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Market Frictions, Interbank Linkages and Excessive Interconnections

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Abstract

This paper studies banks' decision to form financial interconnections using a model of financial contagion that explicitly takes into account the crisis state of the world. This allows us to model the network formation decision as optimising behaviour of competitive banks, where they balance the benefits of forming interbank linkages against the cost of contagion. We use this framework to study various market frictions that can result in excessive interconnectedness that was seen during the crisis. In this paper, we focus on two channels that arise from regulatory intervention—deposit insurance and the too big to fail problem.

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I. INTRODUCTION

The financial crisis that hit the world economy in 2007 has made clear the interconnected nature of the global financial system. What started as difficulties in the US sub-prime mortgage market rapidly escalated and spilled over to markets all over the world. Banks, uncertain of the interconnections and fearful of contagion became less willing to lend money to each other. Interbank lending rates started to rise and soon the market for short-term lending dried up. The credit crunch ultimately triggered a bank run at the British mortgage lender Northern Rock–something not seen in the UK for over 140 years and in Western Europe for the last 15 years.

The mechanics of financial contagion are well studied and the literature broadly takes two approaches–examining direct balance sheet linkages and indirect linkages operating through fire sales and expectations (see Allen, Babus, and Carletti (2009) for a survey of the literature on financial crises). The benefits of financial interlinkages and networks are also well known and stem primarily from better liquidity sharing and diversification. However, the literature on networks and contagion has mostly focused on comparing the social costs and benefits of different network structures as opposed to the banks' decision to form the network.

This paper attempts to fill this gap and develops a simple model to study banks' decision to form financial interconnections. This stylized framework allows us to study various market frictions that can influence this decision and result in a wedge between socially optimal network formation and suboptimal interconnectedness that was seen during the crisis. While such suboptimal interconnections can arise from the several sources such as the underestimation of the crisis probability and disaster myopia–misperception of the probability of rare events that arises from a set of heuristics commonly used to form judgments under uncertainty–the focus of this paper is on two sources of market frictions that arises from regulatory intervention–deposit insurance and the existence of systemically important financial institutions (SIFI). While the former is an explicit government guarantee, the latter is a more subtle implicit guarantee that is embedded in the modern financial system.

Most papers that study contagion in the banking system model the crisis state as an exogenous perturbation of the model. In these models, the crisis state is therefore modeled as a zero probability event. In order to study banks' decision whether or not to form an interconnection, we develop a simplified model of financial contagion with direct linkages through an interbank market that *explicitly* takes into account the crisis state of the world. This allows us to endogenise the network formation decision with banks' recognising the risk of contagion

when forming an interconnection in the form of an interbank market to share liquidity. We model this decision as an optimizing behavior of banks, where they balance the benefit of sharing liquidity against the cost of contagion.

We use this framework to explore some of the market frictions that can lead to suboptimal interconnections. One obvious market friction that can lead to such suboptimal network formation is underestimation of the risk of crisis which can stem from a variety of sources such as disaster myopia. While important, this is not the focus of our paper and we only briefly touch upon this potential source of suboptimal interconnections. We instead focus on deposit insurance and the existence of SIFIs. Deposit insurance and too big to fail protection provided to SIFIs are modeled in a stylized fashion, and while the model does not attempt to fully capture the different aspects of these regulatory interventions–including their role in boosting confidence in the financial system and enhancing financial stability–it nevertheless highlights a channel through which they can have the unintended consequence of incentivizing banks to form suboptimal interconnections. The model shows that explicit deposit insurance and the more implicit too big to fail type perceptions of government guarantees to SIFIs creates a wedge between social and private optimality. In the presence of these implicit and explicit guarantees, competitive banks find it optimal to participate in the interbank market even when the risk of contagion is high and it is socially suboptimal to do so. This results in excessive interconnections in equilibrium.

We model contagion through direct balance-sheet linkages and show how these connections can result in contagious bank runs.[1] Following Bryant (1980) and Diamond and Dybvig (1983), we model bank runs in a setting where depositors have uncertain consumption needs and the long-term investment is costly to liquidate. In these papers, if depositors believe that other depositors will withdraw then all agents find it rational to redeem their claims and a bank run occurs. Another equilibrium exists where everybody believes no panic will occur and agents withdraw their funds according to their consumption needs. The theory is silent on which of the two equilibria will be selected and depositors' belief is coordinated by 'sunspots'.

Allen and Gale (2000) study how the banking system responds to contagion when banks are connected under different network structures. Banks perfectly insure against liquidity shocks by exchanging interbank deposits. The connections created by swapping deposits, however, expose the system to contagion. The authors show that incomplete networks–where banks'

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[1]See Caballero and Simsek (2013) and Zawadowski (2013) for some recent examples of models of financial contagion operating through fire sales and counterparty risk.

do not exchange deposits with every other bank–are more prone to contagion than complete structures. Better connected networks are more resilient since the proportion of the losses in one bank's portfolio is transferred to more banks through interbank agreements. To show this, they take the case of an incomplete network where the failure of a bank may trigger the failure of the entire banking system. They prove that, for the same set of parameters, if banks are connected in a complete structure, then the system is more resilient with regard to contagious effects.

Dasgupta (2004) also explores how linkages between banks, represented by cross holding of deposits, can be a source of contagious breakdowns. However, using the global games techniques developed by Morris and Shin (2000, 2003), Dasgupta isolates an unique equilibrium which depends on depositors private signal about the banks' fundamentals. In the same spirit, Brusco and Castiglionesi (2007) show that there is a positive probability of bankruptcy and propagation of a crises across regions when banks keep interbank deposits and this may lead to excessive risk. Rochet and Vives (2004) and Goldstein and Pauzner (2005) also use global games techniques to study banking crises. Chen, Goldstein, and Jiang (2010) establishes the empirical applicability of the global games approach using a detailed dataset on mutual funds.

Our paper is also related to the literature on financial networks. Allen and Babus (2009) provide a comprehensive survey of this literature, most of which studies network effect rather than network formation. These papers focus on how different network structures respond to the breakdown of a single bank in order to identify network structures that are more resilient. Gale and Kariv (2007) study the process of exchange in financial networks and show that when networks are incomplete, substantial costs of intermediation can arise and lead to uncertainty of trade as well as market breakdowns. Eisenberg and Noe (2001) investigate default by firms that are part of a single clearing mechanism and show the existence of a clearing payment vector that defines the level of connections between firms. The authors develop an algorithm that allows them to evaluate the effects of small shocks on the system. This algorithm produces a natural measure of systemic risk based on how many waves of defaults are required to induce a given firm in the system to fail. Similarly, Minguez-Afonso and Shin (2007) use lattice-theoretic methods to study liquidity and systemic risk in high-value payment systems.

Gai and Kapadia (2010) develop a model of contagion using techniques from the literature on complex networks to assess the fragility of the financial system based on banks' capital buffers, the degree of connectivity and the liquidity of the market for failed banking assets. They find that while greater connectivity reduces the likelihood of widespread default, the

shocks may have a significantly larger impact on the system when they do occur. Moreover, the resilience of the network to large shocks depends on shocks hitting particular fragile points associated with structural vulnerabilities. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015a) show that the magnitude of negative shocks play a role in determining the fragility of a network. If the negative shocks are small, a more densely connected financial network enhances financial stability, but when the shocks become larger, dense interconnections can lead to a more fragile financial system.

There are some models that study endogenous network formation. Leitner (2005) constructs a model where the success of an agent's investment in a project depends on the investments of other agents in the network. Since endowments are randomly distributed across agents, an agent may not have enough cash to make the necessary investment. In this case, agents may be willing to bail out other agents to prevent the collapse of the whole network. Leitner examines the design of optimal financial networks that minimize the trade-off between risk sharing and the potential for collapse. In a related paper, Kahn and Santos (2010) investigate whether banks choose the optimal degree of mutual insurance against liquidity shocks. They show that when there is a shortage of exogenously supplied liquidity, which can be supplemented by bank liquidity creation, the banks generally fail to find the correct degree of interdependence. In aggregate, they become too risky.

Babus (2009) models the decision of the bank to ex-ante commit to ensure each other against the risk of contagion using a network formation game approach and shows that when banks endogenously form networks to respond to contagion risk, financial stability is supported. The model predicts a connectivity threshold above which contagion does not occur, and banks form links to reach this threshold. However, an implicit cost associated with being involved in a link prevents banks from forming more connections than required by the connectivity threshold. Banks manage to form networks where contagion rarely occurs. Castiglionesi and Navarro (2007) are also interested in decentralizing the network of banks that is optimal from a social planner perspective. In a setting where banks invest on behalf of depositors and there are positive network externalities on the investment returns, fragility arises when banks that are not sufficiently capitalized gamble with depositors' money. When the probability of bankruptcy is low, the decentralized solution approximates the first best. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015b) show that even in a setting where banks can lend to each other through interbank contracts that fully reflect counterparty risk, allowing them to internalize all bilateral externalities, banks still fail to take into account the externality that they impose on the rest of the network, and over-lend and under-diversify in equilibrium.

The rest of the paper is organized as follows. Section II sets up our model and outlines the costs and benefits of participating in the interbank market. Section III solves the model for the banks' optimal participation strategy. In the absence of externalities and market failures, competitive banks optimal strategy is socially optimal. Section IV introduces market frictions and explores how these frictions can create a wedge between the socially optimal and privately optimal participation strategy, resulting in excessive interconnectedness in equilibrium. Section V concludes. The proofs are presented in the appendix.

II. MODEL SETUP

We consider an economy which extends over three dates, $t = 0, 1, 2$, with two non-overlapping 'regions' A and B. We assume that each region is populated by a continuum of depositors and a single representative competitive bank. Banks are competitive in the sense that they makes zero profit in equilibrium and offer depositors the highest possible return. There is a single good that is used for both consumption and investment and serves as the numeraire.

Depositors We consider a continuum of *ex-ante* identical depositors who each live for three periods. The depositors have liquidity preferences as in Diamond and Dybvig (1983)–a depositor is either impatient and wishes to consume at date 1 or patient and wishes to consume at date 2. The number of patient and impatient depositors depend on the state of the world.

