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Uncertainty and Investment: The Financial Intermediary Balance Sheet Channel

by Sophia Chen

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Prepared by Sophia Chen[1]

Authorized for distribution by Giovanni Dell'Ariccia

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Abstract

Rollover risk imposes market discipline on banks' risk-taking behavior but it can be socially costly. I present a two-sided model in which a bank simultaneously lends to a firm and borrows from the short-term funding market. When the bank is capital constrained, uncertainty in asset quality and rollover risk create a negative externality that spills over to the real economy by ex ante credit contraction. Macroprudential and monetary policies can be used to reduce the social cost of market discipline and improve efficiency.

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Author's E-Mail Address: YChen2@imf.org

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I Introduction

The reliance on short-term debt by banks and other financial institutions was a prominent feature in the run up to the crisis of 2007 to 2009. Spikes in uncertainty about the quality of their assets caused freezes in the asset-backed commercial paper (ABCP) and overnight sale and repurchase (repo) markets. From a systemic perspective, the inability to roll over short-term debt was a market failure that led to the demise of a substantial part of investment banking and the distress of other financial institutions in the United States, United Kingdom, and other countries. The financial sector distress was followed by the prolonged investment slowdown and economic decline of the Great Recession.

In this paper, I am interested in developing a model of investment contraction caused by uncertainty in the asset quality and rollover risk of banks and other financial intermediaries (hereafter, banks). While there is a growing literature that analyzes the implication of short-term credit market for banks, most existing theories treat investment and the capital structure of banks separately. However, many questions emerged during the crisis are related to banks as a transmission channel between credit and capital markets. For example, how do the investment and financing decisions of banks interact? How does asset quality uncertainty affect the supply of credit? What are the real effects of banks' rollover risk?

These questions motivate a two-sided model of the balance sheet of banks. In the model, a bank separately contracts with a firm to invest in real projects (the asset side) and with investors in the wholesale funding market to finance its assets (the liability side). The investment is long term (e.g. a mortgage) but the financing is short term (e.g. a repo). The bank faces idiosyncratic uncertainty because the quality and rollover decision on an individual project is unknown ex ante. The bank may in addition face aggregate uncertainty because in a bad aggregate state, the quality of projects is more dispersed.

I analyze the interaction between a bank's financing and investment decisions as a channel of uncertainty transmission. The economic mechanism is as follows. On the liability side, a short-term loan with limited debt capacity is the outcome of optimal contracting for a bank that faces a risk-shifting moral hazard problem. However, uncertainty in asset quality and the need to rollover short-term debt can lead to inefficient liquidation in some states of the world, which affects the bank's investment surplus on the asset side. This inefficiency spills over to the real economy causing ex ante credit contraction and underinvestment.

From a social welfare perspective, the use of short-term debt is a mixed blessing. On the one hand, investors in the credit market can use rollover risk as a discipline device to limit the risk of the bank. By forcing the bank into liquidation with a debt run when the signaled project revenue is low, they prevent the bank from taking on riskier but less efficient projects. On the other hand, rollover risk is socially costly. In equilibrium, a debt run can lead to a loss of total surplus (i.e. a deadweight loss) and an ex ante contraction of the economy. This contractionary effect is aggravated by aggregate uncertainty.

The two-sided model has different welfare properties from a standard one-sided model of bank runs (Calomiris and Kahn (1991), Diamond and Rajan (2000)). Here the driving force of uncertainty transmission and welfare loss is the interaction between the bank's investment and financing decisions. In the model, uncertainty in asset quality alone does not lead to inefficiency. If the bank has sufficient capital and all the bargaining power in contract renegotiation, a one-sided bank-firm contract can be renegotiated in a way to fully restore efficiency. In contrast, if the bank relies on short-term lending to finance its balance sheet, the equilibrium investment is inevitably lower than the first-best.

The welfare properties of the model provide a rationale for policy interventions. Ex ante policies such as capital requirements and liquidity requirements in the banking sector can lead to welfare improvement and restore efficiency. Ex post policies such as a monetary intervention can also improve efficiency in crisis time but has no effect on ex ante investment. A subtle point about capital requirement is that it may fully restore efficiency only if there is no equity premium. The reason is that imposing a minimum capital ratio addresses the problem of debt runs; however, it does not address why equity holders require higher returns. The model is not designed to address equity-side frictions.

This paper is related to a growing literature on the effects of aggregate uncertainty on the real economy by driving business cycles (Bloom (2009), Jurado, Ludvigson and Ng (2013)), affecting credit spreads (Gilchrist, Sim and Zakrajsek (2013)), increasing labor wedge (Arellano, Bai and Kehoe (2012)), or deducing credit supply (Valencia (2013)). Like in the last three studies, uncertainty negatively affects investment through the bank's lending decision. But unlike there, the bank's credit supply is a constrained outcome of market discipline. The use of short-term debt is necessary to discipline the bank's risk taking behavior.

The role of deposit or short-run debt runs as a discipline device has been discussed in classical bank run models such as Calomiris and Kahn (1991) and Diamond and Rajan (2000). The difference is that in their models, investment is not affected by bank runs. In Calomiris and Kahn (1991), investment is fixed and welfare loss is ex post. In Diamond and Rajan (2000), bank runs occurs off the equilibrium.

The work in the paper also contributes to the literature on the rollover risk of short-term debt, which is well documented for the ABCP market (Covitz, Liang and Suarez (2013), Kacperczyk and Schnabl (2010)) and the repo market (Martin, Skeie and von Thadden (2014), Gorton and Metrick (2012)). Previous theoretical work focuses on the debt capacity of collateralized assets (Acharya, Gale and Yorulmazer (2011), Adrian and Shin (2013)), separating liquidity and solvency risks (Morris and Shin (2003)), and the interaction between risk-taking behavior and fire sale (Eisenbach (2013)). This literature commonly abstracts away from the equilibrium level of real investment.

The rest of the paper is organized as follows. Section II introduces the model. Section III presents equilibrium financing and investment solutions. Section IV examines several extensions. Section V describes policy options. Section VI concludes.

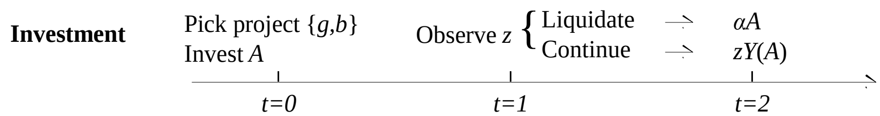

Figure 1: Timing of Events

II Model

Consider an economy with two risky investment projects and three parties. A firm invests in projects and finances its investment through a bank. The bank funds projects and finances its operation through collateralized borrowing. An investor serves as an uninsured wholesale creditor to the bank. All parties are risk neutral. There are three dates $t = 0, 1, 2$. The two-period opportunity cost of capital is $R = 1 + r > 1$.

The firm can invest in one of two projects, a good project (g) and a bad project (b). Each project is represented by a stochastic production technology that transforms A units of capital at date $t = 0$ into $zY(A)$ units of revenue at date $t = 2$, where $z \in \{z_g, z_b\}$ is a random variable with distribution F_g for the good project and F_b for the bad project. The bad project has lower expected revenue

$$\int_0^\infty z dF_g(z) > \int_0^\infty z dF_b(z),$$

but has higher upside risk relative to the good project in the sense of second order stochastic dominance (SOSD). Formally, there is a z^* such that $F_g(z^*) = F_b(z^*)$ and

$$(F_g(z) - F_b(z))(z - z^*) \geq 0$$

for all z. At date $t = 1$, a public signal is observed. The signal can be thought of as a leading economic indicator. It predicts with perfect accuracy the value of z that will be realized at date $t = 2$. After the signal is observed at $t = 1$, a liquidation process can be initiated, in which case the capital is sold at a discount price of $\alpha \in [0, R)$ per unit at date $t = 2$. The timing is summarized in Figure 1.

I refer to *idiosyncratic uncertainty* as the stochastic outcome of an individual project given the project type. It is idiosyncratic in the sense that if the economy is to be replicated by identical projects, their outcomes are independent draws from the same distribution. I use *aggregate uncertainty* to refer to the circumstance in which the distribution may be changed by an exogenous shock. I leave the formal definition of to Section A.

The following assumptions are maintained throughout the paper to ensure interior solutions.

Assumption 1 $Y(A)$ is a strictly concave and increasing function that satisfies the Inada conditions

$$\lim_{A \to 0} Y'(A) = \infty, \lim_{A \to \infty} Y'(A) = 0.$$

In addition, $Y'(A)$ is convex.

