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Optimal Debt Policy Under Asymmetric Risk

by Julio Escolano and Vitor Gaspar

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Optimal Debt Policy Under Asymmetric Risk

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Abstract

In the paper we show that, most of the time, smooth reduction in the debt ratio is optimal for tax-smoothing purposes when fiscal risks are asymmetric, with large debt-augmenting shocks more likely than commensurate debt reducing shocks. Asymmetric risks are a feature of 200 years of data for the U.S. and the U.K.: rare but recurrent large surges of the debt-to-GDP ratio, followed by very gradual but persistent declines over long periods. More informal evidence from many other countries suggests that asymmetry is a general feature of fiscal shocks. The gradual smooth reduction in the public debt to GDP ratio is not a response to past developments. Instead it is optimal given recurrent fiscal risks and the empirical characteristics of fiscal shocks. The behavior of the debt-to-GDP ratio in the U.K. and the U.S. seems roughly compatible with the prescriptions of the tax-smoothing model.

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I. INTRODUCTION

The global financial crisis of 2008 and its aftermath resulted in large increases in public debt ratios to GDP. Ostry, Ghosh, and Espinoza (2015) argue that fiscal authorities should not aim at a primary balance surplus beyond what is necessary to pay the higher interest bill entailed by the additional debt.[2] Rather they should just aim at stabilizing the debt ratio at its current level. The set-up used here, as well as in Ostry, Ghosh, and Espinoza (2015), abstracts from counter-cyclical stabilization policies and rollover risk, and instead focuses on intertemporal considerations associated with government financing and tax smoothing. In this paper, we simplify the framework in Ostry, Ghosh, and Espinoza by disregarding public investment. Instead, we extend long-term optimal debt policy a la Barro (1979 and 1986) to consider stochastic and recurrent shocks to the fiscal position and, hence, to the debt ratio.

The question we ask is whether countries that do not face binding financing constraints should gradually reduce high public debt to GDP ratios. That question is distinct from debt reduction forced by the risk of an immediate loss of market access. Only a few advanced economies, which experienced large debt ratio increases since 2008, faced adverse market conditions for rolling over or increasing their outstanding debt stock. For the remaining advanced countries, including some of the largest economies in the world, debt reduction policies are, at this time, a matter of choice.

The choice of debt policy entails an inter-temporal trade-off. As argued in Ostry, Ghosh, and Espinoza (2015), keeping the debt ratio constant is optimal when the marginal deadweight loss incurred today equals the marginal expected net present value of future gains. Minimizing the net present value of inter-temporal deadweight losses implies tax smoothing: if expenditure is expected to remain constant in the future, tax smoothing implies a tax ratio set permanently at the level that just pays the annual constant expenditure and the interest on the inherited debt. In this case, debt will also be expected to remain at the current level.

Importantly, policy decisions regarding this inter-temporal trade-off must be made in the face of recurrent fiscal risk. As the recent financial crisis showed and the earlier history of debt confirms, public debt ratios have been subject to very substantial disturbances. While the size of the debt accumulation prompted by the crisis of 2008 stands out in post-WWII history, it has certainly not been the first or the largest upward shock to public debt from a broader historical perspective, nor will it be the last.

Thus, to analyze the optimal response to the debt surge after the 2008 global financial crisis, the approach here will be to consider this surge as a (particularly bad) draw from an enduring stochastic process. From this standpoint, we show that whether gradual reduction in the debt ratio is optimal hinges on the distribution of debt shocks. If the distribution of shocks were symmetric, reducing debt now to pay down in advance future adverse shocks would not be more

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[2] Primary balance is used henceforth to denote fiscal revenue less expenditure before interest payments. Thus the primary balance equals the net lending by the government (or overall balance) plus interest payments. The concept is sometimes denoted gross primary balance—since it nets out interest payments from expenditure, but not interest receipts from revenue.

justified than increasing debt now on account of future favorable shocks. Indeed, in the case of symmetric shocks or if no further shocks were expected in the future, the optimal policy would be to stabilize the debt at current levels, making no deliberate attempt to reduce or increase debt ratios—as argued in Ostry, Ghosh, and Espinoza (2015).

However, if the probabilistic distribution of fiscal shocks results in infrequent but large adverse (positive) shocks—as we find it is the case in the data—the optimal policy in normal times is to reduce debt ratios gradually but persistently in anticipation of future large negative events. In the past, the most prominent of these negative events were the outbreak of wars. But in present times, other events can also result in large unexpected fiscal costs. In particular, reducing public debt ratios pre-emptively can be argued on grounds of financial stability (Obstfeld, 2013), or the materialization of public contingent liabilities (Bova et al. 2016). The objective of this policy of reducing debt ratios in good times is to stabilize the (expected) debt ratio in the long run. In years when large adverse shocks materialize, they should be absorbed though borrowing. But in the absence of an emergency—that is, in most years—the optimal policy results in a moderate debt reduction. As we will show, the reduction in the debt-to-GDP ratio is a direct consequence of the distribution of shocks to public debt. Optimal policy entails that both the public debt ratio and the tax ratio behave like random walks. Nevertheless, the empirical characteristics of the distribution of shocks means that it is most likely to witness shocks below average. This bias leads to debt reduction in normal years.

