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Abstract

Senegal's fiscal deficit and public debt have been on the rise in recent years owing partly to an ailing and inefficient oil-based energy sector. In this paper we use a two-sector, open-economy, dynamic general equilibrium model to investigate the effects of varying fiscal policy instruments one at a time and of policy packages that increase public investment in energy and infrastructure in scenarios with varying degrees of debt finance and with different types of supporting fiscal adjustment. Lowering the fiscal deficit by raising taxes and cutting government expenditure has adverse effects on growth, real wages and the supply of public services. Senegal does not need, however, to undertake such difficult fiscal adjustment. A public investment program that coordinates new investment in low-cost hydroelectric, coal or gas-fired power with a phased contraction of the oil-based sector raises the total supply of energy by 70 percent, increases real wages and real GDP, stimulates private investment, and significantly reduces the fiscal deficit in the medium long term. More aggressive investment programs borrow against future fiscal gains to combine new energy investments with either delayed or frontloaded investments in non-energy infrastructure. These programs lead to much higher real wages and real GDP while keeping public debt sustainable and the fiscal deficit low in the medium and long term.

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I. Introduction

Senegal is experiencing higher fiscal deficits just five years after the Heavily Indebted Poor Countries Initiative (HPIC) cut the stock of total public debt stock from 79 percent of GDP in 2000 to 21 percent in 2006. The fiscal deficit has increased from 2.5 percent of GDP in 2001-2006 to an average of 5 percent of GDP in the last five years. Public debt has increased two-fold and is now *higher* than its 2006 level. At the same time, the country is grappling with a recurring energy crisis. It needs to find a way to both reduce the fiscal deficit and finance new public investment in energy and non-energy infrastructure.

Fiscal adjustment is generally difficult and contentious; in practice, easy expenditure cuts and easy ways to raise revenue are hard to find. Senegal has an unusual option, however. The adverse response to higher energy prices a couple years ago led to investment in an inefficient oil-based energy sector. This has created scope for new investments to both improve efficiency in the energy sector and generate fiscal surpluses that can be shared with investments in other types of infrastructure and/or help pay for deficit reduction. Reform can be accomplished by investing in an efficient hydropower, gas-fired or coal-fired energy sector while downsizing the inefficient oil-based sector. The critical issue is whether the potential efficiency gains from downsizing the oil-based inefficient sector and replacing it with a more efficient hydropower, gas-fired and coal-fired sector are big enough to allow the country to both invest more overall in infrastructure and achieve the necessary deficit reduction.

In this paper we employ a variant of the Fund's new tool for debt sustainability analysis to analyze the impact of different adjustment programs on growth, private investment, real wages, debt, and the fiscal deficit. The model developed by Buffie et al. (2012) incorporates sector-specific capital, productivity-enhancing infrastructure, concessional loans and external commercial debt, a consumption VAT and government transfer payments, variable efficiency of public investment, an absorptive capacity constraint, and poor hand-to-mouth consumers. To adapt the framework for Senegal, we add wage and profits taxes, government consumption of traded and nontraded goods, controlled energy prices, regional CFA-zone debt, an inefficient, oil-based energy sector, and a new, low-cost coal, gas and hydropower energy sector. After investigating the effects of varying policy instruments one at a time, we focus on policy packages that increase public investment in energy and infrastructure in scenarios with varying degrees of debt finance and with different types of supporting fiscal adjustment.

Our central finding is that Senegal's extremely inefficient existing oil-based energy sector represents both a problem and an opportunity. A public investment program that coordinates new investment in low-cost hydroelectric power with a phased contraction of the oil-based sector increases the total supply of energy by 70 percent while stimulating private investment and increasing real wages and real GDP. Because technology differs in the oil- and hydro-based sectors, the fiscal deficit increases 1 percent of GDP in the short run. In the medium run, however, the investment program delivers fiscal gains on the order of 4 percent of GDP. A temporary 30 percent increase in energy prices ensures that the fiscal deficit decreases continuously. But this "solution" is problematic – energy prices are already quite high in Senegal. Given the large fiscal gains that accrue over the medium term, a strong case can be made that Senegal should borrow against its fiscal surpluses and strongly scale-up investment in infrastructure at the same time that it invests in a more efficient hydro-based energy sector. This big-push investment program increases real wages and real output by more than 10 percent; moreover, the medium-term fiscal dividend still exceeds 3 percent of GDP. The program can be financed by borrowing either in the regional CFA market or the Eurobond market, but, at current interest rates, borrowing in the Eurobond market is more costly and entails more supporting fiscal adjustment.

The rest of the paper is organized into four sections. Section II summarizes macroeconomic developments in Senegal since 2001. Following this, we lay out the model and calibrate it to the data for Senegal in Sections III and IV. In Section V, the heart of the paper, we investigate the pros and cons of various strategies of fiscal consolidation. Section VI concludes.

II. Macroeconomic Developments since 2001: A Brief Overview

Over the last five years Senegal's fiscal position has deteriorated. The budget deficit averaged 5.1 percent of GDP during the 2007-2012 period, up from an average of 2.6 percent for 2001-2006. The rise in the deficit reflects the fact that revenues have not kept pace with rising public expenditure (Table 1): relative to 2001-2006, revenue increased by 2.3 percent of GDP, while expenditure rose nearly 5 percent of GDP. Despite the increase in deficit in recent years, average GDP growth was only 4.3 percent during 2007-2012 (compared to 4.4 in the 2001-2006 period) and this sluggish growth has created problems in terms of the efficiency of public spending.

A look at the structure of the budget reveals the predominance of current expenditure. Since 2001 current expenditure has remained a steady 60 percent of total expenditure (or 15.4 percent of GDP). Compared to the 2001-2006 period, spending on goods and services (which includes wages and salaries), total subsidies and other transfers as well as their key component — energy related subsidies all increased in percent of GDP during the 2007-2012 period. The latter development reflects an unsettling trend of growing fiscal problems in the energy sector. We elaborate on this point below.

Table 1. Revenue and Expenditure (Period Averages, in percent of GDP)

Period	2001-2006	2007-2012
Total Revenue and Grants	20.1	22.4
Total Expenditure	22.7	27.5
Current Expenditure	14.0	16.7
Goods and services	8.9	10.8
Subsidies and other current transfers	4.2	4.9
Energy Sector related subsidies**	1.1*	1.8
Capital Expenditure	8.6	10.8
Overall Balance	−2.1	−5.1

Source: Senegalese authorities and IMF staff estimates

*Average based on 2005-2006 figures only

**Include fuel, butane and Société Africaine de Raffinage (SAR) subsidies

The challenges facing the energy sector are related to growth as much as they are related to fiscal. These challenges stem from an inefficient mode of electricity generation, transmission and distribution, coupled with controlled prices that mask the true costs of power generation.[1] About 90 percent of the sector's power supply is generated using imported oil, with the remaining 10 percent obtained from hydropower. Benchmark electricity prices, or tariffs, are set below full cost recovery, giving rise to tariff gaps and explicit producer and consumer subsidies. As a result, budgetary compensation from the government to the state-owned electricity company (SENELEC) amounted to CFAF 105 billion or 1.5 percent of GDP in 2012. Despite these large budgetary transfers, SENELEC has run large operating deficits in recent years. Additional budget costs in the power sector include a shortfall in tax collection (0.5 percent of GDP), the cost of renting mobile power generators, and expenditure on the rehabilitation or extension of existing power plants (0.2 and 0.3 percent of GDP, respectively). When these additional costs are taken into account, the total deficit of the power sector rises to 2.5 percent of GDP in 2012.[2] In addition, power outages, resulting from inefficient production and distribution, have been costly to growth. In 2011, outages are thought to have subtracted 1-1.5 percentage point of GDP growth.

The costly subsidies in the energy sector, coupled with the negative growth impacts of the inefficient mode of electricity production and distribution, are partly to blame for the recent rapid rise in Senegal's public debt. Since 2006, when the HIPC initiative reduced the debt to 21.9 percent of GDP, the total public debt increased more than two-fold and currently stands at 45.4 percent of GDP.[3] Higher government spending on goods and services has also contributed to the increase in public debt.

To stabilize the public debt and avert a potential budget crisis, Senegal must tackle the root causes of its fiscal problems. It could cut expenditure on goods and services and/or transfers and subsidies;

[1] See Torres and others (2011).
[2] June 2013 Senegal Staff Report (IMF Country Report No. 13/170).
[3] Stock at end-2013, IMF Country Report No. 14/4.

it could raise revenue by raising taxes, or it could combine both expenditure cuts and tax increases to reduce its fiscal deficit. Over the medium term, however, the best solution is to reform the energy sector by coordinating new investment in a more efficient mode of energy production with gradual disinvestment in the existing inefficient oil-based energy plants.

III. The Model

The core structure of the model is the same as in Buffie et al. (2012). To adapt the model for Senegal, we add a parastatal energy sector, a regional bond market, and more public sector spending and tax variables. The analysis abstracts from money and nominal rigidities in order to focus on the medium/long-run effects of adjustment on the fiscal deficit and growth.

A lot of notation accompanies any large model. In what follows, x and n subscripts refer to the tradables and nontradables sectors; k, L, J, E, and z denote capital, labor, land, energy and infrastructure; P_i is the price of good i; and all quantity variables except labor are detrended by $(1+g)^t$, where g is the exogenous long-run growth rate of real GDP.

Technology

Firms operate Cobb-Douglas production functions in the tradables and nontradables sectors. Infrastructure enters as a public good that enhances productivity in both sectors, while land is specific to the tradables sector:

$$q_{x,t} = (a_x z_{t-1}^{\psi_x}) k_{x,t-1}^{\alpha_x} J^{\alpha_J} E_{x,t}^{\beta_x} L_{x,t}^{1-\alpha_x-\alpha_J-\beta_x}, \tag{1}$$

$$q_{n,t} = (a_n z_{t-1}^{\psi_n}) k_{n,t-1}^{\alpha_n} E_{n,t}^{\beta_n} L_{n,t}^{1-\alpha_n-\beta_n}, \tag{2}$$

Energy is produced by the state. Initially, all plants employ an inefficient, oil-based technology. In the reform scenarios, these plants are replaced by more efficient coal-fired, gas-fired and/or hydroelectric plants.[4] The capital stocks associated with the two technologies are k_e (inefficient) and k_h (efficient). There is no scope for substitution between inputs and production is constrained by the size of the capital stock:

$$q_{e,t} = a_e k_{e,t-1}, \tag{3}$$

$$q_{h,t} = a_h k_{h,t-1}. \tag{4}$$

Leontief technology also characterizes production of capital goods. Factories and infrastructure are built by combining one imported machine with a_j $(j = k, z, e, h)$ units of a nontraded input (e.g.,

[4] It is important to note that given the possibly limited potential in sources of hydropower in Senegal, the actual strategy for investing in an efficient energy sector may focus more on coal- and gas-fired and less on hydropower. What is more important, however, is the move to a more efficient mode of electricity production.

construction). The supply prices of private capital and infrastructure are thus

$$P_{k,t} = P_{mm} + a_k P_{n,t}, \tag{5}$$
$$P_{z,t} = P_{mm} + a_z P_{n,t}, \tag{6}$$
$$P_{ke,t} = P_{mm} + a_{ke} P_{n,t}, \tag{7}$$
$$P_{kh,t} = P_{mm} + a_{kh} P_{n,t}, \tag{8}$$

where P_{mm} is the price of imported machinery.[5]

Factor Demands

Competitive firms maximize profits in the tradables and nontradables sectors by hiring land, labor, and capital up to the point at which the marginal value product of the input equals its price. Labor is intersectorally mobile, but capital is sector specific. Hence

$$P_{n,t}(1 - \alpha_n - \beta_n) q_{n,t} / L_{n,t} = w_t, \tag{9}$$
$$P_{x,t}(1 - \alpha_x - \alpha_J - \beta_x) q_{x,t} / L_{x,t} = w_t, \tag{10}$$
$$P_{x,t} \alpha_J q_{x,t} / J_x = r_{J,t}, \tag{11}$$
$$P_x \alpha_x q_{x,t} / k_{x,t-1} = r_{x,t}, \tag{12}$$
$$P_n \alpha_n q_{n,t} / k_{n,t-1} = r_{n,t}, \tag{13}$$

where w is the wage, r_i is the capital rental in sector i, and r_J is the land rent.