As is standard, the assumption on concavity and Inada conditions ensure that (absent any frictions) there is positive investment for any positive cost of capital. The additional assumption requires that the production technology has diminishing marginal return at an increasing rate. Assumption 1 is satisfied by commonly used production functions including A^{α} and $\ln A$.

Assumption 2 For $F \in \{F_g, F_b\}$, the survival function $1 - F$ is log-concave; that is, the hazard rate $h(z) = f(z) / (1 - F(z))$ is continuous and increasing.

Assumption 2 is satisfied by commonly used distributions including normal, logistic, exponential, chi-squared and certain parameterization of gamma and beta.

A First-Best Investment

I begin the analysis with a characterization of the first-best investment level and composition as a benchmark for the rest of the paper. Suppose there is a hypothetical social planner who can directly allocate funds and operate projects. The planner prefers the good project over the bad project because it generate higher expected revenue for any given level of investment. The planner chooses the capital size A at date $t = 0$ and decides whether to liquidate the project at date $t = 1$ when the signal is observed. Using backward induction, for any given level of investment, the planner liquidates the project if the project revenue is less than liquidation value. In other words, the planner liquidates the project if $z < \underline{z}(A)$, where $\underline{z}(A)$ is a liquidation threshold that satisfies

$$\underline{z} Y(A) = \alpha A. \tag{1}$$

In what follows, I refer to $\underline{z}$ as the *efficient* threshold. This threshold is strictly increasing in A by the strict concavity of $Y(A)$; that is, $\partial \underline{z}(A) / A > 0$. The same assumption also implies that the marginal return of the last completed project is lower than its liquidation value: $\underline{z} Y'(A) < \alpha$. For ease of notation, I suppress the dependence of $\underline{z}$ on A in what follows when no confusion occurs.

The social planner's problem is given by

$$\max_{A} \alpha A F_g(\underline{z}) + \int_{\underline{z}}^{\infty} z Y(A) \, dF_g(z) - RA, \tag{2}$$

with first order condition

$$\alpha F_g(\underline{z}) + \int_{\underline{z}}^{\infty} z Y'(A) \, dF_g(z) - R = 0 \tag{3}$$

This condition simply says that the expected marginal return on investment is equal to the marginal cost R. As a approaches zero, Assumption 1 guarantees that there exists a unique solution. When $\alpha \in (0, R)$, the expected return falls but there is a gain from liquidation because $\underline{z}$ is higher. The following proposition shows that under Assumptions 1 and 2, diminishing marginal returns outweigh the gain from liquidation; hence there is a solution for first-best investment.

Proposition 1 *There exists a unique first-best investment level $A^{fb} \in (0, \infty)$.*

Proof. See Appendix. ■

B Contracting Framework

Bank-Firm Contract

Suppose the firm lacks internal funds and has to finance investment from the bank. Let (A, B) characterize the bank-firm contract, where A is the loan amount and B is the face value of the debt. The contract is implemented as follows. At date $t = 0$, the bank chooses from one of the two projects–good (g) or bad (b)–and invest A in the firm. The firm pays B at $t = 2$ if the project is complete. If the project is terminated at date $t = 1$, the bank takes the full liquidation value of the project αA. I implicitly assume that the debt payment cannot be contingent on z because, perhaps, z is not verifiable in a court. As a benchmark for later analysis on the bank's financing problem, suppose for now that the bank has an arbitrarily large size of capital with opportunity cost R.

The firm has a reservation payoff C. If C is too large, the project may not generate sufficient revenue to be worth the firm's effort. I assume this is not the case, that is,

$$\int_{\underline{z}}^{\infty} \left(zY\left(A^{fb}\right) - \alpha A^{fb}\right) dF_g\left(z\right) > C.$$

This assumption guarantees that the first-best investment is feasible. If the firm receives the first best loan and the entire surplus, it strictly prefers to participate in the contract.

Because investment is determined at date $t = 0$ and there are only two outcomes at date $t = 1$, continuation or liquidation. The firm follows a simple rule. It liquidates the project if the signaled revenue is less than its debt. The liquidation threshold $\bar{z}$ is given by

$$\bar{z}Y\left(A\right) = B. \tag{4}$$

For productivity levels $z \in (\underline{z}, \bar{z})$, liquidation is not efficient because the project generates higher revenue if it is continued. One way to overcome this is to give the bank the power to write off a fraction of the firm's debt and allow the project to continue in these states of the world. If the bank has all the bargaining power in contract renegotiation, the marginally completed project has productivity level $\underline{z}$, which is the same as the ex post efficient threshold in (1). To see this, assume for the moment that the bank can extract the entire surplus after contract renegotiation. Allowing a project to continue yields a surplus of $zY(A) - aA$ in addition to its liquidation value, which is positive if $z \in (\underline{z}, \bar{z})$ and negative if $z < \underline{z}$. So it is optimal to set a liquidation threshold at $\underline{z}$. It remains to show that the bank can indeed implement a contract to extract the entire surplus. The bank can make a take-it-or-leave-it offer to the firm and adjust the debt payment so the firm is indifferent between continuing or liquidating the project. If the firm agrees, the bank takes control of the project and collects all the revenue in the next period. This implies zero debt payment when $z \in (\underline{z}, \bar{z})$. The firm's payoff is the same as in liquidation (i.e. zero) and the bank captures the entire surplus.

Taking into account the liquidation threshold with contract renegotiation, the optimal contract at date $t = 0$ can be solved by backward induction:

$$\max_{(A,B)} \alpha A F_g(\underline{z}) + \int_{\underline{z}}^{\bar{z}} zY(A)\, dF_g(z) + B(1 - F_g(\bar{z})) - RA \tag{5}$$

subject to firm's participation constraint (PCF)

$$\int_{\bar{z}}^{\infty} (zY(A) - B)\, dF_g(z) \geq C. \tag{6}$$

The next result shows that allowing contract renegotiation recovers the first-best solution, provided the bank has all the bargaining power.

Proposition 2 *The bank gives the first-best loan A^{fb} to the firm and the corresponding debt value B^{fb} is set to satisfy the firm's reservation payoff.*

Proof. See Appendix. ■

The conclusion from this section is that absent financing constraints, the bank can assume the role of a social planner and choose the first-best investment as long as it can capture the entire surplus in contract renegotiation. Even though the contract cannot be contingent on project outcome, renegotiation can remedy this ex post inefficiency caused by idiosyncratic uncertainty. This is not the case if the bank cannot capture the entire surplus. In the next section, I explore one such case with frictions on the bank's liability side.

Bank-Creditor Contract

I start by relaxing the previous assumption that the bank has arbitrarily large capital. Suppose the bank starts with limited equity and finances its operation through a collateralized debt arrangement, such as a repo. At $t = 0$, the bank sells its assets A for a price D and agrees to repurchase the asset at $t = 2$ for price $\tilde{D}$ using revenue from the project. The difference $\tilde{D} - D$ resembles the promised interest payment to the creditor and $\tilde{D}/D-1$ can be interpreted as the repo interest rate. The debt is short-term in the sense that the creditor can decide whether to roll it over at $t=1$.[1]

Given the bank's capital structure, I analyze the ex ante contracting problem between the bank and the investor at $t=0$. The bank-creditor contract faces a moral hazard problem because the creditor cannot observe the project type chosen by the bank. As I shall show momentarily, without proper incentive, the bank may choose the riskier and less inefficient project. The optimal financing contract needs to specify the value of debt subject to the bank's incentive constraint. Specifically, for any contract (A,B) the bank might give to the firm, the optimal contract solves the face value $\tilde{D}$ and the market value D of the debt. As noted by Merton (1974), a defaultable debt claim with face value $\tilde{D}$ can be replicated by a portfolio with cash of $\tilde{D}$ and short position in a put option on the assets with strike price $\tilde{D}$. Because the borrower has limited liability, the face value of the debt has the interpretation of the strike price of the embedded option in the contract. If at $t=1$ the signaled project revenue is lower than $\tilde{D}$, the bank will not be able to roll over the debt and will be forced into liquidation. The creditor loses the claim and receives the liquidation value of the assets. If the project revenue is higher than $\tilde{D}$, the creditor is fully repaid at $t=2$.

