WP/16/104

IMF Working Paper

Fiscal Buffers, Private Debt, and Stagnation: The Good, the Bad and the Ugly

by Nicoletta Batini, Giovanni Melina and Stefania Villa

INTERNATIONAL MONETARY FUND

 WP/16/104

IMF Working Paper

European Department and Research Department

Fiscal Buffers, Private Debt, and Stagnation: The Good, the Bad and the Ugly*

Prepared by Nicoletta Batini, Giovanni Melina and Stefania Villa

Authorized for distribution by Christian Mumssen and Prakash Loungani

May 2016

Abstract

We revisit the empirical relationship between private/public debt and output, and build a model that reproduces it. In the model, the government provides financial assistance to credit-constrained agents to mitigate deleveraging. As we observe in the data, surges in private debt are potentially more damaging for the economy than surges in public debt. The model suggests two policy implications. First, capping leverage leads to milder recessions, but also implies more muted expansions. Second, with fiscal buffers, financial assistance to credit-constrained agents helps avoid stagnation. The growth returns from intervention decline as the government approaches the fiscal limit.

JEL Classification Numbers: E44, E62, H63
Keywords: private debt, public debt, borrowing constraints, fiscal limits, DSGE
Author's E-Mail Address: nbatini@imf.org; gmelina@imf.org; stefania.villa@unifg.it

---

* Batini: European Department, IMF; Melina: Research Department, IMF; Villa: University of Foggia & KU Leuven. We are grateful to Olivier Blanchard, Giovanni Callegari, Alessandro Cantelmo, Jacopo Cimadomo, Mark De Broeck, Lorenzo Forni, Vitor Gaspar, Nikolay Guorguiev, Federico Grinberg, Heiko Hesse, Matteo Iacoviello, Prakash Loungani, Racha Moussa, Christian Mumssen, Maurice Obstfeld, Jerome Vandenbussche, Wei Shi, participants to an IMF seminar and the 2015 ECB Conference on "Debt overhang, macroeconomic adjustment and EMU economic governance" for useful comments. All remaining errors are ours.

Contents

List of Tables

List of Figures

1 Introduction

The global financial crisis followed an extraordinary upward swing in the leverage cycle (Geanakoplos et al., 2012).[1] When the bubble burst, the massive debt accumulation in the private sector sparked a typical debt deflation dynamics (Fisher, 1933; Minsky, 1982) that propelled the ratio of public debt-to-GDP very rapidly. This reflected, on one side, the recession-induced decline in government revenues and prices, including those of assets; and, on the other side, governments directly taking over private debt gone sour.

Spurred by such economic developments, late empirical studies have started to focus increasingly more on the relationship between private debt and the macroeconomy. Part of this literature documents the expansion in global credit–especially credit to households–in the advanced world (Jordà et al., 2014) and the links between rapid credit growth and financial crises (Dell'Ariccia et al., 2012; Schularick and Taylor, 2012a; Taylor, 2012; Jordà et al., 2013, 2014). The key messages from this body of research are that credit growth predicts financial crises and that, conditional on having a recession, stronger credit growth predicts deeper recessions (Glick and Lansing, 2010; Dell'Ariccia et al., 2012; IMF, 2012; Schularick and Taylor, 2012b; Jordà et al., 2013; Mian and Sufi, 2014). Mian et al. (2016) take these results a level further, finding, among other things, negative dynamic correlations between global household debt changes and subsequent global growth, contrary to what envisaged by standard macroeconomic models. In addition, analyses in Schularick and Taylor (2012a), Taylor (2012), Jordà et al. (2013), and Mian et al. (2016) have demonstrated that rapid increases in private debt make financial crises more likely, while rises in public debt have no bearing on the probability of a financial crisis (citing Greece as an exception).

Theoretical economic modeling has flanked the empirical research, at least up to a certain point. Building upon the modern model-based literature on collateral and leverage cycles going back to the mid-1990s (pioneered by Bernanke and Gertler, 1995; Bernanke et al., 1999; Kiyotaki and Moore, 1997; Holmstrom and Tirole, 1997; Aoki et al., 2004; and Iacoviello, 2005) a number of recent papers in macro-finance have focused on how to reproduce mechanisms through which excessive indebtedness

[1]The literature defines the leverage cycle as the expansion/contraction of leverage over the business cycle. The existence of procyclical leverage amplifies the effect on asset prices over the business cycle. In turn, a deterioration of the business cycles can accelerate deleveraging. So the leverage and business cycles are distinct, but can reinforce each other over time.

in the private sector can harm the economy (e.g. Eggertsson and Krugman, 2012; Korinek and Simsek, 2014; Martin and Philippon, 2014; Farhi and Werning, 2015; Guerrieri and Lorenzoni, 2015).

None of these models, however, is able to reproduce the macro-financial links between private and public balance sheets observed empirically, nor the dynamic interaction between fiscal and private agents during leverage cycles, which so distinctly characterized both the evolution and the recovery phases of the recent crisis. At the same time, models featuring a fully fledged public sector facing borrowing constraints (such as Corsetti et al., 2013) do not feature the role of the government as a lender of last resort during protracted phases of financial stress. In addition, research so far–notably by Gertler and Karadi (2011)–has focused exclusively on the impact of central bank lending to banks, abstracting from lending to financially-constrained agents and from the government's fiscal limits.

In this paper we want to derive the minimum model structure that reproduces leverage cycles and is suitable to examine a "crisis"-style event featuring high private leverage and government intervention, and then use it to conduct policy analysis. To this end, borrowing the approach of the influential paper by Mian et al. (2016), we start by empirically revisiting the interaction between private and public debt in affecting economic growth. Within a parsimonious specification, we reaffirm the empirical result that public debt does not generally exacerbate recessions. However, we also confirm results in previous literature (Schularick and Taylor, 2012b; Taylor, 2012; Jordà et al., 2013) finding that important nonlinearities are at play between debt and output.This literature finds that the impact of public debt on recessions changes depending on its level. When public debt is high, the negative effects of excessive private debt on growth are harshened. This effect disappears when public debt is medium-low, suggesting that the public sector can (and has likely) alleviate(d) private borrowing constraints during phases of private deleveraging, as long as it still enjoys fiscal space. A rise in private debt, instead, is unambiguously associated with lower output growth.

We then build a parsimonious analytical model that can stylize these interactions by embedding explicit links between private and public debt dynamics.[2] The basic

[2]To be clear, we do not attempt to explicitly model the global financial crisis. This had many channels of shock propagation that made it systemic at the national level, but focusing on the systemic portion of financial risk is beyond the scope of this paper. Besides, in the Iacoviello and Kiyotaki and Moore's tradition, ours is a closed-economy model.

structure follows Kiyotaki and Moore (1997)'s model of credit cycles and it embeds Iacoviello (2005)'s modifications to replicate features of borrowing constraints in the housing market within a New-Keynesian setting. The model is enriched with elements of the literature on government debt and on the sovereign risk premium (along the lines of Corsetti et al., 2013), on one side, and on government intervention in the intermediation of funds (Gertler and Karadi, 2011), on the other side. Thus, the setting accounts explicitly for the two key links between private and public indebtedness that characterize debt deflation dynamics and have played a central role in the recent financial crisis. First, through the financial accelerator, private deleveraging affects output and prices, which in turn depresses government revenues. Second, public debt increases due to government interventions to alleviate private borrowing constraints, and mitigate the consequences of private deleveraging on output and prices. This way we capture how excessive private leverage can infect public finances, and weigh on economic activity; and we can also track the way in which, in turn, increases in public debt associated with financial assistance to the private sector require fiscal consolidation, depressing income and thus potentially aggravating private deleveraging. Shocks and great ratios are calibrated on average euro area data, although its policy lessons are more general.

The model is able to reproduce two main findings of the empirical literature, namely that higher levels of private leverage lead to more severe recessions, with more serious consequences for public finances; and that an initially public high debt level exacerbates the recession because of the lack of fiscal space to stabilize the economy. Our analysis also shows that it is desirable for policymakers to financially assist credit-constrained agents during phases of rapid deleveraging, through targeted interventions aimed at alleviating credit constraints. Crucially, the model is also capable of reproducing realistic caveats to limitless financial assistance related to debt sustainability, on one side; and to the trade-off between costs and benefits of intervention associated with distortionary taxation, the evolution of sovereign risk premia and possible inefficiencies in lending to the private sector directly, on the other side.

The paper is organized as follows. Section 2 sets the context in relation to the data, which we revisit. Section 3 describes the model. Section 4 presents the results. Section 5 concludes and draws policy implications.

2 The link between private and public debt and economic activity revisited

To pin down the basic relationship between total private debt, public debt and subsequent output growth we build on the baseline equation in Mian et al. (2016). This influential paper studies, among other issues, the relationship between the growth in the ratios of private and public debt-to-GDP and GDP growth. Our unbalanced panel annual dataset encompasses the same 30 advanced and emerging market countries, but is slightly lengthier, stretching from 1960 to 2014 (Appendix A.1 reports the sample period available for private and public debt for each country).[3]

Our econometric approach is virtually identical to that in Mian et al. (2016) with the difference that we: (i) focus exclusively on total private debt (rather than also on households' or non-financial corporations' debt individually);[4] (ii) experiment with different measures of debt-to-GDP ratios as regressors, looking also at levels of private and public debt-to-GDP ratios; (iii) focus also on subsamples ordered by the level of the public debt-to-GDP ratio, to explore the nonlinear features that–the literature suggests–underlie the relationship between private and public debt on one side, and output growth on the other side; and (iv) experiment also with different measures of real output as a regressand, namely cyclical deviations from a long-run trend.

Specifically, in our initial panel regression, the dependent variable is future output growth over three years, $\Delta_3 y_{it+3}$, while the two regressors are the change in total private debt-to-GDP ratio in the previous three years, $\Delta_3 \left(\frac{PRD}{Y}\right)_{it-1}$, and the change in public debt-to-GDP ratio, again in the previous three years, $\Delta_3 \left(\frac{PUD}{Y}\right)_{it-1}$. As in Mian et al. (2016), this specification, without trying to prove causality, simply seeks to capture partial correlations between past private and public debt growth and future GDP growth. Using predetermined explanatory variables avoids endogeneity issues.

[3] As in Mian et al. (2016) data on private debt are taken from the BIS dataset. For public debt we combine data from the World Bank World Development Indicators and the IMF World Economic Outlook datasets to maximize the sample size. Real output is taken from IMF International Financial Statistics.

[4] While Mian et al. (2016) also estimate the relationship between total private debt and growth, they find that this relationship is mainly driven by household debt. Both in the empirical analysis and in the DSGE model, we chose to focus on total private debt to be able to derive stylized differences between private and public debt in their relationship with other macro variables.

We estimate the following equation using realized income growth as the dependent variable:

$$\Delta_3 y_{i,t+3} = \alpha_i + \beta_{prd} \Delta_3 \left(\frac{PRD}{Y} \right)_{it-1} + \beta_{pud} \Delta_3 \left(\frac{PUD}{Y} \right)_{it-1} + u_{it}, \tag{1}$$

where $\Delta_3 x_{i,t+3} = x_{i,t+3} - x_{it}$ and i indexes a country.[5]

Table 1 reaffirms in column 1 the negative relationship between changes in private debt and subsequent growth in output. The magnitudes of the relationship are very similar to what Mian et al. (2016) find for total debt: a one standard deviation increase in the change in total private debt-to-GDP ratio (14 percentage points) is associated with a 1.8% lower output growth in subsequent years on average.

