated and calibrated parameters.

fluctuations, financial shocks also play an important role over the cycles. For the longest part of the sample period, TFP shocks account for a large fraction of the consumption and investment fluctuations, and more prominently of GDP. However, the role of financial shocks becomes increasingly important during the recent economic downturn. In particular, financial shocks are found to contribute about 5 percentage points to the drop in investment in 2015Q1 (Figure 20). We find that financial shocks also explain an additional 0.5 percentage points drop in consumption and GDP in 2015Q1. Government spending and financial shocks become more relevant only after the global financial crisis of 2008-09. However, while government spending shocks affect consumption, investment and GDP positively, financial shocks play the opposite role (Figure 23).

However, the estimation results should be interpreted as the lower bound for the relevance of financial shocks for real variables. In the model, financial shocks directly affect investment decisions, and in turn, the amount of capital employed in the production process. Because investment is substantially smaller than capital, it may not be sufficient to generate a large fluctuation in GDP. Considering that the financial frictions affect employment decisions indirectly, through the general equilibrium channel, in the model the relatively limited role of financial shocks for GDP fluctuations is unsurprising. Indeed, it is acknowledged in literature that, in a large class of DSGE models, TFP shocks are one of the most important sources of fluctuations. Financial shocks contribution to GDP fluctuation would substantially increase if our model could allow financial frictions to directly affect firms' employment decisions (Jermann and Quadrini, 2012). This is a complex exercise that we leave for future research.

Figure 23. Brazil: Historical Decomposition Investment, GDP, and Consumption[1]



[1] The series are HP filtered.

IV. Conclusion

In a rapidly changing economic environment, the study of macro-financial linkages must resort to multiple strategies to remain informative for policy design and to help identify mounting risks in a timely way. We have followed this approach in analyzing the intersectoral linkages in Brazil, focusing on the rapidly deteriorating position of the largest borrower sector in the economy—the NFC sector. Our analysis goes beyond the empirical approach, to identify the sources of shocks affecting real variables in a DSGE model. The model provides a formal framework to analyze the propagation of shocks from the financial sector to the real economy through NFC balance sheets.

Corporate leverage in Brazil is a concern, inasmuch as it is accompanied by declining firm profitability and cash buffers. Arguably, the increase in debt liabilities, together with the decrease in the share of equity-to-assets and cash holdings, might be caused by a mitigation strategy rather than by exacerbation of financial shocks. Indeed, since the onset of the global financial crisis, investors' search for higher yields has increased EMs access to financial markets and many of them, including Brazil, have taken advantage of it. However, in Brazil, the increase in leverage is a concern because it is coupled with a profitability-induced decline in equity. The concern is even more real as shocks to NFCs' leverage are found to negatively affect the country's future growth potential. High leverage, and low liquidity and profitability of the three largest firms are worrisome also because the buildup of risks in these entities constitutes a growing contingent liability for the public sector.

Estimation based on the DSGE model suggests that the effect of financial shocks on real variables is non-negligible. We estimate that financial shocks contributed about 5 percentage points to the drop in investment, and 0.5 percentage points to the drop in consumption and GDP in 2015Q1. Financial shocks have become more relevant in explaining fluctuations in GDP during the recent economic slowdown in Brazil.

References

Akinci, O., 2013, Global Financial Conditions, Country Spreads and Macroeconomic Fluctuations in Emerging Economies, Journal of International Economics, vol. 91(2), pages 358-371.

Bates, T. W., Kahle, K. M. and R. M. Stulz, 2009, Why Do U.S. Firms Hold So Much More Cash than They Used To?, The Journal of Finance, 64: 1985–2021

Bernanke, B.S., Gertler, M., and S. Gilchrist, The Financial Accelerator in a Quantitative Business Cycle Framework, Chapter 21, Handbook of Macroeconomics, Elsevier, 1999, Volume 1, Part C, Pages 1341-1393.

Buera, F. J., Kaboski, J. P., and Y. Shin, 2011, Finance and Development: A Tale of Two Sectors, American Economic Review, 101(5): 1964-2002

Caballero, J., Fernandez, A., and J. Park, 2015, On Corporate Borrowing, Credit Spreads and Business Cycles in EMEs, An Empirical Investigation, Mimeo.

