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The Design of Fiscal Reform Packages: Insights from a
Theoretical Endogenous Growth Model

by Andrew Hodge

I N T E R N A T I O N A L M O N E T A R Y F U N D

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**The Design of Fiscal Reform Packages:
Insights from a Theoretical Endogenous Growth Model**

Prepared by Andrew Hodge¹

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Abstract

This paper studies the impact on growth, welfare, and government debt of fiscal reform packages in a theoretical model drawing together three key features of the endogenous growth literature: (i) investment in technology (in the form of human capital) offsets diminishing marginal productivity of private capital, allowing for perpetual growth in output per capita; (ii) changes in investment behavior because of cuts to distortionary tax rates impact long-run growth; and (iii) public capital has a role influencing total factor productivity and growth. A quantitative simulation using reasonable parameter values suggests that modest capital and/or labor income tax cuts and public investment increases have significant positive effects on consumer welfare but small effects on per capita income growth, where fiscal costs are offset by reductions in unproductive government spending. Capital income tax cuts and public investment increases continue to boost welfare when offset by consumption tax rises (rather than spending cuts), although the welfare benefits of modest labor income tax cuts are outweighed by the costs of a compensating consumption tax increase.

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INTRODUCTION

This paper takes a theoretical approach to study the design of fiscal policy reform packages and their potential to boost trend growth and welfare. Specifically, a theoretical endogenous growth model is used to study reform packages involving (i) cuts to distortionary capital and labor income tax rates; and (ii) increases in public investment. Given that these measures can cause fiscal balances to deteriorate, offsetting reductions in unproductive government spending or increases in non-distortionary consumption taxes are included as part of the fiscal reform packages in order to keep public debt sustainable.

The endogenous growth model constructed in this paper combines three key features of the endogenous growth literature: (i) investment in technology (in the form of human capital) offsets diminishing marginal productivity of private capital, allowing for perpetual growth in output per capita; (ii) changes in investment behavior because of cuts to distortionary tax rates impact long-run growth; and (iii) public capital has a role influencing total factor productivity and growth.

First, the theoretical properties of the model are demonstrated mathematically. The relationship between fiscal policy variables (e.g., distortionary tax rates) and the trend (or steady state) economic growth rate is shown. Also, the transition path of the model's macroeconomic variables is described following the implementation of reforms, as the model economy converges to a new steady state with a higher trend growth rate.

In the latter part of the paper, a numerical simulation of the model is presented using reasonable parameter values. Two main types of fiscal reform packages are simulated. The first involves cuts to capital or labor income tax rates (or both), with offsetting reductions to unproductive government expenditure or increases in consumption tax rates, in order to keep public debt sustainable at its pre-reform level. The second type of reform package involves increases to public investment in productive infrastructure, also with the same measures to offset the fiscal cost.

The numerical simulation results indicate that modest cuts to capital income taxation and/or labor income taxation (of 5 percentage points or less) improve welfare significantly, when offset by cuts to unproductive government spending, although the impact on long-run per capita growth rates is small. Raising public investment (e.g., by 1 percentage point of GDP) has similar growth and welfare effects when accompanied by cuts to unproductive spending, under conservative assumptions about the productivity of public capital. Modest capital income tax cuts and public investment increases have smaller welfare benefits when offset by higher consumption tax rates, although the growth effects are the same. It is found that there is a small welfare loss from a fiscal reform package involving modest labor income tax cuts offset by higher consumption taxes.

The key contributions of the paper are (i) to study the impact of tax cuts and public investment increases (both the transition path and new steady state post-reform) in a theoretical growth model integrating endogenous technological progress, productive public capital, several types of distortionary taxation and public debt; (ii) to verify numerically that cuts to capital income taxation can have positive (albeit small) growth effects and non-trivial

welfare effects even when compensated for by increases in consumption taxation, although this is not necessarily the case for labor income tax cuts; and (iii) to verify numerically that increases in public investment can increase per capita growth and produce a non-trivial boost to welfare, even under conservative assumptions about the productivity of public capital.

It should be noted that the model in this paper does not allow for income or wealth inequality, since consumers are homogeneous. The impact on growth and welfare of fiscal reform packages may differ in models with *heterogeneous* agents. Examples of endogenous growth models allowing for heterogeneous agents include Glomm and Ravikumar (1992) and Benabou (2002) which compare the effect of different education systems and redistributive policies.² Heterogeneous agent models of this type could be used to study more complex fiscal reforms, including changes to progressive marginal income tax rates and targeted transfers. However, given the challenges of solving these models numerically, this is left for future research.

This paper is organized as follows. Section II distinguishes between the different types of theoretical models used to study the macroeconomic consequences of fiscal policy, with a particular focus on the difference between neoclassical growth models and endogenous growth models. Section III presents the theoretical model and its key properties, including its unique, locally stable steady state (referred to as a *balanced growth path*) along which all macroeconomic variables grow at a constant rate in the long run. Section IV describes the impact of fiscal reform packages in the model economy, focusing on the transition path of macroeconomic and fiscal variables to the new, post-reform balanced growth path. The results of a numerical simulation are presented as an example and the impact of fiscal reform packages on growth and consumer welfare is quantified.

II. FISCAL REFORM IN THEORETICAL MACROECONOMIC MODELS: NEOCLASSICAL AND ENDOGENOUS GROWTH THEORY

A. Overview

There are several different frameworks which allow the macroeconomic effects of fiscal policy to be studied. The short-run effects of fiscal policy over the business cycle have been studied in Dynamic Stochastic General Equilibrium (DSGE) models, such as the neoclassical models of Uhlig (2010) and Baxter and King (1993), as well as in the New Keynesian models of Christiano, Eichenbaum, and Rebelo (2011) and Cogan, Cwik, Taylor, and Wieland (2009). There are many other examples. In these models, the long-run levels of macroeconomic variables are stationary constants, in a steady state prevailing in the absence of shocks.

² Tamura (1991), Haque and Kim (1995), De Gregorio and Kim (2000), Lucas (2009), and Lucas and Moll (2014) are other examples of endogenous growth models with heterogeneous agents.

The effects of fiscal policy have also been studied in models where macroeconomic variables can grow indefinitely. Theoretical models of macroeconomic growth are of two types: (i) neoclassical or exogenous growth models; and (ii) endogenous growth models.

In neoclassical growth models, the economy's long-run steady state growth rate is exogenous and cannot be affected by fiscal policy changes: the classic example is Solow (1956). Berg et al (2010) uses a neo-classical growth model to study the *short to medium* run effects of scaling up public investment, using different fiscal financing options.

In endogenous growth models, the long-run growth rate is determined by a mechanism within the model, raising the possibility that fiscal policy can impact growth. Simple endogenous growth models have been used to consider the growth effects of various fiscal policies. Barro (1990) and Turnovsky (1997) consider the impact of scaling up public investment in models where public capital is assumed to be the driver of long-run growth. Rebelo (1991), Pecorino (1993), and Stokey and Rebelo (1995) (among others) consider the effect of cuts to distortionary tax rates in models without public capital. Greiner and Semmler (2000) considers the role of government debt in simple endogenous growth models. Caballe and Santos (1993), and Bond, Wang, and Yip (1996) characterize theoretically the transition path of macroeconomic variables to long-run steady state growth in simple endogenous growth models and their results will be utilized in this paper. Devereux and Love (1994) analyses this transition path in a model with distortionary taxation, using numerical solution techniques.

The remainder of this section elaborates on the key differences between neoclassical and endogenous growth models, focusing on the drivers of growth in each case.

B. Neoclassical Growth Theory

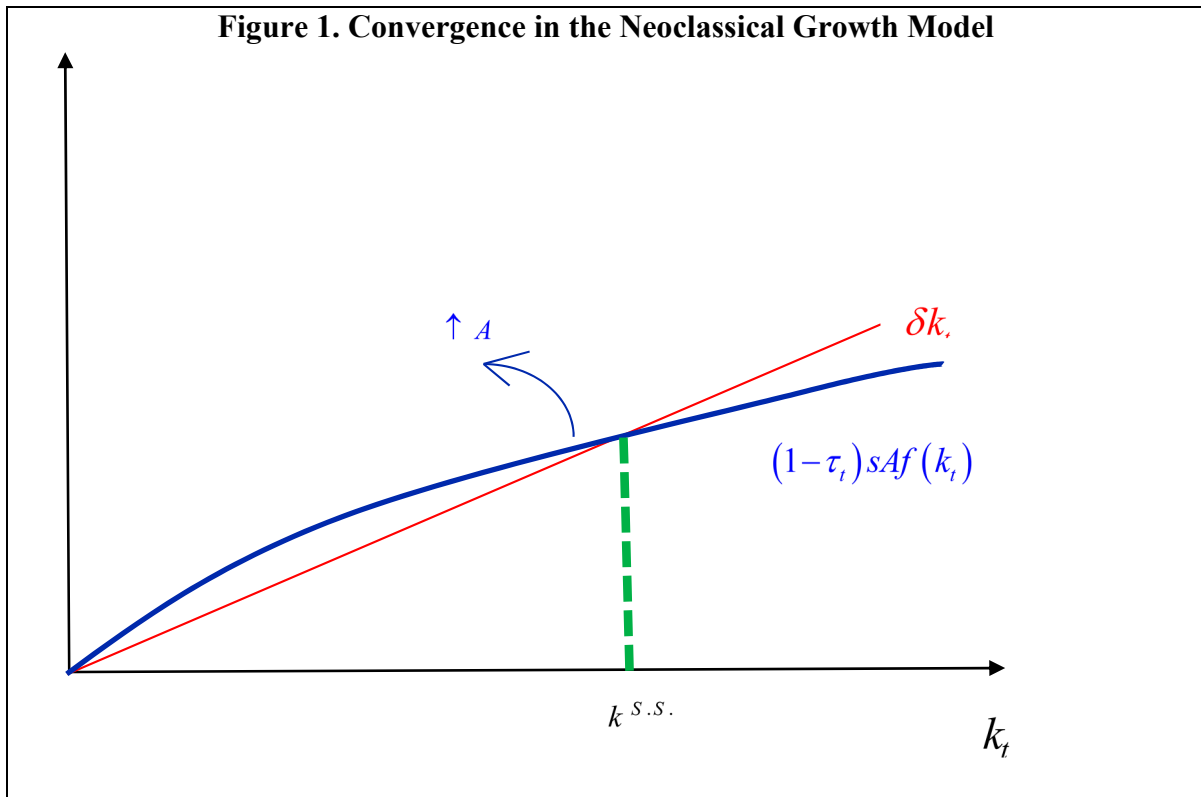
The neoclassical growth model has at its core a standard production function for per capita output $y_t = Af(k_{t-1})$ where A is total factor productivity and $f(k_{t-1})$ is a strictly concave function in per capita physical capital k_{t-1} , such that the production function exhibits diminishing marginal returns to accumulated capital: $\lim_{k \rightarrow \infty} f'(k_{t-1}) = 0$. Assuming that consumers operate the production technology in the economy, the law of motion for per capita capital is described by a difference equation:

$$k_t - k_{t-1} = (1 - \tau_t)s_t f(k_{t-1}) - \delta k_{t-1}$$

where τ_t is a distortionary tax rate, s_t is the fraction of income consumers save each period by investing in physical capital (which Solow (1956) assumes constant), and δ is the rate of depreciation. The concavity of $f(k_{t-1})$ ensures that there is a unique $k^{SS} = k_{t-1}$ such that income per capita remains constant each period at $y^{SS} = Af(k^{SS})$: i.e., there is a unique, stable steady state. Diminishing marginal returns to additional capital investment (relative to

a constant rate of depreciation) ensure that $\frac{\delta \Delta k_t}{\delta k} < 0$ if $k_{t-1} > k^{SS}$ and vice versa (where $\Delta k_t \equiv k_t - k_{t-1}$): please see Figure 1. Given a constant saving rate $s_t = s$, tax rate $\tau_t = \tau$, and depreciation rate $\delta_t = \delta$, per capita income cannot exceed y^{SS} in the long run, *unless* there is growth in total factor productivity A , which in the neoclassical model must be assumed: it is exogenous.

The long-run steady state growth rate of per capita income is invariant to fiscal policy, since the growth rate is exogenous. Beginning in a steady state, *changes* in the tax rate τ_t can affect growth *temporarily* as the economy transitions to a new long-run *level* of income per capita (i.e., because the tax rate affects the *level* of steady state k^{SS}), but cannot affect the long-run growth rate. Neoclassical models can be much richer than the simple Solow (1956) model described here (e.g., Berg et al (2010), which includes a role for productive public capital), but the long-run growth rate of income per capita will still be exogenous so long as the production function exhibits decreasing returns to scale in reproducible inputs, similar to the simple, strictly concave function $f(k_{t-1})$ described here.



C. Endogenous Growth Models

Endogenous growth models have the property that the level of output per capita can grow indefinitely, because of actions taken by agents in the model. Diminishing marginal productivity of accumulated private physical capital is offset by the accumulation of another reproducible input. This allows for non-decreasing returns to scale to reproducible inputs in the final output production function and steady state growth: i.e., a balanced growth path. In Figure 1, this could be shown by rotating the curved line $(1 - \tau_t) sAf(k_t)$ upwards continually over time, so that the levels of capital per capita and output per capita keep rising. The accumulated input offsetting diminishing returns to private capital is usually one or more of (i) ideas or inventions; (ii) public capital; or (iii) human capital. The decision by economic agents to accumulate the reproducible input may be influenced by government policy, such as taxation, in the case of human capital or subsidies in the case of research and development into new ideas.