Given that our analysis is motivated by the actual historical pattern of the debt ratio in the major economies of the time, we start in Section II by summarily presenting the evidence of the last two centuries for the United Kingdom and the United States. We argue that the path of debt ratios is characterized by large and sudden upward shocks that occur infrequently, and by long periods of gradual but persistent debt reduction. We focus on the United Kingdom and the United States because one of the two was the most advanced economy in the world during the historical period studied (1790-present). Moreover, both developed financial markets and institutions early on, and their sovereign debt was, for most of the period, considered as a safe asset. Thus, these sovereigns were least likely to face financing constraints. Also, consistent data on debt and GDP are available for these countries, as well as a well-researched historical economic record. Nevertheless, it is important to stress that there is evidence that positive asymmetry is a general feature of fiscal shocks that applies to a wide cross-country evidence (International Monetary Fund, 2016).

Section III then lays out a basic framework in which government debt is subject to random shocks. The policymaker seeks to smooth the tax ratio over time, subject only to a non-Ponzi game solvency constraint, but is otherwise free to accommodate any amount of debt. The optimal policy in this setting is in line with Barro (1979). After a shock to the debt ratio, the government should adjust the tax ratio and the primary balance up or down just enough to pay the new level of (growth-adjusted) interest, and the expected annual expenditure including the long-term average value of shocks (if different from zero). As a result, the actual paths of the debt and tax ratios (and of the primary balance as a ratio to GDP) will be random walks and the expected path of the debt ratio will be constant. Section IV specializes this optimal debt policy to a skewed distribution of shocks, with infrequent but large adverse shocks. Section V formally analyzes the historical fiscal shocks and test the skewness of the distribution of changes in the debt ratio. It finds that the statistical evidence is consistent with a skewed distribution of shocks. Section VI concludes.

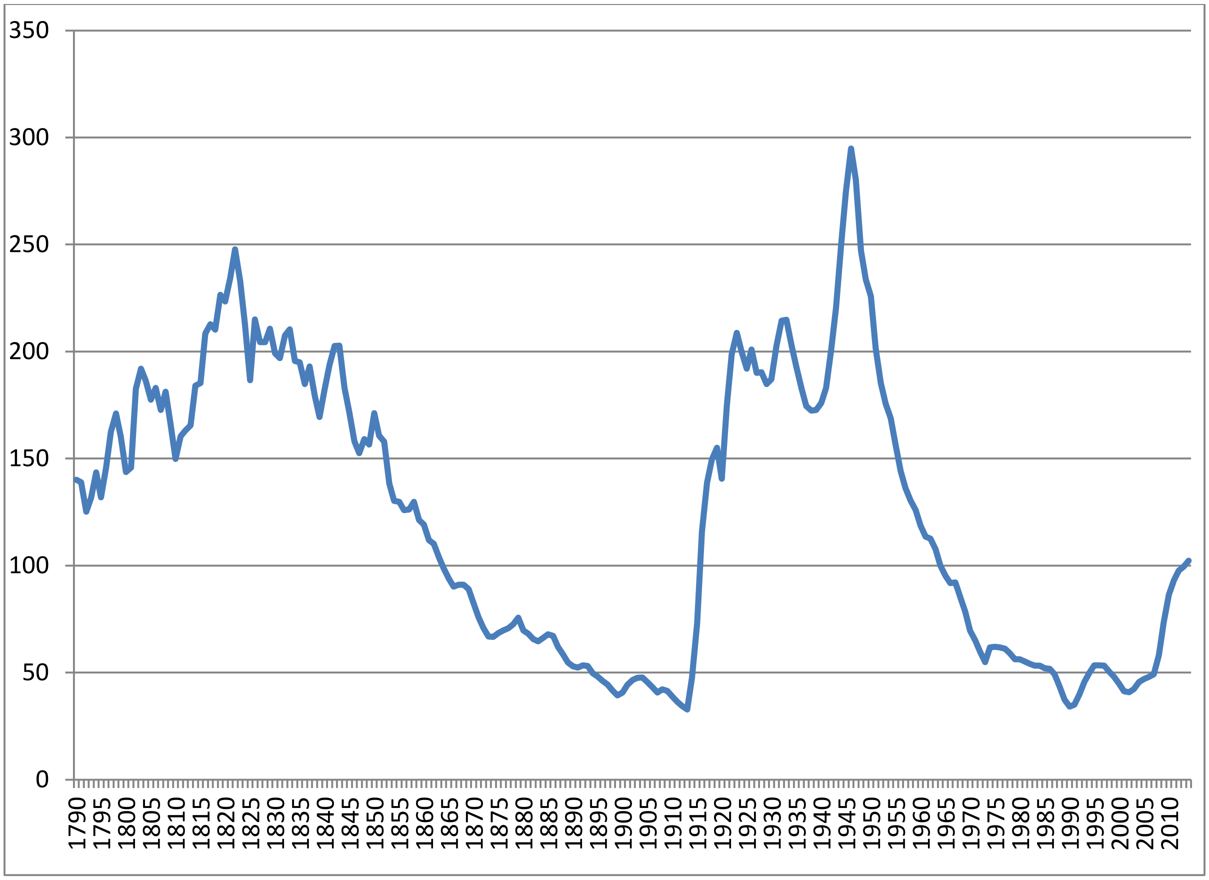
II. Evidence from Debt History

In discussing the appropriate response to recent increases in public debt, it is useful to start from the historical record. Over the past two and a quarter centuries, the path of government debt ratios in the United Kingdom and the United States were characterized by occasional large increases prompted by national emergencies such as wars and large economic crises followed by long periods of sustained reductions in debt ratios.

