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Official Demand for U.S. Debt: Implications for U.S. real interest rates

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Official Demand for U.S. Debt: Implications for U.S. real interest rates

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Abstract

By constructing and estimating a structural arbitrage-free model of demand pressures on US real rates, we find that recent purchases of US government debt securities by the Fed and foreign officials have significantly affected the level and the dynamics of US real rates. In particular, by 2008, foreign purchases of US Treasuries are estimated to have had cumulatively reduced long term real yields by around 80 basis points. The subsequent total impact of Fed purchases in 2008-2012 has been even larger: the quantitative easing (QE) has depressed real 10-year yields by around 140 basis points. Our findings also reveal that the Fed policy interventions and foreign official purchases affect longer term real bonds mostly through a reduction in the bond premium.

JEL Classification Numbers: F31, G10

Keywords: Term structure of interest rates, Large Scale Asset Purchases (LSAP), real yield curve, Bayesian estimation, international reserves

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I. INTRODUCTION

Over the past decade or so, purchases of US Treasury bonds by official investors reached unprecedented levels becoming an increasingly important tool in central banks' policies. Foreign officials were the first to significantly increase their holdings of US Treasury bonds, as part of their reserve accumulation policies (ECB, 2006). Then, during the recent financial crisis, as policy interest rates approached the zero lower bound, the Federal Reserve (Fed) launched the policy of quantitative easing (QE), and as a result the Fed also became an important active investor in the Treasury bond market. The objective of this unconventional policy in fact was to stimulate the economy by reducing longer-term interest rates through a series of asset purchase programs. Specifically, the Fed's Large Scale Asset Purchases (LSAP) mainly focused on longer-term securities including government bonds and on mortgage-backed securities (MBS).

Two main features distinguish foreign and domestic policy makers from a typical US Treasury bond investor. First, official investors stand out for the massive market share of the Treasury market owned.[1] Second, their demand displays relatively low price elasticity, in that, it is only slightly sensitive to risk-return considerations (see, for example, Krishnamurthy and Vissing-Jorgensen, 2012). Of further interest is the rather high duration of foreign officials' portfolio of Treasury securities. Taken together these facts lie behind an emerging consensus among market participants, academics and policy makers that such inelastic, large-scale official purchases of US Treasury bonds have qualitatively contributed to lowering US long-term interest rates.

A natural question though is how big is the impact of official demand on yields? We answer this question by estimating a structural arbitrage-free model of US real rates. Specifically, we employ the preferred-habitat model proposed by Vayanos and Vila (2009), in which equilibrium interest rates are determined by the interaction of two different types of investors: those who trade bonds at different maturities for return considerations (arbitrageurs) and those who buy bonds of specific maturities mainly for reasons other than returns (like official investors).

We estimate this structural model on the term structure of monthly rates, derived from US Treasury Inflation-Protected Securities (TIPS), over the 2001-2012 period. The term structure is determined by two factors: the short-term real interest rate and the (excess) demand factor. We specify the demand factor as a function of a number of observable variables that capture the share of the outstanding Treasury securities held by the Fed and the foreign officials (the so-called official demand pressure) and the relative maturity of the Fed's Treasury portfolio.

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[1] Since the early 2000s, foreign investors have been among the major purchasers of US assets and held an exceptionally large proportion of US long-term debt. For example, in 2007 their holdings of Treasury notes due from three to ten years reached 80 percent of the whole amount outstanding. Subsequently, since the launch of the LSAP in 2009, Fed has held an increasingly important share of the bond market, and, by 2012, Fed kept about 18 percent of the total Treasury securities outstanding. Notably, similarly to foreign officials, the Fed played a particularly important role at the long end of the maturity spectrum, holding about 35 percent of the segment of the Treasury market with maturity over ten years.

We find that recent purchases of US government debt securities by the Fed and foreign officials have indeed affected the level and dynamic of US real rates significantly. The impact of foreign purchases has been particularly important in the period prior to the crisis, while the impact of the Fed has become important since 2009. Specifically, by 2008 foreign purchases of US Treasuries exerted a cumulative negative impact of around 80 basis points on long-term US Treasury yields. The subsequent total impact of Fed purchases is even larger, with QE depressing real 10-year yields by around 140 basis points. Our results also show that not only do the amount of Treasuries held by the Fed matter, but also the maturity pressures exerted by the Fed purchases matter. In other words, by altering the duration (i.e., the riskiness) of the bonds outstanding, even without changing the actual amount of bonds outstanding, the Fed's unconventional policies can be effective. We estimate that during the Maturity Extension Program (MEP) 5- and 10-year yields were depressed by around 48 and 61 basis points respectively. Furthermore, our findings show that what matters is the official demand pressure (i.e., official policy demand relative to the Treasury supply); thus, the overall market impact of official purchases varies with the supply of bonds by the Treasury.

Our study relates to a number of earlier studies on reserve accumulation by foreign central banks and its impact on US interest rates[2] and to those studies trying to quantify the impact of Fed asset purchases on US interest rates.[3] Although these studies agree on the qualitative (i.e., downward) effect of official intervention on US Treasury yields, the evidence is not unanimous in quantitative terms, as the estimates can vary significantly.

In this paper, we bring together these otherwise separate strands of the literature, by jointly analyzing the impact of foreign and domestic official demand pressures on the US Treasury bond markets. Moreover, although a large number of the empirical studies on QE impacts are motivated by the preferred-habitat model of Vayanos and Vila (2009), to the best of our knowledge, this is the first study to structurally estimate the preferred-habitat model including observable measures of demand pressures. Overall, many of the earlier empirical studies investigating the impact of official purchases on the level of yields lack the structural integrity provided by the preferred-habitat model and/or tend to look at the effect of the foreign and domestic demand in isolation. Notably, within the framework proposed here, we can estimate the impact of official demand on real interest rates consistently across maturities and over time. In addition, by using various observable measures of QE, we can shed light on the separate impact of quantity pressures from the impact of duration pressures, which allows us to better understand the transmission channel of the Fed's policies. As a caveat, however , our analysis abstracts from the "local" demand effects on yields due to the lack of maturity-specific data on official

[2] Such as the macroeconomic literature on 'global imbalances' (see, for example, Caballero (2006); Mendoza, Quadrini and Rios-Rull (2007); Caballero, Farhi and Gourinchas (2008); and Caballero and Krishnamurthy (2009)), or empirical, mostly reduced-form, studies (Warnock and Warnock (2009), Sierra (2010), and Krishnamurthy and Vissing-Jorgensen (2012)).

[3] E.g. studies by D'Amico and King (2013), Gagnon, Raskin, Remache, and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011, 2012), Swanson (2011), D'Amico, English, L´opez-Salido, Nelson (2012), Greenwood and Vayanos (2013), Hamilton and Wu (2012), and Meaning and Zhu (2012).

holdings. Analysis of the local supply effects on US interest rates can be found, for example, in D'Amico and King (2013).

The remainder of the paper is organized as follows: Section 2 summarizes recent developments in US Treasury bond markets, and Section 3 reviews recent literature estimating the impact of large Treasury bond investors' demand on interest rates. The model is presented in Section 4, whereas we describe the data and introduce the econometric methodology in Section 5. We subsequently discuss the results in Section 6. Finally, Section 7 concludes our analysis.

II. RECENT DEVELOPMENTS IN US TREASURY MARKETS

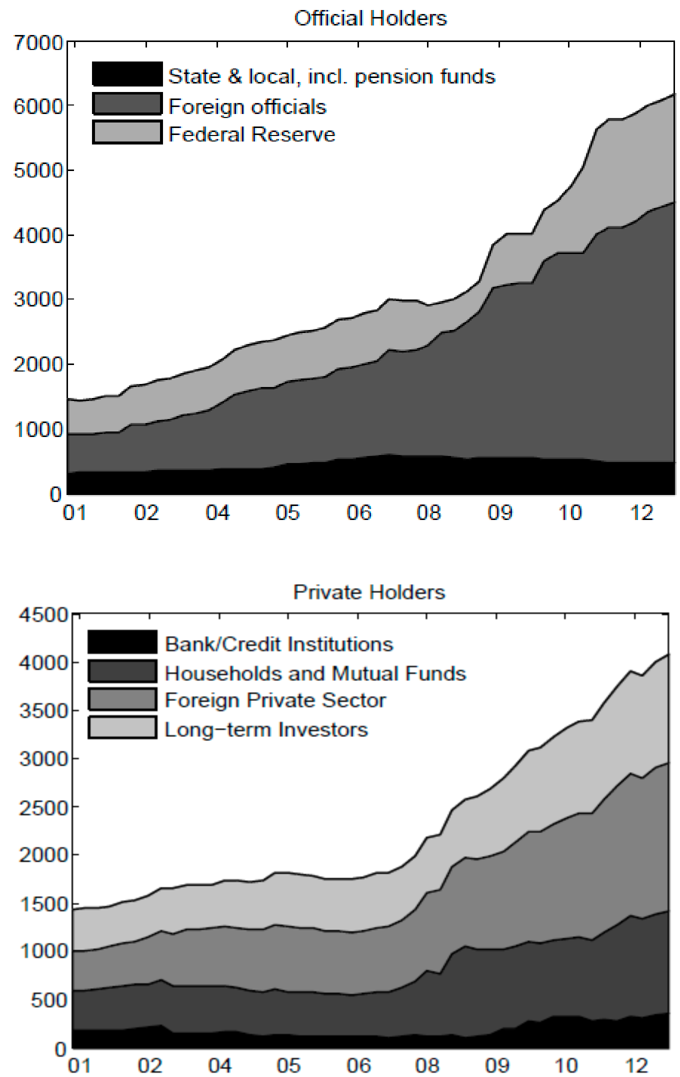
A. Foreign official purchases and interest rate conundrum

The foreign official sector has been playing a key role in the Treasury market since the early 2000s (Figure 1). Indeed, as the Treasury International Capital System (TIC) data show, the US debt market has been growing constantly since the early 2000s and subsequently foreign investors have been major purchasers of US assets. For example, in September 2009 non-Americans held more than 65 percent of the all US government notes and bonds outstanding,[4] with foreign official investors holding roughly 37 percent of the total Treasury supply, making them the largest holder of Treasury debt.

There are several drivers behind such an unprecedented accumulation of US assets by foreign officials. Above all, foreign central banks tend to buy Treasuries because they are highly liquid assets that provide a reliable store of value and also serve as an insurance against future crises. This consideration became particularly important for many emerging market economies (EMEs) in the aftermath of the crises of 1990s and early 2000s. Exchange rate policy management is another key factor. A large chunk of EMEs reserves is invested in US Treasury securities. In 2004, for example, foreign official institutions are estimated to have purchased US Treasury notes and bonds quantifiable in a net face value of 201 billion USD, amounting to 70percent of the total net issuance of US Treasury notes and bonds during that period. Finally, certain features of the domestic financial systems of EMEs, especially in Asia, are likely to have played an important role. These include underdeveloped local financial systems; the resulting tendency towards dollarization of official and/or private cross-border assets; and an excess of domestic savings over investment driven by either a savings glut (e.g. China) or an investment drought (for a more detailed discussion, see ECB, 2006).

[4] Private foreign holdings are usually overstated because TIC data do not capture foreign central banks acquisitions, taking place through a third-country intermediary.

Figure 1. Estimated Ownership of U.S. Treasury Securities



The top panel presents the official holders, in billions of dollars, and the bottom panel shows the private holders, in billions of dollars. Source: Office of Debt Management, Office of the Under Secretary for Domestic Finance, own calculations.

Irrespective of the actual motives behind their financing of a significant share of the US current account deficit, it is important to note that foreign official institutions pursue objectives that are only slightly sensitive to risk-return considerations. Krishnamurthy and Vissing-Jorgensen (2012) argue that foreign officials' demand for US Treasuries is inelastic; when a foreign central bank receives a dollar capital inflow, it accumulates more dollar reserves, buying Treasuries regardless of their prices relative to other assets.

Evidently, the rapid increase in foreign official holdings of US Treasury bonds coincided with the decline in US long-term interest rates in 2004-2006. The prevailing standard macro-financial literature of the time had difficulties in explaining the decline in rates by relying solely on macroeconomic and financial fundamentals,[5] and for this reason, the phenomenon was described as a "conundrum" by Alan Greenspan in 2005. However, he also noted a factor outside the traditional macroeconomic view; namely, heavy purchases of longer-term Treasury securities by foreign central banks may have boosted bond prices and pulled down longer-term yields. Back in 2005, he suggested that the foreign buying of U.S. bonds might have depressed U.S. long rates by ''less than 50 basis points.''

B. Fed unconventional policies: LSAP1, LSAP2, and MEP

Since the beginning of the recent financial crisis, the Federal Reserve played a key role in stabilizing financial markets. To promote liquidity in Treasury and other collateral markets and thus more generally to foster the functioning of financial markets, the Fed announced the Term Securities Lending Facility (TSLF) and conducted the first auction in March 2008. Treasury securities, held by the System Open Market Account (SOMA), were offered for loan, over a one-month term, in exchange for some eligible collateral. Securities loans were awarded to primary dealers based on a competitive single-price auction.

As US policy interest rates reached the lower zero bound later in 2008, the Federal Reserve embarked on the QE program with the aim to reduce long-term interest rates and therefore to stimulate economic activity and facilitate the recovery from the financial crisis. In particular, to support the QE objectives, the Federal Reserve launched several unconventional asset purchase programs, largely focusing on longer-term securities including government bonds and MBS.