In the state-run energy sector, Leontief technology implies that employment and purchases of oil are tied to the capital stocks through constants determined by the fixed input-output coefficients:

$$L_{h,t} = a_2 k_{h,t-1}, \tag{14}$$
$$L_{e,t} = a_3 k_{e,t-1}, \tag{15}$$
$$O_t = a_4 k_{e,t-1}. \tag{16}$$

The price of energy set by the state is far below the notional market-clearing price, P_e^*. We call P_e^* the shadow price and assume efficient rationing of firm demand. The shadow price and the marginal value product of energy are the same therefore in the tradables and nontradables sectors:

$$P_x q_{x,t} \beta_x / E_{x,t} = P_{e,t}^*, \tag{17}$$
$$P_n q_{n,t} \beta_n / E_{n,t} = P_{e,t}^*. \tag{18}$$

Private Sector Optimization Problems

[5] The supply price of capital (the cost of building a factory) is the same in the tradables and nontradables sectors. Once capital is installed, however, it becomes sector-specific. Allowing for separate supply prices of capital does not significantly affect any of the results.

The private sector is populated by two types of agents, savers and non-savers. Labor supply of savers is fixed at L while that of non-savers is $L_1 = aL$. The two agents are identical qua consumers. Their instantaneous utility function is

$$U = \frac{c_i^{1-1/\tau}}{1-1/\tau} + k_o \frac{cE_i^{1-1/\tau}}{1-1/\tau}, \quad i = 1, s,$$

where τ is the intertemporal elasticity of substitution; E is energy consumption; and non-energy consumption

$$c_i = \left[k_1 c_{mi}^{(\epsilon-1)/\epsilon} + k_2 c_{xi}^{(\epsilon-1)/\epsilon} + (1-k_1-k_2) c_{ni}^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)},$$

is a CES aggregate of traded, nontraded, and imported consumer goods, with substitution parameter ϵ and associated price index

$$P_t = [k_1 P_{m,t}^{1-\epsilon} + k_2 P_{x,t}^{1-\epsilon} + (1-k_1-k_2) P_{n,t}^{1-\epsilon}]^{1/(1-\epsilon)}.$$

Non-savers consume all of their income each period. Let h, h_w, and P_{ec} denote the consumption value added tax (VAT), the tax rate on wage income, and the price of energy sold to households. Since the price of energy is artificially low, demand is rationed at the level $\bar{E}_i$. Assuming transfers (T) and remittances ($remit$) are proportional to the agent's share in aggregate employment, the non-savers' budget constraint reads[6]

$$(1+h_t)P_t c_{1,t} + P_{ec}\bar{E}_{1,t} = w_t a L(1-h_{w,t}) + \frac{a}{1+a}(T_t + remit), \tag{19}$$

or

$$c_{1,t} = \frac{w_t a L(1-h_{w,t}) + a(T_t + remit)/(1+a) - P_{ec}\bar{E}_{1,t}}{(1+h_t)P_t}. \tag{19$'$}$$

Savers derive income from profits, land rents, wages, transfers, and remittances. They choose consumption, government bonds, and investment in physical capital to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(c_{s,t})^{1-1/\tau}}{1-1/\tau} + k_o \frac{E_{s,t}^{1-1/\tau}}{1-1/\tau} + a_o \frac{(b_{p,t})^{1-1/\eta}}{1-1/\eta}\right], \tag{20}$$

subject to

$$\begin{aligned} b_{p,t} \;=\; & (1-h_{p,t})[P_x q_{x,t} + P_{n,t} q_{n,t} - P_{e,t}(E_{n,t} + E_{x,t}) - w_t(L_{x,t} + L_{n,t})] + w_t L(1-h_{w,t}) \\ & + \frac{T_t + remit}{1+a} + \frac{1+r_{t-1}}{1+g} b_{p,t-1} - P_t c_{s,t}(1+h_t) - P_{ce,t}E_{s,t} - \mu_t z_{t-1}, \\ & - P_{k,t}\left[i_{x,t} + i_{n,t} + \frac{v}{2}\left(\frac{i_{x,t}}{k_{x,t-1}} - \delta - g\right)^2 k_{x,t-1} + \frac{v}{2}\left(\frac{i_{n,t}}{k_{n,t-1}} - \delta - g\right)^2 k_{n,t-1}\right], \end{aligned} \tag{21}$$

$$(1+g)k_{x,t} = i_{x,t} + (1-\delta)k_{x,t-1}, \tag{22}$$

$$(1+g)k_{n,t} = i_{n,t} + (1-\delta)k_{n,t-1}, \tag{23}$$

$$E_{s,t} \leq \bar{E}_{s,t}, \tag{24}$$

[6] Taxes on energy are included in P_{ec}.

where $\beta = 1/[(1+\rho_1)(1+g)^{(1-\tau)/\tau}]$ is the discount factor; ρ_1 is the pure time preference rate; τ is the intertemporal elasticity of substitution; b_p is bonds purchased by *domestic* residents; h_p is the profits tax; i_j is gross investment in sector j; δ is the depreciation rate; r is the interest rate on tradable bonds sold in the regional WAEMU market; and μ is the user fee charged for infrastructure services. Bonds generate non-pecuniary services, and the terms $v(\bullet)k_{j,t-1}/2$ in the budget constraint measure adjustment costs incurred in changing the capital stock. Observe also that the trend growth rate appears in several places in (21)-(23), reflecting the fact that some variables are dated at t and others at t-1.[7]

The choice variables in the optimization problem are $c_{s,t}$, $b_{p,t}$, $E_{s,t}$, $i_{j,t}$, and $k_{j,t}$. On an optimal path,

$$c_{s,t} = c_{s,t+1}\left[a_o\left(\frac{b_{p,t}}{P_t}\right)^{-1/\eta} c_{s,t+1}^{1/\tau}(1+h_t) + \left(\beta_1\frac{1+r_t}{1+g}\frac{P_t}{P_{t+1}}\frac{1+h_t}{1+h_{t+1}}\right)\right]^{-\tau}, \tag{25}$$

$$\begin{aligned} F\frac{P_{k,t}}{P_{k,t+1}}\left[1+v\left(\frac{i_{x,t}}{k_{x,t-1}}-\delta-g\right)\right] &= \frac{r_{x,t+1}(1-h_{p,t+1})}{P_{k,t+1}}+1-\delta \\ &\quad +v\left(\frac{i_{x,t+1}}{k_{x,t}}-\delta-g\right)\left(\frac{i_{x,t+1}}{k_{x,t}}+1-\delta\right) \\ &\quad -\frac{v}{2}\left(\frac{i_{x,t+1}}{k_{x,t}}-\delta-g\right)^2, \end{aligned} \tag{26}$$

$$\begin{aligned} F\frac{P_{k,t}}{P_{k,t+1}}\left[1+v\left(\frac{i_{n,t}}{k_{n,t-1}}-\delta-g\right)\right] &= \frac{r_{n,t+1}(1-h_{p,t+1})}{P_{k,t+1}}+1-\delta \\ &\quad +v\left(\frac{i_{n,t+1}}{k_{n,t}}-\delta-g\right)\left(\frac{i_{n,t+1}}{k_{n,t}}+1-\delta\right) \\ &\quad -\frac{v}{2}\left(\frac{i_{n,t+1}}{k_{n,t}}-\delta-g\right)^2, \end{aligned} \tag{27}$$

$$k_o E_{s,t}^{-1/\tau} = P_{ce,t}\frac{c_{s,t}^{-1/\tau}}{P_t(1+h_t)} + \lambda_{4,t}, \tag{28}$$

$$\lambda_{4,t}(E_{s,t}-\bar{E}_{s,t}) = 0 \tag{29}$$

where

$$F \equiv \left(\frac{c_{s,t}}{c_{s,t+1}}\right)^{-1/\tau}\frac{P_{t+1}}{P_t}\frac{1+h_{t+1}}{1+h_t}\frac{1+g}{\beta_1}$$

and λ_4 is the multiplier attached to the rationing constraint (24). These are generally familiar conditions. The Euler equations in (25)-(27) govern the paths of non-energy consumption and sectoral investment. Equations (28) and (29) state that the relative price of energy (P_{ec}/P) is less than the marginal rate of substitution between energy consumption and non-energy consumption when demand is rationed (i.e., $\lambda_{4,t} > 0$).

[7] The convention for detrending the capital stocks differs from that for other variables. Because $K_{j,t-1}$ (the capital stock before detrending) is the capital stock in use at time t, we define $k_{j,t-1} \equiv K_{j,t-1}/(1+g)^t$. Under this convention, $i_j = (\delta+g)k_j$ in the long run — as required for the capital stock to grow at the trend growth rate g.

In passing we should say a few words about the assumption that bonds yield non-pecuniary services. This is not to everyone's taste. It is essential, however, for realistic calibration of the model. Note from (25-(27) that

$$\begin{aligned}\frac{a_o b^{-1/\eta} P(1+h)}{c_s^{-1/\tau}} &= 1 - \frac{\beta_1(1+r)}{1+g},\\ \frac{r_i(1-h_p)}{P_k} - \delta &= \frac{1+g}{\beta_1} - 1, \quad i = x, n\end{aligned}$$

at the initial steady state. When bonds do not confer non-pecuniary services, the real after-tax return on private capital is constrained to equal the real interest rate r paid on government bonds. But r is only 3.5 percent , whereas the return on capital is probably on the order of 8-10 percent . If non-pecuniary benefits do not enter as a wedge between the financial returns, then the model would have to be calibrated either with an unrealistically high real interest rate or with an unrealistically low return on private capital (implying, also, absurdly high values for the capital-ouput ratio and the share of investment in GDP).

Exact Price Indices

Accurate measurement of real wages requires exact consumption price indices for saving and non-saving households. So far, all we have is the formula for P, the exact price index for non-energy consumption. To derive the exact price indices for aggregate consumption, we also need the shadow prices of E_1 and E_s. For households that save, the shadow price is the price at which $\lambda_4 = 0$ in equation (28):

$$P^*_{ec,t} = k_o \left(\frac{\bar{E}_{s,t}}{c_{s,t}}\right) P_t(1+h_t). \tag{30}$$

The exact price index paired with P^*_{ec} is

$$CPI^*_{s,t} = \left[P_t^{1-\tau} + k_o^\tau (P^*_{ec,t})^{1-\tau}\right]^{1/(1-\tau)}. \tag{31}$$

With P^*_{ec} and CPI^*_s in hand, it is easy to show (see Appendix A) that

$$CPI_{s,t} = \frac{CPI^*_{s,t}}{1+\gamma_{e,t}(P^*_{ec,t} - P_{ec,t})/P_{ec,t}}, \tag{32}$$

where $\gamma_{e,t} \equiv P_{ec,t}\bar{E}_{s,t}/(P_t c_{s,t} + P_{ec,t}\bar{E}_{s,t})$, the share of energy in aggregate consumption measured at official prices.

Since savers and non-savers are identical qua consumers,

$$CPI_{1,t} = \frac{CPI^*_{1,t}}{1+\gamma_{e1,t}(P^*_{ec,t} - P_{ec,t})/P_{ec,t}}, \tag{33}$$

with

$$
\begin{aligned}
P^*_{ec1,t} &= k_o \left(\frac{\bar{E}_{1,t}}{c_{1,t}}\right) P_t(1+h_t), \\
\Longrightarrow P^*_{ec1,t} &= P^*_{ec,t}\left[\frac{\gamma_{e,t}(1-\gamma_{e1,t})}{\gamma_{e,t}(1-\gamma_{e,t})}\right]^{1/\tau}, \\
CPI^*_{1,t} &= \left[P_t^{1-\tau} + k_o^{\tau}(P^*_{ec,t})^{1-\tau}\right]^{1/(1-\tau)},
\end{aligned}
$$

and $\gamma_{e1,t} \equiv P_{ec,t}\bar{E}_{1,t}/(P_t c_{s,t} + P_{ec,t}\bar{E}_{1,t})$. The data indicate that $\gamma_{e1} < \gamma_e$. Consequently, $P^*_{ec1} > P^*_{ec}$ and $CPI^*_1 > CPI^*_s$ at the initial equilibrium.

Public Investment

The capital stocks in the energy sector increase over time at the rates

$$(1+g)k_{e,t} = i_{e,t} + (1-\delta)k_{e,t-1}, \tag{34}$$

$$(1+g)k_{h,t} = i_{h,t} + (1-\delta)k_{h,t-1}, \tag{35}$$

where i_e and i_h denote gross investment.