The debt ratio of the United Kingdom (Figure 1) reached a peak of 220 percent of GDP in 1822, after the Napoleonic wars.[3] But during the following almost a century, from 1822 to 1913, the debt ratio declined more or less steadily to a historical low of 31 percent—a deliberate and sustained debt reduction policy. The rationale for such a sustained reduction in the debt to GDP ratio was part of the so-called Gladstone doctrine of public finances (Schumpeter, 1956). During these years, roughly covering the Victorian period, there were few severe downturns (Hills, Thomas, and Dimsdale, 2010). Military engagement costs—mainly in the Crimean and Boer wars—relative to the size of the economy were of a scale well below those of the preceding period (Barro, 1987) and also those to come in the 20th century.

[3] Data for the United Kingdom in early years correspond to "national debt" and are from Hills, Thomas, and Dimsdale (2010, database updated in 2014). Data for more recent years correspond to consolidated general government debt and are from the U.K. Office for National Statistics (ONS, series code BKPX). For consistency with the data available for the earliest dates, we use GDP at current factor cost for the whole sample period (calculated like in Hills, Thomas, and Dimsdale (2010) as the difference between the ONS series codes: YBHA-CMVL). This results in a debt-to-GDP ratio of around 100 percent in 2014, which is higher than the debt-to-GDP ratio of about 90 percent obtained by using the headline series of GDP at current market prices.

Figure 1. United Kingdom: National/Government Debt (Percent of GDP)



Sources: Hills, Thomas, and Dimsdale (2010, updated 2014), and UK Office for National Statistics (ONS).

WWI and the recession of 1920 – 1921 prompted a sharp increase in the debt ratio, which was also followed by subsequent attempts at reduction, albeit interrupted by the depression in 1930 – 1933.[4] In turn, these years were followed by a remarkable fall in the debt ratio, in the seven years (1934 – 1940) preceding WWII. As a result of the WWII effort, the debt ratio rose to 270 percent of GDP by 1946. But during 1946 – 1990 it fell again to about 34 percent. The recession in the early 1990s, raised the debt ratio to about 53 percent. And finally, the global financial crisis of 2008 raised it again from about 49 percent in 2007 to 102 percent in 2014. Overall, for the past two centuries, national debt policies in the United Kingdom show persistent reductions in the debt ratio in the wake of a few large increases associated with wars or large recessions.

[4] The increase in the debt ratio during 1930 – 1933 was mainly due to a decline in nominal GDP. The nominal value of debt went from 7.6 billion pounds in 1929 to 8 billion pounds in 1933 or a nominal increase of about 5.7 percent, while nominal GDP declined by more than 8.4 percent (Hills, Thomas, and Dimsdale, 2010).

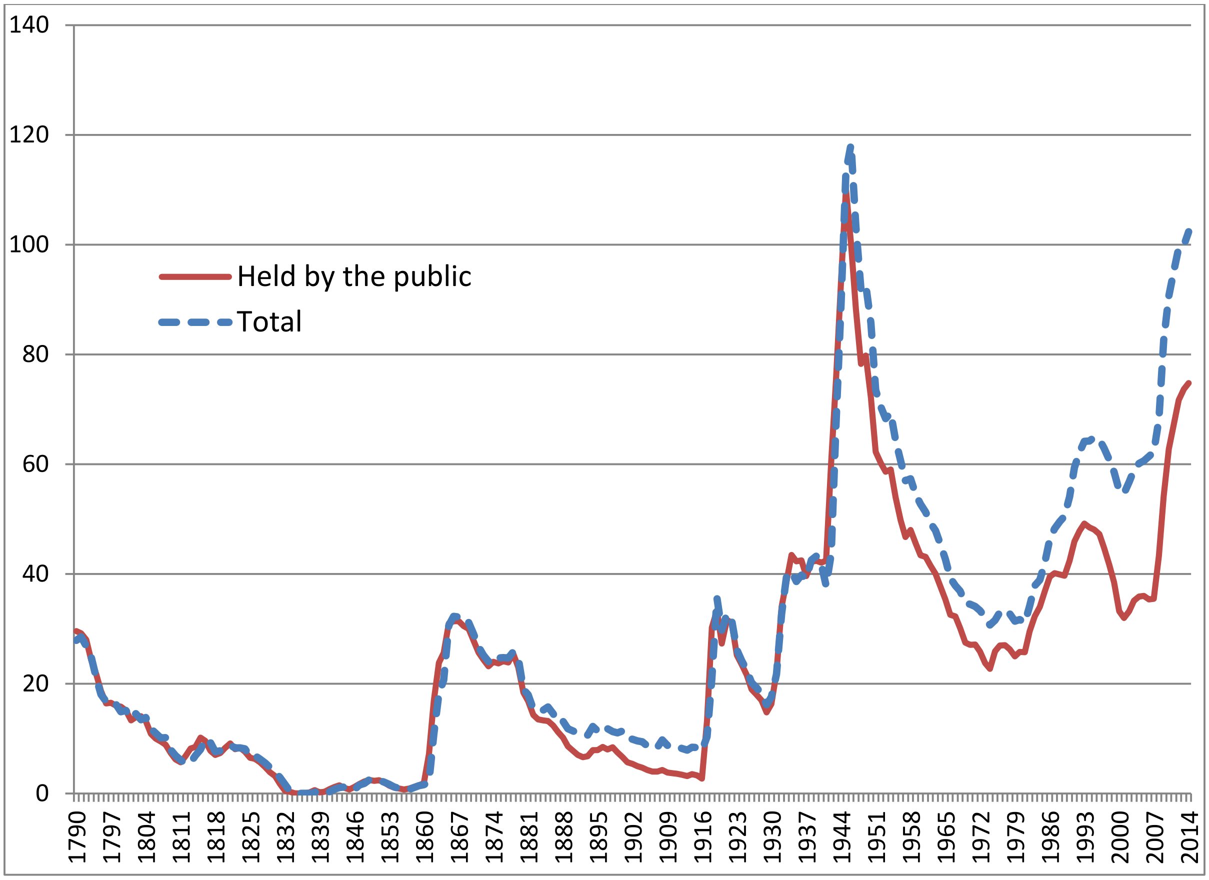
The debt history of the United States (Figure 2) shows a similar pattern.[5] As a result of the revolutionary war, the debt of the federal government was about 30 percent of GDP in 1790 and the new republic started essentially bankrupt.[6] However, building the credit standing of the federal government was a central objective of the first secretary of the U.S. Treasury, Alexander Hamilton (Gaspar, 2015). Hamilton's reforms set the debt ratio firmly on a downward path which continued through the eve of the war of 1812, by which time it was little more than 5 percent of GDP. After rising to about 10 percent as a result of the war, the debt ratio resumed its decline and by 1835 the whole U.S. national debt was fully paid off, with the debt ratio remaining very low until the civil war.[7] The civil war and, later, WWI both prompted steep raises in the debt ratio, which were each clawed back to a large extent in the subsequent years. The depression after 1929 and WWII raised the debt ratio to an unprecedented 121 percent of GDP. After the war, the debt ratio was again brought back down, despite significant military spending pressures during the cold war and Korea and Vietnam wars, to 31 percent by 1974, hovering around that level until the early 1980s.