Caceres, C. and F. Lipinsky, 2014, Switzerland: Technical Note—Stress Testing the Banking System, Select Issues IMF Country Report No. 14/267.

Chang, R. and A. Fernandez, 2013, On the Sources of Fluctuations in Emerging Economies, International Economic Review, Vol. 54, No. 4, November 2013, 1267-1295.

Christiano, L.J., Motto, R., and M. Rostagno, 2014, Risk Shocks, American Economic Review, 104(1): 27-65.

Colla, P., Ippolito, F. and K. Li, 2013, Debt Specialization, The Journal of Finance, 68: 2117–2141

Crouzet, N., 2014, Aggregate Implications of Corporate Debt Choices, Mimeo.

Fernandez, A. and A. Gulan, 2015, Interest Rates, Leverage, and Business Cycles in Emerging Economies: The Role of Financial Frictions, forthcoming American Economic Journal: Macroeconomics.

Garcia-Escribano, M., 2013, Brazil: Technical Note on Consumer Credit Growth and Households Financial Stress, IMF Country Report No. 13/149, International Monetary Fund.

Gilchrist, S., V. Yankov, and E. Zakrajšek, 2009, Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets, Journal of Monetary Economics 56 (4): 471, 93.

Gilchrist, S. and E. Zakrajšek, 2012, Credit Spreads and Business Cycle Fluctuations, American Economic Review, vol. 102(4), pages 1692-1720, June.

Gilchrist, S. and B. Mojon, 2014, Credit Risk in the Euro Area, NBER Working Papers 20041, National Bureau of Economic Research, Inc.

Gray D., Gross M., Paredes J., and M. Sydow, 2013, Modeling Banking, Sovereign, and Macro Risk in a CCA Global VAR, International Monetary Fund.

IMF 2013, “Understanding Housing Market in Brazil: Mark II,” Brazil: Selected Issues Paper for the 2013 Article IV Consultation, IMF Country Report 13/313, International Monetary Fund.

IMF, 2015, “Non-financial Corporate Sector Vulnerabilities,” Brazil: Selected Issues Paper for the 2014 Article IV Consultation, IMF Country Report 15/122, International Monetary Fund.

IMF, 2015b, “Vulnerabilities, Legacies, and Policy Challenges: Risks Rotating to Emerging Markets,” Global Financial Stability Report, October 2015, International Monetary Fund.

IMF, 2015c, Brazil: Staff Report for the 2014 Article IV Consultation, IMF Country Report 15/121, International Monetary Fund.

IMF, 2015d, Balance Sheet Analysis in Fund Surveillance, Policy Paper and Reference Note, International Monetary Fund.

Jermann, U. and V. Quadrini, 2012, Macroeconomic Effects of Financial Shocks, American Economic Review, 102(1): 238-71.

Neumeyer, P.A. and F. Perri, 2005, Business cycles in emerging economies: the role of interest rates, Journal of Monetary Economics, 52, 345-380.

Smets, F., and R. Wouters, 2007, Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, American Economic Review, 97(3): 586-606

Tosoni, G. A., 2014, Wage Share and Economic Growth in Latin America, 1950-2011, CEPAL Review

Townsend, Robert M., 1979, Optimal contracts and competitive markets with costly state verification, Journal of Economic Theory, Elsevier, vol. 21(2), pages 265-293, October.

Uribe, M. and V. Yue, 2006, Country Spreads and Emerging Countries: Who Drives Whom?, Journal of International Economics, Vol. 69, 6-36.

Appendix I. Data Sources for NFC Analysis

S&P Capital IQ—Corporate Balance Sheets

We collect major balance sheet and cash flow statement items of Brazil's non-financial corporates from Capital IQ database, at a quarterly frequency. The original data set is in nominal U.S. dollars but we carry out the analysis on real, local currency data: assessing real values is important to account for the effects of the temporarily high inflation over the past few quarters, mainly driven by a sharp adjustment in regulated prices, while local currency data allows isolating the effect of the pronounced depreciation of the real—60 percent over the past year to October—on financial variables. Although financial intermediaries are central to inter-sectoral linkages in the economy, depository corporations and other financial sector companies are excluded from our sample, because their balance sheet structure is qualitatively different from that of NFCs. We limit the scope of our sample to top 600 firms, based on the average total asset from 2010Q1 to 2015Q1. The variables used in the analysis are asset weighted averages across firms. The sample is an unbalanced panel, thus the number of corporates used in calculating average statistics may vary over time. We drop the top and bottom 0.5 percentile of the observations.