Public investment is seldom perfectly efficient. As noted in the introduction, inefficiency in the energy sector takes the form of excessive reliance on high-cost, oil-based technology. In the case of infrastructure, we allow for the possibility that increases in the stock of physical capital $\tilde{z}$ may not translate into equal increases in the stock of economically valuable capital z.[8] The usual law of motion applies to $\tilde{z}$

$$(1+g)\tilde{z}_t = i_{z,t} + (1-\delta)\tilde{z}_{t-1}, \tag{36}$$

but some of the newly built infrastructure may not enhance productivity:

$$z_t = z_o + s(\tilde{z}_t - \tilde{z}_o), \quad s \leq 1. \tag{37}$$

Fiscal Adjustment and the Public Sector Budget Constraint

Government expenditure comprises transfers, wages, oil imports, investments in energy and infrastructure, interest payments on the debt, supplemental energy imports[9] q_f from Mali at price P_f, and purchases g_n and g_m of nontraded and imported goods. Revenues flow from user fees assessed for infrastructure services, the consumption VAT, taxes on wages and profits, grants (x), and energy sales. When expenditure exceeds revenues, the resulting deficit is financed by issuing additional debt b in the regional bond market:[10]

$$\begin{aligned} b_t - b_{t-1} = {} & P_{z,t}\left[\left(1 + \frac{i_{z,t}}{\tilde{z}_{t-1}} - \delta - g\right)^{\phi}(i_{z,t} - i_{z,o}) + i_{z,o}\right] + P_{ke,t}i_{e,t} + P_{kh,t}i_{h,t} \\ & + P_{n,t}g_{n,t} + P_{m,t}g_{m,t} + \frac{r_d - g}{1+g}d_{t-1} + \frac{r_{dc} - g}{1+g}dc_{t-1} + \frac{r_{t-1} - g}{1+g}b_{t-1} + T_t + w_t(L_{e,t} + L_{h,t}) \\ & + P_{o,t}O_t + P_{f,t}q_{f,t} - P_{ec,t}(E_{s,t} + \bar{E}_{1,t}) - P_{e,t}(E_{x,t} + E_{n,t}) - h_t P_t(c_t + c_{1,t}) - x_t - \mu_t z_{t-1} \\ & - h_{w,t}w_t L(1+a) - h_{p,t}[P_x q_{x,t} + P_{n,t}q_{n,t} - P_{e,t}(E_{n,t} + E_{x,t}) - w_t(L_{x,t} + L_{n,t})]. \end{aligned} \tag{38}$$

Plans to replace the oil-based energy sector with new, more efficient sources of energy have been in the works for years. We assumed they can be implemented efficiently utilizing SENELEC's existing

[8] See Hulten (1996) and Pritchett (2000) for evidence that public investment often fails to increase the supply of productive infrastructure.

[9] These imports are through OMVS (Organization pour la Mise en Valeur du Fleuve Senegal) which is a regional organization that comprises Mali, Mauritania and Senegal).

[10] When concessional and non-concessional borrowing supplement borrowing in the regional bond market, $d_t + dc_t - d_{t-1} - dc_{t-1}$ is added on the left side in equation (38).

personnel. Accordingly, capital outlays in the energy sector are simply the product of the supply price of energy and planned gross investments.

The story for infrastructure investment is different. Due to the scarcity of technical expertise and skilled administrators, there is a risk of large cost overruns in ambitious programs that scale up investment too quickly in too many areas. To capture this, we multiply new investment $(i_{z,t} - i_{z,o})$ by $(1 + i_{z,t}/\tilde{z}_{t-1} - \delta - g)^{\phi}$, where $\phi \geq 0$ determines the severity of the the absorptive capacity constraint in the public sector. The constraint affects only implementation costs for new projects: in a steady state, $(i_{z,t}/\tilde{z}_{t-1} - \delta - g)^{\phi} = 1$ as $i_{z,t}/\tilde{z}_{t-1} = \delta + g$.

Senegal is interested mainly in how alternative programs of investment + fiscal adjustment affect growth and the fiscal deficit. The technical problem in analyzing such scenarios is that the debt dynamics associated with an *exogenously* specified program are unstable. Fortunately, the problem admits of a simple solution. Because transfers are purely lump sum, variations in T do not affect any other variable in the model. Consequently, if we assume that transfers adjust to continuously balance the budget, the paths for the real wage, real GDP, private investment, etc., reflect only the impact of the specified program, while the solution for

$$\begin{aligned}
T_o - T_t = P_{z,t}\left[\left(1 + \frac{i_{z,t}}{\tilde{z}_{t-1}} - \delta - g\right)^{\phi}(i_{z,t} - i_{z,o}) + i_{z,o}\right] + P_{ke,t}i_{e,t} + P_{kh,t}i_{h,t} \\
+P_{n,t}g_{n,t} + P_{m,t}g_{m,t} + \frac{r_d - g}{1+g}d_{t-1} + \frac{r_{dc} - g}{1+g}dc_{t-1} + \frac{r_{t-1} - g}{1+g}b_{t-1} + T_o + w_t(L_{e,t} + L_{h,t}) \\
+P_{o,t}O_t + P_{f,t}q_{f,t} - P_{ec,t}(\bar{E}_{s,t} + E_{1,t}) - P_{e,t}(E_{x,t} + E_{n,t}) - h_tP_t(c_t + c_{1,t}) - x_t - \mu_t z_{t-1} \\
-h_{w,t}w_tL(1+a) - h_{p,t}[P_x q_{x,t} + P_{n,t}q_{n,t} - P_{e,t}(E_{n,t} + E_{x,t}) - w_t(L_{x,t} + L_{n,t})]. \qquad (39)
\end{aligned}$$

mirrors the change in the path of the fiscal deficit.

Immediate deficit reduction is a priority in Senegal.[11] Given the current low cost of borrowing in the regional bond market, however, the government should consider the tradeoffs afforded by temporary deficit-financed programs. In these programs, one or more policy instruments adjust gradually to eliminate the fiscal deficit and stabilize the path of debt. See Appendix B for more details and an illustrative example.

Market-Clearing Conditions

Wages and prices adjust to align demand with supply in the markets for labor and nontraded

[11] The Senegalese authorities have made fiscal reduction a priority and a commitment.

goods:

$$
\begin{aligned}
L(1+a) &= L_x + L_n + L_e + L_h, && (40)\\
q_{n,t} &= (1-k_1-k_2)(P_{n,t}/P_t)^{-\epsilon}(c_{s,t}+c_{1,t}) + g_{n,t}\\
&\quad + a_k\left[i_{x,t} + +\frac{v}{2}(\bullet)^2 k_{x,t-1} + i_{n,t} + \frac{v}{2}(\bullet)^2 k_{n,t-1}\right] + a_{ke} i_{e,t}\\
&\quad + a_{kh} i_{h,t} + a_z\left[\left(1+\frac{i_{z,t}}{\tilde{z}_{t-1}} - \delta - g\right)^{\phi}(i_{z,t}-i_{z,o}) + i_{z,o}\right]. && (41)
\end{aligned}
$$

In the energy sector, where low prices and acute shortages are the norm, rationing constraints purchases to equal available supply:

$$
q_{e,t} + q_{h,t} + q_{f,t} = E_{x,t} + E_{n,t} + E_{s,t} + E_{1,t}. \tag{42}
$$

External Debt Accumulation and the Current Account

Summing the budget constraints of private agents and the government produces the national saving-investment identity

$$
\begin{aligned}
b_{f,t} - b_{f,t-1} &= P_t(c_{s,t}+c_{1,t}) + P_{z,t}\left[\left(1+\frac{i_{z,t}}{\tilde{z}_{t-1}} - \delta - g\right)^{\phi}(i_{z,t}-i_{z,o}) + i_{z,o}\right]\\
&\quad + P_{k,t}\left[i_{x,t} + i_{n,t} + \frac{v}{2}(\bullet)^2 k_{x,t-1} + \frac{v}{2}(\bullet)^2 k_{n,t-1}\right] + P_{ke,t} i_{e,t} + P_{kh,t} i_{h,t}\\
&\quad + \frac{r_d - g}{1+g} d_{t-1} + \frac{r_{dc}-g}{1+g} dc_{t-1} + \frac{r_{t-1}-g}{1+g} b_{f,t-1} + P_{o,t} O_t + P_{f,t} q_{f,t}\\
&\quad + P_{n,t} g_{n,t} + P_{m,t} g_{m,t} - P_{n,t} q_{n,t} - P_x q_{x,t} - x_t - remit, && (43)
\end{aligned}
$$

where $b_f \equiv b - b_p$. Senegal's net foreign debt, b_f, equals its net position in WAEMU tradable bonds. This increases each year by the difference between national spending and national income.

Interest Rate Determination

Senegal is a large player in the regional bond market: when it borrows more, it pushes up the equilibrium interest rate. Rather than build a general equilibrium model of the regional economy, we postulate a simple inverse loan supply curve:

$$
r_t = r_o + \rho\left(\frac{b_{f,t} - b_{f,o}}{b_{f,o}}\right), \quad \rho \geq 0. \tag{44}
$$

IV. Model Calibration

A mix of hard data supplied by the Senegalese government, empirical estimates from the development literature, and judgment were employed to calibrate the model. Below we discuss *seriatim* the rationale and the data sources for the value assigned to each parameter in Table 2:

- *Tax rates on profits and wage income* (h_p, h_w). Revenue from income taxes was 5.6 percent of GDP in 2012, with wage taxes accounting for 60 percent of the total. This and the income shares of wages and profits at the initial equilibrium imply an *effective* tax rate of 5.9 percent on profits and 5.4 percent on wages. The effective rates are much lower than the statutory rates because the base for income taxes is a small fraction of GDP (roughly the share of the formal sector in total output).

- *Consumption value-added tax* (h). The consumption VAT in the model proxies for the average indirect tax rate. Total tax revenue amounted to 18.9 percent of GDP in 2011. Subtracting income taxes and taxes on petroleum products brings the figure down to 10.7 percent . Dividing this by the share of private consumption in GDP yields 14 percent.[12]

- *Consumption shares of energy measured at official prices* (γ_e, γ_{e1}). Household survey data for 2005-2006 put the share of energy in consumption at 2.1 percent overall and at 0.4 percent for the poorest three deciles. Lacking more recent data, we assumed the consumption share for poor, non-saving households is still 0.4 percent. This and the figure for the aggregate consumption share give a share of 2.7 percent for non-poor households.

- *Cost shares of energy in tradables and nontradables production, measured at official prices* (β_{xo}, β_{no}). Data from SENELEC on sales of energy to firms vs. households together with the data for the share of energy in consumption and the ratio of consumption to GDP allow us to back out guesstimates of the cost shares of energy in private production, measured at official prices. Absent any information on the relative energy intensity of tradables vs. nontradables production, we set the cost share at 1.25 percent in both sectors. Sales to firms then comprise 44 percent of total energy sales at the initial equilibrium.[13]

- *Intertemporal elasticity of substitution* (τ). Most estimates of τ for LDCs lie between .20 and .75 (Agenor and Montiel, 1996). The assigned value of .40 is in the middle of this range and close to the estimates for Africa in Ostry and Reinhart (1992) and for Ghana and Kenya in Ogaki et al. (1996).

- *Elasticity of substitution between non-energy consumer goods* (ϵ). Estimates of demand systems with 5-10 goods generally place compensated own-price elasticities of demand in the .15-.50 range.[14] A value of .50 for ϵ produces compensated own-price elasticities in the lower/middle part of this range.

- *Depreciation rate* (δ). There is little hard data on depreciation rates in LICs. Our choice of 5 percent is in line with estimates for developed countries.[15]

[12] Despite the inclusion of revenue from other indirect taxes, the effective rate is well below the statutory VAT rate. This reflects the fact that a substantial part of private consumption escapes the tax net.

[13] The slight upward revision relative to the data (44 percent vs. 37 percent) takes into account energy consumption at owner-occupied businesses.

[14] See Lluch et al. (1977, chapter 3), Deaton and Muellbauer (1980, p.71), Blundell (1988, p.35), and Blundell, Pashardes, and Weber (1993, Table 3b, p.581).

[15] A depreciation rate of 5% is in line with empirical estimates of the depreciation rate for physical capital (Blundell et al., 1992; Nadiri and Prucha, 1996) and with data on service lives of equipment and structures reported by the Bureau of Economic Analysis (Musgrave, 1992). Papers that use a higher depreciation rate of 10% explicitly or implicitly count

- *q-elasticity of investment spending* (Ω). Evaluated at the initial equilibrium, the elasticity of investment with respect to Tobin's q is $\Omega = 1/(\delta+g)v$, where v is the parameter that determines adjustment costs to changing the capital stock. There are no reliable estimates of this elasticity for LDCs. The assigned value, 2, is at the high end of estimates for developed countries. The results do not change substantively when Ω equals .5 or 10.

- *Trend growth rate* (g). The trend growth in per capita income is a modest 1 percent.

