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**Avoiding Dark Corners:  
A Robust Monetary Policy Framework for the United States**

by Ali Aichi, Kevin Clinton, Charles Freedman, Ondra Kamenik,  
Michel Juillard, Douglas Laxton, Jarkko Turunen, and Hou Wang

IMF Working Papers describe research in progress by the author(s) and are published to elicit comments and to encourage debate. The views expressed in IMF Working Papers are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

I N T E R N A T I O N A L M O N E T A R Y F U N D

**IMF Working Paper**

Research Department

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Prepared by Ali Aichi, Kevin Clinton, Charles Freedman, Ondra Kamenik, Michel Juillard,  
Douglas Laxton, Jarkko Turunen, and Hou Wang

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June 2015

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**Abstract**

The Fed has taken several steps towards strengthening its monetary framework over the past several years. Those steps have supported the Fed's efforts to stimulate the economy through forward guidance despite being constrained by having policy rates at zero. We show that an optimal control approach to monetary policy, which includes the publication of a baseline forecast and a description of the uncertainties around that outlook, combined with an improvement in the Fed's communications toolkit, could further enhance the effectiveness of Fed policy. In the current conjuncture, such a risk management approach to monetary policy would result in both a later liftoff of policy rates and a modest, but planned, overshooting of inflation.

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## I. INTRODUCTION AND BACKGROUND

This paper has a number of objectives.<sup>1</sup> One objective is to illustrate the value of an optimal control approach to help policymakers decide on a path for the policy interest rate. At the least, minimization of a loss function that reflects the preferences of policymakers should be added to approaches that use reaction functions as a way of determining this path. Second, given the importance of communication in formulating and implementing monetary policy, it makes a number of suggestions for improvements to the Federal Reserve's communications strategy, expanding on the important changes in this area that the Fed has introduced in the last few years. Many of these proposals are in line with suggestions that have been made by members of the FOMC in recent years. These include the publication of a staff forecast of output, inflation and the policy rate, with the latter based on an endogenous interest rate convention that would bring inflation back towards its target and output back towards potential over the policy horizon. Third, it emphasizes the importance of avoiding "dark corners", outcomes that could potentially lead to very unfortunate movements in inflation and/or output. In particular, it addresses ways of avoiding deflationary spirals and very weak economies in circumstances in which the central bank is limited in the use of its traditional policy tool, the policy interest rate, because of the zero lower bound.<sup>2</sup> Fourth, if the economy (and the way it is modeled) is subject to important nonlinearities, certain results that derive from linear models do not carry over, with major implications for the setting of the path for the policy interest rate. We provide an example of a nonlinear Phillips curve, but the same argument can be made with respect to other nonlinearities that became evident during the global financial crisis.<sup>3</sup>

We would emphasize that the argumentation used in the paper is illustrative and does not try to incorporate all the technical elements that can be found in the literature on the subjects that are addressed. Rather, it tries to reflect the way central banks and their staff approach the challenge of making good policy and communicating it. While its focus is on the United States and the Federal Reserve System, many of its conclusions and recommendations can also be applied to other countries.

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<sup>1</sup> This paper represents an application of our recent work on the practical aspects of implementing inflation-forecast targeting (IFT) or flexible inflation-targeting (IT) regime. For a discussion of the general case and an examples of the model code, see Clinton and others (2015) and Bulir and others (2015). Also for an application to using an FX intervention strategy in the Czech Republic, see Alich and others (2015).

<sup>2</sup> While several central banks have recently lowered policy rates to negative territory, there is a general consensus that there are limits as to how low policy rates can go.

<sup>3</sup> For a discussion of critical nonlinearities that are necessary to explain the vicious interaction between asset prices, financial conditions and the real economy during the global financial crisis, see Benes, Kumhof and Laxton (2014a, b).

The Federal Reserve’s dual mandate—the pursuit of maximum employment and stable prices has provided an adaptable and flexible framework for monetary policy. Over the years, the Fed has been able to modify its operational objectives in line with an evolving understanding of the role of monetary policy, and with the changing economic environment. The Federal Open Market Committee’s (FOMC) statement of longer-run goals and policy strategy of January 25, 2012, announced a 2 percent rate of increase of the PCE deflator as the long-term goal of monetary policy. This represented an important milestone in the articulation of the Federal Reserve’s mandate. It reflects an accumulation of experience, in the United States and many other countries, and economic research, which has demonstrated that such an objective is compatible with a dual mandate. The 2012 announcement also underlined the full employment goal, and noted that while the inflation and employment objectives are generally complementary, the FOMC would take a balanced approach in promoting them. It also reiterated the Fed’s commitment to transparency, noting that it “seeks to explain its monetary policy decisions to the public as clearly as possible.”

All this brings the Fed functionally very close to flexible inflation targeting—or, equivalently, inflation-forecast targeting (IFT)—as the basis for monetary policy, although official statements of the Federal Reserve carefully avoid using such terminology.<sup>4</sup> In this paper, we argue that flexible inflation targeting as now practiced by a number of central banks does indeed provide a useful template also for U.S. monetary policy. The main next step for modifying the existing framework at the Fed would be to publish a staff forecast, showing a future interest rate path consistent with the achievement of the FOMC’s announced longer-run goals. This staff forecast could be endorsed as the majority view of the FOMC through the publication of a regular Monetary Policy Report. The data on the individual forecasts of FOMC members currently released allow for varied interpretations—it is not straightforward to connect the dots to get a coherent vision of the path ahead—and do not provide a clear picture of the FOMC’s majority view. Our argument is not just transparency for its own sake, although there would be nothing wrong with that. Rather, such transparency would raise the effectiveness of monetary policy instruments, especially in unusual circumstances, such as when the zero lower bound (ZLB) constraint on interest rates is binding. In particular, the central bank’s capacity to influence inflation and employment relies, in large part, on the effect it can have on public expectations for the future path of short-term interest rates and inflation. Publication of a complete economic forecast is a good way to strengthen that influence over expectations and support the Fed’s own policy objectives.

A second suggestion is to formulate policy in a way to avoid dark corners. We present results to compare regular linear policy rules for setting the federal funds rate with an alternative

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<sup>4</sup> Svensson (1997) originated the IFT terminology, pointing out that, in principle, the central bank’s inflation forecast made an ideal intermediate target in that it embodied all the relevant information available to the policymakers, including expectations of their own future actions.

approach in which policymakers adjust policy rates to minimize a loss function. In each case, the central bank acts to close, over time, deviations of inflation from the long-run target rate, and gaps between actual and potential output. That is, the bank pursues a dual mandate, or flexible inflation targeting—the two are indistinguishable. The model has a forward-looking component in the expectations of the public and of policymakers. The results, under conditions of weak demand, unduly low inflation, and an interest rate near the lower bound, show a more active response associated with the loss-minimizing strategy rather than following a policy rule. The minimization of the loss function also results in an overshoot of the inflation target<sup>5</sup> and would imply holding the federal funds rate at zero until the inflation and output gaps are almost closed. In effect, the quadratic loss function, by penalizing large deviations from objectives heavily, favors policy actions that will get the economy quickly away from a situation of deflation at the zero lower bound (ZLB), where the main conventional policy instrument loses effectiveness. In the current situation—with near-zero interest rates and low inflation—the loss function approach embodies a form of risk management that has a strong aversion to such dark corners.<sup>6</sup>

Since the loss function approach explicitly weights preferences on the short-run output and inflation trade-off, it can generate alternative scenarios reflecting differences in preferences among the FOMC members. Such scenarios would help the policymakers select their preferred path for the federal funds rate from the array of alternatives consistent with meeting their longer-term objectives. We conclude that this approach is better than linear policy rules and becomes particularly valuable during abnormal times.

The behavior of public expectations is critical to the effectiveness of the loss-minimizing policy. Its impact on aggregate spending, relative to the policy rules, derives from a drop in the real interest rate over an extended horizon. Long-term real interest rates fall as a result of the longer period at the ZLB and a temporary increase in the expected rate of inflation. Given this importance of expectations, and the complexity of describing a quadratic loss function to the public, effective communication of central bank forecasts becomes a key instrument of policy. The publication of a baseline staff forecast, and alternative scenarios based on different policy reaction functions, as well as on different assumptions about the outlook, would be helpful in communicating to the public the rationale for interest rate settings and the majority view of the FOMC, with room to also explain dissenting views of FOMC members.

Section II of this paper briefly reviews developments in the framework of Federal Reserve policy since the global financial crisis. Section III reports experiments with an inflation-forecast based policy rule, as well as a loss-minimizing approach, using a quadratic loss function. Risk avoidance would envisage a modest, planned overshooting of inflation above

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<sup>5</sup> English, López-Salido and Tetlow (2013) has a similar finding.

<sup>6</sup> Blanchard (2014) argues that macroeconomic policy should make a high priority of avoiding dark corners.

the Fed’s longer-term target rate of two percent.<sup>7</sup> Ordinary linear policy rules, e.g., the Taylor rule, do not have this pronounced risk-avoidance property. Section IV argues for publication of a full Federal Reserve staff forecast, including the forecast for the future path of the short-term interest rate, at the same time as policy decisions are announced. This would provide a systematic and specific basis for forward guidance, replacing the current Summary of Economic projections, including FOMC member’s interest rate projections (“dots”). Section V suggests that the published forecast should, therefore, be that of the economic staff, rather than the official forecast of the FOMC. However, the majority of the FOMC could endorse the published forecast and dissenting and opposing members of the committee could have their own forecasts, shown as alternative scenarios in the relevant FOMC publications.<sup>8</sup> Section VI provides concluding remarks. Appendix 1 lists definitions used in the model. Appendix 2 sets out the complete model. Appendix 3 provides a technical description of the loss-function minimization algorithm.

## II. RECENT DEVELOPMENTS AT THE FED

Since 2011 the FOMC has taken several steps to further shed light on its policy actions and strategy, of which the 2012 announcement of the two percent inflation objective was the most striking (see Table 1).<sup>9</sup> The Summary of Economic Projections (SEP), released four times per year shortly after an FOMC meeting, was expanded to provide information about members’ projections for the future path of the federal funds rate, as well GDP growth, the unemployment rate, and inflation. For each variable, the SEP gives range of the individual forecasts. The range for the federal funds rate is presented in more detail, in the form of dots on a chart for each member’s forecast—the so called “dot plot,” which has attracted a lot of attention from Fed watchers in the financial markets. The SEP has been an important element in FOMC communications.<sup>10</sup>

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<sup>7</sup> Dudley (2012) and Evans and others (2015) also use the term “risk management”

<sup>8</sup> A more comprehensive overview of experiences from several countries is discussed in Laxton and others (forthcoming).

<sup>9</sup> English, López-Salido and Tetlow (2013) provide a more detailed description and analysis of these steps. Yellen (2012) discusses non-conventional policy instruments introduced by the Fed since the global financial crisis of 2008. Yellen (2014) outlines developments in the FOMC communications framework.

<sup>10</sup> For example, Yellen (2015) cites the document several times in her discussion of monetary policy “normalization.”

**Table 1. Recent Developments at the Fed**

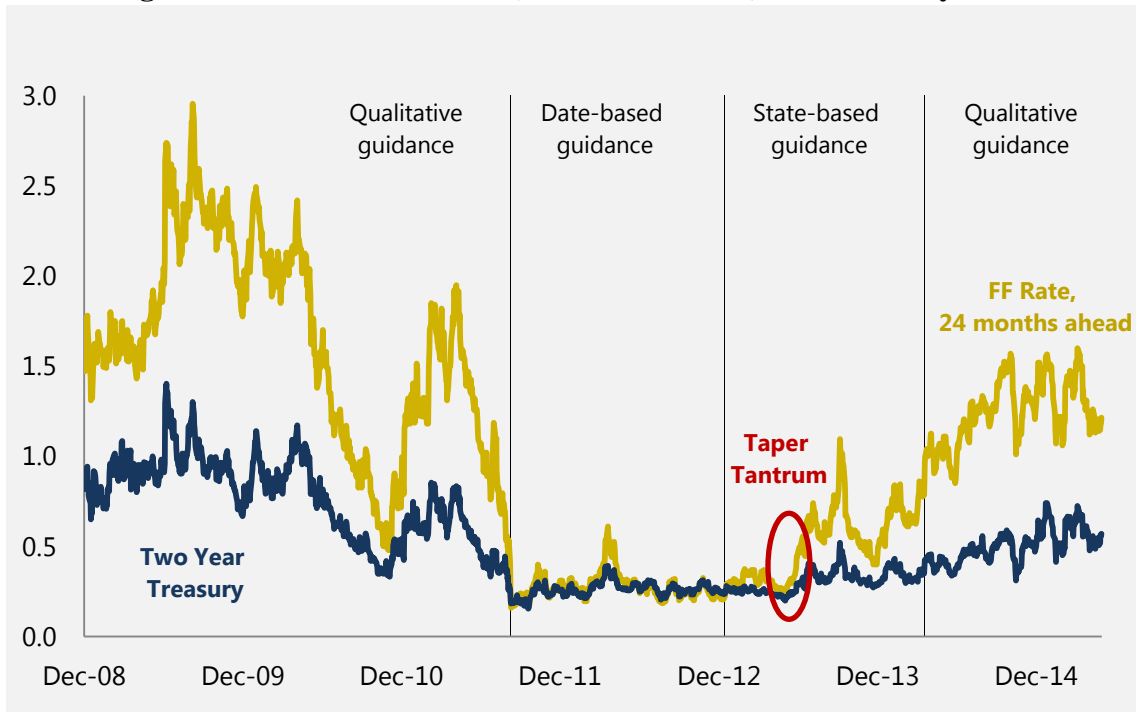
Date	Action	Description
December 2008 to present	Forward guidance	Qualitative (Dec 2008-Aug 2011), date-based (Aug 2011-Dec 2012), threshold-based (Dec 2012-March 2014), qualitative (March 2014-March 2015)
November 2008 to present	Balance sheet guidance	Volume of purchases, pace of purchases, assets purchased, criteria for revising asset purchases, reinvestment and shrinking of the balance sheet
April 2011	Post-meeting press conference	More comprehensive and timely information on the FOMC policy decision and views, including Summary of Economic Projections
January 2012	Statement on longer-run goals and policy strategy	Clarify the Federal Reserve's objectives and policy strategy, including the introduction of a long-run two percent inflation goal.
January 2012	Policy rate projections	Individual FOMC members' policy rate projections were added to the quarterly Selected Economic Projections published following FOMC meetings

