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Inflation and the Black Market Exchange Rate in a Repressed Market: A Model of Venezuela

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Abstract

This paper presents a stylized general equilibrium model of the Venezuelan economy. The model explains how the recent sharp fall in oil revenue combines with foreign exchange rationing to produce a steep rise in inflation. Counterintuitively, a devaluation of the official exchange rate could temporarily *reduce* inflation. The model also explains how the hyper-depreciation of the black market exchange rate reflects prices in the most distorted goods markets.

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I. INTRODUCTION

Venezuela, an oil exporting and import dependent economy with repressed markets for foreign exchange and for intermediate and consumption goods, has experienced a surge in inflation and the black market premium—bordering on hyper-inflation and hyper-depreciation.

A common explanation for hyperinflation draws on Cagan (1956), under which a rise in inflationary expectations is self-reinforcing and imposes a strong negative impact on money demand. This paper argues that the Cagan model is not the best explanation for inflation trends in Venezuela. The Cagan model's assumption that individuals can substitute bond holdings for domestic currency may also be unrealistic for Venezuela due to financial repression. In addition, there is not yet convincing evidence of an increase in velocity or that the elasticity of money demand to inflation has risen (Appendix 1).

This paper argues for an alternative explanation of the surge in inflation and the black market premium. In particular, Venezuela's system of rationing foreign exchange creates a repressed goods market for imports. The plummet in oil receipts, starting in late 2014, led to a massive contraction in the provision of foreign exchange to importers. In turn, the sharp reduction in the supply of goods to retail markets propelled the rise in inflation well beyond money growth. This real shock generates a relative price change that obscures the underlying inflation dynamics. Any Cagan-style de-anchoring of inflation expectations, if they exist, may comprise only a small part of the rise in inflation so far.

The model outlined in this paper shows how a devaluation of the official exchange rate could, counterintuitively, lead to a temporary *decline* in inflation. The intuition is that a devaluation would reduce the fiscal deficit and, all else equal, would also withdraw domestic money supply by requiring more bolivars in payment for each dollar of reserves bought from the central bank. A lower fiscal deficit, following the devaluation, would need to be maintained by reducing expenditure growth, even under a crawling peg. The model also demonstrates recurrent inflation cycles arising from infrequent devaluations.

Over the last few years, the black market exchange rate has risen even faster than inflation. This paper argues that the best way to think of the relationship is that the black market rate reflects prices, not the other way around. When a small share of foreign exchange transactions flows through the black market, its rate reflects prices in the most distorted goods market (i.e., the subset of goods subject to the highest markups over international prices). Depreciation can occur even without expectations of increasing overall inflation.

Portfolio holdings of dollars (i.e., capital flight) could further amplify inflation and the black market premium in a given period. The incentive to hold dollars reflects expectations of higher inflation and black market rates in the future, which in turn would be driven by an expectation of a decline (or further decline) in the provision of foreign exchange for imports. Capital flight could nonetheless *improve* economic welfare if it smooths consumption over time, by saving dollars in a period of relatively high availability of dollars (due to strong export receipts) to a period of low availiability of dollars (due to weak export receipts).

The paper is structured as follows. Section II provides background on the economic developments that motivate this research. Section III presents a one-good general equilibrium model that is used to derive the main results, especially on inflation and the black market rate, while Section IV calibrates the one-good model and conducts simulations related to higher fiscal spending, a devaluation of the official exchange rate, and the impact of an oil price collapse. Then the model is extended to the case of multiple consumption goods in Section V to explain how the black market rate can rise significantly faster than overall inflation. The one-good and multiple-good models can explain the dynamics of inflation and the black market rate, even without resorting to inflation expectations and portfolio choice between dollars and domestic currency. Nonetheless, Section VI adds capital flight to the story. Section VII provides several extensions and Section VIII concludes.

II. Background

Venezuela is an oil exporting country that is highly dependent on imports. Oil exports have constituted about 95 percent of total exports in recent years. Oil receipts accrue to the state-owned company, Petróleos de Venezuela (PDVSA), which transfers some of its earnings to the budget and also engages in quasi-fiscal activities focused on social and developmental objectives. Oil export earnings comprise the main source of foreign exchange, which is used in turn to import many food and consumer goods, as well as intermediate inputs for production.

Over the past several decades, Venezuela's exchange rate regime has undergone many changes, but has frequently included multiple exchange rates. In 2003, the government fixed the exchange rate and introduced foreign exchange controls, including surrender requirements for both current and capital transactions. A government commission administered the controls, with broad discretionary powers to allocate foreign exchange to the private sector for current and other transactions. In the recent few years, the government has introduced alternative systems for allocating or auctioning foreign exchange, but the use of the systems has been sporadic and transaction volumes low.[2]

Monetary aggregates and inflation have surged over the past few years. Broad money grew by about 85 percent on average in 2015 and rose to 100 percent at year end. Headline consumer price inflation averaged 88 percent in 2015 and rose to 180 percent by year end. Some components of the consumer basket with a high dependence on imported goods experienced inflation rates well above the overall index. For example, food inflation ended the year above 300 percent and tobacco and alcohol increased by nearly as much. In the meantime, the black market exchange rate rose 380 percent, ending the year at a rate of 833 bolivars per dollar.

Venezuela recently experienced a sharp decline in oil prices. Its oil basket commanded nearly $100 per barrel in mid-2014. Reflecting the global fall in oil prices in the second half of

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[2] Hausmann (1990), Guerra and Pineda (2000, 2004) treat the macroeconomic dynamics in previous experiences with exchange controls in Venezuela in various episodes during 1960-1996.

2014, the price of Venezuela's oil plunged to $47 per barrel by end-2014 and declined further to $29 per barrel by end-2015. This led to a substantial decline in oil revenues and PDVSA's operating surplus.

Output contracted sharply. While no recent official data on scarcity/diversity has been published, anecdotal evidence suggests that scarcity of basic and intermediate goods continued to worsen. According to the central bank, output has contracted every quarter since the first quarter of 2014. The recession continues to deepen, with output contracting 7 percent in the third quarter of 2014. Anecdotal evidence suggests that many production facilities have had to be shut down due to the lack of necessary imports of intermediate inputs.

III. STYLIZED GENERAL EQUILIBRIUM MODEL FOR VENEZUELA—W/ ONE GOOD AND BALANCED TRADE

A. Contrast with previous literature on dual exchange rates

Models with dual exchange rates were developed in the 1970s and 1980s (Calvo and Rodriguez, 1977; Flood, 1978; Lizondo, 1987a; Lizondo, 1987b), when such exchange rate regimes were prevalent. Many countries have since abandoned dual regimes, especially as they integrated their trade and capital accounts with world markets. But Venezuela and some other countries have maintained capital controls, leading to dual or multiple exchange rates.

A key feature of the earlier models is an economic structure with full current account convertibility. Domestic agents satisfy their import demand by exchanging domestic money at will for the central bank's international reserves. Capital account transactions are satisfied in a (legal) parallel market with a floating exchange rate. When the official rate for current transactions is set at a level inconsistently low given aggregate demand and export supply, there would be a net demand for the central bank's foreign exchange reserves. The excess demand for reserves eventually leads to a balance of payments crisis in which the central bank runs out of reserves. At that time, the exchange rate for current transactions floats and is unified with the rate for capital transactions.

In contrast, the model developed below incorporates a key feature of the Venezuelan economy, namely that the central bank **rations** its allocation of international reserves. In doing so, an outright balance of payments crisis is avoided, but there is an excess demand for reserves and a constraint on imports.

This model shares a feature of the earlier literature in that foreign exchange can also serve as an asset. In line with many other papers and the financial repression in Venezuela, the model does not include any other domestic currency asset other than money. The earlier models imposed ad hoc preferences for money and foreign exchange and the substitution between them. In contrast, the model developed below assumes merely that purchases of consumer goods are limited to the holdings of domestic money (cash-in-advance constraint). This micro-founded model, with relationships derived from optimizing behavior, produces a relationship similar to the quantity theory of money.

B. Summary of the basic structure of the economy

The structure of the model is designed to highlight the key features of the Venezuelan economy that lead to the rising inflation and black market premium. Other elements of the economy will be simplified if they are deemed not essential to the exposition.

A public enterprise produces the only export good (oil), which is not consumed domestically. There are private sector firms that specialize in providing the imported consumption good (food), which is not produced domestically. Other private sector firms specialize in arbitraging foreign exchange between the official market and the black market. The government receives oil revenue from the public enterprise in dollars, and deposits these receipts in the central bank in exchange for domestic currency (bolivars) at the official exchange rate. The government makes lump-sum transfers to households in domestic currency. The central bank allocates its dollar receipts to the private sector at the official rate, through rationing. The fraction $(1-\varphi)$ of the recipients of the central bank's dollar allocations are import firms, but a fraction, φ, of the recipients are arbitrage firms, which in turn sell their dollars to importers in the black market. A holding company consolidates all profits and rents of the import firms and arbitrage firms, and fully distributes them to households each period. A detailed discussion of the model follows below.

C. External, fiscal, and monetary sectors

External sector

Oil production and the dollar price of oil are exogenous, so the dollar value of export receipts, denoted X_t, is also exogenous each period.

The stocks of foreign assets and external debt are initially zero, for both the public and private sector. The public sector fully distributes its export receipts to the private sector each period and does not issue external debt. In sections III-V, there is no accumulation of foreign assets, as the private sector uses the dollar allocations fully for payment of imports; therefore, trade is balanced each period. This assumption is relaxed in section VI to analyze the black market rate when some of the dollar allocation is used to build up portfolio holdings of dollars (capital flight) instead of being fully used for imports.

The foreign price in dollars of the import good is constant at P^*, and the quantity of the good imported is denoted Q_t.

In the case of no capital flows, balanced trade implies that the dollar value of the import good must equal the dollar value of export receipts. Therefore, the quantity of import goods is fixed at the amount determined from the exogenous export value and import price:

(1) $Q_t = X_t / P^*$

Fiscal sector

The government receives export receipts Xt from the state oil company, and deposits them in the central bank at the official exchange rate st (defined as bolivars per dollar). On the expenditure side, the government only makes transfers, Tt, to households in bolivars. The government finances its deficits through an increase in net domestic credit from the central bank, Nt+1-Nt. Thus, the government's budget constraint is:

(2)   Tt - st Xt = Nt+1 – Nt

Central bank

The asset side of the central bank balance sheet consists of international reserves (measured in domestic currency), Rt, and net domestic credit to the government, Nt. The liability side consists of money in bolivars, Mt.

(3)   Rt + Nt = Mt

To simplify the analysis, all money consists of cash in circulation. When the fiscal authority makes transfer payments, it implements this through the central bank, as payments of currency. The assumption of no deposit money banks reflects the high degree of financial repression that sharply weakens the banking channel as a source of monetary dynamics. Indeed the ratio of broad money to reserve money has been broadly stable over many years.

At the beginning of each period, the central bank receives the dollar receipts of the oil revenue from the fiscal authority, and provides an offsetting credit to government deposits in domestic currency.

(4)   (Rt + st Xt) + (Nt - st Xt) = Mt
        Reserves   NDC   Money

When the government makes transfer payments, the central bank debits the government deposits and increases money in circulation. At this stage, the central bank balance sheet appears as follows:

(5)   (Rt + st Xt) + (Nt - st Xt + Tt) = (Mt + Tt)
        Reserves   NDC   Money

Next, the central bank allocates all of the dollars (Xt) to the private sector at the official exchange rate, st, which the private sector pays for from its currency in circulation.

(6)   (Rt + st Xt - st Xt) + (Nt - st Xt + Tt) = (Mt + Tt - st Xt)
        Reserves   NDC   Money

If the central bank fully distributes all dollar receipts and does not accumulate reserves, which it does not need given the lack of convertibility, then the central bank balance sheet

carries no international reserves at the beginning and end of each period ($R_{t-1} = R_t = 0$). Its balance sheet consists only of money and net domestic credit to the government.
Thus at the end of the period, the central bank balance sheet appears as follows:

(7) $R_t = 0$

(8) $M_{t+1} = M_t + T_t - s_t X_t$

(9) $N_{t+1} = N_t + T_t - s_t X_t$

By inspection, we see that money is created from the change in net domestic credit to the government, which is the fiscal deficit:

(10) $\Delta M_{t+1} = \Delta N_{t+1} = T_t - s_t X_t$

However, the equality between the increase in money and the fiscal deficit occurs only because the central bank sells all of its new reserves each period and because the private sector pays for dollars at the same official rate as is credited to the government for export receipts. Money creation will be higher if the central bank accumulates reserves. A higher volume of dollar sales to the private sector or a higher average exchange rate required for dollar purchases will reduce money growth. These relationships are illustrated with numerical examples in Appendix 2.

D. Firms

On the supply side, the market structure consists of a large number of import firms and arbitrage firms. Each firm is small and does not have power to set market prices of goods or the black market rate, but merely takes prices as given, as determined by the overall demand for goods in the retail market and dollars in the black market. For simplicity, I normalize the number of import and arbitrage firms to one representative firm in each market.

Import firm

The representative import firm buys $(1-\varphi) X_t$ dollars from the central bank at the official rate s_t and buys φX_t from the arbitrage firm at the black market rate b_t. The dollars X_t can buy a maximum quantity $Q_t = X_t/P^*$ of import goods at the international price P^* and the firm, serving also as a retailer, sells the import goods in the domestic market at the bolivar price P_t. Therefore, the firm's revenues are $P_t Q_t$ and its costs are the sum of payments to the central bank ($(1-\varphi) s_t P^* Q_t$) and to the arbitrage firm ($\varphi b_t P^* Q_t$).

(11) $\prod^M t = P_t Q_t - [(1-\varphi) s_t + \varphi b_t] P^* Q_t$
         Profits   Rev   Ave ex rt   Cost at world price

Although the representative import firm does **not** have power to set the market price (it is one small firm among many selling that particular good), rents persist because the central bank rations foreign exchange despite excess demand for it. In the model, the central bank does

not allocate dollars to the highest bidder, but makes its rationing decision based on arbitrary criteria. This implies that even though the market price for a good could be well above the marginal cost of supplying the good, firms cannot increase supply. Instead, supply is restricted by the need for foreign exchange, which is rationed. I also assume that the transformation of the import good into a retail good is costless. Therefore, the import firm will want to buy dollars from the central bank and use them to increase imports as long as the bolivar retail price equals or exceeds the domestic currency price of the import good at the official exchange rate:

(12) $P_t \geq s_t P^*$

Likewise, the import firm will want to buy dollars from the arbitrage firm as long as the bolivar retail price equals or exceeds the domestic currency price of the import good at the black market exchange rate:

(13) $P_t \geq b_t P^*$

If the import firm had to incur a transformation cost per unit, ξ_t, (for transportation, distribution, retail service costs), then the domestic retail price would need to exceed the sum of the import cost and transformation cost:

(12′) $P_t \geq s_t P^* + \xi_t$
(13′) $P_t \geq b_t P^* + \xi_t$

For the remaining analysis, I assume $\xi_t = 0$.