[5] Data for the United States in early years are from the U.S. Congressional Budget Office (2010) and U.S. Treasury Department "Historical Debt Outstanding" time series. Later data are from the U.S. Bureau of Economic Analysis, "National Income and Product Accounts (NIPA)" tables and from the U.S. Federal Reserve Bank of St. Louis FRED database. The debt ratio mentioned in the discussion refers to federal government debt held by the public—which excludes debt held by organizations and agencies of the federal government.

[6] For an analysis of debt and public finance policies in the early period of U.S. history, see Sargent (2012) and Hall and Sargent (2013).

[7] In 1835, President Andrew Jackson, who considered debt a "national curse," declared the national debt paid, and distributed the remaining federal surplus to the states. However, the federal government resumed borrowing as a result of the depression of 1837, prompted by the burst of a bubble in land prices.

Figure 2. United States: Federal Government Debt
(Percent of GDP)



Sources: CBO, BEA (NIPA tables), U.S. Treasury, and Federal Reserve Bank of St. Louis FRED database.

The period from 1980 to the start of the first Clinton administration in 1993 represents the most significant departure from the recurrent pattern of swift debt ratio increases followed by sustained debt consolidation. During this period the debt ratio rose steadily from 26 percent in 1980 to 49 percent in 1993, without any obvious exogenous shock to the public finances. Underpinning this drift was a higher interest bill during the high rates of the Volcker years and the expansion of public social insurance costs—primarily health care, pensions, and income support—whose coverage had expanded rapidly over the previous decades. While the tax-to-GDP ratio also rose, it did so more gradually and with a lag. During the Clinton administration, the debt ratio fell again to about 32 percent in 2001. Finally, as in the case of the United Kingdom and other advanced economies, the global financial crisis caused a sharp increase in debt, with the federal debt ratio jumping from 35.5 percent of GDP in 2007 to 75 percent in 2014.

In sum, the historical narrative shows an upside-down Sisyphean debt pattern: long periods of slow but persistent declines in the debt ratio unwound by recurrent but rare large adverse events, after which the gradual decline resumes again from the newly increased debt ratio. Specifically, the sample of annual changes in the debt ratio (its first difference) appears to have an asymmetric distribution with a fat right-hand tail. This will be confirmed below by formal statistical testing.

III. Optimal Debt Policy

We derive in this section an optimal debt policy that is consistent with the broad historical features described above. We will show that the optimal path of the debt ratio follows a random walk if the government seeks to smooth taxes inter-temporally and minimize the present value of dead-weight loss in the presence of debt shocks. The optimal policy sets the tax ratio each year at the level that will pay for the expected annual expenditure and the interest bill on inherited debt. The expected value of the debt ratio for each future period is constant, and equal to the initial debt at the time the policy decision is made. As shocks to the debt ratio occur and the optimal policy is re-evaluated each period, the debt and tax ratios follow a random walk.

Consider the dynamic optimization problem of a government that aims at smoothing the tax ratio over time while maintaining solvency. It faces the usual inter-temporal budget constraint precluding Ponzi games—that is, it cannot systematically roll over principal and due interest by issuing more debt. But otherwise, it faces no immediate constraints as to the level of the debt ratio it can incur or of the tax ratio it can levy to service its debt. That is, we assume away the constraints posed by fiscal limits—such as Laffer curve ceilings on tax revenue or political economy-related upper bounds on primary balances—as an approximation to the policy environment in economies where the lack of fiscal policy space is not a pressing concern.

In particular, we posit that the government minimizes the discounted sum of the square of future tax ratios (Barro, 1979; Sargent, 1987; and Ghosh, 1995). Formulating the loss function in terms of the square of the tax ratio is meant as a stylized representation of several arguments for minimizing variations in the tax ratio around the minimum consistent with inter-temporal solvency.[8] The dead-weight loss associated with taxation is approximately proportional to the square of the tax ratio (Barro 1979; Kay, 1980; Auerbach, 1985; and Afonso and Gaspar, 2007). Moreover, repeated abrupt changes in the tax ratio may cause deleterious disruptions in the institutional fiscal framework, or political costs to the government.