In line with these changes, there has been considerable discussion at the Federal Reserve System on how the ability to affect expectations and communications can strengthen the effectiveness of monetary policy. In normal times, an established pattern of past policy behavior that has proven successful in achieving the central bank's stated policy objectives contributes to credibility and predictability. However, in unusual circumstances, when policymakers have to break from past behavior, forward guidance may require more explicit explanations to be effective (Woodford, 2012). Thus, the profile of communications as a policy tool was raised in the post-global crisis recession, when the interest rate instrument reached its lower bound, near zero. Unconventional instruments—quantitative easing and forward guidance on interest rates—were introduced. *“Beyond the task of describing the new policies, extensive new communication was needed to justify these unconventional policy actions and convincingly connect them to the Federal Reserve's employment and inflation objectives.”* (Yellen, 2013). Thus, the Fed used forward guidance to assure markets that the short-term interest rate would remain low for a while, in order to reduce longer-term rates, underline its commitment to bringing inflation back to 2 percent, and as a result provide

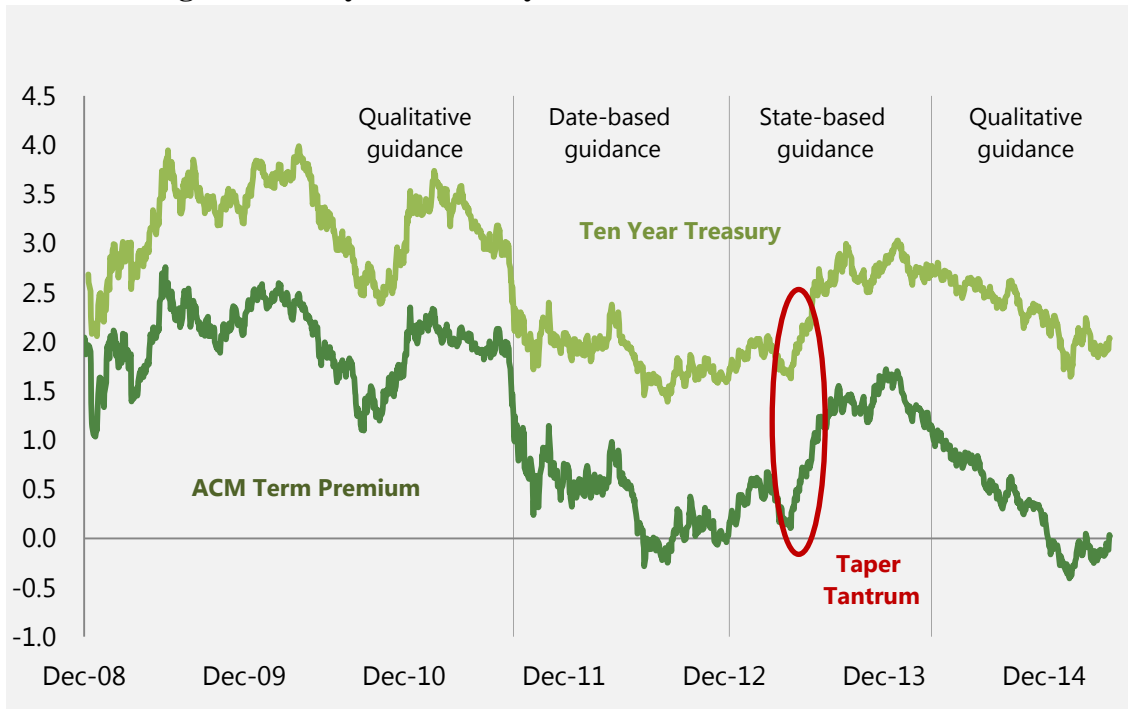


stimulus to the economy.<sup>11</sup> Large-scale asset purchases had the same purpose. The importance of careful communication was underlined by the “taper tantrum” of May 2013, when concerns in the market about a tapering of purchases—following Chair Bernanke’s testimony to Congress’s Joint Economic Committee—caused jumps in the expected future federal funds rate (Figure 1), and in longer-term rates and risk premiums (Figure 2), that were substantially larger than those implied by the modest increases envisaged by FOMC policy rate forecasts.

**Figure 1. Fed Funds Futures (24 months ahead) and Treasury Rates**



<sup>11</sup> Bernanke (2013) provides a more precise description of the evolution of forward guidance at the Fed.

**Figure 2. Ten-year Treasury Yield and ACM Term Premium**

Note: ACM = Adrian, Crump and Moench (2013).

The tantrum took place as the Fed began to consider normalization of policy stance, with the economic recovery at last gathering steam. At the time of writing, the main communications challenge for the FOMC in the months ahead has become how to communicate about progress towards eventual gradual tightening—a gradual upward path in the fed funds rate and a scaling down of the Fed’s large balance sheet—without triggering another market overreaction.

To achieve its objectives, forward guidance should be credible and predictable. That is, the announced path for the policy instrument should be consistent with policy objectives and the economic outlook. This implies that the policy interest rate in the forecast must be endogenously determined along with other key macroeconomic variables. To establish the expectation that over time the inflation rate will converge to the official target, which is the nominal anchor for the system, the policy rate must be free to vary, in a predictable way, to eliminate over time the inflation effect of any shock. In other words, the basis for any subsequent modifications to the policy rate path, in response to changes in the outlook, should be well understood (IMF, 2014).

Moving towards a more transparent policy framework—publishing the inflation objective, providing tentative estimates of the equilibrium (or natural) rate of unemployment, and forecasts of the policy rate path—has supported the Fed’s efforts to stimulate the economy. At the same time, enhancements to the communications toolkit could bring a further gain in effectiveness. Plosser (2014) suggests that the Fed publish a quarterly monetary policy report

to better communicate how the Fed has adjusted, and will adjust, its instruments to achieve its employment and inflation objectives. He would include alternative projections for the short-term interest rate paths deriving from “a few Taylor-like rules,” as well as the FOMC’s central forecast. Such alternative interest rate paths, each consistent with reaching the policy objectives, would provide a menu of options that policymakers could consider before reaching decisions. They might at the same time shed light on the differing risks associated with the various alternatives. In this respect they would be an aid to policy formulation. The alternative policy scenarios could also be used externally, to explain the rationale for, and the risk management aspects of, policy strategy. Mester (2015) discusses the possibility of releasing a complete forecast for each FOMC member. This would, in each case, provide a coherent forecast connecting the projections for economic variables with the expected policy interest rate path. Members might also release individual confidence bands around each variable, to indicate their degree of uncertainty. While the central tendencies in the ranges and “dot plots” of existing SEP do reveal a plausible set of forecast outcomes, they do not constitute a coherent forecast of the kind that would emerge from an explicit model, and they do not accurately convey any notion of the median view of the voting members of the FOMC or the degree of uncertainty that members may feel around their forecasts.

In light of the extensive international experience, over more than two decades, with monetary policy reports at inflation-targeting central banks, we consider that such proposals for increased transparency have a merit.<sup>12</sup> Such reports can help to prepare public expectations for the likely response of monetary policy to evolving economic conditions, and hence increase the effectiveness of policy. In addition, they play an important role in the process of democratic accountability of the central bank. That is, the policymakers provide a justification—which may be challenged—for their past and current actions, in terms of how policy decisions are aiming to meet the announced objectives of monetary policy.

From a technical viewpoint, the inflation forecast in these reports may serve as a keystone for the implementation of a policy of flexible inflation targeting. This is because the central bank’s forecast path for the inflation rate represents, in principle, an ideal intermediate target. It includes all relevant information, including the expected policy actions of the central bank itself. If a disturbance has pushed the actual inflation rate off target, the forecast takes that into account. Targeting the inflation forecast, therefore, means, in general, that monetary policy does not try to get inflation back on track to the announced target as quickly as possible, or within a predetermined number of quarters. The official target, for the purposes of the conduct of policy, is a long-run target. It is up to the central bank to demonstrate that good monetary policy will have an operating target for the short and medium term that

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<sup>12</sup> The empirical evidence suggests long-term inflation expectations are better anchored in IT countries than in non-IT countries. See Levin, Natulucci and Piger (2004), Goretti and Laxton (2005), Gurkaynak and others (2006), and Gurkaynak, Levin and Swanson (2010).

depends on the shocks that have been disturbing the economy, and that may (and generally will) differ from the official target. Flexible inflation targeting, or any good monetary policy, will take account of the short-run impact of interest rate changes on output, as well as on inflation—whether or not there is an explicit dual mandate. Publishing all the key macroeconomic variables from a complete, consistent, forecast, would be an important tool for communicating how the central bank is trading off its short-run output and inflation objectives.<sup>13</sup>

### III. LOSS MINIMIZING MONETARY POLICY BASED ON A SIMPLE MODEL

#### III.1 Background

The Fed Board staff has considerable experience with optimal control approaches to monetary policy (e.g. English, López-Salido and Tetlow, 2013, and Brayton, Laubach and Reifschneider, 2014).<sup>14</sup> We provide an example of this approach applied to a simple model of the U.S. economy that captures the trade-off between inflation and output. We set up the model in a stylized version of the current economic situation. We examine three policy reaction functions, in the same model and context, and compare outcomes. In each case the model derives a projected path for the federal funds rate that will achieve the official target over the medium term, while taking account of the welfare costs of output gaps.<sup>15</sup>

The policy decision frameworks that we consider are:

- a simple Taylor rule;
- an inflation-forecast-based (IFB) reaction function; and
- minimization of a loss function that reflects the preferences of the policymakers.

An interesting result is that the loss minimization approach could, in a situation near the zero lower bound for the interest rate, with a negative output gap, and undesired disinflationary pressure, result in an ex ante planned overshoot of the inflation target. Under the circumstances, this corresponds to a risk management strategy—policymakers would place a high priority on getting away from the dark corner where the deflation spiral lurks. To do

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<sup>13</sup> IMF (2014) has also recommended several enhancements to the communications toolkit, including publishing a quarterly monetary policy report, which is endorsed by the FOMC and which conveys more detail about the majority view of the FOMC on the outlook, policies, and the nature of uncertainties around the baseline. See also Levin (2014).

<sup>14</sup> There is a large technical literature that focuses on robust control of monetary policy, in particular, the analysis and determination of optimal policy under model uncertainty. See Cogley and Sargent (2005), Svensson and Williams (2005), Brock, Durlauf and West (2003), Durlauf and West (2007), Brock and Durlauf (2005). Our paper is meant to be a very practical illustration of how models can be used to improve the policy dialogue, and how different models can provide different policy insights when combined with simple policy rules or optimal-control techniques.

<sup>15</sup> The main equations of the model are presented in Appendices 1 and 2. The model is based on a stripped-down version of the Fund’s Global Projection Model (GPM). See Clinton and others (2015), which provides references for further detail on the GPM family of models.

that, they would raise expectations about future inflation in order to lower the real interest rate and stimulate consumption and investment demand. It is important to note that we do not mean for the model simulations to be taken literally. In practice, any forecast and associated policy analysis by central bank staff is a much more complex undertaking than a simple model projection. The highly-trained economists and sectoral specialists at the Federal Reserve would continue to combine model insights with expert judgment in their policy advice. Rather, the model approach would represent a structure and framework to both consider trade-offs in an internally consistent way and as a means of communicating views.

### III.2 Outline of the model

The simple closed-economy model that we use is based on the standard closed-economy, inflation targeting model, with equations for output (IS curve) and an inflation formation equation (based on an expectations-augmented Phillips curve). The model is forward looking, in that expectations, and policy reactions, are driven in part by the model's own future solved values (in the long run, both expectations and outcomes converge to steady state paths). The output gap equation contains a bank-lending-tightening variable, which captures exogenous changes in credit conditions. Demand shocks are represented by the stochastic term in the output gap equation, and supply shocks by that in the inflation equation.<sup>16</sup>

For present purposes, the only aspect that needs discussion in any depth is the form of the policy decision for the federal funds rate. We experiment with 3 alternatives. Each has in common that the *real* interest rate rises and falls with the inflation rate and the output gap (excess demand).