Let $\Pi_g\left(\tilde{D},A\right)$ denote the price of a put option with strike price $\tilde{D}$ on good assets with face value A. The creditor's initial investment is D and the expected value of its debt claim consists of the payment $\tilde{D}$ and the short position in the put option on the bank's assets with strike price $\tilde{D}$. The creditor's expected gross payoff from the portfolio of the good project is

$$U_g^C(A) \equiv \tilde{D} - \Pi_g\left(\tilde{D},A\right).$$

Given the form of bank-firm contracts derived previously, the firm's payoff from the good project is

$$U_g^F(A) \equiv \int_{\bar{z}}^{\infty} zY(A)\,dF_g(z) - B\left(1-F_g(\bar{z})\right).$$

The bank's payoff from the good project $U_g^B(A)$ is given by the total payoff from the project net of payoffs to the firm and the creditor.

$$U_g^B(A) = \alpha A F_g(\tilde{z}) + \int_{\tilde{z}}^{\infty} zY(A)\,dF_g(z) - U_g^C(A) - U_g^F(A),$$

where $\tilde{z}$ denotes the threshold revenue shock that triggers a liquidation. Lemma 2 shows that $\tilde{z}$ is determined by $\tilde{D}$ and is unique. The bank's payoff from the bad project $U_L^B(A)$ is given analogously by

$$U_b^B(A) = \alpha A F_b(\tilde{z}) + \int_{\tilde{z}}^{\infty} zY(A)\,dF_b(z) - U_b^C(A) - U_b^F(A).$$

[1]Here I implicitly assume that this is the only available debt maturity structure. I relax this assumption In Section C and show that long-term debt is not preferable to short-term debt if endogenous default is allowed. In other words, short-short debt is optimal.

To make the contract incentive compatible so that the bank chooses the good project, the bank's expected payoff from the bad project should not exceed that from the good project:

$$U_g^B(A) \geq U_b^B(A),$$

which gives the incentive compatibility constraint of the bank (ICB)

$$\Delta U^B(A) \geq \Delta\Pi\left(\tilde{D}, A\right), \tag{7}$$

where $\Delta U^B(A) \equiv \int_{\tilde{z}}^{\bar{z}} zY(A)\, d\left(F_g(z) - F_b(z)\right) + \alpha A\left(F_g(\tilde{z}) - F_b(\tilde{z})\right) - B\left(F_g(\bar{z}) - F_b(\bar{z})\right)$ is defined as the difference in expected payoff between good and bad projects for the bank, and $\Delta\Pi\left(\tilde{D}, A\right) \equiv \Pi_b\left(\tilde{D}, A\right) - \Pi_g\left(\tilde{D}, A\right)$ is defined as the difference between the value of put options between these portfolios. The ICB requires that the bank's excess payoff from the good project over the bad project be sufficient to offset the value of the put option granted to the bank as a result of limited liability. The difference in the value of put options $\Delta\Pi\left(\tilde{D}, A\right)$ is analogous to the private benefit of exerting low effort in the moral hazard model of Holmstrom and Tirole (1997).

Lemma 1 *If the liquidation value α is sufficiently small, the bank's incentive compatibility constraint (ICB) binds and there exists a unique solution for the bank's debt value $\tilde{D}^*(A)$.*

Proof. See Appendix. ■

This result implies that the bank's debt capacity is limited by the collateral requirement set by the creditor. The next result shows how this financing constraint affects the liquidation rule.

Lemma 2 *If the liquidation value α is sufficiently small, there exists a unique $\tilde{z} \in (\underline{z}, \bar{z})$ such that $\tilde{D}^*(A) = \tilde{z}Y(A)$.*

Proof. See Appendix. ■

Because the bank's debt value is $\tilde{D}^*(A) = \tilde{z}Y(A)$, for any $z < \tilde{z}$, the project will not generate sufficient payoff to cover the debt. As a result, the bank liquidates any project with $z < \tilde{z}$. A direct implication of this result is that the debt contract leads to welfare loss in some states of the world. For projects with $z \leq \underline{z}$, liquidation does not lead to welfare loss because liquidating yields higher payoff than operating the project. The liquidation for projects with $z \in (\underline{z}, \tilde{z})$ is inefficient because revenue from the project is strictly higher than its liquidation value.

The market value of debt is obtained by the creditor's participation constraint (PCC), which requires that the creditor's expected payoff is large enough to cover the cost of original investment D:

$$U_g^C(A) \equiv \tilde{D}(A) - \Pi_g\left(\tilde{D}, A\right) \geq RD. \tag{8}$$

Competitive lending in the repo market holds down the creditor's payoff so the optimal market value of debt D^* solves

$$RD^* = \tilde{D}^*(A) - \Pi_g\left(\tilde{D}^*, A\right), \tag{9}$$

where $\tilde{D}^*(A)$ is given by Lemma 1. In other words, the interest payment to the creditor $\tilde{D}^* - RD$ is just sufficient to cover the value of the put option granted to the bank. Note the right-hand side of (9) is the payoff to a creditor with a debt claim $\tilde{D}^*(A)$. Therefore, D^* is the solution to the creditor's participation constraint (PCC)

$$RD^* = \alpha A F_g(\tilde{z}) + \tilde{z} Y(A)(1 - F_g(\tilde{z})), \tag{10}$$

where $\tilde{z}$ is given by $\tilde{D}^*(A) = \tilde{z} Y(A)$ following Lemma 2.

A direct implication of Lemma 1 is a limit on the bank's leverage.

Lemma 3 *There is a unique solution to the bank's debt to asset ratio for given asset size A: $d^* \equiv \frac{\tilde{D}^*}{A} < 1$.*

Proof. See Appendix. ■

The result that the bank has to be partially financed by equity is very intuitive. Limited leverage is a device to contain risks to the creditor. Because the creditor is a senior claimant in case of a default, equity buffers the loss of the creditor. If the bank cannot raise enough equity to make up the gap between debt and asset values, no projects will be funded which leads to a trivial equilibrium solution. I assume this is not the case. I also assume for now that equity requires the same return as debt $R^e = R$. This assumption will be relaxed later.

III Equilibrium Financing and Investment

In this section, I describe the banking equilibrium and discuss its implications.

A banking equilibrium is a quadruplet of values $\left(A^*, B^*, D^*, \tilde{D}^*\right)$ such that (A^*, B^*) is the solution to the bank-firm contracting problem and $\left(D^*, \tilde{D}^*\right)$ is the solution to the bank-creditor contracting problem. The equilibrium implies a liquidation threshold $\tilde{z}$. To solve for the equilibrium, first note that the firm's participation constraint (PCF) is not affected by bank financing. The argument in Section B goes through. The firm still liquidates projects with $z \leq \bar{z}$. The bank's problem is to choose the values of asset, loan, debt, and liquidation threshold to maximize payoff:

$$\max_{\left(A,B,D,\tilde{D},\tilde{z}\right)} \int_{\tilde{z}}^{\bar{z}} zY(A)\, dF_g(z) + B(1 - F_g(\bar{z})) - \tilde{D}(1 - F(\tilde{z})) \tag{11}$$

subject to (6), (7), and (10)

Previous results simplify this problem. The argument in Section B goes through because it holds for any contract (A, B) the bank might give to the firm. In particular, Lemma 1 shows that choosing $\tilde{D}^*$ for given (A, B) implies binding incentive constraint for the bank and unique $\tilde{D}^*$, which gives unique $\tilde{z}$ following Lemma 2. Furthermore, changing B does not affect $\tilde{z}$ and the same line of argument leading to (20) carries over. In other words, for given A, the solution to $\tilde{D}, D$, and B are characterized by binding constraints of (7), (10) and (6). The bank's problem can be transformed into an unconstrained problem:

$$\max_{A} \alpha A F_g\left(\underline{z}\right) + \int_{\underline{z}}^{\infty} zY\left(A\right) dF_g\left(z\right) - RA - C - \int_{\underline{z}}^{\tilde{z}} \left(zY\left(A\right) - \alpha A\right) dF_g\left(z\right). \qquad (12)$$

The following result is the first main implication of the model.

Proposition 3 *If the liquidation value a is sufficiently small, there exists a unique banking equilibrium* $\left(A^*, B^*, D^*, \tilde{D}^*\right)$. *In the banking equilibrium, investment is lower than the first-best solution:* $A^* < A^{fb}$.