Type 1: Research and development models

The accumulation of ideas or inventions allows for ongoing technological improvement and growth in output per capita in these models. There can be a detailed microeconomic mechanism determining how ideas are accumulated. Romer (1990) models technological progress as the invention of new varieties of intermediate goods through research and development. The goods are produced under monopolistic competition, which generates sufficient profit to cover the sunk costs of research and development. In this model, government policy such as subsidies to research and development can influence the pace of technological progress and economic growth.³ Other examples of this type of model include Grossman and Helpman (1991), and Aghion and Howitt (1992).

The remaining types of models can be referred to as “investment-based” models.

Type 2: Growth through public capital accumulation

Barro (1990) and Turnovsky (1997) are among a series of papers which assume that accumulated public capital enters the production function and offsets diminishing marginal returns to private capital, allowing for steady-state growth. However, the assumption that public capital alone makes possible long-run growth seems a strong one.

Type 3: Growth through human capital accumulation

Uzawa (1965) and Lucas (1990) present models in which private agents accumulate human capital in a two-sector framework, so that human capital is produced using a separate technology to other goods. Human capital augments the productivity of raw labor, offsetting

³ Jones (1995) demonstrates that Romer (1990) exhibits scale effects in the sense that the growth rate is proportional to the size of the economy, as measured by the stock of ideas or researchers. Jones (1995) modifies the model to rule out these scale effects, but this modification implies that fiscal policy cannot influence long-run growth. Young (1995) also presents a non-scale model of economic growth.

diminishing marginal returns to private physical capital and allowing long-run growth. The two-sector framework allows for a non-trivial transition path of macroeconomic variables to a balanced growth path: see Caballe and Santos (1993), and Bond, Wang, and Yip (1996). Models of this type have been used to study the growth impact of tax cuts, although these models do not usually contain a role for public capital: Rebelo (1991), Ortigueira (1993), Pecorino (1993), de Hek (2006),⁴ and Devereux and Love (1993) are examples of this. Exceptions include Agenor (2008a, 2008b, 2011a, 2011b) which presents two-sector models where human capital accumulation depends entirely on government spending, where there is a simpler, balanced budget tax structure. The papers focus on the mix between different types of spending. Other exceptions are Chen (2007, 2009) which identify the circumstances in which there will be multiple equilibria (indeterminacy) in two-sector models with productive public capital.⁵

III. THE MODEL

The model presented in this paper is in the tradition of investment-based growth models such as Lucas (1990). There are three types of agents: consumers, firms, and the government. Consumers save by accumulating physical capital, as well as holding government bonds. Firms produce final output goods and physical capital goods using a technology that can exhibit constant returns to scale in private reproducible factors (physical and human capital), so that there can be long-run, steady-state growth. Consumers operate a separate technology to produce human capital goods: hence, it is a two-sector model. The level of total factor productivity in the economy is affected by public capital. The government invests in public capital as well as making unproductive expenditures, financed by levying distortionary taxes and issuing government bonds held by consumers. Discrete time periods are denoted by (t) .

A. Consumers

Each consumer is endowed with one unit of raw labor as well as initial endowments of private physical capital K_{-1} , human capital H_{-1} , and government bonds B_{-1} . Human capital augments the consumer's endowment of raw labor in the sense that as the consumer accumulates human capital the consumer's labor becomes more productive.

Consumers earn income by (i) renting a fraction u_t of their human capital augmented labor to firms in return for wage income $u_t w_t H_{t-1}$ (at wage w_t); (ii) renting a fraction v_t of their private capital to firms in return for capital income $v_t r_t K_{t-1}$ (at rate of return r_t); and

⁴ De Hek (2006) does introduce productive public expenditures, although not as the main focus of the paper.

⁵ Glomm and Ravikumar (1997) outlines a two-sector model with productive public capital and then makes simplifying assumptions (shutting down public capital productivity/human capital production) to illustrate the effects of government spending, without a focus on tax issues.

(iii) earning interest income $r_t^b B_{t-1}$ on government bond holdings (at rate r_t^b). The consumer receives income in the form of final output goods. Tax is paid on capital and labor income, as well as on consumption at rates τ_t^K , τ_t^L and τ_t^c , respectively. Consumers use income to finance consumption c_t of final output goods, investment in physical capital I_t^K (accumulated by foregoing consumption), and purchasing government bond holdings. The law of motion for the consumer's stock of private physical capital is given by $K_t = I_t^K + (1 - \delta_K)K_{t-1}$, where δ_K is the rate of depreciation. The budget constraint which consumers must satisfy every period is

$$\begin{aligned} (1 + \tau_t^c)c_t + (K_t - (1 - \delta_K)K_{t-1}) + B_t \\ \leq (1 - \tau_t^K)v_t r_t K_{t-1} + (1 - \tau_t^L)u_t w_t H_{t-1} + (1 + r_t^b)B_{t-1} \end{aligned} \quad (1)$$

Consumers use the remaining fractions $(1 - v_t)$ and $(1 - u_t)$ of physical and human capital to produce additional human capital according to the technology

$$z\left(\frac{G_{t-1}}{K_{t-1}}\right)g((1 - v_t)K_{t-1}, (1 - u_t)H_{t-1}) \quad (2)$$

The assumption of a separate technology for human capital (knowledge) goods seems plausible and is important for allowing a non-trivial transition path for the model's variables following a fiscal reform. The function $g(\cdot)$ is increasing in each of its arguments and satisfies the usual Inada conditions, including $\lim_{x \rightarrow \infty} g'_x(x) = 0$ for each argument x . Total factor productivity in the human capital sector is a function $z(\cdot)$ of the ratio of aggregate public to private capital stocks in the economy, which the individual consumer takes as given. The production function for human capital thus exhibits constant returns to scale to *private* reproducible inputs $(1 - v_t)K_{t-1}$ and $(1 - u_t)H_{t-1}$ at the level of the individual consumer. The law of motion for human capital is given by

$$H_t = z\left(\frac{G_{t-1}}{K_{t-1}}\right)g((1 - v_t)K_{t-1}, (1 - u_t)H_{t-1}) + (1 - \delta_H)H_{t-1}$$

where δ_H is the rate of depreciation. The stocks of public, physical and human capital available for use in time t are determined the previous period.

Formally, given initial conditions $\{K_{-1}, H_{-1}, B_{-1}\}$, an infinite sequence of factor prices and bond prices $\{r_t, w_t, r_t^b\}_{t=0}^{\infty}$, as well as an infinite sequence of fiscal policy variables

$\{G_{t-1}, \tau_t^c, \tau_t^K, \tau_t^L\}_{t=0}^{\infty}$ (where τ_t^c , τ_t^K and τ_t^L are the consumption, capital and labor income tax

rates), the consumer chooses an infinite sequence of consumption, saving and resource allocation across sectors $\{c_t, K_t, H_t, B_t, u_t, v_t\}_{t=0}^{\infty}$ to maximize the discounted present value of utility:

$$\max_{\{c_t, K_t, H_t, B_t, u_t, v_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (3)$$

subject to the consumer's budget constraint each period

$$\begin{aligned} (1 + \tau_t^c)c_t + (K_t - (1 - \delta_K)K_{t-1}) + B_t \\ \leq (1 - \tau_t^K)v_t r_t K_{t-1} + (1 - \tau_t^L)u_t w_t H_{t-1} + (1 + r_t^b)B_{t-1} \end{aligned} \quad (4)$$

and the law of motion for human capital

$$H_t = z \left(\frac{G_{t-1}}{K_{t-1}} \right) g((1 - v_t)K_{t-1}, (1 - u_t)H_{t-1}) + (1 - \delta_H)H_{t-1} \quad (5)$$

where β is the discount factor. The period utility function $u(c_t)$ is strictly concave in c_t and is assumed in this paper to exhibit constant intertemporal elasticity of substitution:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (6)$$

B. Firms

Firms are assumed to be perfectly competitive. Every period, each firm rents physical capital $v_t K_{t-1}$ and human capital $u_t H_{t-1}$ from consumers to produce final output according to the production technology

$$q \left(\frac{G_{t-1}}{K_{t-1}} \right) f(v_t K_{t-1}, u_t H_{t-1}) \quad (7)$$

which exhibits constant returns to scale to private reproducible inputs $v_t K_{t-1}$ and $u_t H_{t-1}$ at the *firm* level. The firm takes as given total factor productivity in the final output sector $q(\cdot)$, which depends on the ratio of the aggregate public to private capital stocks. This assumption implies that public capital is non-excludable but partially *rival* and that it is the level of public capital relative to the size of the private sector economy that matters for total factor productivity. This can be referred to as a ‘‘congestion’’ effect (Eicher and Turnovsky (2000)).

The firm generates revenue by the sale of final output y_t , at a price normalized to 1. Each period the firm chooses the level of produced output y_t and rented inputs $v_t K_{t-1}$ and $u_t H_{t-1}$ to solve the following profit maximization problem:

$$\max_{\{y_t, v_t K_{t-1}, u_t H_{t-1}\}} y_t - r_t v_t K_{t-1} - w_t u_t H_{t-1} \quad (8)$$

subject to the production technology

$$y_t \leq q\left(\frac{G_{t-1}}{K_{t-1}}\right) f(v_t K_{t-1}, u_t H_{t-1}) \quad (9)$$

C. The Government

The government levies distortionary taxes on consumption τ_t^c , capital income τ_t^K and labor income τ_t^L . Unless policy is changed, these tax rates are assumed to be constant over time. The government issues one period, one unit bonds, which pay a market interest rate r_t^b the next period. Revenue and borrowing are denominated in final output goods and finance investment in productive public capital I_t^G and unproductive spending g_t .

The government is assumed to invest so as to target a particular ratio of public to private capital stocks G_{t-1}/K_{t-1} , subject to the law of motion $G_t = I_t^G + (1 - \delta_G)G_{t-1}$. Public capital is accumulated by saving final output goods: i.e., it shares the same production technology as final output and private physical capital goods. A path for unproductive government spending is set that is consistent with public debt sustainability. This is explained further in Section IV. The budget constraint for the government each period is

$$\begin{aligned} & (G_t - (1 - \delta_G)G_{t-1}) + g_t + (1 + r_t^b)B_{t-1} \\ & \leq \tau_t^c c_t + \tau_t^K r_t v_t K_{t-1} + \tau_t^L w_t u_t H_{t-1} + B_t \end{aligned} \quad (10)$$

It will be useful to define the primary balance pb_t as the difference between non-interest revenue and non-interest expenditure:

$$pb_t = \tau_t^c c_t + \tau_t^K r_t v_t K_{t-1} + \tau_t^L w_t u_t H_{t-1} - (G_t - (1 - \delta_G)G_{t-1}) - g_t \quad (11)$$

D. Competitive Equilibrium

The constrained optimization problems solved by firms (8) and consumers (3) imply optimizing behavior that must be followed in competitive equilibrium.

Firms' optimizing behavior

The solution of firms' optimization problem (8) implies that firms hire private capital $v_t K_{t-1}$ and human capital augmented labor $u_t H_{t-1}$ —as inputs into final output production—up to the point where the marginal product of each input equals its market price:

$$r_t = q\left(\frac{G_{t-1}}{K_{t-1}}\right) f'_{vK}(v_t K_{t-1}, u_t H_{t-1}) \quad (12)$$

$$w_t = q\left(\frac{G_{t-1}}{K_{t-1}}\right) f'_{uH}(v_t K_{t-1}, u_t H_{t-1}) \quad (13)$$

Consumers' optimizing behavior

The constrained optimization problem solved by consumers (3) requires three main types of choices: (i) a decision to consume or save current income; (ii) a distribution of new savings across the assets of physical capital K_t and government bonds B_t ; as well as (iii) allocation of existing physical and human capital stocks between human capital production and the markets in which they are rented out to perfectly competitive firms for use in final output production (i.e., a choice of fractions v_t and u_t).

Consumption/saving choice (and allocation of savings): This choice is made according to equations (14)-(16). These equations imply that consumers save by investing in assets up to the point where the marginal return (in terms of utility) of investing an additional unit of income equals the marginal cost of the foregone consumption (also in terms of utility). The first equation (14) is the Euler equation for private physical capital,

$$\frac{u'(c_t)}{(1 + \tau_t^c)} = \beta \frac{u'(c_{t+1})}{(1 + \tau_{t+1}^c)} \left[(1 - \delta_K) + (1 - \tau_{t+1}^K) r_{t+1} \right] \quad (14)$$

while the second (15) is a Euler equation expressed in terms of the human capital good

$$\frac{u'(c_t)}{(1 + \tau_t^c)} p_t = \beta \frac{u'(c_{t+1})}{(1 + \tau_{t+1}^c)} p_{t+1} \left[z\left(\frac{G_t}{K_t}\right) g'_{(1-u)H}((1 - v_{t+1}) K_t, (1 - u_{t+1}) H_t) + (1 - \delta_H) \right] \quad (15)$$

where p_t is the relative price of human capital goods in terms of final output goods; and

$$\frac{u'(c_t)}{(1 + \tau_t^c)} = \beta \frac{u'(c_{t+1})}{(1 + \tau_{t+1}^c)} \left[1 + r_{t+1}^b \right] \quad (16)$$

the third equation (16) is the Euler equation for government bond holdings. In equilibrium, the marginal return on investment in each asset (physical capital K_t and government bonds B_t) must be the same, since the left hand sides of (14), (15), and (16) are equal. This prevents arbitrage across assets.