Depositors are uncertain about their consumption needs at date 0, but learn their preferences privately at date 1. Let ψ denote the fraction of impatient depositors, who wish to consume at period 1. A depositors' utility function is -

$$u(c_1, c_2) = \begin{cases} c_1 & \text{with probability} \quad \psi \\ c_2 & \text{with probability} \quad 1 - \psi \end{cases}$$

where c_t is consumption at date t. Depositors are endowed with a unit of the consumption good at date 0 only, which they can invest in a storage technology or deposit at the bank. If they choose to deposit at the bank, depositors can choose to withdraw at date 1 or date 2. Given depositor preferences, impatient depositors always withdraw at date 1, while patient depositors decide strategically.
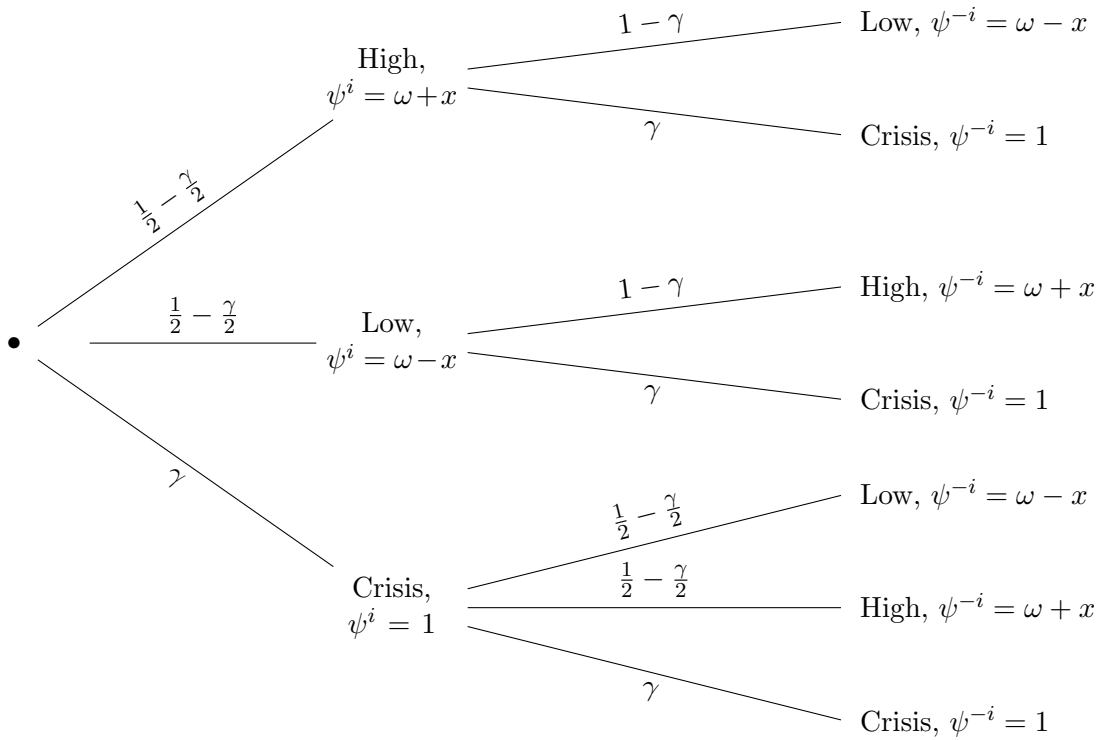
Figure 1. Liquidity States, $i \in \{A, B\}$

States of the World   The distribution of patient and impatient depositors depend on the state of the world. In '*normal*' times, we assume that each region can have high or low demand for liquidity, such that $\psi \in \{\omega + x, \omega - x\}$. These liquidity states are negatively correlated across regions. We further assume that both states are equally likely, implying that aggregate liquidity (ω) is constant in normal times.

Additionally, there exists a '*crisis*' state when the liquidity demand is very high. In order to simplify our calculations, we assume that in the crisis state all consumers are impatient, *i.e.* $\psi = 1$.[2] These crisis states are low probability events and are independently and identically distributed across the two regions, with probability γ, where $\gamma \approx 0$. Figure 1 summaries the possible states of the world.

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[2]Note that for our results to hold, we only require that the liquidity demand be reasonably high during the crisis state, *i.e.* $\psi \gg \omega + x$. This is because when liquidity demand is sufficiently high, the patient depositors also find it optimal to withdraw, effectively making $\psi = 1$.

Assets   Following Allen and Gale (2000), we assume that there are two types of assets. The *liquid asset* represents storage technology. One unit of consumption good invested in the liquid asset at date t produces one unit of consumption good at period $t+1$. The other asset is the *long asset* which takes two periods to mature. Investment in the long asset can take place only at $t=0$, and the asset provides a return $R>1$ in period $t=2$. If the asset is liquidated prematurely at period $t=1$, it produces $0<r<1$ units of the consumption good. Figure 2 summaries the cash flows from the two assets.

t	0	1	2
Liquid Asset	-1	1	
Liquid Asset		-1	1
Long Asset	-1	r *(liquidated)*	R *(not liquidated)*

Figure 2. Asset Cash Flows

Deposit Contracts   At date 0, banks collect deposits from depositors and offers demand deposit contracts. The demand deposit contacts' allow depositors to withdraw either c_1 units of consumption at date 1 or c_2 units of consumption at date 2. Following Dasgupta (2004), we focus on deposit contacts that offer conversion of deposits into cash at par at $t=1$, *i.e.* $c_1=1$. Note that the deposit contract is entered into at date 0 and cannot be contingent on the depositor type since the depositors learn their preferences (patient or impatient) privately.

The role of the banks is to pool the assets of a large number of depositors and invest on their behalf. Such pooling allows the bank to offer insurance to the depositors against their uncertain liquidity demand, by giving them access to the long asset without subjecting them to the full liquidation costs in case they turn out to be impatient. Our model generates multiple equilibria in line with Diamond and Dybvig (1983) and Allen and Gale (2000), *i.e.* even when the bank is solvent, patient depositors may wish to withdraw their deposits at $t=1$, if they believe that enough other patient depositors will do the same, leading to a bank run–the so called 'sunspot' equilibria. Using the global games techniques outlined in Morris and Shin (2000, 2003), Dasgupta (2004) and Goldstein and Pauzner (2005), we can generate unique equilibria in our model by introducing an arbitrarily small amount of uncertainty. However, in order to keep our model simple, we assume a deterministic return R at period $t=2$ and focus on the no-run efficient equilibrium where, as long as the bank remains solvent, patient

depositors wait till period $t = 2$ to withdraw their funds. The existence of deposit insurance ensures that no-run is the dominant strategy.

Banks Competitive banks choose their balance sheets at date t=0 such that they offer depositors highest possible returns. They hold a fraction of their portfolio, y in the liquid asset and invest the remainder $(1 - y)$ in the long asset. They also decide whether or not to participate in the interbank market to insure against regional liquidity shocks. They do this by exchanging interbank deposits at date $t = 0$. Since ex-ante, bank deposits are identical, the banks budget constraint is not affected by the exchange of interbank deposits.

At date 1, the bank uses the liquid asset to service withdrawals by depositors. If the bank holds excess liquidity, *i.e.* $y \geq \psi$, it can invest the surplus in the liquid asset and use it to repay patient depositors in period 2. However, in case liquid assets are not sufficient, *i.e.* $y < \psi$, the bank has to rely on the interbank market or liquidate the long asset.

Given the asset payoffs, there exists an obvious pecking order for banks. Banks uses the liquid asset in the first instance. Next, they meet any shortfall by tapping into the interbank market. This is only possible when the banks collectively decide to participate in the interbank market at $t = 0$ and exchange interbank deposits b. Liquidation is costly and is only used as a last resort. Banks optimally choose their portfolio taking into account the expected withdrawals at date 1.

Interbank Market Since liquidity states are negatively correlated across regions, aggregate liquidity is constant in '*normal*' times. Participation in the interbank market allows banks' to share liquidity and insure against regional liquidity shocks (Allen and Gale, 2000; Dasgupta, 2004). In particular, banks hold the average level of liquidity demand, ω of the short asset and insure against regional liquidity shocks by holding interbank deposits x. This allows banks to avoid costly liquidation of the long asset. However, participation in the interbank market also exposes banks to possible contagion.

In our setting, this contagion risk is modelled by the '*crisis*' state, which occurs with a small probability γ. We assume that the interbank market shuts down in the crisis state, resulting in contagion. While we acknowledge that there are different ways to model the impact of the crisis state on the interbank market, we follow the literature on financial networks and contagion and assume that interbank deposits disappear during the crisis state. We believe that this assumption is consistent with the evidence from the recent financial crisis, where the interbank market shutdown in the aftermath of the collapse of Lehman Brothers in September

2008.[3] Furthermore, in case of insolvency, interbank deposits are likely to be junior to retail deposits and even if the interbank deposits pay-off a fraction of their value after liquidation, this is unlikely to be available at $t = 1$.

We can illustrate the contagion channel through an example. Suppose region A is in the normal state while there is a crisis in region B. In case the banks decide not to participate in the interbank market, region A banks would ex-ante hold enough of the liquid asset to ensure that they have enough liquidity to be viable in both high and low liquidity states. The bank run would therefore be limited to the region B bank.

On the other hand, participation in the interbank market allows banks to share liquidity and optimize their holding of the liquid asset. Since liquidity shocks are negatively correlated, they can use their interbank deposits to address any liquidity shortfalls in the high liquidity demand state. However, this dependence on the interbank market exposes them to contagion risk–the crisis in region B can result in a run on the region A bank. This is because the region A bank, reliant on the interbank market, would now hold insufficient liquidity to pay impatient depositors in the high liquidity state. Therefore, when the interbank market shuts down due to the crisis in region B, the region A bank is forced to liquidate the long asset, resulting in a run on the bank. Note that in our model, contagion takes place only when high liquidity demand coincides with a crisis in the other region. This is because our model abstracts from other channels of contagion such as asset fire sales. If these channels are taken into account, the probability of contagion is likely to increase significantly.

III. MODEL SOLUTION–SOCIAL OPTIMALITY

We can now solve for the banks decision to participate in the interbank market. We do this in 3 stages. First, we look at the case where the bank does not participate in the interbank market. Second, we introduce the interbank market. Finally, we compare the costs and benefits of participation in the interbank market and solve for the bank's optimal participation strategy.