Serasa Experian

We present the index of delinquencies on financial credits and unpaid bills borrowed from the Serasa database tracking the monthly flow of delinquency notifications for debts of individuals and corporates. The published index comprises information on overdue debt to financial institutions as well as to nonfinancial enterprises, the number of bounced checks and notary protests.

Moody's KMV Credit Edge

The database comprises expected default frequencies (EDFs) for publicly listed companies, where EDF measures the probability that a company will default within a given period.

Brazil's Central Bank

All the information on bank credit, banks spreads, maturities of loans, NPLs, households debt and debt service are obtained directly from the central bank web site.

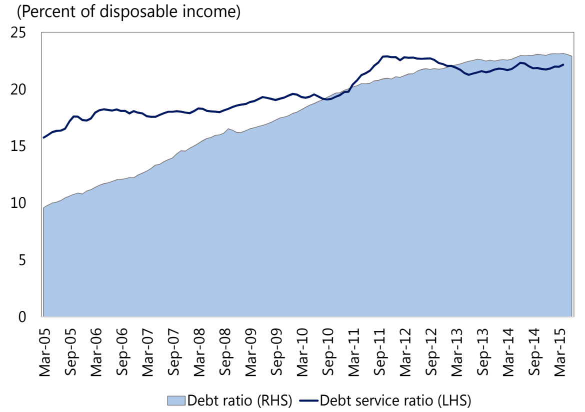
Bloomberg

The EMBI, which is a spread between sovereign bond yields and safe assets, represents sovereign bond market conditions while the CEMBI is a counterpart of EMBI for the corporate sector.

Appendix II. Households Sector Balance Sheets

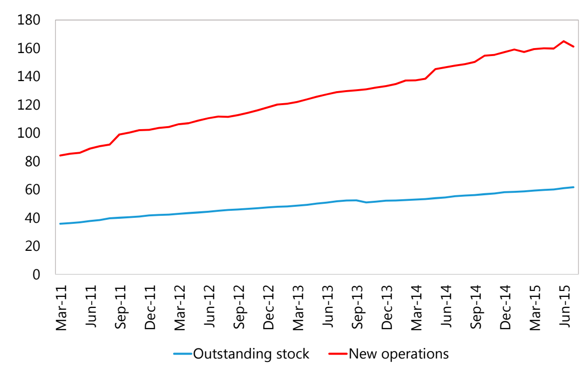
The steady growth in credit over the past decade has contributed to financial deepening and inclusion but also pushed households debt levels up. Households leverage has increased steadily since the pre-crisis period, and amounted to 46 percent of disposable income in June 2015 according to central bank estimates. Households debt servicing cost (interest and principal as a share of disposable income), is high but has stabilized since the peak of 23 percent in late 2011, reflecting lengthening of the average maturity of new operations (Figures 1 and 2). These estimates represent, however, a lower bound of debt burden in Brazil and exclude credit offered by retailers outside of financial institutions and credit extended by real estate developers (Garcia-Escribano, 2013). Mitigating debt service interest rate sensitivity is the overwhelming predominance of fixed-rate loans, while collateral securitization and payroll deduction provide some shelter to banks from defaults. Some signs of financial distress are cropping up more recently, due to declining real income and increasing unemployment, with a tick in bounced checks, higher defaults on credit cards, and default rates on bank loans (Figure 3).[22] Average NPLs on households' debt have, nevertheless, been stable over the last several years and NPLs on mortgages—which account for 15 percent of total loans – are contained, at about 2 percent.

