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The Flexible System of Global Models – FSGM

by Michal Andrle, Patrick Blagrave, Pedro Espaillat, Keiko Honjo, Benjamin Hunt, Mika Kortelainen, René Lalonde, Douglas Laxton, Eleonora Mavroeidi, Dirk Muir, Susanna Mursula, and Stephen Snudden

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Abstract

The Flexible System of Global Models (FSGM) is a group of models developed by the Economic Modeling Division of the IMF for policy analysis. A typical module of FSGM is a multi-region, forward-looking semi-structural global model consisting of 24 regions. Using the three core modules focused on the G-20, the euro area, and emerging market economies, this paper outlines the theory under-pinning the model, and illustrates its macroeconomic properties by presenting its responses under a wide range of experiments, including monetary, financial, demand, supply, fiscal and international shocks.

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I. INTRODUCTION

In recent years, considerable effort has gone into building Dynamic Stochastic General Equilibrium (DSGE) models of the global economy and applying them to policy analysis work at the International Monetary Fund (IMF). Two key examples are the Global Economic Model (GEM) by Laxton and Pesenti (2003)[1] and the Global Integrated Monetary and Fiscal Model (GIMF) by Kumhof and Laxton (2007).[2] These models are rigorously derived from the complete optimization problems facing households and firms.[3]

Both GIMF and GEM are complex structural models with multiple goods and full stock-flow consistency. Because of their structural detail and full tracking of all the bilateral trade flows of multiple goods, there is a constraint on the number of countries/regions that can be described at one time with these models. With both GIMF and GEM, that practical size constraint has turned out to be 6 countries/regions.[4] However, the IMF is being called on more frequently to deliver global macroeconomic analysis that simultaneously covers a much larger number of countries. For example, the analysis that the IMF has been asked to provide to the G-20 to support the Mutual Assessment Program (G20MAP) requires the application of a macroeconomic model with individual blocks for each G20 member country. In addition, the model needs to have sufficient structure to fully capture the global savings and investment decisions underlying the G-20 objective of ensuring strong, sustainable and balanced global growth.

To provide the required support to the G-20, and other similar demands that require the simultaneous analysis of a large number of countries, the IMF has been developing a new suite of macroeconomic models, called the Flexible System of Global Models (FSGM). FSGM contains 3 core modules, each of which fully encompasses the global economy. Each module has 24 countries/regions.[5] The first module that was completed was the G20 Model

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[1] For a more detailed discussion of GEM see IMF Staff Papers Vol. 55 No. 2 – for example, Pesenti (2008). This volume is devoted exclusively to GEM, its structure, properties and several practical applications.

[2] For a more detailed description of the theoretical structure of GIMF see Kumhof and others (2010) and for more details about its simulation properties see Anderson and others (2013).

[3] Rigorous derivation from optimizing foundations also characterizes SIGMA by Erceg and others (2006), QUEST by Ratto and others (2008), and NAWM by Christoffel and others (2008).

[4] Larger versions of the models are technically possible to construct and solve, but it was found that it was too difficult and time consuming to understand the simultaneous interactions between so many countries, and consequently these larger models were abandoned.

[5] For more details, please see Appendix I.

(G20MOD). G20MOD has an individual block for each G20 country and four other blocks that effectively complete the rest of the world. The two other core modules are the Euro Area Model (EUROMOD) and the Emerging Markets Model (EMERGMOD). EUROMOD contains a block for each of the 11 major euro area countries plus 13 other blocks and EMERGMOD has blocks for many more emerging market economies and regions than are contained in G20MOD. In close coordination with Area Department teams, additional modules are also being developed that have more individual country blocks devoted to countries in their respective regions.

To address the computational problem to make it feasible to have a model with 24 blocks, the structure of FSGM modules is substantially simplified relative to GIMF and GEM. FSGM modules are semi-structural, with some key elements, like private consumption and investment, having micro-foundations, with others, such as trade, labor supply, and inflation having reduced-form representations.[6] Giving up structure comes at cost in terms of economic tractability and coherence. However, FSGM has been developed to minimize those costs. The reduced-form components of the model have been designed carefully with a great deal of attention focused on overall system properties using both GIMF and GEM as consistency checks. In addition, FSGM's reduced-form structure has allowed for more empirical content in the determination of its dynamic adjustment properties. This has been an advantage in terms of introducing more heterogeneity into the behavior of individual countries relative to what is feasible with calibrated DSGE models.

This paper provides a description of the structure of a typical FSGM module and presents some simulation experiments that help illustrate the model's dynamic adjustment properties. In section II a brief overview of the theoretical structure used for each FSGM module is presented followed with a more detailed technical description. Those not interested in the specifics of the key model equations can go to section III after reading the overview of the model. In section III, the technique currently used to specify the model's behavioral parameters is described as well as a brief description of the technique that is being developed to apply a system estimation approach to pinning down the model's parameter values in the future. Section IV contains some simulation experiments that illustrate the model's properties in the face of a number of commonly encountered economic shocks, using different regions from G20MOD.

[6] FSGM's semi-structural nature is similar to is spirit to models like FRB/US by Brayton and Tinsley (1996) and G-MUSE by Blagrave and others (2015).

II. THE STRUCTURE OF FSGM

Each FSGM module is an annual, multi-region, general equilibrium model of the global economy combining both micro-founded and reduced-form formulations of various economic sectors. Each country/regional block is structurally identical, but with potentially different key steady-state ratios and behavioral parameters.

A. Model Overview

Real GDP in the model is determined by the sum of its demand components in the short run, and the level of potential output in the long run. The key price in the model is the consumer price index (CPI), and it is modeled by a Phillips curve. What follows is a brief overview of the components of aggregate demand, potential output, the price block, commodities, and finally monetary and fiscal policy. Following this, the key elements of FSGM modules are described in detail, beginning with the household consumption and business investment decisions, based on optimizing behavior. Next potential output and labor, the Phillips curves and relative prices, trade, and commodities are described. Finally, an outline of the government sector, financial sector variables, and the external sector to close the model are provided.[7] Note that when equations are presented, they are all detrended by a steady-state trend in population, productivity, or inflation, as appropriate.

Aggregate Demand

Aggregate demand follows the standard national expenditure accounts identity, where real GDP is the sum of household consumption, private business investment, government absorption and exports of goods and services, less imports of goods and services.

Private Consumption

The consumption block uses a discrete-time representation of the Blanchard-Weil-Yaari OLG model, based on a constant-elasticity-of-substitution utility function containing only consumption. Using OLG households rather than infinitely-lived households results in important non-Ricardian properties whereby the path for government debt has significant economic implications. In the OLG framework, households treat government bonds as wealth since there is a chance that the associated tax liabilities will fall due beyond their expected lifetimes. The OLG formulation results in the endogenous determination of national savings given the level of government debt. The world real interest rate adjusts to equilibrate

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[7] Greater detail on the more structural portions of the model (such as the consumption and investment blocks) can be found in Appendix II.

the global supply of and demand for savings. The use of an OLG framework necessitates the tracking of all the stocks and flows associated with wealth - human wealth (based on labor income) and financial wealth (based on government debt, the private business capital stock, and net foreign assets). It should be noted that financial markets are incomplete, so international financial flows are tracked as net positions (net foreign assets or net foreign liabilities) and denominated in U.S. dollars.

Consumption dynamics are driven not only by OLG households, but also by liquidity constrained (LIQ) households. LIQ households do not have access to financial markets, do not save, and thus consume all their income each period. This feature amplifies the non-Ricardian properties of the basic OLG framework.

Private Investment

Private business investment uses an updated version of the Tobin's Q model, with quadratic real adjustment costs. Investment is negatively correlated with real interest rates. Investment cumulates to the private business capital stock, which is chosen by firms to maximize their profits. The capital-to-GDP ratio is inversely related to the cost of capital, which is a function of depreciation, the real interest rate, the corporate tax rate, and relative prices.

Public Absorption

Government absorption consists of spending on consumption and investment goods. Government consumption spending only affects the level of aggregate demand. It is an exogenous choice determined by the fiscal authority. The level of government investment is also chosen exogenously, but in addition to affecting aggregate demand directly it also cumulates into a public capital stock, which can be thought of as public infrastructure (roads, buildings, etc.). A permanent increase in the public capital stock permanently raises the economy-wide level of productivity.

Net Exports

The real competitiveness index (RCI) is the long-run determinant of the level of net exports that adjust to achieve the current account balance required to support the desired net foreign asset position. Exports and imports, individually, are modeled as reduced-form equations. Exports increase with foreign activity, and are also an increasing function of the depreciation in the RCI. Imports increase with domestic activity, and are an increasing function of the appreciation of the real effective exchange rate (REER).

To keep the dimensionality of the model small enough to allow it to have a large number of individual country blocks the model does not track all the bilateral trade flows among countries. The model has, however, been developed to have exchange rate and export volume properties that are similar to the IMF's multiple-good, structural models. This is accomplished by having time-varying trade shares that are a function of the relative level of tradable and nontradable productivity within each country. Consequently, the model is able reproduce the currency appreciation that results when a country's tradable sector productivity growth exceeds that in the nontradable sector (Balassa-Samuelson effect). Further, even though only the aggregate levels of exports and imports are tracked in each country, there are mechanisms in place that ensure global exports and imports sum to zero**.**

Importantly, the current account and implied net-foreign-asset positions are intimately linked to the saving decision of households. The model can be used to study both creditor and debtor nations as non-zero current accounts can be a feature of the well defined steady-state in the OLG framework.

Aggregate Supply

Aggregate supply is captured by potential output, which is based on Cobb-Douglas production technology with trend total factor productivity, the steady-state labor force, the non accelerating inflation rate of unemployment (NAIRU), and the actual capital stock.

Steady-state population growth is taken as exogenous, although there are cyclical variations in both the participation rate and the unemployment rate. The behavior of the participation rate is based on properties of labor supply observed in other IMF structural models, GIMF and GEM. The unemployment rate varies relative to the NAIRU according to an Okun's law relationship based on the output gap.

Prices

The core price in all regions is the consumer price index excluding food and energy, CPIX, which is determined by an inflation Phillips curve. CPIX inflation is sticky and reflects the expected paths of exchange rates and the economic cycle, as captured by the output gap. In addition, although the direct effects of movements in food and energy prices are excluded, there is a possibility that persistent changes in oil prices can leak into core inflation. The degree of forward looking behavior in inflation is country specific.

Prices mimic the structure of production of consumption, investment, government, and exports goods and services. The headline consumption deflator is the CPI (including the effects of oil and food prices). The investment deflator is a weighted average of the deflators for GDP and imports. The government deflator moves in tandem with the CPIX deflator. The deflator for exports is an estimated equation, with coefficients on the GDP deflator, and a competitiveness-weighted average of the relative price of foreign goods, accounting for real exchange rate movements. The import price deflator is an import-weighted average of all other countries' export price deflators. Finally, the GDP deflator itself is a real-component weighted average of the consumption, investment, government, export, and import deflators.

In addition, there is a Phillips curve for nominal wage growth. Wage inflation exhibits stickiness and allows the real wage to return to its equilibrium only gradually depending on the expected evolution of overall economic activity.

Commodities

The model incorporates three types of commodities – oil, food and metals. This allows for a distinction between core and headline inflation, and provides richer analysis of the macroeconomic differences between commodity-exporting and importing regions.

The demand for commodities is driven by the world demand and is relatively price inelastic in the short run due to limited substitutability of the commodity classes considered. The supply of commodities is also price inelastic in the short run. Countries can trade in commodities, and households consume food and oil explicitly, allowing for the distinction between headline and core CPI inflation.

Commodities can function as a moderator of business cycle fluctuations in the model. In times of excess aggregate demand, the upward pressure on commodities prices from sluggish adjustment in commodity supply relative to demand will put some downward pressure on demand. Similarly, if there is excess supply, falling commodities prices will ameliorate the deterioration. However, shocks originating in the commodities sectors can be highly disruptive to global macroeconomic activity. Also, when countries or regions own business cycle is out of sync with the global business cycle, commodity prices will act to amplify rather than dampen fluctuations in activity.

Monetary and Fiscal Policy

In the short run, the nominal side of the economy is linked to the real side through monetary policy. The behavior of monetary authorities is represented by an interest rate reaction function. The standard form is an inflation-forecast-based rule operating under a flexible exchange rate. However, the form of the interest rate reaction function is such that there is scope for a fixed exchange rate regime, monetary union, or a managed floating exchange rate regime.

The model also contains a 10-year interest rate that is based on the expectations theory of the term structure, plus a term premium. The interest rates on consumption, investment, government debt and net foreign assets are weighted averages of the 1-year and 10-year interest rates, reflecting their differing term structures, and allowing for a meaningful role for the term premium.

The government sector is much broader than government absorption. There is additional spending by the fiscal authority on lumpsum transfers to all households, or targeted exclusively to liquidity-constrained households. The fiscal authority chooses a long-run level of debt relative to GDP (or conversely, a long-run deficit target). In order to meet its debt or deficit targets as well as spending obligations, it can tax, using consumption taxes (VAT), labor income taxes, corporate income taxes and lumpsum taxes. In the face of shocks to the economy under the standard fiscal reaction function, all tax rates remain fixed and spending on general lumpsum transfers adjusts to ensure that the public debt-to-GDP ratio is maintained in the medium term. However, the fiscal reaction function can also be specified to use other instruments besides general transfers.

B. Consumption

Two types of households can consume in the model. There are overlapping generations (OLG) households that can accumulate and draw down from wealth (which is equivalent to future labor income, net foreign assets, government debt, and the private business capital stock at market value). There are also liquidity-constrained (LIQ) households that can only consume out of their current labor income and net transfers from the government. Aggregate consumption, $\check{c}_t$, is the sum of consumption by the OLG households, $\check{c}_t^{OLG}$, and LIQ households, $\check{c}_t^{LIQ}$, and all share a common price deflator, p_t^c:

$$p_t^c \check{c}_t = p_t^c \check{c}_t^{OLG} + p_t^c \check{c}_t^{LIQ}.$$

OLG Households

The theory behind the OLG households is broadly in line with the overlapping generations' model of Blanchard (1985), Buiter (1988), Weil (1987) and Yaari (1965). FSGM uses a slightly different interpretation of the parameters, in line with GIMF. Households have a planning horizon of 20 years, which implies some degree of myopia, θ, in their planning decisions. This is slightly different than the original terminology, framed in terms of households having a working life based on their probability of death. Regardless of how we frame the terminology for OLG households, the qualitative results are the same. OLG households discount their future more than the standard rate of time preference would imply. Therefore any government debt accumulated in the lifetime of an OLG household's lifetime will not necessarily need to be repaid by them, but by future, disconnected, generations, which means government debt holdings become an explicit source of wealth. This gives rise to households deciding their supply of saving as consumers, and their demand for investment as owners of firms. Therefore holdings of net foreign assets are endogenously determined, with the global real interest rate serving as the endogenous market-clearing price.

OLG households' consumption is based on their marginal propensity to consume out of wealth, Θ_t^{-1}:

$$\check{c}_t^{OLG} = \Theta_t^{-1}(p_t^c \breve{w}f_t + \breve{w}o_t + \breve{w}h_t).$$

Wealth, $\breve{w}$, has three components: financial wealth, $\breve{w}f_t$, other wealth, $\breve{w}o_t$, and human wealth, $\breve{w}h_t$. Financial wealth is assumed to be the current value of domestic government bonds, $\check{b}_{t-1}$, net foreign assets, $\check{f}_{t-1}$, converted to domestic currency by Z_{t-1}, and the market value of the private business capital stock, $\breve{w}k_t$. The gross return to these assets is determined by their respective interest rates INT_t^{GB} and INT_t^{NFA} , detrended by the nominal steady-state growth rate of the economy, $\pi_t g_t n$, where π_t is steady-state inflation, g_t is the steady-state productivity growth rate, and n is the steady-state population growth rate.

$$p_t^c \breve{w} f_t = \frac{1}{\pi_t g_t n}\left[INT_{t-1}^{GB}\breve{b}_{t-1} + \ INT_{t-1}^{NFA}\frac{\breve{f}_{t-1}}{Z_{t-1}} + \breve{w}k_t\right].$$

Other wealth, $\breve{w}o_t$, comprises lumpsum transfers from the government (both their share of general transfers, $(1-\lambda^c)\breve{\Upsilon}_t$, and transfers targeted directly to them, $\breve{\Upsilon}_{\mathrm{t}}^{\mathrm{OLG}}$), less lumpsum taxes. Future other wealth is further discounted by the degree of OLG household myopia, θ:

$$\breve{w}o_t = (1-\lambda^c)(\breve{\Upsilon}_t - tax_t^{ls}) + \breve{\Upsilon}_{\mathrm{t}}^{\mathrm{OLG}} + \mathrm{E}_{\mathrm{t}}\frac{\theta g_{t+1}}{\breve{r}_t^c}\breve{w}o_{t+1}.$$

where $\breve{r}_t^c$ is the real interest rate faced by consumers.

The third component, human wealth, is the discounted lifetime value of labor income. Future human wealth is further discounted by both the degree of OLG household myopia and the decline in households' individual labor productivity over their lifetimes, χ:

$$\breve{w}h_t = (1-\lambda^c)(1-\tau_t^L)\breve{w}_t\breve{l}_t + \mathrm{E}_{\mathrm{t}}\frac{\theta\chi g_{t+1}}{\breve{r}_t^c}\breve{w}h_t.$$

where τ_t^L is the labor income tax rate and $\breve{w}_t\breve{l}_t$ is labor income.

The inverse of the marginal propensity to consume, Θ_{t}, depends on, among other things, the consumption tax, τ_t^C, in the short run.

$$\Theta_{\mathrm{t}} = E_t\sum_{s=0}^{\infty}(1+\tau_{t+s}^c)\theta^s\prod_{k=0}^{s-1}\left(\frac{j_{t+k}}{\breve{r}_{t+k}^c}\right),$$

where j_t is the proportion of the utility function which not only defines consumption, $\breve{c}_t$, but defines the relationship of consumption with the endogenous real global interest rate linked to $\breve{r}_{t+k}^c$. j_t, often referred to as the stochastic pricing kernel, is defined as:

$$j_t = (\beta\breve{r}_t^c)^{\frac{1}{\sigma}}\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)^{\frac{1}{\sigma}}\left(\frac{\breve{c}_t^{OLG} g_t n}{\breve{c}_{t-1}^{OLG}}\right)^{\gamma\frac{\sigma-1}{\sigma}},$$

where σ is the inverse of the intertemporal elasticity of substitution, and γ is the degree of external habit persistence, which is why $\breve{c}_{t-1}^{OLG}$ appears as well as $\breve{c}_t^{OLG}$. External habit persistence in OLG households' consumption choice implies that the current consumption by each individual OLG household is based on the previous periods' consumption choice by all OLG households. By acknowledging that OLG households' consumption patterns have a high degree of inertia, there will be a hump-shaped response of OLG households' consumption to any shock.