No Interbank Market

The bank offers deposit contract (c_1, c_2) at date 0 that allows depositors to withdraw either $c_1 = 1$ at date 1 or c_2 at date 2. That bank also chooses its balance sheet and holds a fraction of its portfolio, y in the liquid asset and invests the remainder $(1 - y)$ in the long asset such that it

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[3]See Bank of International Settlements (2009).

maximizes depositor utility. Note that during the crisis state, all consumers are impatient and withdraw their deposits at $t = 1$. Therefore, unless the bank portfolio consists of only liquid asset–which makes it redundant, the bank goes bust. Therefore, when choosing the balance sheet, the bank does not to take into account the crisis state and instead optimally chooses its balance sheet taking into account only the high and low liquidity demand states during normal times.

At date 1, the proportion of impatient depositors, ψ withdraw their deposits. If the bank holds enough liquid assets, *i.e.* $y \geq \psi$, the bank pays out the impatient depositors $c_1 = 1$ and invests the surplus in the short asset. At date 2, it pays patient depositors c_2, which equals the surplus cash from date 1, $(y - \psi)$ and the returns from the long asset, $(1 - y)R$. On the other hand, if banks hold insufficient liquid assets, *i.e.* $y < \psi$, the bank is forced to liquidate a part of its long asset in order to pays out the impatient depositors $c_1 = 1$ at date 1. It then uses the remaining long asset to pay the patient depositors c_2 at date 2. In case the bank is unable to pay patient depositors $c_2 \geq 1$ at date 2, the patient depositors optimally withdraw their deposits at date 1, resulting in a run on the bank. Additionally, if the bank is unable to pay $c_1 = 1$ to all depositors withdrawing at date 1, it is declared insolvent and goes bust.

Banks face uncertain liquidity demand at date 1 as the distribution of patient and impatient depositors depend on the state of the world. Since liquidating the long asset prematurely at date 1 is costly for the bank, $0 < r < 1$, the bank holds liquidity in the form of investment in the liquid asset. However, this liquidity has an opportunity cost–foregone investment in the long asset. Therefore, banks face a trade off and choose their optimal portfolio taking into account the relative costs of liquidity and liquidation.

Proposition 1. *When not participating in the interbank market, the bank optimally holds* $y = \omega + x$ *of the liquid asset.*

Proof. See Appendix A.1

Proposition 1 implies that the bank finds it optimal to hold enough liquidity to pay impatient depositors in the high liquidity demand state, although this has an opportunity cost in terms of surplus liquid asset and foregone investment in the low liquidity demand state. When choosing their balance sheets, bank trade-off the costs of liquidation against the opportunity cost of investing in the long asset. For the rest of this paper, we assume that the relative to the cost of liquidation outweighs the benefit of holding more of the long asset. More formally,

Assumption 1. *We assume that for a given return on the long asset, liquidation is sufficiently costly such that,*

$$r < \frac{R}{2R-1}$$

Using Proposition 1, we can now write down the depositor pay-off's in different states of the world.

Proposition 2. *The depositor's payoff in different states of the world when the bank does not participate in the interbank market is as follows -*

Liquidity State	Probability	c_1	c_2
High Liquidity $(\omega + x)$	$\frac{1}{2} - \frac{\gamma}{2}$	1	R
Low Liquidity $(\omega - x)$	$\frac{1}{2} - \frac{\gamma}{2}$	1	$\frac{2x+(1-\omega-x)R}{1-\omega+x} < R$
Crisis	γ		$\omega + x + (1-\omega-x)r < 1$ *Bank Run, everyone withdraws at date 1*

There is no run on the bank in normal times.

We consider each case in turn. Suppose the demand for liquidity is high, *i.e.* $\psi = \omega + x$. Since the bank hold $y = \omega + x$ of the liquid asset, the withdrawal demand from impatient depositors exactly matches up with the availability of the liquid asset and $c_1 = 1$. The investment in the long asset, $(1-y)$ also matches up with the demand from patient depositors, $(1 - \omega - x)$. Thus, $c_2 = R$. Since $c_2 > c_1 = 1$, patient depositors find it optimal to withdraw their deposits at date 2 and there is no run on the bank.

On the other hand, if the demand for liquidity is low, *i.e.* $\psi = \omega - x$, the bank has excess liquidity at date 1. The demand from impatient depositors at date 1 equals $\omega - x$, while the bank holds $y = \omega + x$ of the liquid asset. The bank invests this surplus cash in the liquid asset at date $t = 1$. At date 2, the bank has a cash flow of $2x$ from the liquid asset and $(1 - \omega - x)R$ from the long asset, which it uses to pay $(1 - \omega + x)$ patient depositors $c_2 = \frac{2x+(1-\omega-x)R}{1-\omega+x}$.

Corollary 1. *In the low liquidity demand state with no interbank market, $1 < c_2 < R$*

Proof. See Appendix A.2

$c_2 > 1$ ensures that there is no run on the bank and patient depositors wait till date 2 to withdraw their deposits. However, since the bank had excess liquidity at date 1, $c_2 < R$. This represents the cost of holding the liquid asset. Banks are forced to hold this excess liquidity in order to avoid liquidating the long asset in the high liquidity demand case.

Finally, in the crisis state, all depositors withdraw their deposit at date 1 and the bank is forced to liquidate the long asset. At date 1, the bank gets $(\omega + x)$ from the liquid asset and $(1 - \omega - x)r$ from liquidating the long asset. Since $r < 1$, $c_1 = \omega + x + (1 - \omega - x)r < 1$ and the bank goes bust.

We can now calculate the expected pay-off of the depositors, taking into account the ex-post distribution of patient and impatient depositors.

Proposition 3. *Expected pay-off of the depositors with no interbank market is -*

$$
\begin{aligned}
\Pi^{NIB} &= \left(\frac{1}{2} - \frac{\gamma}{2}\right)[(\omega + x) + (1 - \omega - x)R] + \gamma[\omega + x + (1 - \omega - x)r] \\
&\quad + \left(\frac{1}{2} - \frac{\gamma}{2}\right)\left[(\omega - x) + (1 - \omega + x) \cdot \frac{2x + (1 - \omega - x)R}{1 - \omega + x}\right] \\
&= R(1 - \omega - x) + \omega + x - (1 - \omega - x)(R - r)\gamma
\end{aligned}
$$

Interbank Market

We now introduce the interbank market. Participation in interbank market expands the bank's sources of liquidity. During normal times, since liquidity shocks are negatively correlated, the interbank market allows banks in different regions to share liquidity and economise on the holding of the liquid asset. Since aggregate liquidity (ω) is fixed during normal times, a bank in a region with high liquidity demand can, through the interbank market, tap the surplus liquidity in the other region with low liquidity demand. This allows banks to offer a higher c_2 to patient depositors in normal times. However, this comes at a cost since it makes the bank dependent on the interbank market. The bank optimally no longer holds enough liquid assets to meet the demands of impatient depositors at date 1. When the interbank market disappears during a crisis, banks are exposed to contagion risk. When faced with high liquidity demand, banks are forced to liquidate the long asset.

Proposition 4. *When participating in the interbank market, the bank optimally holds $y = \omega$ of the short asset and exchanges interbank deposits of size x.*

Proof. See Appendix A.3

Proposition 4 holds as long as liquidity shocks are materially significant, *i.e.* crisis in the other region can lead to contagion and crisis through the disappearance of the interbank market. Formally,

Assumption 2. *We assume that liquidity shock*

$$x > \frac{(1-\omega)(R-1)}{\frac{R}{r}-1} + \varepsilon$$

where $\varepsilon \to 0$.

The above condition is weak and as satisfied for all plausible parameters values. This result is similar to the condition derived in Allen and Gale (2000), but in our setting the bank explicitly take into account the probability of the crisis state and potential contagion. As before, we look at the depositor's payoff on a case by case basis.

Proposition 5. *The depositor's payoff in different states of the world when the bank participates in the interbank market is as follows -*

Liquidity State	Other Region	Probability	c_1	c_2
High Liquidity	No Crisis	$\left(\frac{1}{2}-\frac{\gamma}{2}\right)(1-\gamma)$	1	R
$(\omega + x)$	Crisis	$\left(\frac{1}{2}-\frac{\gamma}{2}\right)\gamma$		$\omega + (1-\omega)r < 1$ *Contagion: Bank Run, everyone withdraws at date 1*
Low Liquidity	No Crisis	$\left(\frac{1}{2}-\frac{\gamma}{2}\right)(1-\gamma)$	1	R
$(\omega - x)$	Crisis	$\left(\frac{1}{2}-\frac{\gamma}{2}\right)\gamma$	1	$\frac{x+(1-\omega)R}{1-\omega+x} < R$
Crisis		γ		$\omega + (1-\omega)r < 1$ *Bank Run, everyone withdraws at date 1*

We consider each case in turn, starting with the normal times, when there is no crisis in either region. Suppose region A banks are in the high liquidity demand state, *i.e.* $\psi = \omega + x$. At date 1, region A bank faces a withdrawal demand of $\omega + x$ from impatient depositors. The bank holds $y = \omega$ of the short asset and can withdraw its interbank deposit of x. Since the liquidity shock in region B is negatively correlated, the bank in this region faces a withdrawal demand of $\omega - x$ and can use the excess cash (arising from $y = \omega$ holding of the short asset) to pay the region A bank. Thus, the cash flows match up and $c_1 = 1$.

At date 2, the region A bank receives $(1 - \omega)R$ from the long asset and has to make an interbank payment of xR to the region B bank. Since it faces a withdrawal demand of $(1 - \omega - x)$

from patient depositors, it offers $c_2 = R$. Region B bank receives $(1 - \omega)$ from the long asset and xR from the region A bank, which matches up with the demand of $(1 - \omega + x)$ from patient depositors and once again $c_2 = R$. Since $c_2 > c_1$, patient depositors find it optimal to wait and withdraw their deposits at date 2 and there is no run on either of the banks.

An analogous analysis holds for the case where there is no crisis and the region A bank faces low liquidity demand, *i.e.* $\psi = \omega - x$. Once again, there is no aggregate liquidity shock and the cash flows match up with $c_1 = 1$ and $c_2 = R$.