- *The pure time preference rate* (ρ_1) *and the real return on capital.* We chose the pure time preference rate jointly with the trend growth rate, the profits tax, and the intertemporal elasticity of substitution so that the after-tax return on capital equals 10 percent.[16]

- *Real interest rate on government debt* (r). Over the past decade, the real interest rate on bonds the Government of Senegal (GOS) sells in the WAEMU market has averaged 3.5 percent .

- *Real interest rates on concessional and non-concessional loans* (r_d, r_{dc}). The average real interest rate on concessional loans is approximately zero. In Senegal's most recent Eurobond issue, the interest rate was 7 percent.

- *Elasticity of non-pecuniary services with respect to the stock of real bonds* (η). The parameter η governs the semi-elasticity of bond demand with respect to the real interest rate, while the ratio η/τ fixes the long-run elasticity of bond holdings with respect to a permanent increase in private income (of saving households).[17] We set η equal to τ on the assumption that unity is a reasonable value for the income elasticity.

- *Cost shares of land and capital in private production* (α_J, α_n, α_x). The cost shares of capital in the nontradables sector and the non-agricultural part of the tradables sector were computed from data in the national income accounts. Unfortunately, no data are available for the agricultural part of the tradables sector. We decided to rely therefore on the factor shares reported for SSA in social accounting matrices assembled by GTAP (Global Trade Assistance Project). The simple average of the shares in smallholder and commercial agriculture is .19 for capital and .14 for land.[18] The capital-labor ratio in agriculture is 54 percent of that in non-agriculture vs. 49.3 percent in nearby Cameroon (Emini et al., 2006). The cost shares in Table 1 for the composite tradables sector are a weighted average of the shares in the non-agricultural and agricultural tradables sectors.

- *Share of poor, non-saving households* $[a/(1+a)]$. There are no estimates of the share of non-saving households in Senegal or other LDCs. Our guesstimate, 60 percent, is at the high end of estimates for developed countries.

---

consumer durables as part of investment. While this is conceptually correct, it is inappropriate in a model that focuses on how changes in the physical capital stock affect GDP growth, tax revenues, and real wages.

[16] Across steady states, the after-tax real return on private capital equals $(1+\rho_1)(1+g)^\tau - 1$. The after-tax return is set directly as r_1 in the computer programs.

[17] Bond demand is a function of consumption and the real interest rate. In the long run, the change in consumption equals the change in real income.

[18] We have converted the GTAP value added shares into cost shares. The labor share is determined residually after computing the cost share of energy at the shadow price of energy.

- *Cost shares in existing oil-based energy production* (α_e, α_o). SENELEC has provided us with data on labor costs, energy costs , and depreciation charges. Straightforward algebra shows that the cost share of oil is
$$\alpha_o = \frac{J\delta/R_e}{1-(1+F)J(1-\delta/R_e)},$$
where J is oil's share in O+M, F is the ratio of wages to oil expenditure, and R_e is the gross return on energy investment.[19] The average values of J and F for 2005-2010 were .79 and .14, respectively. These numbers and the values assigned to δ and R_e give cost shares of .61 for oil, .09 for labor, and .30 for capital.

- *Cost share of capital in planned investments in hydroelectric power* (α_h). The value added share of capital in the inefficient oil-based energy sector is 78 percent . Production of hydroelectric power is even more capital intensive. The Ministry of Energy estimates the cost share of capital in planned investments at 93 percent.

- *Cost share of domestic inputs in the production of private capital, energy capital, and non-energy infrastructure* ($\alpha_k, \alpha_{ne}, \alpha_{nh}, \alpha_z$). The cost share of domestic inputs in the production of capital goods for the private sector is 50 percent , a guesstimate based on observed values in other LDCs. Investment in energy and non-energy infrastructure is much more import intensive. The cost shares α_{ne}, α_{nh}, and α_z come from data on past projects undertaken by the Ministry of Energy and the Ministry of Infrastructure and Transports.

- *Shares of traded goods in total non-energy consumption* (γ_m, γ_x). Traded consumer goods in the model are divided into non-competitive imports and domestically produced tradable goods. Unsurprisingly, the data say that Senegal is a highly open economy: the combined share of tradable goods in total non-energy consumption is 64 percent.[20]

- *Power imported from Mali* (q_f, P_f). Nine percent of total energy supply is imported from a hydropower plant in Mali at a price of 21 CFA/kWh — 17.6 percent of the average domestic tariff and 13 percent of O+M costs at SENELEC plants.[21]

- *Return on infrastructure* ($R_z - \delta$). Estimates of the return on infrastructure are all over the map, but the weight of the evidence in both micro and macro studies points to a high average return. The median rate of return on World Bank projects circa 2001 was 20 percent in SSA and 15-29 percent for various sub-categories of infrastructure investment. In the Bank's recently-completed, comprehensive study of infrastructure in Africa, estimated returns for electricity, water and sanitation, irrigation, and roads range from 17 percent to 24 percent (Foster and

[19] $J = P_oO/(\delta P_{ke}K_e + wL_e + P_o)$. When the capital rental is computed at the shadow price of energy, this can be written as
$$\begin{aligned} J &= \frac{\alpha_o}{(\delta/R_e)\alpha_e + \alpha_o(1+F)}, \\ \Longrightarrow \alpha_o &= \frac{J\delta/R_e}{1-(1+F)J(1-\delta/R_e)}, \end{aligned}$$
as $\alpha_e = 1 - \alpha_L - \alpha_o = 1 - \alpha_o(1+F)$.

[20] The weight in the CPI is 62.7% when energy consumption is measured at official prices.

[21] These imports are through OMVS (Organization pour la Mise en Valeur du Fleuve Senegal) which is a regional organization that comprises Mali, Mauritania and Senegal).

Briceno-Garmendia, 2010, chapter 2). Similarly, the macro-based estimates in Dalgaard and Hansen (2005) cluster between 15 percent and 30 percent for a wide array of different estimators. Hulten et al. (2006), Escribano et al. (2008), Calderon et al. (2009), and Calderon and Serven (2009) supply additional evidence of high returns.

In keeping with this evidence, we assume the return on infrastructure (net of depreciation) equals 25 percent in the base case.

- *Return on investment in the oil-based energy sector* ($R_e - \delta$). The return on investment in the oil-based energy sector is 15 percent. This number might seem too high. It is consistent, however, with the perception that oil-based plants suffer from a high degree of technical inefficiency. The return is a respectable 15 percent only because power is scarce and its shadow price extremely high. *Efficient* energy investments pay a much higher return.

- *Return on investment in coal-fired + hydroelectric power plants* ($R_h - \delta$). The return on investment in efficient coal-fired + hydroelectric power plants is set at 30 percent . This is consistent with the high shadow price of energy in the model and with estimates of cost savings from replacing less efficient with more efficient plants. As noted earlier, the price Senegal pays to purchase power from the jointly operated hydropower plant in Mali is only 13 percent of O+M costs at SENELEC's oil-based plants. In our calibration, O+M costs at new hydropower plants are 26.3 percent of O+M costs at existing oil-based plants. This suggests that an initial return of 30 percent is, if anything, too conservative.[22]

- *Efficiency of public investment in non-energy infrastructure* (s). Casual observation and the empirical estimates in Hulten (1996) and Pritchett (2000) suggest that public investment is inefficient in many LICs. Accordingly, our base case assumes that 30 percent of public investment fails to increase the stock of productive infrastructure ($s = .7$). The *effective* return on infrastructure investment is thus 17.5 percent .

- *Ratio of user fees to recurrent costs per unit of non-energy infrastructure* (μ). On average, user fees cover 43 percent of recurrent costs for non-energy infrastructure. The number was computed by the GOS. It is slightly lower than the average for SSA reported (50 percent) reported in Bricendo-Garmendia (2008).

- *Ratio of revenue from energy sales to operations and maintenance costs in the oil-based energy sector* (*revom*). After adjustment for transmission and distribution losses, the ratio of the average tariff to per unit operations and maintenance costs was .72 in 2011. But inclusion of "uncounted losses" from theft and "inefficient collection of bills" reduces the figure to .55. The resulting revenue shortfall is 2 percent of GDP in the data and 1.97 percent in the model.

- *The shadow price of energy sold to firms* (P_e^*) *and the initial share of oil-based energy investment in GDP*. Values for these two variables are derived residually from the values assigned to the cost share of capital in oil-based energy production (α_e), the depreciation rate (δ), the trend growth rate (g), the gross return on investment in oil-based energy (R_e), the cost shares of energy in

[22] It should be emphasized, again, that 30% is the return measured at the initial high shadow price of energy. The return measured at the official price, which is still quite high compared to prices elsewhere in WAEMU and SSA, is 11.6%.

private production (β_{xo}, β_{no}), the ratio of revenue to operations and maintenance costs (*revom*), and the average power tariff ($P_{e,avg}$). Pen and paper math give

$$i_e = \frac{P_e^*(\delta + g)\alpha_e q_e}{R_e},$$

$$\frac{P_{e,avg}}{P_e^*} = revom[1 - \alpha_e(1 - \delta/R_e)],$$

where q_e is domestically produced energy sales and i_e is gross energy investment. We choose units so that $P_{e,avg} = 1$. Our choices for other variables then return $P_e^* = 2.59$ and a share of energy investment in GDP of .51 percent .[23],[24]

- *Shadow price of energy sold to households* (P_{ec}^*). Needless to say, there is no data that bears on the likely shadow price of energy sold to households. We simply assume the shadow price is the same as for energy sold to firms ($P_{ec}^* = P_e^*$).

- *Slope of the inverse loan supply schedule in the WAEMU* (ρ). The parameter ρ determines how much the interest rate rises when the GOS issues more debt in WAEMU. We assume demand is fairly elastic; the value fixed for ρ says that additional debt sales equal to 6 percent of GDP (a doubling in the stock of debt) would raise the real interest rate from 3.5 percent to 5 percent.

- *Elasticities of sectoral output with respect to the stock of infrastructure* (ψ_x, ψ_n). The ratio ψ_x/ψ_n is set independently. This ratio and other values assigned in calibrating the model — most notably, the return on infrastructure — pin down ψ_n and ψ_x. We assume $\psi_x/\psi_n = 1$ in all runs.

- *Government purchases of imported and nontraded goods* (g_m, g_n)*, non-energy infrastructure investment* (I_z)*, remittances* (*remit*)*, government bonds held by foreign investors* (b_f)*, government bonds held by domestic residents* (b_p)*, concessional debt* (d)*, and non-concessional debt* (d_c)*.* The GOS collects data on all of these variables. The values in Table 1 are for 2012.

[23] The price of energy sold to households and firms equals unity at the initial equilibrium. In fact, households are charged a slightly higher price than firms. All that matters in the simulations, however, is the increase in the price, not its initial level.

[24] Investment at SENELEC was 1.4 percent of GDP in 2012. But purchases from private power companies currently account for 47 percent of SENELEC's sales. If investment at private suppliers is comparable to that at SENELEC, then total energy investment is around 2.8 percent of GDP.

Table 2: Calibration of the Model

Parameter/Variable	**Value**
Tax rates on profits and wage income (h_p,h_w)	h_p =.059, h_w =.054
Consumption VAT (h)	.14
Consumption shares of energy measured at official prices (γ_e, γ_{el})	γ_e=.027, γ_{el}=.004
Cost shares of energy in tradables and nontradables consumption measured at official prices (β_{xo}, β_{no})	β_{xo}= β_{no}= .0125
Intertemporal elasticity of substitution (τ)	.40
Elasticity of substitution between non-energy consumer goods (ϵ)	.50
Depreciation rate (δ)	.05
q-elasticity of investment spending (Ω)	2
Trend growth rate (g)	.01
Pure time preference rate (ρ_1)	.0956
Real interest rate on government debt (r)	.035
Real interest rates on concessional and non-concessional loans (r_d, r_{dc})	r_d= 0, r_{dc}= .07
Elasticity of non-pecuniary services with respect to the stock of real bonds (η)	.40
Capital share of land and capital in private production ($\alpha_J, \alpha_n, \alpha_x$)	α_J= .03, α_n= .44, α_x= .26
Share of poor, non-saving households ($\frac{a}{1+a}$)	.60
Cost shares in existing oil-based energy production (α_e, α_o)	α_e= .30, α_o= .61
Cost share of capital in planned investments in hydroelectric power (α_h)	α_h =.93
Cost share of domestic inputs in the production of private capital, energy capital, and non-energy infrastructure ($\alpha_k, \alpha_{ne}, \alpha_{nh}, \alpha_z$)	α_k= .50, α_{ne}= .30, α_{nh}= .20, α_z= .40
Shares of traded goods in total non-energy consumption (γ_m, γ_x)	γ_m= .29, γ_x= .35
Share of total energy supply imported from Mali ($\frac{q_f}{q_f+q_e}$)	.091
Price of energy imported from Mali (P_f)	.176
Return on infrastructure ($R_z-\delta$)	.25
Return on investment in oil-based energy sector ($R_e-\delta$)	.15
Return on investment in coal-fired and hydroelectric power plants ($R_h-\delta$)	.30
Efficiency of public investment in non-energy infrastructure (s)	.70
Ratio of user fees to recurrent costs for non-energy infrastructure (μ)	.43
Ratio of revenue from energy sales to operations and maintenance costs in the oil-based energy sector ($revom$)	.55
Shadow price of energy sold to firms (P_e^*)	.00
Shadow price of energy sold to households (P_{ec}^*)	.05
Initial share of energy investment in GDP	.0051
Slope of the inverse loan supply schedule in the WAEMU (ρ)	.005
Ratio of non-energy infrastructure investment to GDP	.046
Efficiency of public investment ($\frac{\psi_x}{\psi_n}$)	1

Note: See the calibration discussion in the main text.