Proof. See Appendix. ■

This proposition shows that in the constrained equilibrium, investment is below its first-best level. I leave the technical proof to the Appendix, but discuss the intuition for this result by comparing the planner's problem (2) and the bank's problem (12). The last term of (12) represents surplus loss resulting from the bank-creditor contract. This is illustrated in Figure 2. Recall that if the bank has sufficient capital, idiosyncratic uncertainty can be remedied by contract renegotiation, which fully restores efficiency because the bank can capture the entire surplus after renegotiation. This is not possible when the bank has insufficient capital. The roll-over risk on short-term debt forces some projects into inefficient liquidation, resulting in a loss of surplus. The bank anticipates that the loss will reduce its marginal return from the investment and reduces investment ex ante. This analysis highlights the channel through which uncertainty is transmitted to real investment decision: It is the interaction between the bank's asset quality and debt capacity that leads to the contraction of credit supply and investment.

IV Extensions and Discussions

A Aggregate Uncertainty

Consider an exogenous aggregate shock that leads to higher uncertainty in the economy. How would it affect the equilibrium financing and investment decision? To answer this question, I adopt a simple definition of *aggregate uncertainty* in terms of a mean-preserving spread of the distribution of project productivity. In particular, suppose the

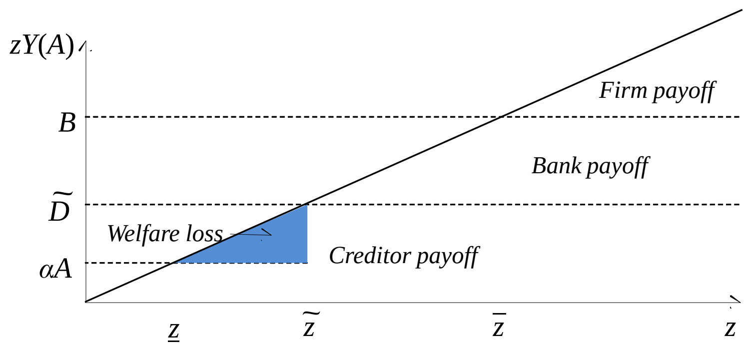

Figure 2: Equilibrium Payoffs and Welfare Loss

distribution of the bad project is unchanged but the good project follows a new distribution F_m such that

$$\int_0^\infty z dF_g(z) = \int_0^\infty z dF_m(z),$$

and there is z^* such that $F_g(z^*) = F_m(z^*)$ and

$$(F_g(z) - F_m(z))(z - z^*) \geq 0. \tag{13}$$

Because both F_m delivers the same expected revenue as F_g, it is still desirable even though it becomes riskier.

The definitions of aggregate uncertainty and idiosyncratic uncertainty (recall Section II) have simple interpretations in the context of my model. Idiosyncratic uncertainty refers to individual project outcome and aggregate uncertainty refers to the aggregate outcome of all projects. Higher uncertainty here resembles a shock to the variance of future productivity as in Bloom (2009) and Jurado et al. (2013). In my model, a representative bank invests in the market portfolio. The model can be easily recasted as a continuum of banks each matched to one project. In this case, idiosyncratic uncertainty shocks can refer to shocks to a bank and aggregate uncertainty shocks can refer to shocks to the banking sector.

The next result presents the second main implication of the model.

Proposition 4 *Conditional on the existence of a banking equilibrium* $\left(A_m^*, B_m^*, D_m^*, \tilde{D}_m^*\right)$, *higher uncertainty leads to lower credit supply and lower investment:* $A_m^* < A^*$.

This result highlights how the contractionary effect of uncertainty is aggravated by an aggregate shock. When uncertainty is high, banks are forced to shed their debt and reduce their balance sheet size. This is consistent with behavior of balance sheet management in financial intermediaries during the 2007-2009 crisis. Adrian and Shin (2010) and Adrian and Shin (2013) document evidence of banks reducing leverage and shrinking balance sheet when uncertainty about the quality of their assets increases. The model implies that this is a market-based mechanism to contain risks of financial intermediaries; however it is socially costly because it investment contraction in the real economy.

B Multiple Assets

In the preceding analysis I assumed that the bank can only invest in one project, thus holds only one (type of) asset. In practice, however, a bank's portfolio consists of multiple assets, whose characteristics jointly affect the bank's financing and investment positions. Allowing the bank to hold multiple assets does not affect the qualitative nature of my results, but does yield some additional implications concerning the transmission of uncertainty on the bank's asset side. A simple way to model this is to introduce two types of good assets . I assume that both assets have the same expected revenue and same variance. This is important to ensure that the bank holds a nondegenerated portfolio of both assets because in equilibrium, the bank only invests in the asset with the higher expected revenue. Other than that, the exact portfolio allocation does not matter for the analysis.

Now consider a shock that increases the correlation of the two types of assets ρ. It is straightforward to establish that all else equal, an increase in ρ increases the variance of the return from the asset portfolio. The same line of argument in Section A applies and I obtain the following result.

Corollary 1 *All else equal, an increase in the return correlation of the bank's asset portfolio leads to lower credit supply and lower investment.*

This inefficient result follows because market discipline is non-discriminative, that is, the short-term creditor funds the bank without discriminating against a certain type of asset.

C Endogenous Default

So far I have assumed that liquidation is involuntary because the short-term creditor is not willing to rolled over the debt. This assumption can be relaxed to allow for debt renegotiation between the bank and the creditor. In this section, I show that all previous results follow by simply allowing the bank to default endogenously. In particular, when the signaled project revenue at $t = 1$ is lower than the bank's debt value (i.e. when $z < \tilde{z}$), the bank may choose to default even if the creditor agrees to partially write off the debt and allow the project to continue. This may happen because equity holders choose not to keep absorbing financial losses by rolling over the debt, in which case the creditor loses the surplus she can potentially gain from debt forgiveness (the equivalence of the welfare loss triangle in Figure 2). In other words, endogenous defaults are costly and the creditor may bear all the cost.

It is also worth noting that the possibility of endogenous default also rules out financing with long-term debt. The intuition is simple. A long-term creditor does not have the power to influence liquidation and payment decisions at $t = 1$. By issuing long-term debt, the creditor forgoes the opportunity to demand payment on her own terms and runs the risk of endogenous default by equity holders.

To see more formally the role of endogenous default in the choice of debt maturity, keep all elements in the model as in previous sections but replace the bank-creditor contract with one that does not need to be rolled over at $t = 1$. I shall leave formal proof on properties of the long-term debt contract to the Appendix and provide a sketch of the argument here. Denote the market value of the long-term debt by D_{long} to distinguish it from the short-term debt value D in the main model. Let $\tilde{D}_{long}$ be its face value. Similar results as in Section B carry over and the optimal debt value implies a liquidation threshold $\tilde{z}_{long}$. If endogenous default is prohibited, the creditor optimally offers to write off a fraction of the debt and roll over the rest when the signal z is such that $z \in (\underline{z}, \tilde{z}_{long})$. The new contract allows the bank to continue the project and pay the creditor all the project revenue $zY(A)$ when $z \in (\underline{z}, \tilde{z}_{long})$ at $t = 2$. I show in the Appendix that the resulting investment level is identical to the first-best solution. In this case, long-term debt has the advantage of increasing the bank's debt capacity and improving efficiency. Now consider the case in which the bank may choose to defaults endogenously in the region $z \in (\underline{z}, \tilde{z}_{long})$. The creditor expects to collect only the liquidation value αA in these states of the world and adjusts the debt value ex ante. The resulting debt value is equal to the debt value of a short-term $\tilde{D}$. In other words, the possibility of endogenous default eliminates the advantage of long-term debt over short-term debt. Short-term debt is an optimal instrument for the creditor in this case.

V Policy Options

When considering policy options, it is important to note that the banking equilibrium described in the last section is constrained optimal. Policy interventions that aim to improve macro efficiency needs to preserve market discipline at the micro level. I first derive an efficiency condition. I then discuss policy options that improves efficiency.

Proposition 3 imply that the source of welfare loss is inefficient runs of short-term debt. The threshold productivity $\tilde{z}$ that triggers a debt run is lower than the efficient liquidation threshold of a planner. Efficiency improves if a policy intervention lowers the debt run trigger, or equivalently lowers the probability of a run. Formally,

Proposition 5 *Absent ex post policy intervention, the banking equilibrium* $\left(A^*, B^*, D^*, \tilde{D}^*\right)$ *incurs no welfare loss if and only if the bank's liquidation value is high enough to fully repay the creditor:*

$$\alpha A \geq \tilde{D}. \tag{14}$$

In other words, for the banking equilibrium (3) to be efficient, the bank's debt value cannot be too high. This result reflects the trade-off between the cost of market discipline and social efficiency. When the moral hazard problem is severe, the cost of discipline is too high and efficiency has to be sacrificed.