We now turn to the cases where there is a crisis in region B. The region A bank gets exposed to this crisis through the interbank market, or more precisely, through the disappearance of the interbank market on which it is now dependant. Suppose the bank is region A faces a high liquidity demand, *i.e.* $\psi = \omega + x$. Since it only holds $y = \omega$ of the short asset and the interbank market disappears due to the crisis, the bank in region A is forced to liquidate a fraction $l = \frac{x}{r}$ of the long asset, even though there is no crisis in region A. As long as $l \leq (1 - \omega)$, the bank is able to pay date 1 depositors c_1. However, if Assumption 2 holds, after liquidating l at date 1, the region A bank does not have enough long assets to offer patient depositors $c_2 \geq 1$ and patient depositors find it optimal to withdraw their deposits at date 1 and there is a run on the bank.

Corollary 2. *If Assumption 2 holds, contagion results in a run on the bank during the high liquidity demand state.*

Proof. See Appendix A.4

Contagion results in a run on the bank and it is forced to liquidate all its assets. The payoff for depositors is $c_1 = \omega + (1 - \omega)r < 1$ and the bank goes bust.

Conversely, consider the case where there is a crisis in region B, but the region A bank faces a low liquidity demand state, *i.e.* $\psi = \omega - x$. In this case $y = \omega$ of the short term asset is sufficient to meet the demands of the impatient depositors and $c_1 = 1$. However, the interbank market disappears due to crisis in region B and the region A bank is forced to invest the surplus cash (x) in the short asset. Therefore, due to the disappearance of the interbank market, it can no longer earn return R on this surplus cash.

At date 2, the region A bank receives $(1 - \omega)R$ from the long asset and has x cash, with with it has to meet the withdrawal demand of $(1 - \omega + x)$ from patient depositors. Therefore, $c_2 = \frac{x + (1 - \omega)R}{1 - \omega + x}$. Clearly, $1 < c_2 < R$. Since we assume that there are no fire sale externalities,

there is no run on the region A bank due to the crisis in region B. Nevertheless, the region A bank still suffers due to contagion since it can no longer pay patient depositors $c_2 = R$.

Finally, we consider the crisis state in region A. The state in region B is now irrelevant. All depositors in region A withdraw their deposit at date 1 and the bank is forced to liquidate the long asset. At date 1, the bank gets (ω from the short asset and $(1-\omega)r$ from liquidating the long asset. Since $r < 1$, $c_1 = \omega + (1-\omega)r < 1$ and the bank goes bust.

As before, we calculate the expected pay-off of the depositors, when the bank decides to participate in the interbank market.

Proposition 6. *Expected pay-off of the depositors when the bank participates in the inter-bank market is -*

$$
\begin{aligned}
\Pi^{IB} = & \left(\frac{1}{2} - \frac{\gamma}{2}\right)(1-\gamma)\left[(\omega+x) + (1-\omega-x)R\right] + \left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma\left[\omega + (1-\omega)r\right] \\
& + \left(\frac{1}{2} - \frac{\gamma}{2}\right)(1-\gamma)\left[(\omega-x) + (1-\omega+x)R\right] + \gamma\left[\omega + (1-\omega)r\right] \\
& + \left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma\left[(\omega-x) + (1-\omega+x)\cdot\frac{x+(1-\omega)R}{1-\omega+x}\right] \\
= & \omega + (1-\omega)R - \frac{3}{2}(1-\omega)(R-r)\gamma + \frac{1}{2}(1-\omega)(R-r)\gamma^2
\end{aligned}
$$

Participation strategy

We are now in a position to derive the bank's participation strategy. Our analysis suggests that the bank faces a tradeoff–participation in the interbank market allows the bank to offer higher deposit contract (higher c_2) to depositors during normal times, but exposes it to contagion during the crisis state. Using Propositions 2 and 5 we can compare depositor payoff from not participating and participating in the interbank market in different states of the world.

The main benefit from participating in the interbank market arises during normal times, when the bank is able to offer higher deposit contracts during the low liquidity demand state. When banks do not participate in the interbank market, they are forced to hold excess liquidity in order to avoid costly liquidation of the long asset. However, in the low liquidity demand state this excess liquidity in the short asset means that the bank has to offer a lower level of consumption to patient depositors at date 2. Thus $c_2 = \frac{2x+(1-\omega-x)R}{1-\omega+x} < R$. On the other hand, participation in the interbank market allows the bank to offer depositors $c_2 = R$ by using the interbank market to earn a return R on its surplus cash.

The bank also gains marginally in the state where there is a crisis in the other region, but low liquidity demand in the bank's own region. This is because, in the low liquidity demand state,

the bank does not need to access the interbank liquidity market at date $t = 1$. Although the bank loses out via-a-vis the normal state in which the bank could access the interbank market at date $t = 2$ and offer depositors $c_2 = R$, it still gains vis-a-vis non participation in the interbank market because participating in the interbank market allows banks to hold lower levels of liquidity (ω as opposed to $\omega + x$), which allows the bank to offer a marginally higher return $\frac{x+(1-\omega)R}{1-\omega+x}$ ($> \frac{2x+(1-\omega-x)R}{1-\omega+x}$, the return the bank can offer when it decides not to participate in the interbank market).

Conversely, the main cost of participation in the interbank market manifests itself through the contagion channel, when the crisis in the other region results in a run on the bank in the high liquidity demand state. When the bank does not participate in the interbank market, it holds enough liquidity to pay impatient depositors in the high liquidity state without liquidating the long asset. This in turn allows the bank to pay patient depositors $c_2 = R$. However, when participating in the interbank market, the bank is dependent on the interbank market to pay impatient date 1 depositors during the high liquidity demand state. When the interbank market disappears, the bank is forced to liquidate its long asset. If assumption 2 holds, this results in a run on the bank, all depositors receive $c_1 = \omega + (1 - \omega)r < 1$ and the bank goes bust.

During the crisis state, there is a run on the bank and it goes bust irrespective of whether or not it decided to participate in the interbank market. However, when participating in the interbank market, depositors face a marginally larger loss, since the bank now holds lower levels of liquidity (ω as opposed to $\omega + x$), and therefore in case of a run, can offer depositors $c_1 = \omega + (1 - \omega)r < \omega + x + (1 - \omega - x)r$, the return the bank can offer when it does not participate in the interbank market.

The bank optimally chooses its participation strategy by comparing the cost and benefit of participating in the interbank market, *i.e* by comparing Π^{NIB} and Π^{IB} (Proposition 3 and 6). This, in turn, depend on the probability of the crisis state, γ.

Proposition 7. $\Pi^{IB} - \Pi^{NIB}$ *is decreasing in crisis probability γ, i.e. the benefits of participating in the interbank market are decreasing in γ.*

Proof. See Appendix A.5

Since the main benefits of participating in the interbank market arise in normal times and the loss arises during the crisis state, the expected gain from participation in the interbank market falls as the probability of crisis increases.

Proposition 8. *There exists a γ^* such that $\Pi^{IB} > \Pi^{NIB}$ for $\gamma < \gamma^*$ and $\Pi^{IB} < \Pi^{NIB}$ for $\gamma > \gamma^*$.*

Proof. See Appendix A.6

As long as the probability of crisis is less than the critical level of γ, γ^*, *i.e.* $\gamma \leq \gamma^*$, the optimal strategy for the bank is to participate in the interbank market. However, as γ rises, the potential costs from contagion increases. When the probability of crisis is relatively significant, $\gamma > \gamma^*$ the bank finds it optimal not to participate in the interbank market.

Proposition 8 gives us the socially optimal threshold level for the crisis probability γ^* beyond which the potential costs of contagion outweigh the liquidity sharing benefits of network formation. In the absence of market frictions, private and social costs are aligned and the privately optimal strategy is also socially optimal.

IV. MARKET FRICTIONS–PRIVATE OPTIMALITY

In this section, we discuss some market frictions that can result in excessive interconnectedness. These market frictions create a wedge between the socially optimal and privately optimal participation strategy, resulting in suboptimal network formation that was seen during the crisis. The frictions can be broadly divided into 2 categories–(i) market frictions that result in underestimation of the probability of crisis; (ii) frictions where the probability of crisis is correctly measured but banks still optimally choose to form suboptimal interconnections.

While the underestimation of the risk of crisis is important and can stem from a variety of sources such as disaster myopia, this is not the focus of our paper and we only briefly touch upon this potential source of excessive interconnectedness. We instead focus on deposit insurance and the too-big-to-fail problem. We show that explicit deposit insurance and more implicit too big to fail type perceptions of government guarantees to SIFIs creates a wedge between social and private optimality. In the presence of these implicit and explicit guarantees, competitive banks find it optimal to participate in the interbank market even when the risk of contagion is high and it is socially suboptimal to do so ($\gamma > \gamma^*$). This results in excessive interconnectedness in equilibrium.

Deposit Insurance

Most jurisdictions require banks to participate in some form of government guaranteed deposit insurance with the intention of protecting bank deposit holders from losses that could occur in the event of a bank failure. While primarily a means of protecting individual (especially small) bank depositors, deposit insurance also serves to protect the financial system

from inefficient bank runs and panics–the so called 'sunspot equilibrium' highlighted in Diamond and Dybvig (1983) and Allen and Gale (2000) where even solvent banks may fail if enough depositors believe that other depositors will withdraw their deposits.

However, deposit insurance has the potential to undermine market discipline as depositors bear little or none of the risk associated with bank failures and therefore seek higher returns, even if it entails contagion risk. This in turn compels competitive banks to offer the highest possible returns–which in our setting is possible by optimizing on the holdings of the short asset and participating in the interbank market. This creates a wedge between socially optimal γ^* and depositors desired γ, leading to excessive suboptimal interconnections in equilibrium.

Since deposit guarantee is contingent on bank failure, in our model, it is only paid out in the crisis state. Furthermore, since the deposit contracts in our model offer conversion of deposits into cash at par at $t = 1$, we model deposit insurance as a guarantee that depositors get $c_1 = 1$ at $t = 1$ even if the bank is insolvent and fails. This effectively means that all depositors withdraw $c_1 = 1$ at $t = 1$, implying that the patient depositors lose out on $c_2 = R$ at $t = 2$.

Put differently, we assumes that the deposit insurance scheme only protects depositor capital and does not guarantee interest. We can make the alternative assumption that the deposit insurance also guarantees interest, which in this setting would imply that the deposit insurance scheme guarantees R to patient depositors at $t = 2$. This alternative assumption strengthens our results.