Arbitrage firm

The representative arbitrage firm receives an allocation of dollars, φX_t, from the central bank and sells it to importers in the black market, at the exchange rate b_t. The arbitrage firm is willing to engage in this transaction as long as the black market rate it receives from the importer equals or exceeds the cost of dollars at the official exchange rate, where the rent is the difference between the black market receipt and the costs incurred.

(14) $\prod^A_t = [b_t - s_t] * \varphi X_t$
        Profits   MR - MC per dollar   Amount transacted

The import firm is willing to buy from the black market as long as the retail price equals or exceeds the black market rate (13). Competition among import firms for the dollars will drive this relationship to equality. Thus, in the case that black market dollars can be used only for imports and not for portfolio holdings (capital flight), the black market rate is bounded by the retail price and the official exchange rate:

(15) $P_t / P^* = b_t \geq s_t$

Holding company

To ensure a complete flow of funds in general equilibrium, a holding company consolidates the profits (rents) of the import firm and the arbitrage firm and distributes these earnings to the representative household each period.

(16) $\prod t = \prod^M t + \prod^A t = Pt \ Qt - st \ P^* \ Qt = [Pt - st \ P^*] \ Qt$

E. Households

The representative household has earnings from profits of the holding company and receives the transfer payment from the government. The household consumes the import good. The government successfully requires that retail sales of goods be purchased using domestic currency. Thus, the household is subject to a cash-in-advance constraint which requires money to pay for goods. The household maximizes a discounted stream of utility of consumption, subject to a budget constraint and the cash-in-advance constraint.

(17) $\max \sum_{t=0}^{\infty} \beta^t U(Ct)$

s.t. $Mt+1 + Pt \ Ct = Mt + (Tt + \prod t)$ budget constraint

and $Mt \geq Pt \ Ct$ cash-in-advance constraint

Appendix 3 sets out the optimizing conditions. The discount rate, β, is slightly below unity (generally around 0.97-0.99) depending on the rate of time preference. Goods market clearing requires that $Ct = Qt$. Therefore, if the import volume is constant over time, the cash-in-advance constraint will be binding with any positive inflation rate. In this case, the price level is proportional to the money supply and inversely proportional to the quantity of imports.

Summing the demand functions across households produces an aggregate demand curve:

 (18) $Pt = Mt \ / \ Qt$ Aggregate demand curve

Figure 1 illustrates the general equilibrium relationship, consisting of a fixed supply of import goods and the aggregate demand curve.

F. Equilibrium exchange rate

The concept of the equilibrium value of the official exchange rate is easy to determine in this framework. Given that the only consumption good is imported, the equilibrium nominal exchange rate is the value of st that satisfies the Law of One Price (LOOP), setting equation (12) to equality.

(19)   Equilibrium exchange rate: $\tilde{s}t = Pt \ / \ P^*$

The equilibrium real exchange rate (= st P* / Pt) equals unity. Any value of st set lower than Pt/P* would imply an overvalued real exchange rate according to LOOP. When the official exchange rate is set at the equilibrium value under LOOP, the black market rate would be the same as the official rate.

Combining equations (1), (18), and (19), assuming no capital flows, the equilibrium nominal exchange rate is related to money and export receipts.

$$(20) \quad \tilde{s}t \ = \ Pt \, / \, P^* \ = \ [\ Mt/ \, (Xt/P^*) \ ] \, / \, P^* \ = \ Mt \, / \, Xt$$

An increase in the money supply has a proportionate increase (depreciation) in the equilibrium nominal exchange rate. Figure 2 illustrates this case. An increase in export receipts, which provides for a greater supply of import goods, leads to a decline (appreciation) in the equilibrium nominal exchange rate.

If the official exchange rate is overvalued, such that (12) does not hold with equality, the import firm will earn rent. A devaluation of the official exchange rate to another level below or equal to the LOOP level would entail a reduction in the rent that the import firm receives. As long as the new level of the official rate is still at or below the LOOP equilibrium level, the retail price Pt won't rise (see Figure 3). The lack of pass-through from the devaluation arises because the retail price (set at the level corresponding to the demand for the import good in the context of a fixed supply of the import good) is above the cost of supplying the good. However, the devaluation will result in lower rents received by the import and arbitrage firms, leading to lower household income and a subsequent reduction in prices.

A very large devaluation of the official exchange rate could lead to an increase in the price level if it significantly raises the import cost of the good such that an increase in the retail price of the good is required to ensure LOOP (Figure 4). Otherwise, if importers started making losses, they would cut back on supply, which would increase prices.

In summary, the demand side of the economy is driven by money growth, which in turn depends mainly on fiscal deficits, as well as the central bank policies for reserve accumulation and allocation of foreign exchange to the private sector. The supply side of the economy is driven by the receipt of dollars from export earnings, which are distributed to importers and arbitrageurs through the central bank.

IV. CALIBRATIONS AND DYNAMIC SIMULATIONS OF ONE GOOD MODEL

A. Calibration of black market exchange rate

Equation 20 provides a simple relationship for the equilibrium exchange rate, given the law of one price and the structure of the Venezuelan economy. The model predicts that the equilibrium exchange rate will equal the domestic money supply divided by the foreign exchange allocated for imports. In the one good model, equation (15) shows that a similar relationship would hold for the black market rate.
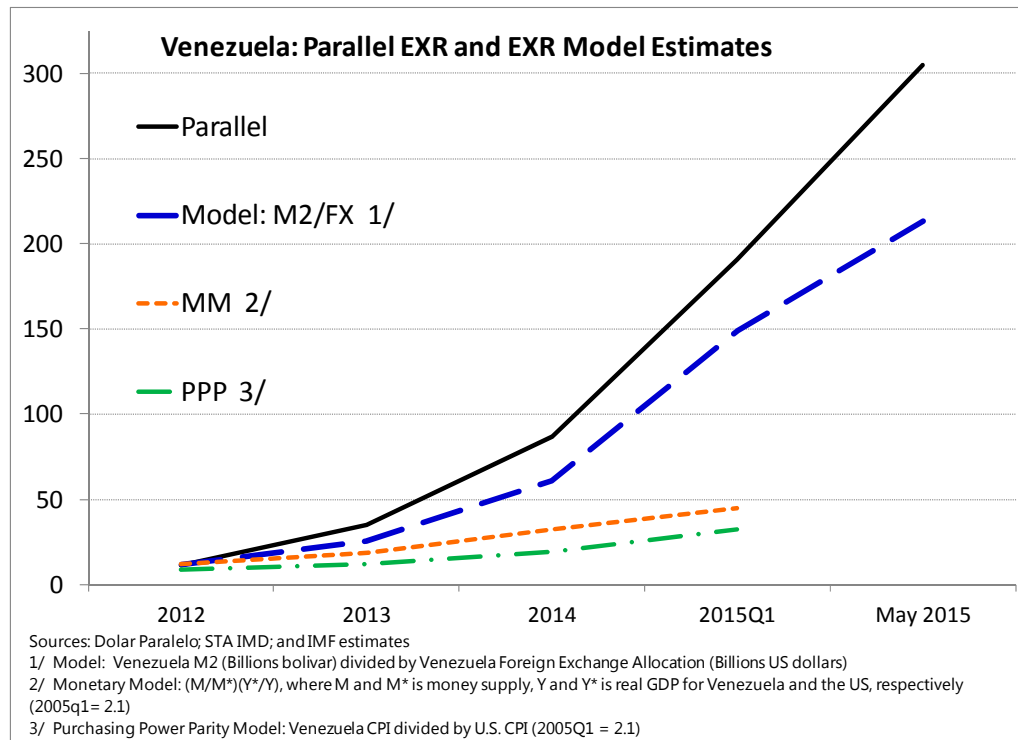
I calibrate the model to Venezuelan data available for mid-2015, based on the following information and sources:

- Venezuela's broad money has risen from 715 billion bolivars at end 2012 to a projected 2,335 billion bolivars (assuming 65 percent growth in broad money y/y).
- GlobalSource Partners estimates that total FX sales peaked in 2012 at $170m per day, falling to $130m per day in 2013, $90m per day in 2014, and $30 million per day in May 2015. These figures imply annual FX allocations of $62 billion in 2012, $48 billion in 2013, $33 billion in 2014, and $11 billion (annualized) in May 2015.

Dividing the money supply by FX allocations, the derived equilibrium exchange rate rises from 11.5 bolivars per dollar in 2012 to 213 bolivars per dollar in May 2015.

As comparisons, two other models of the equilibrium exchange rate are also presented in the figure. The monetary model of the nominal exchange rate was a popular model in the 1970s. It is derived from money demand and purchasing power parity, and reflects relative money supplies and relative real output between Venezuela and the US. A model of purchasing power parity is also presented, which measures the ratios of Venezuela and US CPIs.

Although all three models draw on some of the same behavioral relationships, the model developed in this paper has a much better fit than the standard workhorse exchange rate models. In addition, whereas both the monetary model and PPP model must be normalized (2005Q1=2.1) by the value prevailing at a date in the past in which the actual exchange rate is assumed to be in line with each measure of the equilibrium exchange rate, the model developed in this paper uses raw units (money in bolivars and FX allocation in dollars). The extraordinarily good fit of the model (M2/FX) arises without any normalization. For instance, for May 2015, 2,335 billion bolivars in M2 divided by $11 billion in FX sales directly provides the equilibrium exchange rate estimate of 213 bolivars per dollar.

Venezuela: Parallel EXR and EXR Model Estimates

Sources: Dolar Paralelo; STA IMD; and IMF estimates
1/ Model: Venezuela M2 (Billions bolivar) divided by Venezuela Foreign Exchange Allocation (Billions US dollars)
2/ Monetary Model: (M/M*)(Y*/Y), where M and M* is money supply, Y and Y* is real GDP for Venezuela and the US, respectively (2005q1= 2.1)
3/ Purchasing Power Parity Model: Venezuela CPI divided by U.S. CPI (2005Q1 = 2.1)

B. Dynamic simulations of inflation

The model can also be used to conduct policy experiments to simulate inflation dynamics under alternative fiscal and exchange rate policies. Fiscal policy consists of the growth rate of transfer payments (t), whereas exchange rate policy consists of the devaluation rate (s). The summary of the model equations and initial values for the simulation are presented in Appendix 4.

Impact of fiscal policy

The first scenario assumes that the growth rate of spending and the annual devaluation rate are both constant at **25 percent** ($\tau = \sigma = 25\%$). In this case, money growth and inflation reach a steady state when $\frac{Deficit_t}{M_t} = \frac{Deficit_t}{P_t/Q_t} = \frac{\tau}{1-\tau}$. If the initial deficit to money ratio is higher than the steady state ratio, inflation will be above its steady state. It will fall over time and converge to its steady state rate of 25 percent. If the rate of spending growth and devaluation rises to **95 percent**, the initial deficit to money ratio will be lower than its steady state ratio. Inflation will also be below its steady state and will rise over time until it converges to its steady state of 95 percent.[3]

Impact of Fiscal Policy
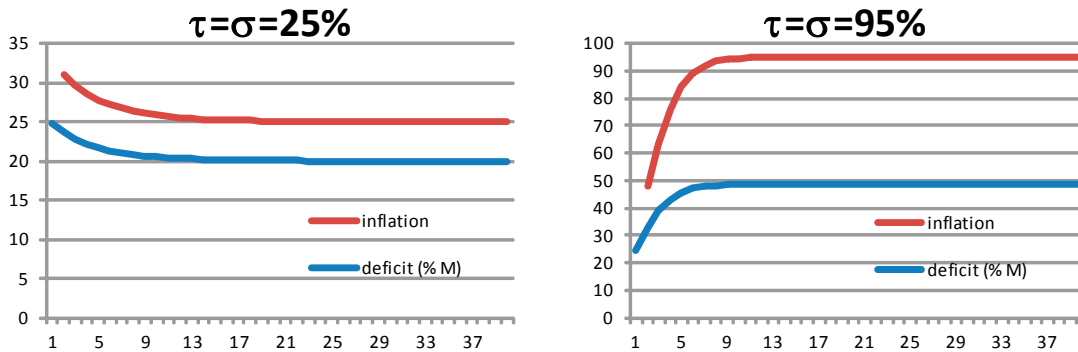
Impact of one-off devaluation of official exchange rate

In this scenario, the growth rate of spending (Tt) is 25 percent, but the official exchange rate is held at 6.3 bolivars per dollars for four years. In year 5, the official exchange rate is devalued from 6.3 to 15, without changing the nominal growth of spending. The devaluation reduces the fiscal deficit by increasing the bolivar value of oil revenues. As a result, the government's need for monetary financing falls. With a lower amount of money supply, aggregate demand and inflation fall relative to the counterfactual case of no devaluation.

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[3] This scenario is an example of fiscal dominance, in which the fiscal deficits drive money creation and inflation (see Sargent and Wallace, 1981).

Thus, the short-run effect of the devaluation is much lower inflation. However, since spending growth remains at 25 percent, inflation still converges to the long-run steady state level of 25 percent. The temporary rebound in inflation in the transition to the steady state is explained by the fact that the real value of government spending increases when inflation falls relative to the no-devaluation scenario.

