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Demand Composition and Income Distribution

David Pothier and Damien Puy

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Prepared by David Pothier and Damien Puy[1]

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Abstract

This paper highlights how changes in the composition of demand affect income dispersion in the short run. We first document how the share of aggregate spending dedicated to labour-intensive goods and services shrinks (expands) during downturns (booms), and argue that this contributes to the observed pro-cyclicality of employment and output in labour-intensive industries. Using a two-sector general equilibrium model, we then assess how this demand composition channel influences the cyclical properties of the income distribution. Consistent with empirical evidence, we find income inequality to be counter-cyclical due to changes in the level of employment and (to a lesser extent) relative factor prices. The model also shows that wealth redistribution policies can potentially involve a trade-off between equality and output, depending on how they affect the composition of aggregate demand.

JEL Classification Numbers: D31, D33, E24, J31

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Author’s E-Mail Address:DPothier@diw.de, DPuy@imf.org

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1 Introduction

How does income inequality vary in the short-run? *A priori*, the impact of the business cycle on inequality is unclear. While unemployment affecting low-income households increases inequality, profits and the return to capital investment also fall during recessions. Since the owners of the capital stock are located at the top of the income distribution, these forces tend to lead to a reduction in inequality. Most empirical studies to date have found income inequality in advanced economies to be counter-cyclical. In the case of the US, evidence from disaggregated data indicate that this effect is mainly driven by employment and wage dynamics affecting the relative position of low income households. Taken together, these studies suggest that labor income dynamics - rather than variations in capital income - are the key factors driving the cyclical properties of the income distribution.

Notwithstanding these well-established empirical results, standard one-sector real business cycles models generally fail to explain the level and cyclicality of income inequality. In the past, this shortcoming has been addressed by assuming that households located at different points in the income distribution face different wage and employment dynamics, thereby generating labor earnings differentials along the cycle (e.g. Castaneda et. al. (1998)). Our paper proposes an alternative theory of income dispersion which does not rely on heterogeneous labor processes across agents. Motivated by a number of new stylized facts, it emphasizes the role played by changes in the composition of demand over the business cycle in explaining income dynamics. In particular, using US industry-level data from the Bureau of Economic Analysis (BEA) over the period 1977-2010, Jin and Li (2012) find that labor-intensive sectors expand disproportionately more than capital-intensive sectors during booms. As a result, the share of production, investment and employment in capital-intensive sectors drops significantly during economic expansions, while the reverse tends to happen during recessions. Using US household consumption data also drawn from the BEA, we present new evidence showing that this pro-cyclicality of labor-intensive sectors is in part attributable to a recomposition of private demand over the business cycle. In particular, during recessions (booms), households tend to cut (increase) spending disproportionately more on labor-intensive goods and services (such as houses, motor vehicles or tourism), thereby generating high pro-cyclicality and volatility of employment and output in labor-intensive industries.

Building on this empirical evidence, we develop a simple model to study how such changes in the composition of demand affect the distribution of income in the short-run. We design a two-sector general equilibrium model with labor market frictions in which the ownership of capital is unequally distributed among the population and consumers have non-homothetic preferences. Building on the hierarchic preferences developed by Matsuyama (2002), we assume that consumers only begin to consume 'secondary' (non-essential) goods after satiating their demand for more 'basic' (essential) goods. As we shall see, this implies that aggregate consumption shares vary with aggregate income, and that aggregate productivity shocks affect the allocation of capital across sectors. In addition, and consistent with empirical evidence presented below, we assume that the factor share of capital is greater in sectors producing more 'basic' goods. Consequently, labor-intensive sectors are particularly sensitive to productivity shocks and experience greater volatility in output and employment.

The theoretical results we obtain go a long way in rationalizing a number of well-established and novel empirical facts. First, aggregate shocks change the composition of demand and lead to a reallocation of capital across sectors. In particular, when TFP increases, a greater share of capital is allocated to the secondary (labor-intensive) sector to match the shift in demand. Second, the counter-cyclicality of the income distribution results from changes in the level of employment and, to a lesser extent, from changes in relative factor prices. The model suggests that two thirds of the variation in the Gini result from changes in the employment rate, and only one third from changes in relative factor prices. Notwithstanding the highly stylized nature of the model, we also find the simulated reaction of income inequality to productivity shocks to be surprisingly close to the estimated value using US data on income dispersion and TFP between 1979 and 2005. In terms of order of magnitude, the model thus succeeds in explaining key features of the observed variation in the

Gini coefficient.