We can now revisit depositor's payoff from participating and not participating in the interbank market respectively. First consider Proposition 2 with deposit insurance.

Proposition 9. *The depositor's payoff in different states of the world when the bank does not participate in the interbank market and is protected by deposit insurance is as follows -*

Liquidity State	Probability	c_1	c_2
High Liquidity $(\omega + x)$	$\frac{1}{2} - \frac{\gamma}{2}$	1	R
Low Liquidity $(\omega - x)$	$\frac{1}{2} - \frac{\gamma}{2}$	1	$\frac{2x + (1 - \omega - x)R}{1 - \omega + x} < R$
Crisis	γ		1 *Bank Run, deposit insurance pays out*

Comparing Proposition 2 and 9, with deposit insurance, depositors receive $c_1 = 1 > \omega + x + (1 - \omega - x)r$ during the crisis state, while their payoff remains the same during the normal state. Note that here we assume that the deposit guarantee is externally funded. This is in line with most deposit insurance schemes, where deposit insurance is government guaranteed, with minimal ex-ante fees. This assumption is also in line with the UK's Financial Services Compensation Scheme (FSCS) which envisages an ex-post 'pay-as-you-go' levy on the banking system to recover the costs of deposit insurance, supplemented by borrowing from the government in case large payouts become necessary.[4] Note that the results are robust to an ex-ante fee, as long as the fee is not contingent on the banks' decision to participate in the interbank market.

Using Proposition 9, we can calculate the expected pay-off of depositors.

Proposition 10. *Expected pay-off of the depositors when the bank does not participate in the interbank market and is protected by deposit insurance is–*

$$\Pi_{DI}^{NIB} = \left(\frac{1}{2} - \frac{\gamma}{2}\right)[(\omega + x) + (1 - \omega - x)R] + \gamma$$
$$+ \left(\frac{1}{2} - \frac{\gamma}{2}\right)\left[(\omega - x) + (1 - \omega + x) \cdot \frac{2x + (1 - \omega - x)R}{1 - \omega + x}\right]$$
$$= R(1 - \omega - x) + \omega + x - (1 - \omega - x)(R - 1)\gamma$$

In addition, the expected cost of deposit insurance is–

$$C_{DI}^{NIB} = (1 - \omega - x)(1 - r)\gamma$$
$$= \Pi_{DI}^{NIB} - \Pi^{NIB}$$

Now, considering Proposition 5 with deposit insurance.

Proposition 11. *The depositor's payoff in different states of the world when the bank partici-pates in the interbank market and is protected by deposit insurance is as follows -*

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[4]For example, following the default of five relatively large banks in 2008 which necessitated payouts to nearly 4 million depositors, the government provided loan facilities of approximately £20 billion to the FSCS. Under the terms of the loan, only the interest on the loan is payable with any recoveries offset against the principal loan amount.

Liquidity State	Other Region	Probability	c_1	c_2
High Liquidity	No Crisis	$\left(\frac{1}{2} - \frac{\gamma}{2}\right)(1 - \gamma)$	1	R
$(\omega + x)$	Crisis	$\left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma$		1
				Contagion: Bank Run, deposit insurance pays out
Low Liquidity	No Crisis	$\left(\frac{1}{2} - \frac{\gamma}{2}\right)(1 - \gamma)$	1	R
$(\omega - x)$	Crisis	$\left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma$	1	$\frac{x+(1-\omega)R}{1-\omega+x} < R$
Crisis		γ		1
				Bank Run, deposit insurance pays out

Once again, the existence of deposit insurance reduces depositors loss during the crisis state since they now receive $c_1 = 1 > \omega + (1 - \omega)r$. Note that the gain from deposit insurance is marginally higher when banks participate in the interbank market since in the absence of deposit insurance, the banks liquidation value during the crisis state is lower when participating in the interbank market. This is driven by the fact that banks optimally invest less in the liquid asset (ω) when participating in the interbank market vis-a-vis when not participating in the interbank market ($\omega + x$), reducing their liquidation values.

As before, participation in the interbank market exposes the bank to contagion and a crisis in the other region results in a run on the bank in the high liquidity demand state. However, with deposit insurance, depositors still get $c_1 = 1 > \omega + (1 - \omega)r$, depositors payoff in the absence of deposit insurance. Using Proposition 11, we can calculate the expected pay-off of depositors.

Proposition 12. *Expected pay-off of the depositors when the bank participates in the interbank market and is protected by deposit insurance is -*

$$\Pi_{DI}^{IB} = \left(\frac{1}{2} - \frac{\gamma}{2}\right)(1 - \gamma)\left[(\omega + x) + (1 - \omega - x)R\right] + \left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma$$
$$+ \left(\frac{1}{2} - \frac{\gamma}{2}\right)(1 - \gamma)\left[(\omega - x) + (1 - \omega + x)R\right] + \gamma$$
$$+ \left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma\left[(\omega - x) + (1 - \omega + x)\cdot\frac{x + (1 - \omega)R}{1 - \omega + x}\right]$$
$$= \omega + (1 - \omega)R - \frac{3}{2}(1 - \omega)(R - 1)\gamma + \frac{1}{2}(1 - \omega)(R - 1)\gamma^2$$

In addition, the expected cost of deposit insurance is–

$$C_{DI}^{IB} = \left(\frac{1}{2} - \frac{\gamma}{2} \right) \gamma(1-\omega)(1-r) + \gamma(1-\omega)(1-r)$$

$$= \frac{3}{2}(1-\omega)(1-r)\gamma - \frac{1}{2}(1-\omega)(1-r)\gamma^2$$

$$= \Pi_{DI}^{IB} - \Pi^{IB}$$

Overall from Propositions 10 and 12, it is clear that for every level of γ, $\Pi^{IB} - \Pi^{NIB}$ is strictly greater with deposit insurance. Therefore, the threshold level of γ up to which depositors find it optimal for banks to participate in the interbank market is greater in the presence of deposit insurance.

Proposition 13. *The critical level of γ is higher with deposit insurance, i.e. $\gamma^{DI} > \gamma^*$.*
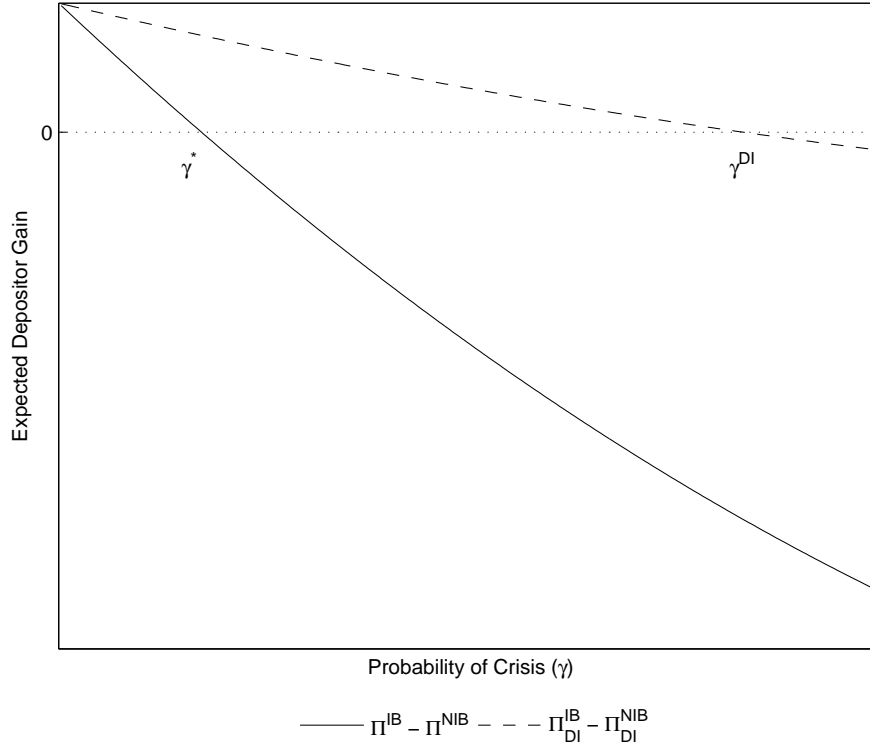
Proof. See Appendix A.7

Figure 3 shows that in the presence of deposit insurance, depositors' expected gain from participating in the interbank market is positive for all $\gamma < \gamma^{DI}$ (dashed line). Thus for all such γ depositors prefer to deposit their endowment with the bank that participates in the interbank market and offers the deposit contract $(c_1 = 1, c_2 = R)$ in the 'normal' state of the world. Although this exposes the depositors to the crisis state in the other region, deposit insurance guarantees that the depositors get $c_1 = 1$ in the crisis state. Since banks are competitive in our setting, in order to attract deposits, they are forced to participate in the interbank market and offer depositors deposit contracts $(c_1 = 1, c_2 = R)$ in the 'normal' state of the world.

In contrast, in the absence of deposit insurance, depositors gain from participation in the interbank market only for $\gamma < \gamma^*$ (solid line). This is because depositors now only receive the liquidation value during the crisis state, *i.e.* $c_1 < 1$ in the crisis state. Thus, for $\gamma^* > \gamma > \gamma^{DI}$, banks optimally participate in the interbank market only in the presence of deposit insurance, *i.e.* deposit insurance incentives banks to form interconnections which banks would otherwise not have formed.

Intuitively, in our model, deposit insurance acts as a state contingent cash transfer from the government to the depositors through the banking system. During the crisis state, depositors loss is limited to $c_1 = 1$ as opposed to the liquidation value of the bank where $c_1 < 1$. This incentivises depositors to take on greater risk as they do not internalize the cost of providing this insurance creating a wedge between social and private optimality.

Figure 3. Expected Gain from Participating in the Interbank Market with and without deposit insurance.

Proposition 14. *For $\gamma^* > \gamma > \gamma^{DI}$, banks participation in the interbank market is socially sub-optimal.*

Proof. See Appendix A.8

In order to compare the welfare effects of deposit insurance, we need to explicitly take into account the cost to the government of providing this insurance. More formally, for $\gamma^* > \gamma > \gamma^{DI}$, we need to compare the expected depositor payoff when the bank participates in the interbank market taking into account the expected cost to the government of providing deposit insurance, *i.e.* $\Pi_{DI}^{IB} - C_{DI}^{IB}$ with the expected depositor payoff in the absence of deposit insurance when the bank does not participate in the interbank market, *i.e.* Π^{NIB}.