One-off Devaluation
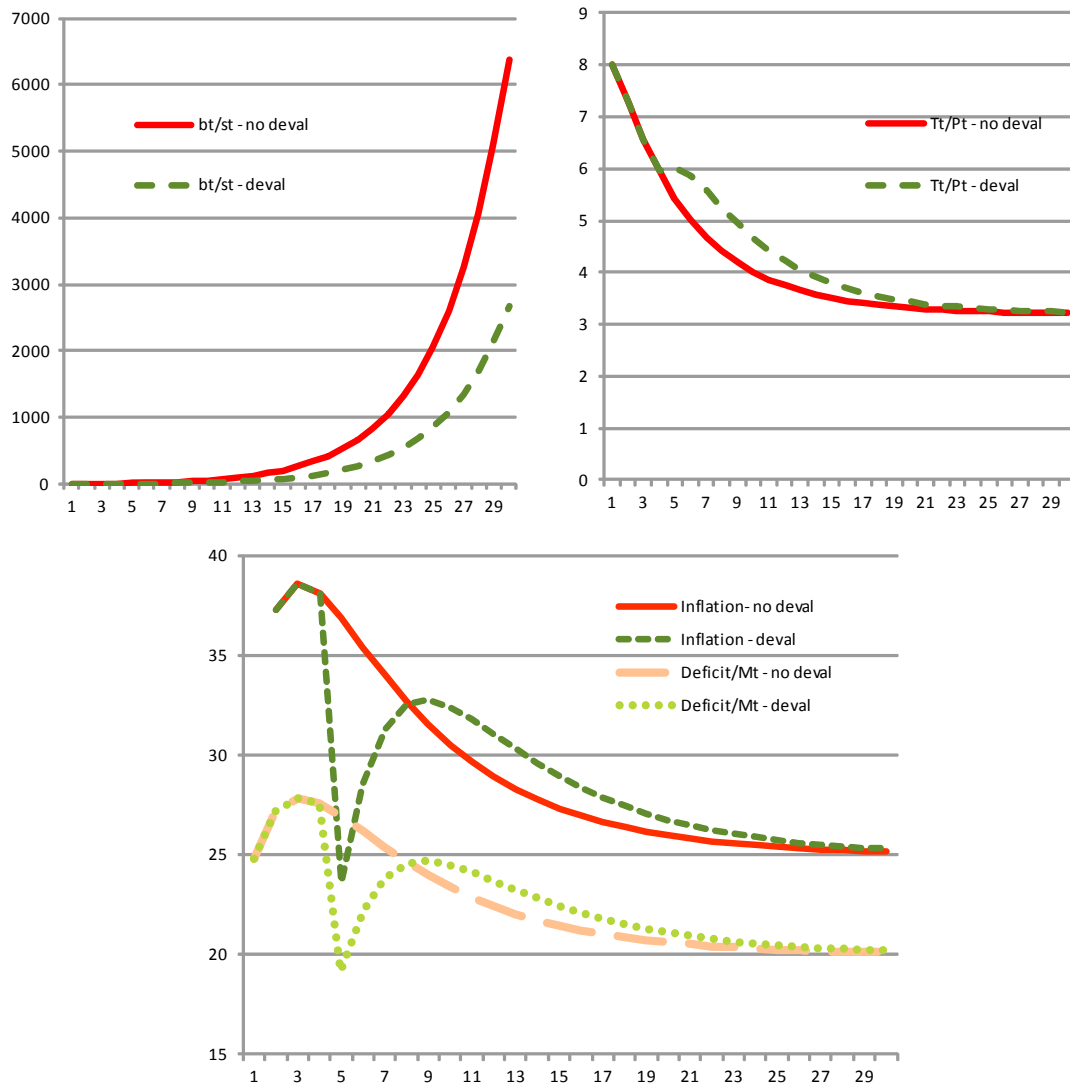
Why doesn't the devaluation pass through to higher prices? The reason is that the market structure is not characterized by perfect competition or monopolistic competition. The market structure is something different—a repressed goods market. Foreign exchange is required to purchase imports and it is allocated through official discretion, not through a market mechanism. Firms fortunate enough to obtain foreign exchange at the official rate are able to extract rents that cannot be competed away. So, they can keep the price of a good well above its cost to import. New firms cannot enter to drive away rents. When the official rate is

devalued, it only reduces rents. It does not necessarily raise prices. In fact, the reduction in rents (subsidies) takes away purchasing power from some domestic agents, reducing aggregate demand. The existence of a black market for foreign exchange does not change this phenomenon. The black market shifts wealth around within the country, but does not change the basic foreign exchange constraint imposed by the government that maintains rents and an excess demand for foreign exchange.

Impact of devaluation cycles

In the past, Venezuela's official exchange rate has repeatedly been devalued after being held fixed for a period of more than a year. This scenario shows how the deficit (relative to the money supply) and inflation evolve when the exchange rate is devalued every five years.

In the first case, fiscal spending grows each year by 25 percent. Every five years, the official exchange rate is devalued by the cumulative growth in spending over five years (205 percent). Each time the exchange rate is devalued, the fiscal deficit and inflation fall temporarily. But since spending continues to growth by 25 percent, the deficit and inflation soon rise again. Inflation cycles around the long-run average of 25 percent. If fiscal spending growth is higher—such as 95 percent—inflation will cycle around the higher level. As above, the deficit and inflation fall in the year of the devaluation and rise again afterward.

Devaluation Cycles
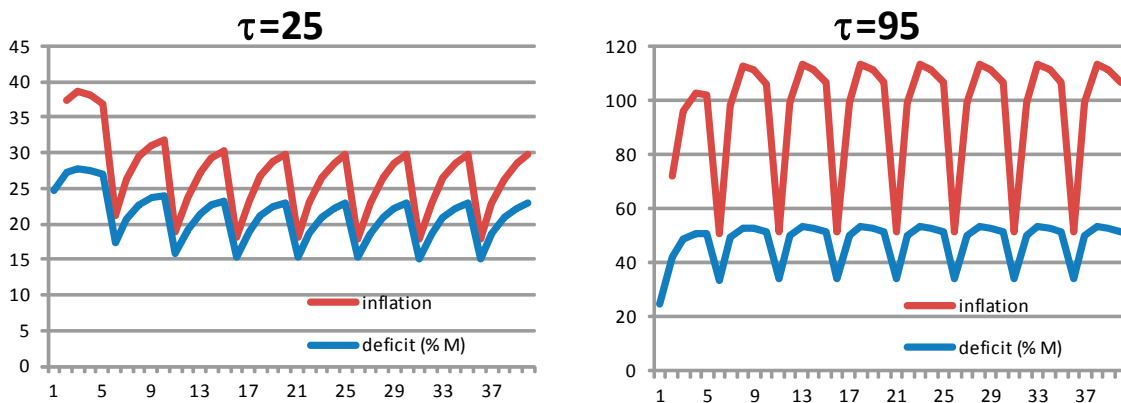
Impact of a real shock: oil revenue decline

The model can also simulate the impact of the sharp decline in oil prices, which has represented a critical real shock to Venezuela. Given the collapse in oil prices, foreign exchange has plummeted, leaving few resources for imports. In fact, as illustrated in Figure 5, the fall in oil revenue combines two effects that increase inflation. Lower oil revenue implies that the government has a higher deficit and must use more monetary financing. In addition to the increase in money, the fall in oil prices requires lower imports and thus a lower supply of goods to the domestic market.

This scenario is calibrated to be similar to recent developments in Venezuela. Values of the money stock, export receipts, the fiscal deficit and inflation are broadly in line with developments from 2014. Going forward, fiscal spending growth is assumed to grow at 95 percent, roughly the observed rate of money growth in 2015. In addition, oil prices are assumed to drop from an average of about $88 per barrel in 2014 to $50 per barrel in 2015 and $30 per barrel in 2016.

Export Revenue Shock

inflation - no X fall

inflation - X fall

Based on these assumptions, the simulation of the model shows that inflation would shoot up to over 200 percent in 2015 and 2016 before beginning to converge back to the steady state level of 95 percent, driven by fiscal spending growth.
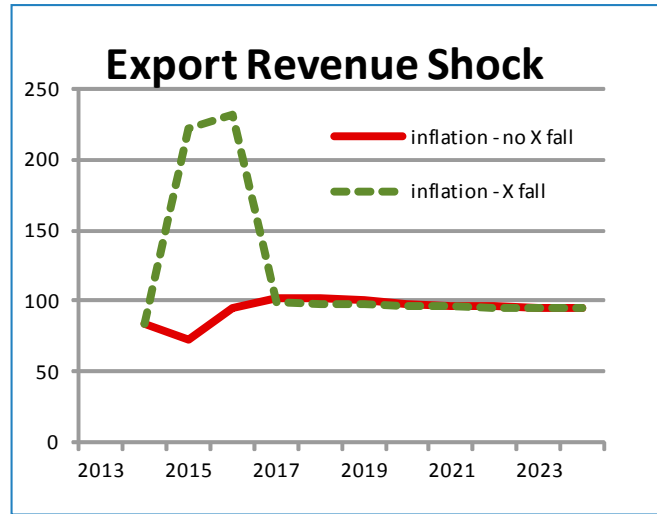
V. BLACK MARKET WITH MULTIPLE CONSUMPTION GOODS

A. The consequence of multiple goods

The one good model has provided a useful framework to understand inflation dynamics in Venezuela. It demonstrates the impact of fiscal spending, oil receipts, and devaluation of the official exchange rate. The model also goes a long way in explaining the rise in the black market rate. However, the rise in the black market rate has been much higher than overall inflation, and the gap with the fitted value of the equilibrium exchange rate has been widening.

In order to fully understand the steep increase in the black market exchange rate in Venezuela in the last few years, it is necessary to consider a model with more than one consumption good. In addition, the black market rate will depend on the amount of foreign exchange transactions taking place in the market relative to the amount of imports. Thus, this section extends the model to multiple goods, with only a small share of transactions taking place in the black market.

Imagine a large number of importing firms competing to buy a portion of the dollars available in the black market. They intend to purchase goods i=1…N from the international markets and sell the goods in the domestic retail market. Each firm is small enough to be a price taker in the domestic market, but all of the firms combined will affect the price of the good. Each firm will consider the profits it could receive by selling good i:

(21) $\pi_i = P_i q_i - b P_i^* q_i$

Profits of an individual firm will therefore increase as long as $P_i > b P_i^*$

An arbitrageur will choose to sell its dollar receipts to the highest bidders among import firms. For any good i, the break even black market rate will set price equal to cost.

(22) $P_i q_i = b P_i^* q_i$ or $b = \dfrac{P_i}{P_i^*}$

 Arranging goods in the order 1…n based on the markups,

(23) $\dfrac{P_1}{P_1^*} > \dfrac{P_2}{P_2^*} > \dfrac{P_3}{P_3^*} > \dots \dfrac{P_n}{P_n^*}$

Importers of good 1 will be willing to buy dollars at the black market rate until P_1 is driven down to $\dfrac{P_2}{P_2^*} P_1^*$ and so on. If goods importers are segmented, the black market rate will reflect the markets with the highest markups over the international price. A reduction in transactions in the black market will raise the black market rate because only the most distorted (highest markups) markets will receive dollar funding.

If a monopolist firm makes all the import decisions, it will use the foreign exchange to import each good until the ratio of marginal revenue to marginal cost is equal across goods.

B. Two goods model

Based on the discussion above, the model is revised to the case of multiple imported consumption goods, where two goods is sufficient to illustrate the points. As before, there are firms that import goods and firms that arbitrage dollars in the black market. The central bank allocates (in an ad hoc manner) foreign exchange at the official exchange rate, st, to import firms that purchase the two import goods from foreign markets at a fixed world price. As long as the domestic price (P_{it}), which constitutes marginal revenue, is above the domestic value of the foreign price (st P_{it}*), which constitutes marginal cost, the importing firm will want to supply the good. The central bank also allocates a share, ϕ, of dollar export receipts to arbitrage firms at the official exchange rate, st. The arbitrageurs sell the dollars in the black market at the highest possible exchange rate.

The household's intertemporal optimization problem now includes utility of two consumption goods.

(24) $\max \sum_{t=0}^{\infty} \beta^t U(C_t^i, C_t^j)$

s.t. $b_t F_{t+1} + M_{t+1} + P_t^i C_t^i + P_t^j C_t^j = b_t F_t + M_t + (T_t + \prod_t)$ budget constraint

and $M_t \geq P_t^i C_t^i + P_t^j C_t^j$ cash-in-advance constraint

Appendix 5 elaborates the solution. For example, if the CIA constraint is binding and utility is logarithmic, then the expenditure shares on each good are equal.

(25) $\quad P_t^i C_t^i = P_t^j C_t^j$

Also, the black market rate grows proportionately to spending on each good.

(26) $\quad \left(\dfrac{b_{t+1}}{b_t}\right) = \dfrac{P_{t+1}^i C_{t+1}^i}{\beta\, P_t^i C_t^i} = \dfrac{P_{t+1}^j C_{t+1}^j}{\beta\, P_t^j C_t^j}$

C. Simulation of black market rate in two goods model

The central bank supplies dollars through rationing in proportion to the export revenues (Xt) deposited by the government from earnings of the state oil company. As before, the central bank allocates a fraction, 1-φ, to importers and a fraction, φ, to arbitrageurs. Since the capital account is restricted, the only capital transactions that occur take place in the black market. Arbitrageurs supply dollars to the black market, which are used for imports of the two goods. Goods markets clear, such that imports, Qit, equal consumption of imported goods, Cit.

The black market rate will mirror price increases in the most distorted good market, i.e., the one with the highest markup (Figure 6). This market is not necessarily the one with the highest inflation always, but rather with the highest margin between cost and price. The arbitrage firms will earn the greatest rent by selling their dollars to import firms that supply goods in the most distorted market.

Table 1 provides a baseline calibration of the model. Fiscal transfers are assumed to grow by 50 percent, which drives money growth and spending of roughly the same magnitude. In 2015 and 2016, export revenues drop sharply, causing a contraction in foreign exchange allocated for imports. Foreign exchange provision is assumed to drop more sharply for good 2, driving up its price by more than three times that of good 1. The black market rate reflects the large markup in good 2 and rises with its higher inflation rate. Thus, the growth of the black market exchange rate is much above overall inflation. Arbitrage and import firms both earn rents from selling dollars and goods, respectively.

If the share of dollars channeled through the black market increases, it could lead to a decline in the black market rate because it will increase the supply of goods in the distorted market and thus reduce their price. The distorted market will become slightly less distorted. If arbitrageurs buy dollars from importers in the less distorted market and sell them to importers in the more distorted market, it would lead to a change in the composition of goods. Supply in the more distorted market would increase and the price would decline, while the opposite would hold in the other market.

Instead of the idea that prices of goods reflect the black market rate (firms pricing at the black market rate), we should think of the converse: the black market rate is based on retail goods prices. But the black market exchange rate can grow faster than overall inflation if price increases for goods in the most distorted markets is higher than goods in less distorted markets.

Figure 8 shows the simulation from an increase in the amount allocated to the black market (in 2016). Dollars are used to substitute additional imports of good 2 (the relatively scarce good) by reducing imports of good 1 (the relatively abundant good). The contemporaneous rate of inflation in good 2 is reduced, while inflation rises somewhat for good 1. Overall inflation is lower than the baseline and the black market rate grows significantly less. Welfare improves since the rise in the utility of consumption of the scarce good is greater than the fall in the utility of consumption of the less scarce good.

D. Simulation of a devaluation

Figure 9 shows the impact of a devaluation of the official exchange rate from 6.3 to 50 (in 2017). The deficit significantly reduces the fiscal deficit, facilitating a decline in money growth. In addition, arbitrageurs and importers incur higher costs of purchasing foreign currency, which reduces their profits. The lower profits transferred as income to households leads them to reduce their subsequent spending on both goods, which leads to lower inflation and lower black market exchange rate than in the baseline.

VI. CAPITAL FLIGHT

Until this point, the model assumed balanced trade, or equivalently, no capital flows or changes in net foreign assets. However, even if the government prevents domestic firms and individuals from establishing financial accounts overseas, it may not be able to prevent capital flight in the form of accumulation of dollar holdings. Import firms may engage in overinvoicing to save some of the foreign exchange allocation. Arbitrageurs can also speculate by choosing to hold the dollars until a later period. The model can be extended to accommodate capital flight. Appendices 6 and 7 present the technical details. Changes in the stock of dollars held by domestic households must equal the difference between the total amount of dollars the central bank distributes from export revenues and the value of imports.

(27) $F_{t+1} - F_t = X_t - P_1 * Q_{1t} - P_2 * Q_{2t}$

This extended model can also be simulated to show the impact of capital flight on inflation, the black market rate, and welfare.

A. Simulation of overinvoicing and dollar savings by importers

An importing firm that sells both goods (or can sell dollars to an importer in the other goods market) has the incentive to overinvoice imports so as to acquire dollars from the central bank at the official exchange rate, and then use the dollars to import goods in the distorted market now or in the future. If FX is expected to become scarcer in the distorted market next period, import firms could try to reduce their imports in the less distorted market this period and use the dollars to increase imports in the distorted market next period. Figure 10 illustrates this scenario and shows that the temporary capital flight that is used to raise supply in the subsequent period of increased scarcity improves overall welfare.

In general equilibrium, the revenue the importers would earn would be the same after the change in quantity supplied because the price would change in inverse proportion. However,

the cost of acquiring dollars from arbitrageurs would fall because the black market rate and the price of the good in the distorted market would decline when its supply increases.