The constraint faced by the government is the inter-temporal budget constraint. This is equivalent to the no-Ponzi game (or transversality) condition in combination with the year-by-year budget accounting identity. The latter states that the debt at the beginning of period *t*+1 (d_{t+1}) is determined by the debt at the beginning of period *t* (d_t), the tax revenue (τ_t) less expenditure (g_t) in *t*, and a shock (ε_{t+1}) which is not known at the time budget plans are drawn

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[8] The desirability of tax smoothing is an old idea. As quoted in Hall and Sargent (2013), Albert Gallatin (U.S. Treasury Secretary during 1801 – 1814) saw the need to borrow when faced with funding emergencies such as the war of 1812 or the Louisiana purchase, while setting the tax ratio to "provide a revenue at least equal to the annual expenses on a peace establishment, the interest on the existing debt, and the interest on the loans which may be raised. (…) losses and privations caused by war should not be aggravated by taxes beyond what is strictly necessary." This view did not prevent Gallatin from following a forceful debt reduction policy in peacetime, devoting up to three quarters of federal fiscal revenue to this end.

and tax and expenditure plans for time *t* are chosen (equation (3) below). The convention used here is that the information available at *t* consists of the variables dated at t or earlier.

We assume that ε_t is an i.i.d. stochastic process with zero expected value and that government primary expenditure is predetermined. Expenditure should be interpreted as expected expenditure: including any planned budgetary outlays identified with certainty as well as the expected value of the stochastic shock, if the latter is different from zero. Thus, ε_t represents the deviation of the budgetary shock with respect to its expected value—however, for short we will refer to ε_t as the shock. For simplicity, we will consider that primary expenditure is constant ($g_t = g$), since the time variation of expenditure plays no role in the analysis. Nevertheless, this assumption is less restrictive that it may appear: g may be interpreted as the annuity value of the net present value of the (possibly, time-varying) stream of (predetermined) future primary expenditure. More restrictive is the assumption that primary expenditure is predetermined rather than an instrument in the government's optimization problem.[9]

Debt, tax revenue, primary expenditure, and shocks will be interpreted as ratios to GDP. Let β^{-1} be $(1+\lambda)$, where λ represents the (risk-free) interest rate-growth differential. That is,

$$\beta^{-1} = 1+\lambda = (1+i)/(1+\gamma) = (1+r)/(1+\alpha) \tag{1}$$

where i (respectively r) and γ (respectively α) represent the nominal (respectively real) interest and GDP growth rates. Alternatively, in an economy without growth, where debt, taxes, expenditure, and budgetary shocks are represented by their (real or nominal) absolute values, λ would represent the (real or nominal) interest rate. With these conventions, the government optimization problem can be laid out as follows.

$$\underset{\tau_t}{Min}\, E_0 \sum_{t=0}^{\infty} \beta^t \tau_t^2 \tag{2}$$

such that

$$d_{t+1} = \beta^{-1} d_t - \tau_t + g + \varepsilon_{t+1} \tag{3}$$

$$d_0, g \text{ given} \tag{4}$$

$$\lim_{t\to\infty} \beta^t d_t = 0 \text{ almost surely} \tag{5}$$

This optimization problem is a version of the stochastic discounted optimal linear regulator (Hansen, Roberds, and Sargent, 1991; and Hansen and Sargent, 2008), with an added no-Ponzi

[9] Notice, however, that even when considering a joint optimization of taxation and expenditure, the taxation path must be optimal when the optimal path of expenditure is taken as given. Since we will be concerned here with necessary conditions for the tax path to be optimal, these must also obtain in the joint optimum.

game condition. The no-Ponzi game (or transversality) condition states that for all realizations of the shocks (except perhaps for a set of realizations with zero probability) debt and interest cannot be systematically paid by issuing more debt. In this formulation, d_t and g play the role of state variables and τ_t is the instrument. The Ricatti equation associated with the first order conditions has two solutions. The first is a trivial solution with $\tau_t = 0$ at all times, which violates the no-Ponzi game transversality condition (5). The second solution meets the no-Ponzi game restriction, and thus it is a solution to the planner's problem given by (2)–(5). Under this solution, the optimal policy and the associated closed loop evolution of the debt ratio are given by the following expressions.

$$\tau_t = \lambda d_t + g \tag{6}$$

$$d_{t+1} = d_t + \varepsilon_{t+1} \tag{7}$$

Combining (6) and (7) results in $\tau_{t+1} = \tau_t + \lambda \varepsilon_{t+1}$, which, like (7), is a random walk.

The policy prescription implied by the solution to this problem, summarized in equations (6)–(7), is clear: Taxes should be set at a level which is enough to pay the (growth-adjusted) interest bill on the debt inherited from the past and the expected expenditure—since g includes the expected value of the uncertain budgetary shock, if different from zero. The resulting expected primary balance ($\tau_t - g = \lambda d_t$) makes the expected next period's debt equal to the current debt ($E_t(d_{t+1}) = d_t$) and thus stabilizes the expected debt ratio. The shocks to the fiscal position should be fully absorbed by the actual debt to be passed on to the next period. In particular, this policy suggests that following an adverse shock and associated increase in the debt ratio, the primary balance should be increased as necessary to pay the higher interest bill and stabilize (in an expected value sense) the newly acquired debt ratio.