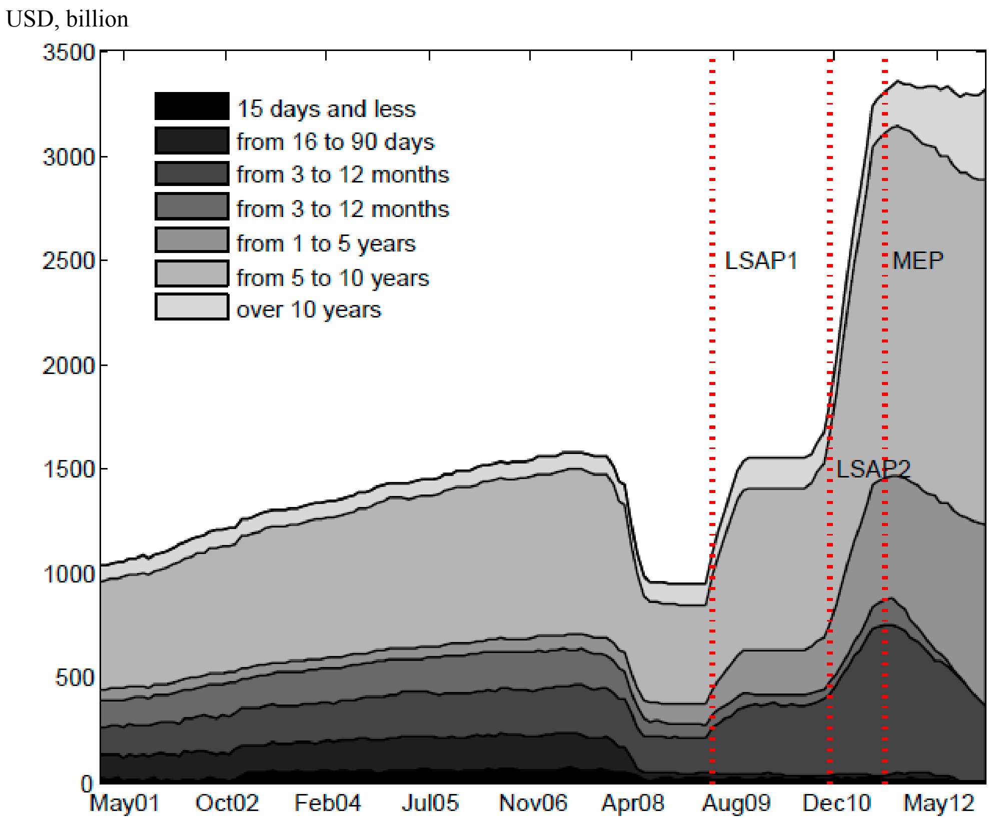
The first large scale asset purchase program took place in the period from March 2009 to March 2010 (LSAP1), when the FOMC committed to purchase \$300 billion of longer term Treasury securities and \$850 billion of agency securities in addition to the \$600 billion in MBS and agency debt announced earlier on in November 2008. As the recovery lost momentum, in November 2010 the FOMC announced \$600 billion in additional purchases of longer-term Treasury securities to be completed by mid-2011 (LSAP2). To further improve financial market conditions and provide support for the economic recovery, in September 2011, the FOMC started the Maturity Extension Program (MEP), which aimed to increase the average maturity of the Federal Reserve portfolio of Treasury securities without further expansion of the Federal Reserves' balance sheet. Under the MEP, the Federal Reserve sold a total of \$667 billion of

[5] In the notable paper examining the conundrum, Rudebusch, Swanson, and Wu (2006) conclude that although the extent of the conundrum was obvious when viewed through a macro-finance lens, much of its exact source remains unexplained.

shorter-term Treasury securities and used the proceeds to purchase longer-term Treasury securities[6].

As a consequence of the unconventional policies undertaken, the Fed's asset holdings expanded rapidly and by 2012 the Fed held about 18percent of the total Treasury securities outstanding. As the focus of the Fed's purchases shifted increasingly towards securities with longer maturities, as documented in Figure 2, the average maturity of its assets has increased and the Fed's holdings in the over-ten-year maturity segment have risen significantly, to account for about 35percent of the market.

Figure 2. Fed Treasury holdings across different maturity segments



Sources: FRED; US Department of the Treasury; BIS.

[6] The FOMC announced a $400 billion program in September 2011 that was to be completed by the end of June 2012. In June 2012, the FOMC continued the program through the end of 2012, resulting in the purchase, as well as the sale and redemption, of an additional $267 billion in Treasury securities.

Although all of these unconventional policies aim at putting downward pressure on interest rates, there are several transmission channels through which they work. To identify the UMP transmission mechanisms we have to start from the observation that returns on long-term bonds are not risk-free and can be decomposed into two components: expectations and bond premia (See for example, Krishnamurthy and Vissing-Jorgensen, 2011).[7]

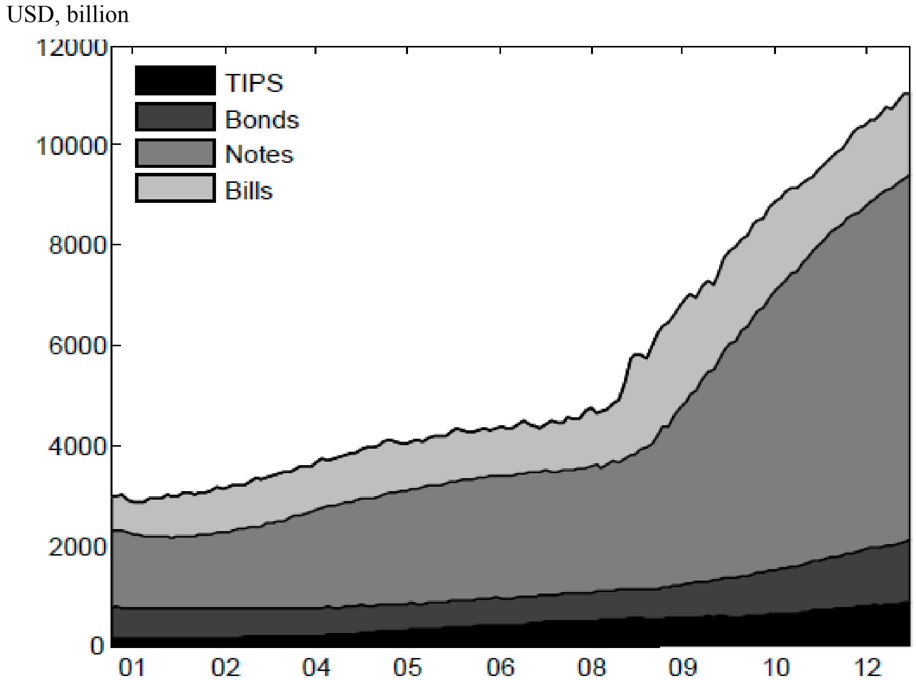
The main UMP channels are duration, scarcity and signaling, which in turn work through different interest rate components. The duration channel works by decreasing the bond risk premium component of Treasury yields, and thus it produces greater effects on longer-maturity bonds, which are more exposed to interest rate risk and thus require larger risk premia. The scarcity channel also works via the term premium component of interest rates, by affecting the price of risk. Treasury bonds trade at a price-premium due to the scarcity of assets with extremely low default risk and exceptionally high liquidity. Therefore, as the relative amount of Treasury bonds outstanding decreases, the bond price premium should go up reflecting a decreased term premium. Instead, via the signaling channel, long-term rates decline due to falls in the expectations' component, because larger and longer lasting asset purchase programs signal an ongoing loose monetary policy stance and therefore lower expected policy rates.

Beside these primary channels, there is another (by-product) channel, through which large asset purchases and quantitative easing might affect interest rates –we call it "inflation channel." With large amounts of money thrown in the economy, LSAPs could eventually increase inflation risk and inflation expectations and therefore undesirably push long-term interest rates up. Thus, there could be an upper limit on the effectiveness of LSAPs, as the downward pressure on interest rates from all the LSAP channels taken together is not unlimited and can be exerted only under monetary policy credibility and anchored inflation expectations.

Importantly, the effectiveness of the Federal Reserve asset purchase programs depends also on Treasury debt management policy (see, for example, Meaning and Zhu, 2012). In the lower interest rate environment created by LSAP programs, the US Treasury has increased the relative supply of longer-term securities (Figure 3). As a result, the average maturity of the Treasury debt outstanding has also increased, which has softened to a certain extent the demand pressures for longer-term bonds created by Fed programs.

[7] Under the pure 'expectations hypothesis' of the term structure, yields on long-term government bonds equal to expectations of average future short-term interest rates. But the expectations hypothesis does not hold in practice, as long-term returns are uncertain and risk-averse investors require a premium for holding long-term bonds relative to rolling over short-term bonds. The premium reflects the investors uncertainty about bond returns (which determines "amount of risk") and their risk aversion (which determines "price of risk").

Figure 3. US Treasury debt outstanding.



Source: US Department of the Treasury

III. LITERATURE REVIEW

Given the significance of foreign official and Fed market shares in the Treasury market, it is not surprising that a large body of empirical literature has been dedicated to study the effects of their purchases on US interest rates.

The impact of foreign demand on US interest rates has been mainly analyzed by the macroeconomic literature on 'global imbalances' (see, for example, Caballero (2006); Caballero, Farhi and Gourinchas (2008); Mendoza, Quadrini and Rios-Rull (2007); and Caballero and Krishnamurthy (2009)). Available empirical studies are mostly reduced-form (Warnock and Warnock (2009), Krishnamurthy and Vissing-Jorgensen (2012); and Sierra (2010)) and do not provide a unanimous estimate of the effect of foreign intervention on US Treasury yields. For example, Warnock and Warnock (2009) find that the fall in the 10-year rate associated with foreign official purchases of US Treasuries is roughly 80 basis points. Krishnamurthy and Vissing-Jorgensen (2012) find that foreign official purchases reduce the supply of safe assets available to the rest of investors and hence drive up the convenience yield. They also find that if foreign officials were to sell their holdings, long-term Treasury yields would be raised by 59 basis points relative to the Baa corporate bond yield. Sierra (2010), through a series of forecasting regressions of realized excess returns on measures of net purchases of treasuries, finds that official flows behave similarly to relative supply shocks. On balance, nonetheless, Sierra (2010) finds little role for foreign investors in reducing 10-year yields on U.S. Treasury bonds. However, this literature lacks structural estimation of the foreign demand effects within a model of the term structure of interest rates.

One paper that estimates foreign demand effects on US bond prices, while imposing the discipline of no-arbitrage, is Kaminska, Vayanos, and Zinna (2011). To explain how demand for US Treasuries can affect the term-structure of interest rates, their paper builds on the "limited arbitrage" model of Vayanos and Vila (2009), in which, similarly to Modigliani and Sutch (1966), investor clienteles with preferences for specific maturities, the so called preferred-habitat investors, could play an important role for bond pricing. Specifically, Vayanos and Vila (2009) set up a formal model of two types of agents: investors with a preferred habitat for specific maturities and risk-averse arbitragers. Nevertheless, in Kaminska, Vayanos, and Zinna (2011), aggregate demand is introduced and estimated as an unobserved factor, and hence it is difficult to disentangle the specific impact of foreign official demand.[8]

Rigorous no-arbitrage term-structure models have appeared to be more popular for the analysis of Fed purchases. For example, to evaluate the effects of LSAPs, Li and Wei (2012) use a no-arbitrage term-structure model with supply factors, in which they introduce observable supply factors derived from the data on private holdings of Treasury debt and agency MBS. To facilitate the estimation, they assume that supply factors influence Treasury yields predominantly through the term-premium channel, thus focusing implicitly on the scarcity and duration channels through which LSAP works to reduce longer term Treasury yields. The vast majority of empirical analyses of the LSAP-style operations, however, are based on either event studies (e.g. D'Amico and King (2013), Gagnon, Raskin, Remache, and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011), Swanson (2011)) or time series regressions of Treasury yields and their components on demand related variables (e.g. D'Amico, English, L´opez-Salido, and Nelson (2012), Greenwood and Vayanos (2013); Krishnamurthy and Vissing-Jorgensen (2011); Meaning and Zhu (2012)). Many of these papers highlight the importance of the preferred-habitat nature of Fed demand, but none of the papers estimate a term-structure model with preferred-habitat demand explicitly. Of particular note are Hamilton and Wu (2012), who derive and estimate a simplified version of the preferred-habitat model by Vayanos and Vila (2009), and then augment the analysis by a forecasting regression exercise to quantify the Fed's impact on the yield curve.

Although all studies document the efficacy of the Federal Reserve's asset purchase programs, the range of the estimated LSAPs' impact on US Treasury yields is quite large, with estimates of the nominal ten-year Treasury yield impacts ranging from 35 to more than 160 basis points. Notably, there is general agreement that the Fed impact on nominal yields is mostly felt through the real rate component, and, in particularly, through the real term premium. Indeed, studies of

[8] Also note that for Kaminska, Vayanos, and Zinna (2011) the only source of preferred-habitat demand comes from foreign official investors, while they do not investigate the impacts of QE policies.

nominal Treasury bond reactions find that the impact of LSAP-style operations on longer-term interest rates is mainly experienced by the nominal term-premium component (see for example, D'Amico, English, L´opez-Salido, and Nelson (2012), IMF (2013), or Li and Wei (2012)), suggesting that Fed's purchases affected long-term yields mainly through the scarcity and duration channels. In addition, recent studies of inflation-protected Treasury securities show that long-term real forwards significantly comove with monetary policy changes and that these effects are primarily due to changes in real term premia around FOMC announcement days. For example, Abrahams, Adrian, Crump, and Moench (2013) indicate that real long-term forward rates move strongly on days of monetary policy announcements and provide strong support for the notion that one transmission channel of interest rate policy is via the equilibrium pricing of risk in the economy. Similarly, Hanson and Stein (2012) find that the nominal term premium has decreased mostly due to the depression of its real term premium component.

IV. MODEL

We build our term-structure model on the limited arbitrage preferred-habitat framework of Vayanos and Vila (2009). In their model, two types of investors integrate maturity markets: *preferred-habitat investors* with strong preferences for specific maturities and *arbitrageurs,* who do not have maturity preferences but trade bonds of any maturity for return considerations, making the term structure arbitrage free. Arbitrageurs not only deal with the disconnect between the short rate and bond yields, but also bring yields in line with each other, smoothing local demand and supply pressures. However, because arbitrageurs are risk-averse, preferred-habitat demand still matters and therefore determines the equilibrium interest rates. The rest of the Section introduces the main elements of the model, while more details are given in Appendix I.