Figure 1. Brazil: Households Debt Burden
(Percent of disposable income)



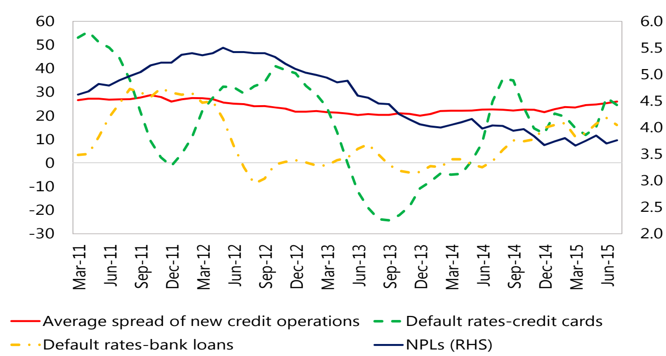
Source: Central Bank of Brazil.

Figure 2. Brazil: Average Term to Maturity of Households Debt
(Months)



Source: Central Bank of Brazil.

Figure 3. Brazil: Households Indicators of Financial Stress
(Percent)



Source: Central Bank of Brazil.

[22] According to Serasa Experian, in August the number of individuals with arrears on at least one obligation was 57.2 million and their total debt amounts to R$246 billion.

Appendix III. DSGE Model

1 Benchmark model - no financial frictions

The representative agent maximizes utility, subject to his budget constraint $c_t + i_t + T_t = y_t$, and a capital accumulation equation (investment is subject to adjustment costs). The agent chooses consumption c_t, capital k_t, and labour n_t. Output is equal to $y_t = k_{t-1}^{\alpha}(z_{y,t}n_t)^{1-\alpha}$. The government finances stochastic government spending through lump-sum taxation $g_{ss}z_{g,t} = T_t$. The Lagrangian for this problem is:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left(z_{c,t} \frac{c_t^{1-\gamma}}{1-\gamma} - \tau_n \frac{n^{1+\phi}}{1+\phi} + \lambda_t \left(k_{t-1}^{\alpha}(z_{y,t}n_t)^{1-\alpha} - c_t - i_t - T_t \right) \right.$$
$$\left. + \lambda_t q_t \left((1-\delta)k_{t-1} + z_{i,t} i_t \left(1 - S\left(\frac{i_t}{i_{t-1}} \right) \right) - k_t \right) \right)$$

The first order conditions with respect to consumption, capital, labour, and investment are:

$$\lambda_t = \frac{1}{c_t^{\gamma}} \tag{1}$$

$$1 = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left(\alpha \frac{y_{t+1}}{q_t k_t} + (1-\delta) \frac{q_{t+1}}{q_t} \right) \right) \tag{2}$$

$$(1-\alpha)\frac{y_t}{n_t} = \tau_n \frac{n_t^{\phi}}{\lambda_t} = \tau_n \frac{n_t^{\phi} c_t^{\gamma}}{z_{c,t}} \tag{3}$$

$$1 = q_t z_{i,t} \left(1 - S\left(\frac{i_t}{i_{t-1}} \right) - S'\left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right) + \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} z_{i,t+1} S'\left(\frac{i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right) \tag{4}$$

The third equation determines q_t, which is equal to one in the steady state. A quadratic adjustment cost function is used:

$$S\left(\frac{i_t}{i_{t-1}} \right) = \frac{\phi_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2$$

The shock processes for TFP, investment efficiency, consumption demand and government spending are:

$$\ln(z_{y,t}) = \rho_{z_y} \ln(z_{y,t-1}) + \varepsilon_{z_y,t} \tag{5}$$

$$\ln(z_{i,t}) = \rho_{z_i} \ln(z_{i,t-1}) + \varepsilon_{z_i,t} \tag{6}$$

$$\ln(z_{c,t}) = \rho_{z_c} \ln(z_{c,t-1}) + \varepsilon_{z_c,t} \tag{7}$$

$$\ln(z_{g,t}) = \rho_{z_g} \ln(z_{g,t-1}) + \varepsilon_{z_g,t} \tag{8}$$

2 Financial frictions

2.1 Preliminaries - the default threshold

The firm purchases capital $q_t k_t$ and raises debt b_t. At the end of the period, it receives a return $\varepsilon_{i,t+1} R_{k,t+1}$ on its capital (consisting of an idiosyncratic and an aggregate component), and pays back the principal plus interest $(1+i_{b,t})b_{t-1} = R_{b,t} b_{t-1}$. As a result, earnings are:

$$\varepsilon_{i,t+1} R_{k,t+1} q_t k_t - R_{b,t+1} b_t$$

As in the standard model (see equation (2)), the aggregate component in the return on capital is equal to:

$$R_{k,t} = \left(\alpha \frac{y_t}{q_{t-1} k_{t-1}} + (1-\delta) \frac{q_t}{q_{t-1}} \right) \tag{9}$$

Firms can default. A firm defaults if the idiosyncratic return component falls below the default threshold ε^*_{t+1}, e.g. if the value of assets falls below liabilities:

$$\varepsilon^*_{t+1} R_{k,t+1} q_t k_t - R_{b,t+1} b_t = 0$$

The default threshold is defined as:

$$\varepsilon^*_t = \frac{R_{b,t} b_{t-1}}{R_{k,t} q_{t-1} k_{t-1}} \tag{10}$$

We can also think of the default threshold as a certain measure of leverage that includes the cost of debt and asset returns.

2.2 Preliminaries - earnings

Suppose, $R_{k,t+1}$ and $R_{b,t+1}$ are given, then firms survive in all states, in which assets are equal or bigger than liabilities. Accordingly, expected earnings are equal to:

$$\int_{\varepsilon^*_{t+1}}^{\infty} (\varepsilon_{i,t+1} R_{k,t+1} q_t k_t - R_{b,t+1} b_t) f(\varepsilon_{i,t+1}) d\varepsilon_{i,t+1}$$

In the following it will be useful to introduce some notation. Beginning with liabilities:

$$\int_{\varepsilon^*_{t+1}}^{\infty} (-R_{b+1,t} b_t) f(\varepsilon_{i,t+1}) d\varepsilon_{i,t+1} = -R_{b+1,t} b_t (1 - F(\varepsilon^*_{t+1}))$$

where $F()$ is the cumulative distribution function of $\varepsilon_{i,t+1}$. We turn now to assets. We assume that the idiosyncratic shock has an unconditional mean of one:

$$E(\varepsilon_{i,t+1}) = \int_{\varepsilon^*_{t+1}}^{\infty} \varepsilon_{i,t+1} f(\varepsilon_{i,t+1}) d\varepsilon_{i,t+1} + \int_{0}^{\varepsilon^*_{t+1}} \varepsilon_{i,t+1} f(\varepsilon_{i,t+1}) d\varepsilon_{i,t+1} \equiv 1$$

And define the second part as delta:

$$\Delta(\varepsilon_{t+1}^*) \equiv \int_0^{\varepsilon_{t+1}^*} \varepsilon_{i,t+1} f(\varepsilon_{i,t+1}) d\varepsilon_{i,t+1}$$

Then, the first part is equal to:

$$\int_{\varepsilon_{t+1}^*}^{\infty} \varepsilon_{i,t+1} f(\varepsilon_{i,t+1}) d\varepsilon_{i,t+1} = 1 - \Delta(\varepsilon_{t+1}^*)$$

And earnings can be written as:

$$(1 - \Delta(\varepsilon_{t+1}^*))R_{k,t+1}k_t - R_{b,t}b_t(1 - F(\varepsilon_{t+1}^*))$$

2.3 Time-varying variance of returns

Suppose $\varepsilon_{i,t+1}$ is log-normally distributed $LN(\mu_t, \sigma_t^2)$.
Next, considering mean preserving shifts only, we set $E(\varepsilon_{i,t+1}) = 1$, then it can be shown that $\varepsilon_{i,t+1}$ is $LN(-\frac{\sigma_t^2}{2}, \sigma_t^2)$. Under this assumption, it can be shown that:

$$F(\varepsilon_t^*) = \Phi\left(\frac{\ln \varepsilon_t^* + \sigma_t^2/2}{\sigma_t}\right) \tag{11}$$