A Ex Ante Policy

Capital Requirement

One way to reduce the cost of market discipline is to impose an ex ante capital requirement. The bank needs to hold sufficient equity to buffer the loss of the creditor in all states of the world. It is straightforward to show that setting (14) to equality implies a minimum equity to asset ratio:

$$\frac{E}{A} = \frac{R - \alpha}{R}. \tag{15}$$

The next result shows that although imposing a debt to equity ratio can eliminate welfare loss, it is not sufficient to fully restore efficiency if there is an equity premium.

Proposition 6 *If there is no equity premium (i.e. $R^e = R$), imposing a debt to equity ratio $\alpha / (R - a)$ achieves the first-best investment. If there is equity premium (i.e. $R^e > R$), investment under such capital requirement is still lower than the first-best level.*

Proof. See Appendix. ■

Although capital requirement leads to a new equilibrium that does not incur welfare loss (and therefore is "ex post efficient"), it nevertheless distorts ex ante investment by increasing its marginal cost if there is an equity premium. The reason is that capital requirement only buffers creditor from default risks. It does not address why shareholders require a higher return. It is likely that the equity premium captures other types of risks not related to the bank's capital structure, for example, agency conflict between shareholders and the manager.

It is worth noting that to eliminate welfare loss in the model, it is not necessary to regulate total debt. Limiting the size of short-term debt would be necessary. This is because short-term debt is the key disciplinary device, while all sources of long-term funding have a similar role as equity in buffering the loss of the short-term creditor. This suggests that an efficient level of long-term funding to asset ratio is also equal to $(R - \alpha) / R$. One way to implement this policy is to impose a net stable funding ratio (NSFR) as proposed in the Third Basel Accord (Basel III), in which case net stable funding includes customer deposits, long-term wholesale funding from the interbank lending market and equity.

Liquidity Requirement

Capital requirements regulate the liability side of the balance sheet. An alternative is to regulate the asset side by imposing liquidity requirements. Suppose the bank is required to hold an amount L of liquid assets funded by equity. If the bank's liquidation value is sufficient to pay back the short-term creditor, inefficient debt run and liquidation can be avoided. An equivalent condition to (14) is

$$L + \alpha A \geq \tilde{D}. \tag{16}$$

Because no surplus is lost, one can use similar argument as Proposition 6 to show that the first best investment can be achieved in the absence of an equity premium. Given $A = A^{fb}$, the debt value can be derived using the creditor's required return because payment to the creditor is guaranteed. Thus $RD = \tilde{D}\left(A^{fb}\right)$.

To achieve the efficiency condition (16), the regulator can impose a minimum level of liquid assets to cover the bank's liquidity needs: $L + \alpha A = \tilde{D}$. The regulatory level of liquid assets $L + \alpha A$ has a nice interpretation as the overall stock of liquid assets after haircut. For example, L may include assets such as cash and central bank reserves that are not subject to a haircut; A may include less liquid assets subject to a haircut of $1 - \alpha$.

Contingent Debt

The model points to the lack of state contingency to the bank's liability as a source of inefficiency. This naturally suggests using contingent debt as a potential remedy but there is a more subtle point. Whether contingent debt can be used as a policy tool to improve efficiency crucially depends on the design of conversion triggers. In order to preserve the bank's incentive, conversion triggers should be based on an aggregate state variable, rather than an individual state variable. The intuition is simple. In the model, idiosyncratic risk is directly linked to debt runs so the bank has the incentive to contain the risk. If individual outcomes are used as conversion triggers to prevent bank runs, this linkage will be eliminated and the bank's incentive to choose the good project will be weakened. Using aggregate outcomes as conversion triggers does not affect the bank's incentive as long as the bank's decision does not influence aggregate outcomes or debt conversions. A simple way to model this is to consider an aggregate shock to productivity z after investment was made and contracts were signed. The shock is unexpected so characterizations of the bank's asset and liability at $t = 0$ remain unaffected as in previous sections. After the aggregate signal is observed at $t = 1$, the regulator can trigger a debt conversion in which the bank-creditor contract is converted to a contingent payment scheme. The bank pays the creditor $\tilde{D}$ if $z \geq \tilde{z}$ and αA if $z < \tilde{z}$. It is straightforward to verify that the creditor's payoff remains the same and the PCC (10) is satisfied. In the region where $z \in (\underline{z}, \bar{z})$, bank runs do not occur because the project will generate sufficient revenue to pay the creditor. As a result, all projects with $z > \underline{z}$ is allowed to continue and welfare loss is eliminated.

The above argument lends support to using regulatory-based trigger to reduce the risk of systemic debt run. It is worth noting that when the banking sector is highly concentrated, the distinction between aggregate state and individual state becomes blurry. In this case, even a policy based on a systemic trigger will violate the incentive of a bank that is "too big to fail".

B Ex Post Policy

Monetary Policy

When there is no ex ante policy to eliminate a debt run, is there still a role for ex post policy? Suppose after the investment is made and financing contracts are signed, the productivity signal at $t = 1$ is lower than the creditor's liquidation threshold in the banking equilibrium (3). One way to prevent a debt run is to reduce the creditor's required return. Recall that the creditor's required return on D satisfies

$$RD^* = \alpha A F_g(\tilde{z}) + \tilde{z} Y(A)(1 - F_g(\tilde{z})),$$

which is based on the opportunity cost of capital R at $t = 0$. If at $t = 1$, the monetary authority announces a new interest rate, which effectively reduces the cost of capital from R to $R^m < R$, the creditor will be willing to roll over the short-term debt for a lower payments of $R^m D$ payable at $t = 2$. The next proposition shows that given asset and debt values committed at $t = 0$, an interest rate can be set to eliminate welfare loss.

Proposition 7 *The banking equilibrium* $\left(A^*, B^*, D^*, \tilde{D}^*\right)$ *incurs no welfare loss if at* $t = 1$ *the effective cost of capital is reduced from* R *to*

$$R^m = \frac{\alpha A^*}{D^*}. \tag{17}$$

Proof. See Appendix. ■

This policy is ex post efficient because it lowers the creditor's liquidation threshold and eliminates inefficient debt run. In other words, in the absence of an adequate ex ante policy, monetary intervention can be used to (partially) remedy the inefficiency caused by undercapitalized banks. To see how ex post monetary intervention is complementary to ex ante capital requirements, note that if the minimum capital requirement (15) is not met, the optimal interest rate (17) at $t = 1$ is indeed higher than ex ante rate R. Proposition 7 says that lowering the interest rate can effectively eliminate welfare loss. Similar to the use of contingent debt, ex post monetary interventions have no impact on ex ante investment decisions. To the extent that capital requirements achieve an ex ante investment level that is strictly higher than the laissez-faire level A^*, it is preferable to monetary interventions.

VI Conclusion

This paper provides a theoretical foundation for the channel of uncertainty transmission through the balance sheet of financial intermediaries. On the liability side, short-term debt contracts are optimal instruments for banks that seek debt financing. But uncertainty and roll over risk can lead to inefficient liquidation and affect investment surplus on the asset side. This interaction between the bank's investment and financing positions leads to lower credit and ex ante underinvestment. From a social welfare perspective, there is an important trade-off between the positive role of short-term debt in limiting excess risk-taking by banks and the cost of this market discipline.

The model lends support to direct interventions in the short-term funding markets. The success of policy intervention depends on the regulators' ability to extract surplus while preserving market discipline at the micro level. Imposing capital and liquidity requirements, and reducing interest rate at crisis times can be welfare-improving. Adding contingent debt can also improve efficiency but may create a moral hazard problem for banks that are "too big to fail".

Appendix

Lemma 4 *The mean-residual-lifetime function $MRL(z) \equiv E(z - \underline{z}|z > \underline{z})$ is decreasing in $\underline{z}$ for any $\underline{z}$, that is,*

$$\frac{\partial \mathbb{E}(z - \underline{z}|z > \underline{z})}{\partial \underline{z}} = h(\underline{z})\, \mathbb{E}(z - \underline{z}|z > \underline{z}) - 1 \leq 0. \tag{18}$$

Proof. Under Assumption 2, the survival function is log-concave. The result follows directly from Theorem 6 of Bagnoli and Bergstrom (2005). ■

Proof of Proposition 1. Dividing the left-hand side of the first order condition (3) by $1 - F_g(\underline{z})$ and rearranging gives

$$Y'(A)\, \mathbb{E}(z|z > \underline{z}) - 1 - \frac{(1-\alpha) F_g(\underline{z}) + R - 1}{1 - F_g(\underline{z})} = 0, \tag{19}$$

where the term $Y'(A)\, \mathbb{E}(z|z > \underline{z}) - 1$ represents the marginal revenue from investment at $t = 0$ conditional on the project not being liquidated and the last term is the conditional marginal cost. In what follows, define $MR(A) \equiv Y'(A)\, \mathbb{E}(z|z > \underline{z}) - 1$ and $MC(A) \equiv [(1-\alpha) F_g(\underline{z}) + R - 1] / (1 - F_g(\underline{z}))$.