B. Simulation of black market dollar savings (capital flight) by arbitrageurs

Figure 11 shows the results of a simulation of capital flight by arbitrageurs. If the black market rate is expected to increase (by enough to offset any time discount factor), arbitrageurs could save a dollar in the current period t—losing bt revenue per dollar—and then sell that dollar in the next period t+1—gaining bt+1 revenue per dollar. They would gain extra profits at the expense of the importing firm. They would have the incentive to do this up until the point in which the black market rate is equalized between t and t+1. This increase in dollar savings constitutes capital flight during the period of saving. It also implies that current high increases in the black market rate could reflect not only lower imports due to a fall in export receipts but also the voluntary withdrawal of dollars for import now because of an expectation that they will become even more restricted in the future. Since dollars are saved this period by reducing imports to the distorted market, this would increase the price of the good this period and reduce it next period. This exchange of imports through time would smooth out imports (and consumption) from the current high level to the future lower level. This would improve welfare.

VII. EXTENSIONS OF MODEL

A. Price controls

Suppose that there are two consumption goods, but the price of one good is constrained by a price control. Assuming that the cash-in-advance constraint holds, as argued above, we can simplify the household's decision on the allocation of consumption between the two goods to a static allocation problem each period.

(28) $\max U = U(C_1, C_2)$

      s.t. $M = P_1\, C_1 + P_2\, C_2$ cash-in-advance constraint

      and $P_2 \leq \bar{P}$ price control

In the case when the price control is not binding, the FOC would give:

(29) $U_1 / U_2 = P_1 / P_2$

For example, if $U(C) = \log(C_1) + \log(C_2)$, then the household would like to spend equal shares of the money budget on each good.

(30) $P_1 C_1 = P_2 C_2$

However, aggregate consumption of each good is constrained by its import supply, which in turn depends on the rationed foreign exchange allocation received by the importers of the goods. If firms are free to set retail prices, the relative prices will be determined by the

relative supply of goods and optimality condition (29). The overall price level will be determined by aggregate household spending, which is given by the money supply. However, if the price control is binding such that the price of good 2, $P_2 = \bar{P}$, is forced below the optimal retail price, then there will be excess demand for good 2 that will produce a shortage (Figure 7). In addition, with restricted supply and controlled price of good 2, households will have unsatisfied spending power that will spill over into additional demand for good 1 and increase its price beyond the level that would prevail in the absence of the price control on good 2. In particular, the price of the non-controlled good, P_1, will be determined by the money supply and import supplies of the two goods.

(31) $\quad P_1 = (M - \bar{P}\, Q_2)/Q_1$

An increase in the controlled price of good 2 (i.e., a relaxation of the binding constraint) or an increase in its import volume would reduce the equilibrium retail price of good 1. In this case, the optimality condition (29) may not hold.

B. Nontraded good

The model can also be extended by adding a non-traded good produced with inelastic labor.
(32) $\quad Y^N = L$

A representative firm maximizes profits $P^N Y^N - w L$
The wage rate is thus tied to the price of the non-traded good: $w = P^N$

The household optimization now includes utility of both consumption goods.

(33) $\quad \max \sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N)$

$\quad$ s.t. $\quad M_{t+1} + P_t^T C_t^T + P_t^N C_t^N = M_t + (T_t + \prod_t + w_t) \qquad$ budget constraint

$\quad$ and $\quad M_t \geq P_t^T C_t^T + P_t^N C_t^N \qquad\qquad$ cash-in-advance constraint

As shown in Appendix 8, the rate of inflation must be at least as high as the discounted marginal rate of substitution in consumption.

(34) $\quad \left(\dfrac{P_{t+1}^T}{P_t^T}\right) \geq \dfrac{\beta U_1(C_{t+1}^T, C_{t+1}^N)}{U_1(C_t^T, C_t^N)}$

In addition, there is another first order condition relating the marginal utility of each consumption good.

(35) $\quad \dfrac{U_1(C_t^T, C_t^N)}{P_t^T} = \dfrac{U_2(C_t^T, C_t^N)}{P_t^N}$

If utility is logarithmic, then the expenditure shares on each good are equal.

(36) $\quad P_t^T C_t^T = P_t^N C_t^N$

Further assume that the inelastic labor supply = 1. Then, $C^N = Y^N = L = 1$ and

(37) $\quad \frac{P_t^N}{P_t^T} = C_t^T$

The relative price of the non-traded good increases when consumption of the traded good increases. In other words, the relative price of the non-traded good must rise in order to induce households to optimally choose a fixed consumption of the non-traded good when consumption of the traded good rises. This result is in line with the usual intuition on the real exchange rate. When national wealth increases—due to higher export receipts in Venezuela's case—the real exchange rate (relative price of non-traded to traded goods) rises. The higher export receipts allow more imports of traded consumption goods. Conversely, when export wealth falls, the equilibrium real exchange rate also falls.

C. Imported intermediate input

Instead of assuming that the retail consumer good is a direct import good, the model can be extended by requiring that the import is an intermediate input that must be combined with domestic labor in order to assemble the retail consumer good. So there is a production function for output.

(38) $\quad Y_i = q_i^{\alpha} L_i^{1-\alpha}$
where q_i is the firm's use of the imported intermediate good. The representative firm is assumed to be a price taker and maximizes its profits, choosing the optimal quantity of labor and the imported intermediate.

(39) $\quad max_{q,L} \quad PY_i - bP^* q_i - wL_i$

The firm will choose labor such that the marginal produce of labor equals the wage,

(40) $\quad (1 - \alpha)P \left(\frac{q_i}{L_i}\right)^{\alpha} = w$

and the imported intermediate such that the domestic currency cost of the input equals its marginal product.

(41) $\quad \alpha P \left(\frac{q_i}{L_i}\right)^{\alpha - 1} = bP^*$

If total labor supply is inelastic, L=1, then aggregating across firms gives

(42) $\quad w = (1 - \alpha)PQ^{\alpha}$

(43) $\quad b = \frac{\alpha P}{P^*} Q^{\alpha - 1}$

The zero-profit black market exchange rate also changes slightly. Instead of reflecting only the ratio of the domestic price to the foreign price of the good, the black market rate also now depends on the overall quantity of the import good. The increase in the black market rate reflects domestic and foreign inflation and the growth of imports.

$$(44) \quad \frac{b_{t+1}}{b_t} = \frac{P_{t+1}}{P_t} \frac{P_t^*}{P_{t+1}^*} \left(\frac{Q_t}{Q_{t+1}}\right)^{\alpha-1}$$

A fall in Q leads to a higher growth in the black market rate compared with inflation. The aggregate demand curve can now be expressed in terms of output

$$(45) \quad M_t = P_t C_t = P_t Y_t = P_t Q_t^\alpha$$

If the intermediate input is reduced because of lack of foreign exchange allocation, it also lowers the output of the retail good. There is a direct link between output, consumption, and the import good. However, consumption and output growth are less volatile than the import good. In fact, this relationship is clearly evident in Venezuela over the past, with output, private consumption, and import growth moving closely together. However, as predicted by this model, import growth rates would be larger in absolute value than output and consumption growth rates.

Venezuela: Real Growth Rates
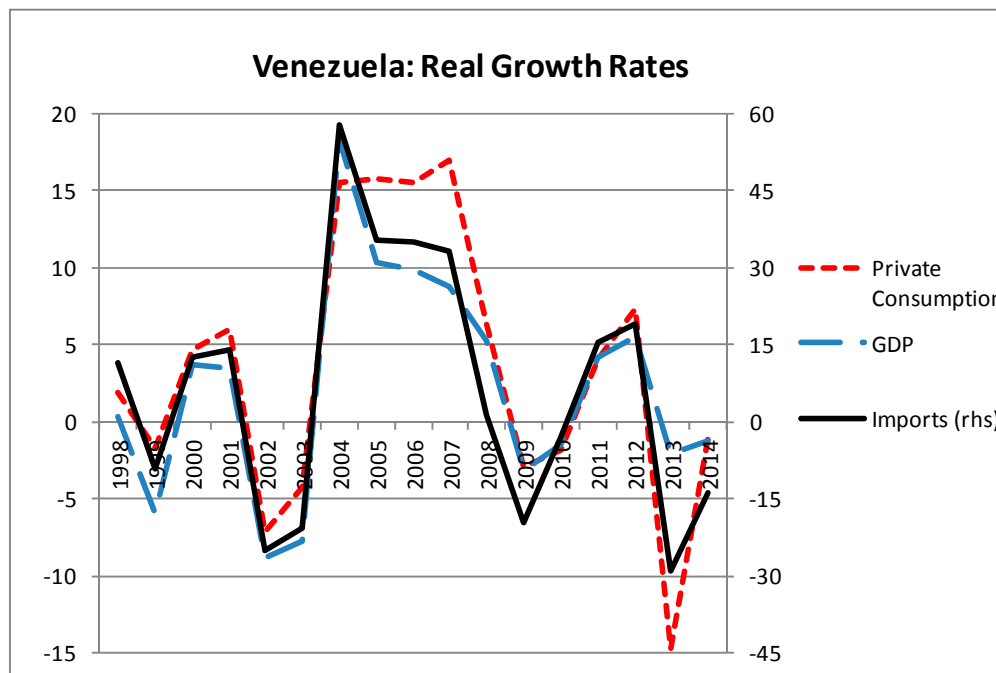
D.   Other macro policies

Government spending in dollars

If the government purchases foreign goods and services using dollars, it will decrease the supply of dollars for import of the consumption good. Thus, Qt will fall. The effects will be

similar to a reduction in Xt, unless government purchases are used to satisfy consumption demand.

Issuance of external debt

If the government issues external debt, it increases the availability of foreign exchange for import, thus raising Qt in the short run. However, it will have to pay back the debt, thus reducing Q in the future. Likewise, payment of external debt service reduces the availability of foreign exchange for import, further driving up inflation and the black market rate.

Issuance of domestic debt

If the government finances its deficit by issuing domestic debt, it reduces the amount of monetary financing required. Thus, the money supply will be lower by the amount of debt sold compared with the counterfactual of no debt issuance. However, the government will have to pay back the debt in the future, so future money supplies will be increased again.

Accumulation of foreign exchange reserves

If the central bank builds foreign exchange reserves instead of selling them, then the money supply will be higher than it would be compared with the sale of reserves. In addition, there will be less foreign exchange available for imports. Likewise, a reduction in foreign exchange reserves through sale to domestic agents would reduce the money supply and increase imports. Thus, a scenario with an accumulation of foreign exchange reserves would show higher inflation and lower consumption relative to a scenario when foreign exchange reserves are fully sold to domestic agents. Conversely, decumulation of reserves would produce more imports and lower inflation.

VIII. CONCLUSIONS

This paper presents a stylized general equilibrium model that captures the key features of the Venezuelan economy. The rationing of foreign exchange for imports, linked to oil export earnings, and a multiple exchange rate regime lead to a repressed market for consumer (and intermediate) goods. In this setting, the demand for consumer goods and foreign exchange availability determine the domestic price of the consumer good. The price of the consumer good then determines the black market exchange rate. In the more realistic situation of multiple goods, the arbitrary allocation to import and arbitrage firms creates varying levels of subsidies, and the black market rate will reflect the subset of goods with the highest markup over international prices. Capital flight (or equivalently, a portfolio choice between dollars and domestic currency) can also contribute to the dynamics, but is not needed to explain the rise in inflation and the even steeper surge in the black market premium. The model also explains two counterintuitive phenomen: (1) a fall in oil revenues can lead to a temporary increase in inflation, and (2) a devaluation can generate a temporary drop in inflation.

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APPENDICES
Appendix 1. The Cagan Model—Not the Best Explanation for Inflation Surge

According to the Cagan model, hyperinflation is driven by severe drops in money demand dominated by rising inflation expectations. This phenomenon could be due to a rise in money growth that feeds inflation expectations and/or an increased elasticity of real money demand to expected inflation. The combination destabilizes inflation expectations and velocity rises.

The Cagan model does not yet appear to fit Venezuela data. At least through 2014, there is no evidence that velocity was increasing or that the elasticity of money demand rose. In fact, velocity has been on a steady decline, irrespective of whether it is measured using GDP, aggregate demand, private consumption, or imports.

The data for 2015 is incomplete. At end-December, the monetary base grew by 111 percent yoy and consumer price inflation reached at least 180 percent by the end of the year. On the surface, the increase in inflation above money growth may show the first signs that velocity has finally started to rise and the Cagan model is relevant. However, national accounts data, including GDP and its components, has not been published yet for 2015, so velocity cannot be updated at this time.
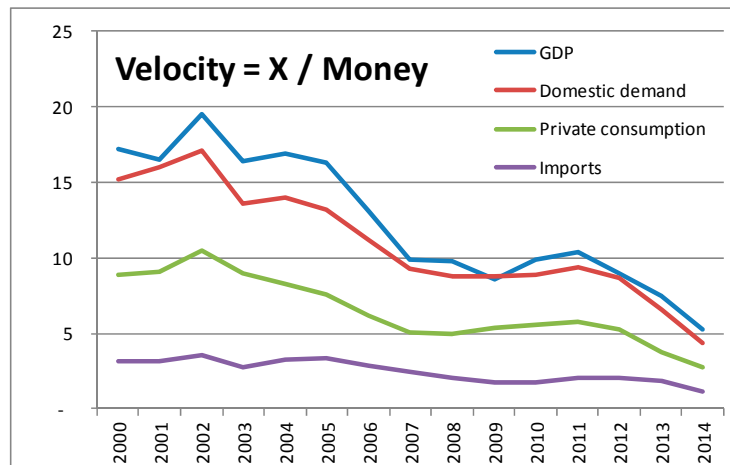
The assumptions of the Cagan model may not fit Venezuela's economic situation. First, the Cagan model assumes that real GDP is constant and simplifies to a relationship between money and expected inflation. This is an unrealistic assumption for Venezuela in 2015, as real GDP is estimated to have fallen sharply (based on indicators of activity). Real private consumption and imports may have fallen even more than output. Since the Cagan model does not account for this important development, the estimates from the model are not reliable.

Second, the Cagan model is based on a money demand equation involving the choice between money and bonds.

$$M = PY\, f(i) \quad \text{where } i \text{ is the nominal interest rate and } i = r + \pi^e$$

The model assumes that Y and r are fixed and derives a relationship between M, P, and π^e. The Cagan model may not fit Venezuela's economic structure because of financial repression. It is likely that households do not have the option of buying bonds as an alternative asset to holding money.

Appendix 2. Numerical example of monetization of deficits, devaluation, and sales of reserves by the central bank.[4]

All scenarios assume that the government earns export revenue equal to 100 dollars and spends 2000 bolivars on transfer payments to households. The official exchange rate is 6 bolivars per dollar, and in some scenarios is devalued to 8 bolivars per dollar.

I. Initial NIR = 0

No devaluation. Central bank builds reserves.

Beginning					End				
Assets		Liabilities			Assets		Liabilities		
0	NIR	0	MB		+600	NIR	+2000		MB
0	DC $_{gov}$	0	Dep $_{gov}$		0	DC $_{govt}$	+600 / -2000	-1400	Dep $_{gov}$

Devaluation. Central bank builds reserves.