The tax on physical capital income τ^K distorts the consumption saving decision, as seen by the “wedge” it imposes in Euler equation (14). The consumption tax τ^c does not distort the consumption/saving decision if it is levied at a constant rate over time, in which case it cancels out in equations (14) and (16). The labor income tax τ^L (levied on human capital augmented labor income) does not distort the consumption/saving decision because deferred consumption cannot be saved directly by investing in human capital. Investment in human capital occurs by allocating a fraction of existing physical and human capital to production of new human capital, as discussed below.

Allocating resources across sectors: Allocating existing physical and human capital stocks between human capital accumulation and final output production (by choosing fractions v_t and u_t) is effectively a choice of how to allocate resources across sectors. The solution to the consumers’ optimization problem (3) implies that the consumer allocates resources so as to equate the marginal product of physical and human capital across sectors (subject to taxation distortions):

$$(1 - \tau_t^K) r_t = p_t z \left(\frac{G_{t-1}}{K_{t-1}} \right) g'_{(1-v)K} ((1 - v_t) K_{t-1}, (1 - u_t) H_{t-1}) \quad (17)$$

$$(1 - \tau_t^L) w_t = p_t z \left(\frac{G_{t-1}}{K_{t-1}} \right) g'_{(1-u)H} ((1 - v_t) K_{t-1}, (1 - u_t) H_{t-1}) \quad (18)$$

noting that in equilibrium the rental rate r_t and wage rate w_t will equal the marginal products of private physical and human capital, according to the firm optimality conditions (12) and (13). Capital and labor income tax rates τ_t^K and τ_t^L impose a wedge between the marginal products in each sector.

Combining the two equations (17) and (18) implies that the marginal rate of technical substitution of physical for human capital must be the same in each sector (subject to taxation distortions). This eliminates the possibility that resources could be used more productively if they were deployed in a different sector.

$$\frac{(1 - \tau_t^K) f'_{vK}(v_t K_{t-1}, u_t H_{t-1})}{(1 - \tau_t^L) f'_{uH}(v_t K_{t-1}, u_t H_{t-1})} = \frac{g'_{(1-v)K}((1 - v_t) K_{t-1}, (1 - u_t) H_{t-1})}{g'_{(1-u)H}((1 - v_t) K_{t-1}, (1 - u_t) H_{t-1})} \quad (19)$$

The combination of capital income tax τ^K and labor income tax τ^L distorts the inter-sectoral allocation decision because tax is paid when resources are used in the final output sector (i.e., tax is paid on income earned in the final output sector), but no tax is paid on resources used in human capital production, since this sector is untaxed in this model.⁶ Labor tax and consumption tax τ^c are *not* equivalent in this model, because of this independent role played by labor income tax τ^L in distorting the inter-sectoral resource allocation.⁷ Consumption tax is non-distortionary in the model because there is no leisure/labor supply choice, which would be distorted by a wedge consisting of consumption and labor income tax.⁸

The equations above describe optimal consumer and firm behavior and form part of the formal definition of competitive equilibrium. Appendix I provides the full derivation of the consumers' and firms' problems.

Definition 1: A *Competitive Equilibrium* is a sequence of endogenous variables

$\{c_t, K_t, H_t, B_t, v_t, u_t\}_{t=0}^{\infty}$, prices $\{r_t, w_t, r_t^b, p_t\}_{t=0}^{\infty}$, fiscal policy variables $\{\tau_t^c, \tau_t^K, \tau_t^L, G_t, g_t\}_{t=0}^{\infty}$

and initial conditions $\{G_{-1}, K_{-1}, H_{-1}, B_{-1}\}$ such that demand equals supply in markets for final output, physical capital, human capital and government bonds every period and the consumer and firm optimality conditions (14), (15), (16), (17), (18), (12) and (13) are satisfied every period. The period budget constraints of the consumer (4) and the government (10) must also hold each period. Finally, the transversality conditions for private physical capital (20), human capital (21), public capital (22), and government bonds outstanding (23) must be satisfied.

The transversality conditions for private physical capital, human capital, and public capital are required for competitive equilibrium since it cannot be optimal behavior by private agents to leave resources un-used in the limit:

$$\lim_{t \rightarrow \infty} \beta^t \frac{u'(c_t)}{(1 + \tau_t^c)} K_t = 0 \quad (20)$$

$$\lim_{t \rightarrow \infty} \beta^t p_t H_t = 0 \quad (21)$$

⁶ Human capital accumulation can have reduced tax liability in reality (e.g., tax deductions for tuition expenses) although is likely still subject to some taxation. Pecorino (1993) describes the human capital sector as being partially taxed in reality.

⁷ This issue is discussed by Milesi-Ferretti and Roubini (1998).

⁸ In this paper, it is assumed that consumers devote all their human capital augmented labor supply to either final output production or human capital accumulation. This simplifying assumption allows for some of the theoretical properties of the economy's transition path (following a fiscal reform) to be analyzed algebraically, rather than only by recourse to numerical solution methods. Also, it helps avoid multiple equilibria: Benhabib and Perli (1994) demonstrates how multiple equilibria can arise relatively easily in endogenous growth models with endogenous leisure/labor supply choices.

$$\lim_{t \rightarrow \infty} \beta^t \frac{u'(c_t)}{(1 + \tau_t^c)} G_t = 0 \quad (22)$$

The transversality condition for government bond holdings rules out Ponzi schemes:

$$\lim_{t \rightarrow \infty} \beta^t \frac{u'(c_t)}{(1 + \tau_t^c)} B_t = 0 \quad (23)$$

E. Steady State Growth: The Balanced Growth Path

Definition 2: A Competitive Equilibrium admits a **Balanced Growth Path (BGP)** if after some $t^* \geq 0$, consumption, private physical capital, human capital, government bond holdings $\{c_t, K_t, H_t, B_t\}$, as well as public capital and unproductive government spending $\{G_t, g_t\}$ all grow at a constant rate for every $t \geq t^*$, given initial conditions $\{G_{-1}, K_{-1}, H_{-1}, B_{-1}\}$. All other variables must be constant. This can also be referred to as **long-run or steady-state growth**.

Normalizing variables

Analysis of a BGP is challenging because the growing variables mentioned in Definition 2 are non-stationary: they grow forever on a BGP. Tractable analysis is made possible by normalizing all of these variables by the value of the human capital stock H_{t-1} available at the beginning of period t . These normalized variables are denoted with a superscript, for example $\tilde{c}_t = c_t / H_{t-1}$.

A unique BGP

Proposition 2: The model has a **unique BGP**.

Proof: Please see Appendix II.

F. The Mechanism Driving Long Run Growth

In order to gain intuition about the drivers of long-run growth, it is useful to obtain a closed form solution for the BGP growth rate, by assuming the following specific functional forms for the production function of final output goods:

$$q\left(\frac{G_{t-1}}{K_{t-1}}\right) f(v_t K_{t-1}, u_t H_{t-1}) = A \left(\frac{G_{t-1}}{K_{t-1}}\right)^\gamma (v_t K_{t-1})^\alpha (u_t H_{t-1})^{1-\alpha} \quad (24)$$

and human capital:

$$z\left(\frac{G_{t-1}}{K_{t-1}}\right)g((1-v_t)K_{t-1},(1-u_t)H_{t-1})=C\left(\frac{G_{t-1}}{K_{t-1}}\right)^\varepsilon\left((1-v_t)K_{t-1}\right)^\phi\left((1-u_t)H_{t-1}\right)^{1-\phi} \quad (25)$$

with $\gamma, \varepsilon \geq 0$ and $0 \leq \alpha, \phi \leq 1$, where A and C are constants.

Proposition 3: the BGP Growth Rate \tilde{H} is given by:

$$\tilde{H} = \beta^{\frac{1}{\sigma}} \left[(1 - \delta_K) + \alpha A \left(\frac{C(1-\phi)}{\alpha A} \right)^{\frac{1-\alpha}{1-\alpha+\phi}} \left(\frac{G}{K} \right)^{\frac{\varepsilon(1-\alpha)+\phi\gamma}{1-\alpha+\phi}} (1 - \tau^K)^{\frac{\phi\alpha}{1-\alpha+\phi}} (1 - \tau^L)^{\frac{\phi(1-\alpha)}{1-\alpha+\phi}} \left(\frac{(1-\alpha)\phi}{\alpha(1-\phi)} \right)^{\frac{\phi(1-\alpha)}{1-\alpha+\phi}} \right]^{\frac{1}{\sigma}} \quad (26)$$

where Φ is a function of parameters associated with the utility function and production function.

Proof: Please see Appendix III.

Steady state growth in income per capita is possible because the accumulation of human capital $u_t H_{t-1}$ offsets diminishing marginal returns to physical capital $v_t K_{t-1}$. Technically, the final output and human capital production functions (24) and (25) exhibit constant returns to scale to reproducible factors (physical and human capital) at the *aggregate level*, assuming that the government sets public investment so as to keep the aggregate ratio of public to private physical capital $\left(\frac{G}{K}\right)$ constant.⁹ The BGP growth rate (26) depends on this ratio of public to private physical capital. In this model, the more abundant is public capital relative to the private capital stock (which proxies for the private sector economy), the higher is total factor productivity in both final output and human capital sectors. This allows for faster growth.

The BGP growth rate is also decreasing in the capital τ^K and labor income τ^L tax rates. As discussed above in Section III.D, the tax on human capital augmented labor income τ^L distorts the decision to allocate resources to human capital production, which can affect growth. The tax on capital income τ^K also distorts this inter-sectoral allocation decision, as well as the decision to consume or save each period, which impacts upon growth. Reducing

⁹ Even if population growth is explicitly assumed, the assumption of a constant *ratio* of aggregate public to private capital prevents the growth rate of the economy rising with population: it rules out scale effects. Glomm and Ravikumar (1994, 1997) contain a discussion of this issue.

these distortions allows for faster growth, since the accumulation of physical and human capital together is the key mechanism driving growth.

As noted in Section III.D, consumption tax τ_t^c is non-distortionary in this model if levied at a constant rate over time, since consumers are assumed to spend all their time working in the final output sector or accumulating human capital: there is no leisure/labor supply choice.¹⁰

G. Stability of the Balanced Growth Path

The BGP is locally stable: there is a single equilibrium path along which normalized macroeconomic variables approach the BGP, given initial conditions $\left\{ \frac{G_{-1}}{K_{-1}}, \widetilde{K}_{-1}, \widetilde{B}_{-1} \right\}$ sufficiently close to the BGP, for the model solved with specific functional forms (6), (24), and (25).

Formally, this is determined by studying the system of equations in the vector of stationary variables $x_t \equiv \left[\widetilde{c}_t \quad \widetilde{g}_t \quad u_t \quad v_t \quad p_t \quad r_t^b \quad \widetilde{B}_{t-1} \quad \widetilde{G}_{t-1} \quad \widetilde{K}_{t-1} \right]^T$, $Ax_{t+1} = Dx_t$, where A and D are square matrices of coefficient parameters. A Jordan decomposition of $D^{-1}A$ identifies the eigenvalues of $D^{-1}A$ ¹¹. The number of eigenvalues less than 1 in absolute value must match the number of non-exogenous, forward looking variables: $\widetilde{c}_t, u_t, v_t, p_t, r_t^b$, for local stability.

IV. FISCAL REFORM PACKAGES: THE TRANSITION PATH TO A NEW BGP

This section considers two main types of fiscal reform packages: (i) a package involving a permanent reduction in the distortionary private capital income tax rate τ_t^K or labor income tax rate τ_t^L ; and (ii) a package involving a permanent increase in public investment, such that the ratio $\frac{G_{t-1}}{K_{t-1}}$ will be at a higher constant level on a post-reform BGP.

Each package will also entail either (a) a reduction in unproductive government spending g_t ; or (b) an increase in the consumption tax rate τ_t^c , in order to keep government debt (as a share of final output) sustainable at its level on the pre-reform BGP.

¹⁰ The consumption tax would not likely have a significant effect on growth, even if a leisure/labor supply choice was allowed. Consumption tax receipts are not rebated to consumers in the form of transfers, so that the income and substitution effects of a consumption tax change on the leisure/labor supply decision are likely to be largely offsetting. This would dampen the effect of consumption tax changes on labor supply and growth. Please see Milesi-Ferretti and Roubini (1998) for further discussion.

¹¹ Alternative techniques exist to handle the situation where D is not invertible.

The analysis of a fiscal reform package begins with the economy on a pre-reform BGP, consistent with a particular fiscal policy comprising constant tax rates $\{\tau^C, \tau^K, \tau^L\}$, constant normalized unproductive spending \tilde{g} and public investment set to target a constant ratio of public to private physical capital G/K .

A fiscal reform package then takes the form of a permanent shock (e.g., a change in tax rates), causing the model economy to transition along a unique transition path to a new, unique BGP consistent with the new fiscal policy.