Column 2 tests the significance of changes in the public debt-to-GDP ratio to determine output growth three years later, conditional on the change in total private debt.[6] Results confirm both Mian et al. (2016) latest panel regression estimates, as well as findings in Schularick and Taylor (2012a), Taylor (2012) and Jordà et al. (2013), that the change in the ratio of public debt-to-GDP is not a significant predictor of changes in future output when the analogous ratio for private debt is included among the explanatory variables.

Columns 3 and 4 in Table 1 take a slightly different tack, replacing growth rates of the debt-to-GDP ratio (private debt only in column 3; and both private and public debt in column 4) with levels of the debt-to-GDP ratio at time $t-1$.[7] The estimates indicate that, for the full sample, the level of private debt expressed in percent of GDP, lagged one year, is significantly correlated in a dynamic way to subsequent changes in GDP (columns 3 an 4), but the level of public debt-to-GDP is not (column 4)–confirming results using lagged differences in the debt ratios. Columns 5 and 6 try to uncover possible nonlinearities in this relationship by looking at whether the negative predictive effect of changes in the private debt-to-GDP ratio on output

[5] Mian et al. (2016) justify the horizon based on the data and also quote, as rationale for this choice: (i) findings of optimal lag in Baron and Xiong (2014) who, similarly to us, use total bank credit to GDP instead of household vs. non-financial corporations' debt separately; and (ii) work by Dell'Ariccia et al. (2012) who show that the median bank credit boom lasts three years.

[6] Looking at raw data and subsequently playing with different lag specifications we find that private debt seems to lead public debt by a three-year lag.

[7] Debt-to-GDP ratios have unit roots in the case of many of the countries in the sample, but their inclusion is justified by the presence of a cointegrating relationships, for which we test using Westerlund (2007)'s panel cointegration tests. For all test statistics the null hypothesis of no cointegration is rejected at a 1 percent confidence level.

Table 1: Private and Public Debt and Subsequent Real GDP Growth

	Dependent variable: $\Delta_3 y_{it+3}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta_3 \left(\frac{PRD}{Y}\right)_{it-1}$	-0.128***	-0.143***				
	(0.014)	(0.016)				
$\Delta_3 \left(\frac{PUD}{Y}\right)_{it-1}$		-0.014				
		(0.019)				
$\left(\frac{PRD}{Y}\right)_{it-1}$			-0.086***	-0.095***	-0.087***	-0.212***
			(0.005)	(-0.006)	(0.006)	(0.030)
$\left(\frac{PUD}{Y}\right)_{it-1}$				-0.008	0.057***	-0.119***
				(0.011)	(0.015)	(0.032)
$\left(\frac{PUD}{Y}\right)_{it-1}$ ≤95%					✓	
$\left(\frac{PUD}{Y}\right)_{it-1}$ >95%						✓
R^2	0.056	0.108	0.131	0.112	0.125	0.034
Country fixed effects	✓	✓	✓	✓	✓	✓
Observations	873	629	898	700	626	74

Notes: Estimates are obtained via panel regressions of real GDP growth from t to $t+3$ on either the change in private and public debt to GDP from $t-4$ to $t-1$ or the level of private and public debt in $t-1$. All specifications include country fixed effects. *,**,*** denote significance at the 0.1, 0.05, 0.01 level, respectively.

growth is stronger when a country's general government accumulates public debt. To do so, we restrict the sample to observations where public debt is below a certain level (we can stretch to 'only' 95 percent of GDP since, beyond this level, the sample becomes too small to conduct reliable statistical inference). Results indicate that, when the level of public debt is low or medium (i.e. below or equal to 95 percent), the level of the ratio between public debt and GDP becomes positively correlated with subsequent growth, suggesting that fiscal expansions can help attenuate the negative impact of deleveraging and, thus, help sustain growth, as long as public debt is contained (column 5). However, the negative relationship between changes in private debt and subsequent GDP growth is exacerbated if public debt has reached a high level (column 6), and the coefficient of public debt itself changes sign, while remaining significant.

The estimated effect of this nonlinearity is non-negligible. In an environment of low-to-medium public debt levels, the negative impact of a change to the level of the private debt-to-GDP ratio is reduced by one tenth; whereas it doubles when debt is high and fiscal buffers have been largely eroded. These results tally with those of Schularick and Taylor (2012a), Taylor (2012) and Jordà et al. (2013) who find that exposure to a credit boom can make recessions painful, but when combined with an adverse fiscal position at the onset of the crash, economies are perhaps even more vulnerable. Such empirical evidence would suggest that countries with more "fiscal space" are better positioned to weather a financial crisis, likely because they have the room needed to allow automatic stabilizers to work fully and/or can offer stabilizing support to the economy in the form of government's financial assistance to borrowing-constrained agents. Both measures help alleviate the impact of deleveraging on the economy, but their effects are likely to be captured endogenously by the behavior of output in response to fiscal policy. On the other hand, when episodes of high public indebtedness are included, such mechanisms are impaired: for a significant part of the sample, high public debt complicates and harshens deleveraging of the private sector, thereby aggravating its consequences on economic growth.

As a final check we rerun the regressions using detrended real GDP (obtained by HP-filtering real GDP) instead of the change in output. This enables us to verify whether the levels[8] of private and public debt-to-GDP also help predict the cyclical component of output, a variable that corresponds more closely to the measure of output in DSGE models. Columns 1 and 2 in Table 2 suggest that indeed, the time-t level of the private debt-to-GDP ratio is inversely correlated in our sample to detrended output three years later, implying that the higher private debt, the larger the distance between real output from its trend level.[9] While the level of the ratio of public debt-to-GDP is insignificantly related to detrended output for the entire sample or for a sample including only levels of public debt-to-GDP below 95 percent, this correlation becomes negative and significant for levels of the ratio above 95 percent. This confirms that a higher level of public debt relative to GDP exacerbates the adverse effect of high leverage on the business cycle.

[8]We use levels at time t for the regressors instead of lags because endogeneity ceases to be a problem once we use detrended output three periods ahead instead of output growth as a dependent variable.

[9]In these regressions, the coefficients on the debt ratios are evidently much smaller in size than those we obtained when regressing ratios onto output growth, mainly because the filter produces a trend which is very close to trend growth. Changes in the trend thus absorb a great portion of the underlying impact between changes in the private or public debt to GDP ratio and output, lessening the residual impact of changes in these ratios on the cyclical residual.

Table 2: Private and Public Debt and Subsequent Cyclical Fluctuations of Real GDP

	Dependent variable: $\hat{y}_{it+3}$			
	(1)	(2)	(3)	(4)
$\left(\frac{PRD}{Y}\right)_{it}$	-0.007***	-0.012***	-0.012***	-0.043***
	(0.003)	(0.003)	(0.003)	(0.013)
$\left(\frac{PUD}{Y}\right)_{it}$		0.006	0.003	-0.029**
		(0.005)	(0.008)	(0.013)
$\left(\frac{PUD}{Y}\right)_{it} \leq 95\%$			✓	
$\left(\frac{PUD}{Y}\right)_{it} > 95\%$				✓
R^2	0.003	0.005	0.003	0.003
Country fixed effects	✓	✓	✓	✓
Observations	972	743	659	84

Notes: Estimates are obtained via panel regressions of deviations of real GDP from HP(100) trend in $t+3$ on the level of private and public debt in t. All specifications include country fixed effects. *,**,*** denote significance at the 0.1, 0.05, 0.01 level, respectively.

3 Model

To reproduce the relationships between debt and output observed in the data, we build a model where the government has a role of lender of last resort and is tasked not just–as conventionally assumed–with providing public goods financed through taxation and help smooth economic cycle, but also by providing financial assistance in the form of loans to borrowing-constrained agents in the aftermath of financial shocks (similarly to Gertler and Karadi, 2011).The backbone of the model presents financial frictions in the Kiyotaki and Moore (1997)-Iacoviello (2005) closed-economy tradition. The basic structure has been extended to account for fiscal policy, government indebtedness, the sovereign risk premium, and private-public debt interlinkages. The economy is populated by patient households (lenders), impatient households (borrowers), entrepreneurs, the government and the central bank. Patient households work, consume, buy housing, invest in riskless private bonds and in government bond holdings. Impatient households work, consume, and borrow subject to collateral constraints. Entrepreneurs also borrow subject to a collateral constraint and produce in monopolistic competition. The government finances its expenditures by raising a mix of lump-sum and distortionary taxes and by issuing government bonds. Holding government debt is subject to sovereign default risk and the fiscal limit is calibrated on real-world default cases, namely Greece, similarly to Corsetti et al. (2013), among others. Finally, to keep the model simple, but without loss of generality, we do not include banks.[10] It is important to note a few but important definitional conventions in the paper. By *leverage cycle* we mean an increase (decrease) in private indebtedness caused by a loosening (tightening) of borrowing constraints when the debt collateral–of either or both impatient households and entrepreneurs–appreciate (depreciate) in value. By *deleveraging* we refer to a reduction in liabilities achieved through cuts to borrowing. The *crisis* occurs when a drop in the value of the collateral reduces the availability of credit to borrow out of future income. In the paper, *public intervention* refers to credit extended to financially-constrained agents to alleviate borrowing constraints that originate in swings in the value of private debt collateral.

Monetary policy follows a Taylor-type rule, while the fiscal rule implies that government expenditures and taxes react to stabilize public debt compatibly with the government's fiscal limits. The sub-sections below provide more details about the model equations. Appendix B reports first order conditions for the optimization problems of patient households, impatient households and entrepreneurs.

3.1 Patient households

Households are infinitely-lived and solve an intertemporal utility maximization problem. Each household's preferences are represented by the following intertemporal utility function:

$$U_t = E_t \sum_{s=0}^{\infty} \beta^{t+s} \left(\ln X'_{t+s} + e_t^H \zeta \ln h'_{t+s} - \frac{\left(L'_{t+s}\right)^{\eta}}{\eta} \right), \tag{2}$$

where $\beta \in (0,1)$ is the discount factor, X'_t is habit-adjusted consumption, e_t^H is a housing shock as in Iacoviello (2015), h'_t are housing holdings, L'_t is labor supply, ζ is a housing preference parameter and η measures the elasticity of labor with respect

[10]Financial intermediaries are essentially intermediaries between the ultimate lenders and borrowers. Their debt reduction does not influence the assessment of sustainability of the debt burden to the economy, which is the focus of this work. Including banks would add financial frictions, and under certain modeling assumptions, could be set in a way as to magnify leverage cycles by allowing a greater mismatch between debt maturities and risk between ultimate borrowers and lenders. Conversely, if at all, it would buttress the economic forces driving our results, not lessen them. This means that, if anything, our policy implications are starker in that we underestimate the financial accelerator effect.

to the real wage. In particular, X_t' is given by:

$$X_t{}' = C_t' - \theta C_{t-1}', \tag{3}$$

where C_t' is the level of consumption and $\theta \in (0,1)$ is the degree of habit formation.