The framework we develop also allows us to study how changes in the ownership of capital assets across households affects the properties of the income distribution. We find that the level of income inequality is largely independent from the concentration of capital ownership. This is because demand composition effects, and the implied changes in relative factor prices and utilization rates, counterbalance the initial effect of redistributive policies. For instance, a redistribution of assets in favor of low income households is neutralized by a fall in the wage-interest rate ratio and employment rate, which itself leads to an increase in the returns to capital compared to labor. In equilibrium, we find that these two effects almost perfectly cancel each other out. Overall, the model suggests that whether or not a trade-off exists between wealth inequality and income inequality fundamentally depends on the way that redistributive policies affect the composition of demand across sectors.

To derive these results, this paper proceeds in three steps. First, we consider a frictionless economy that abstracts from the problem of unemployment in order to analytically characterize how changes in the composition of demand affect relative factor prices and the degree of income inequality. Given the robust theoretical results we obtain from this benchmark economy, we then extend the model and explicitly incorporate labor market frictions in order to study the effects of changes in employment rates. Lastly, we perform some simple numerical exercises that allow us to decompose the relative contribution of changing factor prices and employment rates. These numerical simulations also allow us to study how changes in the concentration of capital ownership affect the income distribution, employment and output.

Recently, there has been a renewed interest in understanding how changes in the distribution of income affect macroeconomic performance. This has been spurred by important empirical studies, notably by Piketty and Saez (2003), (2006) and Atkinson et. al. (2011), that document the long-run trends in income and wealth inequality in the United States. The focus of this paper is different, as we are interested in understanding the cyclical, rather than secular, properties of income inequality. In this sense, it can be seen as complementing recent work studying the possible links between inequality and macro-financial fragility, e.g. Atkinson and Morelli (2011) and Kumhof et. al. (2013). That said, these studies are primarily focused on understanding how changes in the distribution of income affect output, while we focus on the mirror problem; that is, how shocks to output explain the cyclical pattern of inequality.

The model we develop builds on the theoretical literature studying how non-homothetic consumer preferences interact with income distribution effects to explain the sectoral distribution of output and employment. This literature has predominately focused on long-run macroeconomic performance; in particular, issues related to international trade, growth and the process of industrialization.[1] For example, Matsuyama (2002) studies how demand composition and income distribution effects interact to explain the rise of 'mass consumption' societies. As in this paper, a key assumption of Matsuyama's model is that consumer preferences are hierarchic, so that as households' income increases, they expand the range of consumer goods they purchase rather than purchasing greater quantities of the same goods. Among other things, this implies that the market size for each consumption good does not depend only on the level of aggregate income, but also on the distribution of income across households. A similar mechanism is studied by Foellmi and Zweimuller (2006).

Another closely related paper is the one by Foellmi and Zweimuller (2011), who study how inequality affects the level of aggregate employment in an economy in which consumers have non-homothetic preferences and product markets are monopolistically competitive. They consider a model with only labor as a factor of production, and focus on labor income inequality (measured in terms of heterogeneous labor endowments). Our paper, instead, considers a model with both capital and labor as factors of production, and focuses on capital income inequality (measured in terms of heterogeneous ownership shares of the capital stock). Importantly, the introduction of an additional factor of production endogenizes the income distri-

[1] These included papers by Matsuyama (2000), Galor and Zeira (1993), and Banerjee and Newman (1993). See Bertola (2000) for a survey of this literature.

bution through changes in relative factor prices. This, in turn, allows us to explicitly address the counter-cyclical properties of the income distribution, an issue which is absent from Foellmi and Zweimuller's analysis.