The first step is to determine the properties of the pre-reform BGP. The pre-reform BGP growth rate is given by (26), when the utility function and production functions take the specific functional forms assumed in (6), (24), and (25). The values of normalized variables in the pre-reform BGP can be determined by solving the system of equations characterizing a competitive equilibrium (as in Definition 1) and imposing a steady state (e.g. that $\tilde{c}_t = \tilde{c}_{t+1}$ for all t).

The post-reform BGP will eventually be reached following the permanent shock which is the fiscal reform. The properties of the post-reform BGP (the growth rate and values of normalized variables) can be determined using a similar approach.

The transition path of macroeconomic and fiscal variables towards the post-reform BGP can now be examined. The key driver of the transition is a change in the allocation of private physical capital and human capital resources across the final output and human capital sectors. Specifically, consumers change the fraction v_t of physical capital and u_t of human capital rented to firms for final output production, as opposed to being used to generate human capital. It is helpful to note that physical and human capital must move together across sectors in this model.

Proposition 4: *Using the specific functional forms (6), (24), and (25), it is possible to write the share of human capital allocated to final goods production u_t as an increasing function of the share of private physical capital allocated to this purpose v_t :*

$$u(v_t); \quad u'(v_t) > 0 \quad (27)$$

Proof: *Please see Appendix IV.*

This implies that private physical and human capital resources move in the *same* direction across sectors. The *direction* in which resources move across sectors is related to movements in the ratio of private physical capital to human capital $\widetilde{K}_{t-1} = K_{t-1}/H_{t-1}$. This is demonstrated by deriving an expression for u_t , which is done in **Proposition 5** of Appendix IV.

This expression suggests an *inverse relationship* between the ratio of private physical capital to human capital \widetilde{K} and the fraction of resources devoted to the final output sector (i.e., u and v). The intuition for this result is presented later in this section, when each fiscal reform package is discussed.

It is then important to understand how the ratio of private physical capital to human capital \widetilde{K} changes over the transition path following reform. The following proposition indicates how the pre-reform and post-reform ratios of private physical to human capital \widetilde{K}_{t-1} differ.

Proposition 6: *Assuming the specific functional forms (6), (24), and (25), that $\phi > \alpha$ and that $k^h > k^y$ on a BGP (the ratio of physical to human capital must be greater in the human capital sector than in the final output sector, as will be the case in the numerical simulation of Section IV), then:*

1. *An increase in the physical capital income tax rate **lowers** the BGP physical to human*

capital ratio $\frac{\partial \widetilde{K}}{\partial \tau^K} < 0$

2. *An increase in the labor income tax rate has an **ambiguous effect** on the BGP physical to human capital ratio (the effect depends on the choice of parameters, but the effect*

is negative $\frac{\partial \widetilde{K}}{\partial \tau^L} < 0$ *under the arguably reasonable choice of parameters used for the numerical simulation in Section IV)*

3. *An increase in the ratio of public to private physical capital $\widetilde{G} = G/K$ **raises** the BGP*

physical to human capital ratio $\frac{\partial \widetilde{K}}{\partial \widetilde{G}} > 0$, *provided public capital is equally productive in each sector $\gamma = \varepsilon$ and that ε is sufficiently small (these are sufficient but not necessary conditions).*

Proof: *Please see Appendix IV.*

In summary, a *cut* to the capital tax rate τ_t^K (or labor income tax rate τ_t^L under a reasonable calibration) raises the BGP growth rate (according to (26)) and the BGP ratio of physical to human capital \widetilde{K} , all else equal. Initially, resources devoted to final output production $u(v_t)$ are relatively high, but fall as the economy approaches the new BGP. The intuition for this result is discussed in the next sub-section, when numerical simulations of each reform package are presented.

An increase in the ratio of public to private capital $\widetilde{G} = G/K$ targeted by the government raises the BGP growth rate (according to (26)) and the BGP ratio of physical to human

capital \tilde{K} , all else equal (under the conditions in Proposition 6). The increase in the public to private capital ratio boosts productivity in both sectors, leading to increased incentives for investment and faster growth.

A. A Numerical Simulation

In this sub-section, the impact of fiscal policy reform packages is demonstrated numerically, by solving for the numerical values of the economy's variables along the transition path to a post-reform BGP. This will help reveal the intuition behind the results above. The numerical solution is obtained using the specific functional forms (6), (24), and (25), and by assigning values to the parameters of the model, as discussed below. The impact of fiscal reform packages on consumer welfare is also quantified. The pre-reform BGP is the benchmark against which the reforms are assessed.

Parameter values

The chosen parameter values used to solve the model numerically are shown in Table 1. Most values are relatively standard in the growth theory literature, but two are more controversial. First, the parameters γ and ε are respectively the productivity of public capital G_{t-1} in the production functions for final output (24) and human capital goods (25). Both are set to 0.05, which is in the middle of the range of estimates for public capital productivity surveyed by Bom and Ligthart (2014). Second, the parameter attached to private physical capital ϕ in the human capital goods production function (25) is set to 0.55. It is difficult to map empirical estimates about the value of physical capital in human capital production into a chosen value for a parameter. Given this difficulty, a value close to $\frac{1}{2}$ is chosen for the numerical exercise. Different values for this parameter should not change the results qualitatively, provided that $\phi > \alpha$, the parameter attached to private physical capital in the final output production function (24), which is set to 0.36, as is common in the literature. Please see Appendix VI for a discussion of the robustness of some of the numerical simulation results to changes in assumed parameter values.

Welfare metric

Consumer welfare following a fiscal reform is quantified by approximating:

$$W = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad (28)$$

where initial conditions are given by the pre-reform BGP. The welfare effect of the fiscal reform is then expressed as the constant percentage of consumption each period required to make the consumer indifferent between the reform and continuing along the pre-reform BGP. Further details are given in Appendix V.

Table 1. Parameter Values

Parameter	Value	Description	Parameter	Value	Description
β	0.98	Discount factor	δ_H	0.1	Depreciation rate: human capital
σ	2	Elasticity of intertemporal substitution (inverse)	δ_K	0.1	Depreciation rate: private physical capital
γ	0.05	Elasticity of final output to public capital	δ_G	0.1	Depreciation rate: public capital
ϵ	0.05	Elasticity of human capital to public capital			
α	0.36	Capital share in final output	mk	4.63%	Public investment: share of final output
ϕ	0.55	Capital share in schooling			
A	0.36	Scale factor	mg	13.6%	Unproductive government spending: share of final output
			τ^L	17.5%	Labor income tax rate
C	0.36	Scale factor	τ^K	17.5%	Capital tax rate
			τ^C	7%	Consumption tax rate

Fiscal reform packages

Eight reform packages are simulated. Each package has a fiscal cost, either because revenue is lost when tax rates are cut or because expenditure rises when public investment increases. The fiscal cost is compensated for by cutting unproductive expenditure in Reform Packages 1-4. In Reform Packages 5-8, the fiscal cost is offset by increasing consumption taxation.

Reform Packages 1-4: Fiscal cost compensated by cutting unproductive government spending

1. A 5 percentage point cut in the capital income tax rate τ_t^K ;
2. A cut to the labor income tax rate τ_t^L that generates the same discounted present value of total government revenue from the point of reform as the capital income tax cut;
3. Cuts to both the capital income tax rate τ_t^K and the labor income tax rate τ_t^L that are of the same magnitude and that generate the same discounted present value of total government revenue from the point of reform as the 5 percentage point capital tax cut; as well as

4. An increase in public investment by 1 percentage point of final output, sufficient to increase the BGP ratio of public to private capital $\frac{G}{K}$ by around 10 percent.

For each of these reforms, it is necessary to make an assumption about unproductive spending to ensure that public debt remains sustainable on a new BGP. Expressing the government budget constraint in terms of normalized variables and imposing steady state

(e.g. $\widetilde{B}_{t-1} \equiv B_{t-1}/H_{t-1}$, $\widetilde{B}_t = \widetilde{B}_{t-1} = \widetilde{B}$) implies:

$$(\widetilde{r}^b - \widetilde{H})\widetilde{B} = \widetilde{p}\widetilde{b} \quad (29)$$

Where $\widetilde{r}^b \equiv 1 + r^b$ is the gross rate of return on government bonds, \widetilde{H} is the BGP growth rate and $\widetilde{p}\widetilde{b}$ is the normalized primary balance: for further details please see Appendix V. A fiscal reform that causes a permanent reduction in the primary balance (as a share of final output) and an increase in the interest rate—growth differential $(\widetilde{r}^b - \widetilde{H})$, implies that the pre-reform ratio of government debt to final output cannot be sustainable on a new BGP, unless there is fiscal adjustment to prevent the decline in the primary balance.

For reforms 1-4, unproductive government spending \mathcal{G}_t is adjusted along the transition path so that the ratio of government debt to final output remains the same in the post-reform BGP as in the pre-reform BGP.

The properties of the benchmark, pre-reform BGP are given in Table 2, along with information about the effects of each reform, including (i) the post-reform BGP growth rate; and (ii) the welfare impact of the reform.

Reform Package 1: capital income tax cut and government spending cut

A cut to the capital income tax rate τ_t^K reduces distortions, raising the post-tax rate of return to capital investment. This encourages higher investment in physical capital by consumers and the growth rate of physical capital rises above its rate in the pre-reform steady state (Figure 2(a)). Under this calibration, the capital income tax cut provides an initial boost to consumer income, actually allowing for faster consumption growth as well as higher investment. Faster growth in physical capital requires resources to shift to the final output sector, where private capital goods are produced. The share of human capital u_t (and also physical capital v_t , as per equation (27)) in the final output sector is relatively high early in the transition (Figure 2(c)). The growth rate of human capital slows (Figure 2(b)).

The ratio of private capital to human capital $\widetilde{K} = K/H$ rises to be higher in the post-reform BGP compared with the pre-reform BGP, because of the relatively faster growth in physical capital (Figure 2(d)). As the ratio of physical to human capital \widetilde{K} approaches its higher level on the new BGP, resources shift back to the human capital sector, according to the inverse

relationship between u_t and \bar{K} , derived in Proposition 5 of Appendix

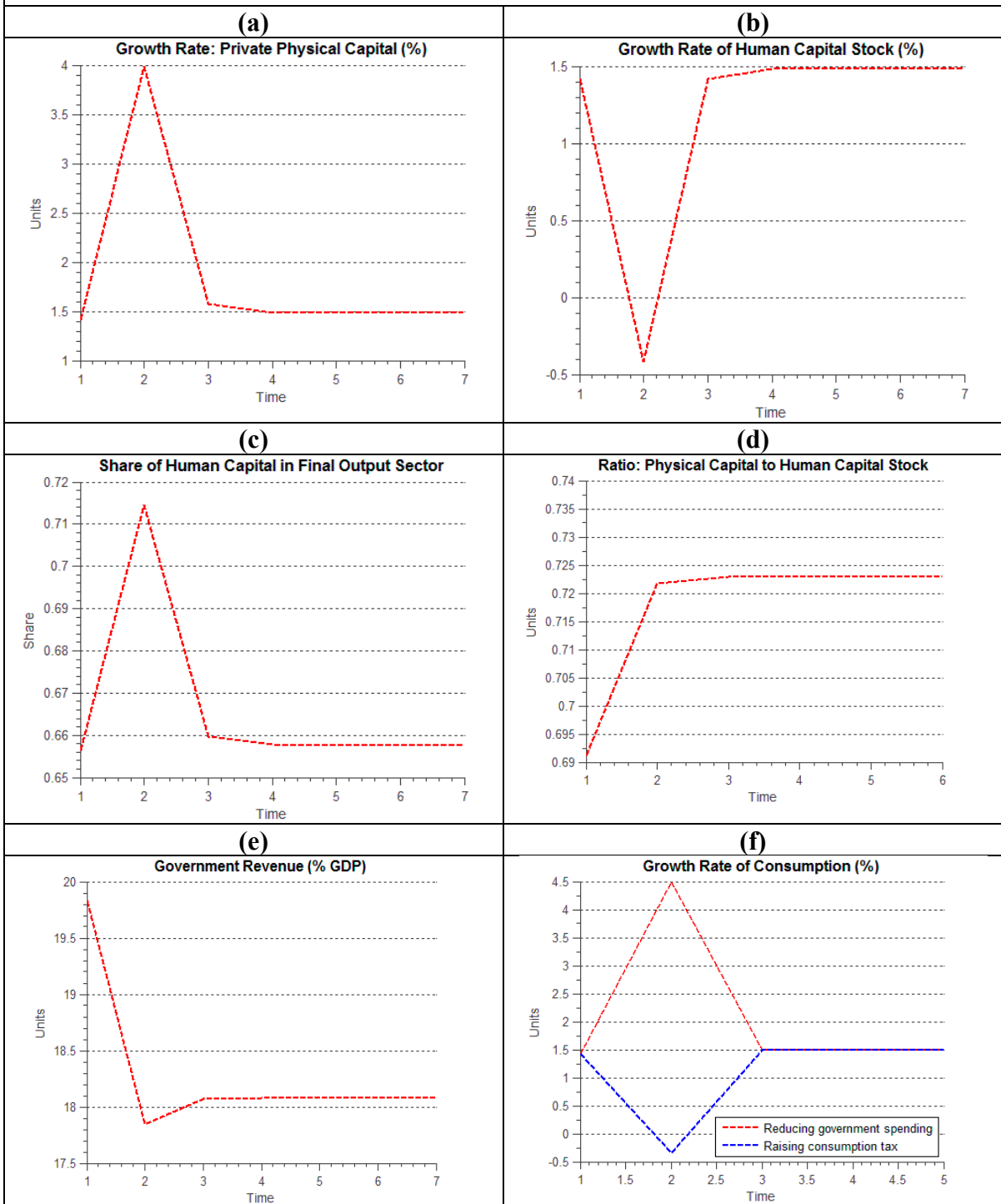
IVError! Reference source not found., Figure 2(c). This raises the growth rate of human capital to be equal to the growth rate of physical capital, which becomes the new BGP growth rate. Faster growth of physical and human capital allows faster growth of output per capita, consumer income and consumption, which boosts welfare by around 4.8 percent (Table 2).¹²

The interest rate paid on government bonds r_t^b increases following the tax cut, to keep pace with the rate of return on physical capital, preventing arbitrage opportunities. This is implied by the Euler equations (14)-(16). It turns out that under this choice of parameter values, the differential between the interest rate on government bonds $(1+r_t^b)$ and the BGP growth rate \bar{H} is larger on the post-reform BGP compared with the pre-reform BGP. Also, the tax cut reduces revenue (as a share of final output) (Figure 2(e)). All else equal, this would lower the primary balance (as a share of final output). Both the lower primary balance and higher interest rate growth differential imply that the government cannot sustain the pre-reform ratio of government debt to final output on the post-reform BGP, as indicated by (29). However, as part of the reform package, the government reduces unproductive spending g_t in order to keep the pre-reform debt ratio sustainable.