Households buy consumption goods, C_t' and housing, h_t'. The relative price of housing is q_t. In addition, they invest in riskless private bonds, B_t, and in nominal government bond holdings, B_t^G; pay a mixture of lump-sum, τ_t^L, and distortionary taxes, τ_t^C and τ_t^W, on consumption and labor income, respectively. Each household receives: (i) the hourly wage, W_t'; (ii) the nominal return on private bond holdings, R_t; (iii) the nominal return on government bond holdings, R_t^G, discounted at the *ex-ante* expected haircut rate, Δ_t^G; and (iv) government transfers, Ξ_t. Therefore, households' budget constraint reads as:

$$\begin{aligned} &\left(1+\tau_t^C\right) C_t' + q_t \Delta h_t' + \frac{B_t'}{P_t} + \frac{B_t^G}{P_t} + \tau_t^L \\ \leq &\left(1-\tau_t^W\right) \frac{W_t'}{P_t} L_t' + \frac{R_{t-1} B_{t-1}'}{P_t} + \left(1-\Delta_t^G\right) \frac{R_{t-1}^G B_{t-1}^G}{P_t} + \Xi_t. \end{aligned} \tag{4}$$

3.2 Impatient households

Impatient households choose consumption, C_t'', housing, h_t'', and labor, L_t'', to maximize the following inter-temporal utility function:

$$E_t \sum_{s=0}^{\infty} (\beta'')^{t+s} \left(\ln X_{t+s}'' + e_t^H \zeta \ln h_{t+s}'' - \frac{\left(L_{t+s}''\right)^\eta}{\eta} \right), \tag{5}$$

where $\beta'' < \beta$ is the discount factor, and the habit-adjusted consumption, X_t'', is given by:

$$X_t{}'' = C_t'' - \theta C_{t-1}''. \tag{6}$$

Impatient households face two constraints in their optimization problem. First, the following flow of funds:

$$\left(1+\tau_t^C\right) C_t'' + q_t \Delta h_t'' + \frac{R_{t-1} B_{t-1}''}{\Pi_t} + \frac{R_{t-1} B_{g,t-1}''}{\Pi_t} \leq \left(1-\tau_t^W\right) \frac{W_t''}{P_t} L_t'' + B_t'' + B_{g,t}'', \tag{7}$$

where B_t'' is what they borrow from patient households, $B_{g,t}''$ denotes the amount of credit received if the government decides to mitigate deleveraging in the private sector, and W_t'' is their wage rate. The interest rate paid to the government is the market rate, R_{t-1}.

Second, as in Kiyotaki and Moore (1997) and Iacoviello (2005), impatient households face a limit on their obligation towards patient households arising from the fact that, if borrowers repudiate their debt obligations, lenders repossess their assets minus a proportional transaction cost. Therefore, they face a borrowing constraint, which limits what they can lend to a fraction of the present discounted value of housing holdings:

$$B_t'' \leq m'' E_t \left[\frac{q_{t+1} h_t'' \Pi_{t+1}}{R_t} \right]. \tag{8}$$

The interesting case is a steady state in which the return to savings is above the interest rate. In such a case, borrowing constraint (8) holds with equality and ensures that private borrowing by impatient households, B_t'', equals the present discounted value of housing holdings. As such, parameter m'' denotes the loan-to-value ratio. Moreover, $\beta'' < \beta$ ensures that impatient households will not postpone consumption and accumulate enough wealth to make the borrowing constraint not binding.

3.3 Entrepreneurs

Entrepreneurs are distributed over the unit interval $e \in (0, 1)$ and produce a differentiated goods $Y_{e,t}$ using households' labor, capital and housing as inputs and operate under monopolistic competition, facing a Dixit-Stiglitz firm-specific demand:

$$Y_{e,t} = \left(\frac{P_{e,t}}{P_t} \right)^{-e_t^P \chi} Y_t, \tag{9}$$

where χ is the intertemporal elasticity of substitution across varieties of goods, and e_t^P is an inflation shock.

Their production function specializes as:

$$Y_{e,t} = e_t^A K_{e,t-1}^{\omega} h_{e,t-1}^{\nu} \left(L_{e,t}' \right)^{\alpha(1-\omega-\nu)} \left(L_{e,t}'' \right)^{(1-\alpha)(1-\omega-\nu)}, \tag{10}$$

where $K_{e,t}$ is capital, $h_{e,t}$ is the real estate input, and $L_{e,t}'$ and $L_{e,t}''$ are the labor inputs provided by patient and impatient households, respectively, and e_t^A is a technology shock. While parameters ω and ν are the elasticities of output to capital and real estate, respectively, α represents the contribution of patient households to the labor share.

Like impatient households, also entrepreneurs discount the future more heavily than patient households. Hence the discount factor of the former is lower than that of the latter, $\gamma < \beta$. This leads to entrepreneurs being borrowers as well. They only care about their own consumption, $C_{e,t}$, and maximize the following inter-temporal utility function:

$$U_t = E_t \sum_{s=0}^{\infty} \gamma^{t+s} \ln \left(X_{e,t+s} \right), \tag{11}$$

where habit-adjusted consumption, $X_{e,t}$, is given by:

$$X_{e,t} = C_{e,t} - \theta C_{e,t-1}, \tag{12}$$

subject to the entrepreneurial flow of funds:

$$\begin{aligned} \frac{P_{e,t}}{P_t} Y_{e,t} + B_{e,t} + B_{ge,t} = \left(1 + \tau_t^C\right) C_{e,t} + q_t \Delta h_{e,t} + \frac{R_{t-1} B_{e,t-1}}{\Pi_t} \\ + \frac{R_{t-1} B_{ge,t-1}}{\Pi_t} + w_t' L_{e,t}' + w_t'' L_{e,t}'' + I_{e,t} + \xi_{K,t} + \xi_{P,t}, \end{aligned} \tag{13}$$

where $w_t' \equiv \frac{W_t'}{P_t}$; $w_t'' \equiv \frac{W_t''}{P_t}$; $B_{e,t}$ represents their debt obligations towards private agents; $B_{ge,t}$ is the credit directly intermediated by the government in case of intervention (analogously to the case of impatient households); $I_{e,t}$ is investment in capital goods following law of motion:

$$I_{e,t} = K_{e,t} - (1 - \delta) K_{e,t-1}, \tag{14}$$

and $\xi_{K,t} \equiv \frac{\psi_K}{2\delta} \left(\frac{I_{e,t}}{K_{e,t-1}} - \delta \right)^2 K_{e,t-1}$ and $\xi_{P,t} \equiv \frac{\psi_P}{2} \left(\frac{P_{e,t}}{P_{e,t-1}} - 1 \right)^2 Y_t$ are quadratic costs of adjusting the capital stock and resetting the price level, respectively.

Also entrepreneurs face a limit on their obligations towards patient households:

$$B_{e,t} \leq m E_t \left[\frac{q_{t+1} h_{e,t} \Pi_{t+1}}{R_t} \right]. \tag{15}$$

The considerations made for impatient households' borrowing constraint apply also to the case of entrepreneurs.

3.4 Government

The government finances its expenditures, G_t, by levying taxes, T_t, and by issuing bonds, B_t^G. It promises to repay one-period bonds the next period and the gross nominal interest rate applied is R_t^G. However, in order to introduce a sovereign risk premium, we assume that government bond contracts are not enforceable. As in Bi and Traum (2014), each period a stochastic fiscal limit expressed in terms of government debt-to-GDP ratio and denoted by Γ_t^*, is drawn from a distribution, the cumulative density function (CDF) of which is represented by a logistical function, p_t^*, with parameters η_1 and η_2:

$$p_t^* = P\left(\Gamma_t^* \leq \Gamma_t\right) = \frac{\exp\left(\eta_1 + \eta_2 \Gamma_t\right)}{1 + \exp\left(\eta_1 + \eta_2 \Gamma_t\right)}, \tag{16}$$

where $\Gamma_t \equiv B_t^G / Y_t$. If government-debt-to-GDP exceeds the fiscal limit, i.e. $\Gamma_t \geq \Gamma_t^*$, then the government defaults. Hence p_t^* represents the probability of default. This occurs in the form of an haircut $\Delta_t^G \in [0,1]$ applied as a proportion to the outstanding stock of government debt. In order to be able to solve the model with perturbation methods, we follow Corsetti et al. (2013) and Cantore et al. (2015) in assuming that agents consider the *ex-ante* expected haircut rate,

$$\Delta_t^G = \begin{cases} 0 & \text{with probability } 1 - p_t^* \\ \bar{\Delta}^G & \text{with probability } p_t^* \end{cases}, \tag{17}$$

where $\Delta^G \in (0,1]$ is the haircut rate applied in the case of default. In other words:

$$\bar{\Delta}_t^G = p_t^* \bar{\Delta}^G. \tag{18}$$

The government has the option of direct intervention in the intermediation of funds towards financially constrained agents as a way to mitigate deleveraging in the face of negative shocks, using a mechanism similar to that proposed by Gertler and Karadi (2011). If government intermediation occurs, the government issues additional bonds $B_t^{int} \equiv B''_{g,t} + B_{g,t}$, that pay the gross nominal interest rate R_t^G, and lends the raised funds to the private sector at the market rate R_t. This operation comes at the cost of an efficiency loss equal to κ per unit supplied due to costs of raising funds through government debt. The total loss affecting the government budget constraint is then $\Upsilon_t \equiv \kappa B_t^{int}$, which is a dead weight loss.

Simple rules define how the government intervention takes place, and link government intervention to deleveraging, to an extent controlled by parameter ϵ:

$$b''_{g,t} = -\epsilon b''_t, \tag{19}$$

$$b_{g,t} = -\epsilon b_t, \tag{20}$$

where lower-case letters indicate deviations of debt variables from their respective steady state, relative to steady-state output, $x_t \equiv \frac{X_t - X}{Y}$. We assume that, at the steady state, no government intervention occurs ($B''_g = B_g = 0$), hence when $\epsilon = 0$ the model collapses to the standard case in which funds are entirely exchanged in the private sector.

A significant departure from the mechanism of Gertler and Karadi (2011) is that here the government is subject to fiscal limits giving rise to a sovereign risk premium. Therefore an additional cost, given by the spread $\left(R_t^G - R_t\right)$ times the units of funds intermediated B_t^{int}, enters the government flow of funds, which reads as:

$$B_t^G = \left(1 - \Delta_t^G\right) \frac{R_{t-1}^G B_{t-1}^G}{\Pi_t} + G_t + \frac{\left(R_{t-1}^G - R_{t-1}\right) B_{t-1}^{int}}{\Pi_t} + \Upsilon_t - T_t + \Xi_t. \tag{21}$$

As in Corsetti et al. (2013), each period, transfers are set in a way that sovereign default does not alter the actual debt level, $\Xi_t \equiv \Delta_t^G \frac{R_{t-1}^G B_{t-1}^G}{\Pi_t}$.[11]

Total government revenue T_t is given by:

$$T_t = \tau_t^C \left(C'_t + C''_t + C_t\right) + \tau_t^W \left(w'_t L'_t + w''_t L''_t\right) + \tau_t^L. \tag{22}$$

In order to reduce the number of tax instruments to one, we impose that τ_t^C, τ_t^W and τ_t^L deviate from their respective steady state by the same proportion (i.e. $\tau_t^C = \tau_t \bar{\tau}^C$, $\tau_t^W = \tau_t \bar{\tau}^W$, $\tau_t^L = \tau_t \bar{\tau}^L$), and that the proportional uniform tax change, τ_t, becomes one of our fiscal policy instruments. As common in the literature, the steady-state

[11] The absence of such transfers would imply lower risk premia prior to default, as the lower post-default debt stock would already be taken into account.

value of the lump-sum tax is treated as a residual to calibrate the government debt at a desired steady-state level.