The model is set in continuous time, so that the term structure is represented by a continuum of zero-coupon bonds, with bond maturities, τ, in the interval $(0; T]$. The risk free instantaneous rate r_t , which is the limit of the spot rate of maturity τ, $R_{t,\tau}$, when τ goes to zero, follows the Ornstein-Uhlenbeck process:

$$dr_t = \kappa_r (\bar{r} - r_t)dt + \sigma_r dB_{r,t} \qquad (1)$$

where $(\bar{r}; \kappa_r; \sigma_r)$ are positive constants and $B_{r,t}$ is a Brownian motion. Preferred-habitat investors form maturity clienteles, with the clientele for maturity τ only buying the bond with the same maturity. The demand for the bond with maturity τ is assumed to be a linear function of the bond's yield $R_{t,\tau}$:

$$y_{t,\tau} = \alpha(\tau)\, \tau(R_{t,\tau} - \beta_t), \qquad (2)$$

where $\alpha(\tau)$ is a positive function of maturity. Thus increases in β_t are associated with a decreasing excess demand $y_{t,\tau}$. Similarly to the original version of the model, we assume that the intercept β_t takes the form

$$\beta_t = \sum_{k=1}^{K} \theta_k \beta_{t,k}, \qquad (3)$$

where $\beta_{t,k}$ are various demand factors. Vayanos and Vila (2009) suggest that these demand factors could capture changes in the needs of preferred-habitat investors (arising because of changes in policies, pension funds' liabilities or regulation, etc.), or changes in the size or composition of the preferred-habitat investor pool, or changes in the available supply of bonds issued by the government, therefore note that increases in β_t can also capture increases in the supply as in Greenwood and Vayanos (2013).[9] Our hypothesis is that these demand factors are influenced by the strong accumulation of reserves by foreign officials and large scale bond purchases by the Fed. Indeed, as Section 2 documents, official demand for long-term US Treasury securities is high and, moreover, not fully elastic. In light of these considerations, we assume that β_t composes of a foreign-specific demand factor, $\beta_{t,For}$, and a separate Fed-specific demand factor, $\beta_{t,Fed}$, so that

$$\beta_t = \theta_1 \beta_{t,For} + \theta_2 \beta_{t,Fed}. \qquad (4)$$

The aggregate demand factor β_t follows the Ornstein-Uhlenbeck process

$$d\beta_t = \kappa_\beta(\bar{\beta} - \beta_t) + \sigma_\beta dB_{\beta,t}, \qquad (5)$$

where $\bar{\beta}$ is the unconditional mean, $(\kappa_\beta, \sigma_\beta)$ are positive constants and $B_{\beta,t}$ is a Brownian motion. We therefore specify the law of motion of the aggregate demand factor, and not of its components. By imposing this assumption we limit the number of parameters entering the pricing of equilibrium bond yields and therefore preserve the model tractability.[10] Thus, the bond pricing depends on the parameters describing the dynamics of two-factors; the short rate of equation (1) and the aggregate demand factor of equation (5). Specifically, the parameters θ_1 and θ_2, though can affect the estimate of β_t, do not enter directly into the bond pricing recursions (presented in Appendix I).

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[9] In the original version of Vayanos and Vila (2009), $\beta_t = \sum_{k=1}^{K} \theta_k^m \beta_{t,k}^m$, where m denotes the maturity, and therefore there are local demand effects. However, we abstract from introducing local demand effects due to the lack of maturity-specific data on official holdings, foreign in particular. Analysis of the local supply effects of QE on bond-by-bond data is provided, for example, by D'Amico and King (2013).

[10] By introducing a third state equation in the model, by separately modeling the dynamics of $\beta_{t,For}$ and $\beta_{t,Fed}$, bond prices would depend on a system of nine non-linear difference equations. The benefit of improving the model fit, resulting from the introduction of a third factor, would come at the cost of solving five additional equations that would seriously compromise the possibility to take the model to the data.

The presence of arbitrageurs guarantees that bonds with maturities in close proximity trade at similar prices and that an equilibrium no-arbitrage price is established. For taking the risk of buying or selling bonds of different maturities, arbitrageurs demand compensation in a form of risk premia. We assume that arbitrageurs' investment strategy follows a mean-variance portfolio optimization, such that the arbitrageurs' optimization problem is given by

$$\max_{\{x_{t,\tau}\}_{\tau\epsilon(0,T]}} \left[E_t(dW_t) - \frac{a}{2} Var_t(dW_t)\right], \qquad (6)$$

with a denoting arbitrageurs' risk-aversion coefficient, $x_{t,\tau}$ denoting their dollar investment in the bond with maturity τ and W_t arbitrageurs time-t wealth. Arbitrageurs' budget constraint is assumed to be

$$dW_t = \left(W_t - \int_0^T x_{t,\tau}\right) r_t dt + \int_0^T x_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}}, \qquad (7)$$

where $P_{t,\tau}$ is the time-t price of the bond with maturity τ that pays $1 at time t + τ. Assuming that equilibrium spot rates are affine in the risk factors r_t and β_t,

$$R_{t,\tau} = A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau), \qquad (8)$$

and imposing equilibrium $x_{t,\tau} = -y_{t,\tau}$, we can solve for $A_r(\tau), A_\beta(\tau)$, and $C(\tau)$ through a system of linear ODEs.

Finally, the expected excess return of inflation-indexed Treasury bond over the risk free rate at any maturity τ is given by

$$\mu_{t,\tau} - r_t = A_r(\tau)\lambda_{r,t} + A_\beta(\tau)\lambda_{\beta,t} \qquad (9)$$

where $\lambda_{r,t}, \lambda_{\beta,t}$ are factor risk premia:

$$\lambda_{r,t} \equiv -a\sigma_r \int_0^T y_{t,\tau}[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)]\, d\tau \qquad (10)$$

$$\lambda_{\beta,t} \equiv -a\sigma_\beta \int_0^T y_{t,\tau}[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)]\, d\tau, \qquad (11)$$

where ρ is a factor correlation between r_t and β_t. Thus, returns in excess of the risk free rate are a linear function of the bond's sensitivities to the risk factors. This result is a general consequence of the no-arbitrage assumption. The economic content of our model is instead in the factor risk premia. Specifically, equations (10) and (11) show that, at any particular maturity τ, the factor risk premia directly relate to the demand for a bond with that maturity. For this reason, the model is able to capture both the scarcity and duration channels that work through changes in the prices of risk.

V. ESTIMATION

A. Data

Previous research shows that demand pressures on interest rates work mostly through the real term premium component (e.g. D'Amico et al (2012)). Thus, if we want to estimate the direct impact of demand, we need to bring the model to real interest rate data.[11] The key source of the data on market real rates is inflation-indexed bonds. In the United States, inflation-indexed bonds are issued by the US Treasury and their principals are adjusted to the consumer prices index (CPI). Since its launch in 1997, the market for Treasury Inflation-Protected Securities (TIPS) has grown considerably and now represents the largest and the most liquid market for inflation-indexed bonds.

The zero-coupon equivalent US real rates we use in this paper are from the Fed's TIPS-yields estimates, where both on-the-run (newly issued) and off-the-run (previously issued) bonds are included in estimation of the TIPS yield curve. We assume that there is no particular liquidity premium in on-the-run TIPS securities.[12] Although data on US real yields are available back to 1999, we have restricted our sample period to start from January 2001 and so we exclude initial years, when TIPS yields were systematically affected by a lack of liquidity.

Our data set spans the period from Jan-2001 to Nov-2012. The inclusion of more maturities improves the precision of the parameter estimates entering the bond pricing; thus, we use real yields of 2-, 3-, 4-, 5-, 7-, 10-, 15- and 20-yr maturities. The lack of short-maturity TIPS prior to 2004 implies that real market yields on two-year TIPS is available only from January 2004. Table 1 presents the summary statistics of the yields (the time series for selected maturities could be found in the top panel of Figure 5). Several observations are worth noting. To start with, the average real yield curve is upward sloping. But from 2001 to 2005 long-term real interest rates fell substantially, flattening and eventually even inverting the curve. After the slight recovery in 2006-07 the rates experienced dramatic swings during the financial crisis starting in the second

[11] One additional benefit of modeling real rates rather than nominal rates is that we can use a more parsimonious specification consisting of two factors. If we instead were to model nominal rates, a third factor capturing the inflation dynamics would be required. Of course one caveat of modeling real rates is that our analysis is silent on the impact of official demand on expected inflation and inflation risk premia, i.e. the so called inflation channel introduced in Section II.

[12] Although TIPS are widely considered to be less liquid than nominal U.S. Treasury bonds, a consensus on the precise level of the TIPS liquidity premium has yet to develop. According to Christensen and Gillan (2012), the liquidity premium is relatively small and does not display significant variation during normal times; the term structures of the TIPS liquidity premiua and their volatilities tend to be downward sloping with maturity.

half of 2007. The odd behavior of real yields at the end of 2008, when real rates spiked dramatically and at some point exceeded nominal rates, coincided with the deflation episode. Around this time the spike in yields also reflects the short-lived drop in liquidity that possibly resulted from tensions in the repo market generated by Lehmans' default (Campbell, Shiller and Viceira, 2009). After 2011 and up to the end of 2012, medium- to long-term real interest rates turned negative and stayed deep in the negative territory. As to the second moments, short-term yields are more volatile than long-term bond yields, with volatility gradually decreasing with maturity.

Table 1. Yield summary statistics and PC analysis

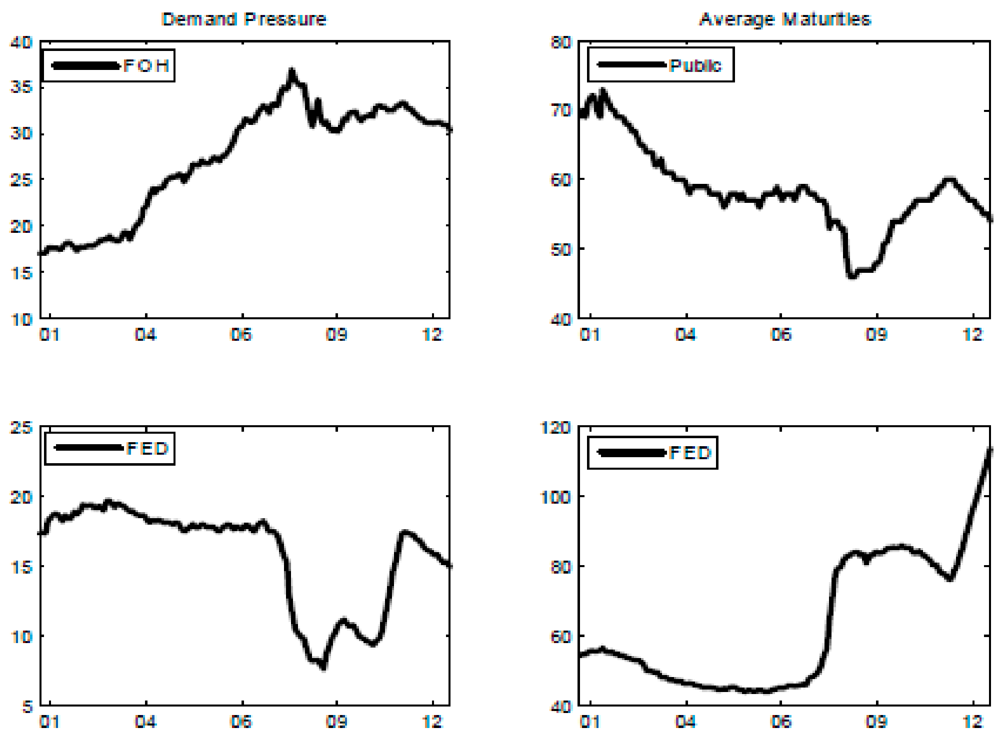
		Yield Summary Statistics							
		2	3	4	5	7	10	15	20
Mean		0.55	0.66	0.80	1.28	1.55	1.83	2.08	2.19
Std.Dev.		1.50	1.36	1.27	1.24	1.15	1.01	0.85	0.77
AC		94.0	94.6	94.6	94.3	94.2	93.8	93.2	92.8
Max		5.01	4.24	3.84	3.62	3.54	3.57	3.59	3.61
Min		-2.06	-1.76	-1.58	-1.50	-1.24	-0.82	-0.24	0.09
		Yield Loadings							
	EV	2	3	4	5	7	10	15	20
PC1	94.5	0.47	0.44	0.41	0.39	0.34	0.28	0.22	0.18
PC2	5.25	0.57	0.26	0.06	-0.09	-0.26	-0.37	-0.44	-0.46
PC3	0.22	0.54	-0.08	-0.32	-0.37	-0.27	-0.03	0.29	0.55
PC4	0.02	-0.29	0.27	0.32	0.16	-0.25	-0.54	-0.19	0.57

Note: Top panel (Yield Summary Statistics) presents mean, standard deviation, autocorrelation, maximum and minimum values of the yields for the 2-, 3-, 4-, 5-, 7-, 10-, 15- and 20-yr maturities for the estimation period from Jan-2001 to Nov-2012. Note that yields were not available for the 2-, 3- and 4-year maturity prior to Jan-2004. Bottom panel (Yield Loadings) presents the principal component (PC) loadings for the term structure of real yields from Jan-2004 to Nov-2012. Column EV shows the proportion of the total variance explained by each PC.

According to the principal component (PC) analysis presented in the bottom panel of Table 1, two factors (PCs) explain more than 99.76percent of the real yields data. The first principal component has large explanatory power for short maturities, while the second principle component's loadings decrease from large and positive values at short maturities to large and negative values at long maturities, which is typical of the slope factor. The findings are consistent with the two-factor model described in Section 4; there the first factor describes movements of the unobserved real short interest rate, while the second factor represents preferred-habitat demand and is related to official demand pressures.

We specify the preferred-habitat demand factor as a function of observable measures capturing the pressure exerted by both foreign officials and the Fed. Figure 4 presents several measures of such demand pressures.

Figure 4. Measures of demand pressures in the US Treasury market.



Note: This figure presents several measures of demand pressure; Foreign Official holdings of long-term Treasuries over the total amount outstanding of Treasury securities (top left panel); Federal Reserve holding of Treasury securities over the total amount of Treasury securities outstanding (bottom left panel); the average maturity (in months) of Treasury securities held by the public (top right panel); and the average maturity (in months) of the Federal Reserve portfolios of Treasury securities (FED).

The top left panel presents the measure of foreign official demand, $\beta_{t,For}$, which is proxied by Foreign Official holdings of long-term Treasuries. Crucially, to account for the role of supply in the model, we scale the foreign holdings by the total amount of outstanding Treasury securities.[13] The adjusted foreign holdings are estimated from the data by Bertaut and Tryon (2007) and address many of the drawbacks of the original TIC system, which provides monthly data on Foreign Official Holdings of US long-term Treasury securities. Most importantly, the TIC data do not account for acquisitions through a third-country intermediary ("indirect transactions"). So the split between foreign officials and foreign investors in the TIC data is blurred (Warnock and

[13] Our proxies of demand pressures with respect to supply are ex-post measures in the sense that we do not address explicitly the concern that Treasury supply might be endogenous to demand. As Greenwood and Vayanos (2013) explain, the government might tailor its debt issuance to fluctuations in investor demand, mitigating and potentially even reversing any positive relationship that would otherwise obtain between supply and yields or expected returns. Estimation of Treasury supply reaction function and isolating ex-ante preferred-habitat demand impacts is outside the scope of this paper.

Warnock (2009)). For example, the reported TIC securities transactions data understate foreign official acquisitions of long-term US Treasuries, whereas UK private holdings are often overstated. By contrast, infrequent benchmark survey of positions provide a more truthful and accurate portrait of foreign official holdings of TIC securities and show that there is often a discrepancy between the measured value of Treasury securities held by foreign official investors as identified in the annual survey, and what results from summing official transactions as-reported in TIC since the last survey. This discrepancy remains even after making several needed adjustments, like taking into account price changes (see Bertaut and Tryon (2007)). By knowing this discrepancy, one can infer the more accurate values of official purchases from the numbers reported in the TIC system.[14] Using the TIC data to analyse real rather than nominal yields presents another caveat, as the TIC includes both Treasury bonds and TIPS, so that disaggregated data on the foreign ownership of TIPS are not available. However, it seems unlikely that this inconsistency should undermine the use of TIC data to construct a proxy for our estimated preferred-habitat demand pressure. In fact, foreign demand at TIPS auctions has been remarkably strong averaging around 39percent (Gongloff (2010)), thus, showing that not only foreign demand for Treasury bonds but also for TIPS has been significant. Furthermore, because one should be able to replicate the pay-offs of Treasury bonds via a combination of TIPS, STRIPS and inflation swaps, the assumption that demand pressure on one Treasury market transmits to the other market seems sensible.