¹² Welfare effects of this magnitude are similar to those reported by Devereux and Love (1994), for distortionary tax cuts of less than 10 percentage points in a two-sector endogenous growth model without productive public capital, although with an endogenous leisure/labor supply choice.

Table 2. Reform Packages 1-4: Compensated by Unproductive Government Spending Cuts						
Baseline Fiscal Policy	Physical Capital Income Tax τ^K Rate (%)	Human Capital Income Tax τ^L Rate (%)	Consumption Tax τ^C Rate (%)	Public Investment (% of final output)	Growth Rate of GDP per Capita (%)	Govt. Debt (% of final output)
	17.5	17.5	7	4.6	1.42	44.5
Tax Policy Reform Packages 1-3						
Revenue Equivalent to a 5 Percentage Point Cut to Physical Capital Income Tax τ^K						
Fiscal Policy Reform	τ^K (%)	τ^L (%)	τ^C (%)	Public Investment (% of final output)	Change in Growth Rate of GDP per Capita (ppts)	Change in Consumer Welfare (%)
Capital Income Tax Cut	12.5	17.5	7	4.7	0.07	4.84
Labor Income Tax Cut	17.5	15.08	7	4.7	0.06	4.31
Cap. and Lab. Tax Cut	15.86	15.86	7	4.7	0.07	4.63
Revenue is 19.8% of final output on the benchmark BGP. Following a 5 percentage point capital income tax cut (or revenue equivalent labor tax cut), revenue is lower on the post-reform BGP by around 2 percentage point of final output. Unproductive government spending must be around 2 percentage points of final output lower on the new BGP, compared with the benchmark BGP, to keep government debt at its level on the original BGP.						
Public Investment Reform Package 4						
Public Investment Reform	Physical Capital Income Tax τ^K Rate	Human Capital Income Tax τ^L Rate	Consumption Tax τ^C Rate	Public Investment (% of Final Output)	Change in Growth Rate of GDP per Capita (ppts)	Change in Consumer Welfare (%)
	17.5	17.5	7	5.6	0.07	3.23

Figure 2. Packages 1 and 5: Capital Income Tax Cut



Reform Package 2: labor income tax cut and government spending cut

The dynamics of a labor income tax cut are more complicated. A cut in the labor income tax rate boosts the post-tax wage. It is necessary that the post-tax rental rate of private capital also rises to prevent arbitrage between physical and human capital investment, as required by Euler equations (14)-(16). Since the capital tax τ^k is unchanged in this experiment, the rental rate of private capital must itself rise. This is achieved by a reduction in the ratio of physical to human capital *in the final output sector* k_{t-1}^y (where physical capital is produced), because the rental rate (12) is decreasing in this ratio (Figure 3(a)). The share of human capital devoted to the final output sector u_t jumps up in the first period of the transition path, so that k_{t-1}^y declines (Figure 3(b)). This also implies that the ratio of physical to human capital *in the human capital output sector* k_{t-1}^h rises (Figure 3(c)).

The shift in human capital resources to the final output sector leads to faster growth in physical capital. Under this choice of parameter values (most importantly that $\phi > \alpha$), the net effect is that the aggregate ratio of private physical to human capital in the economy \bar{K} rises (Figure 3(d)). After jumping up initially, u_t (and the share of physical capital in final output v_t) falls over the transition path as the aggregate \bar{K} rises, consistent with equations (27) and Proposition 5 of Appendix IV (Figure 3(b)(e)). As resources shift back into the human capital sector, the growth rate of human capital rises to stabilize \bar{K} at its value on the new BGP (Figure 3(f)), on which all factors grow faster than on the pre-reform BGP. Faster growth of final output allows faster growth of consumption, boosting welfare by around 4.3 percent (Table 2). Again, government spending is reduced so as to keep government debt sustainable at its original level (as a percentage of final output) on the post-reform BGP.

In this model and under this choice of parameters, a cut in capital income taxation has a slightly larger effect on welfare than a cut in labor income taxation, where the size of the labor tax cut is set so that it has the same effect on the discounted present value of revenue as a 5 percentage point capital income tax cut.

Reform Package 3: capital and labor income tax cut; government spending cut

Cuts to capital and labor income tax of equal magnitude are considered, such that the discounted present value of lost revenue is equal to that of a 5 percentage point capital income tax cut. The growth effect of this policy change is similar to that of the capital income tax cut, while the welfare effect is between those of capital and labor income tax cuts alone (i.e., between the welfare effects of Reform Packages 1 and 2).

Table 3. Reforms 5-8: Compensated by Consumption Tax Rate Increases						
Baseline Fiscal Policy	Physical Capital Income Tax τ^K Rate (%)	Human Capital Income Tax τ^L Rate (%)	Consumption Tax τ^C Rate (%)	Public Investment (% of final output)	Growth Rate of GDP per Capita (%)	
	17.5	17.5	7	4.6	1.4	
Tax Policy Reform Packages 5-7						
Revenue Equivalent to a 5 Percentage Point Cut to Physical Capital Income Tax τ^K						
Fiscal Policy Reform	τ^K (%)	τ^L (%)	τ^C (%)	Public Investment (% of final output)	Change in Growth Rate of GDP per Capita (ppts)	Change in Consumer Welfare (%)
Capital Income Tax Cut	12.5	17.5	12.2	4.7	0.07	0.14
Labor Income Tax Cut	17.5	15.08	13.6	4.7	0.06	-0.48
Cap. and Lab. Tax Cut	15.86	15.86	12.2	4.7	0.07	-0.08
Public Investment and Tax Reform Package 8						
Public Investment Reform	Physical Capital Income Tax τ^K Rate	Human Capital Income Tax τ^L Rate	Consumption Tax τ^C Rate	Public Investment (% of final output)	Change in Growth Rate of GDP per Capita (ppts)	Change in Consumer Welfare (%)
	17.5	17.5	10.33	5.6	0.07	0.22

Reform Package 4: public investment increase and government spending cut

Increased public investment raises the growth rate of the public capital stock (Figure 4(a)). The ratio of public to private physical capital G/K rises, boosting total factor productivity in both the final output and human capital sectors (Figure 4(b)). The growth rates of physical and human capital ultimately increase (after some initial crowding out by higher government borrowing used to finance public investment) (Figure 4(c)). Faster growth in private factor inputs leads to faster growth in output and consumption, boosting welfare by around 3¼ percent. The fiscal cost of increasing public investment spending is offset by a reduction in unproductive government spending, keeping government debt stable (as a share of final output) at its level on the pre-reform BGP.

Reform Packages 5-8: fiscal cost compensated by raising consumption tax rate τ^C

Reform packages 5-8 are the same as packages 1-4, *except* that the fiscal cost of the tax cut or public investment increase is offset by an increase in the rate of consumption taxation τ_t^C . Specifically, the rate of consumption taxation is increased permanently at the same time as the tax cut or public investment increase, so that the discounted present value of tax revenue from the point of reform is the same as on the pre-reform BGP. Additional adjustments in unproductive government spending may be necessary to ensure that the ratio of government debt to final output is the same on the post-reform BGP as under the pre-reform BGP. These adjustments are necessary because of the potential effect of reforms on the interest-rate growth differential ($\tilde{r}^b - \tilde{H}$). Information about the effects of each reform, is given in Table 3, including (i) the post-reform BGP growth rate; and (ii) the welfare impact of the reform.

Reform Package 5: capital income tax cut and consumption tax rise

The key difference between Reform Packages 1 and 5 is that the growth rate of consumption jumps down in response to the permanent consumption tax shock of reform 5 (Figure 2(f)). This is in stark contrast to the acceleration of consumption that occurs in response to the capital income tax cut under Reform Package 1, where the consumption tax rate is unchanged. The slowdown in consumption growth under Reform Package 5 in the early periods of the transition has a significant welfare cost, despite consumption growth eventually picking up as the post-reform BGP growth rate is reached. The welfare gain from Reform Package 5 is just over 0.1 percent, several percentage points lower than that of Reform Package 1.¹³ The growth effects of Reform Packages 1 and 5 are the same, reflecting the non-distortionary nature of the consumption tax in this model.

¹³ Lucas (1990) and Pecorino (1993) consider more drastic experiments in two-sector endogenous growth models (without productive public capital), such as complete abolition of capital income taxation (involving tax cuts of more than 20 or 30 percentage point) and replacement with consumption tax. Those papers report welfare gains of around 3 percent or less for these major reforms.

Reform Package 6: labor income tax cut and consumption tax rise

The increase in consumption tax under Reform Package 6 has an effect similar to that in Reform Package 5. There is a larger slowdown in consumption growth when the permanent consumption tax rise occurs, compared with the situation under Reform Package 2. There is also a much smaller reduction in the growth rate of unproductive government spending and a larger reduction in the growth rate of government borrowing, both reflecting the compensating effect of the consumption tax rise on revenue (Figure 3(g)(h)). The welfare cost of the short-run consumption slow down under Reform Package 6 is sufficiently large so as to offset the benefit of the labor income tax cut, so that there is a small welfare loss from this reform overall.

Reform Package 7: capital and labor income tax cut; consumption tax rise

There is a small, overall reduction in welfare when the combined capital and labor tax cuts are accompanied by a consumption tax rate increase to prevent a loss of revenue. This is consistent with the impact of the consumption tax rise on consumption growth, observed under Reform Packages 5 and 6.

Reform Package 8: public investment increase and consumption tax rise

The public investment increase of Reform Package 8 still leads to an overall welfare gain, despite the increase in the consumption tax, which is used to finance the higher spending. This result is achieved despite the conservative values attached to the parameters γ and ε determining the productivity of public capital.