$$F'(\varepsilon_t^*) = f(\varepsilon_t^*) = \frac{1}{\varepsilon_t^* \sigma_t}\phi\left(\frac{\ln \varepsilon_t^* + \sigma_t^2/2}{\sigma_t}\right) \tag{12}$$

$$\Delta(\varepsilon_t^*) = 1 - \Phi\left(\frac{-\ln \varepsilon_t^* + \sigma_t^2/2}{\sigma_t}\right) \tag{13}$$

$$\Delta'(\varepsilon_t^*) = f(\varepsilon_t^*)\varepsilon_t^*$$

where Φ and ϕ are the cumulative distribution function and the probability density function of the standard normal distribution. We can include now another shock that captures changes in the variance:

$$\sigma_t = (1 - \rho)\sigma_{ss} + \rho\sigma_{t-1} + \varepsilon_{s,t} \tag{14}$$

2.4 Financial accelerator debt contract

The firm maximises expected earnings, choosing debt b_t and the interest rate $R_{b,t+1}$,

$$(1 - \Delta(\varepsilon_{t+1}^*))R_{k,t+1}q_t k_t - R_{b,t+1}b_t(1 - F(\varepsilon_{t+1}^*))$$

taking equity $e_t = q_t k_t - b_t$ as given, subject to a borrowing constraint, the zero profit condition of the lender:

$$R_{b,t+1}b_t(1 - F(\varepsilon_{t+1}^*) + \Delta(\varepsilon_{t+1}^*)R_{k,t+1}q_t k_t(1 - \mu) = R_{d,t}b_t$$

The lender receives the principal plus interest in states when the firm doesn't default. If a firm defaults, the lender receives the assets that he is able to recover. Since the lender lends to a large number of firms, he is able to diversify risk and pays its own creditors in all states of the world what they invested plus the risk-free interest rate $R_{d,t}b_t$. As a result, the Lagrangian, the objective of the firm, takes the following form,

$$L = E_t\Bigg(\frac{1}{R_{d,t}}((1-\Delta(\varepsilon^*_{t+1}))R_{k,t+1}q_tk_t - R_{b,t}b_t(1-F(\varepsilon^*_{t+1})))$$

$$+\lambda^F_{t+1}\frac{1}{R_{d,t}}(R_{b,t+1}b_t(1-F(\varepsilon^*_{t+1}) + \Delta(\varepsilon^*_{t+1})R_{k,t+1}q_tk_t(1-\mu)) - b_t\Bigg)$$

where the default threshold is defined as,

$$\varepsilon^*_{t+1} = \frac{R_{b,t+1}b_t}{R_{k,t+1}q_tk_t} = \frac{R_{b,t+1}b_t}{R_{k,t+1}(e_t+b_t)}$$

where we made use of the accounting identity $q_tk_t = e_t + b_t$. Note for example that:

$$\frac{\partial\varepsilon^*_{t+1}}{\partial R_{b,t+1}}R_{b,t+1} = \varepsilon^*_{t+1}$$

The first order condition with respect to b_t is:

$$\frac{\partial L}{\partial b_t} = E_t\Bigg(\frac{1}{R_{d,t}}\Bigg((1-\Delta(\varepsilon^*_{t+1}))R_{k,t+1} + \lambda^F_{t+1}(\Delta(\varepsilon^*_{t+1})R_{k,t+1}(1-\mu)-1)$$

$$+\lambda^F_{t+1}R_{b,t+1}b_tf(\varepsilon^*_{t+1})\varepsilon^*_{t+1}\mu\frac{1}{q_tk_t}$$

$$-R_{b,t+1}(1-F(\varepsilon^*_{t+1}) + \lambda^F_{t+1}(R_{b,t+1}(1-F(\varepsilon^*_{t+1}) - f(\varepsilon^*_{t+1})\varepsilon^*_{t+1}\mu))\Bigg)\Bigg) = 0$$

The first order conditions with respect to the interest rate $R_{b,t+1}$ is

$$\frac{\partial L}{\partial R_{b,t+1}} = \frac{Prob(R_{k,t+1})}{R_{d,t}}(-b_t(1-F(\varepsilon^*_{t+1})))$$

$$+\frac{Prob(R_{k,t+1})}{R_{d,t}}\lambda^F_{t+1}(b_t(1-F(\varepsilon^*_{t+1}) - f(\varepsilon^*_{t+1})\varepsilon^*_{t+1}\mu)) = 0$$