**Taylor rule.** The variant used in this paper, as shown below, is similar to the rule proposed in Taylor (1993):

$$i_t = \bar{r}_t + \pi 4_t + 0.5 * (\pi 4_t - \pi^*) + 0.5 * y_t + \varepsilon_t^i,$$

where  $i_t$  is the federal funds rate,  $\bar{r}_t$  is the equilibrium real interest rate,  $\pi 4_t$  is average annualized inflation over the past four quarters,  $\pi^*$  is the inflation objective, assumed to be 2 percent, and  $\varepsilon_t^i$  is a policy deviation shock. The nominal federal funds rate is a function of: the equilibrium real interest rate; the inflation rate; the deviation of inflation from target with a coefficient of 0.5; and the output gap, also with a coefficient of 0.5. The exogenous increase over time in the equilibrium real interest rate is based on the assumption that the low level of real interest rates during the crisis were caused partly by contractionary forces and heightened uncertainties in the economy, which will gradually dissipate over time. The

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<sup>16</sup> The detailed model is presented in Appendix II.

Taylor rule results in an increase in the real interest rate following an increase in the rate of inflation – the appropriate monetary policy response.

**Inflation-forecast-based reaction function.** The IFB reaction function used in this study focuses on the forecast for year-on-year PCE inflation three quarters in the future.

$$i_t = 0.71 * i_{t-1} + (1 - 0.71) * [\bar{r}_t + \pi 4_{t+3} + 0.91 * (\pi 4_{t+3} - \pi^*) + 0.21 y_t] + \varepsilon_t^i.$$

Here the nominal federal funds rate is a function of its lagged value (a way of smoothing the reactions to changes in inflation and output), the equilibrium nominal interest rate (as measured by the sum of the equilibrium real interest rate and the projected year-on-year core PCE rate of inflation), the forecast deviation of projected inflation from its target value, and the output gap. This formulation also has the appropriate response over time of the real policy interest rate to off-target forecasts of inflation. Interest rate smoothing is a well known feature of monetary policy, and can be justified on a number of grounds. From the modeling perspective, the key reason for smoothing would be that it increases the predictability of the rate, as the public can see that a rate change is likely to have some persistence (in contrast, a given movement in a highly variable rate would likely be ignored, as noise).<sup>17</sup> In contrast to the Taylor rule, the IFB formulation ignores inflation shocks that are expected to reverse within the three-quarter policy horizon. It also allows the central bank to take account of known developments that might affect inflation over this horizon, including lagged effects of policy itself that are still in the pipeline.

**Minimizing a loss function.** The loss function incorporates the principal objectives of the central bank in policy making.<sup>18</sup>

$$L_t = \sum_{t=1}^{\infty} [\alpha * (\pi 4_t - \pi^*)^2 + \beta * y_t^2 + \gamma * (i_t - i_{t-1})^2]$$

The quadratic formulation implies that large deviations from desired levels weigh disproportionately more in the objective function than small deviations—i.e., that there is a rising marginal cost of inflation deviations from the target, output gaps, and interest rate volatility. Given that policy actions are subject to imprecision and uncertainty, this is reasonable since policymakers should focus principally on avoiding larger errors instead of trying to fine tune the economy. In particular, policymakers would want to keep their economies well away from dark corners where recovery from shocks becomes much more

<sup>17</sup> Woodford (2003) presents a theoretical argument that some policy rate smoothing is optimal for clear signals to the market about the intent of policy.

<sup>18</sup> For example, the January 2012 statement from the Federal Reserve Board refers to the “statutory mandate from the Congress of promoting maximum employment, stable prices, and moderate long-term interest rates.” “Promoting maximum employment” would mean keeping *negative* output gaps small. In terms of the model, this would be achieved by minimizing cumulative gaps symmetrically, in both directions.

difficult, because of nonlinearities such as the ZLB (Blanchard, 2014). In other words, the central bank would very much like to avoid large recessions, or destabilized inflation expectations (and especially, under current circumstances, potential deflationary expectations). A contractionary shock combined with the ZLB is likely to be the main concern at the moment to which policymakers should attach a high weight.<sup>19</sup>

The squared change of the federal funds rate in the equation represents aversion to interest rate volatility. This term smooths the policy response of the federal funds rate, reflecting the behavior of central banks. By taking account of both current and expected future values of output and inflation, this formulation incorporates currently available information about likely future developments into the policy response.

In the baseline calibration, we put equal weight (1.0) on inflation and output gaps, reflecting a dual mandate central bank or, equivalently, an IFT central bank. The coefficient for the change in the nominal interest rate is set to 0.5. It acts to smooth the interest rate path and to prevent the type of abrupt interest rate movements that are rarely seen in reality. In the sensitivity analysis, we consider variants of the baseline calibration where the weights on inflation and output differ from the baseline case.

It is worth briefly summarizing the transmission mechanism in the model, from central bank policy rate actions to output and inflation. An exogenous increase in the policy interest rate leads to an increase in the short-term real interest rate, which feeds into the long-term real rate. The latter, with lags, leads to a decrease in aggregate demand and output via the output gap equation. The more negative output gap (i.e., increased excess capacity) gradually puts downward pressure on inflation. Exogenous shocks in the aggregate demand and inflation equations also have direct and lagged effects on both output and inflation.

The feedback loops are also worth underlining. Any shock of material size and duration, in any equation, reverberates through the whole system, and brings into play the IFT policy response, which eventually, through the transmission mechanism just described, will stabilize inflation at the long-run target rate, and output at its long-run equilibrium, or potential level, as befits a flexible IT central bank with a dual mandate.

### **III.3 Illustrative simulation results**

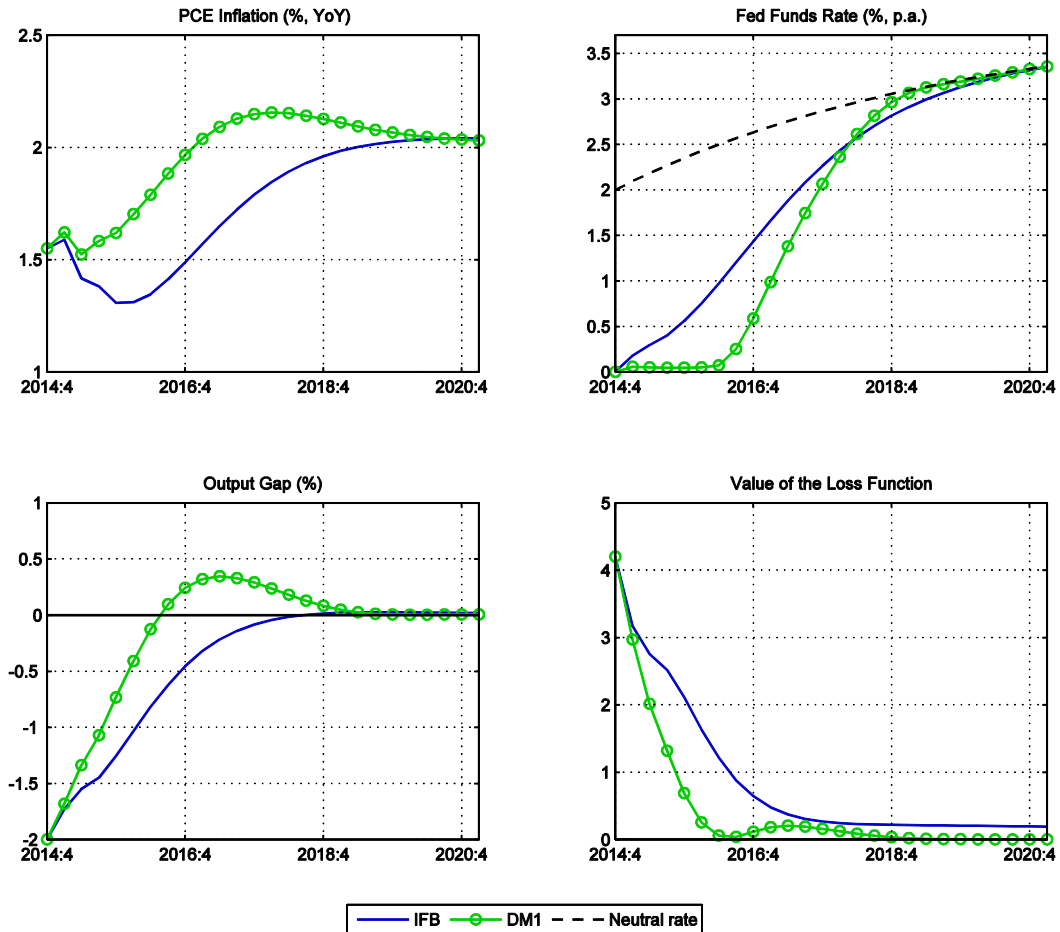
The illustrative simulations of possible policy responses to the current US economic situation are based on assumed initial conditions in the US economy as of 2014Q4, with an output gap of -2 percent (i.e., excess capacity), a rate of PCE inflation of about 1.5 percent, and a federal

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<sup>19</sup> A lesson from endogenous credibility models is that an episode of excessive inflation can result in a costly loss of the nominal anchor (e.g., Argov and others, 2007).

funds rate of about zero. In Figure 3, we compare the policy implications of the IFB reaction function (the blue line) and the loss function DM1 (green circles).

**Figure 3. Illustrative Example of the Current Situation: IFB Reaction Function versus a Dual Mandate CB Loss Function**



Source: IMF staff calculations

The simulations incorporate an estimate of the time-varying *real* equilibrium funds rate (the so-called neutral rate, reported as the dashed line in Figure 3). While there is considerable uncertainty about estimates of the neutral rate, which is unobservable, estimates in Pescatori and Turunen (2015) suggest that it is currently likely to be close to zero and to increase only gradually over time (see also Dudley, 2012, and Yellen, 2015). Consistent with these results, in the simulations we assume that the neutral rate rises from 0 in 2014Q4 to about 1.3 percent in 2020Q4 (close to, but below the median of the FOMC members' forecast of the long term real fed funds rate (about 1.75 percent). We posit the rising trend to depict the return of the equilibrium rate to a more normal value after the damaging effects on investment and



confidence of the Great Recession.<sup>20</sup> Equivalently, with a two percent inflation target, the equilibrium nominal rate rises from 2 percent in 2014Q4 to 3.3 percent in 2020Q4.

The federal funds rate gradually rises starting almost immediately under the IFB reaction function, whereas under the DM1 loss function it remains at the zero lower bound until mid-2016 before beginning to rise. The much earlier increase under the IFB reaction function is related to the anticipated increase in the equilibrium real interest rate in the IFB function specification. The IFB policy tightening means a slower closing of the output gap, and inflation below target for a longer period of time, with the output gap and the rate of inflation moving gradually to the long-run equilibrium. In contrast, the output gap and the inflation rate overshoot somewhat under the DM1 policy, with the output gap moving to a positive 0.3 percent (i.e., modest excess demand) at the maximum and the rate of inflation to 2.2 percent at the maximum. That is, equilibrium is reached via a modest cycle which is optimal given the symmetric weight attached to inflation or the output gap being above or below the long-term goals.

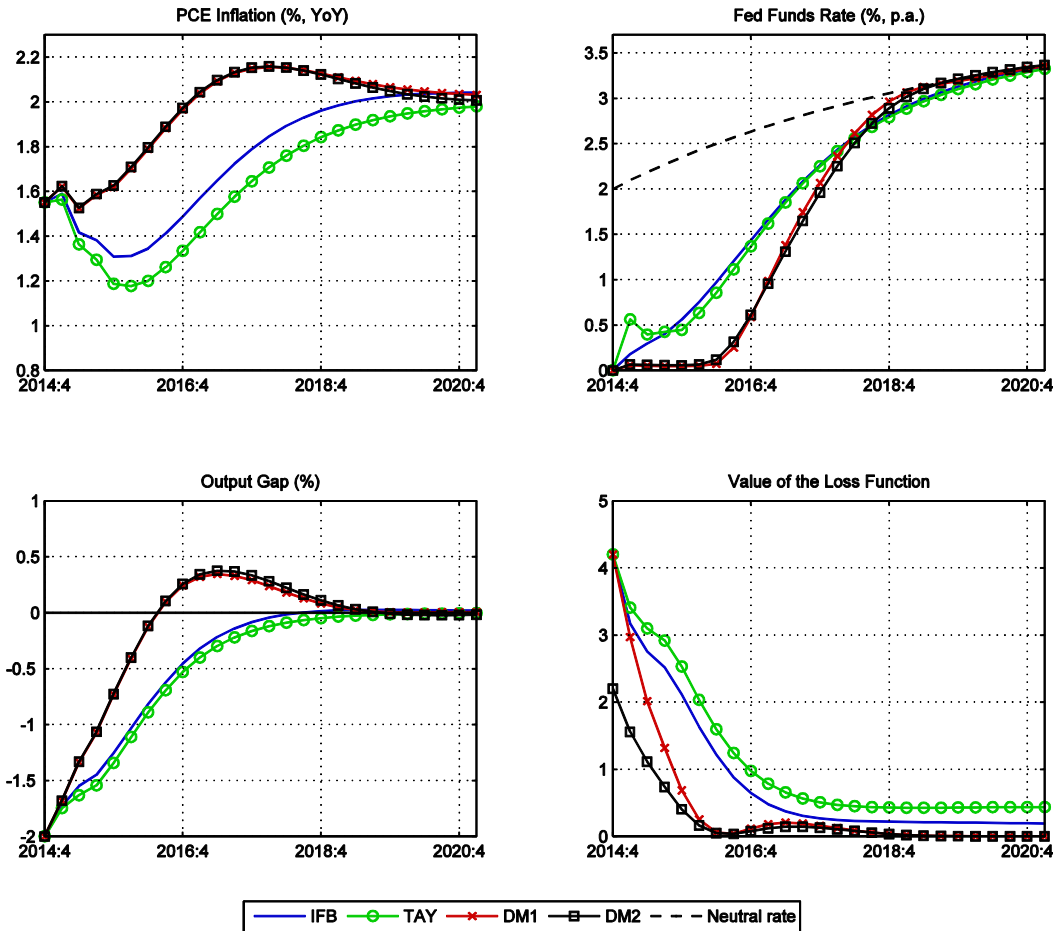
### **III.4 Sensitivity analysis: variants of the model**

The loss function in the baseline analysis puts equal weight (1.0) on inflation and output gaps. To examine the robustness of these results, we consider a second loss function calibration (DM2) which has half the weight on the output gap as that on the inflation gap. Figures 4 and 5 present all 4 policy approaches (Taylor, IFB, DM1, and DM2). Here, DM1 is represented by a red line with crosses, and DM2 by a black line with squares.