LIQ Households

Liquidity constrained households consume from their current income only:

$$p_t^C(1+\tau_t^c)\check{c}_t^{LIQ} = \lambda_t^c\left(\Upsilon_t - tax_t^{ls}\right) + \Upsilon_t^{LIQ} + \lambda_t^c(1-\tau_t^L)\breve{w}_t\check{l}_t,$$

where λ_t^c is the share of liquidity-constrained households, Υ_t is general lumpsum transfers, tax_t^{ls} is lumpsum taxes, Υ_t^{LIQ} is transfers to liquidity constrained households only, $\breve{w}_t\check{l}_t$ is labor income, and $\check{c}_t^{LIQ}$ is the consumption of liquidity-constrained households.

C. Capital and Investment

Firms produce goods and services using labor, defined below, and their holdings of private business capital. $\check{k}_t$, which are defined by the perpetual inventory equation:

$$\check{k}_t = (1-\delta_t)\frac{\check{k}_{t-1}}{g_t n} + \check{I}_t.$$

Where $\check{k}_t$, after accounting for depreciation, δ_t, cumulates over time from annual investment flows, $\check{I}_t$.

Investment demand follows the Tobin's Q (the shadow price of capital) model of investment, where firms accumulate capital based on, q_t:

$$q_t = p_t^i + p_t^i c_1^q\left(\frac{\check{I}_t}{\check{I}_{t-1}} - 1\right)\left(\frac{\check{I}_t}{\check{I}_{t-1}}\right) - E_t\frac{\theta g_{t+1} n}{(\breve{r}_t^{corp})^{c_2^q}} p_{t+1}^i\left(\frac{\check{I}_{t+1}}{\check{I}_t} - 1\right)\left(\frac{\check{I}_{t+1}}{\check{I}_t}\right)^2 + \epsilon_t^q,$$

where p_t^i is the current price of investment, $\breve{r}_t^{corp}$ is the real cost of corporate financing, with its discounting effect magnified by c_2^q, ($p_t^i c_1^q\left(\frac{\check{I}_t}{\check{I}_{t-1}} - 1\right)\left(\frac{\check{I}_t}{\check{I}_{t-1}}\right) - E_t\frac{\theta g_{t+1} n}{(\breve{r}_t^{corp})^{c_2^q}} p_{t+1}^i\left(\frac{\check{I}_{t+1}}{\check{I}_t} - 1\right)\left(\frac{\check{I}_{t+1}}{\check{I}_t}\right)^2$ captures costly adjustment in investment (as found in GIMF, in Kumhof and others, 2010), and ϵ_t^q is a shock term. In steady state, where $\check{I}$ is unchanged from period to period, the shadow price of capital is equal to the price of investment goods, its replacement cost.

Tobin's Q is the ratio of the market value of an additional unit of capital to its replacement costs. If Tobin's Q is above the current price of investment, p_t^i, then this implies that investment is highly profitable.

We can define the market value of the private business capital stock, $\breve{w}k_t$, as:

$$\breve{w}k_t = q_t \breve{k}_{t-1}.$$

The dynamic path of Tobin's Q is defined by:

$$q_t = E_t \frac{\theta}{(\breve{r}_t^{corp})^{C_2^q}} \Big((1-\tau_{t+1}^K) r_{t+1}^K + \tau_{t+1}^K \delta_t q_{t+1} + q_{t+1}(1-\delta_t) + \epsilon_t^i \Big).$$

where q_t is linked to discounted value (by the magnified real cost of corporate financing, $(\breve{r}_t^{corp})^{C_2^q}$) of the return on capital, r_{t+1}^K, the corporate income tax, τ_{t+1}^K, and the depreciation rate, δ_t, plus a shock, ϵ_t^i.

D. Potential Output and the Output Gap

Potential output, which corresponds to production under the full employment equilibrium, is given by Cobb-Douglas technology:

$$\breve{y}_t^{FE} = TFP_t^{FE} COM_t^{FE} \left(\frac{\breve{k}_{t-1}}{g_t n}\right)^{\alpha^{FE}} \left(\left(1 - \frac{U_t^{FE}}{100}\right) \breve{lf}_t^{FE}\right)^{1-\alpha^{FE}},$$

where $\breve{y}_t^{FE}$ is potential output, TFP_t^{FE} is trend total factor productivity, COM_t^{FE} is the effect of permanent shocks to oil and metals, α_t^{FE} is the exogenous trend capital share of income, U_t^{FE} is the exogenous NAIRU (non-accelerating inflation rate of unemployment) and lf_t^{FE} is the equilibrium labor force.

Trend total factor productivity has an exogenous component, but also depends on spillover effects from changes in trend TFP in other regions of the world, and on the deviation of the public capital stock (infrastructure) from its long-run value[8]. The effect from infrastructure is similar to that of GIMF (Kumhof and others, 2011).

Real GDP, $\breve{y}_t$, is determined by the real national expenditure accounts identity and the log of output gap, log ($\breve{y}_t^{gap}$), is defined as a log difference between real and potential output:

$$log(\breve{y}_t^{gap}) = log\left(\frac{\breve{y}}{\breve{y}_t^{FE}}\right).$$

[8] The effect is calibrated based on Ligthart and Suarez (2011).

The output gap is the key variable for determining the effects of both demand and supply in prices and inflation, as indicated below.

E. Labor Sector

Given the output gap and the exogenous NAIRU, unemployment, U_t, is determined based on an Okun's law relationship:

$$U_t = U_t^{FE} + c_1^U(U_{t-1} - U_{t-1}^{FE}) - 100c_2^U(1 - c_1^U)log(\breve{y}_t^{gap}) + \epsilon_t^U.$$

The active labor force is defined by the actual participation rate and the population:

$$\breve{lf}_t = PART_t N_t,$$

The full-employment labor force is defined by the full-employment participation rate and the population:

$$\breve{lf}_t^{FE} = PART_t^{FE} N_t.$$

To capture labor supply effects that would otherwise be absent since household utility is based solely on consumption, there are reduced-form equations governing the properties of both the actual participation rate, $PART_t$, and full-employment participation rate, $PART_t^{FE}$. First, these equations depend on the real wage net of consumption and labor income taxes, detrended by trend total factor productivity. Therefore increases in productivity are neutral in the long run in the labor supply decision, while consumption and labor taxes will reduce labor supply. Secondly, the participation rate equations also depend on the available fiscal instruments, where labor will respond appropriately in the face of temporary, temporary but persistent or permanent fiscal shocks, given the existing link with the detrended after-tax real wage.

The transmission effects from these relationships with the real wage and fiscal instruments are calibrated based on the labor supply reactions in GIMF and GEM, where the labor supply decision is fully articulated as part of a consumption-leisure trade-off in the households' utility function.

The labor input, $\breve{l}_t$, is in turn defined by $\breve{l}_t = \left(1 - \frac{U_t}{100}\right)\breve{lf}_t$, where the active labor force is adjusted by the unemployment rate, U_t.

A condition equating the real wage with the labor share of income can also be postulated, which allows the computation of the actual labor share, $(1 - \alpha_t)$:

$$\frac{\breve{w}_t}{p_t^y} = (1 - \alpha_t)\frac{\breve{y}_t}{\breve{l}_t}.$$

F. Domestic Prices and Inflation

FSGM features a full array of relative prices, which are usually modeled with reduced form equations or identities. They are all related back to two reduced-form Phillips curves, one for core CPI inflation, the other for the wage.

CPIX Inflation Phillips Curve

Core consumer price index (CPIX) inflation is the measure used by the monetary authority when determining the policy interest rate, if they are pursuing an inflation targeting regime. For CPIX inflation, there is the reduced-form Phillips curve:

$$\begin{aligned}\pi_t^{cpix} = {} & c_1^{cpix} E_t \pi_{t+1}^{cpix} + \left(1 - c_1^{cpix}\right)\left(c_2^{cpix} \pi_{t-1}^{cpix} + \left(1 - c_2^{cpix}\right)\pi_{TAR}^{cpix}\right) + c_3^{cpix} log\left(\check{y}_t^{gap}\right) \\ & + c_4^{cpix} \Delta log(REER_t) + c_5^{cpix}\left(c_6^{cpix} \Delta log\left(p_t^{oil}\right) + (1 - c_6^{cpix} \Delta log\left(p_{t-1}^{oil}\right)\right) \\ & + c_9^{cpix} \Delta log\left(p_t^{food}\right) + \epsilon_t^{\pi^{cpix}},\end{aligned}$$

where the inflation rate, π_t^{cpix} is defined by its own expected value, $E_t \pi_{t+1}^{cpix}$, lagged value, π_{t-1}^{cpix}, or targeted value, π_{TAR}^{cpix}, as well as the output gap, $\check{y}_t^{gap}$, the change in the log of real effective exchange rate, $REER_t$, to capture the effects of imported consumption good prices, and the changes in the log of the real global oil price, p_t^{oil}, and in the log of the real global food price, p_t^{food}, both of which aim to capture potential second round effects of persistent movements in oil and food prices on core inflation.

Note that $E_t \pi_{t+1}^{cpix} = \pi_{t+1}^{cpix} + \epsilon_t^{E_t \pi^{cpix}}$, which implies that households can misperceive future inflation, and can expect something different than the *t+1* value of CPIX inflation.

The form of the Phillips curve is consistent with a hybrid New Keynesian Phillips curve, where there are both expectational dynamics from the expected future inflation and inertial dynamics from past inflation.

CPIX inflation is the basis of CPIX, p_t^{cpix}, which gives rise to consumer price index for non-commodity sectors, $p_t^{cpincom}$, through the following equation:

$$p_t^{cpincom} = p_t^{cpix} E_t \pi_{t+1}^{cpix}\left(1 + c_t^{mprop}\right)\left(tariff_t^{im} + quota_t^{im}\right),$$

Where c_t^{mprop} is the marginal propensity to import out of consumption, $tariff_t^{im}$ are tariffs levied on imported goods, , $quota_t^{im}$ are quotas levied on imported goods.

Wage Inflation Phillips Curve

Wage inflation is defined by a Phillips curve that allows for a sluggish response of nominal wages to demand and supply pressures:

$$\pi_t^w = c_1^w E_t \pi_{t+1}^w + (1 - c_1^w)\pi_{t-1}^w - c_2^w log\left(\frac{1-\alpha_t}{1-\alpha^{FE}}\right) + c_3^w log\left(\breve{y}_t^{gap}\right) + \epsilon_t^{\pi^W},$$

where wage inflation, π_t^w, is defined by its own expected value and lag as well as the gap between the current and steady-state labor share of income, $log\left(\frac{1-\alpha_t}{1-\alpha^{FE}}\right)$, and the output gap, $\breve{y}_t^{gap}$. This output gap is used rather than the labor market gap defined by Okun's law (see above) because of the annual nature of the model. With the labor market gap lagging the output gap by one year in the model, there is considerably less empirical support for its use than $\breve{y}_t^{gap}$.

Note that $E_t \pi_{t+1}^w = \pi_{t+1}^w + \epsilon_t^{E_t \pi^w}$, as with consumer price inflation, households can misperceive future wage inflation, expecting something different than the model-consistent *t+1* value.

The gap between the current and steady-state labor share of income is governed by parameter c_2^w, which determines the speed of adjustment of wages back to their steady-state value.

Relative Prices

Once the inflation rates are known, other relative prices can be defined. The headline consumer price index (CPI), p_t^{cpi}, is defined by the identity equating total consumption and the consumption of oil and food:

$$p_t^{cpi}\breve{c}_t = p_t^{cpincom}\left(\breve{c}_t - \breve{c}_t^{oil} - \breve{c}_t^{food}\right) + p_t^{coil}\breve{c}_t^{oil} + pP_t^{cfood}\breve{c}_t^{food}$$

Note that the price of consumption, p_t^c, is identical to p_t^{cpi}.

A similar definition for the non-commodities GDP price deflator also exists, $p_t^{y^{no}}$, based on the production of output, food, oil and metals:

$$p_t^y \breve{y}_t = p_t^{y^{no}}\left(\breve{y}_t - \breve{y}_t^{oil} - \breve{y}_t^{food} - \breve{y}_t^{metal}\right) + p_t^{oil}\breve{y}_t^{oil} + p_t^{food}\breve{y}_t^{food} + p_t^{metal}\breve{y}_t^{metal}$$

Note that the GDP price deflator, p_t^y, is defined from the nominal national expenditure accounts identity:

$$p_t^y \breve{y}_t = p_t^c \breve{c}_t + p_t^i \breve{\imath}_t + p_t^g \breve{g}_t + p_t^x \breve{x}_t - p_t^m \breve{m}_t$$

To complete the system of domestic prices, the model includes behavioral equations for the price of investment goods, as a function of non-commodities GDP price deflator and the price of aggregate imports, allowing for a constant, $c_t^{p^i}$ and a shock term, $\epsilon_t^{p^i}$

$$log\ (p_t^i) = c_1^{p^i} log\ (p_t^{y^{no}}) + (1 - c_1^{p^i}) log\ (p_t^{im}) + c_t^{p^i} + \epsilon_t^{p^i}$$

and the price of government goods, which moves in tandem with CPIX, allowing for a constant, $c_t^{p^g}$ and a shock term, $\epsilon_t^{p^g}$

$$log\ (p_t^g) = log\ (p_t^{cpix}) + c_t^{p^g} + \epsilon_t^{p^g}$$

G. Trade Volumes and Prices

Rather than track all the bilateral flows and prices as is done in GEM and GIMF, the trade sector in the model is greatly simplified by using multilateral measures for prices and volumes for each region. By sharply reducing the number of equations devoted to trade, the number of countries and regions in the typical FSGM module can be greatly expanded relative to what is practical in GEM and GIMF.

For each individual region, we only need to track export and import prices and volumes for the aggregates and the components, with the components being manufactured goods and services, and the three commodities – oil, food and metals. This section explains all the components required for the aggregates and goods and services, so that in the commodities section below we can just note that the three commodities have analogous equations to manufactured goods and services.

The equations for the demand for exports and imports of manufactured goods and services are based on reduced-form equations that impose some long-run restrictions. Export prices are defined in terms of domestic and foreign factors, while import prices are simply multilateral aggregations of other regions' export prices. All the multilateral measures mentioned in the remainder of this section are fully defined in the section on external sector below.

For exports, the long-run model includes a foreign activity variable (foreign import demand based on manufactured goods trade weights), $A\check{C}T_t^F$ and a relative price variable, the real competitiveness index (RCI), RCI_t, reconciled by a constant c_5^x. The speed of adjustment of the long run is governed by c_2^x. There are additional short-run dynamics from their log first differences.

$$\begin{aligned}\Delta log(\check{x}_t^m) = {} & c_4^x \Delta log\left(A\check{C}T_t^F\right) + c_1^x \Delta RCI_t \\ & + c_2^x \left(c_3^x RCI_{t-1} + log\left(\frac{A\check{C}T_{t-1}^F}{g_t}\right) + c_5^x - log\left(\frac{\check{x}_{t-1}}{g_t}\right)\right) + \epsilon_t^{x^m}\end{aligned}$$

For imports, the long-run model includes a domestic activity variable (the domestic components of real GDP weighted by their average import propensities), $A\check{C}T_t$, and a import prices, reconciled by a constant c_6^m. The speed of adjustment of the long run is governed by c_3^m. There are additional short-run dynamics from their log first differences. There is also an additional term for the output gap, $\breve{y}_t^{gap}$, to better capture cyclical movements in import propensity absent in the average import propensities used in the activity variable. The aggregate import propensity will tend to vary over the business cycle because some components of demand are much more variable than others and have different import shares. For examples, demand for durables tends to be much more volatile over the business cycle than other components of consumption and durables have a high import share.

$$\Delta log(\breve{m}_t^m) = c_5^m \Delta log\left(A\check{C}T_t\right) + c_1^m \Delta log(p_t^m) + c_2^m \Delta log\left(\breve{y}_t^{gap}\right) + c_3^m \left(c_4^m log(p_{t-1}^m) + log\left(\frac{A\check{C}T_{t-1}}{g_t}\right) + c_6^m - log\left(\frac{\breve{m}_{t-1}}{g_t}\right) \right) + \epsilon_t^{m^m}$$

Export prices for manufactured goods and services are determined by domestic factors, represented by the non-commodities GDP price deflator, $p_t^{y^{no}}$, and third-country competition effects,[9] p_t^{fm}:

$$\Delta log(p_t^{xm}) = c_1^{pxm} \Delta log\left(p_t^{y^{no}}\right) + \left(1 - c_1^{pxm}\right) \Delta log\left(p_t^{fm}\right) + c_2^{pxm} log\left(\frac{p_{t-1}^{y^{no}}}{p_{t-1}^{xm}}\right) + c_3^{pxm} + \epsilon_t^{p^{xm}}$$

where c_3^{pxm} is a constant, and $\epsilon_t^{p^{xm}}$ is a shock term.

Import prices for manufactured goods and services, $p_t^{im^{UNADJ}}$, at their core, are a weighted average of all other regions' export prices:

$$p_t^{im^{UNADJ}} = -\sum_j S^M(x)(j) \left(\frac{p_t^{xm}(j) Z_t(j)}{E_t^{BASE}(j)}\right) / \left(\frac{Z_t}{E_t^{BASE}}\right) exp\left(c_3^{pim} + \epsilon_t^{p^{im}}\right).$$

However, there is also sluggish adjustment in import prices. This is modeled in much the same way as in GIMF and GEM, with either Rotemberg or Ireland adjustment costs. Both are possible, here the Ireland form is used, which insures that the sticky adjustment is in the rate

[9] This is consistent with pricing to market behavior as in Betts and Devereux (2000).

of change, not in the level itself. The final price of imports, p_t^{im} (described below) is a function of $p_t^{im^{UNADJ}}$.

$$p_t^{im} = \frac{p_t^{im^{UNADJ}}}{\left(1 + p_1^{im} e^{\left(\frac{\pi_t^{pim}}{\pi_{t-1}^{pim}}\right)}\right)} - \theta \frac{g_{t+1}}{r_t^c} p_1^{im} \frac{p_{t+1}^{im}}{p_t^{im}} \frac{\breve{m}_{t+1}^m}{\breve{m}_t^m} e^{\left(\frac{\pi_t^{pim}}{\pi_{t-1}^{pim}}\right)},$$

where θ is the degree of myopia, $g_{t+1}$is the real growth rate, r_t^c is the real consumer interest rate and, p_1^{im} is the adjustment cost parameter. Import price inflation is defined as $\pi_t^{pim} = \frac{p_t^{im}}{p_{t-1}^{im}}$.