IV. Optimal Debt Policy under Skewed Shocks

The optimization problem discussed above and the associated optimal debt policy does not depend on the distribution of the shocks. However, as discussed, the historical record is characterized by prolonged periods during which the debt ratio shows a declining trend punctuated by infrequent sudden increases. This can be studied within the optimization problem given by (2)–(5) by postulating a distribution of the ε_t shocks such that they take a large positive value with very low probability (e.g., the break out of a war) and otherwise take a small negative value in normal times.

Along these lines, we specialize the optimization problem defined by expressions (2)–(5) and the associated optimal policy in (6)–(7) to a "war and peace" model, where war stands in for unusual events that result in large adverse shocks to the public finances, such as wars, financial crisis, or deep recessions. In this setting, government expenditure can take a large value, g^{WAR}, with a low probability, and a low value, g^{PEACE}, with a high probability. In this formulation, the ex post government expenditure (g^{WAR}, g^{PEACE}) includes both the ex ante expected expenditure (g) and the random shock (ε_t). The latter can take the two values $\varepsilon_t = g^{WAR} - g > 0$ and $\varepsilon_t = g^{PEACE} - g < 0$, with a zero expected value. Thus, the distribution of the shocks is asymmetric, with a high value, low probability event. For a small value of the probability of war, the expected expenditure g will be only moderately above the peacetime expenditure g^{PEACE}.

Occasional war periods of high expenditure will interrupt long periods of lower peacetime expenditure. In war as in peace, equation (6) calls for setting the tax ratio at a level sufficient to pay for the long-run average of government expenditure and for the interest bill on legacy debt. In wartime, the associated exceptionally high expenditure will be accommodated through large debt increases: $d_{t+1} = d_t + g^{WAR} - g > d_t$. Conversely, in peacetime, debt will fall gradually since the same optimal primary balance rule will result in debt reduction: $d_{t+1} = d_t + g^{PEACE} - g < d_t$. Since the actual expenditure in peacetime is below the long-run average, the optimal primary balance in peacetime should result in a small excess over the debt-stabilizing primary balance. In sum, in periods of peace, the sustained fall in the debt ratio is the result of a persistent policy which aims at reducing debt steadily—until war breaks out again.

The debt ratio will still follow a random walk, and over the very long term it could drift towards very high values. Hence, the postulated optimality criteria do not provide the basis for any particular debt ceiling, since the existence of fiscal limits (such as a maximum feasible primary balance) is assumed away. However, for most time periods—except as a result of the improbable occurrence of several catastrophic events in short sequence—the optimal debt policy will result in a persistent decline of the debt ratio.

This “war and peace” model is a stylized simplification of a skewed distribution of shocks. In this model there are only two shocks: an extreme debt right-hand shock (war) with very low probability and a small left-hand side shock (peace). The main result, however, obtains for a general skewed distribution: That is, a distribution with zero mean but where the right-hand side tail of the distribution (the debt-augmenting shocks) is longer than the left-hand side tail. In such a distribution, the extreme right-hand shocks have a low probability relative to modal shocks which are below the mean. Hence it is much more likely to witness shocks below the zero average. That being so the debt ratio, under optimal policy, is expected to decline most of the time (Barro, 1979).

V. How Skewed are Debt Shocks?

As discussed above, the historical narrative and the profile of the historical debt ratio point to a skewed distribution of debt shocks. That is, a distribution characterized by a relatively short left tail of debt-reducing shocks and an asymmetrically long right tail of debt-increasing positive shocks.[10] To explore formally the existence of this feature in the data, we use the first difference of the debt ratio as a proxy for the history of debt shocks—which are themselves unobservable. The use of this proxy is motivated by equation (7). This equation indicates that under a tax-smoothing government objective, the change in debt equals the shock. Therefore, the statistical results that follow must be interpreted as being conditional on the government following a tax-smoothing behavior.[11]

The sample distributions of first differences of the debt ratio for the United Kingdom and the United States for the period 1790 – 2014 are shown in Figures 3 and 4. Their corresponding skewness coefficients are 0.83 and 1.9 respectively. Both suggest that the underlying distribution of shocks is skewed to the right, as hypothesized.[12]

Formal testing further reinforces this suggestion. Table 1 presents the results of applying a battery of skewness tests to the first differences of the debt ratio and to the residuals of an AR(1) model fitted to the debt ratio. The skewness tests reported in table 1 include parametric as well as non-parametric tests. Among the former, the tests of Kendall-Stuart (Kendall and Stuart, 1969), Jarque-Bera (Jarque and Bera, 1987) and Jarque-Bera-Urzua (Urzua, 1996, which proposes an adjustment to the Jarque-Bera test to improve its power in small samples) are well-established and widely used tests of skewness. Strictly speaking, however, they test normality as well as symmetry. Therefore, we also apply the test proposed in Bera and Premaratne (2001) whose null hypothesis does not include normality; and the test proposed in Bai and Ng (2005), whose null hypothesis includes neither normality nor independence and which covers the case of correlated shocks. Finally, we also include the non-parametric test proposed in Lisi (2007). This test bootstraps the original sample after subtracting a central tendency measure (we report the test for both the mean and the median), replicating multiple times the sample of these deviations but with randomly assigned signs. The original skewness coefficient is then compared to the actual distribution of a large number of skewness coefficients from the bootstrapped replications (30,000 replications in our case).

All of the tests reject the null hypothesis of symmetry at the 5 percent level of significance. For the United States, significance levels above 1 percent are produced by the Bai-Ng test, which is 3.7 percent for both first differences and AR(1) residuals; and by the Lisi (2007) bootstrap test of symmetry around the median, which is 1.2 percent when applied to the first differences (but not to the AR(1) residuals). For the United Kingdom, significance levels above 1 percent are also

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[10] This is not necessarily related to the existence a deficit (or debt-increasing) bias in government policies. Such bias would result in the shocks having statistically significant non-zero central moments (such as the median or expectation), but not necessarily in asymmetric tails.