Table 2: Calibration of the Model (continued)

Parameter/Variable	Value
Ratios of government purchases of imported and nontraded goods to GDP	g_m= .009, g_n= .043
Ratios of government bonds held by foreign investors (b_f),	
domestic investors to initial GDP (b_p) to initial GDP,	b_f= .06, b_p= .063,
Ratios of concessional debt (d), and non-concession debt to initial GDP(dc)	d= .292, dc= .035
Absorptive capacity parameter (φ)	0.00

Note: See the calibration discussion in the main text.

V. Alternative Methods of Fiscal Adjustment

A. Using Traditional Fiscal Instruments to Reduce Fiscal Deficit

We begin by analyzing the implications of using traditional fiscal instruments to reduce the fiscal deficit. In business-as-usual fiscal adjustment, the government alters tax rates or its level of spending. We look specifically at the effects of (i) raising wage or profit taxes and (ii) reducing government consumption of traded or nontraded goods.

Increasing the wage tax affects only the fiscal deficit. There are no real effects as the change in the tax is equivalent to a one-off lump sum tax.[25] Raising the tax rate by 5 percentage points reduces the fiscal deficit from 5.9 percent of GDP to slightly below 3 percent of GDP (Figure 1). Increasing the profit tax is fiscally inferior to increasing wage tax. The deficit decreases less owing to the fact that profits are a smaller share of GDP than wages. Moreover, a higher profit tax reduces private investment and capital stock, leading to lower GDP and a loss of revenue from other sources (Figure 1).

Although raising the profit tax is distributionally less objectionable than raising the wage tax, it generates a smaller revenue gain and engenders contractionary real effects that are avoided with the wage tax increase. Note in this connection that part of the profit tax is effectively paid by workers: on the transition path, decreases in the capital stock continuously contract the demand for labor; in the long run, this reduces the real wage 2.5 percent.

Cutting government consumption of nontraded goods raises real wages and reduces fiscal deficit. It produces contractionary effects, however, on private investment, the capital stock, and real GDP, thereby reducing revenue from other sources (Figure 2).

[25] The wage tax is a lump-sum tax because labor supply is assumed to be completely inelastic for both saving and non-saving households. Assuming elastic labor supply could change the ranking of wage vs. profits taxes. Although macro and micro estimates of the Frisch elasticity differ greatly, the weight of the evidence supports the view that labor supply is inelastic. Keane and Rogerson (2011) observe, for example, that "the majority of the economics profession has come to the conclusion that labor supply elasticities are small; and, in particular, that labor supply is not very responsive to tax changes".

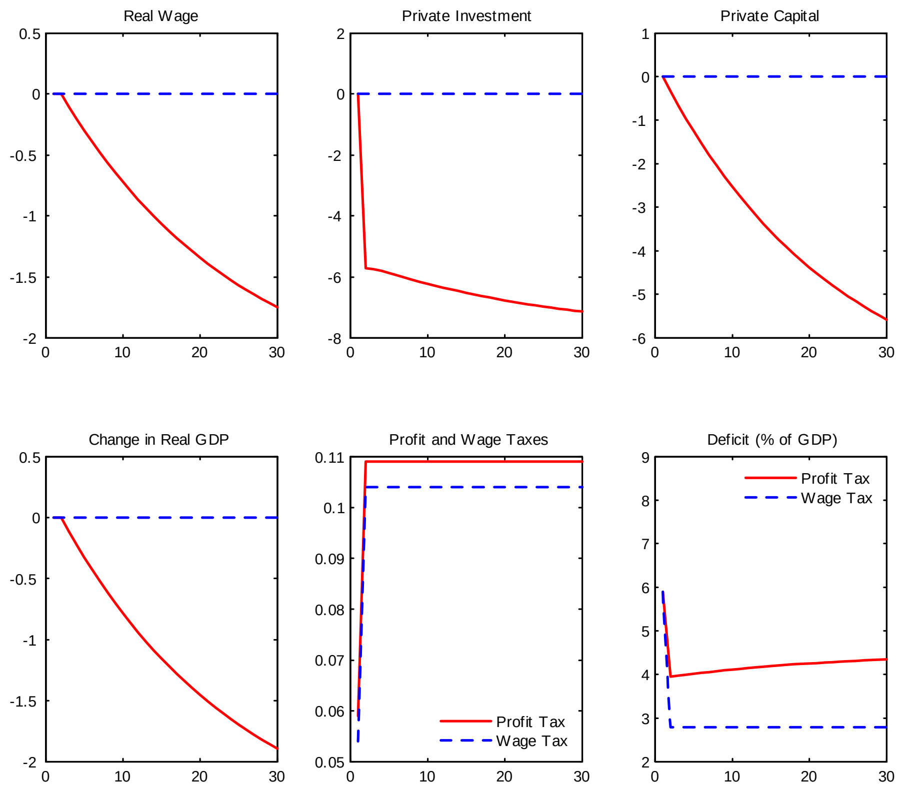

Figure 1: **Increasing Profit or Wage Tax Rate.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

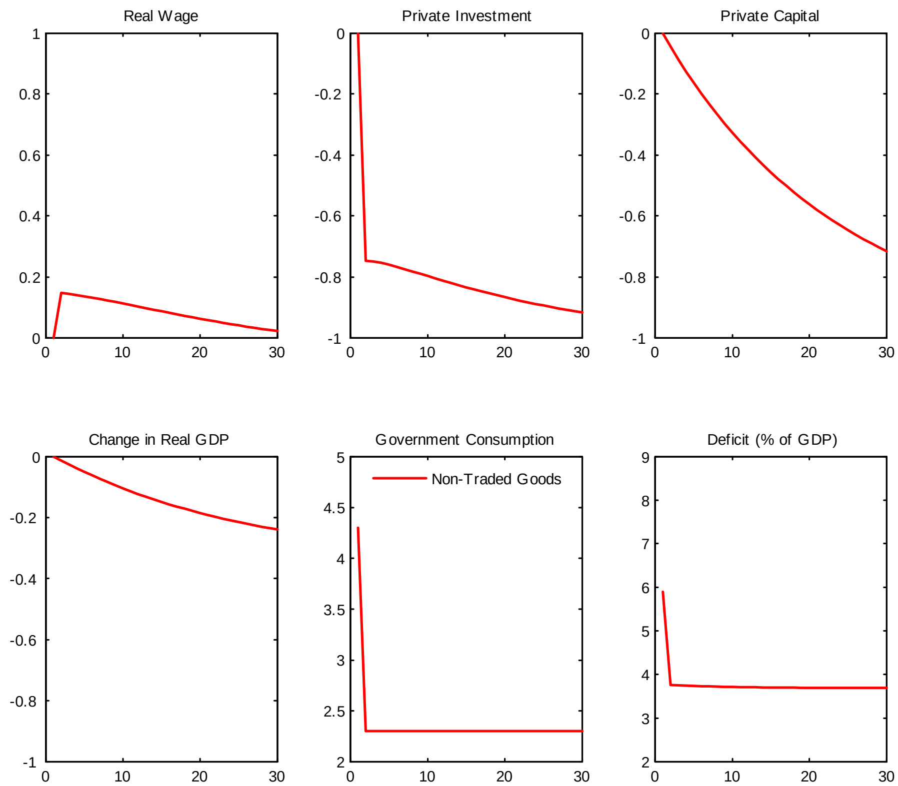

Figure 2: **Cutting Government Consumption of Nontraded Goods.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

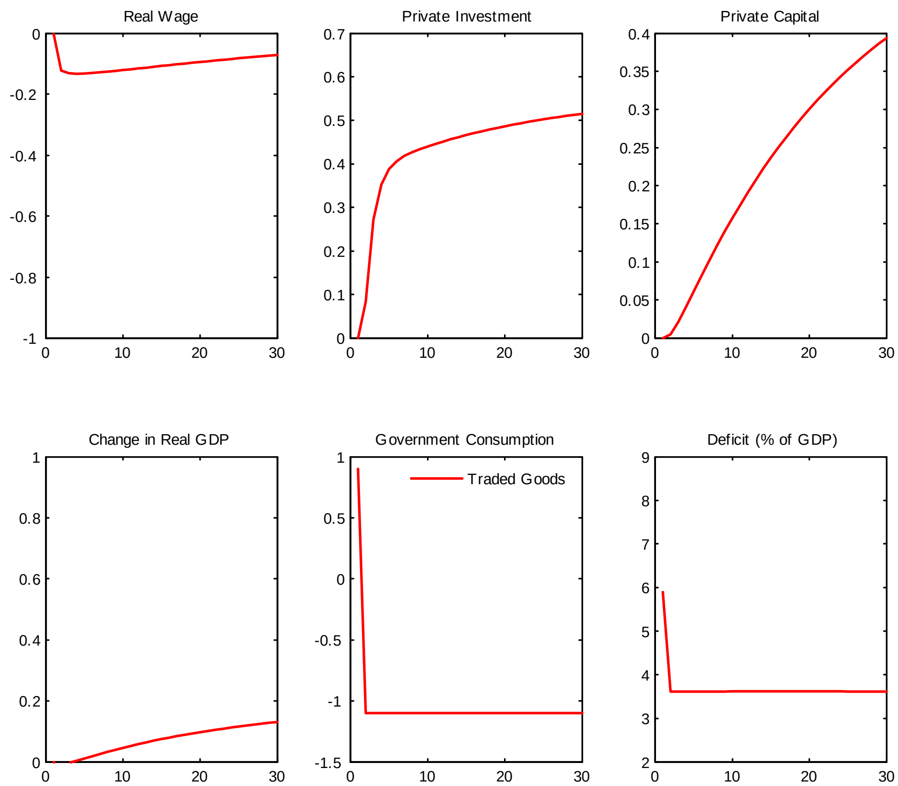

Figure 3: **Cutting Government Consumption of Traded Goods.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

Reducing government consumption of traded goods has opposite effects. Real wages decline, while private investment, the capital stock, and real GDP all rise (Figure 3). The increase in real income enhance the positive effect on the fiscal deficit, which falls from 5.9 percent to 3.5 percent of GDP, slightly more than in the cut of consumption of nontraded goods.

The different results for cuts in government consumption of traded and nontraded goods stem from differences in capital intensity of production. When a cut in government consumption of nontraded goods reduces the relative price of nontraded goods, resources reallocate from the nontradables sector to the tradables sector. The reallocation of resources raises overall demand for labor and lowers overall demand for capital because the tradables sector is labor intensive relative to the nontradables sector.[26] The decrease in overall demand for capital explains the reduction in the private investment and the

[26] Whether the nontradable sector is more or less capital intensive than the tradable sector is strictly an empirical issue. Data on factor shares found in social accounting matrices for Sub-Saharan Africa assembled by the Global Trade and Analysis Project (GTAP) and the International Food Policy Research Institute suggest a capital share of 55-60 percent in the nontradables sector and 20-40 percent for the tradables sector. When setting the share in the tradables sector, a lot depends on the weight of manufacturing in the sector and on whether agricultural production is dominated by smallholders (extremely labor intensive) or large estates.

steady decrease in the capital stock. The real wage increases in the short run while the capital stock is essentially fixed and aggregate labor demand is higher. Over time, however, the decrease in the capital stock reduces labor demand; eventually, this reverses the favorable effect on labor demand, causing the real wage to decline.