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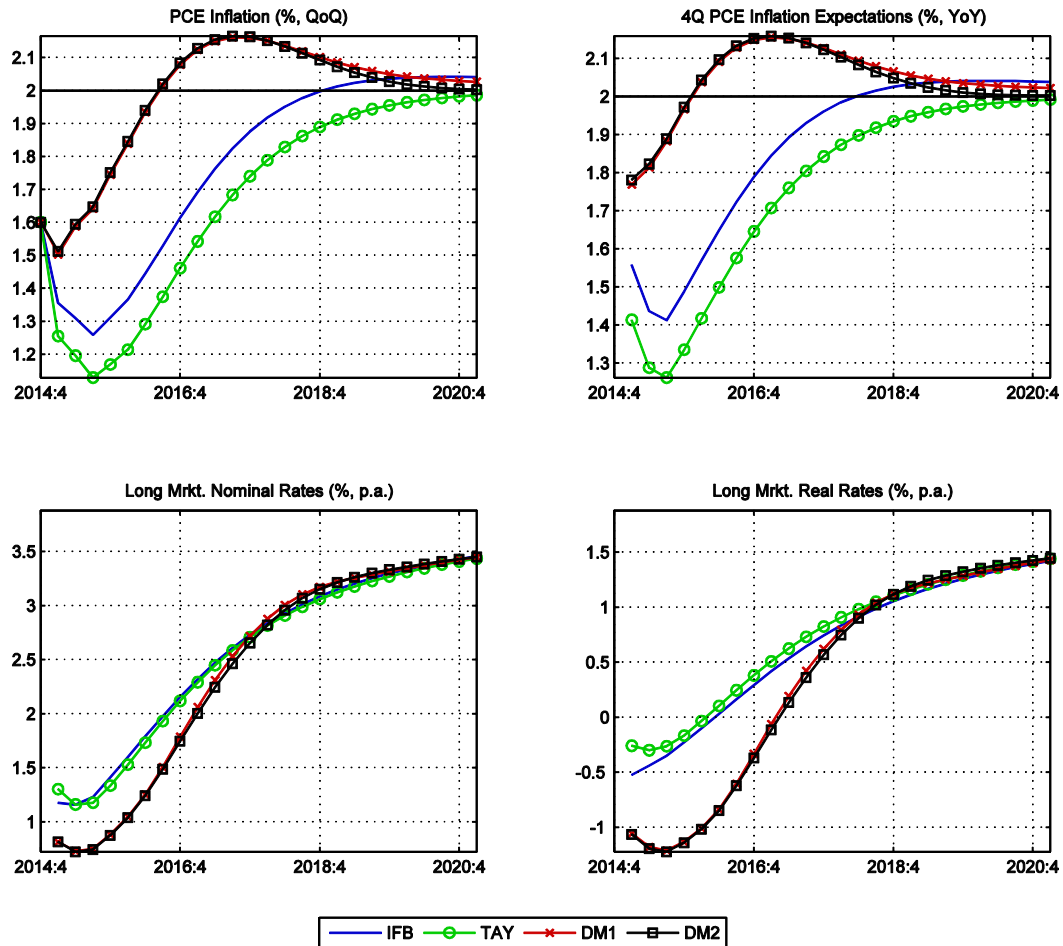
<sup>20</sup> Taylor (1993) assumes a constant 2 percent equilibrium real rate.

Figure 4. Comparison of 4 Reaction Functions for the Current Situation (Part 1)



Source: IMF staff calculations

Figure 5. Comparison of 4 Reaction Functions for the Current Situation (Part 2)



Source: IMF staff calculations

The two DM lines sit almost on top of each other—the halving of the weight on the output gap in DM2 makes hardly any economic difference. The two reaction functions (blue line for the IFB reaction function, green line with circles for the Taylor rule) show very similar results to each other. Thus the major difference is seen in the pair of loss-function minimizations versus the pair of policy rules. To repeat, the much lower market real rate for the loss-function minimization approach has a significant effect on the movements of output and inflation. The higher medium-term inflation in the loss-function cases, with forward-looking expectations, means that the difference is even larger for real long-term rates than for nominal short-term rates—compare the bottom pair of panels of Figure 5. The higher inflation expectations largely result from the market’s understanding of the policy frameworks under the loss-function arrangements and how they lead to lower real interest rates and a stronger economy over time. This, of course, drives stronger growth (as shown in the output gap) and a higher inflation rate, as compared to the reaction functions, since it is

the long-term real interest rate that enters into the aggregate demand function. The planned and predicted overshooting of inflation and output under DM1 and DM2 is the deliberate result of the very stimulative policy stance. From the viewpoint of the loss functions, the starting point for the economy (with output well below potential and inflation below the target rate) is very inefficient. The central bank takes a minor upside risk to insure against a much more costly downside risk, given the rising marginal cost of the negative output and inflation gaps, and given the ZLB, which deprives policy of a stimulative tool in the event of a negative demand shock.

But here we would emphasize the word *planned*. Given the certainty of subsequent random shocks, it is not at all certain that the overshoot would occur. More important, if the forecast path were to aim at an asymptotic approach to the desired equilibrium, monetary policy actions would have to be less stimulative (as in the IFB outcome), and both output and inflation would be below their desired levels for a much longer period of time. Furthermore, as will be seen later in the paper, the bands of uncertainty are much wider than the size of the predicted overshoot in Figure 6. In statistical terms, the overshoot is not significant. Communicating the inflation overshoot in the context of a flexible inflation targeting approach will make it clear to the public that this is what the central bank is aiming to do, thus making the policy more effective.

To illustrate upside risks to inflation, we consider overheating scenarios that incorporate a nonlinear Phillips curve, which causes inflation to increase sharply as slack diminishes, and expansionary shocks that push inflation higher. Figures 6 and 7 present 2 policy approaches (IFB and DM1) assuming non-linear Phillips curve and large expansionary shocks that result in appreciable inflation overshoots. The large expansionary shocks are financial condition shocks that ease credit conditions and stimulate aggregate demand. These shocks hit the economy in 2017q1 and 2017q2. The assumptions for the initial conditions and the real interest rates are the same as in the baseline simulation.

In the baseline analysis we assume a linear Phillips curve as follows.

$$\pi_t = \lambda_1 * \pi_{t+4} + (1 - \lambda_1) * \pi_{t-1} + \lambda_2 * y_{t-1} + \varepsilon_t^\pi$$

$$\lambda_1 = 0.70; \lambda_2 = 0.10$$

The non-linear Phillips curve is as follows.<sup>21</sup>

$$\pi_t = \lambda_1 * \pi_{t+4} + (1 - \lambda_1) * \pi_{t-1} + \lambda_2 * \frac{5y_{t-1}}{5 - y_{t-1}} + \varepsilon_t^\pi$$

$$\lambda_1 = 0.70; \lambda_2 = 0.10$$

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<sup>21</sup> See Laxton, Meredith and Rose (1995) and Laxton, Rose and Tambakis (1999) for discussions on the nonlinear Phillips curve.

This non-linear Phillips curve has the property that inflation responds asymmetrically to negative and positive output gaps. Compared to a linear Phillips curve, the non-linear Phillips curves implies that inflationary pressure is less when there is slack in the economy. Therefore, it is able to generate sluggish downward inflation movements like in the global financial crisis. At the same time, it implies that inflation can rise very quickly when there is excess demand in the economy such as in the 1970s. As a result, allowing excess demand conditions may necessitate significant subsequent tightening to contain inflation and inflation expectations.

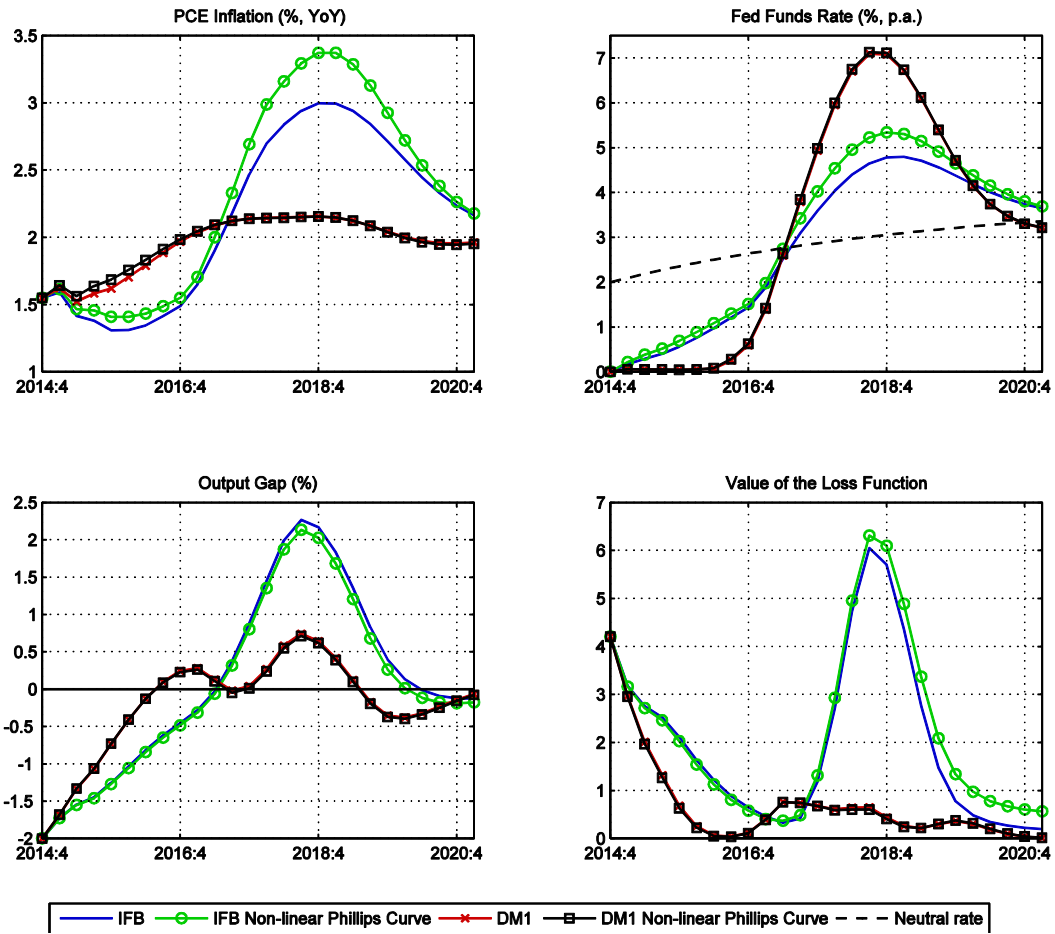
Our calibration of the non-linear Phillips curve implies that inflation cannot go below -5 percent. It also implies that as output gap approaches +5 percent, inflation can go to infinity.

The results suggest the following. First, one of the risks associated with overshooting is that inflation can go very high if there are additional expansionary shocks hitting the economy when inflation overshoots. However, the central bank can always raise the policy rate to combat inflationary shocks and there is no upper limit to how much the policy rate can increase. Under the loss-function approach, the central bank raises interest rate much more aggressively in response to expansionary shocks than under the IFB approach, and therefore, inflation is lower and output gap is smaller. The loss-function approach is good at dealing with the inefficiency associated with both output and inflation gaps being positive.

Second, under IFB, the model with a linear Phillips curve and the one with a non-linear Phillips curve provide different policy reactions and have distinct inflation implications, while under the loss-function approach, the different specifications in the Phillips curve makes hardly any economic difference. The reason is that the loss-function approach “depends on everything”. It sees the inflation implications of the non-linear Phillips curve, and is prepared to adjust interest rates much more aggressively to keep the economy away from those “dark corners” where inflation becomes unanchored. As a result, the economy stays in the region where the non-linearity is much less severe and there is hardly any difference between the two models with respect to policy reactions and inflation implications.