Because the model uses only multilateral trade equations, there is no guarantee that global trade in manufactured goods and services will sum to zero. To insure this, the global real net export surplus / deficit is calculated and then redistributed to each region based on their historical share of global trade (defined as exports plus imports), through each region's exports of manufactured goods and services, such that the adjusted level of exports, $\breve{x}_t^{ma}$, is defined as:

$$\breve{x}_t^{ma} = \breve{x}_t^m + T_t^{SHARE} \frac{w_t^{TRADE}}{E_t^{BASE}}.$$

where T_t^{SHARE} is a region's average share in global trade, w_t^{TRADE} is the U.S. dollar net real global trade surplus or deficit, and E_t^{BASE} is the real exchange rate that converts w_t^{TRADE} into domestic currency.

The calculation is then done again for total nominal trade, with this adjustment distributed in a similar fashion, but this time through the price of imports of manufactured goods and services, such that the adjusted price of imports, p_t^{ima}, is defined as:

$$\breve{p}_t^{ima} = \breve{p}_t^{im} + T_t^{SHARE} \frac{w_t^{TRADEN}}{Z_t \breve{m}_t^m}.$$

where w_t^{TRADEN} is the U.S. dollar net nominal global trade surplus or deficit.

Similar adjustments are done separately in each of the three commodities sectors – oil, food and metals.

Finally, aggregate exports and imports sum up manufactured goods and services, and commodities (more fully defined in the following section on commodities volumes and prices). The aggregate export volume is derived from the real export identity, with the aggregate export price coming from the nominal export identity:

$$\breve{x}_t = \breve{x}_t^{ma} + \breve{x}_t^{oil} + \breve{x}_t^{food} + \breve{x}_t^{metal}$$

$$p_t^x \breve{x}_t = p_t^{xm} \breve{x}_t^m + p_t^{oil} \breve{x}_t^{oil} + p_t^{food} \breve{x}_t^{food} + p_t^{metal} \breve{x}_t^{metal}$$

The parallel formulations exist also for aggregate import volumes and prices.

H. Commodities Volumes and Prices

The model has three commodities, namely oil, food, and metals. This allows the model to better track some economies which either produce (and possibly export) large quantities of natural resources, or which heavily use them. In particular, if the use of natural resources is driving up global commodity prices because of rapidly rising global demand, the increase in aggregate demand will be tempered by higher commodity prices. In addition, disruptions in the supply of these commodities can have major effects on their respective prices as well as in the production of goods and services. Commodities are introduced in a reduced form in several levels of the model.

Consider first the case of oil. In any region there is a domestically produced oil supply, $\breve{y}_t^{oil}$, which can be used domestically or exported. For domestic oil demand, there is consumption demand by households, $\breve{c}_t^{oil}$, and input factor demand by firms (which is not explicitly tracked), $\breve{u}_t^{oil}$.In addition to domestic production, oil demand can also be satisfied with imports $\breve{m}_t^{oil}$. Therefore we impose relationship:

$$\breve{y}_t^{oil} = \breve{c}_t^{oil} + \breve{u}_t^{oil} + \breve{x}_t^{oil} - \breve{m}_t^{oil}$$

It is assumed that the supply of commodities is relatively fixed in the short run but reacts to price signals in the medium-term if oil prices deviate persistently from their expected long-run value. The reduced form of the oil production is given by:

$$log\left(\breve{y}_t^{oil}\right) = log\left(\breve{y}_t^{oil\_exog}\right) + c_1^{prodoil} log\left(\frac{p_t^{oil\_avg}}{\bar{p}_t^{oil}}\right) + \epsilon_t^{YOIL},$$

where $\breve{y}_t^{oil\_exog}$ is the planned production of oil based on the expected equilibrium real price of oil, $p_t^{oil\_avg}$ is a three-year moving average of the real price of oil, and $\bar{p}_t^{oil}$ is the expected equilibrium real price of oil.

The consumption of oil is assumed to be a function of income and prices. The model of oil consumption is given by:

$$\Delta log\left(\check{c}_t^{oil}\right) = c_4^{coil}\Delta log(\check{y}_t) + c_1^{coil}\Delta log\left(\frac{p_t^{oil}}{p_t^{cpi}}\right) + c_2^{coil}\left(c_3^{coil} log\left(\frac{p_{t-1}^{oil}}{p_{t-1}^{cpi}}\right) + log\left(\frac{\check{c}_{t-1}^{oil}}{\check{y}_{t-1}}\right)\right) + c_5^{coil}$$

The reduced form of oil exports is a function of income and prices, with the following model:

$$\Delta log\left(\check{x}_t^{oil}\right) = c_4^{xoil}\Delta log\left(A\check{C}T_t^{OIL,F}\right) + c_1^{xoil}\Delta \log\left(p_t^{oil}\right) + c_2^{xoil}\left(c_3^{xoil} log\left(p_{t-1}^{oil}\right) + log\left(\frac{A\check{C}T_{t-1}^{oil,F}}{g_t n}\right) + c_5^{xoil} - log\left(\frac{\check{x}_{t-1}^{oil}}{g_t n}\right)\right),$$

where $A\check{C}T_t^{oil,F}$ is the foreign activity variable based on oil trade weights.

Oil imports are also a function of prices and income. The model for oil imports is given by:

$$\Delta log\left(\check{m}_t^{oil}\right) = c_5^{moil}\Delta log\left(A\check{C}T_t\right) + c_1^{moil}\Delta \log\left(p_t^{oil}\right) + c_2^{moil}\Delta \log\left(\check{y}_t^{gap}\right) + c_3^{moil}\left(c_4^{moil} log\left(p_{t-1}^{oil}\right) + log\left(\frac{A\check{C}T_{t-1}}{g_t n}\right) + c_6^{moil} - log\left(\frac{\check{m}_{t-1}^{oil}}{g_t n}\right)\right)$$

Since the full demand for oil is not tracked, the market clearing price is supplanted by a an explicit relationship for oil prices. Oil prices are decomposed into a short-run cyclical variation, $p_t^{oil^{cyc}}$, and a long-run trend, $p_t^{oil^{perm}}$:

$$\log\left(p_t^{oil}\right) = \log\left(p_t^{oil^{cyc}}\right) + \log\left(p_t^{oil^{perm}}\right)$$

The long-term component of oil reacts to shocks that can permanently shift both supply-side and demand-side factors:

$$\log\left(p_t^{oil^{PERM}}\right) = \log\left(p_t^{oil^{exog}}\right) + c_1^{poil} log\left(\frac{\check{y}_t^{FE,WRL}}{\overline{\overline{\check{y}_t^{FE,WRL}}}}\right) + c_2^{poil} log\left(\frac{\check{y}_t^{oil,WRL}}{\overline{\overline{\check{y}_t^{oil,WRL}}}}\right) + \epsilon_t^{POIL^{perm}} ,$$

where $log\left(\frac{\check{y}_t^{FE,WRL}}{\overline{\overline{\check{y}_t^{FE,WRL}}}}\right)$ is the log difference between the world equilibrium output after the shock and world equilibrium output before the shock and $log\left(\frac{\check{y}_t^{oil,WRL}}{\overline{\overline{\check{y}_t^{oil,WRL}}}}\right)$ is the log difference between the equilibrium world production of oil after the shock and its value before the shock.

The short-run cyclical component of oil prices, $p_t^{oil^{cyc}}$, is defined as:

$$\log\left(p_t^{oil^{cyc}}\right) = c_3^{poil} log\left(\breve{y}_t^{gap,WRL}\right) + \epsilon_t^{POIL^{cyc}} ,$$

where $\breve{y}_t^{gap,WRL}$ is the global output gap.

Both food and metals have the comparable set of equations as oil, with one exception. There is no equation for the consumption of metals by households, since metals do not enter their consumption basket in any meaningful way in the data.

The commodities sectors affect real economic activity primarily through several direct channels. First, food and oil prices feed into the CPI, which deflates real disposable income and wealth. Second, metal and oil prices affect the cost of producing goods, which is implemented via total factor productivity since oil and metals do not appear directly in the production function. Consequently, commodities prices have an effect on the demands for capital and labor. Finally, to allow for potential second round effects of oil price shocks on inflation, oil prices appear in the CPIX Phillips curve. This implies that monetary policy will need to guard against the risk of persistent oil price shocks leading to persistent deviations of inflation from target.

I. Government

Fiscal Policy

The fiscal policy reaction function is designed to ensure a non-explosive government debt-to-GDP ratio in the medium term by adjusting either tax rates or expenditure so that the government deficit to GDP ratio, $gdef_t^{rat}$, converges to long run target, gss_t^{rat}. The government deficit is given by

$$\widetilde{gdef}_t = -\left(\breve{b}_t - \frac{\breve{b}_{t-1}}{\pi_t gn}\right) = \breve{\tau} - \breve{g}_t^E ,$$

where $\breve{\tau}$ refers to tax revenues, $\breve{g}_t^E$ refers to government expenditure, further defined below, $\breve{b}_t$ refers to government debt and $\widetilde{gdef}_t$ is the government deficit. The ratio of the government deficit to GDP is given by

$$gdef_t^{rat} = \frac{\widetilde{gdef}_t}{p_t^y \breve{y}_t}$$

The current level and the long-run target for the government-debt-to-GDP ratio are $\breve{b}_t^{rat}$ and $\bar{b}_t^{rat}$, respectively. The long-run relationship between the government deficit and government debt-to-GDP ratio in annual model is given by

$$\overline{gdef}_t^{rat} = -\frac{\bar{\pi}_t^{cpix} gn - 1}{\bar{\pi}_t^{cpix} gn} \bar{b}_t^{rat}$$

The long-run inflation rate, $\bar{\pi}_t^{cpix}$, is the inflation target of the central bank. For a given nominal growth rate, $\bar{\pi}_t^{cpix} gn$, choosing a surplus target, $\overline{gdef}_t^{rat}$, implies a finite debt target, $\bar{b}_t^{rat}$, and therefore keeps debt from exploding.

In addition to ensuring long-run stability in the debt-to-GDP ratio, the fiscal policy reaction function also incorporates short-run automatic stabilization. This is achieved by the following deficit rule:

$$gdef_t^{rat} = \overline{gdef}_t^{rat} + d^{debt}\left(\check{b}_t^{rat} - \bar{b}_t^{rat}\right) - d^{gap} log\left(\check{y}_t^{gap}\right)$$

This relationship implies that even if $d^{debt} = d^{gap} = 0$, the rule automatically ensures a non-explosive government-debt-to-GDP ratio of $\bar{b}_t^{rat}$. The convergence speed will be relatively modest depending on the autoregressive coefficient on debt which is the inverse of the nominal growth rate $1/\left(\bar{\pi}_t^{cpix} gn\right)$. This adjustment speed can be enhanced by setting $d^{debt} > 0$ at the expense of more volatile government surpluses. The last term in the right hand side represent the automatic stabilizing response of fiscal policy to the state of the business cycle. Its coefficient d^{gap} has been calibrated using OECD estimates of fiscal rules (Girouard and André, 2005).

Other than the fiscal rule, fiscal policy can be conducted using eight instruments: the consumption tax, corporate income tax, labor income tax, government consumption spending, government investment in infrastructure, government subsidies, general lumpsum transfers, and lumpsum transfers targeted to either LIQ or OLG households. These instruments define the government revenue and spending equations.

Government tax revenues are given by:

$$\check{\tau} = \tau_t^L taxbase_t^L + \tau_t^K taxbase_t^K + \tau_t^C taxbase_t^C + tax_t^{ls} ,$$

where τ_t^L is the labor income tax rate, τ_t^K, is the corporate income tax rate, τ_t^C is the consumption tax rate and tax_t^{ls} is the lumpsum taxes collected by the government.

Government expenditure is given by

$$\check{g}_t^E = p_t^G(\check{g}_t^C + \check{g}_t^I) + \check{g}_t^{SUB} + \check{\Upsilon}_t + \check{\Upsilon}_t^{TARG} + \widetilde{INT}_t^{COST} ,$$

where $\check{g}_t^C$ is government consumption of goods and services, $\check{g}_t^I$ is government investment in infrastructure, $\check{g}_t^{SUB}$ government subsidies for fuel and to the export sector, $\check{\Upsilon}_t$ is general lumpsum transfers to all households, $\check{\Upsilon}_t^{TARG}$ *is* targeted lumpsum transfers to either LIQ or OLG households, and $\widetilde{INT}_t^{COST}$ is the interest costs of the outstanding government debt.

Government investment in infrastructure cumulates to a public infrastructure capital stock, $\check{k}_t^G$, which acts as a positive productivity shock in potential output, as noted above in section II.D.

Monetary Policy

Monetary policy is characterized by an interest rate rule that features interest rate smoothing and potential responses to deviation of a weighted average of the contemporaneous and one-year-ahead CPIX inflation from the inflation target, $\left((1-\delta_w)log\left(\pi_{t+1}^{cpix}\right)+\delta_w log\left(\pi_t^{cpix}\right)-log\left(\bar{\pi}_t^{cpix}\right)\right)$, the output gap, $\check{y}_t^{gap}$, and deviations of current exchange rate depreciation from its long run value, $\left(\frac{\varepsilon_t}{\bar{\varepsilon}_t}\right)$.

$$
\begin{aligned}
log(INT_t^{MP}) = \delta_i \log\,(INT_{t-1}^{MP}) \\
&+ (1-\delta_i)\Big[log\,(r_t^{neut}) + log\left(E_t\pi_{t+1}^{cpix}\right) \\
&+ \delta_\pi\left((1-\delta_w)log\left(\pi_{t+1}^{cpix}\right)+\delta_w log\left(\pi_t^{cpix}\right)-log\left(\bar{\pi}_t^{cpix}\right)\right)+\delta_Y log\left(\check{y}_t^{gap}\right) \\
&+ \delta_\varepsilon log\left(\frac{\varepsilon_t}{\bar{\varepsilon}_t}\right)+\epsilon_t^{INT^{MP}}\Big]
\end{aligned}
$$

For most countries, the rule is calibrated to be an inflation-forecast-based rule with $\delta_\varepsilon = 0$. However, for countries that have either fixed or managed exchange rate regimes, then the monetary policy rate needs to respond to achieve the desired exchange rate behavior, hence $\delta_\varepsilon > 0$.

J. Interest Rates

All interest rates are related to the risk free interest rate, INT_t^{RF}. Its closest parallel is the monetary policy rate, used as an instrument by the monetary authority to achieve its policy goals. All other interest rates in the economy can deviate from the risk-free rate because of a risk premium or because of different maturity.

The risk-free rate is the nominal 1-year interest rate, deflated by a number of risk premia:

$$INT_t^{RF} = \frac{INT_t}{\xi_t^b\,\xi_t^{MKT}\,\xi_t^f} + \epsilon_t^{INT^{RF}},$$

where ξ_t^b is the sovereign risk premium (which applies to the entire domestic economy), ξ_t^{MKT} is the domestic private economy risk premium, and ξ_t^f is the foreign exchange (or country) risk premium, respectively.

The Fisher equation defines the ex-ante one year real interest rate, $INT_t = r_t E_t\pi_{t+1}^{cpix}$.

Note that when there are permanent changes in risk premia, there will be a short-run wedge between the economy-wide 1-year interest rate and the neutral monetary policy rate, as the neutral rate only adjusts gradually to incorporate the permanent change in risk premium. Therefore, there is an equation in the model to facilitate this adjustment of the monetary policy rate to the 1-year interest rate:

$$INT_t = INT_t^{MP}\left(\frac{\xi_t^b\,\xi_t^{MKT}}{(1-c^{RLAG})\xi_t^{b,SM}\,\xi_t^{MKT,SM} + c^{RLAG}\xi_{t-1}^{b\_SM}\,\xi_{t-1}^{MKT,SM}}\right)^{XX^{RR}},$$

where $\xi_t^{b,SM}$ is a moving average of the sovereign risk premium and $\xi_t^{MKT,SM}$ is a moving average of the domestic private economy risk premia.

There is also the 10-year nominal interest rate, defined by the expectation theory of the term structure and an additional term premium:

$$INT_t^{10} = \prod_{i=0}^{9}(INT_{t+i})^{0.1}\,\xi_t^T + \epsilon_t^{R^{10}},$$

where ξ_t^T is the term premium. A proxy for the Fisher equation for the ten year real interest rate is:

$$INT_t^{10} = r_t^{10}\left(\prod_{i=0}^{9}\left(E_t\pi_{t+i}^{cpix}\right)^{0.1}\right).$$

There are also a risk-free 10-year nominal interest rate, INT_t^{RF10} based on the 1-year risk-free interest rate, in much the same fashion as INT_t^{10}.

Interest rates related to consumption, investment, and holdings of government debt and net foreign assets are weighted averages of 1-year and 10-year nominal interest rates, which already include the sovereign and private domestic economy risk premia.

The nominal interest rate for consumption, INT_t^C, is represented by:

$$INT_t^C = (INT_t)^{1-c^{INTC}}INT_t^{10^{c^{INTC}}} + \epsilon_t^{INT^C},$$

where c^{INTC} is determined by the average maturity structure of household debt.

The nominal corporate interest rate, INT_t^{CORP}, is represented by:

$$INT_t^{CORP} = (INT_t\xi_t^R)^{1-c^{INTCORP}}(INT_t^{10}\prod_{j=0}^{9}\left(\xi_{t+j}^R\right)^{0.1})^{c^{INTCORP}} + \epsilon_t^{INT^{CORP}},$$

where $c^{INTCORP}$ is determined by the maturity structure of corporate debt holdings and ξ_t^R is the "external financing" premium for firms. It has both an exogenously specified component, and an endogenous BGG-style[10] component which is driven by the output gap:

$$\xi_t^R = \epsilon_t^{\xi^R} \left(\breve{y}_t^{gap}\right)^{BGG1}$$

where -1 < *BGG1* < 0.

The real corporate and household interest rates are given by similar equations, only substituting real interest rates for nominal.

Interest rates related to government debt and net foreign assets are weighted averages of 1-year and 10-year nominal interest rates, with a sovereign risk premium for government debt. There is a three-year moving average for the 10-year interest rate component, to better capture the maturity structure of the 10-year-denominated debt

The nominal interest rate for domestically-issued government debt, INT_t^{GB}, is:

$$INT_t^{GB} = \left(\frac{INT_t}{\xi_t^{MKT}}\right)^{1-c^{INTGB}} \left(\prod_{i=-2}^{0} \left(\frac{INT_{t+i}^{10}}{\prod_{j=i}^{9+i} \left(\xi_{t+j}^{MKT}\right)^{0.1}}\right)^{1/3}\right)^{c^{INTGB}} + \epsilon_t^{INT^{GB}},$$

where c^{INTGB} is determined by the term structure of the holdings of government debt. Note that the government interest rate is unaffected by the domestic private economy risk premium.

The nominal interest rate for net foreign assets, INT_t^{NFA}, is similar, but is based in every region solely on U.S. interest rates, since the model is built with the assumption of incomplete markets, where all foreign assets are denominated exclusively in U.S. dollars:

$$INT_t^{NFA} = \left(INT_t^{RF,US}\right)^{1-c^{INTNFA}} \left(\prod_{i=-2}^{0} \left(INT_{t+i}^{RF10,US}\right)^{1/3}\right)^{c^{INTNFA}} + \epsilon_t^{INT^{NFA}},$$

where c^{INTNFA} is determined by the term structure of the holdings of net foreign assets.