Proposition 14 formally shows that $\Pi^{NIB} > \Pi_{DI}^{IB} - C_{DI}^{IB}$, but more intuitively it is easy to see from Figure 4 that participation in the interbank market is socially suboptimal for any $\gamma > \gamma^*$.
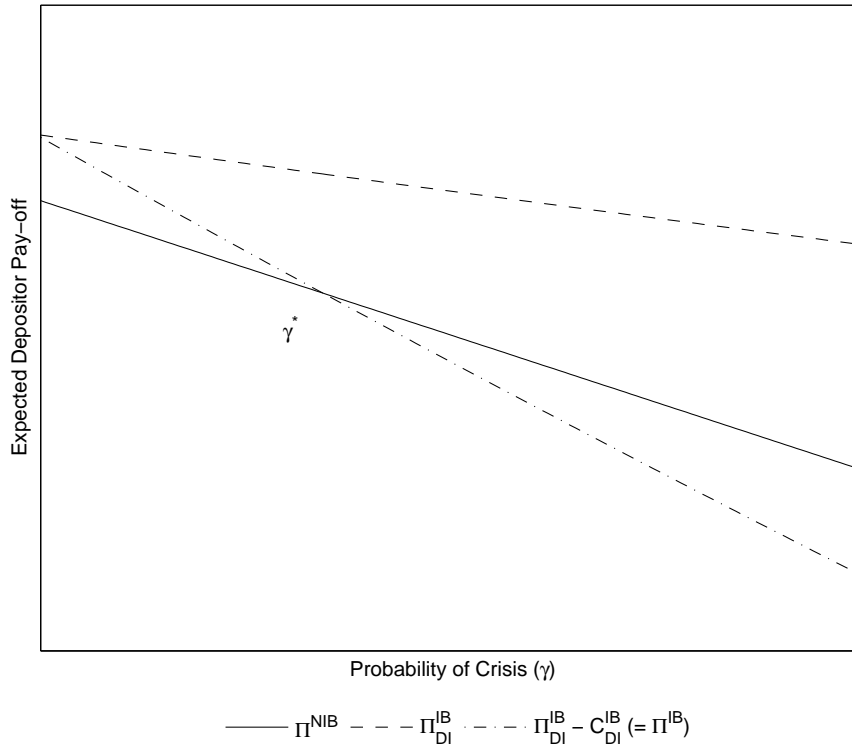
Figure 4. Expected Depositor Pay-off under (i) no interbank market; (ii) interbank market with deposit insurance; and (iii) interbank market with deposit insurance taking into account the expected cost of insurance.

The solid line represents the expected depositor payoff in the absence of deposit insurance when the bank does not participate in the interbank market (Π^{NIB}) while the dashed line shows the expected depositor payoff when the bank participates in the interbank market and the depositor is protected by deposit insurance (Π_{DI}^{IB}). Finally, the dot-dashed line represents the expected depositor payoff when the bank participates in the interbank market taking into account the expected cost to the government of providing deposit insurance ($\Pi_{DI}^{IB} - C_{DI}^{IB}$).

The key thing to note here is that the once the cost of deposit insurance is taken into account, the expected depositor payoff from participating in the interbank market is equal to the depositor payoff in the absence of deposit insurance, *i.e.* $\Pi_{DI}^{IB} - C_{DI}^{IB} = \Pi_{IB}$. This follows directly from Proposition 12, but intuitively, since deposit insurance in our model is essentially a state contingent transfer from the government to the depositors, if its costs are properly taken into

account, it is no coincidence that the depositors gain is exactly offset by the governments cost of providing this insurance.

Now it is easy to see that for all $\gamma^* > \gamma > \gamma^{DI}$, the benefits of participating in the interbank market after accounting for the cost of deposit insurance is less than the benefits of not participating in the interbank market (note that from Propositions 10, $\Pi^{NIB} = \Pi_{DI}^{NIB} - C_{DI}^{NIB}$). Therefore, socially it is not optimal for banks to participate in the interbank for all $\gamma < \gamma^*$. In other words, deposit insurance creates a wedge between the socially optimal γ^* and the privately optimal γ^{DI}.

Systemically Important Financial Institutions

While deposit insurance is an example of an explicit market friction that can result in a wedge between private and social optimality of network formation, the existence of Systemically Important Financial Institutions (SIFI) is an example of an implicit market friction that can result in suboptimal network formation. While there is no consensus on the precise definition of a SIFI, in general terms they can be defined as institutions whose disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity.[5]

Such institutions benefit from an implicit government guarantee that they will not be allowed to fail and in the event of a crisis, they will be bailed out. Interestingly, one of the 'criteria' for being a SIFI is their interconnectedness. Therefore, in the context of our model, we assume that when a bank decides to participate in the interbank market and forms a network, it will be classified as a SIFI./footnote In this setting, it is clear that the optimal strategy for the depositor and the bank would be to participate in the interbank market and form a network, irrespective of the crisis probability.[6]

This result stems from the fact that the SIFI status ensures that during the crisis state, the bank benefits from a government bailout and can continue to function. The key difference from deposit insurance is that when the crisis state materializes, deposit insurance guarantees that in the event of insolvency, depositors get at least $c_1 = 1$ at $t = 1$ when the bank fails. How-

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[5]See Financial Stability Board (2010, 2011)

[6]Note that interconnectedness is one of several criteria used to identify a SIFI. Therefore it is possible for a bank to be highly interconnected and yet not be classified as a SIFI. For the results to go through, we do not require all banks in the network to be SIFIs. If the bank in question is a SIFI, even if it faces contagion from a non-SIFI in the network, it will still have access to government support in the event of a crisis and can honor it commitments under the deposit contract ($c_1 = 1, c_2 = R$).

ever, in the case of a SIFI, the bank is not allowed to fail and receives government support to continue operating as normal and honor all its commitments, including the deposit contract $(c_1 = 1, c_2 = R)$. Furthermore, since there is no insolvency, there is no possibility of a crisis state and therefore no contagion channel.

This implies that our model is restricted to the normal state of the world, with zero probability of the crisis state. Therefore in the case of a SIFI, the bank and the depositors will always find it privately optimal to form a network irrespective of probability of a crisis.

Underestimation of γ

Market frictions such as deposit insurance and the existence of SIFIs can result in excessive interconnections even in cases where the probability of crisis is correctly measured. The other source of excessive and sub-optimal network formation is the underestimation of crisis probability, γ. Such underestimation can stem from a variety of sources.

There is an extensive literature on disaster myopia–misperception of the probability of rare events that arises from a set of heuristics commonly used to form judgments under uncertainty. Tversky and Kahneman (1974) highlight the so called 'availability heuristic', which states that agents base probabilistic assessments on the ease with which an event can be brought to mind: how recently it has occurred, how severe are its effects and how personal is the experience.

Car crashes are the classic example. These often arise from disaster myopia, as drivers systematically under-estimate the probability of a pile-up and drive too fast. The longer the period since the last crash, the greater the risk-taking. After a lengthy stretch of clear motorway, risk appetite may become too healthy, and risk-taking too great, relative to the true probability of disaster. In other words, drivers are disaster-myopic. Kunreuther and others (1978) conducted a widely cited study of disaster protection. As described in Kunreuther (1996), a 1974 survey of more than 1,400 homeowners in hurricane-prone areas of the US found that only 22% of respondents had voluntarily adopted any protective measures. Haldane (2011) discusses the role of such biases in the context of the financial crisis.

Given that before the recent financial crisis, bank runs were not seen in the UK for over 140 years and in Western Europe for the last 15 years, it is possible that such heuristic biases could have result in the underestimation of the probability of the crisis. In the context of our model, this underestimation of γ would have resulted in excessive interconnectedness between banks.

V. Conclusion

In order to study the role of market frictions that can result in the excessive interconnectedness that was seen during the crisis, we develop a simplified model of financial contagion. Departing from the literature which generally models contagion as an exogenous perturbation of the system, our model explicitly take into account the crisis state of the world. This allows us to endogenise the network formation decision with banks' recognising the risk of contagion when forming a network. We model the decision to form a network as an optimizing behavior of banks, where they balance the benefit of forming a network against the cost of contagion.

We use this framework to study how various market frictions can create a wedge between the socially optimal and the privately optimal threshold for network formation, resulting in excessive interconnectedness of the financial system. While such suboptimal network formation can arise from the several sources that can lead to the underestimation of the crisis probability, our results suggest that even in cases where the probability of crisis is perfectly observed, regulatory intervention in the form of deposit insurance and the existence of systemically important financial institutions can result in socially sub-optimal network formation and excessive interconnectedness in equilibrium.

Note however that while this framework is useful for highlighting the channels through which deposit insurance and the existence of systemically important financial institutions can result in excessive interconnectedness and thereby increase systemic risk, by itself it is not suitable for policy analysis. For a fuller discussion on the impact of deposit insurance on bank risk, see Anginer, Demirguc-Kunt, and Zhu (2014), Demirguc-Kunt and Detragiache (2002) and Demirguc-Kunt, Kane, and Laeven (2014); while the issue of the too-important-to-fail problem associated with SIFIs is discussed in more detail in Lim (2013) and Otker-Robe and others (2011).

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APPENDIX A. PROOFS

A.1. Proposition 1

When not participating in the interbank market, the bank optimally holds $y = \omega + x$ of the liquid asset.

Clearly, given the opportunity cost of the liquid asset, banks do not optimally hold liquid asset of size greater than $\omega + x$.[7] But suppose holds $y = \omega + x - \varepsilon$ of the liquid asset, where $\varepsilon \to 0$. We show that given assumption 1, such a deviation is not optimal and the bank optimally holds $y = \omega + x$ of the liquid asset.