Figure 3. Packages 2 and 6: Labor Income Tax Cut

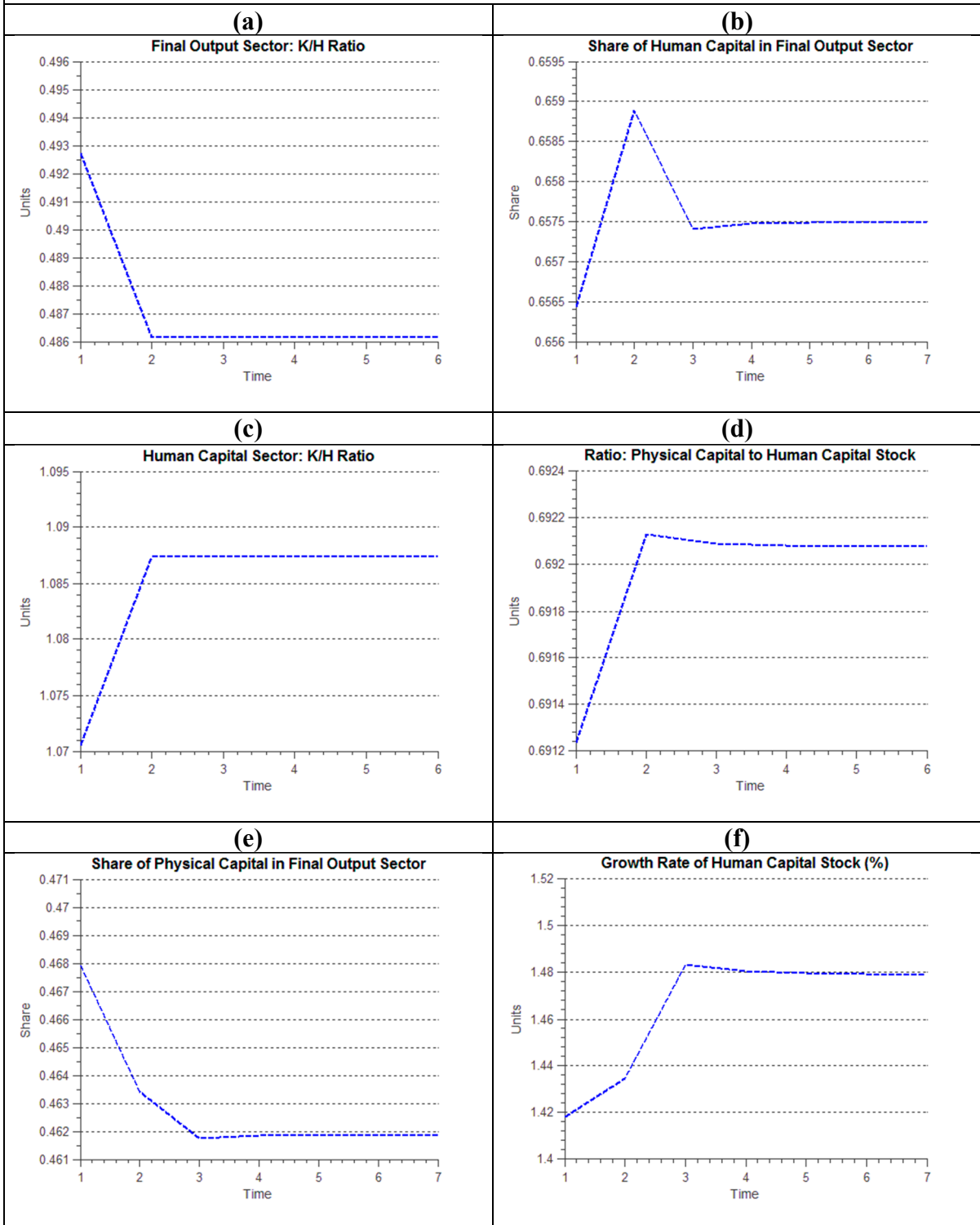


Figure 3. Packages 2 and 6: Labor Income Tax Cut (concl'd)

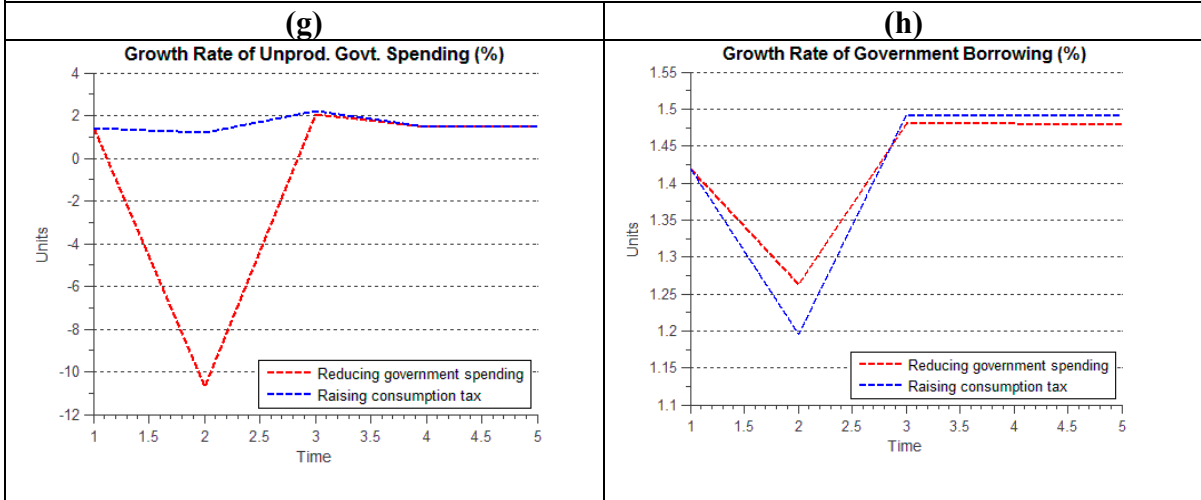
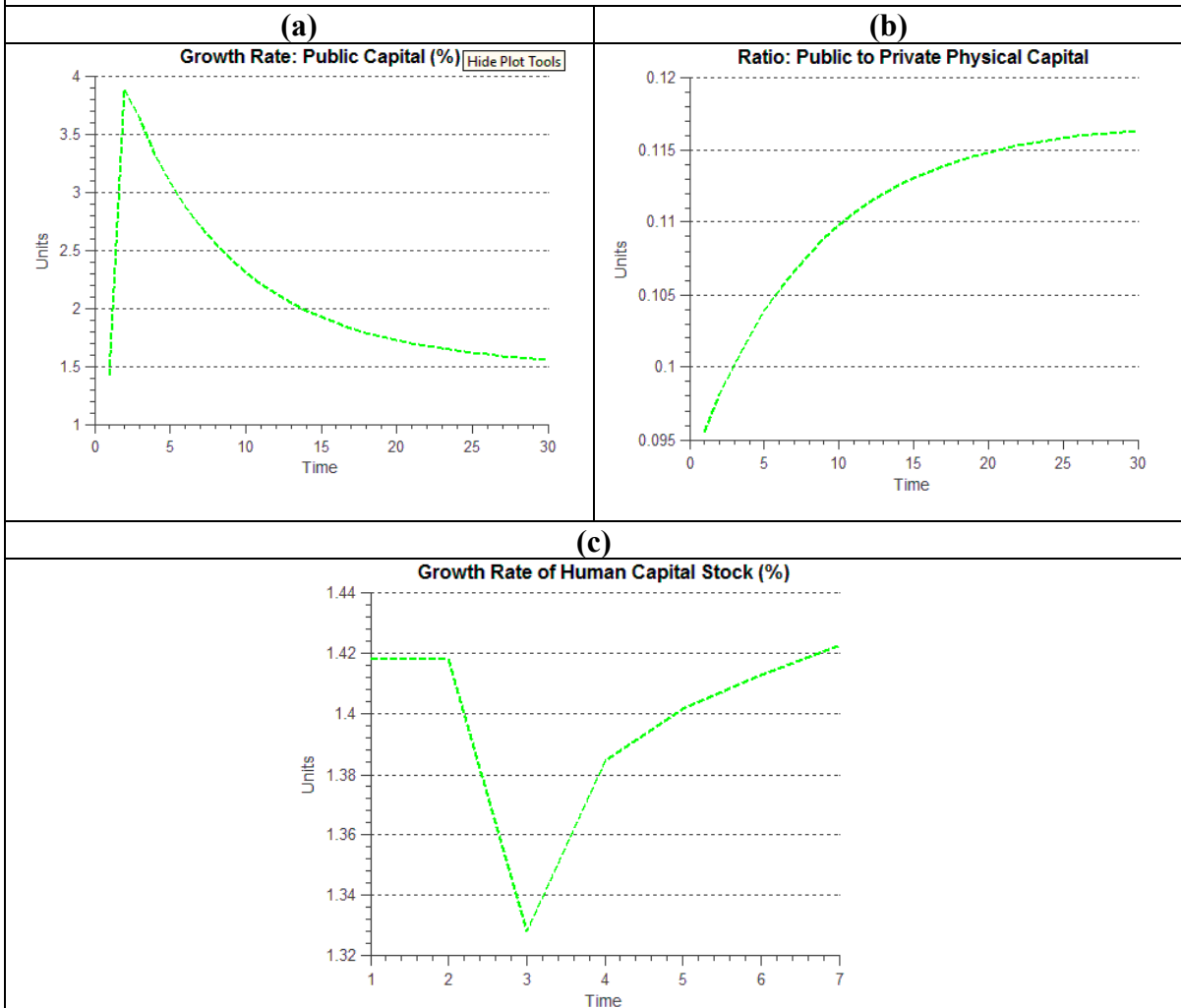


Figure 4. Packages 4 and 8: Public Investment Increase



V. CONCLUSION

The theoretical model in this paper has drawn together several important features of the endogenous growth literature: (i) investment in technology (i.e., human capital) offsets diminishing marginal returns to private capital, so that the accumulation of both physical and human capital allows for perpetual growth in output per capita; (ii) changes in investment behavior because of cuts to distortionary tax rates can impact long-run growth, since growth is linked to endogenous investment decisions; and (iii) public capital has a role influencing total factor productivity and growth. Cuts to distortionary tax rates remove distortions to private investment, spurring faster output growth. Adjustment to changes in tax policy involves resources shifting between final output and human capital sectors, to exploit the opportunities for greater returns from investing in physical or human capital. Higher public investment can increase the ratio of public capital relative to the size of the private economy (proxied by the ratio of public to private physical capital), boosting productivity and growth.

A quantitative simulation of the model using reasonable parameter values suggests that the effects of moderately sized distortionary tax cuts or public investment increases on annual growth of output per capita are small, but are nonetheless of interest given that growth rates increase permanently. Cuts to capital income taxation and increases in public investment can increase welfare substantially. The welfare gain is largest when the fiscal cost of the tax cuts or public investment increases is offset by cuts to unproductive spending, which keep the pre-reform debt ratio sustainable in the long run. The welfare gains are much smaller when consumption tax increases are used instead of spending cuts, although the welfare gains remain positive. Labor income tax cuts improve welfare when the revenue loss is offset by cuts to unproductive government spending, but not when offset by consumption tax increases. Ultimately, the analysis in this paper suggests that fiscal reform packages can boost growth and significantly improve welfare, but the size of these gains depends critically on the design of the reform package.

It should be noted that the effects on growth and welfare of fiscal reform packages may be different in models which allow for income and wealth inequality. These features are missing from the framework considered in this paper, where consumers are homogeneous. A *heterogeneous* agent framework of this type could be used to study and compare more complex fiscal reform packages involving changes to progressive income tax rates and targeted government spending, but this is left for future research.

APPENDIX I. THE COMPETITIVE EQUILIBRIUM

A. Consumers' Problem

Given initial conditions $\{K_{-1}, H_{-1}, B_{-1}\}$, an infinite sequence of factor prices and bond prices $\{r_t, w_t, r_t^b\}_{t=0}^{\infty}$, as well as an infinite sequence of fiscal policy variables $\{G_{t-1}, \tau_t^c, \tau_t^K, \tau_t^L\}_{t=0}^{\infty}$ (where τ_t^c , τ_t^K and τ_t^L are the consumption, capital and labor income tax rates), the consumer chooses an infinite sequence of consumption, saving and resource allocation across sectors $\{c_t, K_t, H_t, B_t, u_t, v_t\}_{t=0}^{\infty}$ to maximize the discounted present value of utility:

$$\max_{\{c_t, K_t, H_t, B_t, u_t, v_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to the consumer's budget constraint each period

$$(1 + \tau_t^c)c_t + (K_t - (1 - \delta_K)K_{t-1}) + B_t \leq (1 - \tau_t^K)v_t r_t K_{t-1} + (1 - \tau_t^L)u_t w_t H_{t-1} + (1 + r_t^b)B_{t-1}$$

and the law of motion for human capital

$$H_t = z \left(\frac{G_{t-1}}{K_{t-1}} \right) g((1 - v_t)K_{t-1}, (1 - u_t)H_{t-1}) + (1 - \delta_H)H_{t-1}$$

The Lagrangian for this optimization problem is:

$$L = \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} u(c_t) - \lambda_t \left\{ (1 + \tau_t^c)c_t + (K_t - (1 - \delta_K)K_{t-1}) + B_t \right. \\ \left. - (1 - \tau_t^K)v_t r_t K_{t-1} - (1 - \tau_t^L)u_t w_t H_{t-1} - (1 + r_t^b)B_{t-1} \right\} \\ - \Omega_t \left\{ H_t - z \left(\frac{G_{t-1}}{K_{t-1}} \right) g((1 - v_t)K_{t-1}, (1 - u_t)H_{t-1}) - (1 - \delta_H)H_{t-1} \right\} \end{array} \right] \quad (30)$$

where λ_t and Ω_t are the lagrange multipliers associated with the period t consumer budget constraint (4) and human capital law of motion (5), respectively.