Beginning					End				
Assets		Liabilities			Assets		Liabilities		
0	NIR	0	MB		+800	NIR	+2000		MB
0	DC $_{gov}$	0	Dep $_{gov}$		0	DC $_{govt}$	+800 / -2000	-1200	Dep $_{gov}$

Punchline: If the central bank does not sell dollars, money creation is determined only by government transfers.

II. Initial NIR = 100 dollars

No devaluation. Central bank sells dollars.

Beginning					End				
Assets		Liabilities			Assets		Liabilities		
+600	NIR	+2000	MB		0	NIR	+2000 / -600	1400	MB
0	DC $_{gov}$	-1400	Dep $_{gov}$		0	DC $_{govt}$	-1400		Dep $_{gov}$

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[4] This illustration decomposes net domestic credit into domestic credit and government deposits, with the latter shown as a liability. "OIN" is other items net.

Devaluation. Central bank sells dollars.

Beginning				End			
Assets		Liabilities		Assets		Liabilities	
+800	NIR	+2000	MB	0	NIR	+2000 ⎱ −800 ⎰	1200 MB
0	DC $_{gov}$	−1200	Dep $_{gov}$	0	DC $_{govt}$	−1200	Dep $_{gov}$

No devaluation for govt, but central bank sells dollars to private sector at devalued rate.

Beginning				End			
Assets		Liabilities		Assets		Liabilities	
+600	NIR	+2000	MB	0	NIR	+2000 ⎱ −800 ⎰	1200 MB
0	DC $_{gov}$	−1400	Dep $_{gov}$	0	DC $_{govt}$	−1400	Dep $_{gov}$
0	OIN			−200	OIN		

Punchline: Sale of dollars to the private sector reduces money supply. Devaluation absorbs more domestic currency from the private sector in exchange for dollars, reducing money creation. Lower money creation is driven by a higher average exchange rate, and is independent of the exchange rate credited to the government (although the budget deficit would be affected by it).

Note, however, that the example assumes that the government maintains a fixed amount of transfers to the public of 2000 bolivars. If, instead, the government were to respond to the lower budget deficit resulting from the devaluation by *increasing* its transfers to the public (as if, for example, it maintained a nominal deficit target), then money creation would rise accordingly.

Appendix 3. Household optimization with one good and CIA constraint

The household maximizes discounted utility of consumption, subject to a budget constraint and a constraint that purchases in a period are limited by money holdings.

$\max \sum_{t=0}^{\infty} \beta^t U(Ct)$

s.t.   Mt+1 + Pt Ct = Mt + (Tt + $\prod$t)   budget constraint

and   Mt $\geq$ Pt Ct   cash-in-advance constraint (CIA)

Letting λt be the Lagrange multiplier associated with the cash-in-advance constraint, the Kuhn Tucker conditions require that:

$$-\beta^{t-1} \frac{U'(C_{t-1})}{P_{t-1}} + \beta^t \frac{U'(C_t)}{P_t} + \beta^t \frac{\lambda_t}{P_t} = 0$$

Rearranging:

$$\lambda_t = \left(\frac{P_t}{P_{t-1}}\right) \frac{U'(C_{t-1})}{\beta} - U'(C_t) \geq 0$$

Since λt $\geq$ 0, it follows that the rate of inflation must be at least as high as the discounted marginal rate of substitution in consumption.

$$\left(\frac{P_t}{P_{t-1}}\right) \geq \frac{\beta \ U'(C_t)}{U'(C_{t-1})}$$

Solving the model with a CIA constraint implies that private consumption spending is either equal to or less than the money stock.   Mt $\geq$ Pt Ct

(Case 1) When the CIA constraint binds, consumption spending equals the money stock
 Mt = Pt Ct

Inflation is proportional to the growth of money and inversely proportional to the growth of consumption.

$$\left(\frac{P_t}{P_{t-1}}\right) = \frac{M_t}{M_{t-1}} \frac{C_{t-1}}{C_t}$$

If money grows at rate μ, then inflation also grows at rate μ adjusted for the inverse of consumption growth.

$$\left(\frac{P_t}{P_{t-1}}\right) = (1 + \mu) \frac{C_{t-1}}{C_t}$$

(Case 2) If the CIA constraint does not bind, consumption spending is less than the money stock, Mt > Pt Ct. In this case, inflation must equal the marginal rate of substitution between consumption this period and last period adjusted for the time preference.

$$\left(\frac{P_t}{P_{t-1}}\right) = \frac{\beta \ U'(C_t)}{U'(C_{t-1})}$$

Under isoelastic utility, $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$, $U'(C) = C^{-\gamma}$ and

$$\left(\frac{P_t}{P_{t-1}}\right) = \beta \left(\frac{C_{t-1}}{C_t}\right)^{\gamma}$$

Under log utility (γ=1), this would reduce to

$$\left(\frac{P_t}{P_{t-1}}\right) = \beta \frac{C_{t-1}}{C_t}$$

Inflation would be inversely related to consumption growth.

When Case 2 holds and consumption (Ct=Qt) is either constant or increasing, inflation would be negative since beta is less than one (β<1). Consumption could be constant or increasing if foreign exchange constraints and foreign borrowing constraints exist. In that situation, an increase in oil revenue could relax the import restriction, allowing an increase in consumption. Note that an increasing path of consumption is not an optimal policy if foreign constraints are not present.

When the CIA constraint does bind,

$$\left(\frac{P_t}{P_{t-1}}\right) = \frac{M_t}{M_{t-1}} \frac{C_{t-1}}{C_t} > \beta \left(\frac{C_{t-1}}{C_t}\right)^{\gamma} \quad \text{or} \quad \frac{M_t}{M_{t-1}} > \beta \left(\frac{C_{t-1}}{C_t}\right)^{\gamma-1}$$

Appendix 4. Summary of model equations for dynamic simulations

The following equations summarize the dynamic equilibrium for the Venezuela model:

(A.1) Mt = Pt Ct   when $\left(\frac{P_t}{P_{t-1}}\right) > \frac{\beta \ U'(C_t)}{U'(C_{t-1})}$ Aggregate demand

or

(A.1′) Mt > Pt Ct   when $\left(\frac{P_t}{P_{t-1}}\right) = \frac{\beta \ U'(C_t)}{U'(C_{t-1})}$

(A.2) Ct = Qt   Goods market clearing

(A.3) ΔMt+1 = Tt – st Xt   Money growth tied to fiscal deficit
                                                    (assumes no change in reserves)

(A.4)   Xt = P*t Qt   Balanced trade

(A.5)   bt = Pt / P*t   Black market under LOOP

(A.6)   Pt ≥ st P*t   Profitability constraint

(A.7)   Mt+1 + Pt Ct = Mt + (Tt + ∏t)   HH budget constraint
                                                             (model with no dollar portfolio holdings)

(A.8) $\left(\frac{T_{t+1}}{T_t}\right) = \tau$ Growth of spending (transfers)

(A.9) $\left(\frac{s_{t+1}}{s_t}\right) = \sigma$ Devaluation rate of official exrt

M is money, P is the domestic price level, C is consumption, β is time preference, Q is the quantity of imported consumption good, T is government spending (transfers), s is the official exchange rate, X is the value of export receipts, P* is the foreign price level, b is the black market rate, P is the income households receive from corporate profits, τ is the growth rate of spending, and σ is the devaluation rate.

The model is simulated under conditions of relatively high inflation as currently experienced in Venezuela. Given this, aggregate demand will be given by (1) rather than (1′). Each policy scenario starts with the same initial values for the simulation, as shown in the table below. These values are chosen to broadly match corresponding data in Venezuela.

					Initial values					
Xt	P*	Qt=Ct	Tt	st	deficit	deficit/Pt	Mt	Pt	bt	Pt/st
80	5	16	1000	6.3	496	4	2000	125	25	20

Appendix 5. Two imported goods and dollar portfolio holdings

The model is extended by assuming that utility depends on the consumption of two imported consumer goods, i and j.

Household problem
The household optimization now includes utility of both consumption goods.

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t^i, C_t^j)$$

s.t. $b_t F_{t+1} + M_{t+1} + P_t^i C_t^i + P_t^j C_t^j = b_t F_t + M_t + (T_t + \prod_t)$ budget constraint

and $M_t \geq P_t^i C_t^i + P_t^j C_t^j$ cash-in-advance constraint

Letting λt be the Lagrange multiplier associated with the cash-in-advance constraint, the Kuhn Tucker conditions require that:

$$-\beta^{t-1}\frac{U_1(t-1)}{P_{t-1}^i} + \beta^t\frac{U_1(t)}{P_t^i} + \beta^{t-1}\frac{\lambda_{t-1}}{P_{t-1}^j} = 0 \quad \text{and}$$

$$-\beta^{t-1}\frac{U_2(t-1)}{P_{t-1}^j} + \beta^t\frac{U_2(t)}{P_t^j} + \beta^{t-1}\frac{\lambda_{t-1}}{P_{t-1}^i} = 0$$

$$-\beta^{t-1}\frac{U_1(t-1)\ b_{t-1}}{P_{t-1}^i} + \beta^t\frac{U_1(t)\ b_t}{P_t^i} = 0$$

$$-\beta^{t-1}\frac{U_2(t-1)\ b_{t-1}}{P_{t-1}^j} + \beta^t\frac{U_2(t)\ b_t}{P_t^j} = 0$$

Since $\lambda_t \geq 0$, it follows that the rate of inflation must be at least as high as the discounted marginal rate of substitution in consumption.

$$\left(\frac{P_{t+1}^i}{P_t^i}\right) \geq \frac{\beta U_1(C_{t+1}^i, C_{t+1}^j)}{U_1(C_t^i, C_t^j)} \quad \text{and} \quad \left(\frac{P_{t+1}^j}{P_t^j}\right) \geq \frac{\beta U_2(C_{t+1}^i, C_{t+1}^j)}{U_2(C_t^i, C_t^j)}$$

The black market exchange rate is also related to the marginal rate of substitution and price growth.

$$\left(\frac{b_t}{b_{t-1}}\right) = \left(\frac{P_{t+1}^i}{P_t^i}\right) \Big/ \frac{\beta U_1(C_{t+1}^i, C_{t+1}^j)}{U_1(C_t^i, C_t^j)} = \left(\frac{P_{t+1}^j}{P_t^j}\right) \Big/ \frac{\beta U_2(C_{t+1}^i, C_{t+1}^j)}{U_2(C_t^i, C_t^j)}$$

In addition, there is another first order condition relating the marginal utility of each consumption good.

$$\frac{U_1(C_t^i, C_t^j)}{P_t^i} = \frac{U_2(C_t^i, C_t^j)}{P_t^j}$$

If utility is logarithmic, then the expenditure shares on each good are equal

$$P_t^i C_t^i = P_t^j C_t^j$$

and the black market rate grows proportionally to spending on each good:

$$\left(\frac{b_{t+1}}{b_t}\right) = \frac{P_{t+1}^i C_{t+1}^i}{\beta\ P_t^i C_t^i} = \frac{P_{t+1}^j C_{t+1}^j}{\beta\ P_t^j C_t^j}$$

Appendix 6. Planner's problem with FX in advance constraint

In order to understand the optimal amount of foreign exchange holdings, the first step is to compute the "central planning" solution. This problem assumes that a benevolent planner optimally chooses resources at the economy-wide level in order to maximize the utility of the representative agent. Foreign exchange must be used to purchase imported goods.

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t)$$

s.t. $F_{t+1} - F_t = X_t - P_t^* C_t$ budget constraint

and $F_t \geq P_t^* C_t$ forex-in-advance constraint

Letting λ_t be the Lagrange multiplier associated with the forex-in-advance constraint, the Kuhn Tucker conditions require that:

$$-\beta^{t-1} \frac{U'(C_{t-1})}{P_{t-1}^*} + \beta^t \frac{U'(C_t)}{P_t^*} + \beta^t \frac{\lambda_t}{P_t^*} = 0$$

Rearranging:

$$\lambda_t = \left(\frac{P_t^*}{P_{t-1}^*}\right) \frac{U'(C_{t-1})}{\beta} - U'(C_t) \geq 0$$

Since $\lambda_t \geq 0$, it follows that the rate of foreign inflation must be at least as high as the discounted marginal rate of substitution in consumption.

$$\left(\frac{P_t^*}{P_{t-1}^*}\right) \geq \frac{\beta \ U'(C_t)}{U'(C_{t-1})}$$

As an example, suppose that the utility function is log utility, so that $U'(C_t) = 1/C_t$, and suppose the international price is constant ($P_t^* = P^* \ \forall t$). Then, the relationship above becomes:

$$\frac{C_t}{C_{t-1}} \geq \beta$$

Households want to smooth consumption over time. In addition, near-term consumption is more valuable than consumption in the future. So, the optimal plan is to have a path of consumption that starts as high as possible and slightly declines over time, subject to meeting the constraints.

If $\lambda_t = 0$, the forex-in-advance constraint is *not* binding, meaning that there is sufficient forex availability to purchase as much foreign goods as desired ($F_t > P_t^* C_t$). In this case, $C_t = \beta C_{t-1}$. In other words, since β is the rate of time preference and is generally calibrated to be around 0.97-0.99, the optimality conditions imply that consumption will be decreasing

slightly over time. In other words, because people are impatient, they prefer to tilt the path of consumption slightly toward higher consumption in the early years. When there are no binding foreign exchange constraints—or they can borrow from abroad—they will prefer higher early consumption.

In the situation when the forex-in-advance constraint *is* binding, such that all available foreign exchange is used to purchase foreign goods ($F_t = P_t^* C_t$), the optimality condition ($C_t > \beta C_{t-1}$) allows that consumption could be constant or increasing over time.

When the forex-in-advance constraint binds, all available foreign exchange in the current period is used to buy foreign goods, $F_t = P_t^* C_t$. Since there is no saving of the current stock of foreign exchange, it implies that the only foreign exchange available in the next period is the amount earned through export receipts during the current period, $F_{t+1} = X_t$. In the case of Venezuela, the FIA constraint can be interpreted as the central bank allocating forex to the private sector (and government import agencies) at the beginning of the period to be used for imports during the period. Export revenues earned by the state oil company after the allocation will be used to restock the central bank's supply of foreign exchange to be used for the next period's allocation to importers.

Returning to the example of log utility and constant international prices, let's further assume that $P^* = 1$, and that export earnings are also constant over time, $X_t = \bar{X}$, then consumption is constant over time and equal to the amount of goods supported by the constant export revenue:

$$C_t = F_t = X_{t-1} = \bar{X} \rightarrow C_t = \bar{C}$$

In a finite horizon framework with a terminal condition on foreign assets ($F_{t+N} = Z$), suppose there is no forex-in-advance constraint. Then, the optimal plan would involve running down foreign assets and borrowing in the first half of the horizon to support the initially higher consumption and then repaying the debt once consumption has diminished.

The following simulation assumes that dollars holdings are initially 370 and the annual income is 10; the time discount factor beta is 0.97; utility is CES with gamma equal to 0.8; the foreign price is constant; and the horizon is 100 years.