Evaluate the left-hand side of (19) as A approaches zero. By Assumption 1, $\lim_{A \to 0} MR(A) = \infty$. Following (1), $\lim_{A \to 0} \underline{z}(A) = 0$ and $\lim_{A \to 0} MC(A) = R - 1$; so the left-hand side of (19) goes to infinity as A approaches zero. Now consider the limit as A approaches infinity. Recall $\underline{z}Y'(A) < \alpha < R$. Multiplying both sides of (18) by $Y'(A) > 0$ and rearranging gives

$$Y'(A)\, \mathbb{E}(z|z > \underline{z}) - 1 \leq \frac{Y'(A)}{h(\underline{z})} - 1 + \underline{z}Y'(A) < \frac{Y'(A)}{h(\underline{z})} + R - 1.$$

Taking the limit as A approaches infinity gives $\lim_{A \to \infty} MR(A) < \lim_{A \to \infty} \left(\frac{Y'(A)}{h(\underline{z})} + R - 1 \right) = R - 1$ by Assumptions 1 and 2. Recall $\lim_{A \to \infty} \underline{z}(A) = \infty$, which implies $\lim_{A \to \infty} MC(A) = \infty$. so the left-hand side of (19) goes to negative infinity as A approaches infinity.

Because all the terms are continuous in A, (19) is satisfied at least once following the Intermediate Value Theorem, proving existence. Define A^{fb} such that $MR\left(A^{fb}\right) = MC\left(A^{fb}\right)$.

To show uniqueness, it suffices to show that $MC(A)$ and $MR(A)$ are both monotone. To show the former holds,

$$\frac{\partial MC(A)}{\partial A} = \frac{(R - \alpha)\, h(\underline{z})}{1 - F_g(\underline{z})} \frac{\partial \underline{z}}{A} > 0.$$

The latter is given by

$$\frac{\partial MR(A)}{\partial A} = Y''(A)\, \mathbb{E}(z|z > \underline{z}) + Y'(A) \frac{\partial \mathbb{E}(z|z > \underline{z})}{\partial \underline{z}} \frac{\partial \underline{z}}{A}$$

By Lemma 4, $0 < \frac{\partial \mathbb{E}(z|z>\underline{z})}{\partial \underline{z}} \leq 1$. Using this and $\partial \underline{z}/\partial A = (\alpha - \underline{z}Y'(A))/Y(A)$ gives

$$
\begin{aligned}
\frac{\partial MR(A)}{\partial A} &\leq Y''(A)\,\mathbb{E}(z|z>\underline{z}) + \frac{\alpha Y'(A)}{Y(A)} - \frac{\underline{z}Y'(A)^2}{Y(A)} \\
&\leq Y''(A)\,\underline{z} + \frac{\alpha Y'(A)}{Y(A)} - \frac{\underline{z}Y'(A)^2}{Y(A)} \\
&= \underline{z}\left(Y''(A) - \frac{Y'(A)^2}{Y(A)}\right) + \frac{\alpha Y'(A)}{Y(A)} \\
&= \underline{z}Y(A)\left(\frac{Y'(A)}{Y(A)}\right)' + \frac{\alpha Y'(A)}{Y(A)} \\
&= \alpha\left[A\left(\frac{Y'(A)}{Y(A)}\right)' + \frac{Y'(A)}{Y(A)}\right]
\end{aligned}
$$

By Assumption 1, $(Y'(A)/Y(A))' < 0$ and $(Y'(A)/Y(A))'' < 0$; in other words, $Y'(A)/Y(A)$ is a strictly convex and decreasing function, which implies

$$
A\left|\left(\frac{Y'(A)}{Y(A)}\right)'\right| > \frac{Y'(A)}{Y(A)},
$$

hence $\frac{\partial MR(A)}{\partial A} < 0$. ■

Proof of Proposition 2. I first use local variational arguments to show that debt renegotiation guarantees a binding PCF. Suppose PCF slacks under the optimal contract. For any given A, the bank can raise B by a small amount $B' = B + \varepsilon$ without violating PCF. By (4), the firm's liquidation threshold $\bar{z}$ increases to $\bar{z}'$ and more projects are forced into renegotiation. Now compare the payoff to the bank under the new contract scheme B' to the original. Changing B does not affect $\underline{z}$. Each project with $z < \underline{z}$ pays the same liquidation value and each project with $\underline{z} \leq z < \bar{z}$ pays the same surplus. Each project with $\bar{z} \leq z < \bar{z}'$ pays B originally but pays all its surplus $zY(A) > B$ under the new scheme with B'. Projects with $z > \bar{z}'$ pays B' higher than B. Combined all types, the bank would have been strictly better off choosing B' rather than B. Therefore, the PCF has to bind under the optimal contract.

Substitute the binding PCF into (5) gives

$$
\max_{A} \alpha A F_g(\underline{z}) + \int_{\underline{z}}^{\infty} zY(A)\,dF_g(z) - RA - C. \tag{20}
$$

This is the same as the planner's problem (2) except the reservation payoff C. Because C does not depend on A, the solution to (20) is the same as the first-best solution A^{fb} chosen by the planner. ■

Proof of Lemma 1. Following Breeden and Litzenberger (1978), the price of an Arrow-Debrew contingent claim that pays 1 at s and zero otherwise is given by the second derivative of the option price with respect to the strike price evaluated at s. With risk-neutral principle and agent, the state price is given by the probability. It follows that for any loan the bank might give to the firm (A, B), the difference in the price of put options is

$$\Delta\Pi(s, A) = \int_0^s (G_b(x, A) - G_g(x, A))\, dx, \tag{21}$$

where $G_g(x, A)$ and $G_b(x, A)$ are the density of revenue from the portfolios of good and bad projects respectively for given A. For ease of notation, I suppress the argument A in $G_g(.)$ and $G_b(.)$ in what follows.

For any given A, the project will be liquidated for any $z \le \tilde{z}$, in which case the liquidation value is $x = aA$. For $z > \tilde{z}$, the project revenue x is a monotone mapping to the productivity z given by $x = zY(A)$. In summary, $G_g(x)$ is given by

$$G_g(x) = \begin{cases} 0, & \text{for } x < \alpha A \\ F_g\left(\frac{\alpha A}{Y(A)}\right) & \text{for } x = \alpha A \\ F_g\left(\frac{x}{Y(A)}\right), & \text{for } x > \alpha A \end{cases}. \tag{22}$$

$G_b(x)$ can be derived analogously. By the assumption of SOSD, there is $z^* > \underline{z}$ such that $F_g(z^*) = F_b(z^*)$ and

$$(F_g(z) - F_b(z))(z - z^*) \ge 0$$

for all z. Monotonicity between x and z for any $x > \alpha A = \underline{z}Y(A)$ implies that there is $x^*(A) = z^* Y(A)$ such that $G_g(x^*, A) = G_b(x^*, A)$

$$(G_g(x) - G_b(x))(x - x^*) \ge 0,$$

for all $x > \alpha A$. Since $G_g(x)$ cuts $G_b(x)$ precisely once from below, $\Delta\Pi(x, A)$ is equal to $F_b(\underline{z}) - F_g(\underline{z})$ for $x = \alpha A$. It increases, is maximized at x^*, then decreases for $x > x^*$.