We estimate Fed demand factor $\beta_{t,Fed}$, as a combination of several observed variables, which should capture different aspects of official demand. The first variable, similarly to the foreign demand pressures, is determined as the Fed's holdings of Treasury securities over the total amount of Treasury securities outstanding. However, the Fed can affect Treasury bond yields not only by changing the size of their bond holdings but also by changing the composition of its bond portfolio. Following Meaning and Zhu (2012), among others, we argue that the maturity structure of the Fed's Treasury holdings is a good indicator of the portfolio's composition and estimate a Fed demand pressure as a combination of their "size pressure" and "maturity pressure", where the latter is captured by the average maturity of the Federal Reserve Portfolio of Treasury securities and the average maturity of Treasury securities held by the public. In essence, by changing the average maturity of its portfolio, the Fed can alter the riskiness of the portfolio of the public (proxying for the arbitrageurs' portfolio) and therefore the bond risk premia.[15]

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[14] Warnock and Warnock (2009) firstly introduced a formula to distribute this error and estimate monthly positions between surveys. But Bertaut and Tryon (2007) have improved even further the estimation technique. Bertaut and Tryon (2007) monthly estimates of TIC data are available as a link from their Discussion Paper: http://www.federalreserve.gov/pubs/ifdp/2007/910/default.htm. They update these data files whenever there is a new survey release.

[15] By including measures of the average maturity of the Fed and public's portfolios we try to capture the duration channel. The mechanism in the preferred-habitat model is as follows; increases in the duration, or maturity, of the preferred-habitat portfolio are matched by decreases in the duration of the arbitrageurs' portfolio, which lead to a drop in the riskiness to which arbitrageurs are exposed and therefore in the compensation required in the form of bond risk premia. In principle, we could have also

B. Econometric methodology

In our model, yields are affine functions of normally distributed factors; and as such, classical statistical method of maximum likelihood could be employed. However, we choose to estimate the model using Bayesian techniques; in particular, we draw on Markov Chain Monte Carlo (MCMC) methods. We choose Bayesian estimation methods for several reasons. First, in our case the likelihood is highly non-linear because bond prices are complex highly nonlinear functions of the parameters, which significantly complicate the numerical optimization. By contrast, Bayesian methods rely on simple block simulations. Second, the draws resulting from the Bayesian algorithm allow us to quantify the uncertainty around post-estimation calculations (e.g. for loadings, term premia, reduce-form regressions etc.), which would be difficult to do by classical methods (Kim and Nelson, 1999). Third, in a Bayesian framework we can easily specify priors and impose constraints on the parameters (Johannes and Polson, 2004). Instead, parameter constraints may compromise further the performance of optimization algorithms needed in maximum likelihood. All these caveats complicate the convergence of the optimization in a frequentist setting, while with the chosen Bayesian method we can easily assess the convergence and incorporate maximum likelihood information into the algorithm (Chib and Ergashev, 2009).

The model is estimated in a state-space framework. In our setting, the state (or, 'transition') equation describes the evolution of the short rate and demand factors under the objective probability measure, while the space (or, 'measurement') equation maps these two factors into the observed real rates of selected maturities. In particular, the state vector is $X_t = [r_t, \beta_t]'$, and the vector of observed yields is $R_t = \left[R_t^{2yr}, yR_t^{3yr}, \dots, R_t^{20-yr}\right]'$, whereas the vector of observed demand factors, described in Section IV, is $D_t=[FO_t, Fed_t, MatFed_t, MatPub_t]'$. Specifically, FO_t is the fraction of foreign officials' holdings of long-term Treasury securities over the total amount of Treasury securities outstanding; Fed_t is the fraction of the Fed's holdings of Treasury securities over the total amount of Treasury securities outstanding; $MatPub_t$ is the log of the average maturity (in months) of Treasury securities held by the public; and $MatFed_t$ is the log of the average maturity (in months) of the Fed's portfolio of Treasury securities. As a result, the state-space representation in discrete time can be written as:[16]

State: $$X_{t+\Delta} = G(\mathrm{P}_1) + F(\mathrm{P}_1) + u_{t+\Delta,} \quad u_t \sim N(0, Q) \qquad (12)$$

Space:
$$Y_{t+\Delta} = f(\mathrm{P}, X_{t+\Delta}) + \varepsilon_{t+\Delta,} \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2 I_n) \qquad (13)$$

standardized the maturity of the Fed's portfolio by the maturity of the public portfolio; however, the public is not a direct proxy of the arbitrageurs at it also includes investors like pension funds that are more alike preferred-habitat investors.

[16] To discretize the continuous-time specification of the factors in equation (1) and (5), we use the Euler scheme.

$$\beta_{t+\Delta} = \gamma D'_{t+\Delta} + \epsilon_{t+\Delta,} \qquad\qquad \epsilon_t \sim N(0, \sigma_\epsilon^2) \qquad\qquad (14)$$

where, to simplify the notation, we group the parameters of our model as $\mathrm{P}_1 = (\rho, \sigma_r, \sigma_\beta, k_r, k_\beta, \bar{r}, \bar{\beta})$, $\mathrm{P}_2 = (a, \delta, \alpha)$, and P=($\mathrm{P}_1$,$\mathrm{P}_2$). The system matrices take the form of $G(\mathrm{P}_1) = \begin{bmatrix} k_r r\Delta \\ k_\beta \bar{\beta}\Delta \end{bmatrix}$, and $F(\mathrm{P}_1) = \begin{bmatrix} 1 - k_r\Delta & 0 \\ 0 & 1 - k_\beta\Delta \end{bmatrix}$, the factors' variance-covariance matrix is $Q = \Delta \begin{bmatrix} \sigma_r^2 & \rho\sigma_r\sigma_\beta \\ \rho\sigma_r\sigma_\beta & \sigma_\beta^2 \end{bmatrix}$; and σ_ε^2 is the common variance of the independent and normally distributed measurement errors, $\varepsilon_{t+\Delta}$. The time step Δ at a monthly frequency is 1/12 **.** Note that the transition equation is a function only of the parameters P_1, while function $f(\cdot)$ is the bond pricing function that maps the parameters and the states into the vector of observed yields. Specifically, the real rates are an affine function of the observed factors with loadings that are complex and highly nonlinear functions of the market price of risk parameters P_2, dynamics parameters P_1, and maturity. Finally, equation (14) is the additional space equation, i.e. empirical counterpart to equation (4), where the vector of coefficients γ links the dynamics of $\beta_{t+\Delta}$ to the observed factors, but does not enter the bond pricing. Also note that $\epsilon_{t+\Delta,}$ may reflect other potential determinants of $\beta_{t+\Delta}$ that are not captured by our foreign and Fed observable factors.

To facilitate the estimation of such a complex model, we rely on MCMC methods. Given that in our model the observation equation is highly nonlinear in the parameters, the functional form of the density is non-analytic. Therefore we use a MCMC algorithm to update the parameters and sample by using Metropolis-Hastings steps within the Gibbs sampler. By combining the prior distribution with the likelihood function we get the posterior distribution. In particular, we sample from the joint posterior distribution of model parameters and latent factors. The details of the estimation are given in the Appendix II.

Also note that we fix some parameters to facilitate model identification. For example, we assume that $\alpha(\tau) \equiv \alpha \exp\{-\delta\tau\}$, where the parameter δ is fixed to 0.1. The unconditional mean of the short rate process, r, is assumed to be 2percent, consistent with the standard assumption on the natural rate. This tells us where the short rates converge in the very long run, rather than where they should be today. (We verified that the estimation results are robust to the choice of this parameter.) Besides, because the arbitrageurs' risk aversion a and the demand elasticity α are not separately identified (see Kaminska, Vayanos, and Zinna, 2011), we estimate the product of the two $(a\alpha)$. Finally, although the priors are uninformative, several parameters are subject to constraints. For example, the factors are stationary and arbitrageurs risk aversion cannot be negative. We discard the draws that do not meet these conditions.

VI. ESTIMATION RESULTS

A. Model fit, estimated factors and bond premia

The model seems to perform rather well. All of the state parameters are statistically significant, and the numerical standard errors and CD diagnostics suggest that the chain has converged.[17] In this section we present the results from the model estimation.

Table 2. Estimated parameters of the model

	mean	lb	ub	nse	CD
κ_r	0.23	0.11	0.38	0.0027	1.52
$\overline{r}$	2.00	-	-	-	-
κ_β	0.15	0.03	0.27	0.0027	1.23
$\overline{\beta}$	2.37	2.09	2.72	0.0064	1.54
σ_r	2.21	2.11	2.31	0.0026	0.42
σ_β	0.72	0.69	0.76	0.0008	0.21
ρ	24.63	17.76	31.02	0.1521	0.58
$a\alpha$	42.56	36.17	48.46	0.1345	1.05
δ	0.1	-	-	-	-
σ_ε	8.41	8.03	8.67	0.0134	0.99

Note: The table presents the posterior mean, the lower and upper bounds of one-standard deviation credible intervals (lb and ub), the numerical standard errors (nse), and the absolute value of the convergence diagnostic (CD), as in Geweke (1992), for the estimated parameters. These estimates result from the Bayesian estimation, described in Section V.B, based on monthly US real rates from Jan-2001 to Nov-2012 for the 2-, 3-, 4-, 5-, 7-, 10-, 15- and 20-yr maturities.

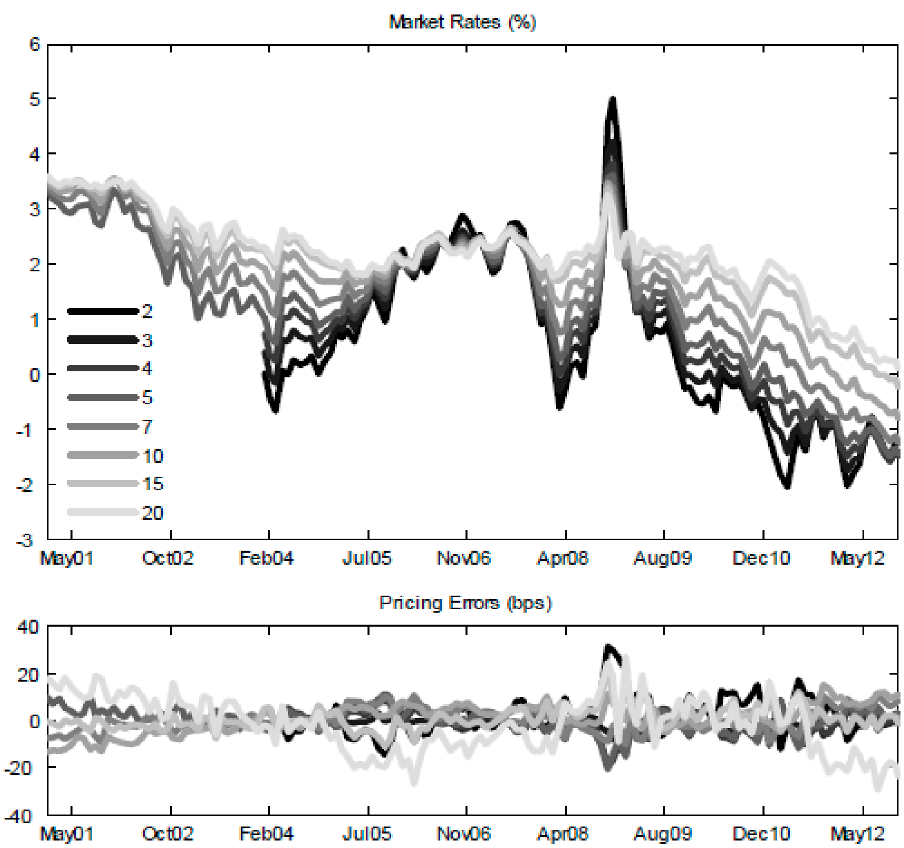
Table 2 presents parameter estimates. The parameters, κ_r and κ_β, determining the factors' speed of mean reversion are significantly larger than zero, meaning that the state variables do not behave as random walks and the real rates are stationary time series. The rate of mean reversion for each factor can be measured by computing mean half-lives. In the model, the mean half-lives are around 3 years for the first factor and around 5 years for the second factor. The estimates are in line with typical estimates in the standard term-structure literature, where the factors with longer mean half-lives are usually needed to determine variation of the longer term bond rates.

[17] CD is distributed as standard normal, thus values of CD less than 1.96, in absolute value, support the convergence of the Markov chain Monte Carlo.

The correlation coefficient between the shocks to two model factors is estimated to be positive (ρ=0.24), which implies that the short-rate factor, which could be interpreted as a real policy rate, and demand pressures are interdependent. In particular, higher excess supply (or, equivalently, lower demand pressures) are associated with higher policy rates. This finding accounts for a plausible reaction function of the Fed, especially during the unconventional monetary policy regime, when longer lasting Fed bond purchases are coupled with the prolonged expectations of depressed policy rates.

In the model, the arbitrageurs' risk aversion a determines bond risk premia. The price of risk parameters, α and a, that determine both the time variation in the term premia and the average excess return, are significantly different from zero ($a\alpha=42.56$). Thus the results are consistent with the earlier findings that real bond term premia are time varying (see e.g. Hanson and Stein, 2012).