As shown in the graph below, consumption declines steadily. Dollar holdings fall in the first decade and then the country borrows heavily for the next three decades. Eventually dollar liabilities are repaid and the economy ends with zero dollar holdings (the terminal constraint). Of course, in the absence of any terminal constraint, the optimal policy would be to borrow infinitely to support infinite consumption.
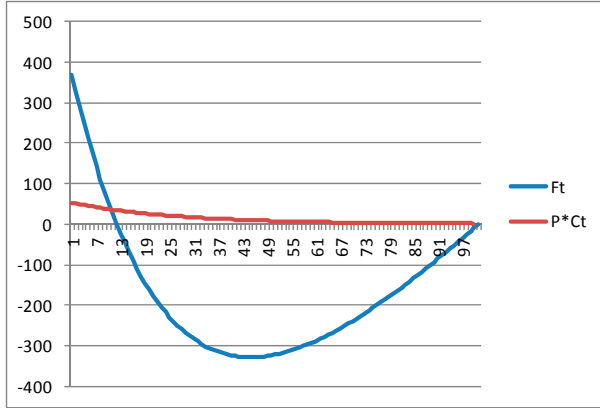
As shown in the graphs below, the longer is the horizon, the more negative dollar holdings would become because there would be more time to pay it back at lower levels of consumption. A longer time horizon would also support a higher level of initial consumption. As the horizon goes to infinity, the amount of dollar liabilities would also be boundless.
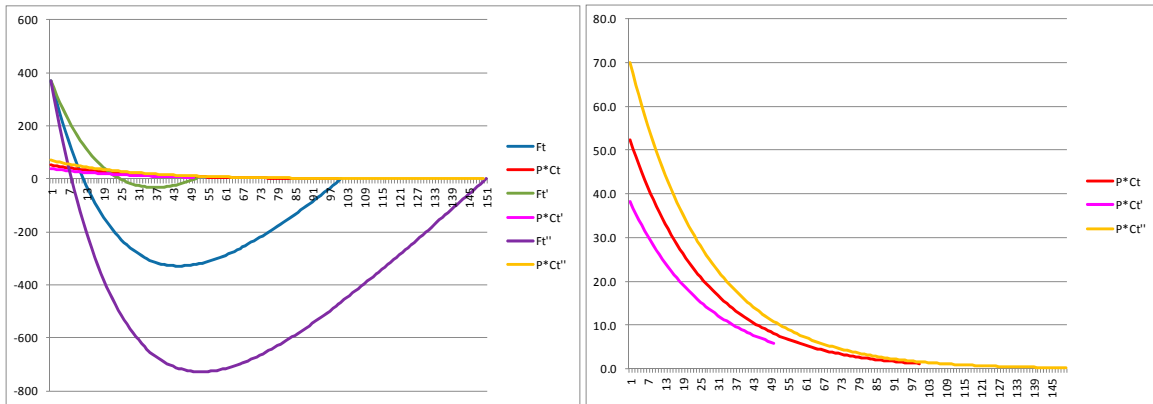
Appendix 7. Household portfolio decision with money and black market dollars

In this model, households maximize discounted utility and make a portfolio choice between holding dollars and domestic money. Purchases of domestic goods is subject to a cash-in-advance constraint with only domestic currency being acceptable for spending.

$$\max \sum_{t=0}^{\infty} \beta^t U(Ct)$$

s.t. $\quad b_t F_{t+1} + M_{t+1} + P_t C_t = b_t F_t + M_t + Y_t \qquad$ budget constraint
$\qquad Y_t = T_t + \prod_t$

and $\quad M_t \geq P_t C_t \qquad\qquad\qquad\qquad$ cash-in-advance constraint

Letting λt be the Lagrange multiplier associated with the cash-in-advance constraint, the Kuhn Tucker conditions require that:

$$-\beta^{t-1} \frac{U'(C_{t-1})}{P_{t-1}} + \beta^t \frac{U'(C_t)}{P_t} + \beta^t \frac{\lambda_t}{P_t} = 0$$

$$-\beta^{t-1}\frac{U'(C_{t-1})\ b_{t-1}}{P_{t-1}} + \beta^t\frac{U'(C_t)\ b_t}{P_t} = 0$$

Rearranging:

$$\lambda_t = \left(\frac{P_t}{P_{t-1}}\right)\frac{U'(C_{t-1})}{\beta} - U'(C_t) \geq 0$$

Since $\lambda t \geq 0$, it follows that the rate of inflation must be at least as high as the discounted marginal rate of substitution in consumption.

$$\left(\frac{P_t}{P_{t-1}}\right) \geq \frac{\beta\ U'(C_t)}{U'(C_{t-1})}$$

The second FOC requires that

$$\left(\frac{b_t}{b_{t-1}}\right) = \left(\frac{P_t}{P_{t-1}}\right)\bigg/\frac{\beta\ U'(C_t)}{U'(C_{t-1})}$$

The right hand side of the expression is > 1 if the cash-in-advance constraint is binding.

$$\left(\frac{b_t}{b_{t-1}}\right) = \frac{1}{\beta}\left(\frac{P_t}{P_{t-1}}\right) \text{ if } C_t = \bar{C}.$$

In other words, in order to be consistent with the desire for constant consumption, the black market rate should be increasing slightly faster than inflation. Households can choose to hold their savings in money assets or dollar assets. Domestic money can be used to purchase consumption immediately, but dollars cannot. Dollars must first be converted into bolivars with a one period lag in order to be used for consumption purchases. That is why the return on the black market rate must be slightly higher than inflation by the rate of time preference. As an optimality condition, the return on holding dollars must compensate the person for having to wait one period to enjoy the consumption. The model does not incorporate a security/evasion cost of using the black market. If there is a cost to purchasing dollars in the black market, then the return on black market dollars would need to be even higher in order for people to choose black market dollars as assets.

These conditions are optimality conditions for the household. If the black market return on dollar holdings is **higher** than given by the FOC above, dollar holdings will dominate bolivar holdings. In that case, people will hold only as much bolivars as needed for next period consumption. The rest will be saved in dollars. But savings in dollars will reduce imports. In aggregate, the quantity of imports must be consistent with aggregate consumption.

If the black market return on dollars is **lower** than the FOC above (such as if the black market return is equal to price inflation rather than higher than inflation), then bolivars will dominate dollars and only domestic money will be used as an asset. Dollars will not be used as assets, but merely to purchase import goods.

When the CIA constraint is always binding, then $M_t = P_t C_t$ for all t. In this case, the budget constraint becomes merely a choice between Ft+1 and Mt+1:

$$b_t F_{t+1} + M_{t+1} + P_t C_t = b_t F_t + M_t + Y_t$$

At time t, every object on the right hand side of the budget constraint is already predetermined. In addition, Pt Ct is predetermined by the CIA constraint. Therefore, the only choice is between Ft+1 and Mt+1. However, Mt+1 will determine next period's consumption since $M_{t+1} = P_{t+1} C_{t+1}$. So money holdings have to be at least as great as the optimal path of consumption spending. If the return on dollars is not high enough (if the black market rate only increases with inflation), households will choose F=0.

If there is no ability to save in dollars, only in money, then the budget and CIA constraints imply that household's only decision is whether to consume up to money holdings or to save. If they spend all their money holdings, their next period money stock is given by their current income.

Appendix 8. Traded and nontraded goods

A non-traded good is produced with inelastic labor: $Y^N = L$ and a representative firm maximizes profits $P^N Y^N - w L$
The wage rate is thus tied to the price of the non-traded good: $w = P^N$

The household optimization now includes utility of both consumption goods.

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N)$$

s.t. $M_{t+1} + P_t^T C_t^T + P_t^N C_t^N = M_t + (T_t + \prod_t + w_t)$ budget constraint

and $M_t \geq P_t^T C_t^T + P_t^N C_t^N$ cash-in-advance constraint

Letting λt be the Lagrange multiplier associated with the cash-in-advance constraint, the Kuhn Tucker conditions require that:

$$-\beta^{t-1} \frac{U_1(t-1)}{P_{t-1}^T} + \beta^t \frac{U_1(t)}{P_t^T} + \beta^{t-1} \frac{\lambda_{t-1}}{P_{t-1}^N} = 0$$

Since $\lambda_t \geq 0$, it follows that the rate of inflation must be at least as high as the discounted marginal rate of substitution in consumption.

$$\left(\frac{P_{t+1}^T}{P_t^T}\right) \geq \frac{\beta U_1(C_{t+1}^T, C_{t+1}^N)}{U_1(C_t^T, C_t^N)}$$

Another first order condition relates the marginal utilities of the consumption goods.

$$\frac{U_1(C_t^T, C_t^N)}{P_t^T} = \frac{U_2(C_t^T, C_t^N)}{P_t^N}$$

Table 1. Baseline

Baseline

time t	Ft	Xt	p1*t	p2*t	initial official allocations		offcl imports	black mrkt dollars	imports (official + bl mrkt)		Mt	π1	π2	π	bt/bt-1	offcl er st	transfers Tt	deficit Tt-stXt
					Q1t	Q2t $			Q1t	Q2t								
2013	0	90	2	2	23	18	81	9	23	22	1220					6.3	955	388
2014	0	80	2	2	26	10	72	8	26	14	2000	45	158	101	158	6.3	1000	496
2015	0	45	2	2	20	0.3	41	5	20	2.5	2496	62	599	331	599	6.3	1500	1217
2016	0	27	2	2	12	0.2	24	3	12	1.5	3713	148	148	148	148	6.3	2250	2080
2017	0	27	2	2	12	0.2	24	3	12	1.5	5792	56	56	56	56	6.3	3375	3205
2018	0	27	2	2	12	0.2	24	3	12	1.5	8997	55	55	55	55	6.3	5063	4892
2019	0	27	2	2	12	0.2	24	3	12	1.5	13890	54	54	54	54	6.3	7594	7424
2020	0	27	2	2	12	0.2	24	3	12	1.5	21313	53	53	53	53	6.3	11391	11221
2021	0	28	2	2	12	0.6	25	3	12	2.0	32534	53	14	34	14	6.3	17086	16910
2022	0	28	2	2	12	0.6	25	3	12	2.0	49443	52	52	52	52	6.3	25629	25453

External sector | Monetary μ(lr)= 52 | Fiscal g_T = 50% ; 10%

Baseline

51

time t	HH bolivar Yt = Tt+Πt	HH bolivar spending	log C1t	log C2t	P1t	P2t	P1t/P1*t	P2t/P2*t	bt	FX rev bt (φ Xt - ΔF^A t)	FX cost st φ Xt	FX profits Π^A t	Rev P1tQ1t+P2tQ2t	Cost st (1-φ)Xt+bt (φ Xt-ΔF^A t)	Profit Π^M t
2013	2000	1220	3.14	3.09	27	28	13	14	14	125	57	68	1220	635	585
2014	2496	2000	3.26	2.64	38	71	19	36	36	286	50	235	2000	739	1261
2015	3713	2496	3.00	0.92	62	499	31	250	250	1123	28	1095	2496	1378	1118
2016	5792	3713	2.48	0.41	155	1238	77	619	619	1671	17	1654	3713	1824	1889
2017	8997	5792	2.48	0.41	241	1931	121	965	965	2607	17	2590	5792	2760	3033
2018	13890	8997	2.48	0.41	375	2999	187	1500	1500	4049	17	4032	8997	4202	4795
2019	21313	13890	2.48	0.41	579	4630	289	2315	2315	6250	17	6233	13890	6403	7486
2020	32534	21313	2.48	0.41	888	7104	444	3552	3552	9591	17	9574	21313	9744	11569
2021	49443	32534	2.48	0.69	1356	8133	678	4067	4067	11387	18	11369	32534	11546	20988
2022	74896	49443	2.48	0.69	2060	12361	1030	6180	6180	17305	18	17288	49443	17464	31979

Household | Firms (Arbitrageur | Importer)

FIGURES
Figure 1. Typical market for (imported) good
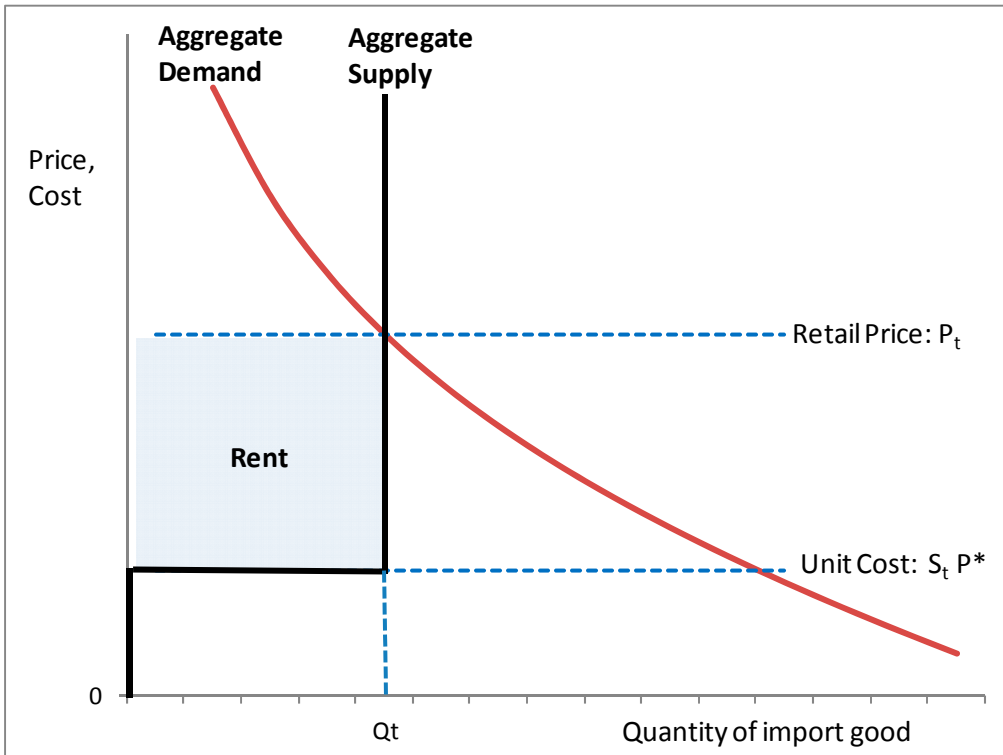

The Aggregate Demand Curve is: Pt = Mt / Qt
Aggregate supply equals the fixed quantity of imports that can be financed from the fixed quantity of foreign exchange allocated to the importers of the good. Aggregate is derived from household preferences and income. The unit cost of supplying the good equals the foreign price, P*, converted to domestic prices at the official exchange rate, st.