Note that this is a state-contingent condition, that for each future macro state the interest adjusts such that the lender can pay back its creditors. As a result, we get:

$$\lambda^F_t = \frac{1-F(\varepsilon^*_t)}{1-F(\varepsilon^*_t) - f(\varepsilon^*_t)\varepsilon^*_t\mu} > 1 \qquad (15)$$

implying that the lending constraint is always binding:

$$R_{b,t}b_{t-1}(1-F(\varepsilon^*_t)) + \Delta(\varepsilon^*_t)R_{k,t}q_{t-1}k_{t-1}(1-\mu) = R_{d,t-1}b_{t-1} \qquad (16)$$

Using the first order condition with respect to $R_{b,t+1}$, we can rewrite the first order condition with respect to b_t, in its final form:

$$E_t\left(\frac{R_{k,t+1}}{R_{d,t}}\left(1-\Delta(\varepsilon^*_{t+1})-\varepsilon^*_{t+1}(1-F(\varepsilon^*_{t+1}))\right.\right.$$
$$\left.\left.+\lambda^F_{t+1}(\varepsilon^*_{t+1}(1-F(\varepsilon^*_{t+1}))+\Delta(\varepsilon^*_{t+1})(1-\mu))\right)\right)=E_t(\lambda^F_{t,+1}) \tag{17}$$

This equation, a key equation of the debt contract, replaces equation (2), and determines the capital stock k_t in the model with financial frictions.
Last, we have to define a law of motion for equity. A fraction z_w of earnings is distributed as dividends, and shareholders inject equity (wealth) equal to w. Consequently, the law of motion for equity is:

$$e_t = z_w(R_{k,t}q_{t-1}k_{t-1}(1-\mu\Delta(\varepsilon^*_{t+1}))-R_{d,t-1}b_{t-1})+w \tag{18}$$

Aggregating all agents, the resource constraint includes now the cost of default:

$$c_t + i_t + R_{k,t}q_tk_{t-1}\mu\Delta(\varepsilon^*_{t+1}) = y_t \tag{19}$$

Equations (1) to (18) determine the model equations for the RBC model with financial frictions (with equation (16) replacing equation (2)).

3 Summary

3.1 RBC without firm balance sheet

First order condition with respect to capital (k_t):

$$1=\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\left(\alpha\frac{y_{t+1}}{q_tk_t}+(1-\delta)\frac{q_{t+1}}{q_t}\right)\right) \tag{1}$$

First order condition with respect to risk-free bond ($R_{d,t}$):

$$1=\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}R_{d,t}\right) \tag{2}$$

First order condition with respect to labour (n_t) (CRRA preferences):

$$(1-\alpha)\frac{y_t}{n_t}=\tau_n\frac{n_t^\phi c_t^\gamma}{z_{c,t}} \tag{3}$$

First order condition with respect to investment (q_t):

$$1=q_tz_{i,t}\left(1-S\left(\frac{i_t}{i_{t-1}}\right)-S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right)+\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}z_{i,t+1}S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right) \tag{4}$$

Law of motion for capital (i_t):

$$k_t = (1-\delta)k_{t-1} + i_t z_{i,t}\left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) \tag{5}$$

Resource constraint (c_t), and definition of output (y_t):

$$c_t + i_t + g_t = y_t = k_{t-1}^{\alpha}(z_{y,t}n_t)^{1-\alpha} \tag{6}$$

Government spending (g_t):

$$g_t = g_{ss}z_{g,t} \tag{7}$$

3.2 RBC with firm balance sheet

First order condition with respect to capital (k_t):

$$\begin{aligned} E_t\Bigg(\frac{R_{k,t+1}}{R_{d,t}}\Big(1 - \Delta(\varepsilon_{t+1}^*) - \varepsilon_{t+1}^*(1 - F(\varepsilon_{t+1}^*)) \\ + \lambda_{t+1}^F(\varepsilon_{t+1}^*(1 - F(\varepsilon_{t+1}^*)) + \Delta(\varepsilon_{t+1}^*)(1-\mu))\Big)\Bigg) = E_t(\lambda_{t,+1}^F) \end{aligned} \tag{1}$$

where λ_t^F:

$$\lambda_t^F = \frac{1 - F(\varepsilon_t^*)}{1 - F(\varepsilon_t^*) - f(\varepsilon_t^*)\varepsilon_t^*\mu}$$