[10] This is aimed to provide a shortcut and quick approximation to the effects of the financial frictions described in fuller detail in Bernanke and others (1999) and Christiano and others (2008).

K. External Sector

The trade balance, TB_t, is defined as:

$$TB_t = p_t^x X_t - p_t^m M_t$$

The current account, CA_t, is also defined in nominal terms as a sum of the trade balance, TB_t, and the interest income from net foreign assets, NFA_t:

$$CA_t = TB_t + (INT_{t-1}^{NFA} - 1)\frac{NFA_{t-1}}{\varepsilon_t Z_{t-1}\pi_t g_t n},$$

The net foreign asset position, , is the accumulation of the current account balances:

$$\frac{NFA_t}{Z_t} = CA_t + \frac{NFA_{t-1}}{\varepsilon_t Z_{t-1}\pi_t g_t n}$$

The uncovered interest rate parity condition is defined in the conventional manner, where the risk-free domestic return on assets must be equal to that of the foreign bond (in this case with incomplete markets, a U.S. dollar-denominated bond), accounting for exchange rate risk:

$$INT_t^{RF} = \frac{INT_t^{US}}{\varepsilon_{t+1}}$$

The change in the real U.S. dollar bilateral exchange rate is thus defined as

$$\frac{Z_t}{Z_{t-1}} = \varepsilon_t \frac{\pi_t}{\pi_t^{US}}$$

However, the key exchange rate measures in the domestic economy are not the U.S. dollar bilateral exchange rate, but multilateral measures that account for relationships with all trading partners. Price effects on imports can be summarized by the real effective exchange rate (REER), defined as:

$$log\ (REER_t) = -\sum_j S^M(x)(j) log\left(\frac{Z_t(j)}{E_t^{BASE}(j)}\right) + log\left(\frac{Z_t}{E_t^{BASE}}\right)$$

where $S^M(x)(j) = \frac{S(x)(j)TFP_t^{effect}(j)}{\sum_l S(x)(l)TFP_t^{effect}(l)}$ is a modified imports weight. $S(x)(j)$ is the fixed weight of imports of the home country *x* from country *j*, currently based on data from 2012 The modification increases the weight of a particular foreign country in the domestic market's imports, if that foreign country faces an increase in its trend total factor productivity. The modification is calibrated to mimic increases in the foreign countries' trend total factor productivity in either the entire economy, the tradable sector or the nontradable sector (noting that the latter two are not explicitly defined in the model).

Exports, on the other hand, depend on another multilateral measure, the real competitiveness index (RCI). It is defined as the ratio of manufacturer's goods export prices to foreign prices:

$$RCI_t = log\left(\frac{p_t^{xm}}{p_t^{fm}}\right)$$

Where, further, foreign prices are defined as:

$$log\ (p_t^{fm}) = \sum_j WM_t(x)(j) log\left(\frac{p_t^{pxm}(j)Z_t(j)}{E_t^{BASE}(j)}\right) - log\left(\frac{Z_t}{E_t^{BASE}}\right),$$

where $WM_t(x)(j) = \sum_l \frac{S^M(l)(j)L(x)(j)}{1-S^M(l)(j)}$ is related to $L(x)(j)$, the fixed export weight from the home country *x* to country *j*, based on data from 2012. It also uses the modified import weights, $S^M(l)(j)$, to accounts for competition effects from other countries *l* in the export market *j*.

L. Productivity Spillovers

The model also incorporates an important productivity spillover channel whereby improvements in productivity in one country are transmitted to the productivity in its trading partners. The spillover structure attempts to capture the direct effects of technology spillovers from importing technology embodied capital, higher intensity of capital imports as a percent of total imports from advanced economies, as well as the indirect effects of the dissemination of technological advances. The functional form of the productivity shock spillovers is as follows:

$$TFP_t^{SPILL^N}(x) = \alpha_{spillm1}^N \sum_j \sqrt[\varphi_M]{\rho_M(j)}\ \frac{S^M(x)(j)M_t(x)}{Y_t(x)}\ \epsilon_t^{TFP^N}(j)$$
$$+ \alpha_{spillx1}^N \sum_j \sqrt[\varphi_x]{\rho_X(x)}\ \frac{L(x)(j)X_t(x)}{Y_t(x)}\ \epsilon_t^{TFP^N}(j)\ ,$$

where $TFP_t^{SPILL^N}$ is the productivity spillovers on sector *N* (tradable or nontradable) on home country *x* at time *t*; $\epsilon_{t,j}^{TFP^N}$ is the productivity shock in sector *N* in foreign country *j* at time *t*; $s(x)(j)M_t(x)$ is the bilateral imports of manufactured goods of country *x* from country *j* at time *t*; $L(x)(j)X_t(x)$ is the bilateral exports of manufactured goods of country *x* to country *j* at time *t*; and $Y_t(x)$ is the real GDP of country *x* at time *t*. The strength of the spillovers is governed by a pooled parameter, $\alpha_{spillx1}^N$ for exports and $\alpha_{spillm1}^N$ for imports. In addition, absorption capacity parameters determine country specific spillovers in imports from the country of origin j based on the TFP ratio, $\rho_M(j)$ and spillovers in exports from the home country x based on the ability of export markets to adapt new technology, $\rho_X(x)$. The dispersion of the absorption capacity ratios are controlled by φ_M and φ_X , for imports and exports respectively.

The strength of the import spillovers, $\alpha^{\mathrm{N}}_{spillm1}$, is based on empirical estimates in Franco and others (2011) for the 1995–2005 period using a date set of 20 OECD advanced economies. The estimates use bilateral imports as a percent of country x's real GDP as per Lumenga-Neso and others (2005) which reduces aggregation bias (see Lichtenberg and others, 1998). The spillovers from an increase in the productivity in the tradable sector are given a higher weight than nontradables sector, reflecting a higher foreign capital import intensity. Hence the magnitudes of the spillovers are influenced by the relative size of the tradable and nontradable sectors.

The import absorption capacity parameter $\rho_M(j)$ is the ratio of TFP of the country of origin j to that of the United States. For example, since the United States is assumed to be at the technological frontier, $\rho_M(USA)$ is equal to 1, whereas $\rho_M(CHN)$ is equal to 0.2. Hence, there are larger technological spillovers from trading partners which are closer to that frontier. The value of $\rho_{M,j}$ is the calculated ratio of TFP of the country of origin to that of the United States. When $\alpha^{\mathrm{N}}_{spillm1}$ was estimated $\sqrt[\varphi_M]{\rho_M(j)}$ was not present since the sample includes only advanced economies, meaning $\rho_{M,j}$ was close to 1, so φ_M would have no discernible impact. $\sqrt[\varphi_M]{\rho_M(j)}$ generalizes spillovers across a wider set of economies. φ_M has a notable impact for emerging market spillovers on other regions because $\rho_M(j)$ is often substantially less than unity in emerging markets.

The parameters governing the strength of the export spillovers, $\alpha^{\mathrm{N}}_{spillx1}$, are set identical to those in the import spillover channel. The export absorption capacity parameter $\rho_X(x)$ is the ratio of total high technology exports as a percent of total exports of the home country to that of China on average over the last ten years. The import channel is found to provide sufficient spillovers for advanced economies. Thus, $\rho_X(x)$ is set close to zero for advanced economies. Hence, this channel is only potentially large for emerging markets. This captures the indirect effects of the dissemination of technological advances in emerging markets based on the ability of the home country export market to adapt new technology. A decrease in φ_X increases the dispersion of the absorption export capacity ratios across countries.

Full-equilibrium total factor productivity, TFP_t^{FE}, is the sum of trend total factor productivity, TFP_t^T, own productivity shocks, $\epsilon_{\mathrm{t}}^{TFP^N}$, and spillover effects, $TFP_t^{SPILL^N}$, given by:

$$TFP_t^{FE}{}_t = TFP_t^T + \sum\nolimits_N \epsilon_{\mathrm{t}}^{TFP^N} + \sum\nolimits_N TFP_t^{SPILL^N} .$$

III. Calibration

We use a two-step approach to determine the parameter values in most of the equations of the modules of FSGM. First, we set initial values for the parameters based on single equation estimation for all the regions and countries covered by G20MOD, usually in panels. In addition, especially for coefficients that have a more structural interpretation, we fix the calibration in accordance with the microeconomic and empirical literature. For some coefficients we introduce country- or region- specific values based on their unique characteristics. This first step of the calibration approach is incomplete and insufficient because it does not take into account the properties and the information of the model as a system. Therefore, in the second step of the approach, we use the full model to make adjustments to the initial parameter values to obtain sensible system-wide properties. We compare the system properties of the model with those of other structural and semi-structural models such as GIMF, GEM, FRB/US, GMUSE and SIGMA.[11] The next two sections explain the approach in more detail.

A. First Step: Choosing Initial Values

Most of the equations of the model are estimated separately for all the regions of G20MOD in panel. To assist with global consistency in trade (that is, global exports and imports sum to zero), the panel estimation of the trade volumes and prices equations assumes that the coefficients of the ECM are the same across regions or countries. The initial values of households' inter-temporal elasticity of substitution are fixed by estimating reduced-form equations in panel that allow for country specific values. The estimation of the inflation Phillips curve also permits country or region differentiation. Finally, for sake of simplicity, we suppose that the coefficients for the monetary policy reaction function are the same across all the regions that pursue a flexible exchange rate regime.

We calibrate some key structural parameters consistently with the calibration of other models like GIMF and GEM and by taking into account country specific characteristics. First, like most global models, we assume that the share of liquidity-constrained households is smaller for advanced economies (35 percent) than for emerging market economies (60 percent). Second, we suppose that European countries have larger investment real adjustment costs. Third, consistent with the properties of GEM, we assume that the coefficients driving the

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[11] For GIMF and GEM, please see the references from the introduction. FRB/US is the Board of Governors of the U.S. Federal Reserve System's large-scale polynomial-adjustment-cost (PAC) model of the U.S. economy (Brayton and Tinsley, 1996). GMUSE is the global large-scale estimated PAC model at the Bank of Canada (Blagrave and others, 2015). SIGMA is a global DSGE model developed at the Board of Governors of the U.S. Federal Reserve System (Erceg and others, 2006).

effect of the price of oil (and metals) on consumption, investment and TFP are negative for oil (and metals) net importers and positive for oil (and metals) net exporters. This is mostly explained by the wealth effect from the shift in the terms of trade. For each of the regions, the initial values of the size of these coefficients are primarily based on the size of their ratios of net exports of oil (and metals) to GDP.

The initial values of price and income elasticities of demand and supply for oil, metals, and food are based on microeconomic and empirical literature. More precisely we make the following assumptions:

- A very low short-term price elasticity of oil demand and a slightly higher price elasticity in the long run.
- A short-term income elasticity of oil demand that is lower for advanced economies than for emerging market economies.
- Inelastic short-term supply curves for oil and metals and very low medium- and long-run price elasticities of oil and metal supply.
- A more elastic short-term demand and supply curve for food.

B. Second Step: Calibration Adjustments Based on the Properties of the Full Model

Using the initial calibration a variety of shocks are simulated in the United States and other regions and the properties of the model's responses are analyzed, comparing them with one of the other aforementioned models. Based on this exercise, common or country-specific adjustments are applied to the calibration of some of the coefficients to achieve sensible properties. In particular:

- Based on the properties from changes in the monetary policy rate and risk premiums, adjustments are made to the calibration of the adjustment cost for Tobin's Q and the intertemporal elasticity of substitution of consumption, proportionately for all regions.
- Based on the response of real GDP to an aggregate demand shock observed in GIMF (which has a fully articulated BGG financial accelerator), the coefficients that govern the reduced-form BGG financial accelerator mechanism for investment is adjusted.
- Based on the properties of aggregate demand shocks in the United States and other regions, country specific adjustments are introduced to the elasticities in the import equations related to real domestic demand growth and the change of the output gap, in order to achieve plausible properties for spillovers and domestic demand shocks

simultaneously. These adjustments are partly a function of the trade openness of each region, according to the data.

- Based on the effect of exchange rate shocks between regions adjustments are made to the coefficient governing the response of CPIX to the real effective exchange rate and to the adjustment costs on the price of imports, assuming that the exchange rate pass-through is positively related with trade openness.

- Common and country-specific adjustments are made to the coefficients driving the effect of the price of oil on real activity, in order to reproduce the effects on global and regional real GDP observed in GEM of an increase in the price of oil driven by a change in oil supply.

Most of the calibration work for FSGM is complete once the G20MOD module is finished. Then, adjustments to the calibration for EUROMOD and EMERGMOD are implemented.

C. Calibration Techniques Going Forward

There is an on-going effort to further formalize the parameterization of the models equations by system estimation of the key behavioral parameters. The approach chosen is a Bayesian Simulated Method of Moments with System Priors (BSMM/SP). The work initially applying the technique is being done on SIXMOD, the six-region module of FSGM. Currently only preliminary results are available for coefficients governing dynamics coefficients of the U.S. block –dynamics of the core inflation Phillips curve, wage inflation dynamics, adjustment cost of investment, and intertemporal elasticity of substitution. The initial conditions for estimation are the calibrated coefficients, which get updated using the moments-matching criterion function and 'system priors'– a priori restrictions on selected properties of the model.[12] The plan is to eventually apply this technique to all FSGM modules.

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[12] A useful reference for background on this methodology is Andrle and Benes (2013).

IV. MODEL PROPERTIES

This section presents some simulation experiments that help illustrate the dynamic adjustment properties of FSGM. First, some standard properties are considered– a temporary increase in the monetary policy rate in the United Kingdom, temporary increases in aggregate demand in Canada, increases in trend factor productivity in the United States, and a permanent reduction in private saving in China. Second, the most often-used fiscal consolidation measures are examined using Japan for the examples. Third, the impact of temporary increases in the various premia are illustrated (sovereign risk premium, domestic private risk premium, corporate risk premium, and the term premium) using the United States as an example. Finally, a permanent increase in the real global oil price is presented.

A. Temporary Increase in the Monetary Policy Rate in the United Kingdom

Figure 1 shows the effects of a one-year 100 basis point increase in the U.K. monetary policy rate that comes as a surprise to firms and households. On impact, U.K. real GDP decreases by almost ½ percent, due to fall in domestic absorption and export demand. Inflation decreases by over ¼ percentage point at its trough in the first year.

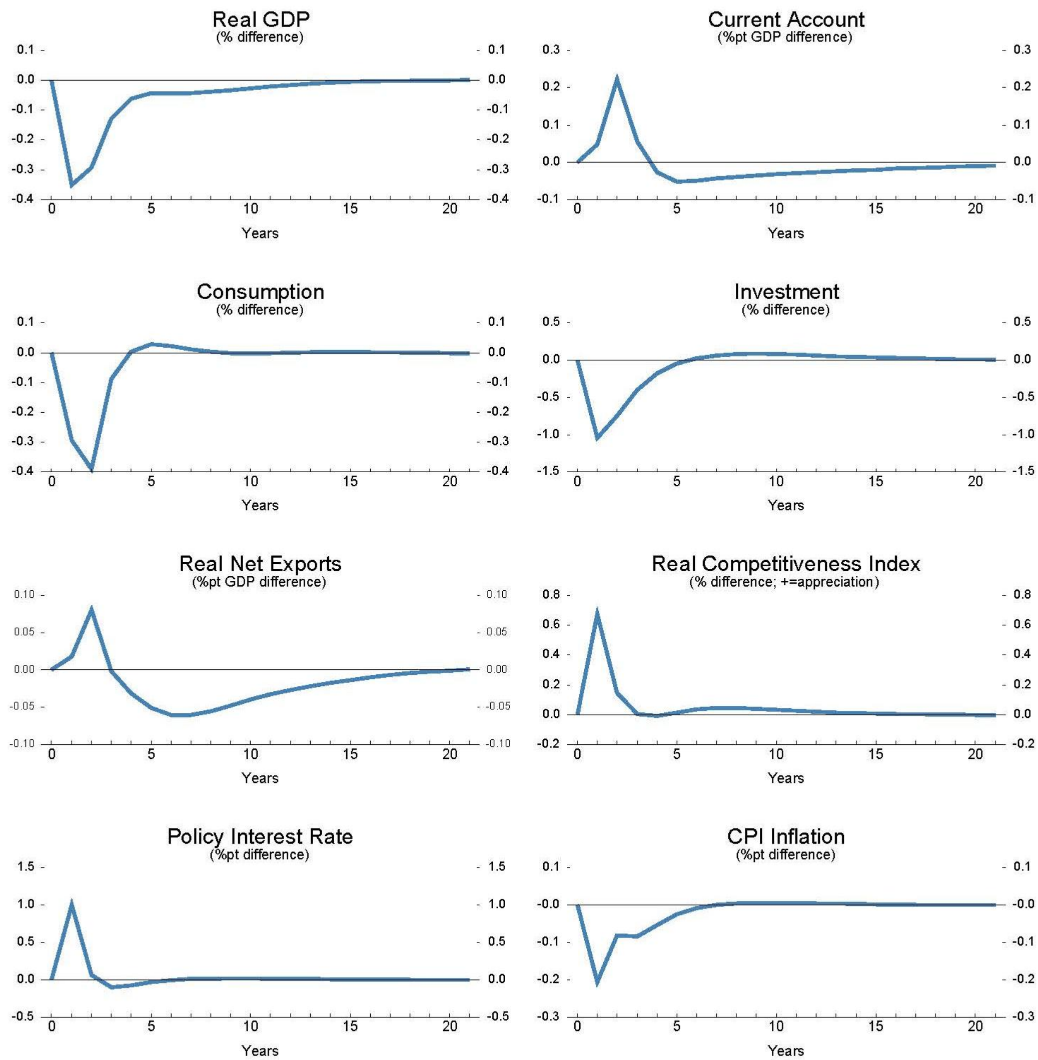
In the U.K. economy, higher real interest rates reduce both private investment and household consumption expenditure. Business investment falls due to the higher cost of capital. Therefore the profitability of firms decreases. This reduces the household wealth. The contraction in production lowers the demand for factor inputs leading to fall in labor income. The fall in wealth, the decline in labor income, and the increase in the cost of current consumption relative to future consumption arising from the increase in interest rates all lead households to reduce consumption expenditure.

With real U.K. interest rates higher than foreign interest rates the pound appreciates. The resulting appreciation of the RCI helps to dampen foreign demand for U.K. exports. The appreciated currency reduces import prices for U.K. households. Although U.K. households substitute toward foreign goods and away from domestic goods, the weaker level of domestic demand still results in a decline in imports that is larger than the fall in exports. The net impact is a small increase in the current account surplus in the short run.