Figure 2. Money growth raises prices and rents
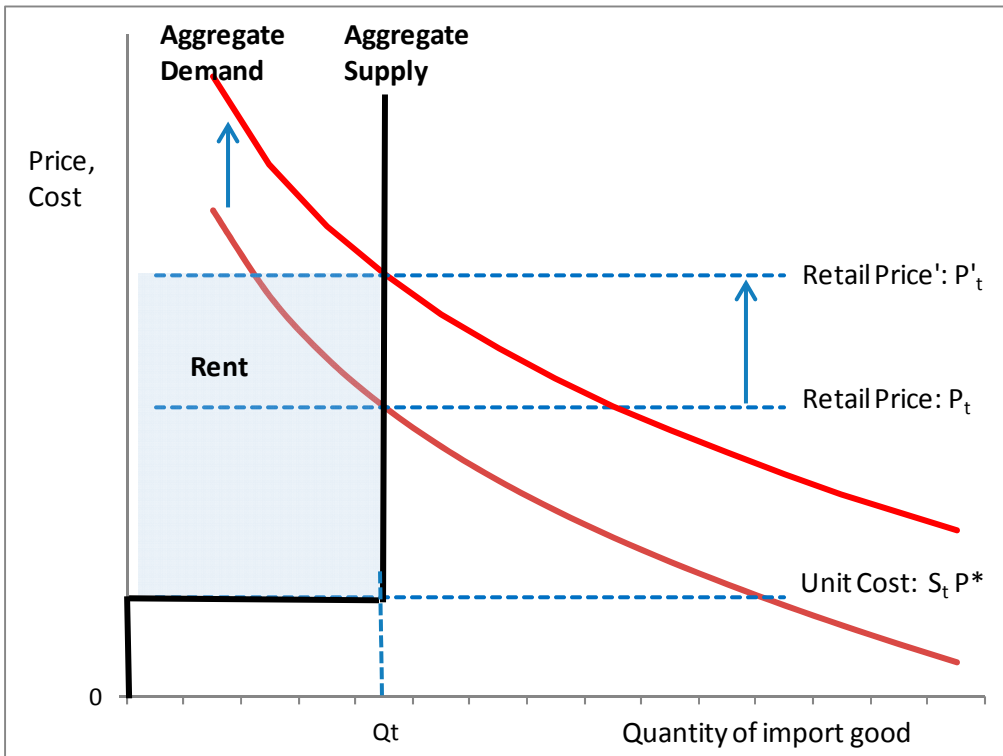
The increase in the price is proportionate to the increase in money supply given that the aggregate demand curve comes from the binding cash-in-advance constraint, $P_t = M_t / C_t$, where the goods market must clear: $C_t = Q_t$. Thus, money growth raises the domestic price of the good and also increases rents of the importing firms (assuming no change in the official exchange rate s_t).

Figure 3. A devaluation of the official rate squeezes rent
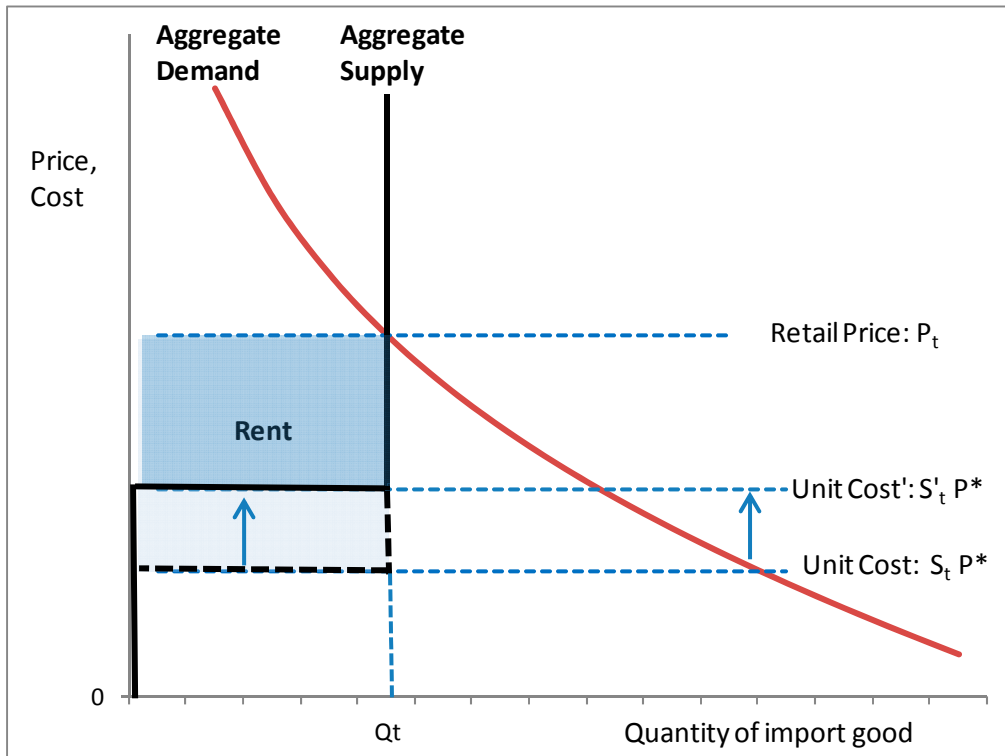
A devaluation of the official exchange rate raises the unit cost of supplying the imported good. With the domestic market price, Pt, still above the unit cost, the import firms still earn rent. But the devaluation reduces the amount of rent they receive.

Figure 4. Large devaluation to a level above LOOP
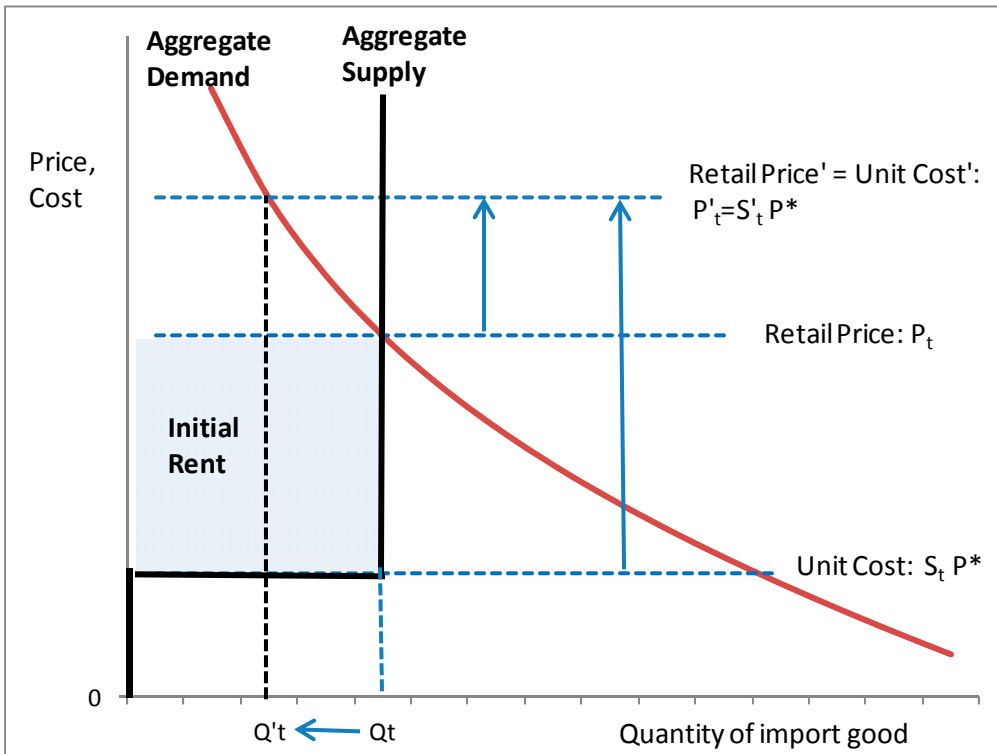
If the exchange rate were to be devalued to a level above LOOP (law of one price), there would be an impact on the retail price and the quantity of the good sold. In this case, since the devaluation moves the exchange rate above the level required for LOOP to hold at Qt, there is pass-through of the devaluation to the domestic price level, and there is also import compression.
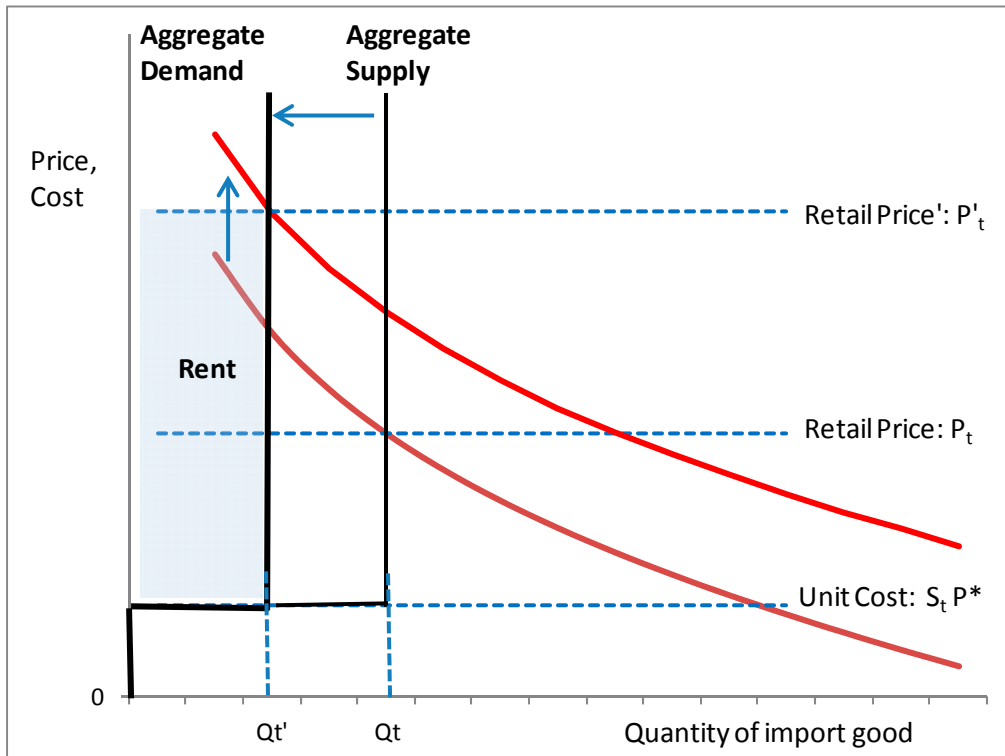
Figure 4 (continued). Large devaluation to a level above LOOP
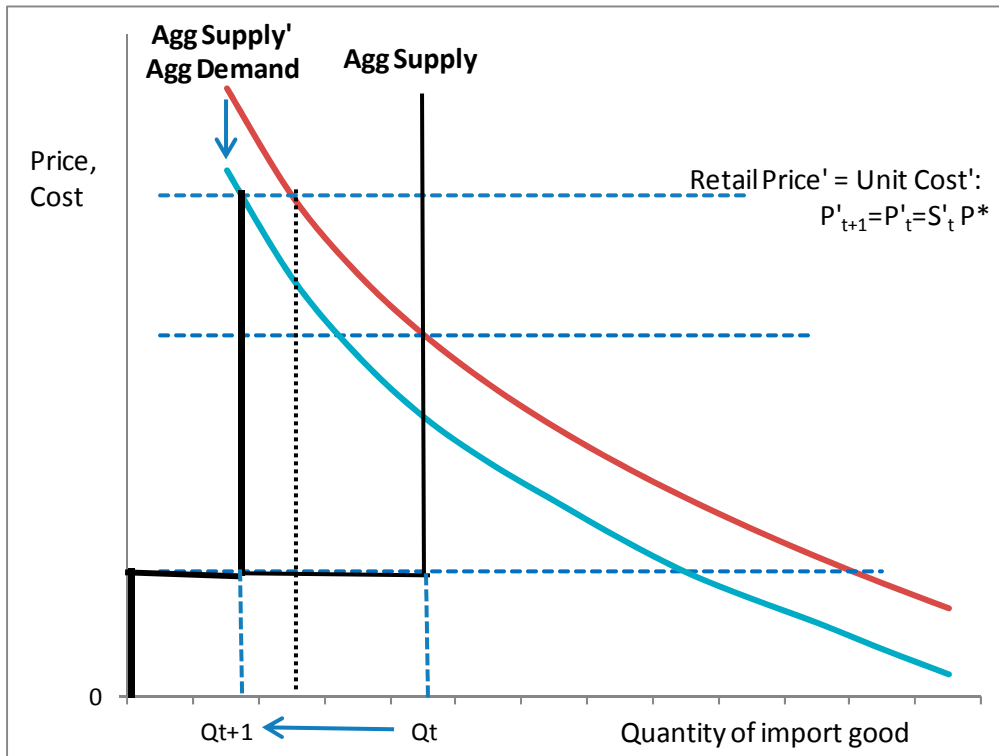
The devaluation increases the domestic value of government revenue from export receipts and reduces the fiscal deficit. There is less need for money creation to finance the fiscal deficit, so the lower money reduces aggregate demand

$P_{t+1} = M_{t+1} / C_{t+1} = (M_t + T_t - s_t X_t) / C_t$

Figure 5. Oil revenue decline

When oil export receipts decline, the supply of foreign exchange is reduced, which in turn reduces the supply of imported goods. The decline in oil receipts also widens the fiscal deficit, which requires higher money creation and higher aggregate demand. So, the aggregate demand curve increases and the aggregates supply curve contracts, both of which lead to higher inflation.

Figure 6. Two goods economy
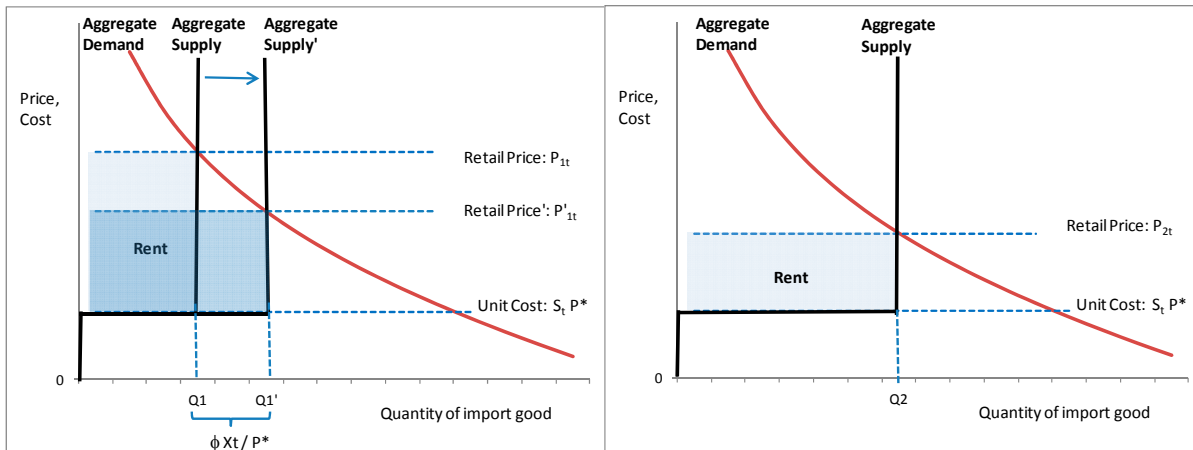
Extending the model to an economy with multiple goods (two in this example), we find that the amount of rents can vary in each of the markets for goods if foreign exchange is allocated in a random (e.g. political, non-economic) way.

In the illustration above, suppose that $(1-\varphi)$ Xt dollars are distributed to import firms in goods markets 1 and 2, which at the international price per unit of P* (let's assume the same world price for both goods) is used to import quantities Q1 and Q2, respectively. Because very little FX is provided to importers of good 1, the price and rent is higher than in the market for good 2.