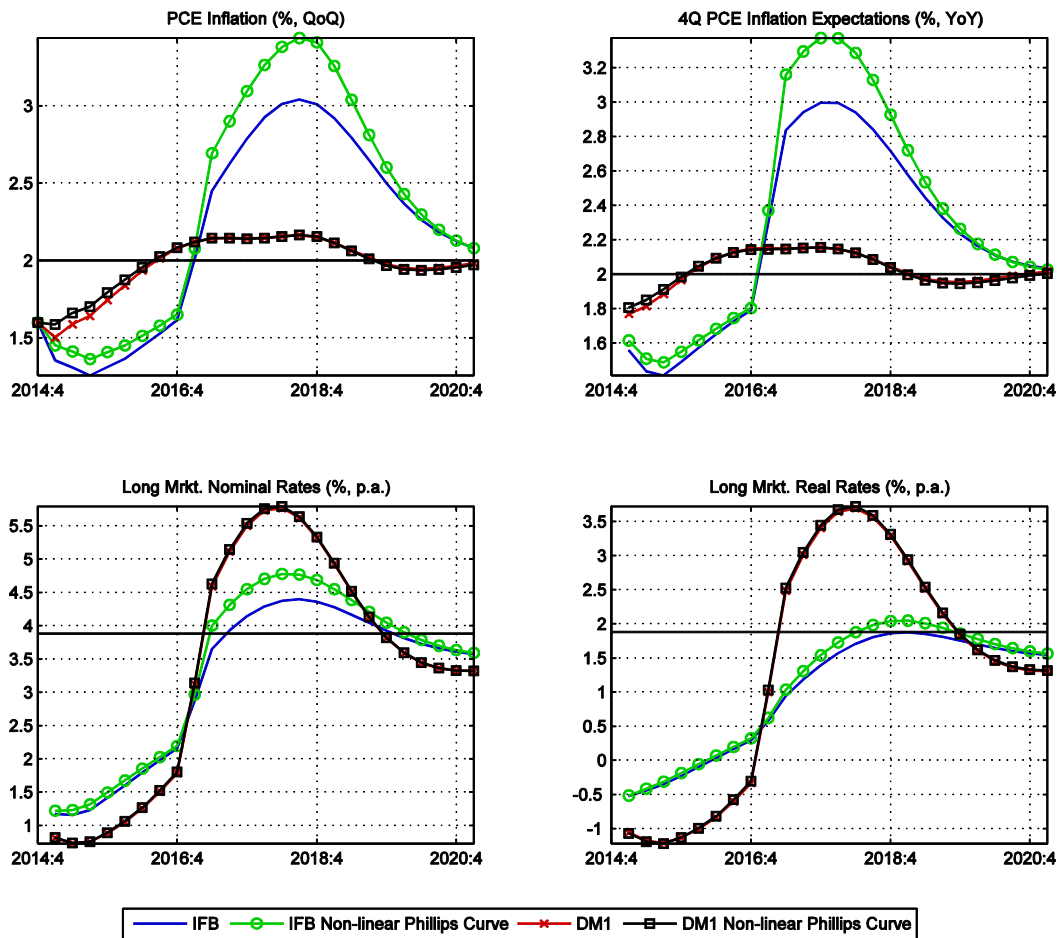
Third, under the IFB approach, because policy responses are much less aggressive, both output and inflation increase significantly after the expansionary shock. The loss-function approach keeps the output gap closer to zero and inflation closer to its target. This also explains why the forecast confidence bands under the loss-function approach are much narrower compared to those under the IFB approach (see Section III.5).

**Figure 6. Two Different Reaction Functions with Non-linear Phillips Curve (Part 1)**



Source: IMF staff calculations

Figure 7. Two Different Reaction Functions with Non-linear Phillips Curve (Part 2)



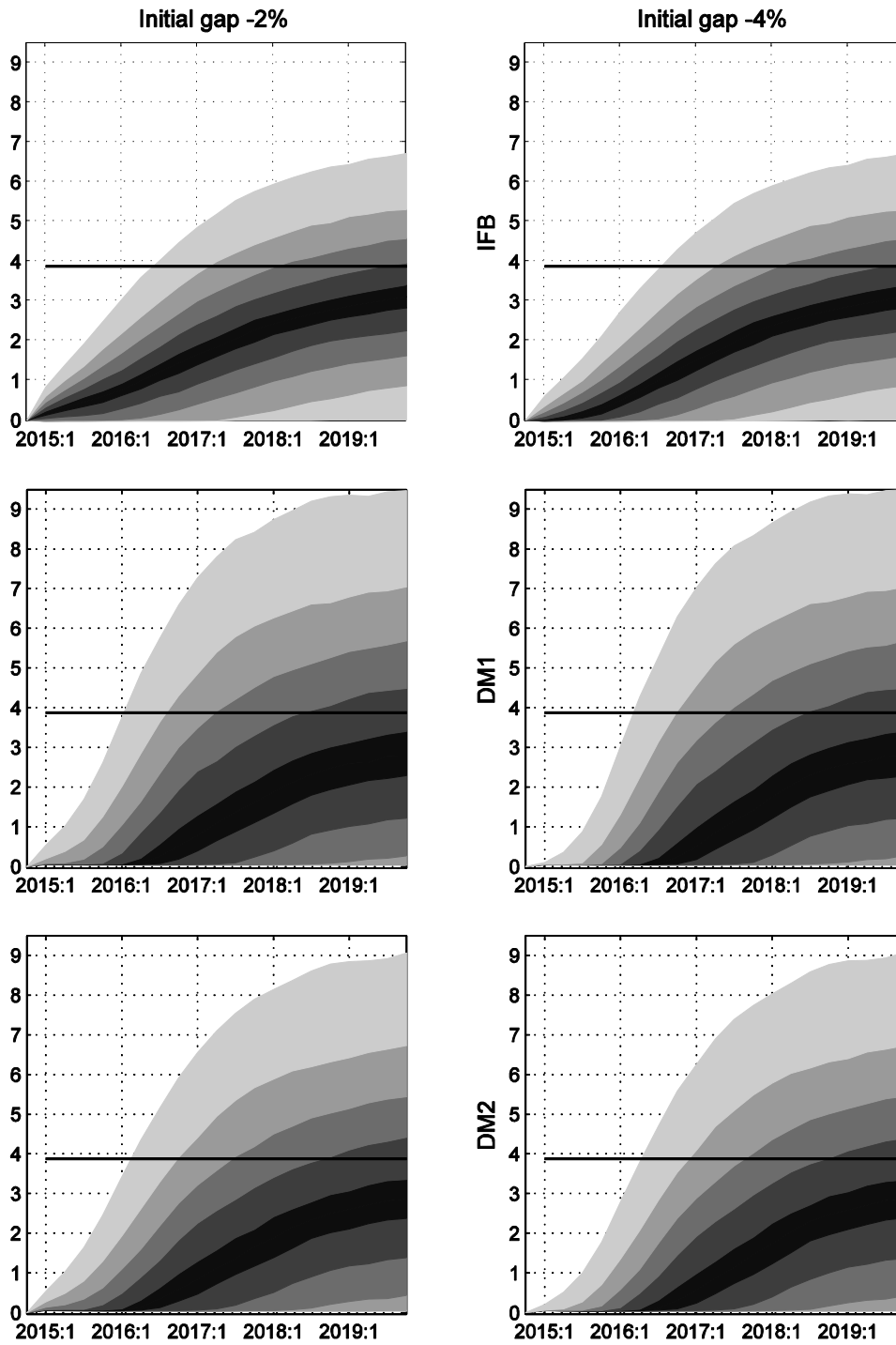
Source: IMF staff calculations

### III.5 Forecast confidence bands and alternative simulations

Every forecast is fraught with risk and it is important that the most relevant risks be communicated to financial markets and the public. Two techniques that are very helpful in such communications are computation of confidence intervals and the simulation of alternative scenarios.

The left hand panels of Figures 8, 9 and 10 show confidence intervals for the forecast federal funds rate, output gap, and year-on-year inflation rate for the monetary policy responses to the current economic situation under IFB, and for the two calibrations of the loss function.

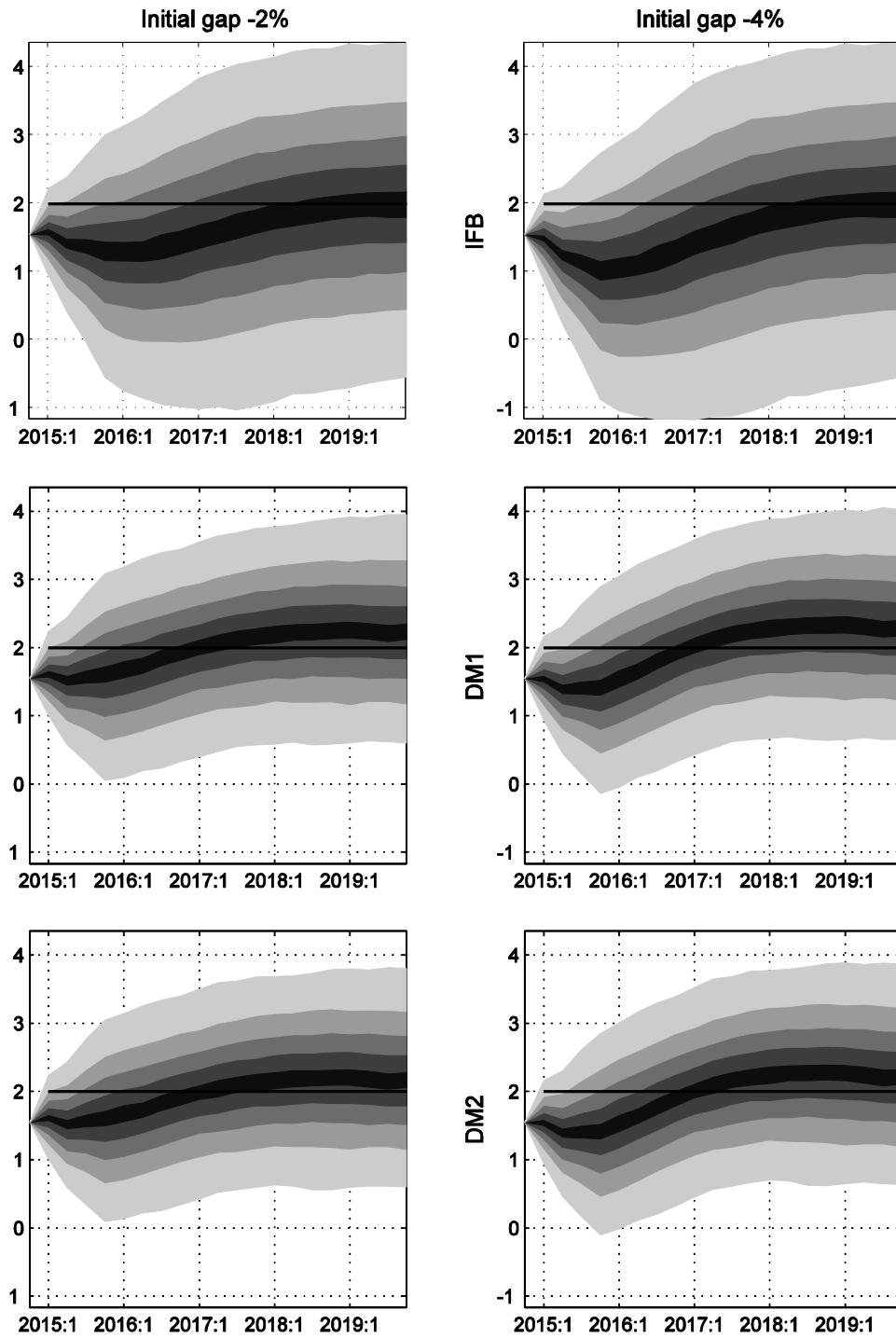
**Figure 8. Illustrative Examples of the Current Situation with Confidence Bands  
(Fed Funds Rate, Percentage, p.a.)**