With demand falling below the economy's supply potential, a negative output gap opens up, putting downward pressure on CPI inflation that is exacerbated by declining import prices owing to currency appreciation.

Fiscal policy strives to stabilize activity through increasing transfers to households. This, and the higher debt servicing costs because of higher interest rates, increase fiscal deficit in the short run. Eventually, the fiscal authority reduces transfers to households to return the public debt-to-GDP ratio back to its long-run target level. Following the exogenously induced tightening in the monetary policy rate, monetary policy must ease to return inflation to target. The monetary authority reduces the monetary policy rate to achieve a period with real interest rates below their long-run level, temporarily stimulating aggregate demand and re-anchoring inflation at the target.

Figure 1: Temporary Increase in the Monetary Policy Rate in the United Kingdom



B. Temporary Increase in Aggregate Demand in Canada

Figure 2 presents the impact of a one-year shock to private domestic demand in Canada that is calibrated to raise real GDP by one percent relative to baseline, such that an increase in private investment is roughly four times larger than the increase in household consumption.

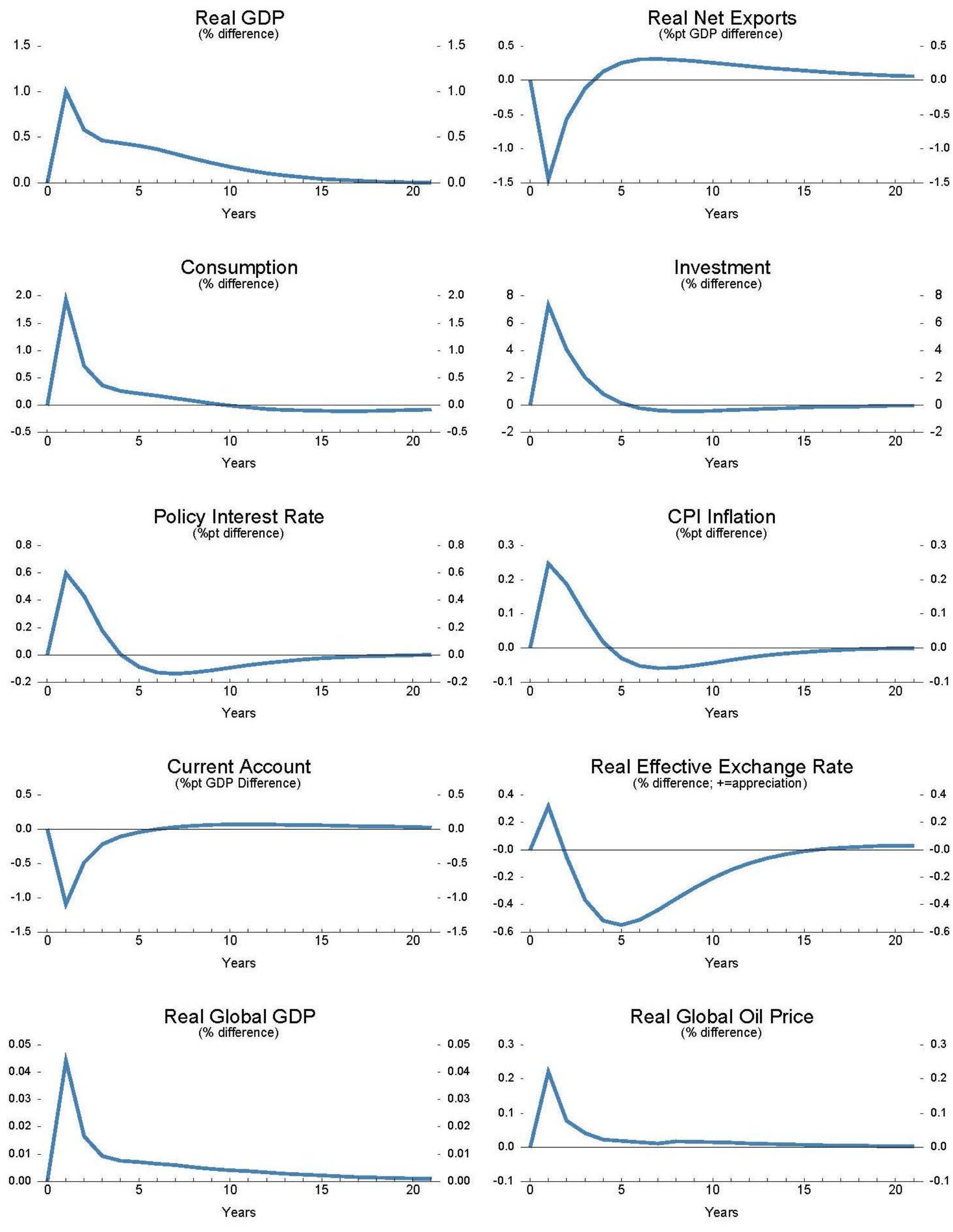
The increase in private investment accumulates into a higher stock of private capital, which contributes to a persistent increase in both actual and potential output, with the latter adjusting more sluggishly. In the short run, output still rises above potential, and the excess demand causes inflation to rise above target, by approximately ¼ of a percentage point. In response, the monetary authority raises its policy rate to close the output gap and return inflation to target. This increase flows through into real interest rates, which raises the cost of capital, thereby exerting downward pressure on private investment demand. In addition, higher real interest rates have an impact on the intertemporal consumption decision of OLG households, enticing them to increase private saving to the detriment of near-term consumption.

As a result of the higher real interest rates relative to foreign real interest rates, the exchange rate appreciates. This appreciation reduces the price of imports, boosting import demand (which is complemented by the increase in domestic activity), while raising the cost of Canadian exports in foreign markets, weakening the net export position.

On the fiscal side, the increase in domestic activity raises tax revenues and thus improves the fiscal balance. The fiscal authority responds to in the short run to close the output gap (the "automatic stabilizer") by decreasing general lumpsum transfers, thereby acting as a dampening influence on private consumption. However, in the medium run they must stabilize the deficit and gradually restore the debt-to-GDP ratio to target by increasing general lumpsum transfers, which puts upward pressure on private consumption.

This shock has the effect of raising real global GDP by less 0.1 percent, which reflects the direct impact of higher real GDP in Canada, as well as the spillover effects associated with this increase in demand. Higher global demand boosts the price of oil temporarily, by about 0.2 percent. In this case, the higher oil price also acts as an automatic stabilizer, dampening demand slightly in Canada and foreign economies that benefit from increased Canadian demand. The very small increase in global metals and food prices has no significant macroeconomic impact.

Figure 2: Temporary Increase in Aggregate Demand in Canada

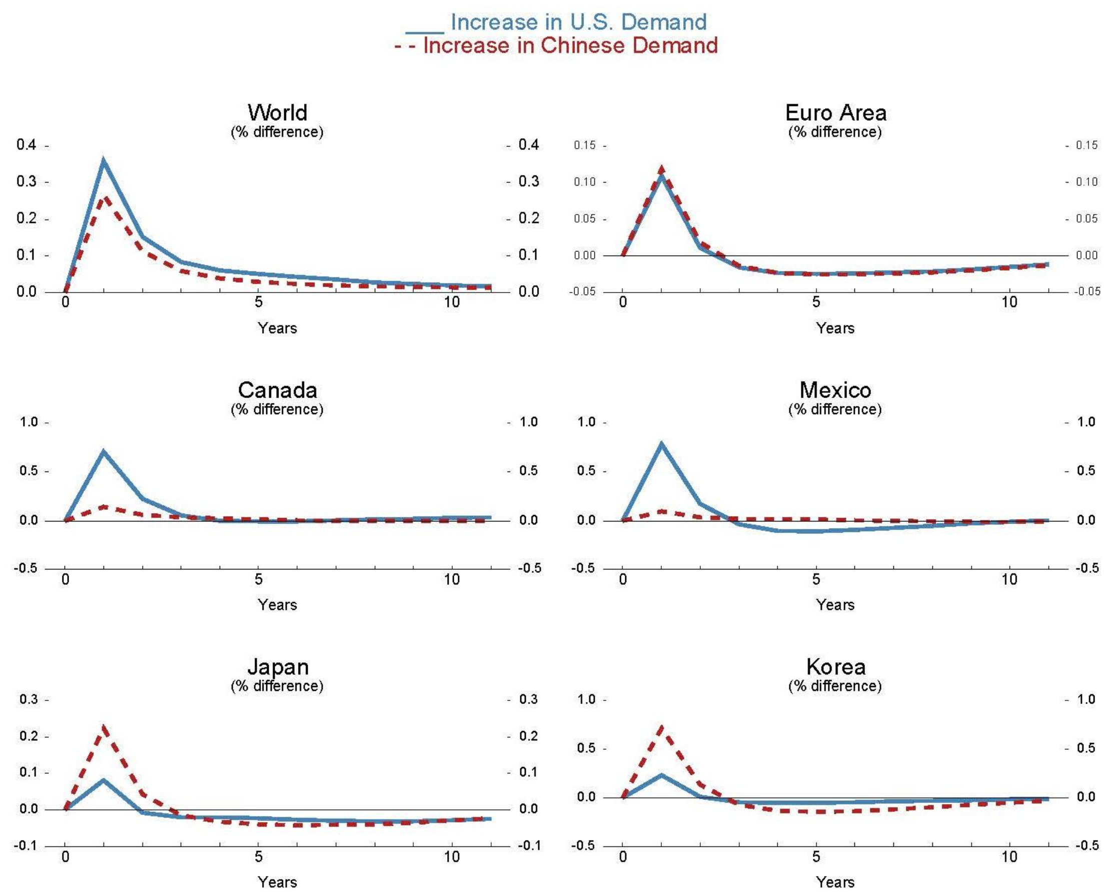


C. Trade Spillovers

Figure 3 presents the impact of a temporary one-year increase in aggregate demand in either the United States or China to illustrate how FSGM can capture the empirical fact that trade spillovers differ across countries according to where the shock originates. The shock is calibrated to raise real GDP by one percent relative to baseline, such that an increase in private investment is roughly four times larger than the increase in household consumption.

Domestic effects in the United States and China are roughly identical, and are qualitatively much like the results for the aggregate demand shock presented for Canada in Figure 2. Globally, U.S. spillovers are somewhat larger than those of China because of their relative sizes, but China is close because it is a more open economy. However, their spillovers to other countries differ because their historical trading relationships, which are often based on geographical proximity. So if we consider Canada, Mexico, Japan, and Korea we see that spillovers from the United States have a much greater effect on Canada and Mexico, whereas in the case of China, it has a much stronger relationship with Korea and Japan. By contrast, a region such as the euro area has almost identical spillovers from both countries.

Figure 3: Spillovers from a Temporary Increase in Aggregate Demand in either the United States or China



D. Permanent Increase in Trend Total Factor Productivity in the United States

In Figure 4, three increases in trend total factor productivity are considered, each of which raises the level of real GDP by 1 percent in the long run. The difference among the three is the sector in which the increase in productivity is assumed to occur. In one case the increase is assumed to be economy wide (blue line in Figure 4), in another case it is assumed to occur only in tradable goods (red), and in the last case it is assumed to occur only in the nontradable goods sector (gold). The distinction between tradable and nontradable goods does not literally exist in the model. The different behavior of foreign demand that underlies the distinction between tradable, nontradable, and economy wide productivity increases is implemented with time-varying trade shares that are a function of the relative levels of tradable and nontradable productivity across countries.

In the long run, the increase in productivity leads to more investment and a higher capital stock in all three cases as higher productivity increases the return to capital. The interesting difference is in the behavior of the RCI. When the productivity increase is located only in the tradable goods sector, the RCI appreciates in the long run. This is the standard Balassa-Samuelson result given the increase in competitiveness of U.S. tradable goods. When U.S. tradable sector productivity rises relative to its trading partners, demand for U.S. goods increases, as all other countries share of U.S. goods in their import bundles rise. The relationship between U.S. tradable sector productivity increases and the increase in the U.S. share of foreign import demand is calibrated so that the resulting appreciation broadly replicates the appreciation that occurs in GIMF under a tradable sector productivity shock of the same magnitude.[13]

When the increase in productivity is economy wide, the RCI depreciates in the long run. This arises because when the increase in productivity is economy wide, the relative price of U.S. tradable goods does not fall. However, higher productivity leads to increased demand for goods and services by U.S. households and firms, some of which they want to import. However, to maintain external balance, the United States also needs to export more to pay for those imports and the only way to increase foreign demand is for the dollar to depreciate.

When the increase in productivity is in the nontradable sector, the currency depreciation is even larger than when the increase is economy wide. Under a nontradable sector productivity shock, the relative price of U.S. tradable goods actually rises and thus to increase foreign demand sufficiently to maintain external stability, the depreciation must be greater.

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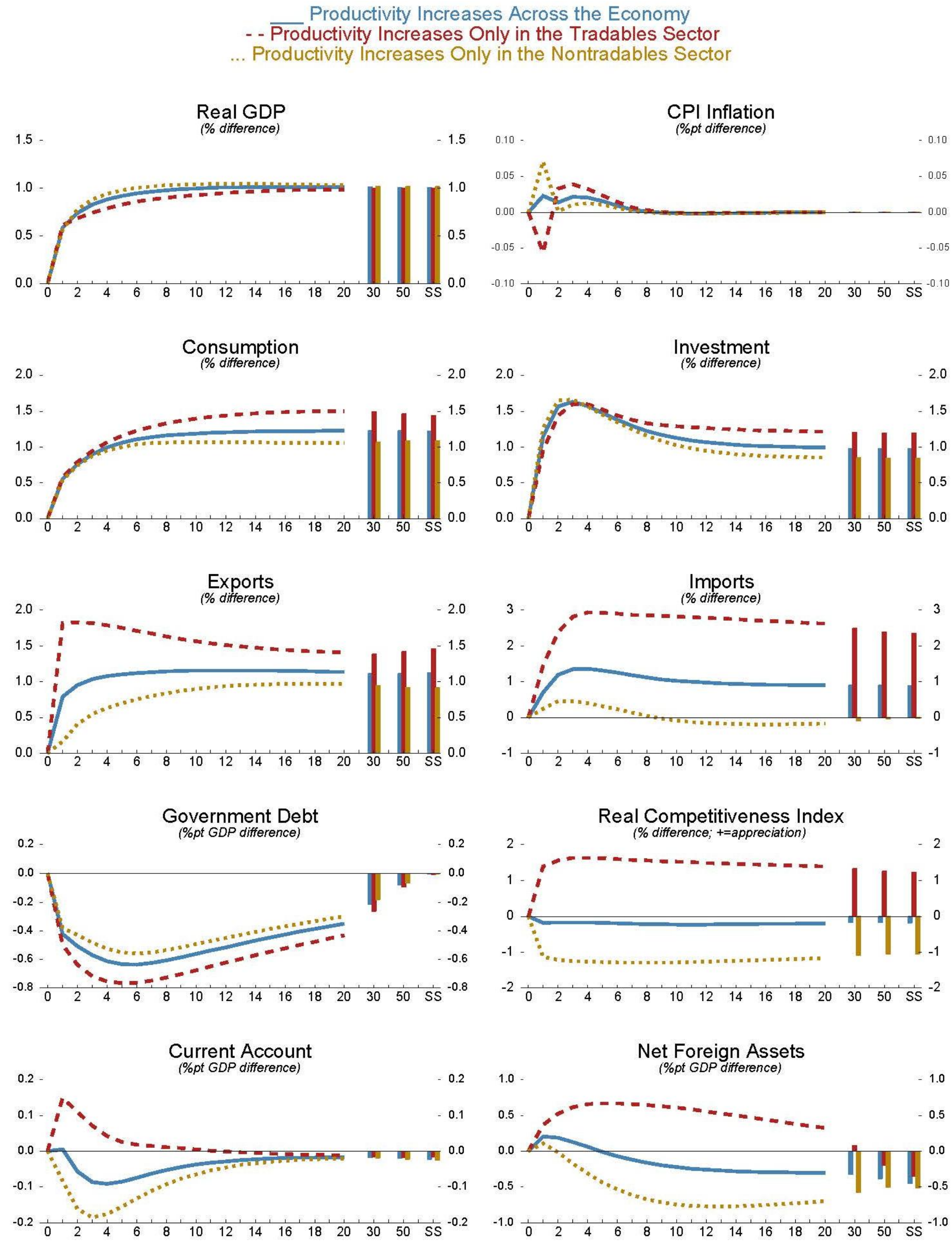
[13] As part of the FSGM development process, multiple 6-region versions of GIMF were calibrated to cover all the G-20 economies. These versions of GIMF were used to calibrate this Balassa-Samuelson effect for the respective blocks in FSGM, assuming an elasticity of substitution between home and foreign goods of 1.5.

Along the dynamic adjustment path, most key components of real GDP reach their new steady-state levels within ten years.

Figure 4: Permanent Increase in Trend Total Factor Productivity in the United States



E. Productivity Spillovers

As discussed in Section II.K above, an increase in productivity in one country spills over to the level of productivity in other countries. The amount of this spillover is based on the difference of the level of productivity *vis-à-vis* the level of U.S. productivity (how far away the country with the productivity increase is from the technological frontier) and the degree of trade openness (where openness is considered as a proxy for a country's commitment to growth-friendly policies).

The experiment considers increases in trend total factor productivity that raise real GDP by one percent in the long run in three countries – the United States, Germany, and China. The global impacts are shown in Figure 5. The increase in U.S. trend total factor productivity is the same as shown earlier, but here we concentrate on the effects on the rest of the world. The scale of the spillover effects depend on two factors. First, it depends on the size of the country experiencing the shock as a share of global real GDP, and second, on the elasticity of rest-of-the-world productivity to this shock. Since the United States is calibrated in the steady-state to be the largest economy, the size of this shock is bigger in absolute terms than any shock conducted in any other economy in G20MOD. The elasticity of the response of the rest of the world hinges on the distance from the technological frontier of the source country and the strength of the recipient country's trade links to the source country.