Our paper is also naturally related to the literature studying the cyclical properties of the income distribution. Lindquist (2004) studies the role played by capital-skill complementarity in explaining the cyclical behavior of wage inequality. His model successfully accounts for both the volatility and the cyclical behavior of the skill premium in the United States. While we consider changes in the wage distribution to be an important component explaining the observed movements of the overall income distribution, we do not explicitly account for these changes in this paper. Instead, the mechanism we develop does not rely on cyclical variations in the skill premium, but rather on the interaction of ex-ante dispersion in wealth and demand composition effects. Our analysis should thus be thought as complementing the existing work studying the cyclical properties of wage inequality.[2] Our paper is also very closely related to Castaneda et. al. (1998), who build an extension of the stochastic neoclassical growth model with heterogeneous agents to explore to what extent unemployment spells and cyclically moving factor shares can account for the counter-cyclical properties of the Gini coefficient. Overall, authors find that (i) cyclically moving factor shares play a small role in explaining the counter-cyclicality of income inequality, and (ii) the cyclical properties of the income distribution are essentially independent from the wealth distribution. The model developed below confirms this result, as we find that the properties of the income distribution are only marginally affected by changes in capital income, even for substantial changes in the wealth distribution. We also clearly identify the general equilibrium effects that explain this seemingly paradoxical result.

[2]In our model, employment probabilities and wages do not depend on the position of the agent in the wealth distribution. Relaxing this assumption would be one way to account for the dynamics of wage inequality and magnify the results shown below. However, this paper shows that such a channel is not necessary to generate counter-cyclical income dispersion.

2 Empirical Motivation

The key empirical claims of the model developed below can be summarised as follows:

1. The Gini coefficient for income is counter-cyclical, increasing during recessions and diminishing during booms. The counter-cyclicality of income inequality is driven by employment and wage dynamics affecting the relative position of households at the bottom of the distribution.

2. Aggregate spending on labor-intensive goods and services is strongly pro-cyclical. In downturns (booms), the share of spending dedicated to labor intensive goods and services (e.g. construction, motor vehicles or tourism) is decreasing (increasing). This recomposition of private demand generates a high pro-cyclicality and volatility of employment and output in labor-intensive industries.

To what extent are these claims supported by empirical evidence? Below, we provide a cursory overview of existing empirical work suggesting that both claims are largely confirmed by the data. We also bring new evidence supporting the *demand composition channel* driving labor intensive sectors' volatility.

2.1 Counter-Cyclical Gini Coefficient

The counter-cyclical properties of income inequality is now a well established empirical fact. In the case of the US, Castaneda et. al. (1998) document the cyclical properties of income shares decomposed by quintile for the US between 1948 and 1986 using Current Population Survey (CPS) data. The correlations, which are reported in Table 1 below, show that the income share earned by the lowest quintile is both the most volatile and the most pro-cyclical. Moreover, the pro-cyclicality of the income shares is monotonically decreasing up to the fifth percentile. Using alternative income data from the Panel Study of Income Dynamics (PSID) between 1969 to 1981, Blank (1989) also confirms that the income distribution narrows during economic expansions.[3] More recently, Maestri and Roventini (2012) generalize this finding by showing that almost all inequality series in OECD countries are counter-cyclical at business cycle frequencies.

	Correlation with Output	Volatility
1st Quintile (0-20%)	0.53	1.07
2nd Quintile (20-40%)	0.49	0.48
3rd Quintile (40-60%)	0.31	0.26
4th Quintile (60-80%)	-0.29	0.17
Next 15% (80-95%)	-0.64	0.36
Top 5% (95-100%)	0.00	0.74

Table 1: Cyclical Behavior of Income Share by Quintile for US 1948-1986.
Source: Castaneda et. al. (1998)

Why is income inequality pro-cyclical? Existing evidence clearly points to a strong effect of labor and wage dynamics affecting the position of low income households. As put by Mocan (1999), citing several previous empirical studies, "the consensus so far is that inequality rises during recessions because unemployment worsens the relative position of low-income groups." In their systematic empirical study of cross-sectional inequality in the United States between 1967 and 2006, Heathcote et. al. (2010) find that recessions are times when earnings inequality widens sharply and that the root of such fluctuations is unemployment. In addition to the employment effect, the heterogeneous response of wages along the cycle seems to be is additional source of variation in labor income: according to Blank (1989), inequality tends to narrow in expansions because both wages and hours are pro-cyclical, in particular among low-income groups. Bonhomme and Huspido (2012) recently illustrated the combined effect of earnings and employment dynamics in driving income inequality in Spain[4]. Overall, the existing literature thus suggests

[3]Note that in the case of the US, Jonghyeon (2013) uses more recent CPS data and confirms that income inequality was countercyclical in the US between 1980 to 2004.