Everything runs in reverse when the government cuts consumption of traded goods. A cut in government consumption of traded goods operates like an exogenous increase in foreign aid. The relaxation of the national budget constraint is associated with higher private spending and a Dutch-disease driven increase in the relative price of the nontraded good. As resources move from the labor-intensive tradables sector to the capital-intensive nontradables sector, overall demand for capital rises and overall demand for labor declines.

Summing up, raising tax rates and cutting government consumption of traded/nontraded goods lowers the fiscal deficit but suffers from numerous drawbacks. It is easy to reduce government consumption of traded and nontraded goods in a theoretical model; in the real world, the adjustment imposes difficult and painful cuts to government services that people value. Raising taxes also involves difficult tradeoffs. Both profit and wage taxes reduce workers' real income: the wage tax does so directly, the profit tax by reducing the capital stock and labor demand. And while labor loses less under the profits tax, the reduction in the fiscal deficit is smaller and GDP growth declines.

But Senegal may not need to confront these difficult fiscal adjustments. We show in the next section that reforming the energy sector, by investing in a more efficient mode of energy production can give the government everything it wants: higher GDP growth, higher real wages, and large medium- and long-term reductions in the fiscal deficit.

B. Efficient Energy Investment Program and Fiscal Adjustment

In this subsection we explore the implications of gradually increasing investment in the efficient gas-fired, coal-fired and hydropower energy sector while reducing investment in the inefficient oil-based energy sector. Table 3 presents the paths of investment in the two sectors, measured as a percentage of initial GDP at initial prices of capital (Pkh_o for the more efficient sector and Pke_o for the oil-based sector). The motivation for the chosen paths derives from very different factor intensities associated with the two technologies: the output capital ratio is much lower for hydropower than for oil-based energy because the oil-based production uses a lot more variable inputs (see the cost shares in Table 2); consequently, an investment program that increases the supply of energy smoothly and continuously entails higher investment in the efficient sector relative to the absolute decrease in investment in the oil based sector.[27] The increase in total energy investment increases for a few years; beyond the short run, however, the saving from lower variable costs greatly exceeds the increase in investment spending.

[27] The impact on supply of energy is simply the product of the change in capital stock multiplied by the output capital ratio ($\Delta K * \frac{Y}{K}$).

Table 3. Efficient Energy Investment Program

Inefficient sector (Ie) and more efficient sector (Ih)

Year	1	2	3	4	5	6	7	8	9	10...	15..	20...
$\frac{Pkh_o*(Ih-Ih_o)}{y_o}$	0	1.0	1.4	1.8	2.1	2.5	2.5	2.5	2.4	2.3	1.5	1.5
$\frac{Pke_o*(Ie-Ie_o)}{y_o}$	0	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5

Note: Numbers are in percent of initial GDP ($y_o = 100$)

Implementation of the energy investment program in Table 3 raises energy supply by 70 percent. Relaxing the energy bottleneck has positive effects on real GDP and real wages, which increase 5 percent and 2 percent, respectively (Figure 4). As the total supply of energy increases, the return on efficient energy investment declines. Shadow prices of household and firm energy consumption also decline but stay above the official price (excess demand and rationing persist). Private investment and private capital stock fall in the short run and then recover slowly — the capital stock does not regain its pre-reform level until year 18. The fiscal deficit rises for 5 years, but then decreases rapidly, dropping to 4.5 percent of GDP at the end of the first decade.

The temporary decline in private investment stems from two effects. First, the increase in public investment in the energy sector drives up the supply price of capital by bidding up the prices of capital inputs purchased from the nontradables sector (construction, for example). Second, the anticipation of higher income in the future leads the private sector to consume more today. Since income does not increase in the short run, the counterpart of consumption smoothing is lower saving and a temporary decrease in private investment.

As noted earlier, the temporary increase in fiscal deficit reflects the fact that in the short run total spending on energy mirrors higher investment in the energy sector. The increase in investment is driven, to repeat, by the difference in factor intensities in the two energy sectors: production in the inefficient energy sector involves much higher recurrent costs (mainly oil), so the fiscal gain from reducing investment in that sector does not fully materialize at the outset. It materializes only after decreases in the capital stock bring about large reductions in expenditure on variable inputs (used in fixed coefficient proportions with the capital stock). By contrast, in the efficient energy sector recurrent costs are labor costs, which are a very small part of total cost. Since the fiscal cost of new investment in the efficient sector is roughly constant over time while the fiscal saving from downsizing the inefficient sector grows steadily, the reform program increases spending in the short run but generates large costs savings in the medium run. When the cost savings combine with higher tax revenue from greater GDP growth and more revenue from energy sales, the total fiscal gain rises to 3 percent of GDP in year fifteen. When investment in the inefficient energy sector is reduced and investment in the efficient energy sector is raised, the fiscal gains occur more gradually overtime leading to a short term increase in deficit. The fiscal deficit rises initially from about 6 to 6.8 percent of GDP and then it declines very strongly to about 2 percent of GDP in the long run as you reap the benefits from lower cost and more efficient energy production, and expansion in output which brings in other tax revenues.

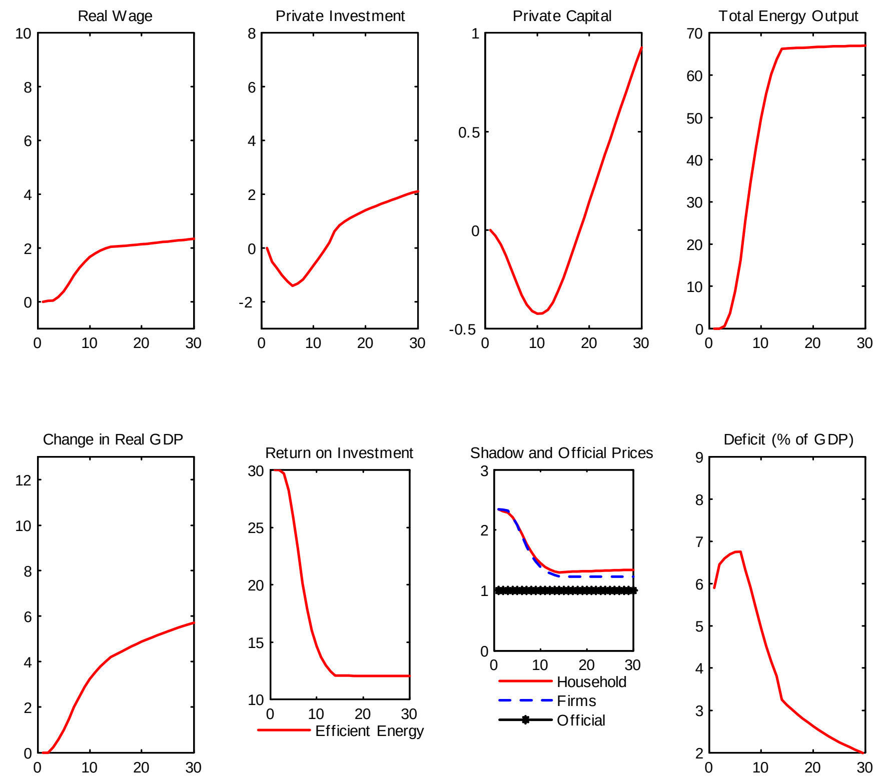

Figure 4: **Efficient Energy Investment Program.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

The large fiscal gains that accrue over the medium run allow the government to consider broader-based, more pro-growth investment programs. We examine several scenarios of this type. In Figure 5 the government initiates large increases in non-energy infrastructure investment once the fiscal deficit drops below its pre-reform level (Table 4). This program slows the reduction in the fiscal deficit but produces much larger gains in real income, real wages, and private investment. Real wages rise 7 percent, while private investment and real GDP increase 6 percent and 10 percent, respectively. The corresponding numbers in Figure 4 are 2.4 percent, 2 percent, and 5.8 percent. Note also that the return on infrastructure investment does not fall nearly as much as the return on efficient energy investment;[28] even at year twenty, the return is well above 20 percent. This suggests that further gains could be reaped by shifting more of the fiscal savings from energy reform to non-energy infrastructure investment.

[28] Because the percent increase in infrastructure investment is not as much as that of efficient energy investment.

Table 4. Efficient Energy Program and Delayed Investment in other Infrastructure
Inefficient sector (Ie), more efficient sector (Ih), and other infrastructure (Iz)

Year	1	2	3	4	5	6	7	8	9	10...	15..	20...
$\frac{Pkh_o*(Ih-Ih_o)}{y_o}$	0	1.0	1.4	1.8	2.1	2.5	2.5	2.5	2.4	2.3	1.5	1.5
$\frac{Pke_o*(Ie-Ie_o)}{y_o}$	0	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5
$\frac{Pz_o*(Iz-Iz_o)}{y_o}$	0	0	0	0	0	0	0	0	0.4	0.8	2.5	1.7

Note: Numbers are in percent of initial GDP ($y_o = 100$)

Perhaps the most important result in Figure 5 is that the fiscal cost of purchasing more growth is *very small*. Although the permanent increase in non-energy infrastructure investment is 1.7 percent of initial GDP, the fiscal deficit in year twenty rises only .7 percent of GDP (3.2 percent in Figure 5 vs. 2.5 percent in Figure 4). In general equilibrium, additional revenue from user fees and taxes pay for 60 percent of the extra investment.

We remind the reader that the runs in Figures 4 and 5 sidestep the issue of how to pay for the temporary increase in total energy investment.[29] If we reject the business-as-usual approach of raising taxes or cutting public sector consumption, then the government is left with two choices: (i) temporarily raise official energy prices or (ii) borrow more in the regional bond market or the Eurobond market. We examine these scenarios in the next three sections.

C. Temporarily Raising Official Energy Prices

Since user fees from energy consumption do not cover even recurrent costs, one can argue that both households and firms should pay higher prices to help finance the broad-based energy + infrastructure investment program. This is done in Figure 6, where energy prices increase 35 percent for 4 years and then drop back to their previous level. The temporary price increase fully covers the bill for higher investment until the cost saving and revenue gains from the reform kick in at year 6, after which the fiscal deficit decreases rapidly. The impact on real variables is similar to that in Figure 5, except private investment falls more in the short run as savers bear more of the tax burden when high energy prices replace cuts in transfer payments.[30]

[29] To get a solution, the runs assume that ransfer payments adjust to balance the budget. The change in transfer payments that balance the budget is then added to the initial fiscal deficit to track the time-varying fiscal effects of the program.

[30] Transfers are proportional to each group's share in total labor supply. In addition, the share of energy in total consumption is 2.7 percent for saving households versus 0.4 percent for poor, non-saving households. Saving households smooth the effects on consumption by cutting investment in the short-run. Hence investment cuts to smooth the path of consumption are larger in the case of energy price increases.

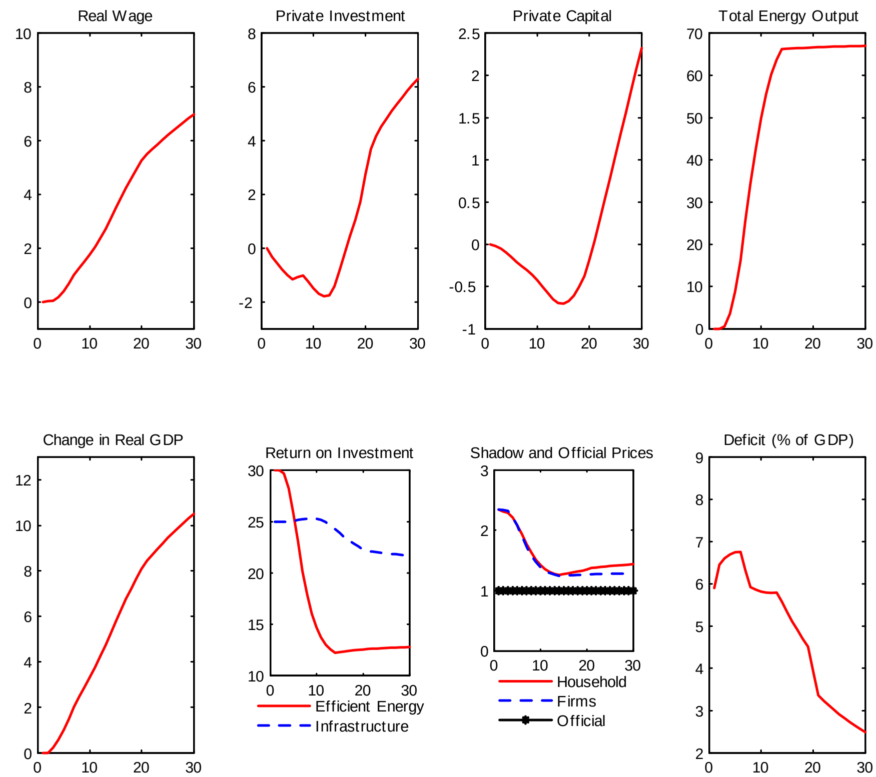

Figure 5: **Efficient Energy Program and Delayed Non-Energy Investment.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

It is hard to judge whether the price increases would be socially controversial. The public might be willing to pay more for a few years if it understands that higher prices are part of a package deal — that they pay for increases in the supply of energy that the public wants and values. But this rationale might be a hard sell. Because existing energy production is so inefficient, energy prices are already very high in Senegal. Raising them further is unlikely to prove popular.