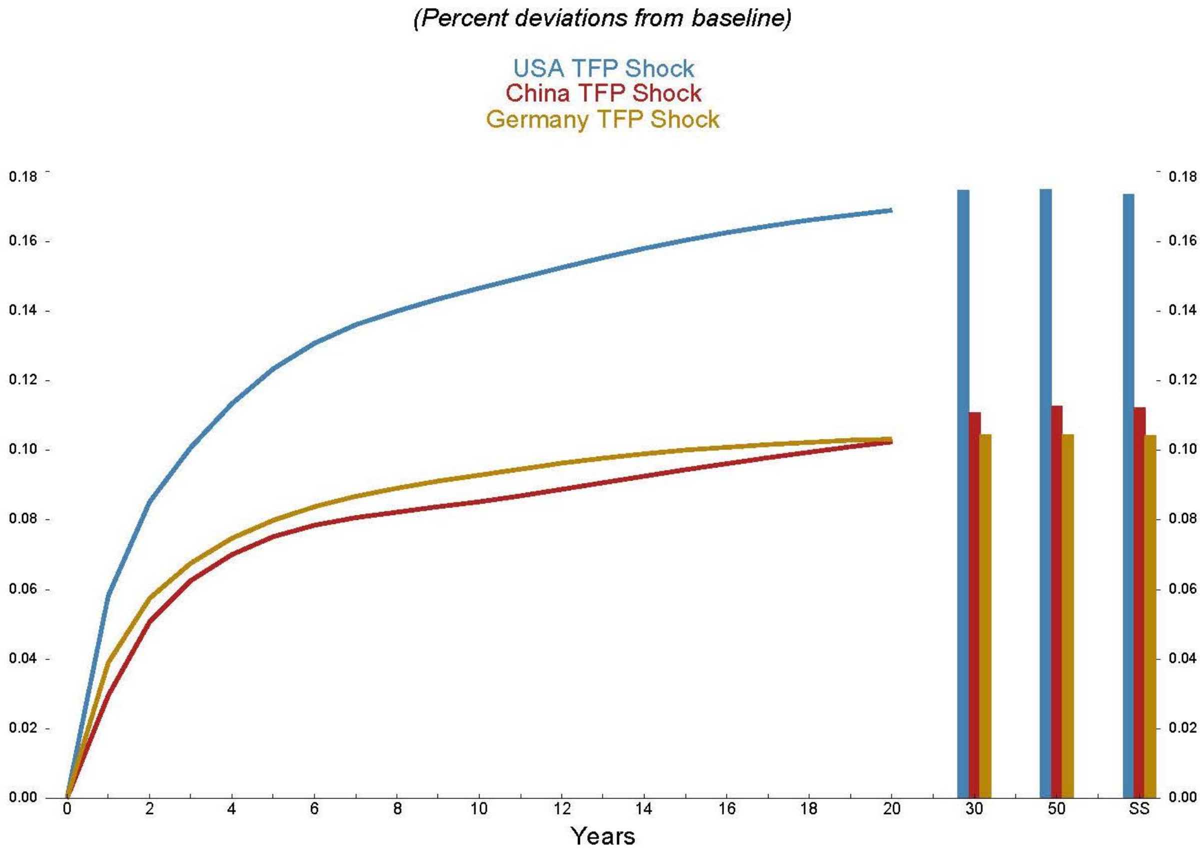
The increases in rest-of-world GDP in Figure 5 illustrate that in the first year, about 1/3 of the long-run benefit is realized. After 5 years, about 3/4 of the total long-run impact has materialized. In the long run, the spillover of the U.S. productivity increase to rest-of-world GDP is about 1/6 of the magnitude of the associated increase in U.S. GDP. German and Chinese productivity increases have smaller spillovers to the rest of the world than do those arising in the United States.

The U.S. productivity increase results in the largest spillover effects for two reasons. First, the United States is the largest economy in the world in absolute terms, so productivity increase in the united States have a tremendous positive impact on the rest of the world. Second, the United States is at the technological frontier and thus productivity shocks arising there have the largest outward spillover to the rest of the world.

Germany's spillovers are notable, but smaller than those from the United States. This arises because Germany is much smaller than the United States. However, its spillovers are still notable because it is both close to the technological frontier and very open.

The spillover effects from the Chinese economy are broadly similar in magnitude to those from Germany. This reflects the fact that although China is a larger economy that Germany and also open, it still far from the technological frontier.

Figure 5: The Effect on the Rest of the World Real GDP by Separate Permanent 1 Percent of GDP Increases in U.S., German and Chinese Trend Total Factor Productivity



F. Permanently Lower Household Saving Rate in China

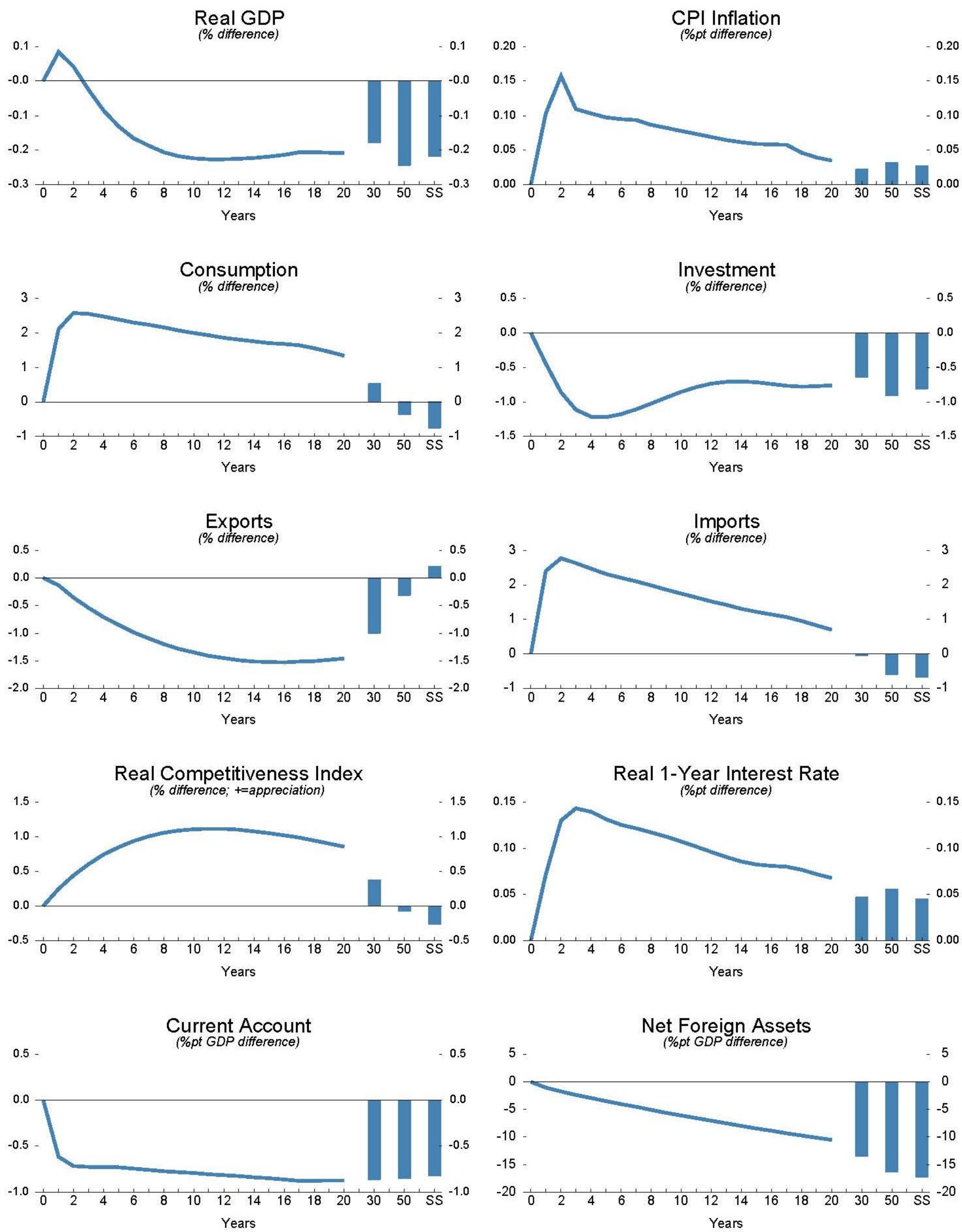
Figure 6 shows a permanent 1 percentage point reduction in the rate of household saving (or private saving) in China. Although, the short-run impact on output is mildly positive, in the long run, the permanently lower savings rate reduces real GDP by over 1/4 percent. The shock has little impact on inflation.

Since this is a permanent shock it is useful to first consider what is the new steady state. With households in China reducing their saving rates they hold fewer assets in the new steady state. Assuming no change in the quantity of domestically available assets, foreigners need to hold more Chinese assets and thus China's net foreign asset position is lower in the new steady state. With lower net foreign assets, China's net export position improves to service the relative increase in liabilities and the RCI depreciates in the long run. Further, with lower global savings the equilibrium global real interest rate rises to reduce demand for savings and increase the supply of savings outside China. With a higher equilibrium real interest rate, the cost of capital rises permanently and the desired level for the capital stock falls.

Consequently, real GDP in China (and throughout the world) is lower in the new steady state. In addition, with fewer assets, Chinese households also consume less in the long run.

However, to reach the new steady state, the dynamic adjustment path looks quite different. Initially, consumption rises as households consume a larger share of their income, in order to help reduce their holdings of net foreign assets in the long run. To further help facilitate the required adjustment in net foreign assets, the RCI initially appreciates, stimulating imports and dampening demand for Chinese exports. This deteriorates the current account and the net foreign asset position declines gradually to its new steady state. Forward-looking firms see the higher cost of capital and quickly start to reduce investment to gradually adjust the capital stock to its new lower desired level. The consumption boom is not large enough to offset both the reduction in investment and the deterioration in net exports and, consequently, real GDP falls below baseline in the third year and then continues to gradually decline to its new steady state supported by a lower level of the capital stock.

Figure 6: Permanently Lower Household Saving Rate in China



G. Permanent Fiscal Consolidation in Japan

In Figure 7, three consolidation experiments are considered, each of which reduces Japan's fiscal deficit permanently by 1 percent of GDP, which in turn results in a permanent reduction in the ratio of public debt to GDP of roughly 20 percentage points. In this example, households and firms fully believe the government's plan to permanently reduce the public debt-to-GDP ratio. The blue line traces out the implications when a cut in public consumption reduces the deficit and debt. The impact from higher labor income taxes is shown by the red line and the gold line traces out the impact when the VAT is raised. In each experiment, as the reduction in government debt leads to lower debt-service costs, the same instrument that was initially tightened is allowed to ease to maintain the improvement in the overall fiscal balance at 1 percent of GDP.

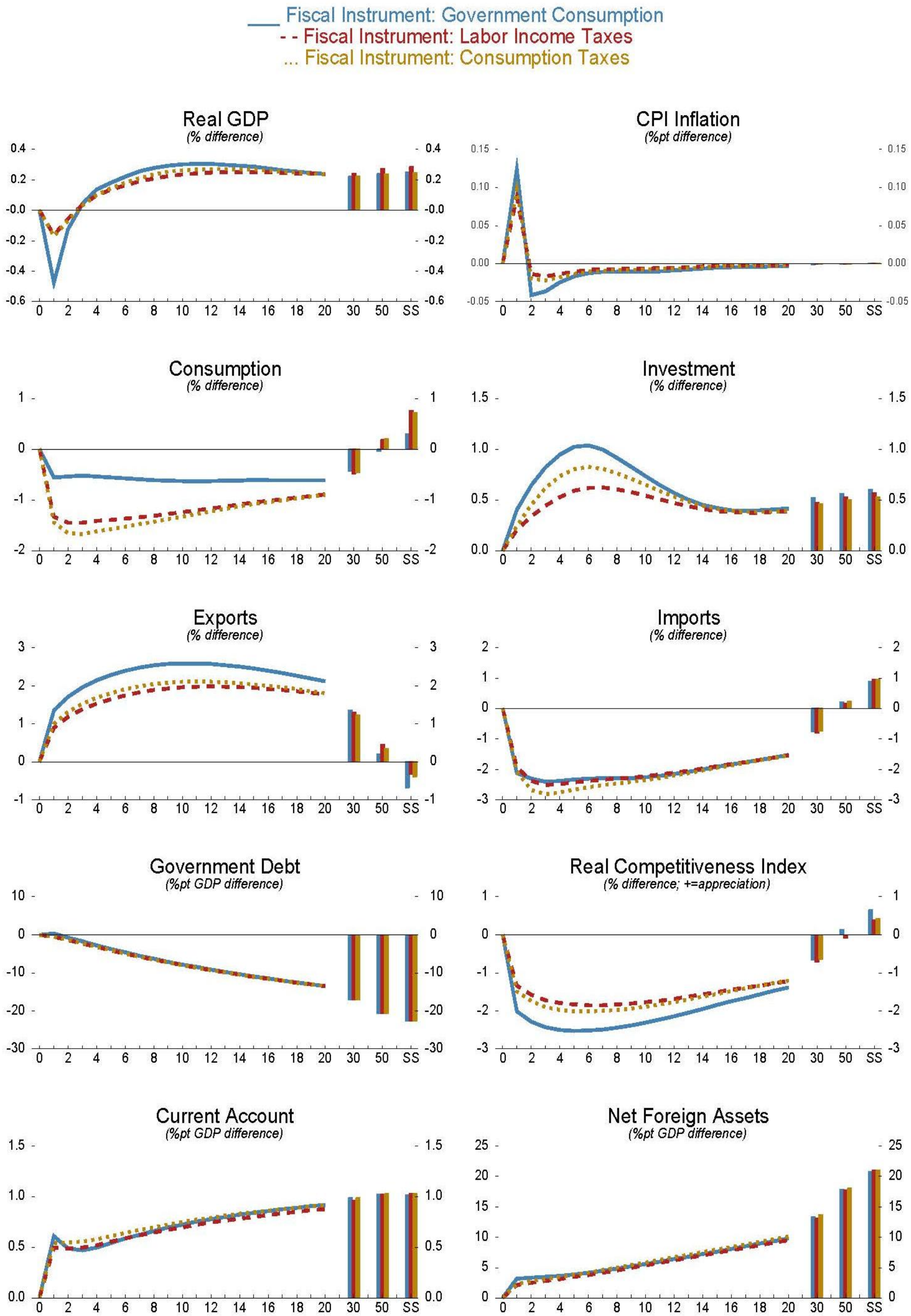
Thinking about the new steady state first is useful. Under all consolidations, real GDP is higher in the new steady state. This comes from two sources. First, with higher public savings in Japan, the global supply of savings is higher and the global real interest rate falls to equilibrate demand for and supply of savings. The lower global real interest rate raises the desired level of capital and potential output rises. Further, the reduced debt burden results in permanently lower debt-service costs and eventually the tightening in fiscal policy can be more than unwound.

With the government permanently increasing saving, the stock of domestic public bonds that households can hold in their asset portfolios declines. With households' desired wealth positions largely unchanged, they need to substitute foreign assets for government bonds and the net foreign asset position improves markedly in the new steady state. The higher net foreign asset position implies that the net export position can deteriorate (given the improvement in the income balance), and the RCI appreciates in the long run.

Along the transition path to the new steady state, real GDP initially falls in all cases. The largest decline occurs when public consumption is reduced to improve the deficit, followed by the cases where the VAT or labor income taxes rise. In all cases, investment and net exports are driving the bounce back as firms' respond to permanently lower real interest rates and net exports respond to the currency depreciation.

In all three cases, to facilitate the accumulation of net foreign assets, the RCI initially depreciates to reduce imports and raise exports. The stronger is the negative effect of the consolidation on domestic demand, the greater the depreciation needs to be to engineer the accumulation of net foreign assets. The largest depreciation is required when government consumption falls, while the least depreciation occurs when labor income taxes rise. The resulting paths for the current account and net foreign assets are similar in all three cases.

Figure 7: Permanent Fiscal Consolidation in Japan



H. Temporary Increase in Interest Rate Risk Premiums in the United States

Figure 8 shows the effects of a temporary but persistent 100 basis point increase in the three interest rate risk premiums in the United States. The first is the corporate risk premium (blue line), the second is the private sector risk premium (yellow line), and the third is the sovereign risk premium (red line). The corporate risk premium affects only firms, the private risk premium affects firms and households, and the sovereign risk premium affects firms, households and the government. In response to higher risk premiums, and thus market interest rates, real GDP temporarily falls. The decline in output is quite similar under all the increases in different risk premium. However, the composition of output varies notably. Lower output also leads to a small decline in inflation in all cases. However, when the corporate risk premium alone increases inflation falls by less.

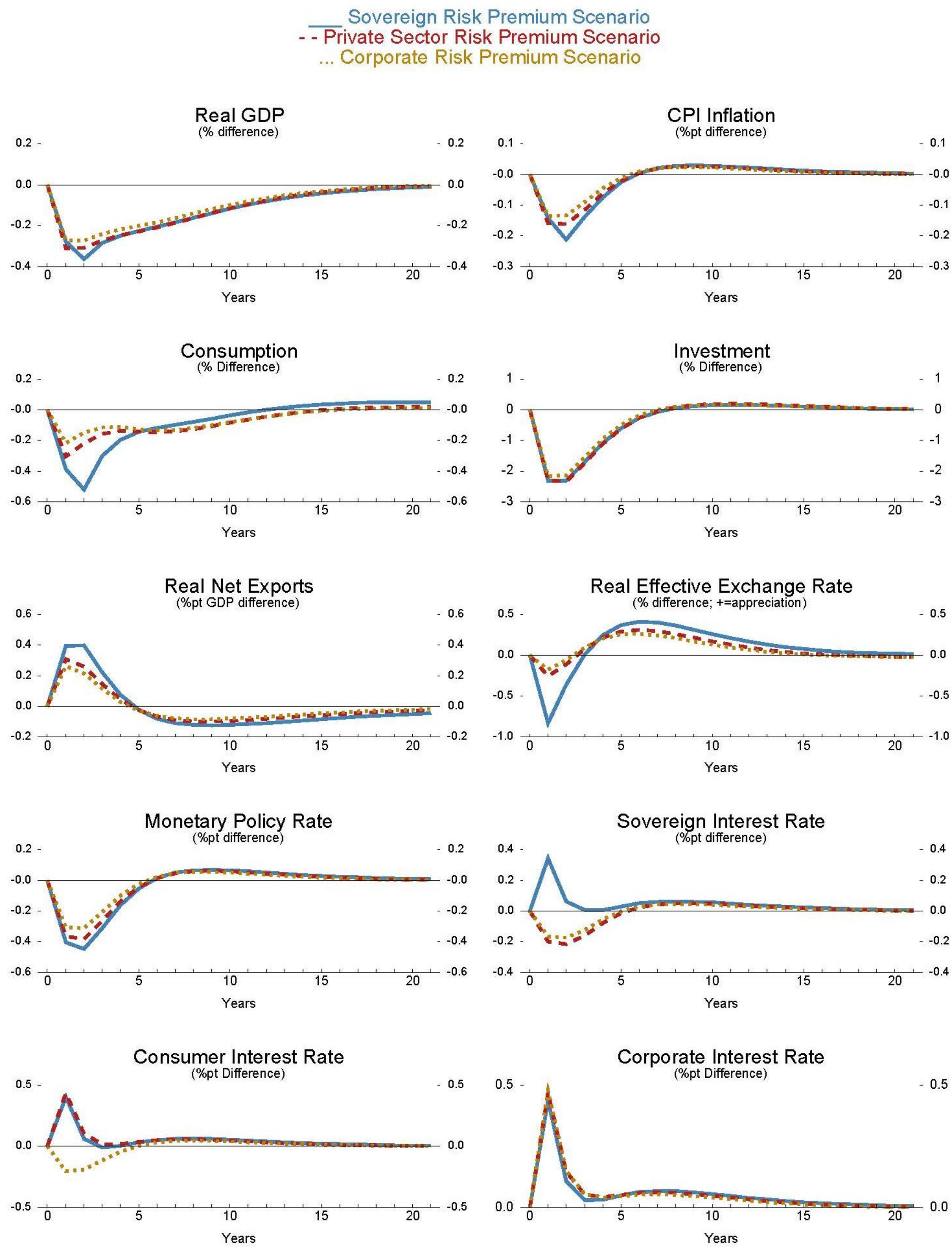
When the corporate risk premium alone increases, the decline in real GDP is primarily driven by lower investment, owing to the higher cost of capital that firms face. This leads to a reduction the capital stock that causes firms to demand less labor, thereby reducing employment as well as the real wage. Less income and lower wealth lead households to reduce consumption in the short run.