We allow the tax and government spending instruments to be adjusted according to the following feedback rules:

$$\log\left(\frac{\tau_t}{\tau}\right) = \rho\log\left(\frac{\tau_{t-1}}{\tau}\right) + (1-\rho)\left[e^{\phi\frac{B^G}{Y}}\rho_B\log\left(\frac{B^G_{t-1}}{B^G}\right)\right], \tag{23}$$

$$\log\left(\frac{G_t}{G}\right) = \rho\log\left(\frac{G_{t-1}}{G}\right) - (1-\rho)\left[e^{\phi\frac{B^G}{Y}}\rho_B\log\left(\frac{B^G_{t-1}}{B^G}\right)\right], \tag{24}$$

where ρ implies persistence in the fiscal policy instruments; ρ_B is the responsiveness of the instruments to the percent deviation of government debt from its steady state; and $e^{\phi\frac{B^G}{Y}}$ is an exponential factor augmenting the fiscal policy stance for increasing steady-state levels of the government debt-to-GDP ratio, in order to expand the model's stability region for high levels of government debt (which imply high sovereign risk premia). Although in practice the government may exhibit different degrees of inertia and elasticities for different instruments, assuming the same parameters for all fiscal instruments greatly simplifies the exercises presented in the following sections without loss of generality.

3.5 Central bank

Monetary policy is set according to a Taylor-type interest-rate rule,

$$\log\left(\frac{R_t}{R}\right) = \rho_\pi\log\left(\frac{\Pi_t}{\Pi}\right) + \rho_y\log\left(\frac{Y_t}{Y}\right), \tag{25}$$

where ρ_π and ρ_y are the monetary responses to inflation and output relative to their steady-state values.

3.6 Equilibrium

Equilibrium in the goods market, the loans market, and the housing market implies that $Y_t = C_t + C'_t + C''_t + I_t + G_t + \Upsilon_t + \xi_{P,t} + \xi_{K,t}$; $B_t + B'_t + B''_t = 0$; and $h + h' + h'' = 1$. This last equilibrium condition in turn implies that housing is in fixed supply, which we normalize to one. The model is completed by autoregressive processes for the shocks, $\log\left(\frac{e^{\varkappa}_t}{\bar{e}^{\varkappa}}\right) = \rho_\varkappa\log\left(\frac{e^{\varkappa}_{t-1}}{\bar{e}^{\varkappa}}\right) + \epsilon^{\varkappa}_t$, where $\varkappa = \{A, H, P\}$, $\rho_\varkappa$ are autoregressive parameters and $\epsilon_t^{\varkappa}$ are mean zero, i.i.d. random shocks with standard deviation $\sigma^{\varkappa}$.

4 Parameter Values

Table 3 reports the parameter values used to simulate the model. For the baseline scenario, to the extent possible, we choose parameters to match stylized facts in line with the average euro area experience. For a few parameters, the estimates of which are not available for the euro area, we borrow estimates for the United States. Shocks are calibrated to match key moments in euro area data. The time period in our model corresponds to one quarter in the data.

We borrow the following parameter values from Iacoviello (2005): agents' discount factors, $\beta = 0.99$, $\beta'' = 0.95$, and $\gamma = 0.98$; the labor supply elasticity, $\eta = 1.01$; capital depreciation rate, $\delta = 0.03$; capital share, $\omega = 0.30$; patient households' wage share, $\alpha = 0.64$; and capital adjustment costs, $\psi_K = 2$.

The value of habit persistence, $\theta = 0.592$, is taken from Smets and Wouters (2003), while for the Taylor rule parameters we choose values that satisfy the Taylor principle $\rho_\pi = 1.5$ (Taylor, 1993), and assign a small reaction to output $\rho_y = 0.1$, in line with Smets and Wouters (2003). For the steady-state values of the share of government spending in GDP, $\bar{G}/\bar{Y} = 0.23$, and the two distortionary tax rates, $\bar{\tau}^C = 0.20$ and $\bar{\tau}^W = 0.45$, as well as the degree of price stickiness, $\psi_P = 41.667$, we rely on the values used by Christiano et al. (2010) for the euro area.[12] Then, in line with the data, we make fiscal instruments persistent ($\rho = 0.90$). We set the degree of fiscal stance, $\rho_B = 0.01$, and its responsiveness to government debt, $\phi = 1.4$, to approximately the minimal value needed to stabilize public debt in the range of government debt-to-GDP ratios explored. The elasticity of substitution across different varieties, χ, is equal to 6 in order to target a steady state gross mark-up equal to 1.20.

The steady-state stock of residential housing over annual output, $\bar{q}\left(\bar{h}' + \bar{h}''\right) / \left(4\bar{Y}\right) = 1.34$, is taken from the the OECD database on balance sheet for non-financial assets on households dwellings in France and Germany between 2000 and 2013.[13] Such a value is matched through an appropriate choice of ζ. The steady-

[12] The value of ψ_P is chosen to match the same slope of the linearized New-Keynesian Phillips curve of Christiano et al. (2010) where prices are set as in Calvo (1983).

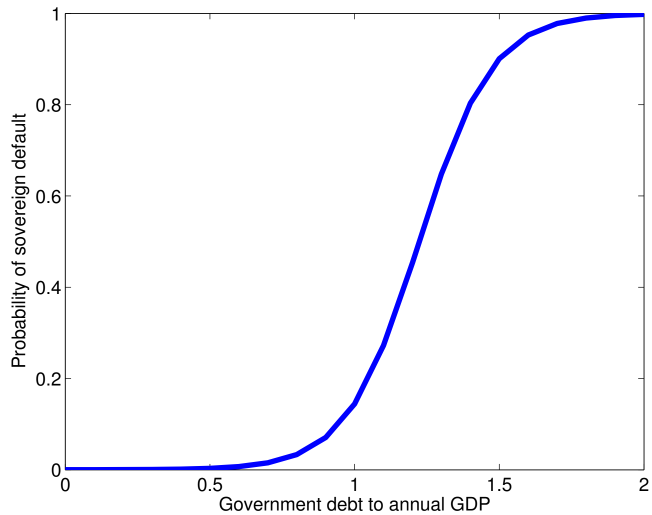
[13] The steady-state stock of residential housing over annual output has a similar value when considering the average of euro area countries.

Table 3: Baseline Parameter Values

Parameter		Value
Patient households' discount factor	β	0.99
Impatient households' discount factor	β''	0.95
Entrepreneurs' discount factor	γ	0.98
Labor supply elasticity	η	1.01
Habits in consumption	θ	0.592
Capital depreciation rate	δ	0.03
Capital share	ω	0.30
Patient households' wage share	α	0.64
Capital adjustment costs	ψ_K	2.00
Elasticity of substitution in goods	χ	6.00
Price stickiness	ψ_P	41.667
Inflation -Taylor rule	ρ_π	1.5
Output -Taylor rule	ρ_y	0.1
SS stock of residential housing over annual output	$\bar{q}\left(\bar{h}' + \bar{h}''\right) / \left(4\bar{Y}\right)$	1.34
SS commercial real estate over annual output	$\bar{q}\bar{h}/\left(4\bar{Y}\right)$	0.65
SS share of government spending in GDP	$\bar{G}/\bar{Y}$	0.23
SS consumption tax rate	$\bar{\tau}^C$	0.20
SS labor income tax rate	$\bar{\tau}^W$	0.45
Persistence of fiscal instruments	ρ	0.90
Fiscal responsiveness to government debt	ρ_B	0.01
Responsiveness of the fiscal stance to government debt	ϕ	1.4
Scaling factor in default probability	η_1	-8.5527
Slope parameter in default probability	η_2	1.8261
Government intervention	ϵ	0.10
Efficiency costs	κ	0.10
SS impatient households loan-to-value ratio	m''	0.80
SS entrepreneurs loan-to-value ratio	m	0.375
SS debt-to-GDP ratio	$\bar{\Gamma}$	0.68
Persistence of housing shock	ρ_H	0.9890
Persistence of inflation shock	ρ_P	0.8171
Persistence of technology shock	ρ_A	0.0421
Standard deviation of housing shock	σ^H	0.0098
Standard deviation of inflation shock	σ^P	0.0015
Standard deviation of technology shock	σ^A	0.0233

state commercial real estate over annual output, $\bar{q}\bar{h}/\left(4\bar{Y}\right) = 0.65$, is taken from the OECD database on balance sheet for non-financial assets on dwellings of non-financial corporations in France and Germany between 2000 and 2013. Such a value is matched through an appropriate choice of ν. In the baseline case, the households'

Figure 1: Cumulative Density Function of the Fiscal Limit



LTV ratio, m, is equal to 0.80, the typical LTV ratio for a new mortgage in the majority of the euro area countries in 2007 (ECB, 2009). The entrepreneurial LTV, $m = 0.375$, is taken from data on corporate indebtedness in the Euro Area (ECB, 2012). Last, the debt-to-GDP ratio $\bar{\Gamma} = 0.68$ corresponds the average of euro area countries between 1999 and 2007. Given that the parameters related to government and private indebtedness are crucial for the results, we explore sensitivity to a wide range of values in Section 5.

Moreover, the baseline scenario exhibits a small degree of government intervention, ϵ, equal to 0.10 and an efficiency cost, κ, set at 0.1 in line with Gertler and Karadi (2011). We nonetheless show how alternative values of these two parameters affect the results.

To calibrate the CDF of the fiscal limit, depicted in Figure 1, we fix two points on the function in a way consistent with empirical evidence. Given two points (Γ_1, p_1^*) and (Γ_2, p_2^*), with $\Gamma_2 > \Gamma_1$, parameters η_1 and η_2 are uniquely determined by

$$\eta_2 = \frac{1}{\Gamma_1 - \Gamma_2} \log \left(\frac{p_1^*}{p_2^*} \frac{1 - p_2^*}{1 - p_1^*} \right), \tag{26}$$

$$\eta_1 = \log \left(\frac{p_1^*}{1 - p_1^*} \right) - \eta_2 \Gamma_1. \tag{27}$$

Let us assume that when the ratio of government debt to annual GDP is Γ_2, the probability of exceeding the fiscal limit is almost unity, i.e. $p_2^* = 0.99$. We set the fiscal limit at $\Gamma_2 = 4 \times 1.8$, broadly in line with the Greek experience. Let us fix $\Gamma_1 = 4 \times 0.6$, the average general government consolidated gross debt in the United States over the period 1980-2007. Before the financial crisis the U.S. sovereign risk premium was very small–around 15 annual basis points (ABP) for sovereign default swap spreads (see e.g. Austin and Miller, 2011). Hence we assume that for $\Gamma_1 = 4 \times 0.6$, $ABP_1 = 15$. At the onset of the Greek sovereign debt crisis, the sovereign risk premium skyrocketed to an order of magnitude of around 1,000 annual basis points, hence we fix $ABP_2 = 1,000$. The haircut rate, $\bar{\Delta}$, consistent with ABP_2 and p_2^* is obtained as $\bar{\Delta} = \left[1 - \frac{1}{\frac{ABP_2}{40000}+1}\right] / p_2^*$.[14] At this point, we can recover the probability of default when $\Gamma = \Gamma_1$,

$$p_1^* = \frac{1 - \frac{1}{\frac{ABP_1}{40000}+1}}{\bar{\Delta}},$$

which is $p_1^* = 0.0152$, and parameters η_1 and η_2 of the fiscal limit CDF can be recovered by using equations (26) and (27), i.e. $\eta_1 = -8.5527$ and $\eta_2 = 1.8261$. As shown in Figure 1, this parametrization implies that the probability of default remains moderate (below 20%) until the government debt-to-annual-GDP is below 100% and then increases at an expedited rate. This captures the fact that problems related to sovereign default may mount at a very fast pace as public debt accumulates.