Let $\bar{A}$ denote the notional value of assets. Substituting for the expressions for $G_g(.)$ and $G_b(.)$ and using a change-of-variable approach gives

$$\begin{aligned} \Delta\Pi(\bar{A}, A) &= \int_0^{\bar{A}} (G_b(x) - G_g(x))\, dx \\ &= \int_0^{\bar{A}} x dG_g(x) - \int_0^{\bar{A}} x dG_b(x) \\ &= \alpha A F_g(\tilde{z}) + \int_{\tilde{z}}^{\infty} zY(A)\, dF_g(z) - \alpha A F_b(\tilde{z}) - \int_{\tilde{z}}^{\infty} zY(A)\, dF_b(z) \\ &= \alpha A F_g(\tilde{z}) + \int_{\tilde{z}}^{\bar{z}} zY(A)\, dF_g(z) + (1 - F_g(\bar{z}))\, B \\ &\quad -\alpha A F_b(\tilde{z}) - \int_{\tilde{z}}^{\bar{z}} zY(A)\, dF_b(z) - (1 - F_b(\bar{z}))\, B \\ &= \Delta U^B(A), \end{aligned} \tag{23}$$

where the second line follows from integration by parts and the fourth line follows from binding PCF. This result says that $\Delta\Pi(x, A)$ approaches $\Delta U^B(A)$ from above as x

approaches $\bar{A}$. It is easy to rule out $\tilde{D} = \bar{A}$ as a solution because the bank's expected payoff is always less than $\bar{A}$, thus smaller than the debt value. In this case, no contract will be signed and no investment will be made. Therefore, there is a unique $\tilde{D}^*(A)$ such that $\Delta\Pi\left(\tilde{D}^*, A\right) = \Delta U^B(A)$. Note that this solution implies positive equity value of the bank..

Now consider the limit of $\Delta\Pi(\alpha A, A)$. As a approaches 0

$$\lim_{\alpha\to 0} \Delta\Pi(\alpha A, A) = \lim_{\alpha\to 0} \int_0^{\alpha A} (G_b(x) - G_g(x))\, dx = 0,$$

following (22) and (21). As a approaches R, $\Delta\Pi(\alpha A, A)$ approaches $\Delta\Pi(RA, A)$, which is positive and well defined. Because $\Delta\Pi(\alpha A, A)$ is continuous in a, there exists a $\bar{a} \in [0, \bar{R})$ such that

$$\forall\alpha \leq \bar{a} : \Delta\Pi(\alpha A, A) < \Delta\Pi\left(\bar{A}, A\right). \tag{24}$$

I refer to α being sufficiently small when $\alpha \leq \bar{a}$. ■

Proof of Lemma 2. Because the function $z \to zY(A)$ is continuous, it suffices to show that $\tilde{D}^*(A) \in (\underline{z}Y(A), \bar{z}Y(A))$. When α is sufficiently small, $\Delta\Pi(\alpha A, A) < \Delta\Pi\left(\bar{R}A, A\right) = \Delta\Pi\left(\tilde{D}^*, A\right)$ by (24); therefore $\tilde{D}^*(A) > \alpha A = \underline{z}Y(A)$. To see $\tilde{D}^*(A) < \bar{z}Y(A)$, suppose otherwise so $\tilde{D}^* \geq \bar{z}Y(A) = B$ following solution to B of the bank-firm contract. In this case, the bank's payoff from lending to the firm is insufficient to cover its debt obligation, which violates the bank's participation constraint. Uniqueness follows the monotonicity of $z \to zY(A)$. ■

Proof of Lemma B. As shown in the proof of Lemma 1, $\tilde{D}^* < RA$. Combining with (9) gives

$$RD^* = \tilde{D}^* - \Pi_g\left(\tilde{D}^*, A\right) < \tilde{D}^* < RA.$$

■

Proof of Proposition 3. The first order condition of (12) is

$$\alpha F_g(\underline{z}) + \int_{\underline{z}}^{\infty} zY'(A)\, dF_g(z) - R - \int_{\underline{z}}^{\tilde{z}} (zY'(A) - \alpha)\, dF_g(z) - \frac{\partial\tilde{z}}{\partial A}(\tilde{z}Y(A) - \alpha A) f_g(\tilde{z}) = 0. \tag{25}$$

The first three terms correspond to the first order condition of the planner's problem (3). The last two terms are the marginal welfare loss.

I first show that the left-hand side of (25) is negative when evaluated at the first best solution $A = A^{fb}$. By (3), the first three terms are zero. Evaluate the first term of marginal welfare loss at A^{fb}. Define $l(\alpha) \equiv \int_{\underline{z}}^{\tilde{z}} \left(zY'\left(A^{fb}(\alpha)\right) - \alpha\right) dF_g(z)$. As shown in Proposition 1, for any $a \in [0, R)$, there exists a unique $A^{fb}(\alpha)$ that is continuous and increasing in a and $A^{fb}(0) > 0$. Lemma 2 show that as a approaches 0, $\underline{z}$ approaches 0 while $\tilde{z}$ is positive and well defined if a is sufficiently small. Also, $\tilde{z}$ approaches $\underline{z}$ as

a approaches R. So $\lim_{\alpha \to 0} l(\alpha) > 0$ and $\lim_{\alpha \to R} l(\alpha) = 0$. Because $l(\alpha)$ is continuous, $l(\alpha)$ is positive for any $a \in [0, R)$.

Next consider the second term of marginal welfare loss. Define

$$J(\tilde{z}) \equiv \alpha A F_g(\tilde{z}) + \tilde{z} Y(A)(1 - F_g(\tilde{z})) - R(A - E).$$

Recall that $D = A - E$ and E is exogenous. Following (10), $J(\tilde{z}) = 0$. By Implicit Function Theorem,

$$\begin{aligned} \frac{\partial \tilde{z}}{\partial A} &= -\frac{\partial J/\partial A}{\partial J/\partial \tilde{z}} \\ &= -\frac{\alpha F(\tilde{z}) + \tilde{z} Y'(A)(1 - F_g(\tilde{z})) - R}{\partial J/\partial \tilde{z}}. \end{aligned}$$

The numerator is negative following (10) and Assumption 1. The denominator is given by

$$\begin{aligned} \partial J/\partial \tilde{z} &= \alpha A f_g(\tilde{z}) + Y(A)(1 - F_g(\tilde{z})) - \tilde{z} Y(A) f_g(\tilde{z}) \\ &= [(\alpha A - \tilde{z} Y(A)) h_g(\tilde{z}) + Y(A)](1 - F_g(\tilde{z})) \end{aligned}$$

where $h_g(z) \equiv f_g(z) / (1 - F_g(z))$ is the hazard rate. The sign of $\partial J/\partial \tilde{z}$ is determined by the sign of $\mu(z) \equiv (\alpha A - zY(A)) h_g(z)$. First consider the limits of $\mu(z)$. As z approaches $\underline{z}$, $aY(A)$ approaches aA, so $\mu(z)$ is positive. As z approaches ∞, $\mu(z)$ approaches $-\infty$ because $h_g(z)$ is increasing by Assumption 2. Because $\mu(z)$ is continuous, there exists a unique $z^{\max}$ such that $\mu(z^{\max}) = 0$ and for any $z < z^{\max}$, $\mu(z) > 0$. Now consider $J(z)$. It is easy to check that $J(\underline{z}) = \alpha A - RD < 0$, $J(z)$ is maximized at $z^{\max}$ and approaches $-\infty$ as z approaches ∞. Lemma 2 establishes the existence and uniqueness of $\tilde{z}$. This also implies that $\tilde{z}$ is the unique $z \in (\underline{z}, \bar{z})$ that satisfies (10). By the monotonicity of $J(z)$ for $z \in (\underline{z}, z^{\max})$, $J(z)$ cross zero precisely once at $\tilde{z}$ and $0 = J(\tilde{z}) < J(z^{\max})$. In other words, $\tilde{z} < z^{\max}$, $\mu(\tilde{z}) > 0$, so $\partial J/\partial \tilde{z} > 0$. Put together, the second term of marginal welfare loss is positive and the left-hand side of (25) evaluated at A^{fb} is negative.