Source: IMF staff calculations

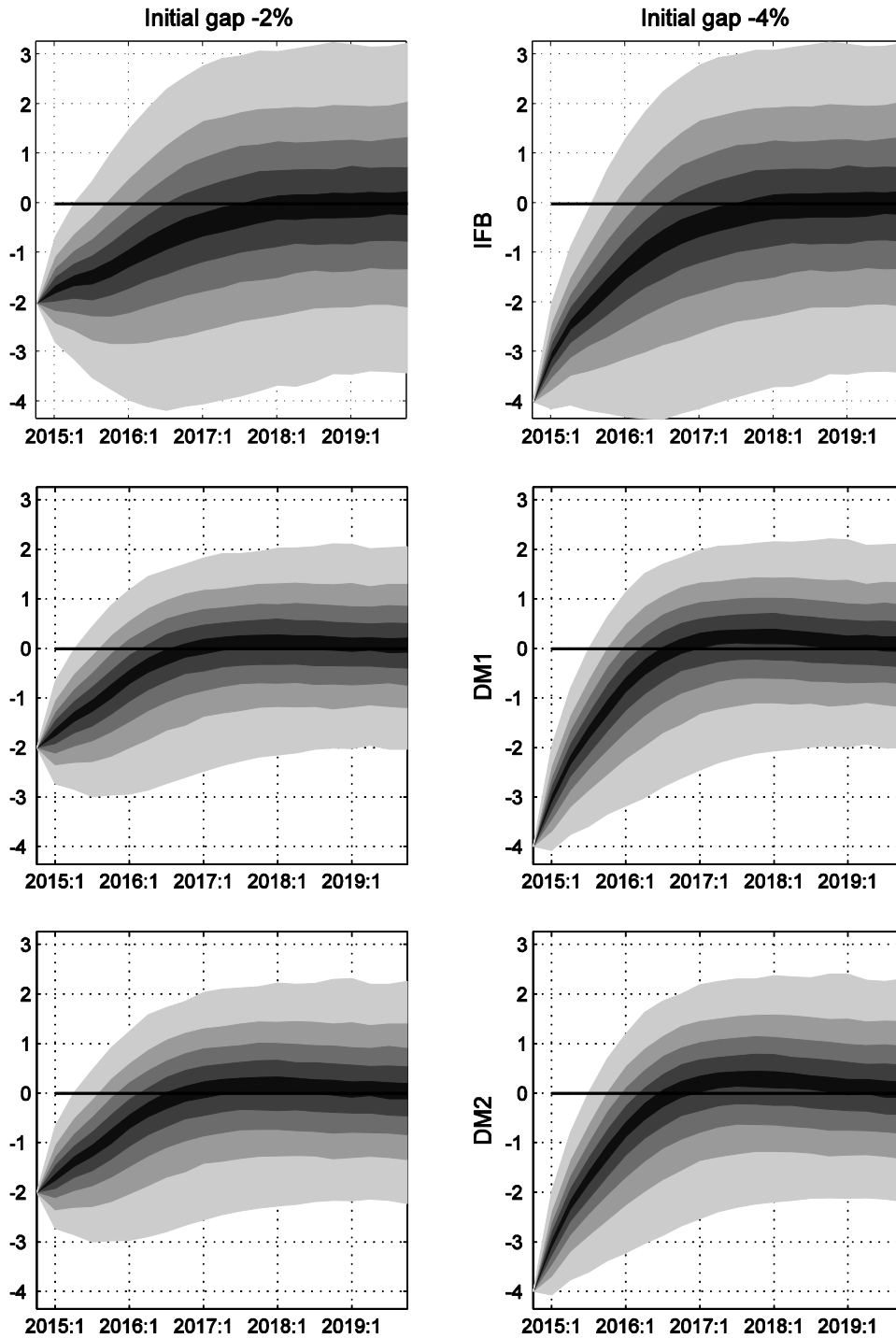


**Figure 9. Illustrative Examples of the Current Situation with Confidence Bands (YoY PCE Inflation, Percentage)**



Source: IMF staff calculations

**Figure 10. Illustrative Examples of the Current Situation with Confidence Bands (Output Gap, Percentage)**



Source: IMF staff calculations

The right hand panels show the results for the current economic situation if the initial output gap were believed to be -4 percent rather than -2 percent. The shaded bands show the confidence intervals from 10 to 90 percent in 10 percentage-point increments.

These confidence intervals show a wide range of uncertainty in the projections. In particular, the overshoots discussed in the preceding subsections are very small relative to, say, the 30 to 70 percent confidence bands. This gives one a better perspective on the challenges facing policymakers in the current economic circumstances.

Since the loss-function approach implies more aggressive interest rate responses to negative demand shocks, given the initial conditions, it results in wider confidence bands for the interest rate, and much narrower bands for output and inflation, than the IFB reaction function. Moreover, under the latter, there is a 15 to 20 percent probability of deflation one year into the future.

Confidence bands for the policy rate under the loss-function approach are skewed because of the zero-interest rate floor. And the volatility of the economy is somewhat greater under the -4 percent output gap case versus the -2 percent case, because of the higher likelihood that the floor will impede an effective stabilizing policy response.

### **III.6 A summing up**

While the reaction functions may give reasonable results in normal times, they have difficulty in abnormal times. When policy interest rates are at or very near the zero lower bound, the quadratic loss-function approach appears to give better results because its response to disinflationary conditions involves an extended commitment to keep the rate at the floor. As the ZLB approaches, a policy that avoids extreme outcomes (the quadratic loss function) provides ever stronger policy reactions to contractionary shocks, to keep the economy away from the deflation dark corner. At the ZLB, this is reflected in a clearly communicated commitment to hold the short-term interest rate to zero for long enough that inflation will rise, perhaps for a while above the long-run target rate. The boost to inflation expectations—since the credibility of the policy commitment is high and, as a result, economic agents believe it and react accordingly—reduces the real medium-term interest rate, even though the nominal short rate can go no lower. Under the circumstances, where there is very little risk of sustained inflationary pressure, but a high risk of getting stuck in the deflation trap, such a policy reaction represents prudent risk management.<sup>22</sup> This is consistent with Evans and others (2015), who show that monetary policy, under uncertainty and close to the ZLB, is better placed to deal with risks from inflation temporarily increasing above the central bank's objective than from a scenario where inflation falls short of its

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<sup>22</sup> Isard, Laxton and Eliasson (1999) makes a more general argument for stronger policy reactions to avoid bad outcomes in the presence of model uncertainty, especially where non-linearities may be involved.

objective. The desire for continuity in policy would argue that more attention should therefore be paid to the loss-minimization approach, in both normal and abnormal circumstances. Nonetheless, this does not imply that information from Taylor rule and IFB approaches should be ignored. Indeed, they can serve as useful crosschecks when there is concern that the specification of the model used to implement the loss-function approach is less than satisfactory.

This conclusion is even stronger if there are other nonlinearities over and above the ZLB. This is well illustrated by the example of the nonlinear Phillips curve and also holds for a situation in which central bank credibility responds in a nonlinear way to inflation outcomes. Thus, the benefits of focusing discussion on a model with quadratic loss function show up even more clearly in cases where there are multiple nonlinearities in the economy.

#### **IV. PUBLICATION OF ENDOGENOUS INTEREST RATE FORECASTS**

As monetary policy has become more forward-looking and more pre-emptive, the central bank's views on how future output and inflation will evolve have become a central element in its decision making and in communications. The projected medium-term outlook forms the basis for the discussion around short-term decision making and around the medium-term strategy of the central bank. That is, given the objectives of the central bank, given the current view of the central bank with respect to the country's economic and inflation prospects, and given its understanding of the transmission mechanism between its actions and economic and inflation developments, the forecast allows the Monetary Policy Committee (or its equivalent) and the central bank staff to determine the appropriate path (or paths) for the policy instrument so as to achieve the central bank's objectives. These paths are of course conditional on what is known at the time that the forecast is prepared and will change as new information becomes available and as the understanding of economic relationships evolves.

The alternative is to base a forecast on the assumption of unchanged interest rates or the market forecast of interest rates. There are a number of problems with this approach. Perhaps the most important one is that such forecasts are internally inconsistent. This problem becomes very evident when working with models where expectations include model-consistent elements—see Isard and Laxton (2000). In the limit, longer-term forecasts based on exogenous policy rates (whether unchanged rates or the market forecast) could fail to converge since there is no anchor to the system. That is, inflation could increase explosively or decrease explosively without leading to an offsetting monetary policy reaction. And while short-term forecasts are not likely to explode, an increase or decrease in inflation over the forecast horizon that does not give rise to an increase or decrease in real interest rates is clearly logically inconsistent with the stated policy approach of an inflation targeting central bank. Thus, from the point of view of logical coherence, it is clearly better to have a fully consistent forecast with endogenous interest rates.

Of course, for the Fed the endogenous interest rate path will be determined by its dual mandate, i.e., the requirement that inflation returns to its 2 percent objective and the economy reaches full employment over the policy horizon. Within the family of paths that satisfy this condition, the FOMC will choose a path that results in output being close to potential at the end of the policy horizon, inflation approaching its target, and movements of key economic variables not being unduly volatile along the path.

There are also benefits from the perspective of transparency and ease of communications to having an endogenous interest rate convention. Whether the Fed chooses to describe its policy path qualitatively or quantitatively, it is able to tell a logical and coherent story as to the appropriate monetary policy reaction to pressures on output and inflation.

Publishing the Fed's outlook along with an endogenous interest rate scenario would have perceived advantages and disadvantages. Publication of its interest rate scenario can be very helpful for the Fed in managing expectations of future interest rate movements. Svensson (2007) explained the Riksbank's decision to publish its interest rate forecast as follows.

*“Monetary policy works by affecting expectations about the future interest rate. It is the entire interest rate path that is important for future inflation and resource utilization, not merely the interest rate over the coming weeks. The Riksbank has therefore come to the conclusion that the only right thing is to explicitly discuss the interest rate path and to choose a particular path as the main forecast, as well as publishing the interest rate path and justifying its selection. This is in my opinion the most effective way of conducting monetary policy. Not to discuss and select a particular interest rate path as a main forecast would be an incomplete decision-making process. Not to publish the interest rate forecast would be to hide the most important information.”*

The Fed's summary of economic projections, including FOMC member's interest rate projections which summarize their views on appropriate policy interest rates over the next few years and over the long-term, currently provides useful information about likely path of policy looking forward. However, not publishing a single baseline endogenous interest rate scenario that is consistent with the baseline economic outlook leaves the Fed with the awkwardness of talking about a particular outlook for output and inflation but leaving the public unclear or not very clear about what interest rate path underlies it. While the “dots” provide information about individual policymakers expectations, they do not necessarily represent the FOMC's majority view of the expected policy rate path. As such, they could be counterproductive in obfuscating (or at time contradicting) the majority view of the FOMC. Also, it is hard to judge internal consistency since for each “dot” there is presumably a consistent view of the growth, interest rate, and inflation paths. The public cannot identify which dots belong to which individual and the median of the dots may be far from internally consistent with the median of the macroeconomic forecast.

The principal perceived risk in publishing an explicit path for projected interest rates appears to be that at least some market participants might believe that the Fed is making a commitment to bring about the projected interest rate path regardless of changing developments. Indeed, one of the most important messages that the Fed has to communicate to the public is that the forecast or outlook in the Report is conditional upon information available at the time of writing and will almost certainly change as new information (and new interpretations) become available. Of course, it may take some time for the market to learn, and it is important for a central bank deciding to publish the interest rate path underlying the forecast to be very diligent about explaining the conditional nature of the path. For example, when the Riksbank, decided to publish the interest rate scenario underlying its forecast in February 2007, the Governor noted (Ingves, 2007) that *“it is important to emphasize . . . that we are talking about a forecast for the repo rate. This is the repo rate development that currently appears most likely given the information available. We are not making any promises. The fact that the Riksbank is presenting its own interest rate path does not mean that we are laying down a policy that we will commit ourselves to following.”* While it appears that the markets learn over time how to interpret the conditionality of the interest rate path, there may be an awkward period while they are on the learning curve. As Svensson (2007) put it in the context of the Swedish situation, *“it is natural that it will take some time for the new system to become established.”* In view of these cautionary statements from policymakers, it is somewhat surprising that the experience—in New Zealand since June 1997, Norway since November 2005, Sweden since February 2007, Israel since July 2007 and the Czech Republic since February 2008—suggests that the financial market participants learn fairly quickly about the conditional nature of the forecast interest rate path.

There are also largely practical difficulties in forging agreement on an expected path for policy rates when the decision-making committee is based on a large and diverse committee, as in the Fed case. Mishkin (2004) and Goodhart (2001), for example, have argued that attempting to agree on a path in a committee setting could complicate the process of deciding on the next change in the policy rate. Diverse views on the economic outlook and the appropriate policy rate path among committee members would likely make it difficult to come to agreement on an FOMC endorsed policy rate path. Additional complications for formulating model-based policy stem from the decentralized nature of the Federal Reserve system.

## V. WHOSE FORECAST SHOULD BE PUBLISHED?

There are two options. In option one, the baseline forecast as well as alternative scenarios are supplied by FOMC members and are somehow aggregated by the staff. This approach would be technically demanding and would require transparency in the assumptions behind each FOMC member's forecast. As Mester (2015) has pointed out, this could be achieved by releasing a complete forecast for each FOMC member.

The second option, which has worked quite well in central banks that make policy decisions by committee, is for the staff to produce and publish the baseline forecast that the majority of the FOMC could then endorse through the monetary policy report as a representation of the views of the committee. The report could also include alternative scenarios (favoured by some FOMC members) to give a sense of the dispersion of views around this baseline.<sup>23</sup> Central banks that produce and publish such a forecast have emphasized that the staff baseline provides an important input into the monetary policy decision making, but represents only one input as some policymakers may disagree with the staff forecast. In practice, the staff baseline forecast does not usually wander too far away from the consensus view of the committee through the governor's (or deputy governor) coordination.