[11] Notice that while actual variations in revenue, expenditure, or output may provide a more detailed analysis of the origin of the shocks, they would not provide a proxy for the shocks that is independent of the hypothesis that the government follows a tax-smoothing behavior. In the end, these magnitudes determine the actual changes in the debt ratio, used here. Independent measurement of the fiscal shocks would require reliable information on the expectations of policymakers at the time budget decisions were made.

[12] A positive (negative) skewness coefficient indicates right-side (left-side) skewness. A symmetric distribution has a zero skewness coefficient.

produced by the Bai-Ng test (1.5 percent) and the bootstrap test of symmetry around the median (1.04 percent). All other tests produce significance levels below 1 percent, often very close to zero. We interpret these results for both the United States and the United Kingdom as providing sufficient basis to reject symmetry in favor of right-skewness.

Figure 3. United Kingdom: Histogram of the First Differences of the Debt Ratio

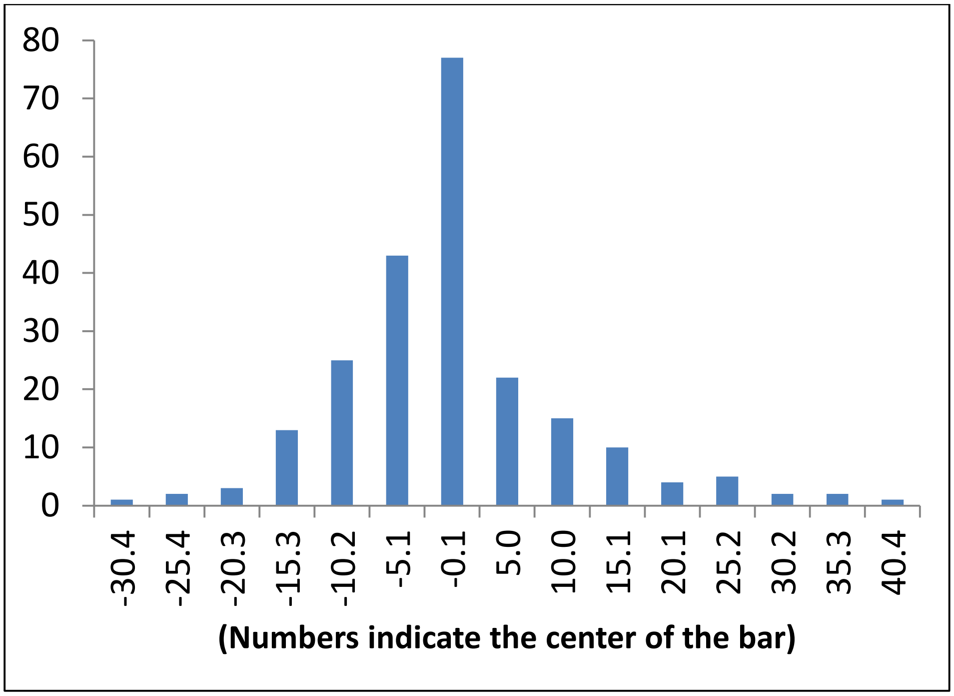

Figure 4. United States: Histogram of the First Differences of the Debt Ratio

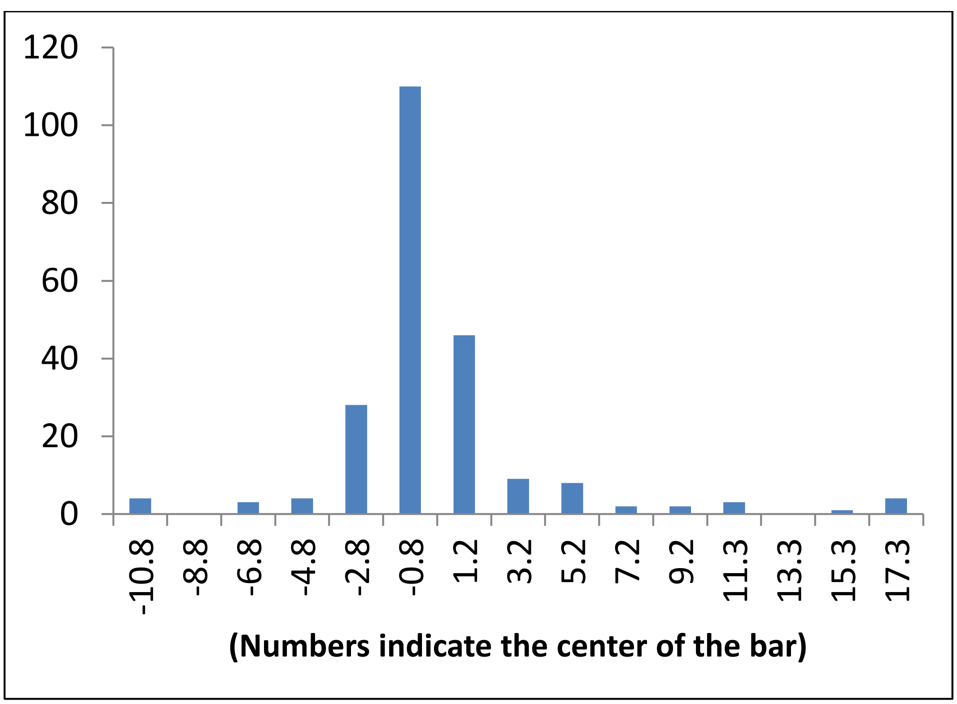

Table 1. Symmetry Tests Results

Test	Null hypothesis, comments	Basis for reported significance	1st difference of the debt ratio				Residuals of an AR(1) of the debt ratio			
			US		UK		US		UK	
			Statistic	Significance	Statistic	Significance	Statistic	Significance	Statistic	Significance
sc = skewness coefficient; kc = kurtosis coefficient.										
Kendall-Stuart. [Kendall and Stuart (1969)]	H0: sc=0, data i.i.d. normal.	1-tail	1.8897	0.0000	0.83182	1.75E-07	1.9696	1.17E-33	1.0055	0.0000
Jarque-Bera test [Jarque and Bera (1987)]	H0: Data i.i.d. normal, sc=0, kc=3 (as in the normal). Note: Monte Carlo-based significance estimate (more accurate for small sample).	2-tail	727.6	0.0000	75.836	0.0000	760.752	0.0000	87.824	0.0000
Jarque-Bera-Urzua test for normality [Urzua (1996, 1997)]	H0: Data i.i.d. normal, sc=0, kc=3 (as in the normal).	2-tail	776.5	0.0000	81.197	0.0000	811.515	0.0000	93.509	0.0000
Bera-Premaratne symmetry test. [Bera and Premaratne (2001)]	H0: sc=0, data may not be normal, and may have excess kurtosis relative to normal.	1-tail	2.52	0.0059	2.341	0.0096	2.571	0.0051	2.855	0.0022
Bai-Ng symmetry test. [Bai and Ng (2005)]	H0: sc=0, data may not be normal, and may not be independent.	1-tail	1.78	0.0372	2.169	0.0151	1.779	0.0377	2.718	0.0033
Bootstrapping test of symmetry around the MEAN. [Lisi (2007)]	H0: Data are from a distribution that is symmetric around the mean.	1-tail	1.89	0.0078	0.832	0.0077	1.970	0.0057	1.006	0.0013
Bootstrapping test of symmetry around the MEDIAN. [Lisi (2007)]	H0: Data are from a distribution that is symmetric around the median.	1-tail	1.89	0.0122	0.832	0.0104	1.970	0.0098	1.006	0.0034

A by-product of the statistical analysis of the first differences of the debt ratio is the identification of outlier years, their frequency, and the relationship between the average debt ratio increases in those years relative to normal years. We identify the outliers by dropping sequentially from the sample the years with the largest increases in the debt ratio, which reduces the skewness coefficient of the remaining sample, until this skewness coefficient reaches the value that is closest to zero. The dropped values are classified as outliers, and the remaining sample as normal years.[13]