As shown in the proof of Proposition 1, the first three terms of (25) approach ∞ as A approaches 0. Following Assumption 1, both $\underline{z}$ and $\bar{z}$ approaches 0 as A approaches 0, implying $\underline{z}$ approaches 0 as well because $\underline{z} \in \{\underline{z}, \bar{z}\}$. So the limit of the left-hand side of (25) is ∞ as A approaches 0. Following continuity of the left-hand side of (25) in A, there exists a $A^* < A^{fb}$ such that (25) is satisfied. This gives A^* as a function of $\left(B, D, \tilde{D}\right)$. Once A^* is given, $\left(B^*, D^*, \tilde{D}^*\right)$ solve PCF, ICB and PCC. Three equations with three unknowns give unique solution. ■

Proof of Lemma 4. The approach is similar to the proof of Proposition 3. The bank's problem becomes:

$$\max_A \alpha A F_m(\underline{z}) + \int_{\underline{z}}^{\infty} zY(A)\, dF_m(z) - RA - C - \int_{\underline{z}}^{\tilde{z}} (zY(A) - \alpha A)\, dF_m(z). \quad (26)$$

The first order condition is

$$\alpha F_m(\underline{z})+\int_{\underline{z}}^{\infty} zY'(A)\,dF_m(z)-R-\int_{\underline{z}}^{\tilde{z}}(zY'(A)-\alpha)\,dF_m(z)-\frac{\partial\tilde{z}}{\partial A}(\tilde{z}Y(A)-\alpha A)\,f_m(\tilde{z})=0. \tag{27}$$

To prove $A_m^* < A^*$, it is sufficient to show that the first order condition (27) evaluated at A^* is negative, that is

$$\alpha F_m(\underline{z})+\int_{\underline{z}}^{\infty} zY'(A^*)\,dF_m(z)-R-\int_{\underline{z}}^{\tilde{z}}(zY'(A^*)-\alpha)\,dF_m(z)-\frac{\partial\tilde{z}}{\partial A}(\tilde{z}Y(A^*)-\alpha A^*)\,f_m(\tilde{z})<0. \tag{28}$$

Comparing the left-hand side of (28) and (25) gives

$$\begin{aligned}&\alpha(F_m(\underline{z})-F_g(\underline{z}))+\int_{\underline{z}}^{\infty} zY'(A^*)\,d(F_m(z)-F_g(z)) \\ &-\left[\int_{\underline{z}}^{\tilde{z}}(zY'(A^*)-\alpha)\,d(F_m(z)-F_g(z))\right] \\ &-\frac{\partial\tilde{z}}{\partial A}(\tilde{z}Y(A^*)-\alpha A^*)(f_m(\tilde{z})-f_g(\tilde{z})).\end{aligned} \tag{29}$$

Because the left-hand side of (25) is 0, showing (28) is equivalent to showing (29) is negative. Consider the first two terms. The part of proof of Lemma 1 showing binding ICB is still valid when F_g is changed to F_m for given $A = A^*$, that is, $\Delta U^B(A^*) = \Delta\Pi(RA, A; F_g) = \Delta\Pi(RA, A; F_m)$, which by substituting (23) and PCF gives

$$\alpha(F_m(\underline{z})-F_g(\underline{z}))+\int_{\underline{z}}^{\infty} z\frac{Y(A^*)}{A^*}d(F_m(z)-F_g(z))=0. \tag{30}$$

Recall $\underline{z}Y(A)=\alpha A$ for given A. Evaluating at A^*. The strict convexity of $Y(A)$ implies $\underline{z}Y'(A^*)<\alpha=\underline{z}Y(A^*)/A$ and $zY'(A^*)<zY(A^*)/A$ for any $z>\underline{z}$. So

$$\int_{\underline{z}}^{\infty} zY'(A^*)\,d(F_m(z)-F_g(z))<\int_{\underline{z}}^{\infty} z\frac{Y(A)}{A}d(F_m(z)-F_g(z)).$$

Combined with (30), this implies the first two terms of (29) are negative.

Define the third term of (29) as $l_{mg}(\alpha)\equiv\int_{\underline{z}}^{\tilde{z}}(zY'(A^*)-\alpha)\,d(F_m(z)-F_g(z))$. As a approaches 0, $zY'(A^*(0))>0$, $\underline{z}$ approaches 0 and $\tilde{z}$ is positive and well defined. The proof of Proposition 1 shows that $\tilde{z}<z^*$, it following that $F_m(z)-F_g(z)>0$ for any $z\in(\underline{z},\tilde{z})$. Put together, as a approaches 0, $l_{mg}(\alpha)>0$. As a approaches R, $l_{mg}(\alpha)$ approaches zero because $\tilde{z}$ approaches $\underline{z}$. Because $l_{mg}(\alpha)$ is continuous, $l_{mg}(\alpha)$ is positive for any $a\in[0,R)$. To show the last term of (29) is positive, first note that $f_m(\tilde{z})-f_g(\tilde{z})>0$ because $\tilde{z}<z^*$, where z^* is defined in (13). $\frac{\partial\tilde{z}}{\partial A}(\tilde{z}Y(A^*)-\alpha A^*)>0$ as shown in the proof of Proposition 3. ■

Proof (long-term contract). I shall prove long-term contract properties summarized in Section C. To distinguish the long-term debt value from the short term debt value, let

D_{long} and $\tilde{D}_{long}$ be the market value and face value of the long-term debt, respectively, and $\tilde{z}_{long}$ be the corresponding liquidation threshold. For simplicity and with abuse of notation, I keep the other notations as the main model. I omit the prove for the uniqueness of $\tilde{D}_{long}$ and $\tilde{z}_{long}$ which follows the same line of argument as Lemma 1 and 2.

Case 1. To show debt value in the absence of endogenous default, note that the renegotiated contract promises the long-term creditor $zY(A)$ in the region where $z \in (z, \tilde{z}_{long})$. The PCC becomes

$$RD^{*} = \alpha AF_{g}(\underline{z}) + \int_{\underline{z}}^{\tilde{z}_{long}} zY(A)\,dz + \tilde{z}_{long}Y(A)\left(1 - F_{g}(\tilde{z}_{long})\right). \tag{31}$$

Using backward induction, the bank's problem is given by

$$\max_{\left(A,B,D_{long},\tilde{D},\tilde{z}_{long}\right)} \int_{\tilde{z}_{long}}^{\bar{z}} zY(A)\,dF_{g}(z) + B\left(1 - F_{g}(\bar{z})\right) - \tilde{D}_{long}\left(1 - F(\tilde{z}_{long})\right)$$

subject to (6), (31), and

$$\Delta U^{B}(A) \geq \Delta\Pi\left(\tilde{D}_{long}, A\right).$$

All constraints are binding as in the proof of Proposition 3, which transform the bank's problem into

$$\max_{A} \alpha AF_{g}(\underline{z}) + \int_{\underline{z}}^{\infty} zY(A)\,dF_{g}(z) - RA - C.$$

This is the same as the planner's problem (2) except for the firm's reservation payoff C. Because C does not depend on A, the solution to this problem is the same as the first-best solution A^{fb}.

Case 2. To show debt value in the case of endogenous default, note that default may happen for any z such that $z \in (z, \tilde{z}_{long})$, in which case the creditor receives the liquidation value aA. Taking this into account, the creditor's requires payoff is given by

$$RD_{long} = \alpha AF_{g}(\tilde{z}_{long}) + \tilde{z}_{long}Y(A)\left(1 - F_{g}(\tilde{z}_{long})\right).$$

Note that this is identical to the PCC of a short-term contract (10). It is straightforward to show that the bank's problem is also identical to the case of a short-term contract (11). It follows from the uniqueness of the equilibrium (3) that the value of a long-term contract is identical to that of a short-term debt. ■

Proof of Proposition 5. It is easy to see from (1) and Lemma 2 that $\alpha A = \tilde{D}$ when $\tilde{z} = \underline{z}$. Substituting $\tilde{z} = \underline{z}$ and (1) into (10) gives

$$RD = \alpha AF_{g}(\underline{z}) + \underline{z}Y(A)\left(1 - F_{g}(\underline{z})\right) = \alpha A.$$

The maximal debt to asset ratio follows directly. ■

Proof of Proposition 6. Because capital requirement ensures no surplus will be loss, investment chosen by the bank also maximizes total surplus:

$$\max_{A} \alpha A F_g\left(\underline{z}\right) + \int_{\underline{z}}^{\infty} zY\left(A\right) dF_g\left(z\right) - R^t A - C, \tag{32}$$

where the total return on investment R^t is a weighted average of debt and equity return

$$R^t = R + \left(R^e - R\right) \frac{R - \alpha}{R}.$$

The only difference between (32) and the planner's problem (20) is the higher marginal cost of investment when $R^e > R$, in which case lower investment level follows directly from the convexity of the its marginal revenue. ■

Proof of Proposition 7. Suppose the creditor is willing to roll over debt for projects with $z \geq \tilde{z}^m$ under the reduced cost of capital. The break-even payment for the creditor is

$$R^m D = \alpha A F_g\left(\tilde{z}^m\right) + \tilde{z}^m Y\left(A\right)\left(1 - F_g\left(\tilde{z}^m\right)\right).$$

Eliminating welfare loss requires $\tilde{z}^m = \underline{z}$, which in turn gives

$$R^m D = \alpha A F_g\left(\underline{z}\right) + \underline{z} Y\left(A\right)\left(1 - F_g\left(\underline{z}\right)\right) = \alpha A.$$

■

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