First order condition with respect to risk-free bond ($R_{d,t}$):

$$same as above \tag{2}$$

First order condition with respect to labour (n_t) (CRRA preferences):

$$same as above \tag{3}$$

First order condition with respect to investment (q_t):

$$same as above \tag{4}$$

Law of motion for capital (i_t):

$$same as above \tag{5}$$

Resource constraint (c_t) includes now cost of default (definition of output (y_t) unchanged):

$$c_t + i_t + g_t + R_{k,t}q_t k_{t-1}\mu\Delta(\varepsilon_{t+1}^*) = y_t \tag{6}$$

Equation determining corporate bond interest rate ($R_{b,t}$):

$$R_{b,t}b_{t-1}(1 - F(\varepsilon_t^*)) + \Delta(\varepsilon_t^*)R_{k,t}q_{t-1}k_{t-1}(1-\mu) = R_{d,t-1}b_{t-1} \tag{7}$$

Default threshold (ε_t^*):

$$\varepsilon_t^* = \frac{R_{b,t} b_{t-1}}{R_{k,t} q_{t-1} k_{t-1}}$$

Law of motion for equity (e_t):

$$e_t = z_w(R_{k,t} q_{t-1} k_{t-1}(1 - \mu\Delta(\varepsilon_t^*)) - R_{d,t-1} b_{t-1}) + w \tag{8}$$

Equation for debt (b_t):

$$q_t k_t = e_t + b_t \tag{9}$$

Table 1. Estimated Parameters

Parameter	Description	Prior Distribution			Posterior Distribution	
		Prior Distribution	Mean	SD	Mode	SD
ρz_y	Autocorrelation, Productivity shock	beta	0.800	0.100	0.773	0.079
ρz_i	Autocorrelation, Investment shock	beta	0.800	0.100	0.665	0.103
ρz_c	Autocorrelation, Preference shock	beta	0.800	0.100	0.846	0.100
ρz_g	Autocorrelation, Government shock	beta	0.800	0.100	0.936	0.024
ρ	Autocorrelation, Financial shock	beta	0.800	0.100	0.544	0.080
ϵ^*	Steady state default threshold	beta	0.500	0.050	0.500	0.051
σ_{ss}	Steady state standard deviation of financial shock	beta	0.450	0.050	0.329	0.011
μ	Fraction of out loss in case of default	beta	0.400	0.100	0.080	0.008
η	Habit persistence	beta	0.900	0.080	0.972	0.049
γ	Risk aversion	gamma	4.000	2.500	0.344	0.061
ϕ	GHH preference parameter	gamma	0.550	0.070	0.493	0.009
ϕ_k	Capital adjustment cost	gamma	3.000	2.000	0.138	0.033
$SD(\epsilon z_y)$	Standard deviation of productivity shock	inverse gamma2	0.005	0.005	0.010	0.001
$SD(\epsilon z_g)$	Standard deviation of gov. spending shock	inverse gamma2	0.050	0.005	0.043	0.003
$SD(\epsilon_s)$	Standard deviation of financial shock	inverse gamma2	0.070	0.005	0.059	0.004

Table 2. Other Parameters (calibration)

Parameter	Description	Value	Source
n_{ss}	Steady state labor	0.330	Standard
zw_{ss}	Fraction of earnings distributed	0.900	Standard
g_{ss}	Steady state gov. spending-to-output ratio	0.170	Data
α	Capital share of income	0.360	Standard
β	Discount factor	0.980	Standard
δ	Capital deprication	0.025	Standard
$SD(\epsilon z_i)$	Standard deviation of investment shock	0.007	Minimize investment shock
$SD(\epsilon z_c)$	Standard devtation of preference shock	0.000	Virtually shutdown consumption shock