Last, we set (i) the standard deviations, and (ii) the persistence of the shocks via moment-matching of (a) the empirical standard deviations and (b) the persistence of real output, inflation and the real house price.

Given the difficulty in matching exactly all moments, we construct a quadratic loss function $L = \sum_{j=1}^{6} \left(x_j^m - x_j^d\right)^2$, where x_j^m is the j-th moment in the model and x_j^d is its analogue in the data, and we numerically search for those parameters that minimize L. This procedure leads to persistent housing and inflation shocks, $\rho_H =$

[14]To see this, note that equations (B.3) and (B.4) imply the following steady-state sovereign risk premium:

$$\frac{R^G}{R} = \frac{1}{(1 - \Delta^G)} = 1 + \frac{ABP}{40000},$$

using which Δ^g can be written as a function of a chosen premium expressed in annual basis points, $\Delta^g = 1 - \frac{1}{1+\frac{ABP}{40000}}$. Finally, from equation (18) $\bar{\Delta}^G = \Delta_t^G / p_t^*$.

Table 4: Moments of Key Macroeconomic Variables

Moment	Data	Model
Standard deviations		
Real output	0.0138	0.0094
Inflation	0.0061	0.0046
Real house prices	0.0158	0.0175
Autocorrelations		
Real output	0.8779	0.9511
Inflation	0.2386	0.2685
Real house prices	0.8614	0.8441
Cross-correlations with output		
Investment	0.8221	0.9826
Private consumption	0.9218	0.9952

0.9843 and $\rho_P = 0.8431$; while, as in Iacoviello (2005), the technology shock exhibits a small persistence $\rho_A = 0.0301$, as the model produces significant endogenous persistence. The standard deviations of the shocks are of magnitudes of around 1% and 2%.

Table 4 shows the volatilities, persistences and correlations of variables in the data and in the model that we directly target, as well as two other important moments.[15] Overall, the model replicates reasonably well the moments in the data and gets close to the cross-correlation of investment and private consumption with output.

Table 5 reports dynamic correlations between private and public debt/GDP ratios and the output gap calculated on simulated data from the model. Correlations show that the model behaves in line with historical data and our panel regressions. First, in line with the standard behavior of the leverage cycle, time-t private debt is positively correlated with the output gap at time t while public debt displays an inverse contemporaneous correlation with the output gap. Second, under the baseline calibration, private debt is negatively correlated with the future output gap, and more strongly so three years out. In contrast, simulated data do not exhibit a

[15]Data on euro area countries are taken from the Statistical Data Warehouse of the ECB and the International Financial Statistics database of the IMF. They refer to the period 1999Q1-2015Q1 (or shorter where observations are not available). Time series of GDP components and real house prices are detrended using the HP filter.

Table 5: Dynamic Correlations Between Private/Public Debt/GDP Ratios and the Output Gap in Simulated Data

	$corr\left(\frac{B_t^{TOT}}{4Y_t}, Y_{t+i}\right)$		$corr\left(\frac{B_t^{G}}{4Y_t}, Y_{t+i}\right)$	
	Baseline	High private debt	Baseline	High public debt
$i=0$	0.5421***	0.5039***	-0.2057***	-0.3363***
$i=4$	0.3057***	0.2814***	-0.0632	-0.2044***
$i=6$	0.1329***	0.0832*	0.0006	-0.1318***
$i=8$	-0.0061	-0.0590	0.0422	-0.0744*
$i=10$	-0.0714	-0.0986**	0.0588	-0.0383
$i=12$	-0.1005**	-0.0917**	0.0635	-0.0063

Notes: Correlations are computed on simulated time series of length 500 quarters. B_t^{TOT} is total private debt. High private debt refers to LTV ratios in the high range of the distribution in the euro area experience, $m'' = 0.99$ and $m = 0.44$; high government debt refers to $\Gamma = 1$. *,**,*** denote significance at the 0.1, 0.05, 0.01 level, respectively.

significant correlation between public debt and the future output gap. Third, using a higher steady-state level of the private debt/GDP ratio ($m'' = 0.99$ and $m = 0.44$), the simulated data imply once more a negative correlation between private debt and future levels of the output gap (the correlation peaks slightly sooner). Finally, if the steady-state level of the public debt/GDP ratio is set to a high value ($\Gamma = 1$), the dynamic correlation between public debt and the output gap becomes negative and significant from a year out.

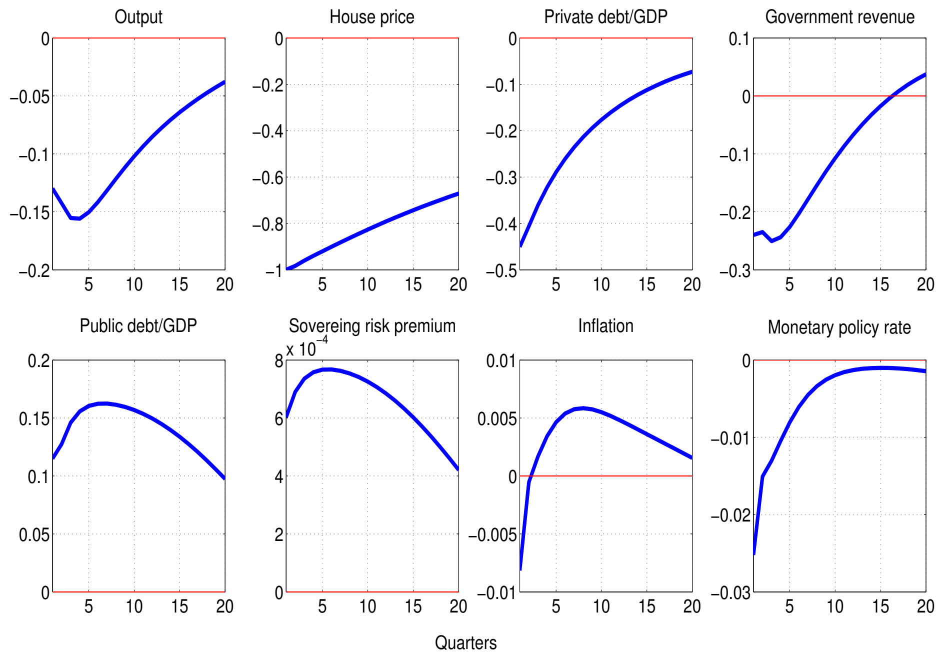
5 Results

5.1 Do the levels of private and public debt amplify swings in economic activity over the leverage cycle?

This section first analyzes the macroeconomic consequences of deleveraging and then discusses the role of private and public debt overhangs in affecting the response of key variables in the model. We trigger a downward phase of a leverage cycle with a temporary negative house price shock, which depresses the value of the housing collateral. In the experiments discussed throughout, the shock is such that house prices fall by one percent.

In Figure 2 the protracted decline in house prices, and the consequent fall in the value of constrained agents' collateral make borrowing constraints tighter. This

Figure 2: Impulse Responses to a Negative One-Per-Cent House Price Shock



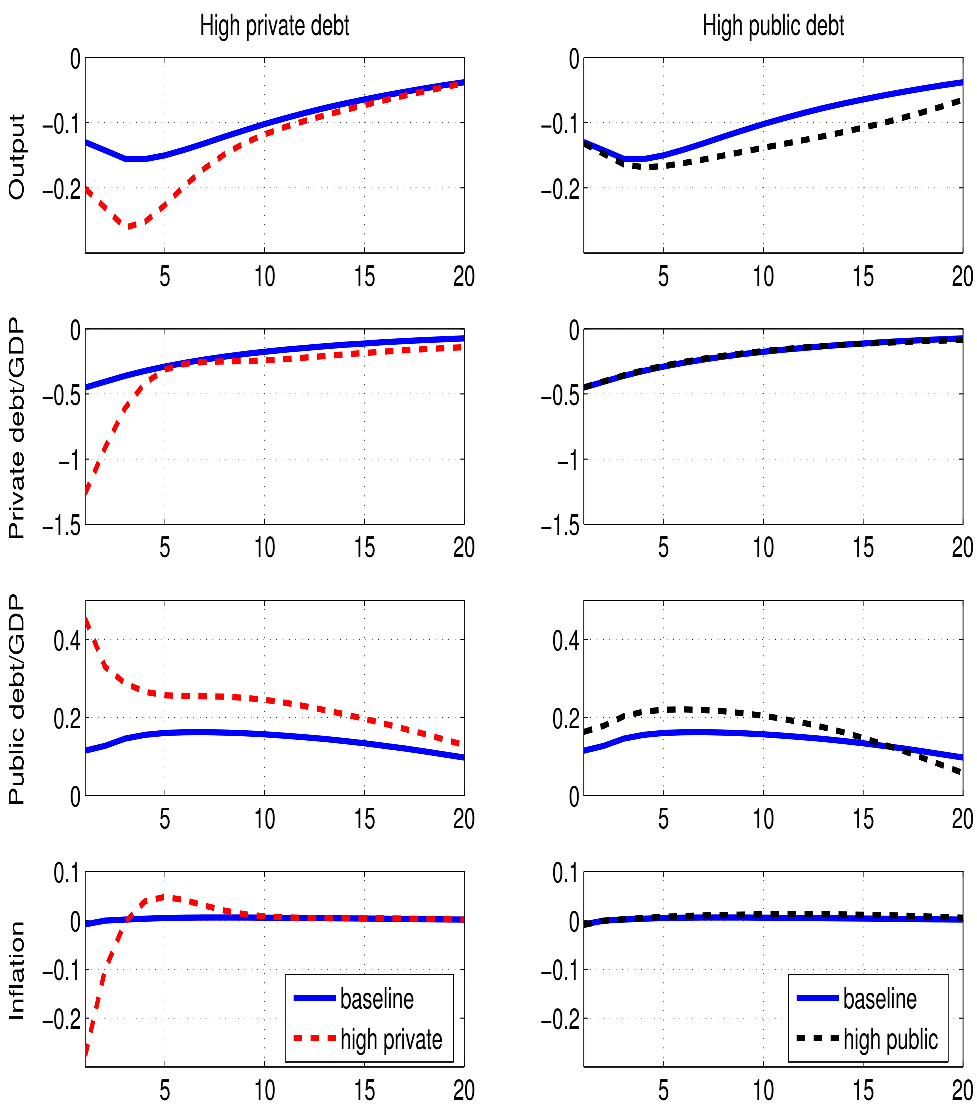
Notes: X-axes in quarters; Y-axes are in percent deviations from steady state, except for private and public debt to GDP ratios where deviations are absolute.

forces private agents to deleverage by cutting consumption and investment. In turn, this fall in private demand implies a protracted output contraction and a deflation. The size of the response matches well the observed relationship between changes in house prices and the output gap in advanced economies.[16] The worsened economic outlook spills over to public finances: the fall in output induces a reduction of government revenues and the public debt-to-GDP ratio unambiguously rises. This mechanism is enhanced (i) by debt deflation; (ii) by the fact that higher public indebtedness boosts the sovereign risk premium, causing higher government's financing costs; and (iii) by the response of the government–which, we assume, reacts endogenously via equations (19) and (20)–to partially mitigate the private sector deleveraging itself, entailing the payment of premium $R_t^G - R_t$ in the financial market and efficiency losses (intervention is small in the baseline calibration, and its effects are disentangled in Subsection 5.2).