[4]Using information on labor earnings and employment from social security records between 1988 to 2010, the authors find that male earnings inequality was strongly countercyclical over that period, and that this evolution went in parallel with the cyclicality of employment in the lower-middle part of the wage distribution.

that both unemployment and wage effects drive income dispersion in the short run, even though the former seems to dominate the latter.

2.2 Pro-Cyclical Labor-Intensive Sectors

Although inequality dynamics in the short run are rather well documented, the underlying mechanism driving this result is not well understood. We argue that wage and employment dynamics have a significant impact on income distribution because labor-intensive sectors are both more volatile and more pro-cyclical than capital intensive sectors.[5] Using US industry-specific data on employment and output in several advanced economies, Jin and Li (2012) show that labor-intensive sectors' output is significantly more volatile than that of capital-intensive sectors - more than twice as volatile in the US and on average more than 60% as volatile among 12 OECD countries.[6] Figures 1 and 2, taken from Jin and Li (2012), illustrate the compositional change in US output and employment over the business cycle. Both figures clearly show the counter-cyclicality of (de-trended) output and employment shares of capital-intensive sectors. In particular, the correlation of the share of value added in capital-intensive sectors with GDP is 0.87, while the correlation of the share of employment in capital-intensive sectors with GDP is 0.58. This pattern is also found to be robust across the majority of countries in the sample, with the average corresponding correlation in a group of OECD countries being -0.53 and -0.63.

We argue that demand composition effects are partly responsible for the excessive volatility of labor intensive sectors. Traditionally, heterogeneous responses to

[5] In their study using Spanish data, Bonhomme and Huspido (2012) find that the counter-cyclical behavior of inequality over the last cycle was related to changes in employment composition, especially with regards to the (labor-intensive) construction sector. We argue that the pro-cyclicality of labor intensive industries is in fact a more general feature of advanced economies, i.e. not limited to Spain over the last cycle.

[6] In the case of the US, these figures were generated with data taken from the US Bureau of Economic Analysis (BEA) Industry Economic Accounts at the NAICS 2-4 digit level from 1977 to 2009. Statistics for OECD countries are based on 2-3 digit ISIC level taken from the STAN database from 1992 to 2010. Capital shares at the industry level were constructed as follows: (capital share) = 1 - (compensation of employees)/(value-added)-(taxes less subsidies). Capital-intensive sectors are then defined as all sectors where the capital share is greater than the median.

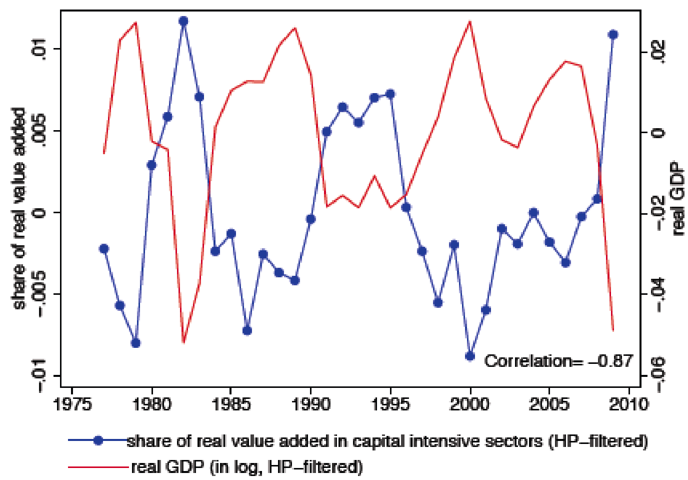

Figure 1: Correlation between the share of value added in capital-intensive sectors and GDP for US 1977-2009. *Source: Jin and Li (2012)*