In our view, in many cases it would be better for the staff to have ownership of the projection. This view is not based on any theoretical argument but, rather, it is a practical judgment that would apply to many central banks, although not necessarily to all. This judgment is particularly relevant for central banks with relatively large policymaking bodies and those in which policymakers are likely to have particularly divergent views about future economic outcomes.

Constructing a consistent and coherent projection that reflects both good economic reasoning and the circumstances of a specific economy is a challenge at the best of times. In situations of considerable uncertainty about both real and financial external and domestic developments, it is even more challenging. Maintaining the mechanics of the projection within the staff creates an efficient mechanism for producing a coherent projection, which is crucial to ensuring the credibility of the central bank among expert observers of monetary policy developments, such as the specialist media and economists in financial institutions (whose opinions are solicited by the mass media). However, insofar as the projection is itself a (contingent) statement of future policy, it should have the weight of the FOMC behind it.

The production of the Monetary Policy Report is also simplified when the forecast is presented as an important staff input to decision making, albeit one among a number of inputs. While the Report would underline that the staff forecast plays a key role in this regard, the policymakers would not be required to defend any particular forecast, beyond stating their confidence in the process that produced it.

The alternative approach, in which the committee takes direct ownership of the projection, would be much more difficult at a central bank where decisions are made by vote, rather than

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<sup>23</sup> A significant investment has been made by Fed policymakers and staff to develop the scenarios that are presented in the "dots." The scenarios could serve as the basis for constructing a baseline scenario, highlighting key risks and communicating the width of the confidence bands around the baseline forecast.

by consensus (or by the Governor alone), such as the Fed. Voting committee members may have divergent views about the forecast. When there is no consensus, The Fed with twelve voting members might have to publish up to twelve (or more if non-voting FOMC member's projections are also included) complete projections. And how would all this be accommodated in the write-up explaining policy? Such an approach could be inefficient internally and confusing externally.

Ownership of the projection by the staff would avoid the problem of trying to adjust the projection to a mixture of views of committee members. But noting that the projection is the key, but not the only, input in decision-making gives the committee or individual committee members the scope to choose a different interest rate decision or path from that advocated by the staff in its projection. And the views of the members of the policymakers, whether unanimous or not, with a properly functioning forecasting and policy analysis system, would be reflected in alternative scenarios prepared by the staff, if not in the baseline, and in the discussion of uncertainties in the monetary policy report.

However, it is important that members of the committee do have solid economic arguments for their views in circumstances when they differ from those of the staff. For example, at times it may be uncertain whether recent inflation pressures are persistent or transitory. In the former case, central bank action may well be required to counter the pressures, while in the latter case, the problem unwinds itself without any central bank action. Another example would be where the baseline projection sees a slowdown, and hence a reduction in the rate of inflation, but the projected slowdown is not yet reflected the data. Whether the central bank should ease now, or wait-and-see, can be the subject of legitimate disagreement.

Clear and transparent explanations of such differences in the monetary policy report would help financial market participants understand the action (or absence of action) by the central bank. It might even increase the credibility of the central bank, since the debate would shed light on how the central bank would react when future data reveal which of the opposing views was more valid.

## VI. CONCLUSION

Over the past several years, the Federal Reserve has effected major reforms to its approach to monetary policy, under the dual mandate framework. It has adopted an explicit inflation objective (two percent). It has increased the transparency of policy in other ways too, notably through the timely publication of FOMC member forecasts, including projections for policy interest rates (the "dots"), and more open communications. These steps, like the unconventional instruments that were introduced during and after the global financial crisis, may have been primarily motivated by the need to stimulate the economy, and the constraint of the zero lower bound on the federal funds rate. The recent acceleration of the economy provides evidence that they have been successful in this regard.



The changes, however, amount to more than a set of measures to boost policy effectiveness during an abnormal period. They have brought about a more transparent framework for monetary policy that is appropriate for normal times, as well as for the unusual post-global crisis situation of chronic excess capacity, undesirably low inflation, and a rock bottom federal funds rate. We regard the revised Fed arrangements as equivalent, for all intents and purposes, to an inflation-forecast targeting approach. And from this perspective, there is room for further enhancements to the monetary policy framework.

We use a simple model to show that loss-function minimization approach to monetary policy, which includes publication of a baseline forecast and related uncertainties, replacing the current “dot” plot, can enhance the effectiveness of Fed’s policy actions. A simulation under conditions of a negative output gap, below-target inflation, and a binding zero interest floor, results in a modest, planned overshooting of inflation. The monetary stimulus comes from expectations that under this policy the short-term policy interest rate will be held at zero for an extended period, and from the boost that this gives to the expected rate of inflation, which reduces the current and expected real interest rate. Since this policy gets the economy away quite quickly from the dark corner of the deflation spiral, it can be viewed as a risk management strategy.

Explicit differences could exist in preferences among policymakers with respect to the short-run trade-off between inflation and output. Consideration of a range of alternative scenarios embodying different preferences and assumptions, as well as of a baseline forecast produced by Fed staff, would help the FOMC to decide upon a “good” path for the policy interest rate. Publications of the forecast, which could be endorsed by the majority of the FOMC, would be helpful for communicating to the public the reasons for differences in views among FOMC members, and the rationale for the federal funds rate decisions. It would help guide expectations of economic agents over the future prospects for inflation and policy rates which, in turn, would strengthen the effectiveness of monetary policy and allow the Fed’s goals to be reached more quickly and with smaller and shorter-lasting deviations over time.

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**Appendix 1. Definitions**

$i_t$	Federal funds rate (%)
$r_t$	Real interest rate (%) $r_t = i_t - \pi_{t+1}$
$r^*$	Steady-state level of real interest rate (%)
$\bar{r}_t$	Equilibrium real interest rate (%) $\bar{r}_t = 0.95 * \bar{r}_{t-1} + 0.05 * r^* + \varepsilon_t^r$
$r4_t$	Four-quarter average real interest rate (%) $r4_t = (r_t + r_{t+1} + r_{t+2} + r_{t+3}) / 4$
$\bar{r}4_t$	Four-quarter average equilibrium real interest rate (%) $\bar{r}4_t = (\bar{r}_t + \bar{r}_{t+1} + \bar{r}_{t+2} + \bar{r}_{t+3}) / 4$
$lr_t$	Long-term real interest rate (%) $lr_t = 0.1 * r_t + 0.35 * r4_t + 0.35 * (r4_t + r4_{t+4} + r4_{t+8}) / 3$ $+ 0.2 * (r4_t + r4_{t+4} + r4_{t+8} + r4_{t+12} + r4_{t+16}) / 5$
$\bar{lr}_t$	Equilibrium long-term real interest rate (%) $\bar{lr}_t = 0.1 * \bar{r}_t + 0.35 * \bar{r}4_t + 0.35 * (\bar{r}4_t + \bar{r}4_{t+4} + \bar{r}4_{t+8}) / 3$ $+ 0.2 * (\bar{r}4_t + \bar{r}4_{t+4} + \bar{r}4_{t+8} + \bar{r}4_{t+12} + \bar{r}4_{t+16}) / 5$
$P_t$	PCE price index
$p_t$	Log of PCE price index $p_t = 100 * \log(P_t)$
$\pi$	Quarterly PCE inflation (%) $\pi_t = p_t - p_{t-1}$

$\pi 4_t$	<p>Year-on-year PCE inflation (%)</p> $\pi 4_t = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) / 4$ $\pi 4_{t+3} = \underbrace{(\pi_{t+3} + \pi_{t+2})}_{\substack{\text{model forecast of inflation} \\ \text{(depend on all inputs into} \\ \text{forecast including monetary} \\ \text{policy reactions)}}} + \underbrace{(\pi_{t+1} + \pi_t)}_{\substack{\text{partly observable} \\ \text{or monitored}}} / 4$ $\pi 4_{t+4} = (\pi_{t+4} + \pi_{t+3} + \pi_{t+2} + \pi_{t+1}) / 4$
$\pi^*$	Inflation objectives (2%)
$y_t$	Output gap
$BLT_t$	Bank Lending Tightening condition
$\varepsilon_t^i$	Deviation from the interest rate reaction function
$\varepsilon_t^\pi$	Supply shock
$\varepsilon_t^y$	Shock to the output gap

## Appendix 2. The model

We employ a closed-economy model, which is simple but rich enough to study monetary policy issues.<sup>24</sup> In the main text, we presented the main elements of the model. In this appendix, we present the main equations of the model in more detail.

### a) The IS Curve

The first equation of the model is an aggregate demand function known as the IS curve, or the output gap equation:

$$y_t = \beta_1 * y_{t-1} + \beta_2 * y_{t+1} + \beta_3 * (lr_{t-1} - \bar{lr}_{t-1}) - \eta_t + \varepsilon_t^y$$

$$\beta_1 = 0.57; \beta_2 = 0.23; \beta_3 = -0.19$$

The output gap  $y_t$  (the deviation between the level of actual and potential output) is a function of a one-period lag and a one-period lead of the output gap, and the deviation of the lagged long-term real interest rate ( $lr_t$ ) from its equilibrium value ( $\bar{lr}_t$ ). It is also a function of financial conditions ( $\eta_t$ ) that will be explained later.

The short-term real interest rate ( $r_t$ ) is equal to the nominal federal funds rate ( $i_t$ ) minus the projected rate of core Personal Consumption Expenditures (PCE) inflation over the next quarter ( $\pi_{t+1}$ ):

$$r_t = i_t - \pi_{t+1}.$$

The equilibrium short-term real interest rate is a weighted average of the steady-state level of the short-term real interest rate, the lagged equilibrium short-term real interest rate, and an error term:

$$\bar{r}_t = \rho * r^* + (1 - \rho) * \bar{r}_{t-1} + \varepsilon_t^{\bar{r}}$$

$$\rho = 0.05$$

The long-term real interest rate is a weighted average of the current short-term real interest rate and the four quarter average short-term real interest rate over the current year and the next 16 quarters while the equilibrium long-term real interest rate is a weighted average of the short-term equilibrium real interest rate and the four quarter average equilibrium real interest rate, with the same weights as appear in the equation for the long-term real interest rate:

$$lr_t = 0.1 * r_t + 0.35 * r4_t + 0.35 * (r4_t + r4_{t+4} + r4_{t+8})/3$$

$$+ 0.2 * (r4_t + r4_{t+4} + r4_{t+8} + r4_{t+12} + r4_{t+16})/5$$

$$r4_t = (r_t + r_{t+1} + r_{t+2} + r_{t+3})/4$$

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<sup>24</sup> Different versions of this model have been used as part of the multi-country Global Projection Model (see, e.g., Blagrove and others, 2013).



The bank lending tightening variable ( $\eta_t$ ) reflects bank lending tightening conditions and thereby influences the output gap. The bank lending tightening (BLT) variable is constructed on the basis of the responses in Senior Loan Officer Survey of the Federal Reserve to the question of whether lending conditions have tightened or eased. Thus, an easing of bank lending tightening conditions, a reduction in  $\eta_t$ , would lead to an increase in aggregate demand and in the output gap.

Finally, an error term ( $\varepsilon_t^y$ ) allows for shocks to the output gap.

### b) Bank Lending Conditions

BLT not only affects the output gap, but also is affected by it. In other words, we also need to model how the real economy affects financial conditions. Deviations of the BLT from its equilibrium level are modelled proportional to future output gap ( $y_{t+4}$ ) and adjusted for a shock ( $\varepsilon_t^{BLT}$ ):

$$BLT_t - \overline{BLT}_t = -\kappa_1 * y_{t+4} + \varepsilon_t^{BLT}$$

$$\kappa_1 = 20.1;$$

In turn, the output gap is affected by a distributed lag of past  $\varepsilon_t^{BLT}$ , denoted by  $\eta_t$ , which takes the following form:

$$\eta_t = \theta_t * (0.04 * \varepsilon_{t-1}^{BLT} + 0.08 * \varepsilon_{t-2}^{BLT} + 0.12 * \varepsilon_{t-3}^{BLT} + 0.16 * \varepsilon_{t-4}^{BLT} + 0.20 * \varepsilon_{t-5}^{BLT}$$

$$+ 0.16 * \varepsilon_{t-6}^{BLT} + 0.12 * \varepsilon_{t-7}^{BLT} + 0.08 * \varepsilon_{t-8}^{BLT} + 0.04 * \varepsilon_{t-9}^{BLT})$$

$$\theta_t = 1.0708$$

This weighting (with a peak effect at the 5- and 6-quarter lags) is intended to reflect a pattern in which an increase in  $\varepsilon_t^{BLT}$  is expected to negatively affect spending by firms and households in a hump-shaped fashion, with an initial buildup and then a gradual rundown of the effects.<sup>25</sup> If lending conditions are easier than might have been anticipated on the basis of expectations of future economic behavior, the effect will be a larger output gap and a stronger economy, also in a hump-shaped fashion.<sup>26</sup>

### c) The Phillips Curve

The next model equation is the expectations-augmented Phillips curve:

<sup>25</sup> See Carabenciov and others (2008, 2013) for more discussion on the incorporation of the bank lending conditions into the Global Projection Model.