D. Temporary Borrowing in the Regional Bond Market

The natural alternative to raising energy prices is to borrow against future fiscal gains to finance the temporary increase in the fiscal deficit. One possibility is to issue CFA franc bonds in the regional market comprised by countries belonging to the West African Economic and Monetary Union (WAEMU).[31]

[31] When Senegal issues CFA Franc denominated bond, holders can be Senegal residents as well as individuals and commercial banks in the remaining parts of the WAEMU region. In 2012, CFA franc denominated debt was composed of 40 percent domestic (i.e. Senegal residents) and 60 percent regional (i.e. WAEMU region's residents excluding

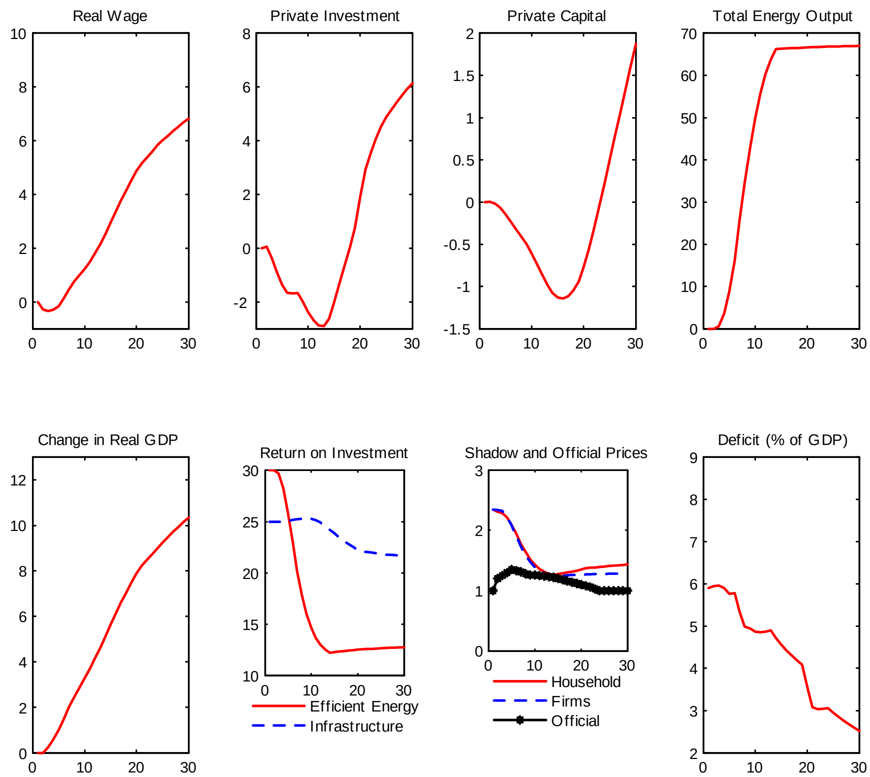

Figure 6: **Efficient Energy Program and Delayed Non-Energy Investment: Raising Energy Prices.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

Regional borrowing requires temporary supporting fiscal adjustment to prevent explosive growth in the public debt. In Figure 7 the cut in transfer payments is limited to .5 percent of initial GDP. Aided by this small amount of fiscal support, the scheme works very well. The paths for the real wage, real GDP, and private investment are broadly similar to those in Figure 6. Domestic (total) debt increases only slightly, rising from 12 (45) percent to 15 (48) percent of GDP. As expected, the sale of more debt increases the real interest rate. But the increase is small and short-lived. After rising to 3.64 in year 6, the interest rate declines continuously; in the long run, the rate decreases to 3.1 percent .

The program with delayed infrastructure investment and temporary borrowing is safe and effective. Arguably, however, it is *too* safe. Given the large fiscal gains that energy reform delivers in the medium run, Senegal may wish to consider more aggressive pro-growth programs that frontload increases in infrastructure investment.

Senegalese).

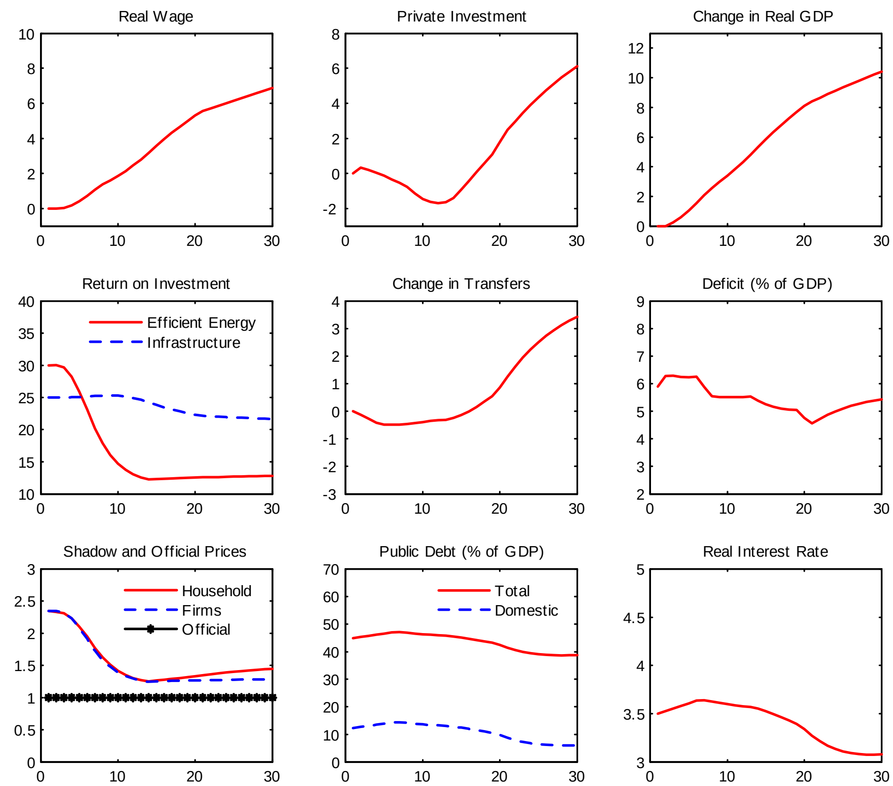

Figure 7: **Efficient Energy Program and Delayed Non-Energy Investment: Regional Borrowing.** Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

E. Pushing Harder for Growth: Frontloaded Infrastructure Investment and Aggressive Borrowing in the Regional versus Eurobond Market

Figure 8 shows the outcome when the government increases infrastructure concurrently with the energy reform program. Infrastructure investment rises by 0.4-0.5 percent of GDP until the cumulative increase reaches 2.1 percent of initial GDP in year 6 (Table 5). By year 15, the stock of infrastructure has increased 17 percent vs. 11 percent in the runs with delayed infrastructure investment.

Table 5. Efficient Energy Program and Frontloaded Investment in other Infrastructure

Inefficient sector (Ie), more efficient sector (Ih), and other infrastructure (Iz)

Year	1	2	3	4	5	6	7	8	9	10...	15..	20...
$\frac{Pkh_o*(Ih-Ih_o)}{y_o}$	0	1.0	1.4	1.8	2.1	2.5	2.5	2.5	2.4	2.3	1.5	1.5
$\frac{Pke_o*(Ie-Ie_o)}{y_o}$	0	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5	−0.5
$\frac{Pz_o*(Iz-Iz_o)}{y_o}$	0	0.4	0.8	1.3	1.7	2.1	2.1	2.1	2.1	2.1	2.1	2.1

Note: Numbers are in percent of initial GDP ($y_o = 100$)

Frontloading generates significant additional benefits. In contrast to the runs with delayed infrastructure investment, there is very little crowding out of of private investment in the short run. This together with faster growth in the stock of infrastructure add 2-3 percentage points to the increases in the real wage, private investment, and real income at the 10- and 20-year horizons.

Senegal pays a price for these gains. Ramping up infrastructure investment right away requires much more borrowing. Domestic debt rises to nearly 30 percent of GDP, while total debt peaks at 60 percent in year 11. There is also an adverse effect on the country's financial terms of trade. Because Senegal is big in the regional bond market, the extra borrowing pushes the real interest rate up from 3.5 percent to 4.5 percent. The marginal cost of debt is thus several percentage points higher than the average cost.

In Figure 9, the government taps the Eurobond market instead of the WAEMU market. The interest rate on domestic debt rises much less, but since Eurobond debt carries a yield of 7.5 percent the overall cost of borrowing rises.[32] This shows up in a longer period of depressed transfer payments (20 years in Figure 9 vs. 17 years in Figure 8). Eurobond borrowing is competitive with borrowing in the regional only if the government is anxious to tie down the interest rate.[33]

[32] The interest rate rises in the regional market because the private sector borrows more to finance temporary dissaving.

[33] These results point to regional borrowing being supperior to Eurobond issuance. In 2012, real yields on Eurobonds hovered around 7.5 percent while that on the Regional bond was about 3.5 percent. Regional borrowing may involve rollover risks but it is not clear as to which way these risks could go. Recently, the yield on the Eurobond has decline in light of macroeconomic developments in the OECD countries. However this decline (although it may call for locking-in lower rates now) may not last long because we may see a reversal once the Federal Reserve Board begins tapering off. One of the benefits of relying on the regional bond market is that it may help deepen the financial sector in the region. In our analysis we assume that regional borrowing is less expensive relative to Eurobond borrowing because the rate differential in 2012. We are aware that the costs and benefits of each financing method are open to discussion.

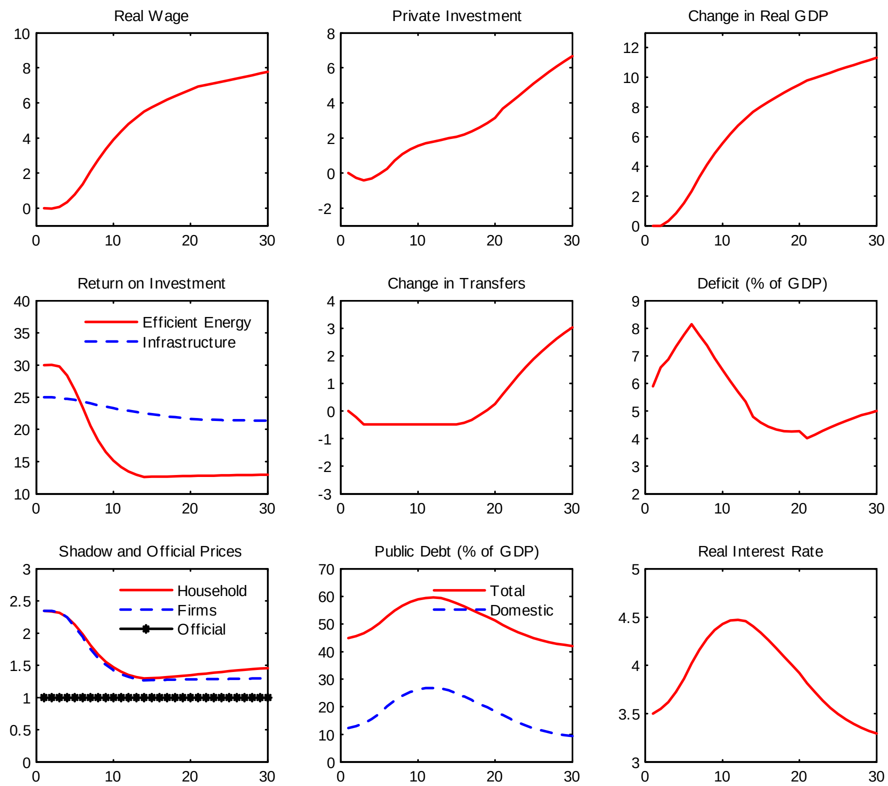

Figure 8: **Efficient Energy and Frontloaded Non-Energy Investment: Regional Borrowing.**Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