The first order conditions are:

$$L_{c_t} = u'(c_t) - \lambda_t(1 + \tau_t^c) = 0 \quad (31)$$

$$\begin{aligned} L_{K_t} &= -\lambda_t + \beta \lambda_{t+1}(1 - \delta_K) + \beta \lambda_{t+1}(1 - \tau_{t+1}^K)r_{t+1}v_{t+1} \\ &+ \beta \Omega_{t+1} z \left(\frac{G_t}{K_t} \right) g'_K((1 - v_{t+1})K_t, (1 - u_{t+1})H_t) = 0 \end{aligned} \quad (32)$$

$$\begin{aligned}
L_{H_t} &= -\Omega_t + \beta\Omega_{t+1}(1-\delta_H) + \beta\lambda_{t+1}(1-\tau_{t+1}^L)w_{t+1}u_{t+1} \\
&+ \beta\Omega_{t+1}z\left(\frac{G_t}{K_t}\right)g'_H((1-v_{t+1})K_t, (1-u_{t+1})H_t) = 0
\end{aligned} \tag{33}$$

$$L_{v_t} = \lambda_t(1-\tau_t^K)r_t K_{t-1} - \Omega_t z\left(\frac{G_{t-1}}{K_{t-1}}\right)g'_v((1-v_t)K_{t-1}, (1-u_t)H_{t-1}) = 0 \tag{34}$$

$$L_{u_t} = \lambda_t(1-\tau_t^L)w_t H_{t-1} - \Omega_t z\left(\frac{G_{t-1}}{K_{t-1}}\right)g'_u((1-v_t)K_{t-1}, (1-u_t)H_{t-1}) = 0 \tag{35}$$

$$L_{B_t} = -\lambda_t + \beta\lambda_{t+1}(1+r_{t+1}^b) = 0 \tag{36}$$

B. Firms' Problem

The firm chooses the level of produced output y_t and rented inputs $v_t K_{t-1}$ and $u_t H_{t-1}$ to solve the following profit maximization problem:

$$\max_{\{y_t, v_t K_{t-1}, u_t H_{t-1}\}} y_t - r_t v_t K_{t-1} - w_t u_t H_{t-1}$$

subject to the production technology

$$y_t \leq q\left(\frac{G_{t-1}}{K_{t-1}}\right)f(v_t K_{t-1}, u_t H_{t-1})$$

The price of output y_t is normalized to 1. The production technology (9) can be substituted into the firms' objective function (8) so that the firm can be treated as solving the following unconstrained optimization problem each period:

$$\max_{\{v_t K_{t-1}, u_t H_{t-1}\}} \pi_t = q\left(\frac{G_{t-1}}{K_{t-1}}\right)f(v_t K_{t-1}, u_t H_{t-1}) - r_t v_t K_{t-1} - w_t u_t H_{t-1} \tag{37}$$

where π_t denotes profit. The first order conditions are:

$$\frac{\partial \pi_t}{\partial v_t K_{t-1}} = q\left(\frac{G_{t-1}}{K_{t-1}}\right)f'_{vK}(v_t K_{t-1}, u_t H_{t-1}) - r_t = 0 \tag{38}$$

$$\frac{\partial \pi_t}{\partial u_t H_{t-1}} = q\left(\frac{G_{t-1}}{K_{t-1}}\right)f'_{uH}(v_t K_{t-1}, u_t H_{t-1}) - w_t = 0 \tag{39}$$

APPENDIX II. SOLVING FOR THE BALANCED GROWTH PATH

Simplifying manipulations: factor price equalization

The analysis is simplified by defining two new variables: the ratios of private physical and human capital inputs in the final output sector $k_{t-1}^y \equiv \frac{v_t K_{t-1}}{u_t H_{t-1}} = \frac{v_t \widetilde{K}_{t-1}}{u_t}$ and human capital

sector $k_{t-1}^h \equiv \frac{(1-v_t)K_{t-1}}{(1-u_t)H_{t-1}} = \frac{(1-v_t)\widetilde{K}_{t-1}}{(1-u_t)}$. The consumer optimality conditions regarding the

allocation of resources across sectors, (17) and (18) can be combined with firm optimality conditions (12) and (13) to solve for k_{t-1}^y and k_{t-1}^h as a functions $k_{t-1}^y \left(p_t, \tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right)$ and

$k_{t-1}^h \left(p_t, \tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right)$ of the relative price of human capital p_t , the capital τ_t^K and labor τ_t^L

tax rates and total factor productivity in the final output $q \left(\frac{G_{t-1}}{K_{t-1}} \right)$ and human capital $z \left(\frac{G_{t-1}}{K_{t-1}} \right)$

sectors, observing that total factor productivity depends on the ratio $\frac{G_{t-1}}{K_{t-1}}$, which can be

targeted/controlled by the government. Substituting the functions for k_{t-1}^y and k_{t-1}^h into the expressions (12) and (13) for factor prices, leads to the following proposition:

Proposition 1: Factors prices can be expressed in terms of only the relative price of human capital p_t and fiscal variables τ_t^K, τ_t^L , as well as $\frac{G_{t-1}}{K_{t-1}}$:

$$r_t \left(p_t, \tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right) \quad (40)$$

$$w_t \left(p_t, \tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right) \quad (41)$$

Proof: The first order conditions of the consumers' problem (34) and (35) can be combined with the first order conditions of the firms' problem, (38) and (39), to form equilibrium conditions which require the marginal product of private physical capital and human capital to be equal across sectors (subject to taxation distortions):

$$(1-\tau_t^K)q\left(\frac{G_{t-1}}{K_{t-1}}\right)f'_{vK}(v_t K_{t-1}, u_t H_{t-1}) = p_t z\left(\frac{G_{t-1}}{K_{t-1}}\right)g'_{(1-v)K}((1-v_t)K_{t-1}, (1-u_t)H_{t-1}) \quad (42)$$

$$(1-\tau_t^L)q\left(\frac{G_{t-1}}{K_{t-1}}\right)f'_{uH}(v_t K_{t-1}, u_t H_{t-1}) = p_t z\left(\frac{G_{t-1}}{K_{t-1}}\right)g'_{(1-u)H}((1-v_t)K_{t-1}, (1-u_t)H_{t-1}) \quad (43)$$

It is possible to solve these two equations for the variables k_{t-1}^y and k_{t-1}^h in terms of the relative price of human capital goods p_t , the capital and labor tax rates τ_t^K and τ_t^L and the ratio of public to private physical capital $\frac{G_{t-1}}{K_{t-1}}$ targeted by the government. The solutions for the variables k_{t-1}^y and k_{t-1}^h can then be substituted into the first order conditions of the firms' problem, (38) and (39) to solve for factor prices.

A unique BGP

Proposition 2: The model has a **unique BGP**.

Proof: Using the factor price expressions (40) and (41), the Euler equations for private physical capital (14) and human capital (15) can be combined to form the following implicit difference equation in the relative price p_t of human capital in terms of final goods, assuming that the government holds tax rates τ_t^K and τ_t^L constant, as well as setting public investment to hold $\frac{G_{t-1}}{K_{t-1}}$ constant:

$$p_{t+1}(1-\delta_H) - p_t(1-\delta_K) = r_{t+1}(p_{t+1})(1-\tau_{t+1}^K)p_t - w_{t+1}(p_{t+1})(1-\tau_{t+1}^L) \quad (44)$$

This equation implies that the relative price of human capital goods p_t will adjust so as to equate the post-tax, net of depreciation rates of return on private physical and human capital investment, as required by (14) and (15), to eliminate opportunities for arbitrage. It can be shown under weak assumptions that there is a unique p^* such that $p_{t+1} - p_t = 0$ and p must be constant on a BGP. Given constant τ_t^K and τ_t^L , as well as constant $\frac{G_{t-1}}{K_{t-1}}$, it follows that

there are unique quantities $k^y\left(p^*, \tau^K, \tau^L, \frac{G}{K}\right)$ and $k^h\left(p^*, \tau^K, \tau^L, \frac{G}{K}\right)$ consistent with the

value of p^* . These quantities can be substituted into (40) and (41) to identify a unique, constant rental rate r and wage w . Using the production function for final output (7) and the law of motion for human capital (5), the unique, constant normalized levels of final output

$\tilde{Y}_t \equiv \frac{Y_t}{H_{t-1}}$ and human capital $\tilde{H}_t = \frac{H_t}{H_{t-1}}$ can be identified:

$$\tilde{Y} = q \left(\frac{G}{K} \right) f(k^y) \quad (45)$$

and

$$\tilde{H} = z \left(\frac{G}{K} \right) g(k^h) + (1 - \delta_H) \quad (46)$$

Constant \tilde{Y} implies that output per capita Y_t is growing at the same rate as human capital H_t per capita, with this growth rate defined by \tilde{H} . A combination of the consumer and government budget constraints, (4) and (10), then implies a constant normalized $\tilde{c} = c_t / H_{t-1}$ so that consumption also grows at the rate \tilde{H} , assuming that the government sets unproductive spending to grow at rate \tilde{H} , with the normalized value $\tilde{g}_t \equiv \frac{g_t}{H_{t-1}}$ being constant. Given constant \tilde{c} and assuming the government sets a constant consumption tax rate τ^c , the Euler equation for government bonds (16) implies a constant interest rate on government bonds r^b . The government budget constraint (10) implies a normalized value of government bond holdings $\tilde{B} = B_{t-1} / H_{t-1}$, so that government bond holdings also grow at the rate \tilde{H} .

APPENDIX III. THE GROWTH MECHANISM

Proposition 3: The BGP growth rate \widetilde{H} is given by:

$$\widetilde{H} = \beta^{\frac{1}{\sigma}} \left[(1 - \delta_K) + \alpha A \left(\frac{C(1-\phi)}{\alpha A} \right)^{\frac{1-\alpha}{1-\alpha+\phi}} \left(\frac{G}{K} \right)^{\frac{\varepsilon(1-\alpha)+\phi\gamma}{1-\alpha+\phi}} (1 - \tau^K)^{\frac{\phi\alpha}{1-\alpha+\phi}} (1 - \tau^L)^{\frac{\phi(1-\alpha)}{1-\alpha+\phi}} \left(\frac{(1-\alpha)\phi}{\alpha(1-\phi)} \right)^{\frac{\phi(1-\alpha)}{1-\alpha+\phi}} \right]^{\frac{1}{\sigma}} \quad (47)$$

where Φ is a function of parameters associated with the utility function and production function.

Proof: Using the specific functional forms (6), (24), and (25), the Euler equations for private physical capital (14) and human capital (15) can be re-written as:

$$\frac{c_t^{-\sigma}}{(1 + \tau_t^c)} = \beta \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \left[(1 - \delta_K) + \alpha A \left(\frac{G_t}{K_t} \right)^\gamma (k_t^y)^{\alpha-1} (1 - \tau_{t+1}^K) \right] \quad (48)$$

and

$$\frac{c_t^{-\sigma} p_t}{(1 + \tau_t^c)} = \beta \frac{c_{t+1}^{-\sigma} p_{t+1}}{(1 + \tau_{t+1}^c)} \left[(1 - \delta_H) + (1 - \phi) C \left(\frac{G_t}{K_t} \right)^\varepsilon (k_t^h)^\phi \right] \quad (49)$$

The next step is to impose a steady state (BGP) where p_t (the relative price of human capital), $k_{t-1}^y \equiv \frac{v_t K_{t-1}}{u_t H_{t-1}}$, $k_{t-1}^h \equiv \frac{(1-v_t) K_{t-1}}{(1-u_t) H_{t-1}}$, tax rates and the ratio $\frac{G}{K}$ targeted by the government are all constant. Assuming also for simplicity that the depreciation rates are equal $\delta_K = \delta_H = \delta$, (48) and (49) can be combined to yield:

$$\alpha A \left(\frac{G}{K} \right)^\gamma \left(\frac{v\widetilde{K}}{u} \right)^{\alpha-1} (1 - \tau^K) = (1 - \phi) C \left(\frac{G}{K} \right)^\varepsilon \left(\frac{(1-v)\widetilde{K}}{(1-u)} \right)^\phi \quad (50)$$

The consumer optimality condition (19) requiring that the marginal rate of technical substitution of private physical capital for human capital be equal across sectors (subject to taxation distortions) takes the following form on a BGP:

$$\frac{v(1 - \tau^L)(1 - \alpha)\phi}{u(1 - \tau^K)\alpha(1 - \phi)} = \frac{(1 - v)}{(1 - u)} \quad (51)$$

Combining (51) with (50) yields:

$$\left(\frac{v}{u}\right)^{\alpha-1-\phi} \widetilde{K}^{\alpha-1-\phi} = \frac{C(1-\phi)}{\alpha A} \left(\frac{G}{K}\right)^{\varepsilon-\gamma} (1-\tau^K)^{-\phi-1} (1-\tau^L)^\phi \left(\frac{(1-\alpha)\phi}{\alpha(1-\phi)}\right)^\phi \quad (52)$$

The normalized private capital Euler equation is:

$$\widetilde{c}_t^{-\sigma} = \beta \frac{(1+\tau_t^c) \widetilde{h}_t^{-\sigma} \widetilde{c}_{t+1}^{-\sigma}}{(1+\tau_{t+1}^c)} \left[(1-\delta_K) + \alpha \left(\frac{G_t}{K_t}\right)^\gamma (k_t^y)^{\alpha-1} (1-\tau_{t+1}^K) \right] \quad (53)$$

Imposing steady state (i.e., a BGP) on (53) with $\widetilde{c}_t = \widetilde{c}_{t+1}$ and $\tau_t^c = \tau_{t+1}^c$ and using (52) to substitute for $k_t^y = \frac{v_{t+1} \widetilde{K}_t}{u_{t+1}}$ yields the BGP growth rate:

$$\widetilde{H} = \beta^{\frac{1}{\sigma}} \left[(1-\delta_K) + \alpha A \left(\frac{C(1-\phi)}{\alpha A}\right)^{\frac{1-\alpha}{1-\alpha+\phi}} \left(\frac{G}{K}\right)^{\frac{\varepsilon(1-\alpha)+\phi\gamma}{1-\alpha+\phi}} (1-\tau^K)^{\frac{\phi\alpha}{1-\alpha+\phi}} (1-\tau^L)^{\frac{\phi(1-\alpha)}{1-\alpha+\phi}} \left(\frac{(1-\alpha)\phi}{\alpha(1-\phi)}\right)^{\frac{\phi(1-\alpha)}{1-\alpha+\phi}} \right]^{\frac{1}{\sigma}}$$

APPENDIX IV. THE TRANSITION PATH

Proposition 4: Using the specific functional forms (6), (24), and (25), it is possible to write the share of human capital allocated to final goods production u_t as an increasing function of the share of private physical capital allocated to this purpose v_t :

$$u_t(v_t); \quad u_t'(v_t) > 0 \quad (54)$$

Proof: Using the specific functional forms (24) and (25), the consumer optimality condition requiring that the marginal rate of technical substitution of private physical for human capital be equal across sectors (subject to taxation distortions) (19):

$$\frac{(1-\tau_t^K) f'_{vK}(v_t K_{t-1}, u_t H_{t-1})}{(1-\tau_t^L) f'_{uH}(v_t K_{t-1}, u_t H_{t-1})} = \frac{g'_{vK}((1-v_t) K_{t-1}, (1-u_t) H_{t-1})}{g'_{uH}((1-v_t) K_{t-1}, (1-u_t) H_{t-1})}$$

can be rewritten as:

$$\frac{v_{t+1}}{(1-v_{t+1})} = \frac{u_{t+1} (1-\tau_{t+1}^K) \alpha (1-\phi)}{(1-u_{t+1}) (1-\tau_{t+1}^L) (1-\alpha) \phi} \quad (55)$$

implying that an increase in v_t must be accompanied by an increase in u_t , provided that tax rates are held constant.