First consider the high liquidity demand case, *i.e.* $\psi = \omega + x$. Since the bank only holds $y = \omega + x - \varepsilon$ of the liquid asset, in order to pay $\omega + x$ of impatient depositors $c_1 = 1$ at date $t = 1$, it has to liquidate $l = \frac{\varepsilon}{r}$ of the liquid asset. Assuming that the bank can do so without causing a run on the bank, it can no longer pay patient depositors $c_2 = R$ at date $t = 2$.

$$
\begin{aligned}
c_2 &= \frac{\left(1 - \omega - x + \varepsilon - \frac{\varepsilon}{r}\right)R}{1 - \omega - x} \\
&= R - \underbrace{\frac{\varepsilon R}{1 - \omega - x}\left(\frac{1}{r} - 1\right)}_{\text{Loss due to liquidation}}
\end{aligned}
$$

Next, we consider the low liquidity demand case, *i.e.* $\psi = \omega - x$. After paying out impatient depositors $c_1 = 1$ at date $t = 1$, the bank has surplus cash of $2x - \varepsilon$. However, this surplus cash is now less compared to the case where the bank holds $y = \omega + x$ of the liquid asset. Therefore,

$$
\begin{aligned}
c_2 &= \frac{2x - \varepsilon + (1 - \omega - x + \varepsilon)R}{1 - \omega + x} \\
&= \underbrace{\frac{2x + (1 - \omega - x)R}{1 - \omega + x}}_{c_2 \text{ when } y = \omega + x} + \underbrace{\frac{\varepsilon(R - 1)}{1 - \omega + x}}_{\text{Gain due to smaller } y}
\end{aligned}
$$

Comparing the loss and the gain, taking into account the ex-post distribution of patient and impatient depositors, we can show that the loss due to liquidation outweighs the gain due to

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[7]Note that the bank can increase its liquidation value during the crisis state by holding more of the liquid asset. However, since by assumption all consumers are impatient and withdraw their deposits at $t = 1$, the bank will always go bust unless it only holds liquid assets–in which case it becomes redundant.

lower opportunity cost of investing in the liquid asset.

$$(1-\omega-x)\cdot\frac{\varepsilon R}{1-\omega-x}\left(\frac{1}{r}-1\right) > (1-\omega+x)\cdot\frac{\varepsilon(R-1)}{1-\omega+x}$$

$$\implies R\left(\frac{1}{r}-1\right) > (R-1)$$

$$\implies r < \frac{R}{2R-1}$$

which is true by Assumption 1.

A.2. Corollary 1

In the low liquidity demand state with no interbank market, $1 < c_2 < R$, *where* $c_2 = \frac{2x+(1-\omega-x)R}{1-\omega+x}$

$$c_2 > 1$$

$$\implies \frac{2x+(1-\omega-x)R}{1-\omega+x} > 1$$

$$\implies 2x+(1-\omega-x)R > 1-\omega+x$$

$$\implies (1-\omega-x)(R-1) > 0$$

Since $(1-\omega-x) > 0$ and $R > 1 \implies c_2 > 1$.

$$c_2 < R$$

$$\implies \frac{2x+(1-\omega-x)R}{1-\omega+x} < R$$

$$\implies 2x+(1-\omega-x)R < (1-\omega+x)R$$

$$\implies x < R$$

Since $x < 1$ and $R > 1 \implies c_2 < R$.

A.3. Proposition 4

When participating in the interbank market, the bank optimally holds $y = \omega$ *of the liquid asset and exchanges interbank deposits of size x.*

We show this in two stages. First, we show that the bank hold just enough liquidity (in the form of investment in the liquid asset and interbank deposits) to pay impatient depositors

$c_1 = 1$ at date $t = 1$ during '*normal*' times. Next, we show that banks find it optimal to hold this liquidity in the form of ω of the liquid asset and interbank deposits of size x.

Given the opportunity cost of liquid assets, it is obvious that the bank would not hold liquidity greater than $\omega + x$, the demand from depositors in the high liquidity demand state. But suppose they hold less. They can do this either by holding smaller interbank deposits or by investing less in the liquid asset. We consider both cases in turn.

Case 1(a): Suppose the bank invests $y = \omega$ in the liquid asset and holds interbank deposits of size $x - \varepsilon$, where $\varepsilon \to 0$. We can show that such a deviation is not optimal and banks are better off holding interbank deposits of size x. Since interbank deposits disappear during the crisis state, their size does not matter in such states and it is sufficient to compare the payoffs during '*normal*' times.

Suppose the bank faces high liquidity demand, *i.e.* $\psi = \omega + x$ and has to meet the withdrawal demand of $\omega + x$ from impatient depositors at $t = 1$. The bank holds ω of the liquid asset and can withdraw the interbank deposit of $x - \varepsilon$. This leaves the bank with a deficit of ε which it must meet by liquidating a fraction $l = \frac{\varepsilon}{r}$ of the long asset. However, due to early liquidation of the long asset, the bank can no longer pay date 2 depositors $c_2 = R$. At date 2, the bank gets $(1 - \omega - \frac{\varepsilon}{r})R$ from its remaining long assets and has to pay $(x - \varepsilon)R$ for the interbank deposits and meet withdrawal demand of $(1 - w - x)$. This implies that

$$c_2 = \frac{\left(1 - \omega - \frac{\varepsilon}{r} - (x - \varepsilon)\right)R}{(1 - \omega - x)}$$
$$= R - \underbrace{\frac{\varepsilon R}{1 - \omega - x}\left(\frac{1}{r} - 1\right)}_{\text{Loss due to liquidation}}$$

Clearly, this is decreasing in ε and the bank can get $c_2 = R$ if it sets $\varepsilon = 0$.

Next, when the bank faces low liquidity demand, *i.e.* $\psi = \omega - x$, it has a cash surplus of ε at date 1, after paying out $x - \varepsilon$ for interbank deposits and $(\omega - x)$ to the impatient depositors. At date 2, it uses this surplus cash, $(1 - \omega)R$ return from the long asset, and $(x - \varepsilon)R$ from interbank deposits to pay $(1 - \omega + x)$ of patient depositors. This implies that

$$c_2 = \frac{\varepsilon + (1 - \omega)R + (x - \varepsilon)R}{(1 - \omega + x)}$$
$$= R - \underbrace{\frac{\varepsilon(R - 1)}{1 - \omega + x}}_{\text{Loss due to cash surplus}}$$

Once again, this is decreasing in ε and the bank finds it optimal to set $\varepsilon = 0$ and increase $c_2 = R$. Therefore, given $y = \omega$ holdings of the liquid asset, the bank optimally holds interbank deposit of size x.

Case 1(b): Suppose instead the bank invests $y = \omega - \varepsilon$ in the liquid asset and holds interbank deposits of size x. During '*normal*' times, the analysis remains the same as above. However, since the bank now holds only $y = \omega - \varepsilon$ of the liquid asset, its liquidation value decreases. During the crisis state, as well as in the high liquidity demand state when there is a run on the bank, the banks liquidation value,

$$
\begin{aligned}
c_1 &= (\omega - \varepsilon) + (1 - \omega + \varepsilon)r \\
&= \omega + (1 - \omega)r \underbrace{-\varepsilon(1 - r)}_{\text{Loss due to smaller y}}
\end{aligned}
$$

Therefore, the bank can increase its liquidation value by setting $\varepsilon = 0$. Finally, consider the case where the bank faces low liquidity demand, *i.e.* $\psi = \omega - x$ and there is a crisis in the other region. Interbank deposits disappear and the bank has surplus cash of $x - \varepsilon$ at date 1 after paying out $c_1 = 1$ to $(\omega - x)$ impatient depositors. At date 2, it uses this surplus cash and $(1 - \omega + \varepsilon)R$ return from the long asset to pay $(1 - \omega + x)$ of patient depositors. This implies,

$$
\begin{aligned}
c_2 &= \frac{(x - \varepsilon) + (1 - \omega + \varepsilon)R}{(1 - \omega + x)} \\
&= \frac{x + (1 - \omega)R}{(1 - \omega + x)} \underbrace{+ \frac{\varepsilon(R - 1)}{1 - \omega + x}}_{\text{Gain due to smaller y}}
\end{aligned}
$$

Although the bank can increase c_2 in this case, it is easy to see that for all $\gamma < \frac{1}{2}$, in expectation this gain is less than the loss during the no crisis state. Therefore, banks cannot do better by holding less liquidity, either by holding smaller interbank deposits or by investing less in the liquid asset.

We now show that banks find it optimal to hold this liquidity in the form of ω of the liquid asset and interbank deposits of size x.

Case 2(a): Suppose the bank holds $\omega - \varepsilon$ of the liquid assets and interbank deposits of size $x + \varepsilon$. When the bank faces high liquidity demand, the cash flows match up during normal times and $c_1 = 1$ and $c_2 = R$. However, during the low liquidity demand state, the bank has a liquidity shortfall of 2ε, even during normal times–$(\omega - \varepsilon)$ cash inflow from the investment in the liquid asset vs the outflow of $(x + \varepsilon)$ on account of interbank deposits and $(\omega - x)$ to pay

impatient depositors. The bank is forced to liquidate $l = \frac{2\varepsilon}{r}$ of the long asset. This implies that

$$
\begin{aligned}
c_2 &= \frac{\left(1 - \omega + \varepsilon - \frac{2\varepsilon}{r} + (x + \varepsilon)\right)R}{(1 - \omega + x)} \\
&= R - \underbrace{\frac{2\varepsilon R}{1 - \omega + x}\left(\frac{1}{r} - 1\right)}_{\text{Loss due to liquidation}}
\end{aligned}
$$

Clearly, this is decreasing in ε and the bank can get $c_2 = R$ if it sets $\varepsilon = 0$. During the crisis state in the other region, the interbank deposits disappear, but there is no bank run in the low liquidity demand case and the bank has a smaller cash surplus of $(x - \varepsilon)$ at date 1. Therefore,

$$
\begin{aligned}
c_2 &= \frac{(x - \varepsilon) + (1 - \omega + \varepsilon)R}{(1 - \omega + x)} \\
&= \frac{x + (1 - \omega)R}{(1 - \omega + x)} + \underbrace{\frac{\varepsilon(R - 1)}{1 - \omega + x}}_{\text{Gain due to smaller y}}
\end{aligned}
$$

As in Case 1(b), although the bank can increase c_2 in this case, for all $\gamma < \frac{1}{2}$, in expectation this gain is less than the loss during the no crisis state. Finally, in the event of a bank run, the banks liquidation value falls.

$$
\begin{aligned}
c_1 &= (\omega - \varepsilon) + (1 - \omega + \varepsilon)r \\
&= \omega + (1 - \omega)r \quad \underbrace{-\varepsilon(1 - r)}_{\text{Loss due to smaller y}}
\end{aligned}
$$

Therefore, the bank is better off with $\varepsilon = 0$.