What roles do private/public debt overhangs have in amplifying swings in eco-

[16]For example, in 2009q1, the S&P/Case-Shiller Home Price Index fell by about 24% from its trend and by the end of 2009 the U.S. output gap had reached 3.2%, a level close to what the model would suggest, $\approx 0.15 \times 24 = 3.7$.

Figure 3: Impulse Responses to a Negative One-Per-Cent House Price Shock: Effects of High Private and Public Debt



Notes: High private debt refers $m'' = 0.99$ and $m = 0.44$; high government debt refers to $\Gamma = 1$; X-axes in quarters; Y-axes are in percent deviations from steady state, except for private and public debt to GDP ratios where deviations are absolute.

nomic activity over the leverage cycle? To answer this question, in Figure 3 we compare the baseline results against alternative scenarios obtained assuming higher private or public debt at the steady state.

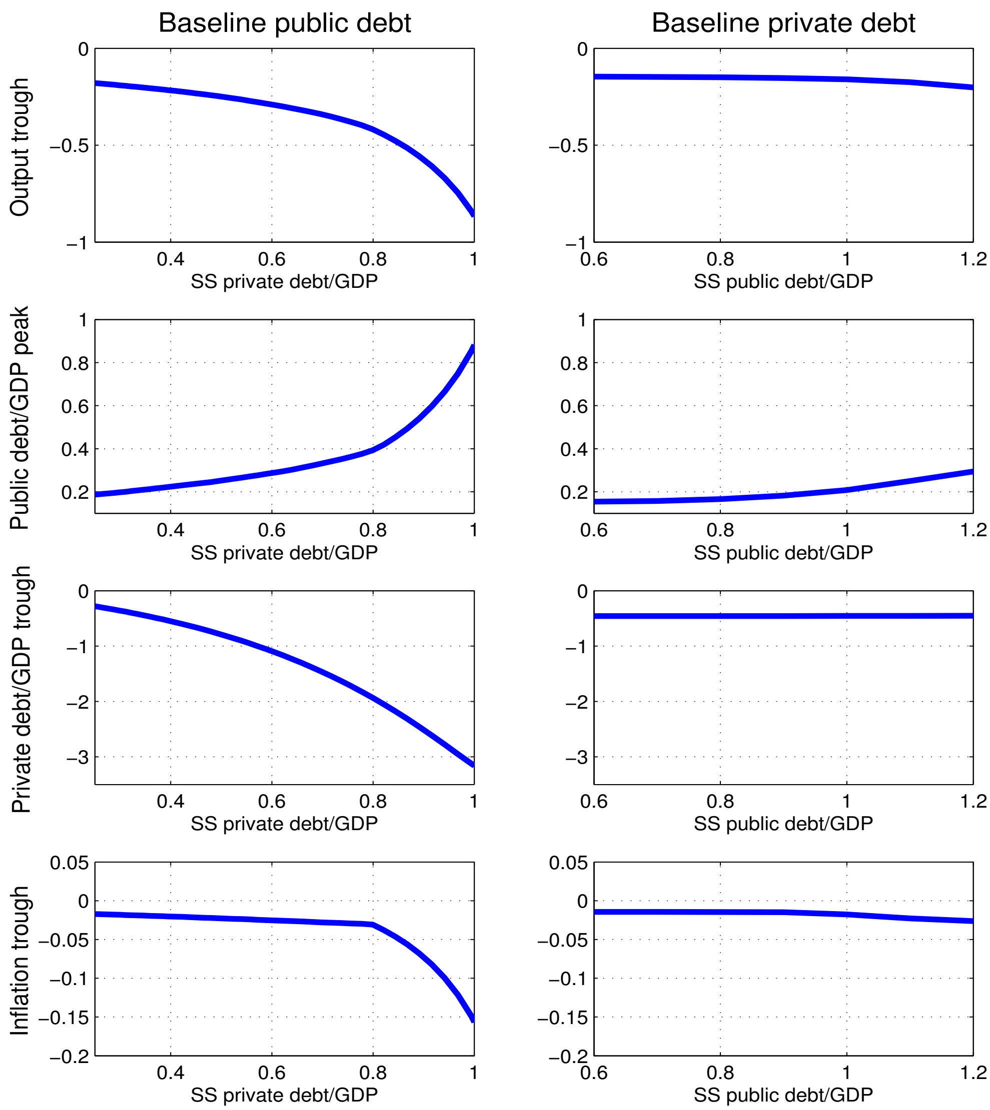
In particular, the first column considers an economy where, at the steady state, public debt stays at the baseline value while private debt is higher because the LTV ratios are set at levels in the high range of the distribution in the euro area experience (ECB, 2012)–$m'' = 0.99$ and $m = 0.44$. Because of the powerful financial accelerator effect at play, such an economy experiences a deeper and stronger deleveraging than an economy with baseline private debt and, consequently, a more severe fall in aggregate demand that ultimately triggers a deeper GDP contraction and deflation. The resulting stronger fall in government revenues, combined with the collapse in output and the debt deflation effect, also leads to a more pronounced increase in public debt as a fraction of GDP.

The second column shows an economy with baseline private debt, but a public debt that, as a fraction of GDP, is high (i.e. set from values in the top percentiles of cross-country averages reported in Table A.1), but still well below the fiscal limit, $\bar{\Gamma} = 1$. In this case, the recession is milder, yet more persistent, relative to the case of high private debt, while the effects on deleveraging and inflation are negligible compared to the baseline scenario. In fact, in response to the negative shock–unlike the private sector who is facing borrowing constraints–the government resorts to more borrowing and can partially absorb the shock itself, despite smaller fiscal buffers and higher financing costs than in the baseline scenario. The more protracted recession is due to a higher-than-baseline sovereign risk premium, leading to higher interest rate payments, in turn demanding higher tax rates in the future. This case is reminiscent of the point made by Ostry et al. (2015), whereby if public debt is sufficiently below that implied by the fiscal limit–the government is still better off increasing its debt further to absorb a negative shock.

These results are not confined to the specific parameter choice adopted in Figure 3, but they hold true across plausible ranges of the LTV ratio and debt/GDP ratios. This conclusion emerges by looking at Figure 4, where we plot how, following an identical negative house price shock, the severity of the contraction in output, private and public debt-to-GDP ratios, and inflation vary with (i) different caps on the LTV ratio (for ease of comparison with public debt, on the x-axis we report the resulting private debt/GDP ratio); and (ii) different long-run (steady-state) targets of the public debt-to-GDP ratio.[17] Four important additional findings emerge. First, the

[17] Specifically, first we keep the steady-state level of public debt/GDP at the baseline value of $\Gamma = 0.68$ (left column), let the LTV ratios vary by the same amount ($m = m'' \in [0.375, 0.95]$) and we plot the corresponding peak responses of output, public debt/GDP, private debt/GDP

Figure 4: Peak Responses to a Negative One-Per-Cent House Price Shock for Different Loan-to-Value (LTV) Ratios and Different Steady-State (SS) Public Debt/GDP Ratios



Notes: In the left column the LTV ratios, m and m'', vary between 0.375 and 0.95; for ease of comparison with public debt, on the x-axis we report the resulting private debt/GDP ratio. In the right column, the steady-state government debt-to-GDP ratio, Γ, varies between 0.6 and 1.2; Y-axes are in percent deviations from steady state except for private and public debt to GDP ratios where deviations are absolute.

---

and inflation. Second (right column), we keep the steady state of private debt the baseline level ($m = 0.375$ and $m'' = 0.80$), let the steady-state level of GDP vary in the interval $\bar{\Gamma} \in [0.6, 1.2]$, and plot the same variables.

economic contraction is increasingly worse the higher the LTV ratio. In contrast, the initial level of public debt has no bearing on the severity of the contraction if public debt is below a certain level (somewhere about 100% of annual GDP in our calibrated model, but potential at higher/lower levels depending on country-specific conditions), in line with our empirical results based on panel regressions.[18] Second, the public debt/GPD ratio resulting after a shock is positively correlated with the initial level of private debt. The larger private liabilities before the shock hits, the worse the public debt legacy afterwards, because the private sector will be facing a faster deleveraging from a more adverse starting point, which will also activate greater government support, other things equal. Third, higher caps on the LTV ratio cause more deleveraging, while the amount of deleveraging that takes place after the shock marginally depends on the level of public debt. Fourth, the deflationary effects of the negative house price shock are stronger the higher the LTV ratio, while the inflation rate is barely affected by the steady state level of public debt.

5.2 Should governments extend financial assistance to credit-constrained agents at times of financial stress?

In our model, the government can lend money to private sector borrowers (i.e. impatient households and entrepreneurs) at times when swings in the value of their debt collateral and their binding borrowing constrains would force a pronounced deleveraging. This captures real world policy measures taken during the crisis to facilitate mortgage payments by agents in distress (e.g. in the United States), government credit (either in cash or tax credit form) for home renovation, or other initiatives to spur spending on consumer durables (e.g. the program "Cash-for-Clunkers" launched in the United States in 2009-10), in addition to more widespread practices of financial assistance to private borrowers vehicled indirectly via direct support to financial intermediaries.

For the government there is an obvious merit in relaxing the private sector's borrowing constraints at times of stress: by allowing them to smooth spending through a deleveraging phase, the government is *de facto* indirectly supporting economic activity, which in turn prevents a drop in government revenues that would other-

[18]This result is present but not strongly apparent in the figure due to the same scaling of the y-axis of the charts on the left and right-hand side columns.

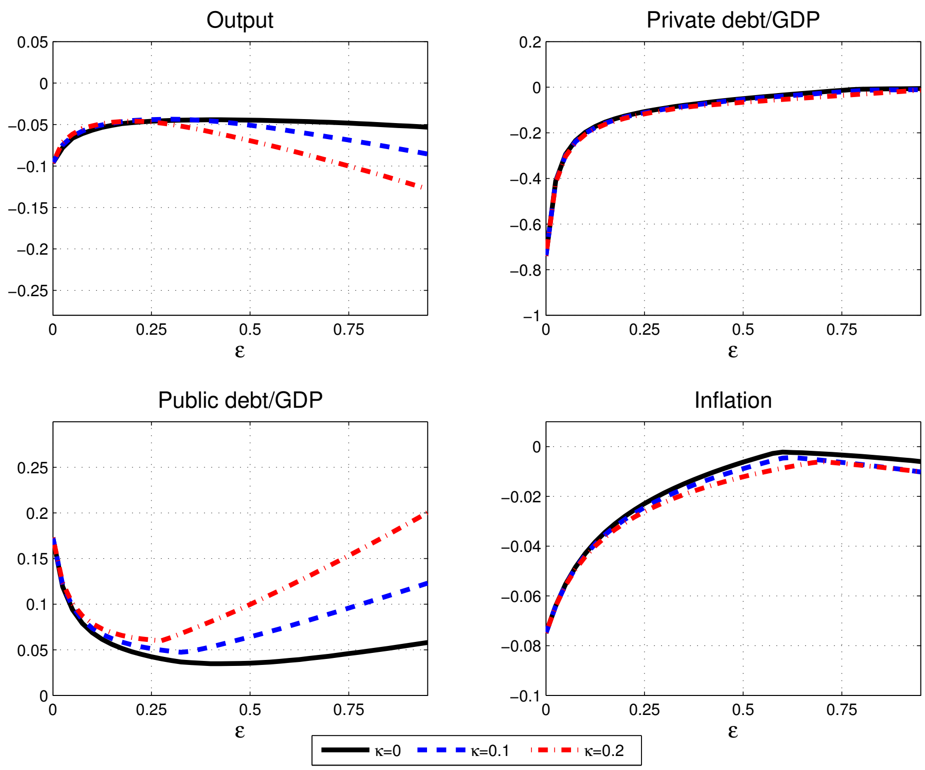
wise permanently lost. There are two obvious trade-offs. The first has to do with intervention itself. To be worthwhile, the output/fiscal revenue support of the intervention must be large enough to outweigh the adverse impact on output (and hence fiscal revenues) of subsequent fiscal consolidations to rein in spending on intervention (the government financial assistance pushes up public debt). Second, to intervene the government must have sufficient fiscal space. Like in the real world, in our model this is given by the distance between the initial stock of government debt outstanding and the fiscal limit. The larger the public debt before the shock hits, the narrower the room of maneuver for public intervention as well as the harsher the first type of trade-off mentioned above.