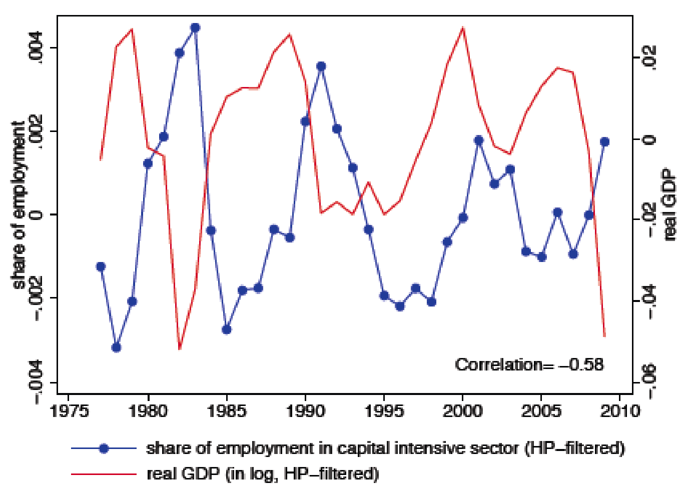

Figure 2: Correlation between the share of employment in capital-intensive sectors and GDP for US 1977-2009. *Source: Jin and Li (2012)*

business cycle fluctuations at the sectoral level have been thought to result from sector-specific productivity shocks. Instead, we argue that such sectoral dynamics reflect changes in the composition of aggregate demand, i.e. private consumption

Recession	2008			1991			1981/1982			1980	
Phase	drop	rebound		drop	rebound		drop	rebound		drop	rebound
Total PCE change (in%)	**-10,31**	**8,1**		**-4,36**	**5,5**		**-3**	**2,7**		**-9,2**	**10,0**
of which											
Durable Goods	**-4,6**	**3,0**		**-2,7**	**1,0**		**-3,7**	**1,6**		**-6,6**	**3,8**
Motor vehicles and parts	-1,8	0,4		-2,6	0,8		-2,7	2,0		-4,1	2,2
Furnishings and durable household equipment	-1,4	0,8		-0,1	0,0		-0,5	-0,3		-1,1	0,5
Recreational goods and vehicles	-1,0	1,6		0,1	0,5		-0,3	-0,1		-0,7	0,8
Other durable goods	-0,5	0,2		-0,1	-0,2		-0,2	0,0		-0,7	0,3
Non-Durable Goods	**-3,2**	**2,0**		**-1,0**	**0,8**		**0,5**	**0,3**		**-1,8**	**0,2**
Food and beverages purchased for off-premises consumptio	-1,0	0,5		-0,3	0,3		0,6	-0,1		0,2	-1,3
Clothing and footwear	-1,1	0,7		0,0	0,2		-0,1	0,3		-0,2	1,1
Gasoline and other energy goods	-0,2	-0,1		-0,5	0,4		0,0	0,3		-1,4	-0,4
Other nondurable goods	-0,9	0,8		-0,1	-0,1		0,0	-0,2		-0,4	0,8
Services	**-2,9**	**2,8**		**-0,6**	**3,6**		**0,2**	**0,9**		**-0,9**	**5,8**
Housing and utilities	0,7	0,8		0,2	1,3		0,4	0,4		1,4	1,3
Health care	1,3	0,5		0,1	0,7		-0,4	-0,3		-0,2	1,8
Transportation services	-1,4	-0,2		-0,6	-0,1		-0,2	-0,2		-0,8	-0,1
Recreation services	-0,7	0,1		-0,2	0,0		0,0	0,1		0,0	0,4
Food services and accommodations	-1,3	0,3		-0,6	0,5		-0,2	0,0		-0,6	0,5
Financial services and insurance	-0,7	0,7		1,3	1,0		0,2	0,4		-0,3	1,0
Other services	-1,0	0,5		-1,3	-0,2		0,0	0,2		-0,5	0,4
Not reported (services)	0,2	0,2		0,4	0,4		0,4	0,3		0,2	0,5

Figure 3: PCE change decomposition during US recessions and recoveries, 1980-2010. *Source: BEA*

Note: "Drop" periods are defined as the number of quarters during which the PCE fell below zero. "Rebound" periods are defined as the (same) number of quarters following the drop period. Figures report the cumulative drop (or increase) over a given period. As an example, the PCE fell below zero over 4 quarters in 2008/2009, for a total cumulative drop of 10.3% (at annual rates). Therefore, the "rebound" column reports the cumulative increase in PCE (or sub-category) over the 4 quarters following the drop period, *i.e* from 2009 Q2 to 2010 Q2. Data series are all seasonally adjusted.

and investment. Jin and Li (2012) also measure the (de-trended) share of investment in capital-intensive sectors in total investment over the 1977 to 2009 period in the US. Their results indicate that the correlation between the share of investment in capital-intensive sectors and output is -0.70. The magnitude of this investment reallocation is deemed to be economically significant, as the share of investment in capital-intensive sectors increases by about 5% during recessions.