In the United Kingdom, there is on average an outlier year every 28 years with an average increase in the debt ratio of 30.8 percentage points of GDP. The corresponding numbers for the United States are an outlier every 32 years on average with an average increase in the debt ratio of 15.4 percentage points of GDP. A corollary of the optimal debt policy represented in equation (7) is that in the long run, the sample average annual increase in the debt ratio should go to zero as the sample size grows, with the average debt ratio decline in normal years compensating the average debt ratio increase in outlier years. For the average annual change in the debt ratio to be zero, the average decline in normal years in the United Kingdom should be 1.1 percentage points of GDP; the actual average decline in normal years is 1.3 percentage points of GDP. The corresponding values for debt ratio reduction in normal years for the United States are 0.5 (model prediction) and 0.3 (actual) percentage points of GDP. Policies followed by the U.K. and the U.S. seem roughly compatible with the implications from the optimal tax-smoothing model.

VI. CONCLUSIONS

This paper places the large accumulation of debt in advanced economies since the 2008 crisis in historical perspective. It argues that the latest increase in debt ratios is part of a pattern of infrequent but large adverse debt shocks, which have typically been followed by policies of gradual but sustained debt consolidations. This pattern broadly corresponds to the optimal debt policy of a government that aims at minimizing the net present value of deadweight losses by smoothing tax rates inter-temporally, when the debt dynamics is affected by such asymmetric shocks. An analysis of the annual changes in the debt ratio in the United Kingdom and the United States supports the hypothesis that fiscal shocks are asymmetrically distributed, with large debt-increasing shocks being more likely than commensurate debt-reducing shocks.

Optimal debt policy essentially requires that each year tax revenues be set at the level that funds the (growth-adjusted) interest bill on the inherited debt and expected expenditure—including planned regular expenditure and the long-run expected value of fiscal shocks. Under this optimal policy, the tax ratio follows a random walk. The implied path for the debt ratio is also a random walk.

Optimal policy also implies that during economic or national emergencies, when expenditure far exceeds its regular level, the debt ratio will rise as necessary—spreading the temporary unexpected costs over the indefinite future and avoiding large temporary tax increases in the present. In turn, in normal times, when the shocks to the debt ratio are below their long-run

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[13] For the United Kingdom, the outlier years are 1916, 1802, 1921, 1944, 1826, 1915, 1945, and 1922. For the United States they are 1944, 1942, 1918, 1945, 1943, 1932, and 2009.

average, the debt ratio will fall slowly but enduringly, year after year. In the model, debt reduction comes not from a reaction to past developments but instead as a rational precautionary response to expected future disturbances. Given the empirical distribution of shocks to debt, optimal policy implies that in most years the debt ratio will be falling gradually to compensate for the few years in which the debt ratio will increase sharply.

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