Case 2(b): Suppose the bank holds $\omega + \varepsilon$ of the liquid assets and interbank deposits of size $x - \varepsilon$. As before, during normal times, the cash flows match up in the high liquidity demand state. But during the low liquidity demand state, the bank has surplus liquidity of 2ε - $(\omega + \varepsilon)$ cash inflow from the investment in the liquid asset vs the outflow of $(x - \varepsilon)$ on account of interbank deposits and $(\omega - x)$ to pay impatient depositors. This implies that

$$
\begin{aligned}
c_2 &= \frac{2\varepsilon + (1 - \omega - \varepsilon)R + (x - \varepsilon)R}{(1 - \omega + x)} \\
&= R \quad - \underbrace{\frac{2\varepsilon(R - 1)}{(1 - \omega + x)}}_{\text{Loss due to cash surplus}}
\end{aligned}
$$

This is decreasing in ε and the bank can get $c_2 = R$ if it sets $\varepsilon = 0$.

During the crisis state in the other region, the interbank deposits disappear, but there is no bank run in the low liquidity demand case and the bank has a surplus cash of $(x + \varepsilon)$ at date 1. Therefore,

$$
\begin{aligned}
c_2 &= \frac{(x + \varepsilon) + (1 - \omega - \varepsilon)R}{(1 - \omega + x)} \\
&= \frac{x + (1 - \omega)R}{(1 - \omega + x)} \underbrace{- \frac{\varepsilon(R - 1)}{1 - \omega + x}}_{\text{Loss due to cash surplus}}
\end{aligned}
$$

Once again we see that the optimal strategy for the bank is to set $\varepsilon = 0$, *i.e.* hold $y = \omega$ of the liquid asset and exchange interbank deposits of size x

However, in the event of a bank run, the liquidation value of the bank increases when it holds $y = \omega + \varepsilon$ of the liquid asset.

$$
\begin{aligned}
c_1 &= (\omega + \varepsilon) + (1 - \omega - \varepsilon)r \\
&= \omega + (1 - \omega)r \underbrace{+ \varepsilon(1 - r)}_{\text{Gain due to larger y}}
\end{aligned}
$$

Based on this, we can compute the expected gain during liquidation and the expected loss when there is no bank run.

Expected gain during bank runs: $\left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma\varepsilon(1 - r) + \gamma\varepsilon(1 - r)$

Expected loss when no bank run: $\left(\frac{1}{2} - \frac{\gamma}{2}\right)(1 - \gamma)2\varepsilon(R - 1) + \left(\frac{1}{2} - \frac{\gamma}{2}\right)\gamma\varepsilon(R - 1)$

As long as $\frac{1-r}{R-1} < \frac{\gamma^2 - \gamma + 2}{\gamma(1-\gamma)}$, the expected loss is greater than the expected gain and banks find it optimal to hold ω of the liquid asset and interbank deposits of size x. The above condition is relatively weak and holds for most parameter values. Intuitively, it says that the probability of the crisis should not be too high relative to the liquidation cost and the return of the long term assets. For practical purposes, it is only violated when the liquidation cost is close to 0 and/or R is very close to 1 and at the same time, the probability of the crisis is very high.

A.4. Corollary 2

If Assumption 2 holds, contagion results in a run on the bank during the high liquidity demand state.

In the case of a crisis in region B, the region A bank is forced to liquidate a fraction $l = \frac{x}{r}$ of the long asset to meet the withdrawal demand of $\psi = \omega + x$ during the high liquidity state and pay impatient depositors $c_1 = 1$. Note that this implicitly assumes that $l \leq (1 - w)$. In case $l > (1 - w)$, the bank cannot pay $c_1 = 1$ to date one depositors even after it liquidates all its long assets. It pays $c_1 < 1$ and therefore goes bust.

Assuming this does not hold, the bank has $(1 - \omega - l)$ of the long asset left, with which it has to pay $(1 - \omega - x)$ patient depositors. Thus,

$$(1 - \omega - l)R = (1 - \omega - x)c_2$$
$$\implies c_2 = \frac{(1 - \omega - \frac{x}{r})R}{1 - \omega - x}$$

For there to be a run on the bank, we need to show,

$$c_2 < 1$$
$$\implies (1 - \omega - \frac{x}{r})R < 1 - \omega - x$$
$$\implies (1 - \omega)R - x\frac{R}{r} < 1 - \omega - x$$
$$\implies (1 - \omega)(R - 1) < x\left(\frac{R}{r} - 1\right)$$
$$\implies x > \frac{(1 - \omega)(R - 1)}{\frac{R}{r} - 1}$$

Given Assumption 2, this is always true.

A.5. Proposition 7

$\Pi^{IB} - \Pi^{NIB}$ *is decreasing in crisis probability* γ*, i.e. the benefits of participating in the inter-bank market are decreasing in* γ*.*

Using Propositions 3 and 6, we have

$$\Pi^{IB} - \Pi^{NIB} = \omega + (1 - \omega)R - \frac{3}{2}(1 - \omega)(R - r)\gamma + \frac{1}{2}(1 - \omega)(R - r)\gamma^2$$
$$- R(1 - \omega - x) + \omega + x - (1 - \omega - x)(R - r)\gamma$$
$$= x(R - 1) - \frac{1}{2}(1 - \omega + 2x)(R - r)\gamma + \frac{1}{2}(1 - \omega)(R - r)\gamma^2$$

Taking the partial derivative with respect to γ,

$$\frac{\partial}{\partial \gamma}(\Pi^{IB} - \Pi^{NIB}) = -\frac{1}{2}(1 - \omega + 2x)(R - r) + (1 - \omega)(R - r)\gamma$$

$$\text{Now, } \frac{\partial}{\partial \gamma}(\Pi^{IB} - \Pi^{NIB}) < 0 \text{ if } \gamma < \frac{1 - \omega + 2x}{2(1 - \omega)}$$

This is always true for all $\gamma < \frac{1}{2}$, implying that $\Pi^{IB} - \Pi^{NIB}$ is decreasing in the crisis probability γ.

A.6. Proposition 8

There exists a γ^* such that $\Pi^{IB} > \Pi^{NIB}$ for $\gamma < \gamma^*$ and $\Pi^{IB} < \Pi^{NIB}$ for $\gamma > \gamma^*$.

We solve for γ^* such that $\Pi^{IB} - \Pi^{NIB} = 0$.

$$\Pi^{IB} - \Pi^{NIB} = x(R - 1) - \frac{1}{2}(1 - \omega + 2x)(R - r)\gamma + \frac{1}{2}(1 - \omega)(R - r)\gamma^2 = 0$$

Solving for γ^*, we have

$$\gamma^* = \frac{\frac{1}{2}(1 - \omega + 2x)(R - r) - \sqrt{\frac{1}{4}(1 - \omega + 2x)^2(R - r)^2 - 2(1 - \omega)(R - r)x(R - 1)}}{(1 - \omega)(R - r)}$$

Using Proposition 7, it is clear that $\Pi^{IB} > \Pi^{NIB}$ for $\gamma < \gamma^*$ and $\Pi^{IB} < \Pi^{NIB}$ for $\gamma > \gamma^*$.

A.7. Proposition 13

The critical level of γ is higher with deposit insurance, i.e. $\gamma^{DI} > \gamma^*$.

From the proof of Proposition 8, we have the gain from participating in the interbank market as a function of γ in the absence of deposit insurance.

$$\Pi^{IB} - \Pi^{NIB} = x(R - 1) - \frac{1}{2}(1 - \omega + 2x)(R - r)\gamma + \frac{1}{2}(1 - \omega)(R - r)\gamma^2$$

Similarly, we can calculate the gain from participating in the interbank market as a function of γ in the presence of deposit insurance.

$$\Pi^{IB}_{DI} - \Pi^{NIB}_{DI} = x(R - 1) - \frac{1}{2}(1 - \omega + 2x)(R - 1)\gamma + \frac{1}{2}(1 - \omega)(R - 1)\gamma^2$$

Comparing these two equations, we can show that,

$$(\Pi_{DI}^{IB} - \Pi_{DI}^{NIB}) > (\Pi^{IB} - \Pi^{NIB})$$
$$\implies (\Pi_{DI}^{IB} - \Pi_{DI}^{NIB}) - (\Pi^{IB} - \Pi^{NIB}) > 0$$
$$\implies \frac{1}{2}(1 - \omega + 2x)(1 - r)\gamma - \frac{1}{2}(1 - \omega)(1 - r)\gamma^2 > 0$$
$$\implies (1 - \omega + 2x) > (1 - \omega)\gamma$$

Since $0 < \gamma < 1$ and $x > 0$, this condition holds.

This implies that for every level of γ, the gains from participating in the interbank market are greater in the presence of deposit insurance. This in turn implies that $\gamma^{DI} > \gamma^*$.

Using the expression for $\Pi_{DI}^{IB} - \Pi_{DI}^{NIB}$, we can explicitly solve for the value of γ^{DI}. Solving, we have

$$\gamma^{DI} = \frac{2x}{1 - \omega}$$

Note that we ignore the other root $\gamma^{DI} = 1$ as we are interested in small γ.

A.8.   Proposition 14

For $\gamma^* > \gamma > \gamma^{DI}$, banks participation in the interbank market is socially sub-optimal.

For $\gamma^* > \gamma > \gamma^{DI}$, in the absence of deposit insurance the bank optimally does not participate in the interbank market and the expected depositor payoff is Π^{NIB} (Proposition 3). On the other hand, with deposit insurance, it is privately optimal for the bank to participate in the interbank market and the expected depositor payoff is Π_{DI}^{IB} (Proposition 12). In order to assess the social optimality of participation in the interbank market, we need to explicitly take into account the costs of deposit insurance, C_{DI}^{IB}. Participation is socially sub-optimal when $\Pi^{NIB} > \Pi_{DI}^{IB} - C_{DI}^{IB}$.

From Proposition 12, we have $C_{DI}^{IB} = \Pi_{DI}^{IB} - \Pi^{IB}$. Substituting, it is sufficient to show that $\Pi^{NIB} > \Pi^{IB}$. This always holds for all $\gamma > \gamma^*$ (Proposition 8). Therefore, for $\gamma^* > \gamma > \gamma^{DI}$, banks participation in the interbank market is socially sub-optimal.