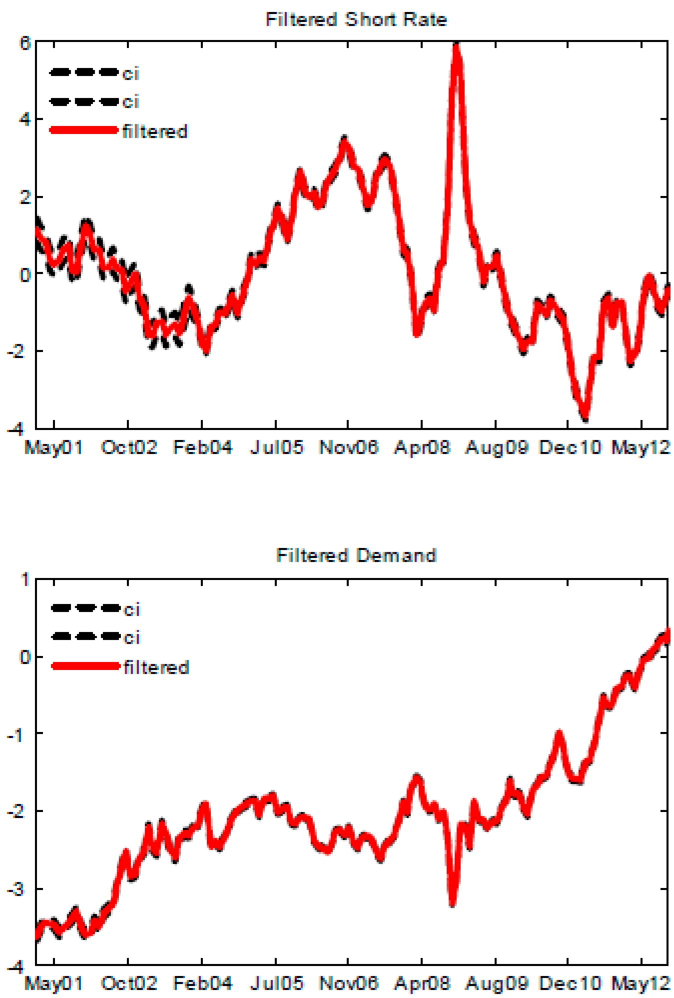
Figure 5. Observed yields and measurement errors



Note: Top panel presents the term structure of the US real rates for the 2-, 3-, 4-, 5-, 7-, 10-, 15- and 20-yr maturities at a monthly frequency spanning the period from Jan-2001 to Nov-2012. The data are obtained from the Federal Reserve and are TIPS-yields estimates. Bottom panel presents the measurement errors, which are computed as observed rates minus model implied rates. Model implied rates are computed using parameter estimates and smoothed estimates of the factors resulting from the MCMC model estimation described in Section V.B.

The standard deviation of yield fitting errors is small (σ_{ε}^{2}=8.41). However, although the model fits the yield curve reasonably well, the very long end (20-year maturity) is not fully captured (See Figure 5). One plausible explanation is that very long-term real bonds are subject to strong local demand pressures by institutional investors such as pension funds (Greenwood and Vayanos, 2010). The measurement errors are also large for all yields during the 2008-2009 crisis, when inflation protecting securities suffered from deflation and illiquidity issues. But overall, given that at any point in time and for any fitted maturity the measurement errors do not exceed 30 basis points in absolute values, we find that this model delivers an accurate empirical performance.

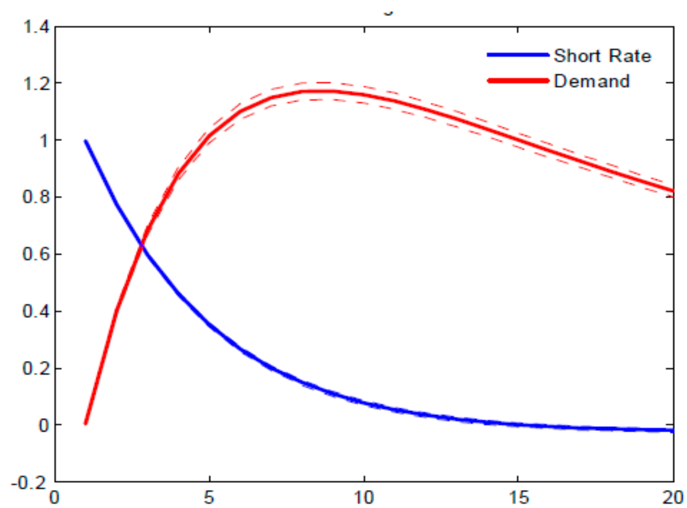
Figure 6. Estimated factors



Note: Smoothed factors with one-standard deviation credible intervals. Top plot refers to the short-term real interest rate (first unobservable factor, r_t). Bottom panel presents the smoothed demand factor ($d_t = -\beta_t$).

More importantly, the model estimation enables us to generate the time series of the two factors, which are presented in Figure 6. The factors behave quite differently throughout the sample. During 2002-2005, the demand factor was increasing, while the short-rate factor was decreasing. Subsequently, the short-rate factor picked up and then grew steadily until the financial crisis started in 2007, at the same time when the demand factor seemed to stabilize. The sharp movements in the short-rate factor during the crisis, when rates vacillated between 6percent and -4percent, coincided with the deflation episode, heightened illiquidity in the markets and sharp adjustments in policy rates. Since then, the short-rate factor remained below zero. In contrast, the demand factor resumed its growth around the outset of the crisis, though at a higher rate than over the earlier part of the sample.

Figure 7. Factor loadings



Note: This figure shows the effect of a rise in the short-term real rate (blue) and the effect of a decrease in demand (red) on the term structure of spot rates for maturities from 1 to 20 years. Dotted lines denote the credible intervals.

The analysis of the factor loadings helps us shed more light on the response of rates at different maturities to changes in the estimated factors. In this model, as shown in equation (8), the yields are assumed to be a linear function of the two unobserved factors with maturity-specific time-invariant factor loadings. Figure 7 shows the factor loadings for different maturities. It is apparent that the coefficients on the short-rate factor decrease quickly as time to maturity increases and are negligible for maturities longer than 15 years. In contrast, the coefficients associated with a drop in demand, i.e. with an increase in β_t, are basically zero at very short maturities, reach their maximum around 10 years and are an approximate unit for subsequent maturities. Taken together these results suggest that the short-rate factor has a strong influence on short term rates and a diminishing effect on long-term rates, while longer rates are mostly driven by the demand factor. This result, coupled with the evidence in Christensen and Gillan (2012), showing that liquidity premia decrease with the maturity, explains why the short-lived episode of increased illiquidity in the TIPS market is largely picked up by an increase in the short-rate in our model. Our estimated demand factor displays a more subdued response to this short-lived illiquidity episode than the short rate.

This framework postulates that demand pressures may affect interest rates mainly through the scarcity and duration channels, because demand factor enters explicitly the risk premium specification (see equations (9)-(11)). However, to the extent that the demand and short-rate factors are correlated, shocks to demand factors could also affect the expected short rate. As a result, the signaling channel could also play a role—albeit indirect and of second order—in our model. The channels, through which the demand factor affects long-term rates, become more obvious when we look closer at the decomposition of interest rates on expectations and risk premia components.[18] Figure 8 shows this decomposition for the five- and ten-year rates. The estimated term premia averaged above 100 basis points in the period up to 2004, while they fall below zero in 2005. Then they remained around zero until the crises started, and fell dramatically into negative territory in the aftermath of the crisis. The falls in the term premium coincide with two important episodes in the recent history of the US long-term rates: the so-called conundrum period (2004-2005) and the zero policy rate environment, when the Fed's interventions aimed to depress longer-term interest rates.

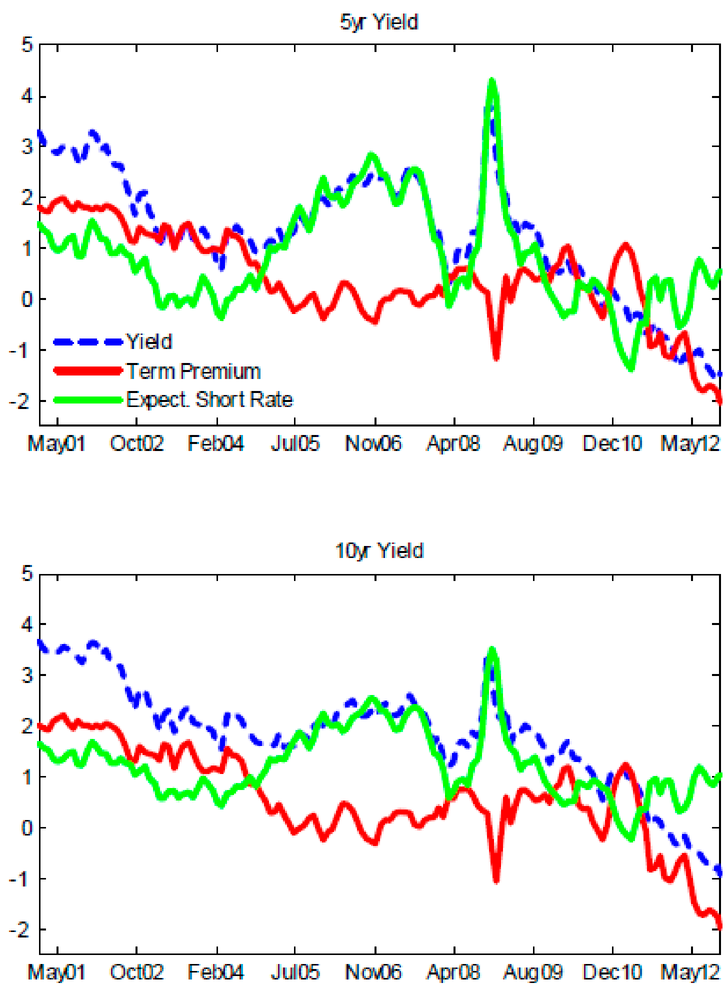
The decrease in long rates during 2004-05 period, the conundrum period, was associated with increasing average expected short rates. And this upward trend continued until the start of the crisis. Therefore, the behavior of expected risk-free rates is not puzzling: they indeed followed the rising policy rates. In contrast, the long-term real rates did not follow the increase in the expected real policy rates as they were driven by a rising demand and the associated drop in real bond premia. Thus, on the basis of the preferred-habitat model, it looks as if the conundrum was never there. The puzzle in the behavior of US real rates was a result of the wrong beliefs that long-term real rates reflect only the expectations of future real policy rates.

Moreover, it is apparent from Figure 8 that the post-2009 decrease in the long-term rates is attributed mainly to lower expectations of policy rates. In contrast, the consequent decrease in the long-term rates, especially from 2011 onwards, is mainly due to a decreasing term premium, while policy rates remained at zero and expectations stabilized. Thus the model shows that the term premium on real bonds is extremely important in explaining movements in long real rates during the Fed's policy interventions. In turn, as it is evident from a comparison of Figure 6 (bottom chart) and Figure 8, the term premia on long real rates strongly co-move with our estimated demand factor. In fact, the correlation between the ten-year term premium and the demand factor is around 70 per cent, suggesting that the evolution of the term premia is largely

[18] To make the analysis simpler, we abstract from the convexity term. The impact of convexity increases with maturity and would affect interest rates even in the case of zero market prices of risk, but is constant across time. Therefore our analysis of the yields decomposition dynamics is robust to including the convexity term.

driven by the demand factor.[19] The following subsection provides a detailed analysis of the estimated demand factor.

Figure 8. Real Interest Rate Decomposition



Note: This figure.presents the decomposition of the five- and ten-year model implied rates into the term premium and the average expected short rate over a five and ten year horizon.

B. The demand factor analysis

We now turn to analyzing the link between the filtered excess supply factor (β_t), presented in the previous section, and our observable measures of demand (D_t). Our main hypothesis is that both the size and maturity structure of official holdings can explain the filtered excess supply factor. In essence, we expect the γs coefficient of equation (14) to be statistically significant.

[19] The unprecedented and short-lived spike in TIPS yields in the fall of 2008 reflecting a drop in liquidity due to a combination of technical factors linked to the repo market (Campbell, Shiller and Viceira, 2009) is mainly captured by an increase in the short rate. As a result, an increase in the expected rate is compensated by a short-lived but substantial fall in the term premium. Thus, in this circumstance the term premium seems to reflect largely changes in the short rate rather than in the demand factor.

Specifically, the filtered excess supply factor should decrease as i) the fraction of official purchases relative to the securities outstanding increases; and ii) the average maturity of the Fed's portfolio increases, while the maturity of the public's portfolio decreases.[20]

Table 3. Demand factor parameters

	mean	lb	ub	nse	CD
const	0.020	-0.006	0.046	0.0006	0.98
FO DP	-0.071	-0.081	-0.061	0.0002	0.35
FED DP	-0.276	-0.298	-0.255	0.0005	0.52
Pub Avg Mat	0.044	0.038	0.051	0.0002	0.99
FED Avg Mat	-0.028	-0.030	-0.027	0.0001	0.39

Note: The table presents estimates of coefficient c_i resulting from the following regression:

$$\beta_t = \gamma_0 + \gamma_1 FOdp_t + \gamma_2 FEDdp_t + \gamma_3 PubAvgMat_t + \gamma_4 FEDAvgMat_t + \varepsilon_t \, ,$$

where β_t is minus the demand factor, i.e. the excess supply in the model, which we assume to be a linear combination of official foreign and Fed demand pressures, $\beta_t = \theta_1 \beta_{t,For} + \theta_2 \beta_{t,Fed}$, and regress on observable measures of demand pressure. The coefficients are estimated within the MCMC algorithm. We report the posterior mean, the one-standard deviation credible intervals, the numerical standard errors (nse), and the absolute value of the convergence diagnostic (CD).

Table 3 presents the γ estimates resulting from regressing the aggregate demand factor on following observable measures of demand pressure: Foreign Official holdings of long-term Treasuries over the total amount outstanding of Treasury securities (FO DP); Federal Reserve holding of Treasury securities over the total amount of Treasury securities outstanding (FED DP); average maturity of Treasury securities held by the public (Pub Avg Mat); average maturity of the Federal Reserve Portfolio of Treasury securities (FED Avg Mat). Note that the coefficients are estimated jointly with all the other parameters and states and are not subject to any constraint. Therefore the model is free to determine whether the observable demand variables are consistent with our filtered excess supply factor.

Our estimates indicate that both the size and the maturity structure of official holdings matter for the demand factor and hence also for long-term Treasury bond yields. Indeed, all the estimated coefficients, but the constant, are significantly different from zero and correctly signed. Increases in the relative shares of Treasury bonds held by foreign officials and the Fed significantly decrease excess supply and hence increase the preferred-habitat demand for bonds. In addition, as the average maturity of the Fed holdings increases excess supply of Treasuries drops ($\hat{\gamma}_4$ =-

[20] We introduce the maturity of the Fed's portfolio and the maturity of the Public's portfolio as separate regressors. In principle, we could simply use the ratio of the two, so that an increase of the maturity of the Fed's portfolio relative to the maturity of the Public's portfolio should be associated with a drop in excess supply. However, the Public may include other potential preferred-habitat investors, such as pension funds. For this reason, we prefer a more general specification where the Fed and Public maturities are included as separate regressors.

0.028), while a similar increase in the average maturity of the public holdings' has the opposite effect ($\hat{\gamma}_3$ =0.044).