In addition to the direct allocation of FX to the importing firms, the central bank also sells some of its dollars, φ Xt, to arbitrage firms at the official rate st. In this case, the arbitrage firms would prefer to sell their dollars in the black market to importers of good 1 because the black market price would be higher (bt $= P_1'$) due to the higher rents in goods market 1 compared with goods market 2. The sale of dollars to importers of good 1 would also increase the quantity supplied and reduce the price of good 1 somewhat.

Thus, in a multiple good economy, the black market rate will tend to reflect the price in the most distorted goods markets (i.e., those with the highest rents or highest relative prices P_i/P_i^*).

Figure 7. Price controls and scarcity

Figure 7. Price controls and scarcity
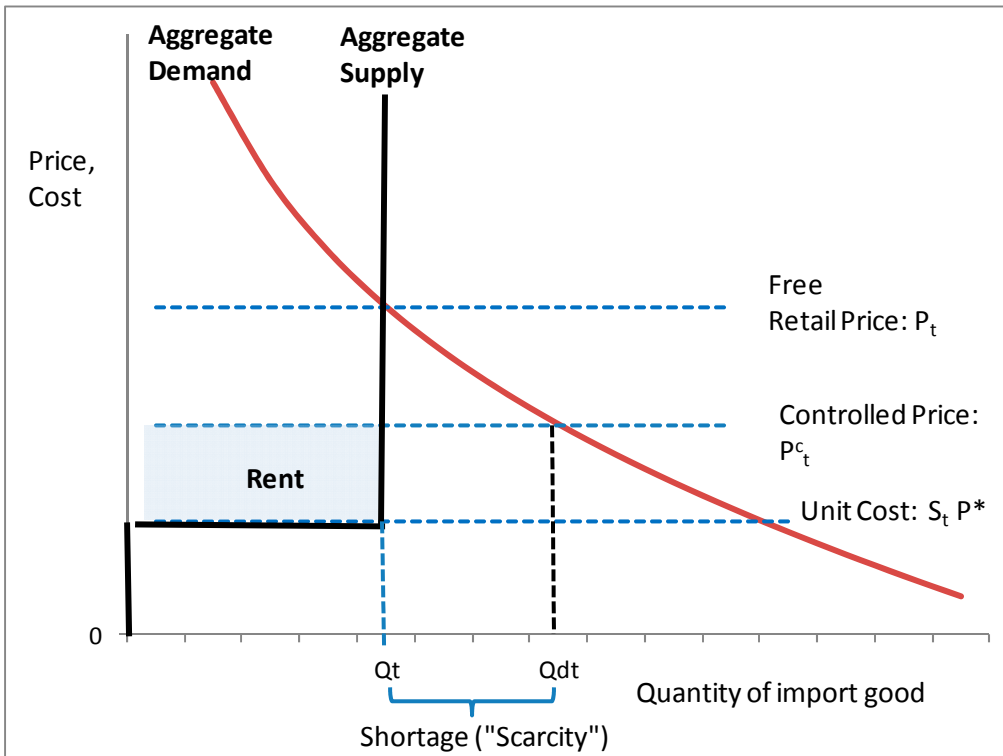
Q_t^D is the amount of the good demanded at the controlled price P_t^C
$\bar{Q}_t$ is the fixed supply of the import good, given the FX allocation from export receipts

Price controls reduce the price of the good below the market-clearing free price, and also reduce the rents received by the importing firm. However, at the lower price, there is excess demand for the good, which generates "scarcity" (or shortages in the parlance of economists).

Figure 8. Simulation of an increase in allocation to black market (Changes from baseline)
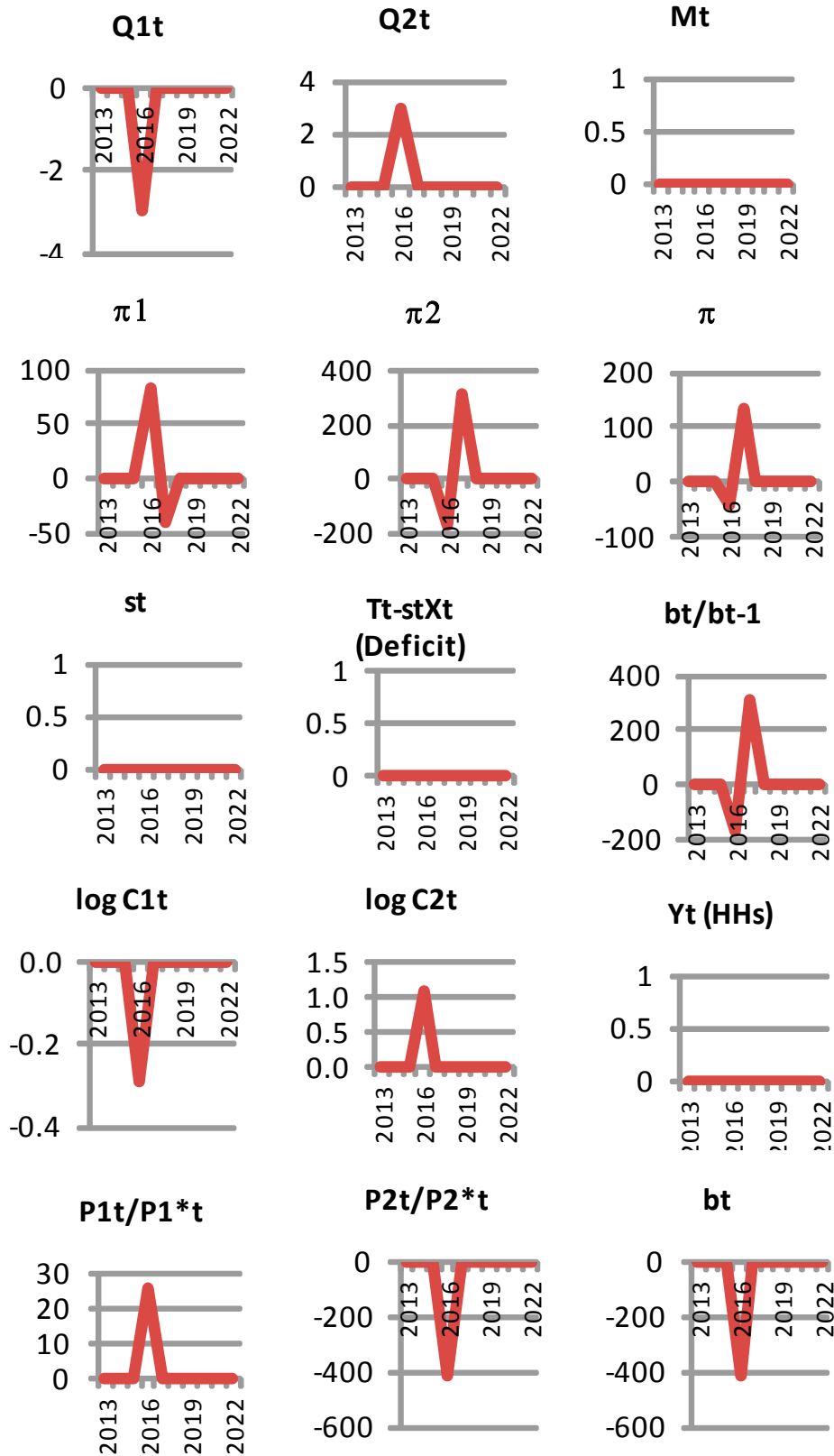
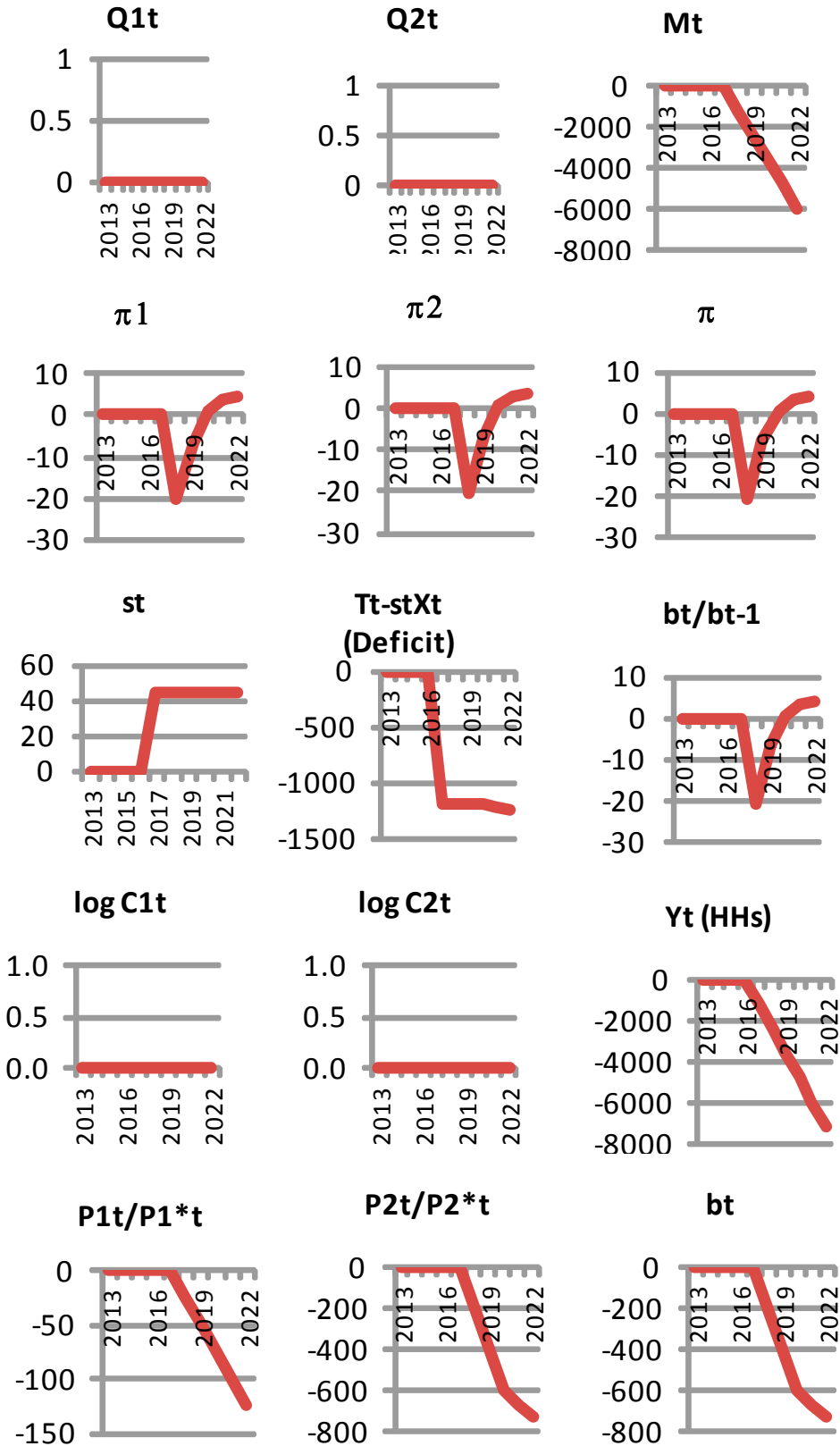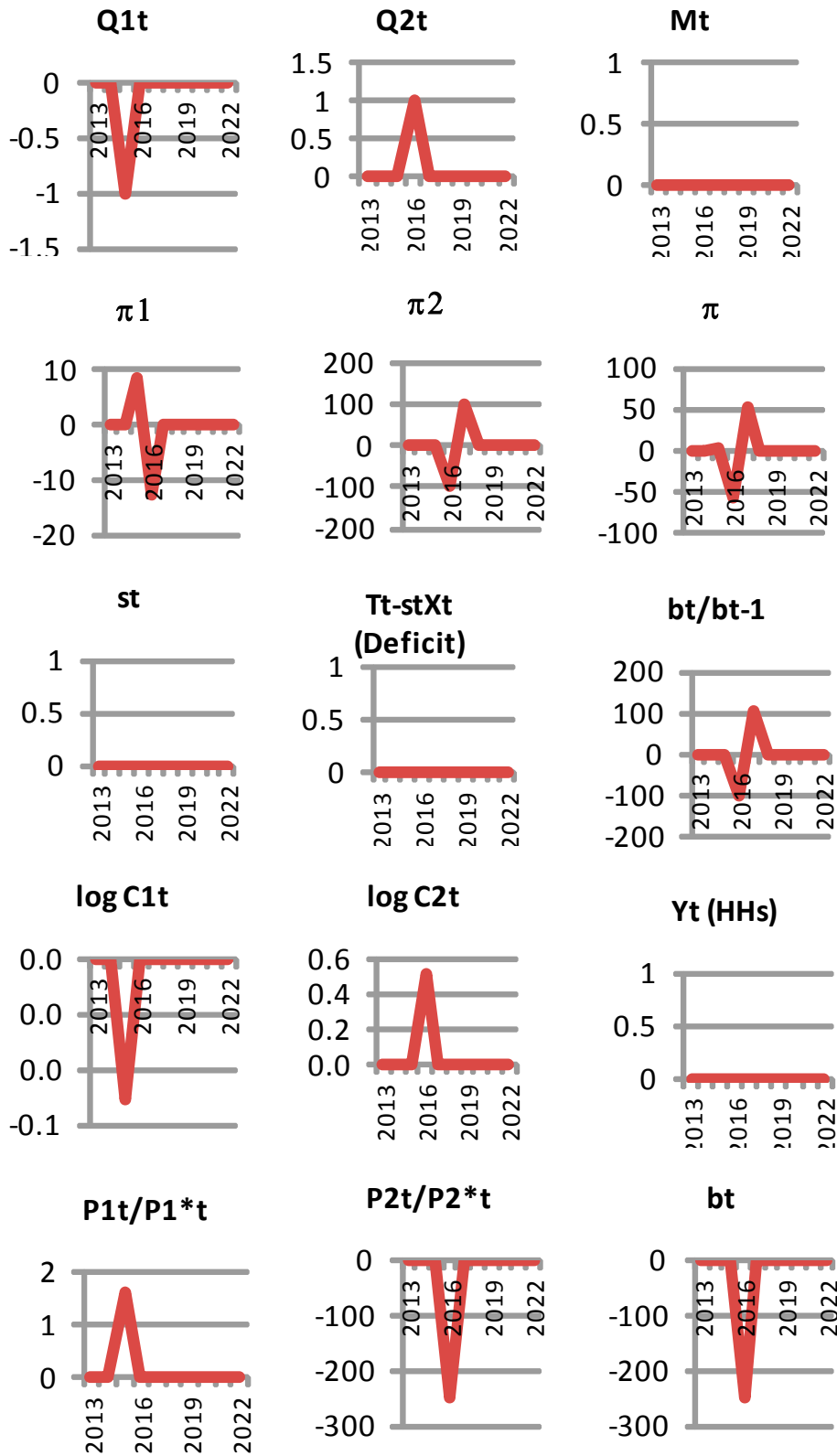**Figure 9. Simulation of devaluation**

(Changes from baseline)

Figure 10. Simulation of overinvoicing good 1 to supply good 2

(Changes from baseline)

Figure 11. Simulation of capital flight used to supply market in the future

(Changes from baseline)