<sup>26</sup> This reduced form for the effects of financial conditions on the real economy captures insights from structural models, e.g., Benes, Kumhof and Laxton (2014a, b).

$$\pi_t = \lambda_1 * \pi 4_{t+4} + (1 - \lambda_1) * \pi 4_{t-1} + \lambda_2 * y_{t-1} + \varepsilon_t^\pi$$

$$\lambda_1 = 0.70; \lambda_2 = 0.10$$

The rate of inflation ( $\pi$ ) is defined as quarterly core PCE rate of inflation and is a function of the weighted average of a one quarter lag in year-on-year core PCE inflation ( $\pi 4_{t-1}$ ) and a weighted average of future year-on-year core PCE inflation over the subsequent four quarters ( $\pi 4_{t+4}$ ). These future rates of inflation are partly based on observable or monitored inflation and partly on the model forecast of inflation. The latter in turn depends on all inputs into the forecast including the functions and determines monetary policy actions. Thus, there are both backward-looking and forward-looking elements to expected inflation. In addition, there is an output gap term and an error term ( $\varepsilon_t^\pi$ ) in this equation. The latter can be interpreted as a supply shock.

#### d) Policy Reaction

The model is closed with an assumption about the process for setting the short-term interest rate. We use 3 options. The first is an inflation-forecast-based policy reaction function. The second derives policy reactions by minimizing a loss function. The third is a standard Taylor rule.

##### d1) Inflation-Forecast Based Reaction function

The inflation-forecast-based reaction function (IFB) is used by a number of central banks (e.g. the Bank of Canada, the Czech national Bank). The variant used in this study focuses on the forecast 4 quarter rate of inflation three quarters in the future.

$$i_t = \gamma_1 * i_{t-1} + (1 - \gamma_1) * [\bar{r}_t + \pi 4_{t+3} + \gamma_2 * (\pi 4_{t+3} - \pi^*) + \gamma_3 * y_t] + \varepsilon_t^i$$

$$\gamma_1 = 0.71; \gamma_2 = 0.91; \gamma_3 = 0.21.$$

In this equation the nominal fed funds rate ( $i_t$ ) is a function of its lagged value (a way of preventing overly sharp movements in the policy interest rate), the equilibrium nominal interest rate, as measured by the sum of the equilibrium real interest rate ( $\bar{r}_t$ ) and the projected year-on-year core PCE inflation ( $\pi 4_{t+3}$ ), the forecast deviation of projected inflation from its target value, and the output gap. This formulation has the appropriate property for the real policy interest rate of responding to an increase in inflation. However, this may not occur in the very short term because of the distributed lag nature of the response of the nominal short-term interest rate to an increase in inflation.

In contrast to the so-called “Taylor rules” used in some studies, the IFB formulation has the advantage of ignoring shocks to the system that are expected to reverse within the three-quarter policy horizon and also allows the central bank to take account in the setting of the

policy interest rate of expected future developments to the economy over the three-quarter forecast policy horizon on which they currently have information.

### **d2) Minimization of a loss function**

The loss function incorporates the principal objectives of the central bank in policy making – taking the appropriate actions to bring inflation back to its target over time and closing the output gap.

$$L_t = \sum_{t=1}^{\infty} [\omega_1 * (\pi_t - \pi^*)^2 + \omega_2 * y_t^2 + \omega_3 * (i_t - i_{t-1})^2]$$

The loss function or objective function typically associated with inflation targeting uses a quadratic formulation, implying that large errors or deviations are more important in the thinking of central banks than small errors or deviations.

The loss function includes a term in the change of the federal funds rate. This term has the effect of preventing very sharp movements in the federal funds rate, which could otherwise occur on a regular basis, and which would not reflect the behavior of central banks in practice. By taking account of both current and expected future values of output and inflation, this formulation enables the central bank to incorporate into its decision making any information currently available to it about likely developments in the system over the next few quarters.

The loss function used in this study has 2 variants. Both can be viewed as reflecting the objectives of a dual mandate central bank or, equivalently, a flexible inflation targeting central bank. That is, the central bank focuses on both the deviation of inflation from its target and the deviation of output from potential. In the first variant of the loss function (DM1), there is equal weight on the deviation of inflation from its target and the deviation of output from potential. In the second (DM2), the weight on the output gap is half that on the gap between inflation and its target. Both have a term with the weight of 0.5 on the change in the nominal interest rate.

The coefficients of the 2 variants are as follows:

DM1: Dual Mandate Central Bank (equal output and inflation weights)

$$[\omega_1, \omega_2, \omega_3] = [1.0, 1.0, 0.5]$$

DM2: Dual Mandate Central Bank (lower weight on output)

$$[\omega_1, \omega_2, \omega_3] = [1.0, 0.5, 0.5].$$

### **d3) Taylor rule**

The variant used in this paper is based on Taylor (1993).

$$i_t = \bar{r}_t + \pi 4_t + 0.5 * (\pi 4_t - \pi^*) + 0.5 * y_t + \varepsilon_t^i$$

The federal funds rate is a function of the equilibrium real interest rate ( $\bar{r}_t$ ), the year-on-year core PCE inflation ( $\pi 4_t$ ), the deviation of the inflation rate from the inflation objective ( $\pi^*$ ), and the output gap ( $y_t$ ). Deviations from the rule (e.g. policy experiments), are represented by  $\varepsilon_t^i$ .

### Appendix 3: Minimizing the loss function

This appendix provides a technical description of the algorithm used in the loss-function minimization approach, based on a simpler version of the model.

Assume that the economy has the IS curve and the Phillips curve as follows.

$$\begin{aligned} y_t &= \beta_1 y_{t-1} + \beta_2 \mathbb{E}_t y_{t+1} + \beta_3 (i_t - \mathbb{E}_t \pi_{t+1}) + \epsilon_{y_t} \\ \pi_t &= \lambda_1 \mathbb{E}_t \pi_{t+1} + (1 - \lambda_1) \pi_{t-1} + \lambda_2 y_t + \epsilon_{\pi_t} \end{aligned}$$

The policymaker is choosing the whole path of the policy rate  $i_t$  ( $t=1, 2, \dots$ ) to minimize an intertemporal quadratic loss function that takes into account the distance of inflation to its target,  $(\pi_t - \bar{\pi})$ , and the size of the output gap,  $y_t$ , with equal weights on both of them, and a discount factor of 1. The intertemporal loss function is written as

$$L_t = \sum_{t=1}^{\infty} \mathbb{E}_t ((\pi_t - \bar{\pi})^2 + y_t^2).$$

The Lagrangean of the problem is

$$\begin{aligned} \mathbb{L}_t &= \sum_{t=1}^{\infty} \mathbb{E}_t \{ (\pi_t - \bar{\pi})^2 + y_t^2 \\ &+ \mu_{1_t} (y_t - \beta_1 y_{t-1} - \beta_2 \mathbb{E}_t y_{t+1} - \beta_3 (i_t - \mathbb{E}_t \pi_{t+1}) - \epsilon_{y_t}) \\ &+ \mu_{2_t} (\pi_t - \lambda_1 \mathbb{E}_t \pi_{t+1} - (1 - \lambda_1) \pi_{t-1} - \lambda_2 y_t - \epsilon_{\pi_t}) \}, \end{aligned}$$

where  $\mu_{1_t}$  and  $\mu_{2_t}$  are Lagrange multipliers.

The first-order conditions are obtained by computing the partial derivatives of the Lagrangean with respect to  $i_t$ ,  $y_t$ ,  $\pi_t$ ,  $\mu_{1_t}$  and  $\mu_{2_t}$  and setting them to zero. Note that the derivation is different for  $t = 1$  and for  $t > 1$ , but the conditions can be made formally equivalent by setting the initial value of the Lagrange multipliers to zero, ( $\mu_{1_0} = \mu_{2_0} = 0$ ):

$$\begin{aligned} \frac{\partial \mathbb{L}_t}{\partial i_t} &= \mu_{1_t} \beta_3 = 0, \\ \frac{\partial \mathbb{L}_t}{\partial y_t} &= 2y_t + \mu_{1_t} - \mu_{2_t} \lambda_2 - \mathbb{E}_t \mu_{1_{t+1}} \beta_1 - \mu_{1_{t-1}} \beta_2 = 0, \\ \frac{\partial \mathbb{L}_t}{\partial \pi_t} &= 2(\pi_t - \bar{\pi}) + \mu_{2_t} - \mu_{1_{t-1}} \beta_3 - \mu_{2_{t-1}} \lambda_1 + \mathbb{E}_t \mu_{2_{t+1}} \lambda_1 = 0, \\ \frac{\partial \mathbb{L}_t}{\partial \mu_{1_t}} &= y_t - \beta_1 y_{t-1} - \beta_2 \mathbb{E}_t y_{t+1} - \beta_3 (i_t - \mathbb{E}_t \pi_{t+1}) - \epsilon_{y_t} = 0, \\ \frac{\partial \mathbb{L}_t}{\partial \mu_{2_t}} &= \pi_t - \lambda_1 \mathbb{E}_t \pi_{t+1} - (1 - \lambda_1) \pi_{t-1} - \lambda_2 y_t - \epsilon_{\pi_t} = 0. \end{aligned}$$

From the first condition, it is obvious that  $\mu_{1_t} = 0$  in each period and the system simplifies to

$$\begin{aligned}
2y_t &= \mu_{2_t} \lambda_2, \\
2(\pi_t - \bar{\pi}) &= -\mu_{2_t} + \mu_{2_{t-1}} \lambda_1 - \mathbb{E}_t \mu_{2_{t+1}} \lambda_1, \\
y_t &= \beta_1 y_{t-1} + \beta_2 \mathbb{E}_t y_{t+1} + \beta_3 (i_t - \mathbb{E}_t \pi_{t+1}) + \epsilon_{y_t}, \\
\pi_t &= \lambda_1 \mathbb{E} \pi_{t+1} + (1 - \lambda_1) \pi_{t-1} + \lambda_2 y_t + \epsilon_{\pi_t}.
\end{aligned}$$

In this particular case,  $\mu_{2_t}$  could be also substituted out using  $\mu_{2_t} = 2y_t/\lambda_2$ , but, in general, it is not possible to eliminate all the Lagrange multipliers.

When the zero interest rate floor is taken into account, the problem is transformed into a mixed complementarity problem (MCP):

$$\begin{aligned}
\lambda_2 \mu_{2_t} &\geq 2y_t \perp i \geq 0, \\
2(\pi_t - \bar{\pi}) &= -\mu_{2_t} + \mu_{2_{t-1}} \lambda_1 - \mathbb{E}_t \mu_{2_{t+1}} \lambda_1, \\
y_t &= \beta_1 y_{t-1} + \beta_2 \mathbb{E}_t y_{t+1} + \beta_3 (i_t - \mathbb{E}_t \pi_{t+1}) + \epsilon_{y_t}, \\
\pi_t &= \lambda_1 \mathbb{E} \pi_{t+1} + (1 - \lambda_1) \pi_{t-1} + \lambda_2 y_t + \epsilon_{\pi_t}.
\end{aligned}$$

where  $\lambda_2 \mu_{2_t} \geq 2y_t \perp i \geq 0$  means that either  $\lambda_2 \mu_{2_t} = 2y_t$  and  $i \geq 0$ , or  $\lambda_2 \mu_{2_t} < 2y_t$  and  $i_t = 0$ .

Dynare uses the LMMCP algorithm to solve mixed complementarity problems. Below is the Dynare code corresponding to the above example.<sup>27</sup>

The simulation is done under perfect foresight as if no shocks were to hit the economy in the future. The optimal policy is computed *under commitment*, meaning that the policymaker is not going to reoptimize the policy in the future. In technical terms, the Lagrange multipliers are not reset to zero after the first period of the simulation.

As discussed in the paper, when the output gap is largely negative and conventional monetary policy being constrained by the zero interest rate floor, the policy recommendation derived from minimizing the loss function is to keep the interest rate at the floor for a relative long time, which would result in inflation modestly overshooting the target and output gap being positive for some periods before it closes.

One can imagine that in circumstances when the outlook starts to improve, it would become difficult for the policymaker to stick to his guns and he may decide to reconsider his optimal plan (to reoptimize) and raise interest rate earlier than warranted under the original contingent plan. This would be a departure from commitment and may damage the credibility of the policymaker. As a result, the commitment and credibility of the policymaker is crucial to the success of such strategy.

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<sup>27</sup>This example of the code and a working version of Dynare with these features are available at [www.douglaslaxton.org](http://www.douglaslaxton.org).

```
var i y pi;
varexo e_y e_pi;

parameters beta1 beta2 beta3 lambda1 lambda2 pi_bar;
beta1 = 0.6;
beta2 = 0.25;
beta3 = -0.2;
lambda1 = 0.7;
lambda2 = 0.1;
pi_bar = 2.0;

model;
[mcp = 'i > 0']
y = beta1*y(-1) + beta2*y(+1) + beta3*(i-pi(+1)) + e_y;
pi = lambda1*pi(+1) + (1-lambda1)*pi(-1) + lambda2*y + e_pi;
end;

planner_objective (pi-pi_bar)^2 + y^2;

ramsey_model(planner_discount=1.0);

histval;
y(0) = -2.0;
pi(0) = 1.0;
end;

steady;

perfect_foresight_setup(periods=50);
options_.stack_solve_algo = 7;
options_.solve_algo = 10;
perfect_foresight_solver;

rplot i;
```