So far we showed that our measures of demand pressures are qualitatively consistent with the filtered excess supply resulting from the preferred-habitat model. Next step is to quantify the impact of foreign official holdings and Fed's unconventional monetary policy related variables on the level of interest rates at different maturities. We do this by re-estimating the preferred-habitat model by imposing that the demand factor is now observed and is equal to the fitted demand:

$$\beta_t^{obs} = \hat{\gamma} D'_{t+\Delta}\,, \tag{15}$$

where $\hat{\gamma}$s are the parameters from Table 3. In this way we abstract from factors other than official demand pressures that could possibly affect the unobserved demand factor. More fundamentally, by doing this, we ensure that the model-implied interest rates are consistent with the dynamics of the observed demand factor. Specifically, the resulting parameters $(\rho, \sigma_\beta, k_\beta, \bar{\beta})$ are consistent with the dynamics of β_t^{obs}. As a result, we can quantify accurately, in a model consistent fashion, the impact of the demand components on the term structure of interest rates.

This exercise effectively consists of replacing in the estimation the measurement equation (14) with equation (15). Interestingly, we find that even when forcing the model to perfectly match the observed (fitted) official demand factor, the model performs well, with average pricing errors (not reported) of comparable magnitude with those reported earlier in Table 2. We now turn to analyzing the impact of each of the separate components of official demand pressures.

Foreign official demand pressures

We first look at the impact of foreign officials' purchases on the term structure of real rates. For a generic period that goes from time t_0 to t_1, we quantify the impact of the foreign official demand on the yield at maturity ↲ such as:

$$iFOdp_{\tau,t_{0},t_1} = \frac{A_\beta(\tau)}{\tau} \times \widehat{\gamma_1} \times (FOdp_{t1} - FOdp_{t0}) \tag{16}$$

where $\widehat{\gamma_1}$ is the loading of foreign official demand pressure as in Table 4, while $A_\beta(\tau)$ is the factor loading for maturity ↲ associated with the demand factor β_t^{obs}.

The model implied estimates of foreign officials' demand for Treasuries' impact on the term structure of real rates are given in Table 4. Column 1percent denotes the impact of a 1percent increase of foreign official demand pressure for rates of 2-, 5-, 10- and 20-year maturities. The remaining columns show the impact over the following periods: 2004-05 is the year from May-04 to May-05 as in Warnock and Warnock (2009); 2001-08 is the period in which foreign official demand pressure moves from its minimum value (January 2001) to its maximum value (August 2008); LSAP1 and LSAP2 are from March 2009 to November 2009 and from November 2010 to June 2011, respectively.

Table 4. Impact of foreign officials' demand on real rates

	Foreign Official Demand Preassure						
	1%	2001-12	2004-05	2001-08	LSAP1	LSAP2	MEP
2yr	-2.4	-31.6	-5.1	-43.5	-1.1	-1.1	3.6
5yr	-4.1	-54.8	-8.9	-75.5	-2.0	-2.0	6.2
10yr	-4.9	-65.2	-10.6	-80.8	-2.4	-2.4	7.4
20yr	-4.4	-58.6	-9.6	-80.8	-2.1	-2.1	6.7

Note: The table presents the impact of the demand by Foreign Officials on the term structure of real rates over different periods. Column 1percent (1std) denotes the impact of a 1percent (1std) increase of foreign officials' demand pressure (public and Fed average maturity) for rates of 2-, 5-, 10- and 20-year maturities. The rest of the columns show the impact for the following periods: 1) 2004-05, as in Warnock and Warnock (2009); 2) 2001-08 ; 3) LSAP1 from Mar-09 to Nov-09; 4) LSAP2 from Nov-10 to Jun-11; 5) MEP from Sep-11 to Jun-12.

The results indicate that purchases of US government debt securities by foreign officials indeed have affected the level and dynamics of US real rates significantly. In particular, the impact of foreign purchases has been more important before the crisis, and by 2008 foreign purchases of US Treasuries have had a cumulative negative impact of around 81 basis points on long term US Treasury yields. This number is consistent with the reduced-form estimates by Warnock and Warnock (2009). However, our estimate refer to the much longer 2001-2008 period, whereas we find that over the 2004-05 period the drop was more contained.[21] Our estimates are possibly more in line with the Greenspan opinion, who back in 2005 suggested that the foreign buying of U.S. bonds could have depressed U.S. long rates by "less than 50 basis points".

Of further interest is the finding that during the LSAP1 and LSAP2 periods the effect of foreign official demand was negligible. This is explained by the fact that, in the post-crisis period, the rapid increase in foreign official holdings of US Treasury bonds (Figure 1) coincided with a comparable increase in the Treasury supply (Figure 3). Thus the overall market impact of foreign official pressures has been muted (top left panel of Figure 4). As Greenwood and Vayanos (2013) explain, the coincidence could be endogenous and the effects on the Treasury yields coming from changes in Treasury supply are significant. Therefore, in the absence of such increases in supply, the drop in long-term yields due to foreign demand pressures could have

[21] Note that what distinguish our estimates from Warnock and Warnock (2009) is not only the model used, but also the fact that we look at real rates.

been substantially more pronounced. However, in this study we estimate the effect of demand conditional on changes in supply by using observable measures of demand pressure rather than studying separately demand and supply effects.

Federal Reserve Demand Pressure

Table 5. Impact of Federal Reserve asset purchase policies on real rates

PANEL A: FED Demand Preassure					
	1%	LSAP1	LSAP2	MEP	QE
2yr	-9.3	-30.3	-65.6	13.6	-77.2
5yr	-16.2	-52.6	-113.7	23.6	-133.9
10yr	-19.3	-62.6	-135.4	28.1	-159.3
20yr	-17.3	-56.3	-121.7	25.2	-143.3
PANEL B: Average Maturity					
	1std	LSAP1	LSAP2	MEP	QE
2yr	-12.6	10.3	11.6	-36.6	8.7
5yr	-21.8	17.8	20.1	-63.4	15.1
10yr	-26.0	21.2	23.9	-75.5	17.9
20yr	-23.4	19.1	21.5	-67.9	16.1
PANEL C: Total Effect					
	1	LSAP1	LSAP2	MEP	QE
2yr	-21.9	-20.0	-54.0	-23.0	-68.5
5yr	-38.0	-34.8	-93.7	-39.8	-118.8
10yr	-45.3	-41.4	-111.5	-47.4	-141.4
20yr	-40.7	-37.2	-100.2	-42.6	-127.1

Note: The table presents the impact of changes in the demand by the Federal Reserve (Panel A) and in the average maturity held by the public and the FED (Panel B) on the term structure of real rates over different periods. Panel C displays the total impact as Panel A plus Panel B. Column 1percent (1std) denotes the impact of a 1percent (1std) increase of Fed demand pressure (public and Fed average maturity) for rates of 2-, 5-, 10- and 20-year maturities. The rest of the columns show the impact for the following periods: 1) LSAP1 from Mar-09 to Nov-09; 2) LSAP2 from Nov-10 to Jun-11; 3) MEP from Sep-11 to Jun12; 4) 2009-12 from the start of QE in Mar-09 to the end of the sample Nov-2012.

Table 5 presents the impact of changes in the relative demand by the Federal Reserve (Panel A) and in the average maturity held by the Federal Reserve versus that of the public (Panel B) on the term structure of real rates over different periods. In particular, for a generic period that goes from time t_0 to t_1, we quantify the impact on the yield at maturity ↲ such as:

- for Fed demand pressure (Panel A):

$$iFEDdp_{\tau,t_0,t_1} = A_\beta(\tau)/\tau \times \widehat{\gamma_2} \times (FEDdp_{t1} - FEDdp_{t0}); \tag{17}$$

- for average maturity pressure (Panel B):

$$iAvgMat_{\tau,t_0,t_1} = A_\beta(\tau)/\tau \times [\widehat{\gamma_3}(PubAvgMat_{t1} - PubAvgMat_{t0}) + \widehat{\gamma_4}(FEDAvgMat_{t1} - FEDAvgMat_{t0})], \tag{18}$$

where $\widehat{\gamma_2}, \widehat{\gamma_3}, \widehat{\gamma_4}$ are the coefficients associated with the components of the Fed's demand pressure, as presented in Table 5. As for the foreign official exercise, $A_\beta(\tau)$ for maturity ↵ associated with the demand factor β_t^{obs}. Panel C displays the total Fed's demand impact as the sum of the two separate demand and maturity pressures' impacts, so that Panel C results from summing the entries of Panels A and B.

We estimate the impact of the LSAP 1 purchases on the 10-year yield to be -63 basis points. Though, if we were to include the associated maturity effects, displayed in panel C, the impact would decrease to -41 basis points. These results are comparable to previous estimates available in the literature on the impact of LSAP. For example, D'Amico and King (2013) estimated the impact to be roughly equal to -45 basis points, Bomfim and Meyer (2010) to be -60 basis points and Gagnon et al. (2011) to be between -58 and -91 basis points.

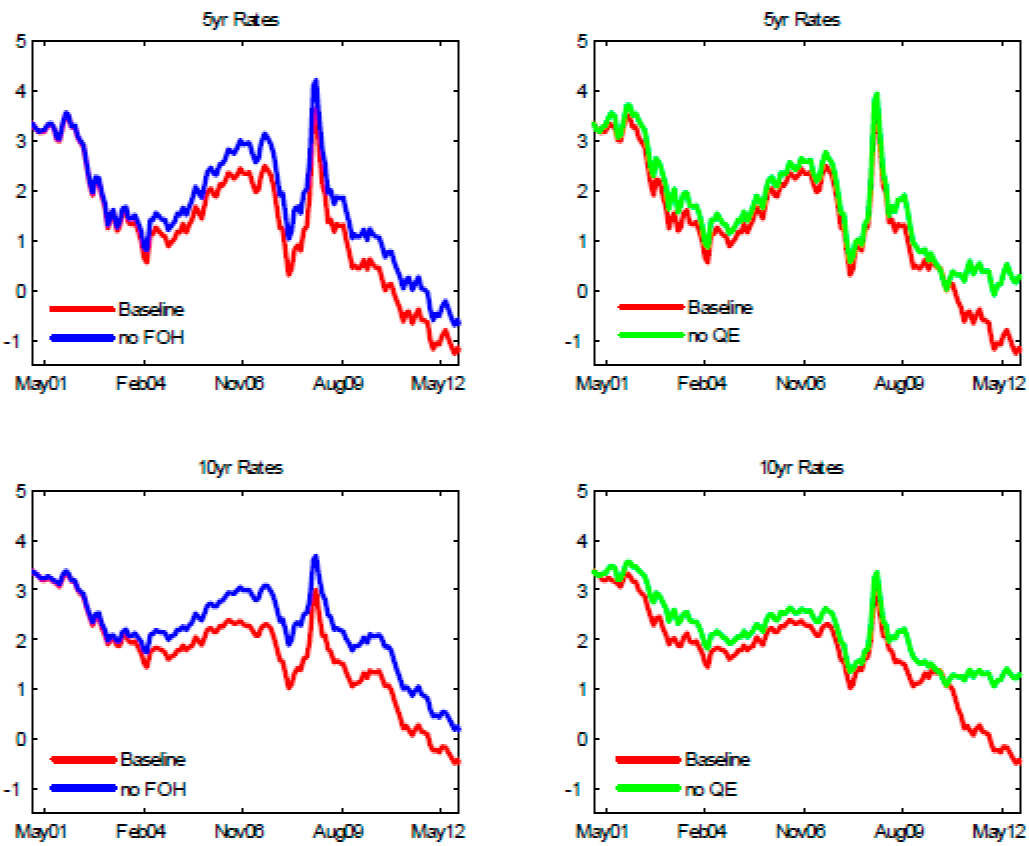
Our estimates of the LSAP2 impacts on 10-year yields are slightly higher than those from earlier studies. We estimate that 10-year yields would have been at least 111 basis points higher in the absence of the second round of asset purchases by the Fed. The estimates by Krishnamurthy and Vissing-Jorgensen (2011) and D'Amico et al. (2011) instead are in the order of -33 and -55 basis points, respectively. Of course one reason explaining such different results lies in the different methodologies used. In addition, our study focuses on the impact of LSAPs on the real rates, whereas other empirical studies focus on nominal rates. Their estimates therefore not only reflect the impact of the Fed's policies on the real component of the nominal yields, but also on inflation expectations and inflation risk premia. And, according to the various sources, both these components slightly increased during the LSAPs programs, and, as a result, the impact on nominal yields is smaller than the impact on real rates. For example, according to Krishnamurthy and Vissing-Jorgensen (2011), inflation expectations increased as a result of both QE1 and QE2. More specifically, according to the Cleveland Fed estimates, expected inflation at 10-year horizon has increased by more than 20 bps in the period from November 2010 to July 2011.

By controlling for both aspects of the Fed's demand pressure, i.e. the size and the maturity profile of Treasury issuance, we can quantify the direct impact of the new Maturity Extension Program (MEP). Notably, we find that the reduction in yields during the MEP program is due to changes in the maturity profile of the Fed's portfolio, relative to the public portfolio, rather than to changes in the size of the Treasury portfolio, relative to the amount of Treasury securities outstanding. We find that the impact of the MEP on long-term yields is of comparable magnitude to the impacts resulting from the LSAP programs. In particular, the estimated impact of the MEP on the 10-year real yield is around -47 basis points. This estimate is larger than the -15 basis points suggested earlier by Swansson (2011), but smaller than the -80 basis points estimated by Meaning and Zhu (2012).

Official demand pressures over time

In this section, we present the counterfactual analysis, that is, the interest rates that would have prevailed in the absence of changes in our observed measures of demand pressures. Specifically, we do this by fixing first the foreign official demand pressure at the initial 2001 level, and then similarly the Fed demand pressure at the initial 2001 level. The corresponding cumulative effects are presented by Figure 9 both for the five- and ten-year interest rates.

Figure 9. The cumulative effects of demand pressures on real rates



Note: The figure presents model-implied rates in red (Baseline) and the counterfactual rates, when Foreign Official Demand Pressure is fixed to its Jan-2001 starting value in blue (no FOH), and when Fed holdings, public and Fed average maturity are jointly fixed to their Jan-2001 starting values in green (no QE). The distance between the Baseline and the counterfactual line denotes the cumulative effect since Jan-2001. Top (bottom) panels refer to the five- (ten-) year rates. The counterfactual estimates result from the model with observed demand.