The second of these trade-offs, i.e. the relationship between the fiscal space and the magnitude of the government's financial intervention, is characterized by the model's regions of instability. In practice, the two main mechanisms via which government debt may become unstable are: (i) increasingly higher sovereign risk premia associated with higher public debt stocks and; (ii) the government's direct intermediation of funds towards the private sector to mitigate deleveraging. Both features cause additional expenditures for the public sector: the former via greater borrowing costs per unit of funds borrowed (R_t^G); the latter via the cost the government bears from borrowing funds (at rate R_t^G) to lend it to the private sector (at rate $R_t < R_t^G$), and the efficiency loss (κ) this operation entails.

Let us suppose that the private sector is highly indebted ($m'' = 0.98$ and $m = 0.44$), but the government has indeed fiscal space to intervene with direct intermediation of funds, without having to compensate this off through a more aggressive fiscal stance ($\bar{\Gamma} = 0.68$). To check whether and to what extent it is desirable for the government to intervene, we compare the peak responses to a contractionary one-per-cent house price shock for different degrees of government reaction to private deleveraging, $\epsilon \in [0, 1]$, and for alternative levels of inefficiency losses created by direct government intermediation of funds, κ (Figure 5).[19] A number of results emerge from this exercise: (i) there is a non-zero level of government intervention that minimizes output losses; (ii) the more efficient is government intervention (the lower the value of κ) the bolder is the output-loss-minimizing degree of intervention (higher ϵ); (iii) private sector's deleveraging and deflation are mitigated by a stronger intervention (virtually irrespective of the value of κ); (iv) there is a non-

[19]We use the value for the fiscal stance, ρ_B, equal to 0.05 to guarantee public debt stability in all cases examined in the figure.

Figure 5: Peak Responses to a Negative One-Per-Cent House Price Shock for Different Degrees of Government Intervention to Private Deleveraging, ϵ, and Alternative Levels of Inefficiency Created by Direct Government Intermediation of Funds, κ



Notes: Private debt is high ($m'' = 0.99$ and $m = 0.44$); government indebtedness is base ($\Gamma = 0.68$); Y-axes are in percent deviations from steady state for output and inflation and absolute deviations for private and public debt to GDP ratios.

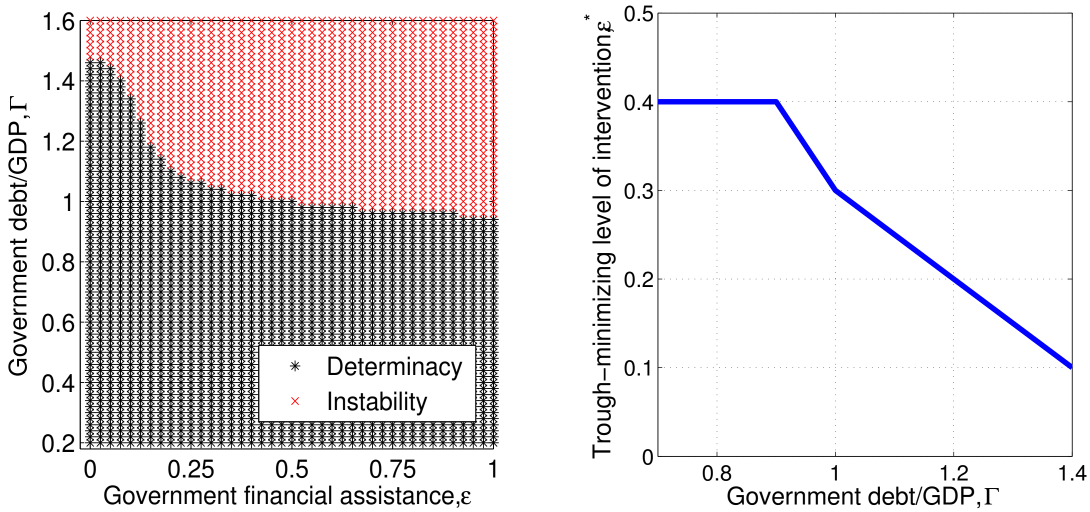
zero level of intervention that minimizes the surge in government debt/GDP and this is a positive function of its efficiency.

In the case of higher and higher public indebtedness, intervention can still mitigate output losses, but the government has much less room for maneuver. Figure 6-(a) shows that, given the baseline fiscal stance, the model's region of stability shrinks as government debt increases above values around 100 percent of GDP, and as financial assistance becomes bolder. At high levels of government debt the scope for financial assistance becomes extremely limited, because, even assuming small efficiency losses, the sovereign risk premium paid to directly intermediate funds towards the private sector is large, which makes the operation very costly and government debt prone to instability. Figure 6-(b) shows the trough-minimizing level

Figure 6: Fiscal Space and Level of Government Intervention via Financial Assistance

(a) Model's Determinacy and Instability Regions

(b) Trough-Minimizing Government Intervention



Notes: Private debt is high ($m'' = 0.99$ and $m = 0.44$); all other parameters are as in the baseline calibration.

of government intervention, ϵ^*, as a function of government debt, conditional on stability. For levels of debt below 90% of GDP, the desirable level of intervention stays virtually constant, then it monotonically declines as debt becomes higher and higher; and, at a certain point, it coincides with the maximum level allowed by the stability condition (from 110% of GDP onwards).

In sum, if there is fiscal space–and abstracting from moral hazard considerations–the trade-off between the additional fiscal costs created by government intervention and its ability to mitigate the private sector's deleveraging, the deflation and, ultimately, the recession suggests intervening. A moderate intervention has also beneficial effects on government debt through its boost on output, government revenues and inflation. On the contrary, excessive intervention (especially if inefficient) is detrimental and self-defeating because it creates a fiscal burden requiring pronounced consolidations. If fiscal space is limited intervention may become either too costly or unfeasible.

6 Conclusion

Do the levels of private and public debt amplify swings in economic activity over the leverage cycle? Should governments extend financial assistance to credit-constrained agents at times of financial stress?

This paper attempts to answer these fundamental, and yet largely unanswered, policy questions in the context of a general equilibrium model that can reproduce the observed empirics regarding private/public debt overhangs and output.

Our answer to the first question is yes, with some caveats. In line with common priors, our model reaffirms the empirical evidence that private debt booms raise the severity of a recession, and make it worse the larger the boom is. Yet, we also find in the data, and are able to replicate in our model, that public debt only exacerbates a downturn when its level is especially high, precisely because high levels of public debt impair fiscal accommodation during phases of private deleveraging. From this we arrive at the less obvious conclusion that accelerations in private debt are as, or possibly more, worrisome than accelerations of public debt. We also deduct, somewhat innovatively, that one of the key benefits of having fiscal buffers is the greater macroeconomic resilience to financial shocks particularly after phases of high leverage: under normal or more muted leverage cycles, fiscal buffers remain important but are not as valuable.

Our answer to the second question is also yes, but critically depends on two qualifications. First, financial assistance should not be confused with blanket fiscal stimuli: we explore a targeted policy, i.e. lending to financially-constrained agents during phases of credit deleveraging, and not standard spending. Second, as we expose numerically, based on realistic assumptions, there are limits to unbounded financial assistance related to debt sustainability. And, even before these limits kick in, there is a clear trade-off between costs and benefits of intervention. This is because the economic costs of financial assistance rise (i) with the level of public debt–as taxes need to increase by more, causing greater output losses, while endogenous sovereign risk premia aggravate debt servicing; and (ii) with the inefficiency of public intervention in aid of financially-constrained agents.

Results also support some policy actions taken since the global financial crisis. For instance, it was right to bring LTV ratios to more appropriate levels internationally–levels that greatly reduce macro financial vulnerabilities associated with excessive credit booms.

On the other hand, results also ring three alarming bells. First, several countries considered "safe" by financial markets, may in fact be more vulnerable than countries which are seen as less safe from a macro-fiscal sustainability point of view. This calls for modifications to implicit practices entrenched in macro-fiscal and macro-financial surveillance in order to give equal attention to the risks posed by the evolution and levels of private indebtedness relative to those traditionally believed to be associated with public indebtedness in isolation.

Second, fiscal consolidation in some parts of the world has become more neutral, but before doing so, may have been set in a way that prolonged deleveraging and magnified its costs. Inasmuch as this is still ongoing, and thinking of future shocks, fiscal rules should be modified to account explicitly for the quintessential mitigating role of government as a lender of last resort during protracted phases of financial stress. This implies that debt consolidations should become more gradual when economies are in the midst of a deleveraging phase: by extending financial assistance to credit-constrained agents, the government *de facto* provides a targeted fiscal stimulus that reduces any planned structural adjustment.

Third, while LTVs have been internationally capped down at safer levels, above-safe levels LTV loan options exist and remain common around advanced and emerging market economies alike. Ruling out these options would likely greatly limit the realizations of deep and prolonged recessions.

References

Aoki, K., Proudman, J., and Vlieghe, G. (2004). House prices, consumption, and monetary policy: A financial accelerator approach. *Journal of Financial Intermediation*, 13(4):414 – 435.

Austin, D. A. and Miller, R. S. (2011). Treasury securities and the US sovereign credit default swap market. In *Congressional Research Service, CRS Report for Congress*, number 7-5700.

Baron, M. and Xiong, W. (2014). Credit expansion and financial instability: Evidence from stock prices. *Working paper.*

Bernanke, B. and Gertler, M. (1995). Inside the black box: the credit channel of monetary policy transmission. *Journal of Economic Perspectives*, 9,4:27–48.

Bernanke, B., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle model. *Handbook of Macroeconomics*, 1:1341–1393.

Bi, H. and Traum, N. (2014). Estimating fiscal limits: The case of Greece. *Journal of Applied Econometrics*, 29(7):1053–1072.

Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Cantore, C., Levine, P., Melina, G., and Pearlmanl, J. (2015). Optimal fiscal and monetary policy, debt crisis and management. *Discussion Papers in Economics, University of Surrey*, 05/15.

Christiano, L., Motto, R., and Rostagno, M. (2010). Financial factors in economic fluctuations. *ECB Working Paper Series*, 1192.

Corsetti, G., Kuester, K., Meier, A., and Müller, G. J. (2013). Sovereign risk, fiscal policy and macroeconomic stability. *Economic Journal*, 123(566):F99–F132.