Proposition 5: The relationship between u_t and the ratio of private physical capital to human capital \widetilde{K}_{t-1} is given by:

$$u_t(\widetilde{K}_{t-1}, p_t, \widetilde{G}_{t-1}, \tau_t^K, \tau_t^L) = \frac{k_{t-1}^h(p_t, \widetilde{G}_{t-1}, \tau_t^K, \tau_t^L) - \widetilde{K}_{t-1}}{k_{t-1}^h(p_t, \widetilde{G}_{t-1}, \tau_t^K, \tau_t^L) - k_{t-1}^y(p_t, \widetilde{G}_{t-1}, \tau_t^K, \tau_t^L)} \quad (56)$$

Proof: Recall that fraction v_t of private physical capital is allocated to final goods production, with the remainder to human capital production, so $K_{t-1} = v_t K_{t-1} + (1-v_t) K_{t-1}$.

Normalizing by H_{t-1} and multiplying and dividing the right hand side terms by u_t and $(1-u_t)$ respectively implies $\widetilde{K}_{t-1} = u_t k_{t-1}^y + (1-u_t) k_{t-1}^h$. Rearranging gives the result. Please recall that k_{t-1}^y is the ratio of private physical to human capital in the final output sector, while k_{t-1}^h is the ratio in the human capital sector.

In the numerical simulations conducted in Section IV, the relative price of human capital p_t and fiscal variables $\{\widetilde{G}_{t-1}, \tau_t^K, \tau_t^L\}$ adjust very quickly to their new BGP values following a

fiscal reform (of course, in the case of tax rates, this is immediate). This then suggests an inverse relationship between the ratio of private physical capital to human capital \widetilde{K} and the fraction of resources devoted to the final output sector (i.e., u and v). The intuition for this result is presented in Section IV, when each fiscal reform package is discussed.

The following proposition indicates how the pre-reform and post-reform ratios of private physical to human capital \widetilde{K}_{t-1} differ.

Proposition 6: Assuming the specific functional forms (6), (24) and (25), that $\phi > \alpha$ and that $k^h > k^y$ on a BGP (which will be the case in the numerical simulation of Section IV), then:

1. An increase in the physical capital income tax rate **lowers** the BGP physical to human capital ratio $\frac{\partial \widetilde{K}}{\partial \tau^K} < 0$.

2. An increase in the labor income tax rate has an **ambiguous effect** on the BGP physical to human capital ratio (the effect depends on the choice of parameters, but the effect is negative $\frac{\partial \widetilde{K}}{\partial \tau^L} < 0$ under the arguably reasonable choice of parameters used for the numerical simulation in Section IV).

3. An increase in the ratio of public to private physical capital $\widetilde{G} = G/K$ **raises** the BGP physical to human capital ratio $\frac{\partial \widetilde{K}}{\partial \widetilde{G}} > 0$, provided public capital is equally productive in each sector $\gamma = \varepsilon$ and that ε is sufficiently small (these are sufficient but not necessary conditions).

Proof: Using specific functional forms (24) and (25) for final output and human capital production functions, it is possible to solve for

$$k_{t-1}^y \left(p_t, \tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right); \quad k_{t-1}^h \left(p_t, \tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right) \quad (57)$$

as shown in **Proposition 2**. Again, using specific functional forms, it is possible to solve the difference equation (44) exactly for the unique BGP $p^* \left(\tau^K, \tau^L, \frac{G}{K} \right)$ as a function of fiscal variables. This result can be combined with (57) to yield:

$$k_{t-1}^y \left(\tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right); \quad k_{t-1}^h \left(\tau_t^K, \tau_t^L, \frac{G_{t-1}}{K_{t-1}} \right) \quad (58)$$

as functions only of fiscal variables. It follows from the algebraic solution that $\frac{\partial k_{t-1}^y}{\partial \tau^K} < 0$ and

$\frac{\partial k_{t-1}^h}{\partial \tau^K} < 0$. Recall that the solution for the BGP growth rate \tilde{H} (26) implied that $\frac{\partial \tilde{H}}{\partial \tau^K} < 0$,

$\frac{\partial \tilde{H}}{\partial \tau^L} < 0$ and $\frac{\partial \tilde{H}}{\partial \tilde{G}} > 0$.

The human capital accumulation equation (5) on a BGP (using specific functional forms) is:

$$\tilde{H} = (\tilde{G})^\varepsilon (1-u)(k^h)^\phi + (1-\delta_H) \quad (59)$$

where u takes the value in, evaluated on a BGP:

$$u = \frac{k^h - \tilde{K}}{k^h - k^x}$$

The expression (59) can then be rearranged and solved for \tilde{K} in terms of fiscal variables $\{\tilde{G}, \tau^K, \tau^L\}$. The statements in **Proposition 6** then follow, provided $\phi > \alpha$: the parameter associated with private capital in the human capital production function must be greater than the corresponding parameter in the final output production function, which is often assumed to be approximately 1/3. It is difficult to know precisely what the value of ϕ should be. In that case, assuming it to be approximately 1/2, implies $\phi > \alpha$. It should also be the case then that $k^h > k^x$ for sensible values of tax rates.

APPENDIX V. A NUMERICAL EXAMPLE

Welfare metric

Consumer welfare following a fiscal reform is quantified by approximating:

$$W = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad (60)$$

where initial conditions are given by the pre-reform BGP. The welfare effect of a reform is expressed as the constant percentage of consumption each period required to make the consumer indifferent between the reform and continuing along the pre-reform BGP.

Quantitatively, consumer welfare following a reform is approximated in two steps. First, the transition path is truncated at some time T^* , sufficiently large such that normalized variables are close to the stationary values on the new BGP and the following quantity is evaluated:

$$W_1 = \sum_{t=0}^{T^*} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad (61)$$

The second step is to evaluate welfare as an infinite geometric sum assuming that the new BGP has been reached, so that consumption grows every period at the BGP growth rate:

$$W_2 = \sum_{j=0}^{\infty} \beta^{T^*+j} \frac{\left(c_T \widetilde{H}^j\right)^{1-\sigma}}{1-\sigma} \quad (62)$$

recalling that \widetilde{H} is the BGP growth rate (26). The total welfare following a reform is then $W = W_1 + W_2$.

The government budget constraint on a BGP

The budget constraint for the government each period is

$$\begin{aligned} & (G_t - (1 - \delta_G)G_{t-1}) + g_t + (1 + r_t^b)B_{t-1} \\ & \leq \tau_t^c c_t + \tau_t^K r_t^K v_t K_{t-1} + \tau_t^L w_t u_t H_{t-1} + B_t \end{aligned}$$

It will be useful to define the primary balance pb_t as the difference between non-interest revenue and non-interest expenditure:

$$pb_t = \tau_t^c c_t + \tau_t^K r_t^K v_t K_{t-1} + \tau_t^L w_t u_t H_{t-1} - (G_t - (1 - \delta_G)G_{t-1}) - g_t$$

Normalizing non-stationary variables $\{G_{t-1}, B_{t-1}, K_{t-1}, H_t, pb_t, c_t\}$ by H_{t-1} produces

$$(1 + r_t^b) \widetilde{B}_{t-1} \leq \widetilde{pb}_t + \widetilde{B}_t \widetilde{H}_t \quad (63)$$

where normalized variables are denoted with a superscript: e.g. $\widetilde{B}_{t-1} \equiv B_{t-1} / H_{t-1}$.

All variables in (63) are stationary. Imposing a steady state (e.g. $\widetilde{B}_t = \widetilde{B}_{t-1} = \widetilde{B}$), yields:

$$(\widetilde{r}^b - \widetilde{H}) \widetilde{B} \leq \widetilde{pb} \quad (64)$$

where $\widetilde{r}^b \equiv 1 + r^b$ is the gross rate of return on government bonds. It is also possible to express \widetilde{B} and \widetilde{pb} as ratios of final output. Equation (64) establishes a link between the government debt ratio, the primary balance and the interest rate-growth differential on a BGP.

APPENDIX VI. ROBUSTNESS OF NUMERICAL SOLUTIONS

Table 4 presents the results from some of the numerical simulations of Section IV, under different assumptions about the values of key parameters. First, increasing the tax rates prevailing on the pre-reform BGP does not alter significantly the long-run growth impact of a 5 percentage point cut to the capital income tax rate τ^K (compensated for by a cut to unproductive government spending), although the welfare gain is larger than reported in Table 2, Section IV (5.29 percent compared with 4.84 percent). Under the higher tax rates on the pre-reform BGP, a 5 percentage point capital tax cut compensated for by an increase to the rate of consumption tax τ^C produces a much smaller welfare gain than when compensated for by a spending cut, as found in the simulations presented in Table 3, Section IV.

Assuming a lower intertemporal elasticity of substitution of consumption (by assuming a coefficient of relative risk aversion σ of 4, as opposed to σ of 2, Table 2, Section IV) does not change significantly the long-run growth impact of a 5 percentage point capital tax cut (compensated for by lower government spending), although the welfare gain is over 1 percentage point smaller than reported in Table 2, Section IV (3.29 percent compared with 4.84 percent). Changing the exponent parameter ϕ in the human capital production function (25) has a relatively small effect on the long-run growth impact of a 5 percentage point capital income tax cut, although more significant effects on the welfare impact.¹⁴ Finally, increasing both the exponents γ, ε on public capital in the production functions for final output (24) and human capital (25) from 0.05 to 0.15 can boost the long-run growth impact of a 1 percentage point increase in public investment by around 0.2 percentage points, as well as significantly increasing the positive welfare impact. Values of 0.15 for γ, ε are still well within the range found in the literature on the productivity of public capital (Bom and Ligthart (2014)).

¹⁴ Setting the parameter ϕ in the human capital production function (25) below the value of the parameter α in the final output production function (24) may have significant effects on the direction in which endogenous variables move over the transition path to a new BGP following a reform.

Table 4. Robustness Tests: Effect of Fiscal Policy Reforms under Different Assumptions (government debt 45 percent of final output on pre-reform and post-reform BGP)						
Cuts to Capital Income Tax Rate τ^K						
	Physical Capital Income Tax τ^K Rate (%)	Human Capital Income Tax τ^L Rate (%)	Cons. Tax τ^C Rate (%)	Total government revenue and spending (% of final output)	Growth Rate of GDP per Capita (%)	Change in Consumer Welfare (%)
Change of Initial Fiscal Policy						
Baseline	25	25	15	29.24 & 27.66	1.42	--
<i>Reform (offset by unprod. spending cut)</i>	20	25	15	27.54 & 25.93	1.5	5.29
Change of Initial Fiscal Policy						
Baseline	25	25	15	29.24 & 27.66	1.42	--
<i>Reform (offset by cons. tax rise)</i>	20	25	21.25	29.04 & 27.42	1.5	0.05
Change of Coefficient of Relative Risk Aversion to $\sigma = 4$						
Baseline	17.5	17.5	7	20.78 & 17.85	1.42	--
<i>Reform (offset by unprod. spending cut)</i>	12.5	17.5	7	19.05 & 16.06	1.47	3.29
Change of Parameter in Human Capital Production Function ($\phi = 0.85$)						
Baseline	17.5	17.5	7	19.58 & 18	1.42	--
<i>Reform (offset by unprod. spending cut)</i>	12.5	17.5	7	17.84 & 16.21	1.51	5.55
Change of Parameter in Human Capital Production Function ($\phi = 0.25$)						
Baseline	17.5	17.5	7	20.34 & 18.76	1.42	--
<i>Reform (offset by unprod. spending cut)</i>	12.5	17.5	7	18.58 & 16.98	1.46	3.75
Increase in Public Investment (from 4½ to 5½ % of Final Output) when $\gamma, \varepsilon = 0.15$						
Baseline	17.5	17.5	7	19.83 & 18.25	1.42	--
<i>Reform (offset by unprod. spending cut)</i>	17.5	17.5	7	19.91 & 18.21	1.66	10.7

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