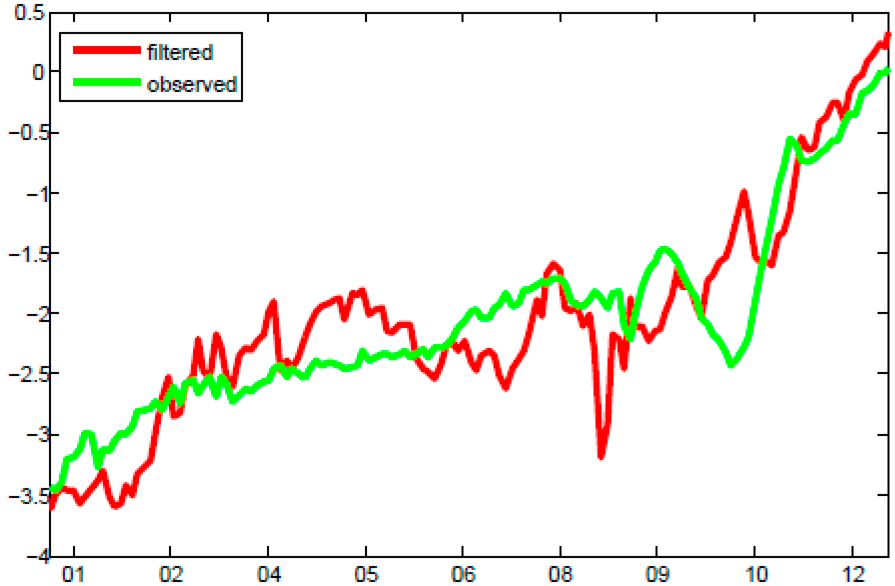
It is apparent that the Fed's asset purchase programs have been effective. In particular, the visual inspection of the right panels in Figure 9 shows that in the absence of any Fed purchases/sells of Treasury securities both the five- and ten-year real yields would have been significantly higher from 2010 onwards. It is not surprising that the impact of the Fed's demand pressure is substantial with the introduction of the Fed's policies. In contrast, the cumulative impact of the foreign officials' demand pressure was concentrated during the conundrum period, whereas it was negligible during the crisis.

Other potential determinants of preferred-habitat demand

We use a limited set of observable demand variables to explain the evolution of preferred-habitat demand, as our main focus is on official investors. Moreover, our findings are generally consistent with a number of previous studies either on foreign official investors or Fed's policies. However, it is worth noting that other variables may also help explain the filtered preferred-habitat demand. In principle, investors that behave similarly to official investors - being large holders with a relatively inelastic demand - may also be included. To this end, natural candidates are institutional investors, such as pension funds, which can affect both the level and the dynamics of the preferred-habitat demand. In addition, there might be other investors that at times display an inelastic demand for Treasuries, such as for example regulated commercial banks, and can therefore also explain temporary differences between the filtered factor β_t and the fitted factor β_t^{obs}. Furthermore, variables that capture changing market conditions, may be important in explaining short run variations in the demand for Treasuries, but their effects are unlikely to be captured by our measures of demand pressures that in contrast are rather persistent. Figure 9 shows the estimated filtered ($-\beta_t$) and observed demand (β_t^{obs}) factors. It is evident that, although the fitted factor tracks the filtered demand rather well, a significant fraction of the filtered demand evolution remains unexplained. The divergence between the two factors is somehow persistent and is particularly strong in 2010. In what follows we review the potential omitted candidates, in no specific order, that could account for this difference.

First, we should consider variables capturing market portfolio rebalancing effects. In a recent paper, Malkhozov et al. (2013) show, both theoretically and empirically, that there are supply effects resulting from the hedging activity of financial intermediaries. Specifically, at times when the duration of outstanding MBS drops, financial intermediaries rebalance their hedging portfolio by buying Treasuries. In this way, they can put further pressure on the interest rates. This hedging demand, which may represent an additional demand pressure in the Treasury market, could be captured by the low duration of outstanding MBS and can potentially explain the difference between the filtered and observed demand.

Figure 10. Filtered and observed demand factors



Note: The figure shows the filtered demand factor and the projected (observed) demand factor, which is based on several observable measures of demand pressure from equation (15).

Second, in explaining temporary variations in the demand factor, variables that capture changes in market sentiment could also be important. Larger fear in financial markets should create additional demand pressures for Treasuries. In periods of market turmoil, not only Treasuries are perceived as a safe haven, but also specialized investors have fewer possibilities to trade away arbitrage opportunities possibly created by preferred-habitat demand investors. Following a number of recent studies, like, for example, Bollerslev, Marrone, Xu, and Zhou (2012), or Londono (2011, 2012), we use the US variance risk premium to capture changes in risk sentiment and macroeconomic uncertainty risk.

Third, in our analysis we abstracted from pension funds and life insurers that are natural candidates to be considered as preferred-habitat investors. Their investment in index-linked securities, and Treasuries securities in general, is largely driven by considerations other then return (e.g. liability matching), often resulting from regulatory changes. As a result, these institutional investors have price inelastic preferences and are usually regarded as long-term investors. Moreover, pension funds and (life) insurers are part of the third largest group, together with mutual funds, of holders of US government debt (around 11 per cent, as of 2012).[22]

Finally, despite low interest rates on US Treasury bonds, major commercial banks have been increasingly hoarding Treasuries during the crisis, reflecting their preference for safe and liquid

[22] The data on the estimated ownership of U.S. Treasury Securities can be found in TABLE OFS-2 of the quarterly issued Treasury Bulletin.

assets. Commercial banks demonstrated record purchases and hoarding of Treasuries in the 2008-2010 period. As a result, commercial banks demand can also account for this wedge between our observed (fitted) and unobserved (filtered) demand factors.

Table 6. Explaining differences between observed and filtered demand factors

	[1]	[2]	[3]	[4]
Const	0.0004	0.0052	-0.0056	0
(t-statistics)	(-0.99)	(-2.64)	(-3.69)	(-0.02)
VRP*	0.0347	-	-	0.0347
(t-statistics)	(-2.15)			(-2.28)
Commercial banks	-	-	0.0501	0.042
			(-3.78)	(-3.12)
MBS duration	-	-0.0013	-	-0.0009
(t-statistics)		(-2.73)		(-2.07)
Adj R-squared	0.03	0.03	0.09	0.13

Note: The table presents coefficient estimates of three versions of regression (12). Sample: Jan. 2001-Nov. 2012. Each of the versions relates unexplained filtered demand pressures to [1] variance risk premia (VRP), or [2] MBS duration, or [3] index of commercial banks holdings (from Federal Reserve Bank of St. Louis), or [4] all of above.

Therefore, to shed light on the difference between filtered and estimated observed demand pressures, we estimate the following regressions:

$$d_t^{fil} - d_t^{obs} = \varsigma_0 + \varsigma X'_t + \eta_t, \qquad (19)$$

where $d_t^{fil} = -\beta_t$; $d_t^{obs} = -\beta_t^{obs}$ and X_t denotes the explanatory variables of market based demand movements discussed above.[23]

We present the estimation results in Table 6. Interestingly, we find that growing market fear, lower MBS duration and commercial bank investment in Treasury securities are qualitatively consistent with additional demand pressures. In the univariate regressions commercial banks' activity delivers the highest R-square. Notably, all three variables are significative also when they are included all together. In sum, the results illustrate that other variables, in addition to official investors' demand, can also be important in explaining demand pressures for Treasuries.

[23] We omit pension fund holdings from this reduced form analysis due to unavailability of the data at monthly frequency. For the UK case study on the importance of pension funds see Greenwood and Vayanos (2010).

VII. CONCLUDING REMARKS

This paper provides a structural estimation of the impact of official demand pressures on the term structure of US real interest rates. A distinguishing feature of our analysis is that we have explicitly estimated a two-factor no-arbitrage model with preferred-habitat preferences using data on US real rates and official holdings of US Treasury bonds.

We find that the Fed's asset purchase programs have been effective. In particular, we estimate that in the absence of Fed purchases, the 10-year real yields would have been up to 140 basis points higher. Foreign official investors have pushed US rates down by around 80 basis points, and their impact has not changed significantly in the aftermath of the crisis. Our findings also reveal that the Fed policy interventions and foreign official purchases affected real bonds mostly through the bond premium channels.

Although our analysis ends in 2012, our model set up can be used to assess the recent Fed policy events, such as the Federal Reserve's first tapering announcement in May 2013. The policy of tapering is likely to result in a weakening of the Fed's demand pressure, as well as in an increase in the duration in Treasuries held by the public. Our results suggest that this policy in turn would imply an increase in interest rates through higher bond premia. However, studying the interplay between changes in official demand for Treasuries and the forward guidance policy remains important.

Moreover, the two-factor model estimated here is clearly a simplified representation of a complex behavior and interdependence of various players acting on the Treasury market. To better represent the Treasury market, one could enrich the model by introducing other large players, such as pension funds and regulated commercial banks. Alternatively, rather than focusing on an aggregate demand within a two factor model, a more complex dynamic of individual demand factors could be considered. Lastly, in this paper we modelled the real rates, whereas to quantify impacts of the official demand pressures on nominal rates, a joint term-structure model on nominal and real rates is needed. All these extensions constitute promising avenues for original research on the nexus of central bank policies and financial markets.

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APPENDIX I. MODEL DETAILS

We conjecture equilibrium spot rates that are affine in the risk factors, i.e. the short rate (r_t) and the demand factor (β_t), so that the equilibrium bond price takes the following exponential form

$$P_{t,\tau} = e^{-(A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau))}, \tag{A1}$$

for three functions $A_r(\tau)$, $A_\beta(\tau)$, $C(\tau)$ that depend on maturity τ. Applying Ito's Lemma to (A1) and using the dynamics (1) of r_t and (5) of β_t , we find that the instantaneous return on the bond with maturity τ is

$$\frac{dP_{t,\tau}}{P_{t,\tau}} = \mu_{t,\tau}dt - A_r(\tau)\sigma_r dB_{r,t} - A_\beta(\tau)\sigma_\beta dB_{\beta,t} \tag{A2}$$

where

$$\mu_{t,\tau} \equiv A'_r(\tau)r_t + A'_\beta(\tau)\beta_t + C'(\tau) - A_r(\tau)\kappa_r(\bar{r} - r_t) - A_\beta(\tau)\kappa_\beta(\bar{\beta} - \beta_t) + \frac{1}{2}A_r^2(\tau)\sigma_r^2 + \frac{1}{2}A_\beta^2(\tau)\sigma_\beta^2 + \rho A_r(\tau)A_\beta(\tau)\sigma_r\sigma_\beta \tag{A3}$$

is the instantaneous expected return. Substituting (A2) into the arbitrageurs' budget constraint (7), we can solve the arbitrageurs' optimization problem.

Next we show how to derive bond risk premia (or excess returns) of eq. (9). Using (A2), we can write

$$\begin{aligned} dW_t &= \left(W_t - \int_0^T x_{t,\tau}\right) r_t dt + \int_0^T x_{t,\tau}\frac{dP_{t,\tau}}{P_{t,\tau}} \\ &= \left(W_t r_t - \int_0^T x_{t,\tau}\,(\mu_{t,\tau} - r_t)d\tau\right)dt - [\int_0^T x_{t,\tau}\,A_r(\tau)d\tau]\,\sigma_r dB_{\beta,t} \\ &\quad - [\int_0^T x_{t,\tau}\,A_\beta(\tau)d\tau]\,\sigma_\beta dB_{\beta,t} \end{aligned}$$

and (6) as

$$\max_{\{x_{t,\tau}\}_{\tau\epsilon(0,T]}} \left[\int_0^T x_{t,\tau}\,(\mu_{t,\tau} - r_t)d\tau - \frac{a\sigma_r}{2}\left[\int_0^T x_{t,\tau}\,A_r(\tau)d\tau\right]^2 - \frac{a\sigma_\beta}{2}\left[\int_0^T x_{t,\tau}\,A_\beta(\tau)d\tau\right]^2 - a\rho\sigma_\beta\sigma_r\left[\int_0^T x_{t,\tau}\,A_r(\tau)d\tau\right]\int_0^T x_{t,\tau}\,A_r(\tau)d\tau\right], \tag{A4}$$

Point-wise maximization of (A4) yields (9).

Next we derive the factor loadings $A_r(\tau), A_\beta(\tau)$. By imposing market clearing, so that $x_{t,\tau} = -y_{t,\tau}$, and using eq. (2), (A1) and the definition of $R_{t,\tau}$, we find

$$x_{t,\tau} = \alpha(\tau)\{\beta_t\tau - (A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau))\} \tag{A5}$$

Substituting $(\mu_{t,\tau}, \lambda_{r,t}, \lambda_{\beta,t}, x_{t,\tau})$ from (A3), (10), (11) and (A5) into (9), we find an affine equation in r_t, β_t. Setting linear terms in (r_t, β_t) to zero gives

$$A_r'(\tau) + \kappa_r A_r(\tau) - 1 = M_{1,1}A_r(\tau) + M_{1,2}A_\beta(\tau) \tag{A6a}$$
$$A_\beta'(\tau) + \kappa_\beta A_r(\tau) - 1 = M_{2,1}A_r(\tau) + M_{2,2}A_\beta(\tau) \tag{A6b}$$

where the matrix M is given by

$$M_{1,1} \equiv -a\sigma_r \int_0^T \alpha(\tau)A_r(\tau)\left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right]d\tau$$
$$M_{1,2} \equiv -a\sigma_\beta \int_0^T \alpha(\tau)A_r(\tau)\left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right]d\tau$$

$$M_{2,1} \equiv a\sigma_r \int_0^T \alpha(\tau)\left[\tau\theta(\tau) - A_\beta(\tau)\right]\left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right]d\tau$$

$$M_{2,2} \equiv a\sigma_\beta \int_0^T \alpha(\tau)\left[\tau\theta(\tau) - A_\beta(\tau)\right]\left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right]d\tau$$

The solution to the system of (A6a) and (A6b) is given by equations (A7) and (A8):

$$A_r(\tau) = \frac{1-e^{-\nu_1\tau}}{\nu_1} + \gamma_r\left(\frac{1-e^{-\nu_2\tau}}{\nu_2} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right) \tag{A7}$$
$$A_\beta(\tau) = \gamma_\beta\left(\frac{1-e^{-\nu_2\tau}}{\nu_2} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right) \tag{A8}$$

To determine $(\nu_1, \nu_2, \gamma_r, \gamma_\beta)$, we substitute (A7) and (A8) into (A6a) and (A6b), and identify terms in $\frac{1-e^{-\nu_1\tau}}{\nu_1}$ and $\frac{1-e^{-\nu_2\tau}}{\nu_2}$. This yields

$$(1-\gamma_r)\left(\nu_1 - \kappa_r + M_{1,1}\right) - \gamma_\beta M_{1,2} = 0 \tag{A9}$$
$$\gamma_r\left(\nu_2 - \kappa_r + M_{1,1}\right) + \gamma_\beta M_{1,2} = 0 \tag{A10}$$

in the case of (A6a) and

$$\gamma_\beta\left(\nu_1 - \kappa_\beta + M_{2,2}\right) - (1-\gamma_r)M_{2,1} = 0 \tag{A11}$$
$$-\gamma_\beta\left(\nu_2 - \kappa_\beta + M_{2,2}\right) - \gamma_r M_{2,1} = 0 \tag{A12}$$

in the case of (A6b). Combining (A9) and (A10), we find the equivalent equations

$$\nu_1 + \gamma_r(\nu_2 - \nu_1) - \kappa_r + M_{1,1} = 0 \tag{A13}$$

$$(1-\gamma_r)\gamma_r(\nu_1 - \nu_2) - \gamma_\beta M_{1,2} = 0 \tag{A14}$$

and combining (A11) and (A12) we find the equivalent equations

$$\gamma_\beta(\nu_2 - \nu_1) + M_{2,1} = 0 \tag{A15}$$

$$\kappa_\beta - \nu_2 - \gamma_r(\nu_1 - \nu_2) - M_{2,2} = 0 \tag{A16}$$

Equations (A13)-(A16) are a system of four scalar non-linear equations in the unknowns (ν_1, ν_2, γ_r , γ_β).