Although inflation declines, the fall is quite small because weaker investment means the economy’s supply potential is declining quickly along with demand. The monetary authority responds to lower inflation by reducing the policy rate.

When the increase in the risk premiums also includes the sovereign, in addition to facing higher interest rates, the government cuts lumpsum transfers to households, in response to the deterioration in the fiscal balance from higher debt service costs. This leads to lower domestic demand, a larger decline in inflation more easing in the monetary policy rate and more currency depreciation. The resulting stronger net export position helps offset weaker private consumption relative to when only the private sector risk premium increases.

The reduction in domestic demand associated with these shocks has the effect of compressing imports, which improves the real net export position. The reduction in the policy rate results in the depreciation of the real effective exchange rate, reinforcing the improvement in the net export position. The result is a current account surplus, relative to baseline. This surplus implies a temporary improvement in the net foreign asset position.

Figure 8: Temporary Increase in Interest Rate Risk Premiums in the United States



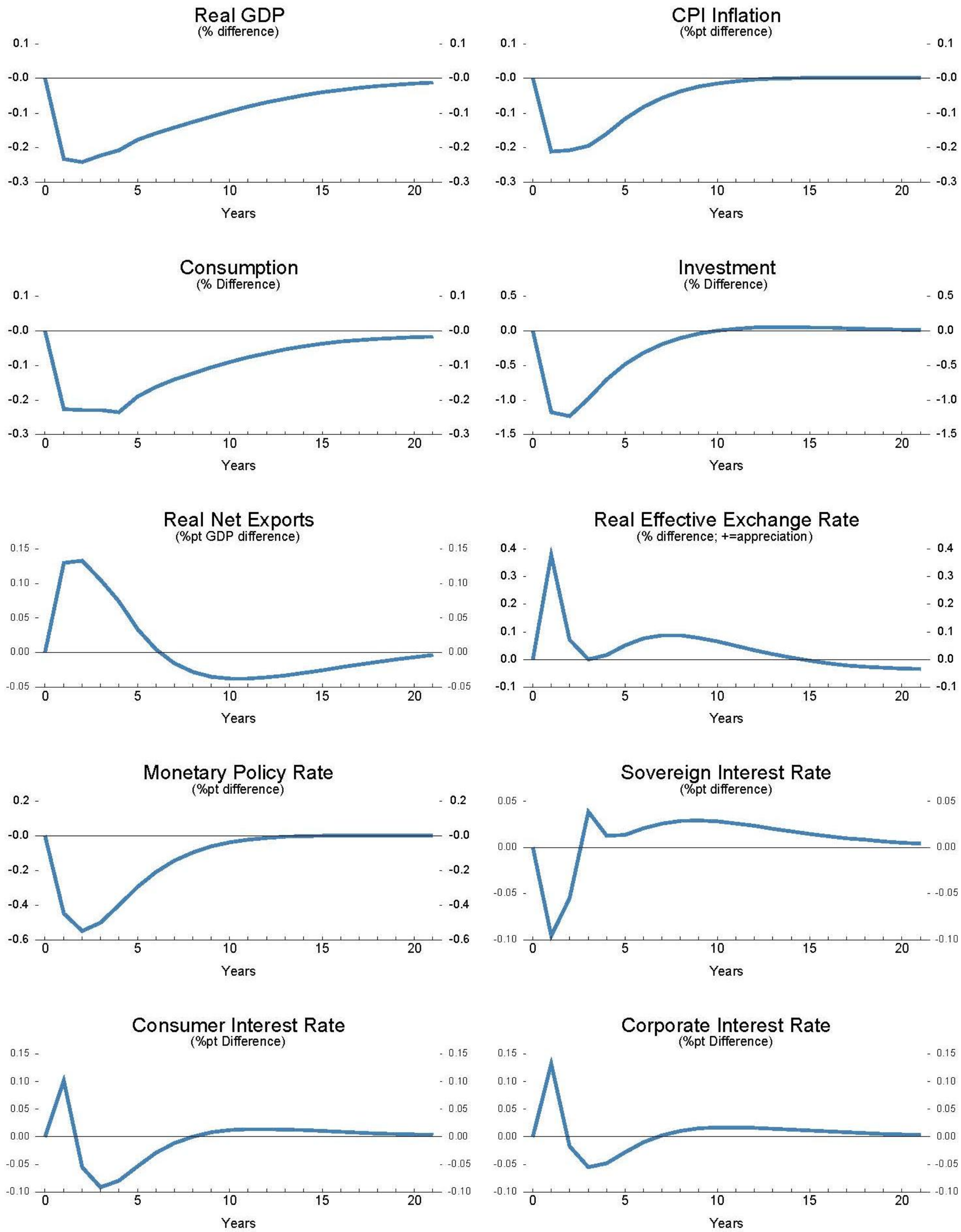
I. Temporary Increase in the Term Premium in the United States

Figure 9 shows the impact of a 100 basis point increase in the term premium on 10-year debt instruments in the United States. This raises the effective interest rate faced by households, firms, and the government. Real GDP falls by roughly ¼ percent at its trough and inflation declines by slightly less than ¼percentage points.

With firms facing a higher cost of capital, they reduce investment, labor demand and wages. Households, facing a higher consumption interest rate and lower income, reduce consumption. The impact on households is amplified by the response of the fiscal authority. With revenues falling, owing to weaker domestic demand, and debt service costs rising, the resulting deterioration in the fiscal balance prompts the fiscal authority to cut lumpsum transfers to households, further reducing their consumption.

Weaker domestic demand leads to a fall in inflation and the monetary authority responds by easing its policy rate. However, the currency appreciates since the risk adjusted real interest rate on U.S. assets still rises relative to foreign assets owing to the increase in the term premium. With a higher real effective exchange rate, the improvement in the real net export position is smaller compared to the case of rising risk premiums on U.S. assets. This implies that despite a much smaller impact on domestic demand from an increase in the term premium, the net impact on real GDP is not that much smaller than under increases in risk premiums.

Figure 9: Temporary Increase in the Term Premium in the United States



J. Permanent Increase in the Real Global Price of Oil

Figures 10 and 11 illustrate a permanent reduction in the supply of oil by OPEC countries leading to a long-run increase in the real global price of oil of 10 percent. First the impact on the German economy is considered in detail in Figure 10 and the impacts on real GDP for the global economy and several key countries/regions are presented in Figure 11. The long-run impact is a permanent reduction in German real GDP of almost ¾ percent, which is reached after just a few years. There is an initial pickup in headline CPI inflation, but no material change in core inflation along the adjustment path.

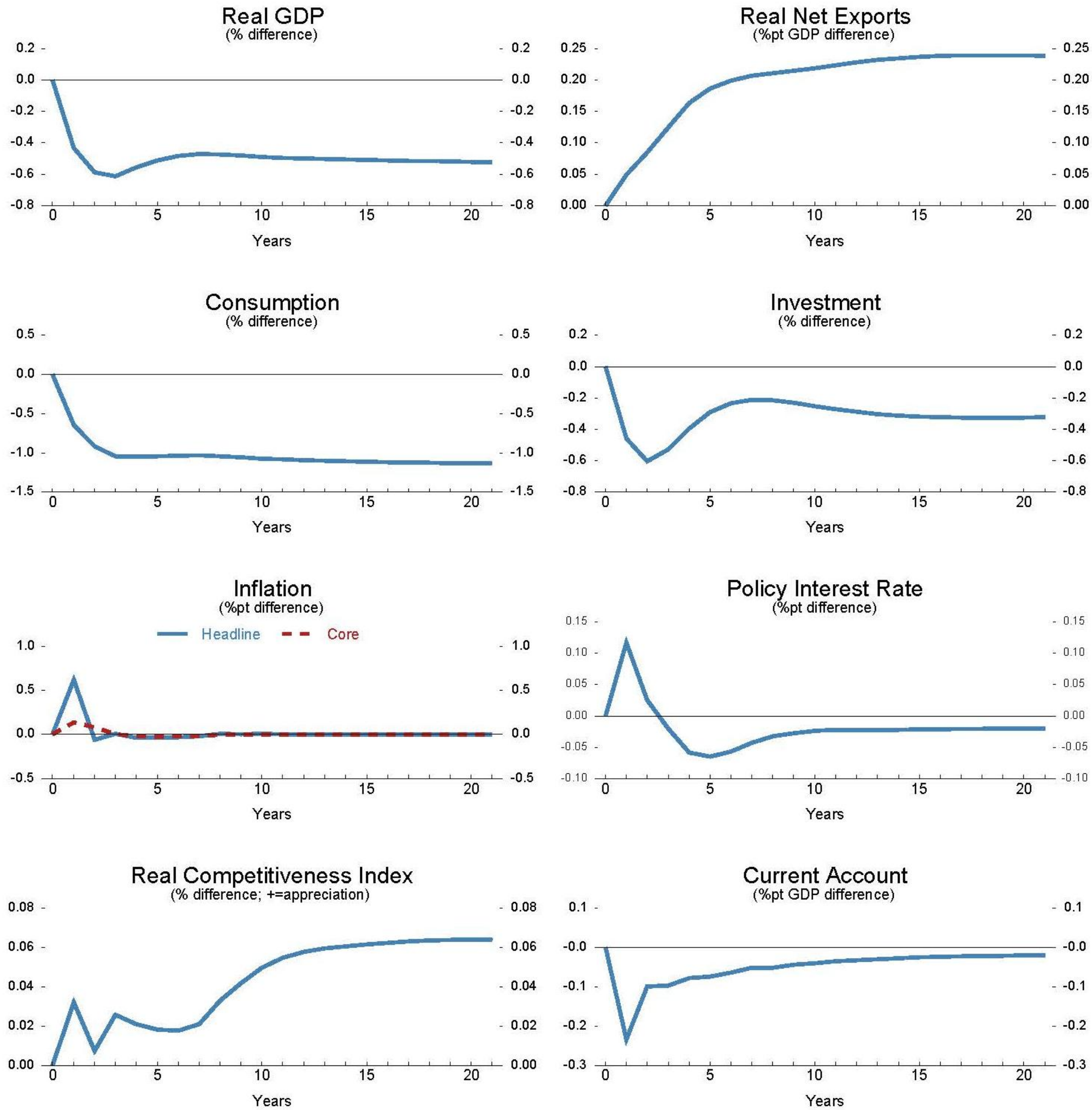
The increase in the real global price of oil feeds immediately into German headline CPI inflation increasing by roughly ¾ percentage points. For households, this immediately reduces their real income and wealth. Consequently, consumption falls with liquidity-constrained households getting hit harder than the overlapping-generations households. Firms see both their inputs cost rising and demand for their goods falling, reducing the return to capital. A lower return to capital induces firms to cut investment and lower their desired level of capital. Consequently, firms also reduce their demand for labor, further reducing household income. Lower investment and consumption demand feed through to lower imports, which actually fall more than exports.

Export demand is supported by a reduction in Germany’s RCI owing to the negative terms-of-trade shock implied by higher oil prices. Although Germany’s real trade balance improves, the current account deteriorates in the medium term owing to the higher price of imported oil. This leads to a decline in Germany’s net foreign asset position. The permanent currency depreciation is necessary for Germany to be able to export enough to pay the higher cost of its import bundle and cover the fall in its foreign income position.

Because the CPIX inflation Phillips curve allows only limited scope for oil prices to feed through into persistent inflation and a small negative output gap opens initially, core inflation rises very little in Germany.

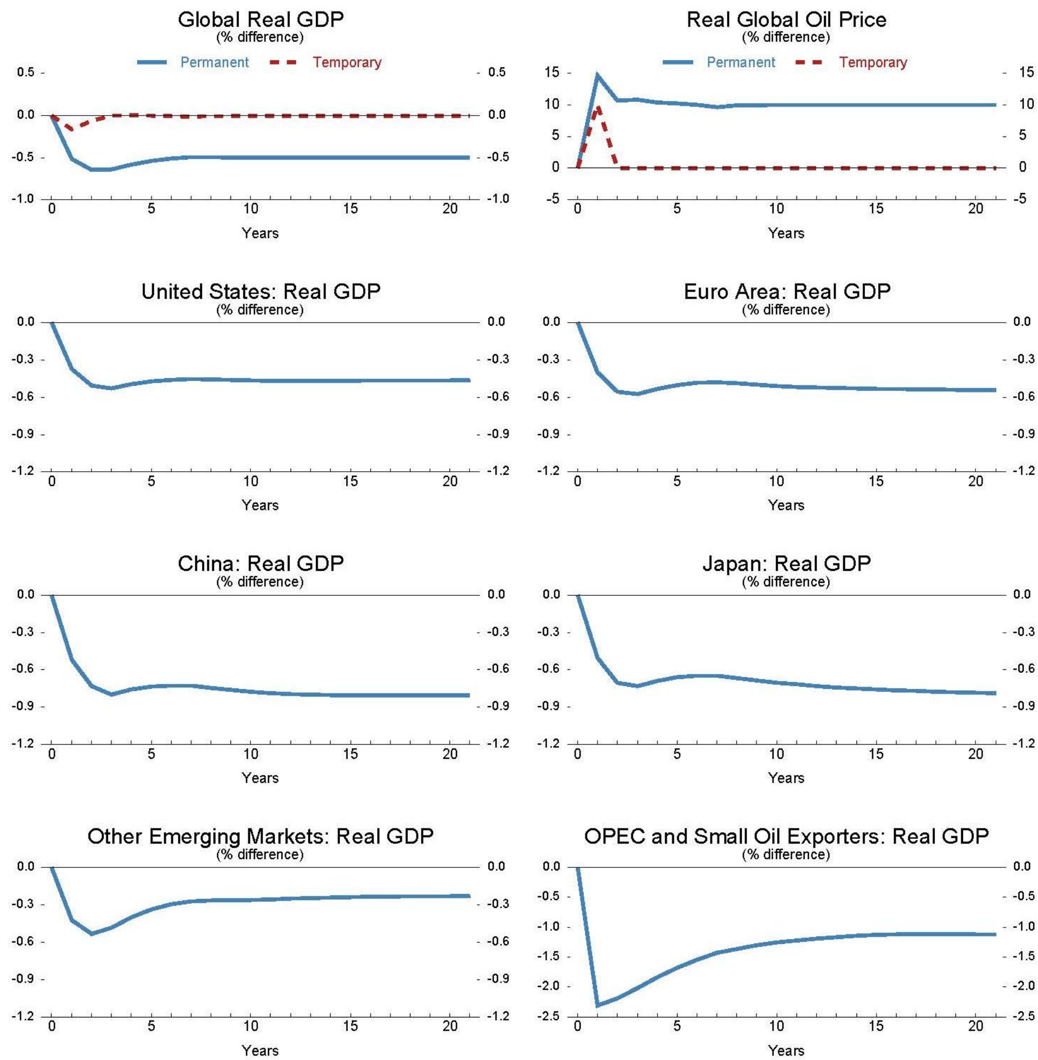
Turning to the effects elsewhere in the world (Figure 11), it can be seen that a permanent 10 percent increase in the price of oil takes roughly 1/2 percent off of real global GDP. For the G-3 economies, Japan is the hardest hit because of its large net oil import position. The impacts on the United States and the euro area are similar despite a higher oil cost share in U.S. GDP. This reflects the fact that the United States also produces a large amount of the oil it uses and U.S. energy firms and the households that own them benefit from the higher oil price. Turning to emerging market economies, both oil exporters and importers suffer. For oil exporters, the reduction in supply along with the impact of higher oil prices on their domestic economies lead to larger impact effects than for oil importers. The negative impact on China is large because of its high net oil import position.

Figure 10: Impact on Germany of a Permanent 10 Percent Increase in the Real Global Price of Oil



For reference, Figure 11 also contains the impact of a temporary 1-year 10 percent increase in the real price of oil on real global GDP. The impact of the temporary oil price rise is notably less than that of a permanent increase, as firms and households understand the temporary nature of the increase, and only adjust their current behavior temporarily to absorb the temporary increase in input costs and the cost of the consumption bundle, as opposed to making long-run changes in behavior.

Figure 11: Impacts on Real GDP Worldwide of a Permanent 10 Percent Increase in the Real Global Price of Oil



V. CONCLUSIONS

Global macroeconomic models are playing an increasingly important role in IMF multilateral and bilateral surveillance and, consequently, there is increased demand for a very high level of individual country granularity. Unfortunately, it is not feasible to expand the coverage so that a large number of individual countries can be analyzed simultaneously in the IMF's multiple-goods, fully structural models like GEM and GIMF.

Given the demand for extensive country coverage, the rich set of topics to be addressed in surveillance, the state of economic theory, and computing constraints, the design of a new global macroeconomic model faced important trade-offs. An eclectic approach has been chosen and the new framework, the Flexible System of Global Models (FSGM), is a semi-structural model where practical functionality of the framework has been prioritized, rather than theoretical rigor.

Although FSGM deviates in many ways from theoretical dynamic stochastic general equilibrium (DSGE) models, it draws from those heavily and the system's properties are constantly benchmarked against the state-of-the-art DSGE models like GEM and GIMF. The availability of such strict guardians has facilitated the design of both FSGM's structure and its dynamic adjustment properties.

This paper has documented the structure of a typical FSGM module and illustrated the rich array of issues that it can be used to analyze. While there is still considerable work left to do in terms of further enhancing the framework's data coherence and extending it so that it can be used for medium-term forecasting applications, it has already proven to be a highly useful tool.

Appendix I: Composition of the Modules of FSGM

Here are complete lists (3 letter country and region codes) for the 9 modules of FSGM.