Dell'Ariccia, G., Igan, D., Laeven, L., Tong, H., Bakker, B., and Vandenbussche, J. (2012). Policies for macrofinancial stability: How to deal with credit booms. *IMF Staff Discussion Note.*

ECB (2009). Housing finance in the euro area. Technical report, European Central Bank, Structural Issues Report.

ECB (2012). Corporate indebtedness in the euro area. Technical report, European Central Bank, Monthly Bulletin.

Eggertsson, G. B. and Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach. *The Quarterly Journal of Economics*, 127(3):1469–1513.

Farhi, E. and Werning, I. (2015). A theory of macroprudential policies in the presence of nominal rigidities. *Harvard University OpenScholar Working Paper*, (95131).

Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica*, 1(4):337–57.

Geanakoplos, J., Axtell, R., Farmer, D. J., Howitt, P., Conlee, B., Goldstein, J., Hendrey, M., Palmer, N. M., and Yang, C.-Y. (2012). Getting at systemic risk via an agent-based model of the housing market. *The American Economic Review*, 102(3):53–58.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.

Glick, R. and Lansing, K. J. (2010). Global household leverage, house prices, and consumption. *FRBSF Economic Letter*.

Guerrieri, V. and Lorenzoni, G. (2015). Credit crises, precautionary savings, and the liquidity trap. *Working paper*.

Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–91.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 95(3):739–764.

Iacoviello, M. (2015). Financial business cycles. *Review of Economic Dynamics*, 18(1):140–164.

IMF (2012). Dealing with household debt. In *World Economic Outlook*, pages 89–124.

Jordà, Ò., Schularick, M., and Taylor, A. M. (2013). When credit bites back. *Journal of Money, Credit and Banking*, 45(s2):3–28.

Jordà, Ò., Schularick, M., and Taylor, A. M. (2014). Private credit and public debt in financial crises. *FRBSF Economic Letter*.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105,2:211–248.

Korinek, A. and Simsek, A. (2014). Liquidity trap and excessive leverage. *NBER Working Papers*, 19970.

Martin, P. and Philippon, T. (2014). Inspecting the Mechanism: Leverage and the Great Recession in the Eurozone. *NBER Working Papers*, 20572.

Mian, A. R. and Sufi, A. (2014). *House of debt: how they (and you) caused the Great Recession and how we can prevent it from happening again*. University of Chicago Press.

Mian, A. R., Sufi, A., and Verner, E. (2016). Household debt and business cycles worldwide. *Working paper*.

Minsky, H. P. (1982). *Can it happen again? Essays on Instability and Finance*. Routledge.

Ostry, J. D., Ghosh, A. R., and Espinoza, R. A. (2015). When should public debt be reduced? *IMF Staff Discussion Notes*, 15/10.

Schularick, M. and Taylor, A. M. (2012a). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review*, 102(2):1029–61.

Schularick, M. and Taylor, A. M. (2012b). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review*, 102(2):1029–61.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175.

Taylor, A. M. (2012). Global financial stability and the lessons of history: A review of Carmen M. Reinhart and Kenneth S. Rogoff's this time is different: Eight centuries of financial folly. *Journal of Economic Literature*, 50(4):1092–1105.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214.

Westerlund, J. (2007). Testing for error correction in panel data. *Oxford Bulletin of Economics and Statistics*, 69(6):709–748.

Appendix

A Countries in Panel Regressions and Descriptive Statistics

Table A.1: Countries in Panel Regressions and Descriptive Statistics

	Private debt (% of GDP)			Public debt (% of GDP)		
	Years	Average	Std. dev.	Years	Average	Std. dev.
Australia	1960-2014	110.62	47.21	1989-2014	21.75	8.03
Austria	1960-2014	92.29	38.36	1988-2014	68.27	9.28
Belgium	1970-2014	120.52	45.38	1980-2014	111.33	16.01
Canada	1955-2014	126.49	36.49	1980-2014	78.12	14.39
Czech Republic	1993-2014	77.25	10.07	1995-2014	27.92	11.11
Denmark	1966-2014	162.54	48.05	1992-2014	51.00	13.89
Finland	1970-2014	120.34	31.20	1980-2014	36.61	16.67
France	1969-2014	125.86	25.73	1980-2014	54.93	22.21
Germany	1960-2014	100.24	19.14	1991-2014	62.37	11.61
Greece	1970-2014	62.28	33.17	1980-2014	91.52	45.06
Hong Kong	1978-2014	163.28	48.74	2001-2014	1.18	1.00
Hungary	1989-2014	81.49	33.96	1997-2014	66.83	10.04
Indonesia	1976-2014	35.96	15.05	2000-2014	40.31	20.11
Ireland	1971-2014	135.17	85.22	1995-2014	61.43	33.97
Italy	1960-2014	79.55	21.70	1988-2014	109.02	11.61
Japan	1964-2014	169.53	30.98	1980-2014	132.25	66.72
Korea, Rep.	1962-2014	107.75	52.29	1990-2014	21.63	9.58
Mexico	1980-2014	28.30	10.07	1980-2014	34.92	23.07
Netherlands	1961-2014	141.13	70.35	1980-2014	63.21	10.56
Norway	1960-2014	144.30	36.05	1980-2014	36.63	8.53
Poland	1992-2014	50.17	21.06	1995-2014	46.61	5.70
Portugal	1960-2014	124.69	49.46	1990-2014	72.27	28.40
Singapore	1970-2014	98.78	19.75	1963-2014	67.57	25.84
Spain	1970-2014	123.53	44.86	1980-2014	52.09	19.57
Sweden	1961-2014	138.07	44.74	1993-2014	50.70	12.56
Switzerland	1960-2014	156.31	33.05	1983-2014	48.33	11.11
Thailand	1970-2014	86.46	40.24	1996-2014	43.65	9.64
Turkey	1986-2014	31.74	18.82	1987-2014	40.45	12.38
United Kingdom	1963-2014	110.19	46.68	1980-2014	49.87	17.12
United States	1952-2014	110.83	29.93	1947-2014	85.54	15.25

B Equilibrium conditions

B.1 Patient households

Intertemporal maximization yields the following first-order conditions with respect to C'_t, L'_t, B'_t, B^G_t and h'_t:

$$\mu'_t = \frac{1}{\left(1+\tau^C_t\right) X'_t}, \tag{B.1}$$

$$\left(1-\tau^W_t\right) \frac{W'_t}{P_t} = \left(L'_t\right)^{\eta-1} \left(1+\tau^C_t\right) X'_t, \tag{B.2}$$

$$\frac{1}{\left(1+\tau^C_t\right) X'_t} = \beta E_t \left[\frac{R_t}{\left(1+\tau^C_{t+1}\right) X'_{t+1} \Pi_{t+1}} \right], \tag{B.3}$$

$$\frac{1}{\left(1+\tau^C_t\right) X'_t} = \beta E_t \left[\frac{\left(1-\Delta^G_{t+1}\right) R^G_t}{\left(1+\tau^C_{t+1}\right) X'_{t+1} \Pi_{t+1}} \right], \tag{B.4}$$

$$\frac{q_t}{\left(1+\tau^C_t\right) X'_t} = \frac{\zeta e^H_t}{h'_t} + \beta E_t \left[\frac{q_{t+1}}{\left(1+\tau^C_{t+1}\right) X'_{t+1}} \right], \tag{B.5}$$

where μ'_t is the Lagrange multiplier associated to the budget constraint and $\Pi_{t+1} \equiv P_{t+1}/P_t$ represents the gross inflation rate. Equations (B.3) and (B.4) imply a non-arbitrage condition between the riskless interest rate and that on government bonds, whereby a sovereign risk spread arises, i.e. $R^G_t = E_t \left[\left(1-\Delta^G_{t+1}\right)^{-1} \right] R_t$.

B.2 Impatient households

Intertemporal maximization yields the following first-order conditions with respect to C''_t, L''_t, B''_t and h''_t:

$$\mu''_t = \frac{1}{\left(1+\tau^C_t\right) X''_t}, \tag{B.6}$$

$$\left(1-\tau^W_t\right) \frac{W''_t}{P_t} = \left(L''_t\right)^{\eta-1} \left(1+\tau^C_t\right) X''_t, \tag{B.7}$$

$$\frac{1}{\left(1+\tau^C_t\right) X''_t} = \beta'' E_t \left[\frac{R_t}{\left(1+\tau^C_{t+1}\right) X''_{t+1} \Pi_{t+1}} \right] + \lambda''_t R_t, \tag{B.8}$$

$$\frac{q_t}{\left(1+\tau^C_t\right) X''_t} = \frac{\zeta e^H_t}{h''_t} + E_t \left[\frac{\beta'' q_{t+1}}{\left(1+\tau^C_{t+1}\right) X''_t} + \lambda''_t m'' q_{t+1} \Pi_{t+1} \right], \tag{B.9}$$

where μ''_t is the Lagrange multiplier associated to the flow of funds and λ''_t is the Lagrange multiplier associated with the borrowing constraint.

B.3 Entrepreneurs

Maximization of function (11) subject to (9), (10), (12), (13), (14), (15) and the two quadratic adjustment costs yields the following first-order conditions with respect to $X_{e,t}$, $B_{e,t}$, $I_{e,t}$, $K_{e,t}$, $h_{e,t}$, $L'_{e,t}$, $L''_{e,t}$, and $P_{e,t}$ which, evaluated at the symmetric equilibrium, read as:

$$\mu_t = \frac{1}{\left(1+\tau_t^C\right) X_t}, \tag{B.10}$$

$$\mu_t = \lambda_t R_t + \gamma E_t \left[\mu_{t+1} \frac{R_t}{\Pi_{t+1}}\right], \tag{B.11}$$

$$u_t = \mu_t \left[1 + \frac{\psi_K}{\delta}\left(\frac{I_t}{K_{t-1}} - \delta\right)\right], \tag{B.12}$$

$$u_t = \gamma E_t \left\{ \begin{array}{c} \mu_{t+1}\left[\frac{\psi_K}{\delta}\left(\frac{I_{t+1}}{K_t} - \delta\right)\frac{I_{t+1}}{K_t} - \frac{\psi_K}{2\delta}\left(\frac{I_{t+1}}{K_t} - \delta\right)^2\right] \\ + \left[\mu_{t+1} MC_{t+1} \frac{\omega Y_{t+1}}{K_t} + (1-\delta)\, u_{t+1}\right] \end{array} \right\}, \tag{B.13}$$

$$\mu_t q_t = E_t \left\{\gamma \mu_{t+1}\left[q_{t+1} + MC_{t+1}\frac{\nu Y_{t+1}}{h_t}\right] + m\lambda_t q_{t+1}\Pi_{t+1}\right\}, \tag{B.14}$$

$$w'_t = MC_t \frac{\alpha\,(1-\omega-\nu)\,Y_t}{L'_t}, \tag{B.15}$$

$$w''_t = MC_t \frac{(1-\alpha)\,(1-\omega-\nu)\,Y_t}{L''_t}, \tag{B.16}$$

$$\begin{aligned} 0 &= 1 + e_t^P \chi\,(MC_t - 1) - \psi_P\,(\Pi_t - 1)\,\Pi_t \\ &+ \psi_P E_t\left[\gamma \frac{\mu_{t+1}}{\mu_t}\,(\Pi_{t+1} - 1)\,\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right], \end{aligned} \tag{B.17}$$

respectively, where λ_t is the Lagrange multiplier associated with the borrowing constraint, MC_t is the the firm's marginal cost and u_t is Tobin's q.