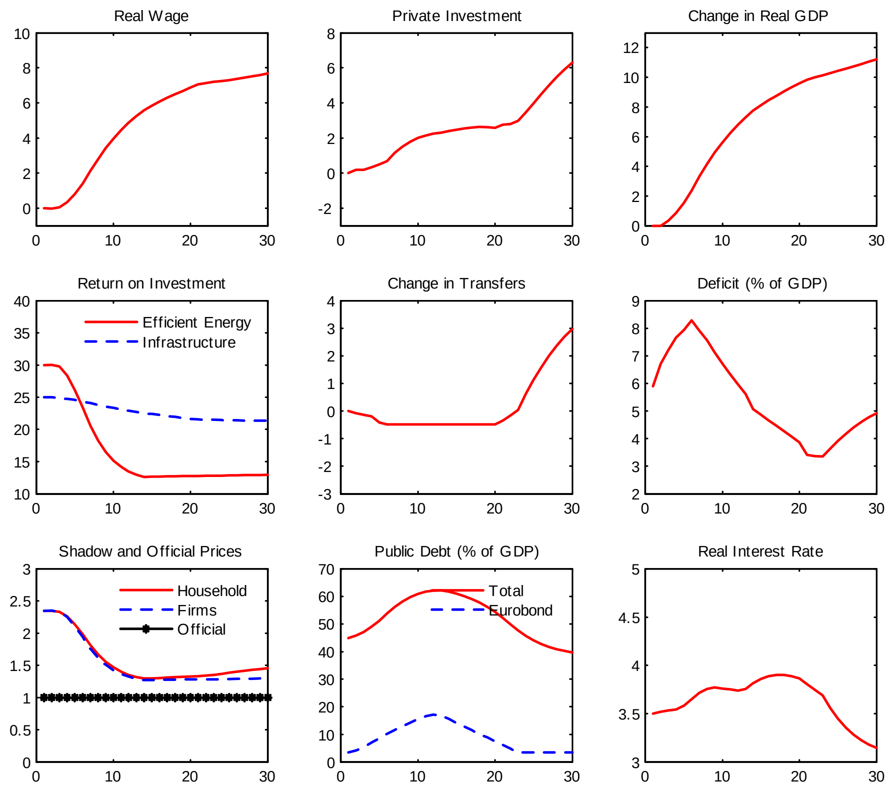

Figure 9: **Efficient Energy and Frontloaded Non-Energy Investment: Eurobond Borrowing.**Variables are expressed as percentage deviations from the initial steady state, unless otherwise indicated.

VI. Concluding Remarks

Senegal needs to reform its ailing oil-based energy sector in order to lower costs and increase the supply of energy. Difficulties in the existing inefficient mode of energy production have slowed growth and contributed significantly to rising fiscal deficits and rapid growth of the public debt since 2006.

To analyze the impacts of different adjustment programs on growth, private investment, real wages, debt and the fiscal deficit, we use a variant of the two-sector, open-economy dynamic general equilibrium model developed by Buffie et al. (2012). This model incorporates sector-specific capital, productivity-enhancing infrastructure, external concessional and commercial debt, a consumption value added tax and government transfers payments, variable efficiency of public investment, an absorptive capacity constraint, and non-savers consumers. We adapted the model for Senegal by adding wage and profit taxes, government consumption of traded and nontraded goods, controlled energy prices, regional and domestic currency debt, an inefficient, oil-based energy sector, and a new, low-cost gas, coal and hydropower energy sector. After calibrating the model for Senegal, we investigated the effects of varying traditional fiscal policy instruments one at time before focusing on policy packages that increase public investment in energy and infrastructure in scenarios with varying degrees of debt finance and with different types of supporting fiscal adjustment.

Raising taxes and cutting expenditure lowers the fiscal deficit at all horizons. Traditional fiscal adjustment involves difficult tradeoffs, however. Higher wage and profit taxes reduce workers' real income, while cuts in government consumption reduce the supply of public services; the profit tax also lowers the capital stock and real GDP.

Reforming the energy sector is the best way to adjust. An energy program that invests in a new, low-cost coal-fired and hydropower energy sector while downsizing the inefficient oil-based sector can give the government everything it wants: higher GDP growth, higher real wages, and large medium- and long-term reductions in fiscal deficits. Real GDP and real wages increase even more when some of the fiscal gains from the energy reform program are used to finance delayed investments in non-energy infrastructure. The only potential drawback is that the fiscal deficit increases in the short run. The temporary increase in the fiscal deficit is easily managed, however, by borrowing in the regional bond market.

Given the large fiscal gains that result from implementation of the efficient energy investment program, the government should cautiously give serious consideration to a "big-push" program that immediately supplements higher energy investments with a substantial increase in infrastructure investment. Our analysis argues that the benefits of a big push outweigh its costs. The program, when underpinned by an efficient public investment process, generates significant additional gains in real GDP, real wages, and private investment. The public debt grows rapidly for a few years, but in the medium run the government reaps a large fiscal dividend on the order of three percent of GDP.

Appendix A: The Exact Consumer Price Index With Rationing

Consider a household that chooses E and c to minimize the cost X of achieving the level of utility u_o when the price of energy equals its shadow price:

$$\underset{\{c,\,E\}}{Min}\, X = Pc + P^*_{ec}E, \tag{A1}$$

subject to

$$\frac{c^{1-1/\tau}}{1-1/\tau} + k\frac{E^{1-1/\tau}}{1-1/\tau} = u_o \tag{A2}$$

The optimal choices for c and E are

$$c = \left[\frac{X/P}{u_o(1-1/\tau)}\right]^{\tau}, \tag{A3}$$

$$E = \left[\frac{kX/P^*_{ec}}{u_o(1-1/\tau)}\right]^{\tau}. \tag{A4}$$

Substituting these solutions into (A1) gives

$$X = \frac{\left[P^{1-\tau} + k^{\tau}(P^*_{ec})^{1-\tau}\right]^{1/(1-\tau)}}{[u_o(1-1/\tau)]^{\tau/(1-\tau)}}. \tag{A5}$$

The expenditure function is of the general form

$$X = f(u_o)g(P, P^*_{ec}), \tag{A6}$$

where $g(P, P^*_{ec})$ is the exact consumer price index, viz.:

$$g(P, P^*_{ec}) = CPI^* = \left[P^{1-\tau} + k^{\tau}(P^*_{ec})^{1-\tau}\right]^{1/(1-\tau)}. \tag{A7}$$

Actual energy consumption is rationed at $\bar{E}$ and the actual energy price is P_{ec}. Define X^* and X^a to be expenditure measured at the shadow price of energy versus the actual, artificially low price. Trivially,

$$X^* = \underbrace{Pc + P_{ec}\bar{E}}_{X^a} + (P^*_{ec} - P_{ec})\bar{E},$$

$$\Longrightarrow \frac{X^*}{X^a} = 1 + \left(\frac{P^*_{ec} - P_{ec}}{P_{ec}}\right)\frac{P_{ec}\bar{E}}{X^a}.$$

Note also that[34]

$$\frac{X^*}{X^a} = \frac{f(u_o)g(P, P^*_{ec})}{f(u_o)g(P, P_{ec})} = \frac{g(P, P^*_{ec})}{g(P, P_{ec})} = \frac{CPI^*}{CPI},$$

[34] Since energy consumption is the same for X^* and X^a, non-energy consumption must be the same. Hence X^* and X^a are associated with the same level of utility u_o.

where CPI is the exact consumer price index associated with the rationed level of consumption. It follows that

$$\begin{aligned}\frac{CPI^*}{CPI} &= 1 + \left(\frac{P^*_{ec} - P_{ec}}{P_{ec}}\right)\frac{P_{ec}\bar{E}}{X^a}, \\ \Longrightarrow CPI &= \frac{CPI^*}{1 + \gamma_e(P^*_{ec} - P_{ec})/P_{ec}},\end{aligned} \tag{A8}$$

where $\gamma_e = P_{ec}\bar{E}/X^a$ is the consumption share of energy measured at the official price P_{ec}.

Appendix B: Policy Reaction Functions and the Path of Government Debt

The government budget constraint is

$$
\begin{aligned}
b_t - b_{t-1} = {} & P_{z,t}\left[\left(1+\frac{i_{z,t}}{\tilde{z}_{t-1}}-\delta-g\right)^{\phi}(i_{z,t}-i_{z,o})+i_{z,o}\right]+P_{ke,t}i_{e,t}+P_{kh,t}i_{h,t} \\
& +P_{n,t}g_{n,t}+P_{m,t}g_{m,t}+\frac{r_d-g}{1+g}d_{t-1}+\frac{r_{dc}-g}{1+g}dc_{t-1}+\frac{r_{t-1}-g}{1+g}b_{t-1}+T_t+w_t(L_{e,t}+L_{h,t}) \\
& +P_{o,t}O_t+P_{f,t}q_{f,t}-P_{ec,t}(\bar{E}_{s,t}+E_{1,t})-P_{e,t}(E_{x,t}+E_{n,t})-h_tP_t(c_{s,t}+c_{1,t})-x_t-\mu_t z_{t-1} \\
& -h_{w,t}w_tL(1+a)-h_{p,t}[P_xq_{x,t}+P_{n,t}q_{n,t}-P_{e,t}(E_{n,t}+E_{x,t})-w_t(L_{x,t}+L_{n,t})]. \qquad \text{(B1)}
\end{aligned}
$$

In the short run, the deficit can be financed by issuing more debt. Eventually, however, some subset of policy variables must adjust to cover the ex ante financing gap. The latter is the size of the revenue shortfall when T, h, h_w, h_p, g_n, and g_m in (B1) equal their initial values:

$$
\begin{aligned}
GAP_t = {} & P_{z,t}\left[\left(1+\frac{i_{z,t}}{\tilde{z}_{t-1}}-\delta-g\right)^{\phi}(i_{z,t}-i_{z,o})+i_{z,o}\right]+P_{ke,t}i_{e,t}+P_{kh,t}i_{h,t} \\
& +P_{n,t}g_{n,o}+P_{m,t}g_{m,o}+\frac{r_d-g}{1+g}d_{t-1}+\frac{r_{dc}-g}{1+g}dc_{t-1}+\frac{r_{t-1}-g}{1+g}b_{t-1}+T_o+w_t(L_{e,t}+L_{h,t}) \\
& +P_{o,t}O_t+P_{f,t}q_{f,t}-P_{ec,t}(\bar{E}_{s,t}+E_{1,t})-P_{e,t}(E_{x,t}+E_{n,t})-h_oP_t(c_{s,t}+c_{1,t})-x_t-\mu_t z_{t-1} \\
& -h_{w,o}w_tL(1+a)-h_{p,o}[P_xq_{x,t}+P_{n,t}q_{n,t}-P_{e,t}(E_{n,t}+E_{x,t})-w_t(L_{x,t}+L_{n,t})]. \qquad \text{(B2)}
\end{aligned}
$$

Policy makers specify the share of fiscal adjustment borne by each policy variable. This determines the long-run target values of the adjusting variables. For example, if transfers and government consumption of traded goods adjust, then

$$
\begin{aligned}
T_{\text{target,t}} &= T_o-\lambda GAP_t, \quad 0\le\lambda\le 1, && \text{(B3)} \\
g_{m,\,\text{target,t}} &= g_{m,o}-(1-\lambda)\frac{GAP_t}{P_{m,t}}, \quad 0\le\lambda\le 1. && \text{(B4)}
\end{aligned}
$$

Equations (B3) and (B4) are paired with a target for the long-run level of government debt. The *reaction functions* that govern the paths of h and g_m incorporate these targets as well as socio-political constraints on how much and how fast fiscal policy can change:

$$
\begin{aligned}
g_{m,t} &= Max\left\{g_{m,t-1}+\lambda_1(g_{m,\,\text{target,t}}-g_{m,t-1})+\lambda_2(b_{t-1}-b_{\text{target}}),\ \bar{g}_m\right\}, && \text{(B5)} \\
T_t &= Max\left\{T_{t-1}+\lambda_3(T_{\text{target,t}}-T_{t-1})-\lambda_4(b_{t-1}-b_{\text{target}}),\ \bar{T}\right\}. && \text{(B6)}
\end{aligned}
$$

$\bar{g}_m$ and and $\bar{T}$ are lower bounds on g_m and T. Inside the bounds, the parameters λ_1- λ_4 determine whether policy adjustment is fast or slow. Under "slow" adjustment, the debt instrument that varies endogenously to satisfy the government budget constraint may rise above its target level in the time it takes g_m and T to reach $g_{m,\,\text{target}}$ and T_{target}. When this happens, the transition path includes a phase in which $T < T_{\text{target}}$ and $g_m < g_{m,\,\text{target}}$ to generate the fiscal surpluses needed to pay down the debt.

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