CORE MODULES

1 - G20MOD:

19 countries (80-85 percent of global GDP)

ARG,AUS,BRA,CAN,CHN,FRA,DEU,IND,IDN,ITA,JPN,KOR,MEX,RUS,SAU,ZAF, TUR,GBR,USA

Plus 5 regions:

EA1 = Other Euro Area
EU1 = Other European Union
OA1 = Other Advanced Economies (HKG,ISL,ISR,NZL,NOR,SGP,CHE,TWN)
OX1 = Other Oil Exporters (mostly OPEC)
RC1 = Rest of the World

2 - EUROMOD:

18 countries

AUT,BEL,CHN,FIN,FRA,DEU,GRC,IND,IRL,ITA,JPN,NLD,PRT,ESP,SWE,CHE,GBR, USA

Plus 6 regions:

AS2 = Newly Industrialized Asia excluding China and India (HKG,IDN,KOR,MYS,PHL,SGP,THA,TWN)
EU2 = Other European Union (BGR,CZE,DNK,HUN,POL,ROM,SVK,SVN,CYP,EST, LVA,LTU,LUX,MLT)
LA2 = Latin America (ARG,BRA,CHL,COL,MEX,PER)
OA2 = Other Advanced Economies (AUS,CAN,ISL,ISR,NZL,NOR)
OX2 = Oil Exporters (mostly OPEC, Russia)
RC2 = Rest of the World

3 - EMERGMOD:

9 countries

USA,JPN,CHN,HKG ,SGP,CHL,VEN ,NGA,ZAF

Plus 15 regions:

ASE = ASEAN-3 (MYS,PHL,THA)
AS3 = Rest of Emerging and Newly Industrialized Asia (including IND,IDN,KOR)
CCM = Caucasus and Central Asia Oil Importers
CCX = Caucasus and Central Asia Oil Exporters
CEE = Central Europe (CZE,HUN,POL)
EAR = Euro Area Region (excludes EST,LVA,LTU,SVK,SVN)
EA3 = Emerging Euro Area (EST, LVA,LTU,SVK,SVN)
ESE = Eastern and Southeastern Europe
LA3 = Other Latin America (including ARG,BRA,MEX)
MNM = Middle East and North Africa Oil Importers
MNX = Middle East and North Africa Oil Exporters
OX3 = Other Oil Exporters (mainly NOR,RUS)
SSM= Sub-Saharan Africa Oil Importers
SSX = Sub-Saharan Africa Oil Exporters
RC3 = Rest of the World (AUS,CAN,DNK,ISL,ISR,NZL,SWE,CHE,GBR,MNG,TUR)

SATELLITE MODULES

4 - SIXMOD:

2 countries

JPN, USA

Plus 4 regions:

AS4 = Emerging and Newly Industrialized Asia
EUA = Euro Area
LA4 = Latin America 5 (BRA, CHL, COL, MEX, PER)
RC4 = Rest of the World

5 - WHDMOD (Western Hemisphere Department):

18 countries

ARG,BRA,CHL,COL,CRI,DOM,GTM,MEX,PAN,PER,TTO,URY,CAN,CHN,IND,JPN,
RUS,USA

Plus 6 regions:

CA5 = Other Central America (SLV,HND,NIC)
EUA = Euro Area
LA5 = Other Latin America
OA5 = Other Advanced Economies (AUS,DNK,HKG,ISL,ISR,KOR,NZL,NOR,SGP,CHE,
SWE,TWN, TUR,GBR)
OX5 = Oil Exporters
RC5 = Rest of the World

6 - APDMOD (Asia and Pacific Department)

18 countries

AUS,BGD,KHM,CHN ,HKG,IND,IDN,JPN,KOR,MYS,MNG,NZL,PHL,SGP,LKA,THA,
VNM, USA

Plus 6 regions:

EUA = Euro Area
LAT = Latin America
OA6 = Other Advanced Economies (CAN,ISL,ISR,ZAF,CHE,TWN,TUR,GBR + Eastern
Europe, Scandinavia)
OX6 = Oil Exporters
PIC = Pacific Island Countries
RC6 = Remaining Countries

7 - MCDMOD (Middle East and Central Asia Department)

12 countries

CHN,JPN,RUS,USA,ARM,EGY,GEO,IRN,KAZ,MAR,PAK,SAU

Plus 11 regions:

AS7 = Emerging Asia
CM7 = Central Asia and Caucasus Oil Importers
CX7 = Central Asia and Caucasus Oil Exporters
EUA = Euro Area

EU7 = Other European Union
GCX = Other GCC Oil Exporters
LAT = Latin America
MN7 = Other Middle East and North Africa Oil Importers
OA7 = Other Advanced Economies (AUS,CAN,GBR,HKG,ISL,ISR,KOR,NZL,NOR,SGP, CHE,TWN)
OX7 = Other Oil Exporters
RC7 = Remaining Countries

<u>8 - EEUMOD (European Department):</u>

13 countries

CHN,JPN,USA,BGR,HRV,CZE,HUN,POL,ROM,RUS,SRB,TUR,UKR

Plus 11 regions:

AS8 = Emerging Asia
CIS = CIS excluding Russia (BLR,MDA)
EA8 = Emerging Euro Area (EST,LVA,LTU,SVK,SVN)
EC8 = Core Euro Area (AUT,BEL,DEU,FIN,FRA,IRL,NLD,PRT)
EP8 = Euro Area Periphery (GRC,ITA,ESP,CYP)
ES8 = Eastern and Southeastern Europe (ALB,BIH,KOS,MKD,MNE)
EU8 = Other European Union (DNK,SWE,GBR)
LAT = Latin America
OA8 = Other Advanced Economies (AUS,CAN,HKG,ISL,ISR,KOR,NZL,NOR,SGP, CHE,TWN)
OX8 = Oil Exporters
RC8 = Rest of the World

<u>9 - AFRMOD (African Department)</u>

13 countries

USA,CHN,FRA,DEU,IND,ITA,JPN,GBR,AGO,GHA,NGA,ZAF,ZMB

Plus 11 regions:

EA9 = Other Euro Area
EAF = Eastern Africa (KEN,RWA,TZA,UGA)
FAF= Fragile Africa (BFA,BDI,CIV,CAF,COD,COM,ERI,GIN,GNB,STP,SLE,TGO,LBR, ZWE)
LAT = Latin America
LIA = Low Income Africa (ETH,GMB,MDG,MWI,MOZ)
MIA = Middle Income Africa (BWA,CPV,LSO,MUS,NAM,SEN,SYC,SWZ)
OA9 = Other Advanced Economies (AUS,CAN,DNK,HKG,ISL,ISR,KOR,NZL,NOR,SGP,

SWE,CHE,TWN)
OX9 = Other Oil Exporters
SX9 = Sub-Saharan Africa Oil Exporters
WMU = West African Economic and Monetary Union, WAEMU (BEN,MLI,NER)
RC9 = Remaining Countries

Core Models

G20MOD, EUROMOD, EMERGMOD

34 unique countries

19 unique regions

Satellite Models

WHDMOD,APDMOD,MCDMOD,EEUMOD,AFRMOD

35 additional unique countries

13 additional unique regions

Appendix II: Structural Interpretation

This appendix further expands the exposition found in Section II, on the structure of a typical module in FSGM, particularly those sectors which are not just reduced forms, but can be derived from a more complete optimization problem. Most of this appendix is described in a similar fashion to the corresponding sections in Kumhof and others (2010), which explains the IMF's Global Integrated Monetary and Fiscal model (GIMF).

Household Behavior

Household behavior is based on the stochastic lifetime model, as found in Blanchard (1985). This effectively means that there is some positive birth rate which generates non-Ricardian behavior as observed by Weil (1987) and Buiter (1988). The excess wealth left over due to the stochastic death of households is redistributed to the surviving generations using the Yaari (1965) insurance scheme.

We further separate households into the overlapping generation (OLG) households and into the liquidity constrained (LIQ) households. OLG households in this Blanchard-Yaari model live typically, on the one hand, a longer time than the two periods in Samuelson OLG model and, on the other hand, a shorter time than infinitity as in the representative agent model by Ramsay and Sidrauski. OLG households are not wealth constrained. The existence of the liquidity-constrained agents enhances the non-Ricardian behavior in the model.

Furthermore, it is assumed that the productivity of a household's labor declines throughout its lifetime. This affects the discounting of human wealth, thus creating additional non-Ricardian behavior.

Moreover, the economy has a fiscal authority which levies several distortionary taxes on both households and firms creating additional sources of non-Ricardian behavior in the model.

The OLG household behavior described here is a simplified version of the specification applied in GIMF. In particular, the labor supply is not part of the households' optimization problem.

Definitions

The world economy experiences a constant positive trend technology growth rate $g = T_t/T_{t-1}$, where T_t is the level of labor augmenting world technology. n is the constant positive population growth rate. N indexes absolute population sizes in period 0. λ_t^c is the share of liquidity constrained households at time *t*. In each period $Nn^t(1-\lambda_t^c)\left(1-\frac{\theta}{n}\right)$ is the number of overlapping generation households are born. Furthermore, $Nn^t\lambda_t^c\left(1-\frac{\theta}{n}\right)$ is the number of liquidity constrained households born each year.

Overlapping Generations Households

Overlapping generations (OLG) households that can accumulate wealth maximize their expected discounted utility:

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s U_{a+s,t+s}^{OLG} ,$$

where E_t, β, θ, and $U_{a+s,t+s}^{OLG}$ are the conditional expectations held at time t, the subjective discount factor, the degree of myopia, and the utility function of household born in time a respectively.

The constant elasticity of substitution (isoelastic) utility function is

$$U_{a,t}^{OLG} = \frac{\left(\frac{c_{a,t}^{OLG}}{(H_t)^{\gamma}}\right)^{1-\sigma} - 1}{1-\sigma} ,$$

where $c_{a,t}^{OLG}$ is the consumption at time t of agent born at time a, H_t is the external habit stock of consumption, γ is the habit share, and σ is the inverse of the elasticity of substitution. External habit is further defined as follows:

$$H_t = c_{t-1}^{OLG} ,$$

where c_{t-1}^{OLG} is the aggregate consumption of OLG households in the previous period.

The periodic budget constraint in nominal terms is:

$$\begin{aligned}(1+\tau_t^c)P_t^c c_{a,t}^{OLG} + B_{a,t} + \varepsilon_t F_{a,t} \\ = \frac{1}{\theta}\left[INT_{t-1}^{GB} B_{a-1,t-1} + INT_{t-1}^{NFA} \varepsilon_t F_{a-1,t-1} + WK_{a,t}\right] + P_t \Upsilon_{a,t}^{OLG} \\ + (1-\tau_t^L) W_t L_{a,t} \Phi_{a,t} + \Omega_{a,t},\end{aligned}$$

where τ_t^c, is the sales/value-added tax rate, τ_t^L is the labor income tax rate, $\Upsilon_{a,t}^{OLG}$ is government general lumpsum transfers, P_t^c is the price of consumption, $B_{a,t}$ is nominal value of domestic bonds owned by households born at time a, $F_{a,t}$ is the nominal value of net foreign assets owned by households born at time a, $WK_{a,t}$ is the nominal market value of the capital stock owned by households born at time a, ε_t is nominal exchange rate (domestic currency per unit of foreign currency), INT_{t-1}^{GB} is the gross nominal interest rate paid by the domestic government, INT_{t-1}^{NFA} is the gross nominal interest rate paid by the foreign government, $W_t L_{a,t}$ is nominal labor income per household, $\Phi_{a,t}$ is the productivity of the household, and $\Omega_{a,t}$ is the dividends rebated back as lumpsum income to the household by firms and other agents in the economy.

The productivity of a household's labor declines throughout her lifetime, with productivity $\Phi_{a,t} = \Phi_a$ of a age group is given by $\Phi_a = \kappa\chi^a$, where $\chi < 1$. This assumption enhances the non-Ricardian behavior of the model.

Aggregation takes account of the size of each age cohort at the time of birth, and of the remaining size of each generation. For example, the aggregation of overlapping generation households' consumption is as follows: $c_t^{OLG} = Nn^t(1-\lambda^c)\left(1-\frac{\theta}{n}\right)\sum_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a c_{a,t}^{OLG}$, where λ_t^c is the share of liquidity constrained households.

The first order conditions yield the following consumption Euler equation:

$$E_t c_{a,t+1}^{OLG} = E_t j_t c_{a,t}^{OLG} ,$$

where

$$j_t = (\beta \check{r}_t^c)^{\frac{1}{\sigma}} \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)^{\frac{1}{\sigma}} \left(\frac{\check{h}_{t+1} g_{t+1} n}{\check{h}_t}\right)^{\gamma\frac{\sigma-1}{\sigma}},$$

where $\check{r}_t^c$ is the real interest rate for consumption and $\check{h}_t = \frac{H_t}{T_t n^t}$ is habit persistence.

The aggregated consumption plan of OLG households is:

$$\check{c}_t^{OLG} = \frac{1}{\Theta_t}(p_t^c \check{w}f_t + \check{w}h_t + \check{w}o_t),$$

$$p_t^c \check{w}f_t = \frac{1}{\pi_t g_t n}\left[INT_{t-1}^{GB}\check{b}_{t-1} + INT_{t-1}^{NFA}\left(\frac{\varepsilon_t}{\varepsilon_{t-1}}\right)\check{f}_{t-1}\left(\frac{\varepsilon_{t-1}P_{t-1}^*}{P_{t-1}}\right) + \check{w}k_t\right],$$

$$\check{w}o_t = (1-\lambda^c)\left(\Upsilon_t - tax_t^{ls}\right) + \check{\Upsilon}_t^{OLG} + E_t \frac{\theta g_{t+1}}{\check{r}_t^c}\check{w}o_{t+1},$$

$$\check{w}h_t = (1-\lambda^c)(1-\tau_t^L)\check{w}_t\check{l}_t + E_t \frac{\theta\chi g_{t+1}}{\check{r}_t^c}\check{w}h_t,$$

$$\Theta_t = (1+\tau_t^c) + E_t \frac{\theta j_t}{\check{r}_t^c}\Theta_{t+1},$$

$$j_t = (\beta \check{r}_t^c)^{\frac{1}{\sigma}} \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)^{\frac{1}{\sigma}} \left(\frac{\check{h}_{t+1} g_{t+1} n}{\check{h}_t}\right)^{\gamma\frac{\sigma-1}{\sigma}},$$

$$\check{h}_t = \frac{\check{c}_{t-1}^{OLG}}{g_t n},$$

where $\Theta_t$is the inverse of marginal propensity to consume out of wealth, $\breve{w}f_t$ is financial wealth, $\breve{w}h_t$is human wealth, and $\breve{w}o_t$ is other wealth. In the financial wealth equation we have domestic bonds, $\breve{b}_{t-1}$, net foreign assets, $\breve{f}_{t-1}$, and the market value of capital, $\breve{w}k_t$, all in real scaled terms. The human wealth equation contains the expected discounted after-tax labor income. The discounting term for human wealth includes the assumption of falling productivity of individual households, χ. Other wealth includes dividends/other income and net government lumpsum transfers (both general and targeted).

Liquidity-Constrained Households

The periodic budget constraint for liquidity-constrained (LIQ) households in nominal terms is:

$$(1+\tau_t^c)P_t^c c_{a,t}^{LIQ} = P_t\lambda_t^c\big(\Upsilon_{a,t} - tax_{a,t}^{ls}\big) + \Upsilon_{a,t}^{LIQ}) + (1-\tau_t^L)W_t L_{a,t}\lambda_t^c\Phi_{a,t}\ ,$$

Where λ_t^c is the share of LIQ households in all households, $\tau_{a,t}$ is general lumpsum transfers, $tax_{a,t}^{ls}$ is lumpsum taxes, $\Upsilon_{\mathrm{a,t}}^{LIQ}$ is lumpsum transfers targeted to LIQ households only, and $c_{a,t}^{LIQ}$ is consumption of LIQ households, all at time *t* for households born at time a.

Aggregated consumption of LIQ households is:

$$(1+\tau_t^c)p_t^c\check{c}_t^{LIQ} = \lambda_t^c\big(\Upsilon_{a,t} - \tau_{a,t}^{ls}\big) + \Upsilon_{a,t}^{LIQ} + \lambda_t^c(1-\tau_t^L)\breve{w}_t\check{l}_t.$$

Aggregate Household Sector

Aggregate consumption is the sum of OLG and LIQ household consumption:

$$\check{c}_t = \check{c}_t^{OLG} + \check{c}_t^{LIQ}.$$

Capital Goods Producers

Capital goods producers produce the end of period t physical private business capital stock $\bar{K}_t$ on the behalf of OLG households. They rent out capital $\bar{K}_{t-1}$ inherited from previous period to manufacturers. They are competitive price takers and are subject to a corporate income tax. Capital goods producers are owned by OLG households who receive dividends as lumpsum transfers. The accumulation of physical capital stock is given by:

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + I_t\ ,$$

where δ is the depreciation rate, and investment I_t^J are subject to adjustment costs:

$$G_t^I = \frac{c^q}{2} I_t \left(\frac{\frac{I_t}{g_t n} - I_{t-1}}{I_{t-1}} \right)^2 .$$

The nominal price level of capital is denoted by Q_t. The optimization problem of the capital goods producer is to maximize the present discounted value of dividends by choosing the level of new investment, I_t, and the level of the physical capital stock, K_t:

$$\underset{\{I_{t+s}, K_{t+s}\}_{s=0}^{\infty}}{Max} E_t \sum_{s=0}^{\infty} R_{t,s} D_{t+s}^K ,$$

where dividends are defined as:

$$D_t^K = \left((1-\tau_t^K) R_t^K + \tau_t^K \delta Q_t\right) K_{t-1} - P_t^i I_t + Q_t \left((1-\delta) K_{t-1} + I_t - K_t\right).$$

where R_t^K is the return on capital, P_t^i is the price of investment goods, and τ_t^K is the corporate income tax rate.

The first order conditions after scaling and aggregation can be written as

$$q_t = p_t^i + p_t^i c^q \left(\frac{\check{I}_t}{\check{I}_{t-1}} - 1 \right) \left(\frac{\check{I}_t}{\check{I}_{t-1}} \right) - E_t \frac{\theta g_{t+1} n}{r_t^{corp}} p_{t+1}^i \left(\frac{\check{I}_{t+1}}{\check{I}_t} - 1 \right) \left(\frac{\check{I}_{t+1}}{\check{I}_t} \right)^2 ,$$

and

$$q_t = E_t \frac{\theta}{\check{r}_t^{corp}} \left((1-\tau_{t+1}^K) r_{t+1}^K + \tau_{t+1}^K \delta q_{t+1} + q_{t+1} (1-\delta) \right).$$

Furthermore, the capital accumulation equation yields in equilibrium:

$$\check{k}_t = (1-\delta) \frac{\check{k}_{t-1}}{g_t n} + \check{I}_t .$$

Manufacturers

Manufacturing firms maximize the present discounted value of dividends by choosing capital:

$$\underset{\{K_{t+s}\}_{s=0}^{\infty}}{Max} E_t \sum_{s=0}^{\infty} \tilde{R}_{t,s} D_{t+s}^M ,$$

where dividends are:

$$D_t^M = P_t^y Y_t - W_t L_t - R_t^k K_t,$$

where W_t is the wage.

The first order condition after scaling and aggregation can be written as

$$r_t^k = \alpha \frac{\check{y}_t}{\check{k}_{t-1}/(g_t n)}.$$

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