To solve the system of (A13)-(A16), we must assume functional forms for $\alpha(\tau)$,$\theta(\tau)$. Many parametrizations are possible. A convenient one that we adopt from now on is $\alpha(\tau)\equiv\alpha\exp\{-\delta\tau\}$ and $\theta(\tau)=1$ (i.e., the demand factor affects all maturities equally in the absence of arbitrageurs). We also set $\alpha=1$, which is without loss of generality because α matters only through the product αa.

Next, we show how to determine the function $C(\tau)$. Setting $x_{t,\tau} = -y_{t,\tau}$ in (10) and (11), and using $R_{t,\tau} \equiv -\frac{\log(P_{t,\tau})}{\tau}$, (2) and (A1) we find

$$\lambda_{r,t} \equiv a\sigma_r^2 \int_0^T \alpha(\tau)\big[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\big]A_r(\tau)\,d\tau + a\rho\sigma_r\sigma_\beta \int_0^T \alpha(\tau)\big[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\big]A_\beta(\tau)\,d\tau, \tag{A17}$$

$$\lambda_{\beta,t} \equiv a\sigma_\beta^2 \int_0^T \alpha(\tau)\big[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\big]A_\beta(\tau)\,d\tau + a\rho\sigma_r\sigma_\beta \int_0^T \alpha(\tau)\big[\beta_t\tau - A_r(\tau)r_t + A_\beta(\tau)\beta_t + C(\tau)\big]A_\beta(\tau)\,d\tau, \tag{A18}$$

Substituting $\mu_{t,\tau}$ from (A3), $\lambda_{r,t}$ from (A17), $\lambda_{\beta,t}$ from (A18), we find

$$C'(\tau) - \kappa_r\bar{r}A_r(\tau) + \tfrac{1}{2}\sigma_r^2 A_r(\tau)^2 + \tfrac{1}{2}\sigma_\beta^2 A_\beta(\tau)^2 + \rho\sigma_\beta\sigma_r A_r(\tau)A_\beta(\tau) = a\sigma_r A_r(\tau)\int_0^T \alpha(\tau)\big[\bar{\beta}\tau - C(\tau)\big]\big[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\big]\,d\tau + a\sigma_\beta A_\beta(\tau)\int_0^T \alpha(\tau)\big[\bar{\beta}\tau - C(\tau)\big]\big[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\big]\,d\tau \tag{A19}$$

The solution to (A19) is

$$C(\tau) = z_r\int_0^T A_r(u)du + z_\beta\int_0^T A_\beta(u)du - \frac{1}{2}\sigma_r^2\int_0^T A_r(u)^2du - \frac{1}{2}\sigma_\beta^2\int_0^T A_\beta(u)^2du - \rho\sigma_r\sigma_\beta\int_0^T A_r(u)A_\beta(u)du,$$

where

$$z_r \equiv \kappa_r \bar{r} - a\sigma_r \int_0^T \alpha(\tau)C(\tau)\left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right]d\tau \quad \text{(A20)}$$

$$z_\beta \equiv \kappa_\beta \bar{\beta} - a\sigma_\beta \int_0^T \alpha(\tau)C(\tau)\left[\rho\sigma_r A_r(\tau) + \sigma_\beta A_\beta(\tau)\right]d\tau \quad \text{(A21)}$$

Substituting $C(\tau)$ into (A20) and (A21), we can derive (z_r, z_β) as the solution to a linear system of equations.

APPENDIX II: MCMC ALGORITHM

We estimate the term structure model by means of Bayesian methods. Ang, Dong and Piazzesi (2007), Feldhutter (2008) and Chib and Ergashev (2009) among others, have estimated multi-factor affine yield curve models using a MCMC with a Gibbs sampling algorithm. Although, our approach is similar in nature, we apply it here to estimate the specific model of Vayanos and Vila (2009), which departs from more traditional no-arbitrage affine models.

We decompose joint posterior density $\pi\left(P, \sigma_\varepsilon^2, \sigma_\epsilon^2, X^T \middle| Y^T, D^T\right)$ into a number of independent full conditional densities, such as those for parameters $\pi\left(\theta \middle| P_{-\theta}, \sigma_\varepsilon^2, X^T, Y^T\right), \forall\theta \in P$, for state variables $\pi\left(X^T \middle| P, \sigma_\varepsilon^2, Y^T\right)$ and for pricing error variance $\pi\left(\sigma_\varepsilon^2 \middle| P, X^T, Y^T\right)$, for the demand coefficients $\pi\left(\gamma \middle| X^T, D^T\right)$ and measurement error variance $\pi\left(\sigma_\epsilon^2 \middle| X^T, D^T\right)$. To implement the MCMC algorithm we iteratively sample from these conditional densities. We use the Metropolis step for drawing from $\pi\left(\theta \middle| P_{-\theta}, \sigma_\varepsilon^2, X^T, Y^T\right)$ that consists of drawing a candidate parameter from a proposal distribution, and then accepting or rejecting the draw based on the information in the yields, state evolution and priors. The step of drawing the factors is standard, because the Vasicek model is linear and Gaussian and we can use the forward-filtering backward sampling by Carter and Kohn (1994). The remaining densities are known in closed form so that we draw from these densities using standard Gibbs sampling steps.

Likelihood Functions

The density of the factors is

$$\pi\left(X^T \middle| P_1\right) \propto \prod |Q|^{-1} \exp\left(-\frac{1}{2} u'_t Q^{-1} u_t\right)$$

where u_t are the transition equation errors from (12) with a full variance-covariance matrix Q. Conditional on a realization of the parameters and latent factors, the likelihood function of the data is

$$L\left(Y^T \middle| P, \sigma_\varepsilon^2, X^T\right) \propto \prod |\sigma_\varepsilon^2 I_n|^{-1} \exp\left(-\frac{1}{2} \varepsilon'_t (\sigma_\varepsilon^2 I_n)^{-1} \varepsilon_t\right) \prod_{t=1}^{T-1} \pi\left(X_t \middle| X_{t+1}, \sigma_\varepsilon^2, X^T\right)$$

where the measurement errors ε_t are given by (13). Finally, the joint posterior distribution of the model parameters and the latent factors is given by

$$\pi\left(P, \sigma_\varepsilon^2, X^T \middle| Y^T\right) \propto L\left(Y^T \middle| P, \sigma_\varepsilon^2, X^T\right) \pi\left(X^T \middle| P_1\right) \pi(P),$$

i.e. the product of the likelihood of the observation, the density of the factors and the priors of the parameters. Next, we present the block-wise Metropolis-Hastings (MH) algorithm within Gibbs sampler that allows us to draw from the full posterior, $\pi\left(P, \sigma_\varepsilon^2, X^T \middle| Y^T\right)$. In principle, we approximate the target density by repeatedly simulating from the conditional distributions of each block in turn. If the conditional distributions were known, this algorithm then consists of a series of

Gibbs sampler steps. But in our case most of these conditional distributions are not recognizable, so we replace Gibbs sampler steps with MH steps.

Drawing Factors and Parameters

The term structure model is linear and has a Gaussian state-space representation. The measurement and transition equations are linear in the unobserved factors, X^T. And both equations have Gaussian distributed errors. So we use the Carter and Kohn (1994) simulation smoother to obtain a draw from the joint posterior density of the factors, which is

$$\pi\left(X^T \middle| P, \sigma_\varepsilon^2, Y^T\right) \propto \pi\left(X_T \middle| P, \sigma_\varepsilon^2, X^T\right) \prod_{t=1}^{T-1} \pi\left(X_t \middle| X_{t+1}, \sigma_\varepsilon^2, X^T\right)$$

In short, a run of the Kalman filter yields $\pi\left(X_T \middle| P, \sigma_\varepsilon^2, X^T\right)$ and the predicted and smoothed means and variances of the states, while the simulation smoother provides the updated estimates of the conditional means and variances that fully determine the remaining densities. (See Kim and Nelson, 1999).

Although in the discretized case, VAR parameters have conjugate normal posterior distribution given the factors X^T, in our model the *drift parameters* also enter the pricing of yields. Thus, their conditional posteriors are unknown. We draw the drift parameter of the latent factors using a Random-Walk Metropolis (RWM) algorithm (see Johannes and Polson, 2004). Let denote with $\theta^{(g)}$ the $(g)^{th}$-draw of the parameter. At the (g + 1)-iteration we draw a candidate parameter $\theta^{(c)}$from the proposal normal density

$$\theta^{(c)} = \theta^{(g)} + v_\theta \epsilon,$$

where $\epsilon \sim N(0,1)$ and v_θ is the scaling factor used to tune the acceptance probability around 10-50percent. Let define $P_{-\theta}$ as all the P parameters but θ. We accept the candidate draw with probability

$$ap = min\left\{\frac{L(Y^T|P_{-\theta},\theta^{(c)},\sigma_\varepsilon^2,X^T)\pi(X^T|P_{1,-\theta})\pi(P_{1,-\theta},\theta^{(c)})}{L(Y^T|P_{-\theta},\theta^{(g)},\sigma_\varepsilon^2,X^T)\pi(X^T|P_{1,-\theta})\pi(P_{1,-\theta},\theta^{(g)})}, 1\right\}.$$

Because the proposal density is symmetric it does not impact on the acceptance probability. We perform this RWM step for each of the individual drift parameters (k_r; k_β; $\bar{r}$;$\bar{\beta}$).

The *posterior of variance-covariance matrix*, Q, of the transition equation (12) takes the form of

$$\pi(Q|Y^T) \propto L\left(Y^T|P, \sigma_\varepsilon^2, X^T\right)\pi\left(X^T|P_1\right)\pi(Q) \quad ,$$

where $\pi(Q)$ is a prior distribution. Specifying an inverse Wishart prior, we can easily draw from the inverse Wishart proposal distribution:

$$q(Q) = \pi\left(X^T|P_1\right)\pi(Q),$$

and the acceptance probability simplifies to

$$ap = min\left\{\frac{L(\mathrm{Y^T}|\mathrm{P}_{-\theta},\theta^{(c)},\sigma_\varepsilon^2,\mathrm{X^T})}{L(\mathrm{Y^T}|\mathrm{P}\theta_{-\theta},\theta^{(g)},\sigma_\varepsilon^2,\mathrm{X^T})}, 1\right\}.$$

Note that we perform the "accept/reject" step for each individual candidate draw, i.e. for $\theta^{(c)}$ equal to ρ, σ_r, σ_β, because otherwise too few draws would be accepted.

Arbitrageurs' risk aversion, a, and excess demand elasticity, α, are not separately identified, so we estimate their product aα. The estimation is similar in spirit to that of market price of risk parameters in traditional no-arbitrage models. These parameters are notably difficult to estimate because they only enter the measurement equation (bond pricing). We again use a RWM algorithm, but the acceptance probability simplifies to

$$ap = min\left\{\frac{L(\mathrm{Y^T}|\mathrm{P}_{-a\alpha}, a\alpha^{(c)}, \sigma_\varepsilon^2, \mathrm{X^T})}{L(\mathrm{Y^T}|\mathrm{P}_{-a\alpha}, a\alpha^{(g)}, \sigma_\varepsilon^2, \mathrm{X^T})}, 1\right\}$$

because $a\alpha$ does not enter the transition equations.

We simply use a Gibbs sampler to draw *the variance of the bond pricing errors*. Conditional on the other parameters, P, the factors and the observed yields, we get the measurement errors, ε_t. And because we assume a common variance for all the maturities, we implicitly pool the n vectors of residuals into a single series. So the inverse Gamma distribution becomes the natural prior for the variance, σ_ε^2.

Finally, we get the *demand factor loadings, γ, and the variance of the demand measurement errors*, σ_ϵ^2, conditional on the factors and the observed Fed and foreign demand proxies. In a simple Gibbs sample step, we draw the γ parameter from $\pi(\gamma| \mathrm{X^T}, \mathrm{D^T})$, which is the Gaussian distribution with mean$(\mathrm{X^T}'\mathrm{X^T})^{-1}(\mathrm{X^T}'\mathrm{D^T})$. Therefore, conditionally on γ, the factors and demand variables, we get the measurement errors ϵ_t and draw the variance of the demand measurement errors, σ_ϵ^2.

Priors. We set the priors such that they are proper but only little informative. The prior on the transition equation covariance matrix is inverse Wishart, and the one on the measurement error variance is inverse Gamma. The rest of the parameters have normal or, in a few cases, truncated normal distributions. For example, we impose arbitrageurs risk aversion and the mean reversion parameters to be positive (to insure factors' stationarity). We discard the draws that do not fall within the desired region, and we keep drawing a proposal parameter until it respects the constraint. But to avoid that the chain gets stuck we specify a maximum number of draws, otherwise we retain the old draw (also see Mikkelsen (2002)).

Implementations Details. We perform 80,000 replications, of which the first 40,000 are "burned" to insure convergence of the chain to the ergodic distribution. We save 1 every 20 draws of the last 40,000 replications of the Markov chain to limit the autocorrelation of the draws. The RWM algorithm converges for an around 20-40percent acceptance level (Johannes and Polson (2004)). If the variance is too high we reject nearly every draw, and the opposite is true for a variance that is too low. In order to reach reasonable acceptance ratios we follow the method of Feldhutter (2007). The variance is tuned over the first half of the burn-in period and we check the acceptance ratio every 100 draws. If we accepted more than 50 draws over the last 100, we double the standard deviation. If, instead, we accepted less than 10 percent of draws we half the standard deviation.

Convergence Check. In order to check the convergence of the Markov chain we use two convergence diagnostics: the numerical standard error (NSE), and the convergence diagnostic (CD) of Geweke (1992).[24] The NSE is a widely used measure of the approximation error. A good estimate of NSE has to compensate for the correlation in the draws (Koop, 2003). The second diagnostic, CD, relies on the idea that an estimate of the parameter based on the first half of the draws must be essentially the same to an estimate based on the last half. If this was not the case, then either the number of replications is too small, or the effect of the starting value has not vanished.

[24] To compute the NSE and CD we use the codes of James P. LeSage.