Examining household consumption, we also find strong evidence that the composition of aggregate consumption adjusts over the business cycle. Consumer spending in the US, as measured by the BEA Personal Consumption Expenditure (PCE), decreased in only four occasions over the last 30 years, namely during recessions in 1980, 1981/1982, 1991 and 2008/2009. Table 3 reports the respective contributions to the change in PCE during these events, using quarterly data provided by the BEA and distinguishing consumption by major type of product. Although the recessions were different in nature and magnitude, it appears that important consumption adjustments were systematically made on outlays involving high labor intensity. For instance, recessions in 1980, 1982 and 1991 were mainly characterized by adjustments in durable goods consumption, in particular in “Motor Vehicles and Parts” and to a lesser extent in “Furnishings and Durable Household Equipments.” On the other hand, the 2008/2009 recession was not limited to durables and impacted both non-durables (“Clothing and Footwear”) and Services (“Food services and Accommodation” and “Transportation”). Following the methodology outlined in Jin and Li (2012), we find that these goods and services all display high labor intensity, ranging on average from 70.6% to 81.6%. Building on these empirical results, the remainder of this paper studies how such demand recomposition effects can rationalize the counter-cyclical properties of income inequality.

Type of Goods/Services in PCE	Underlying Industry	Absolute Labor Intensity	Rank (among 61 sectors)
Motor Vehicles and Parts	Motor vehicles, bodies and trailers, and parts	81.6%	8th
Furnishings and Durable household equipments	Furniture and related products	74.3%	19th
Clothing and Footwear	Apparel and leather and allied products	75.3%	15th
Food Services and Accomodation	Food services and drinking places & Accommodation	70.6%	18th & 32nd
Transportation	Air Transportation & Rail Transportation	74.9%	10th & 23rd

Table 2: Labor intensity of the main underlying industry

Note: Figures for 'Absolute labor intensity' were generated using the same methodology as in Jin and Li (2012). Sectoral data are taken from the US Bureau of Economic Analysis (BEA) Industry Economic Accounts at the NAICS 2-4 digit level from 1977 to 2009. Capital shares at the industry level are constructed as follows: (capital share) = 1 - (compensation of employees)/(value-added)-(taxes less subsidies). The rank column reports the rank of the given industry out of these 61 sectors. The list of labor/capital intensities calculated using this method for all 61 sectors is provided in Appendix C. Note that although they come from the same source (BEA), there is no direct equivalence between categories in the PCE and Industry-specific output tables used to compute capital/labor intensity. As a result, the "underlying industry" column reports the equivalent or closest industry to the PCE outlet category among the 61 sectors listed in the NAICS 2-4 digit classification.

3 The Model

3.1 Preferences and Endowments

The economy we consider is populated by a continuum of risk-neutral households (agents), indexed by $i \in N$ and of measure $\mathcal{N} = 1$. The production-side of the economy consists of two sectors $s \in \{1, 2\}$, with sector 1 producing basic goods (e.g. food) and sector 2 producing secondary goods (e.g. cars). Consumers have identical non-homothetic preferences over these two goods, represented by the following 'hierarchic' utility function

$$u(c_1, c_2) = \begin{cases} c_1 & \text{if} \quad c_1 \leq \bar{c}_1 \\ \bar{c}_1 + c_2 & \text{if} \quad c_1 = \bar{c}_1 \end{cases}$$

where $\bar{c}_1 > 0$ denotes the satiation point for consumers' demand of the basic good. The structure of preferences implies that agents only increase their consumption of the basic good until they reach this satiation point. After this point, agents continue to consume a fixed amount of the basic good, and spend all additional income on the secondary good.

Agents are endowed with one unit of labor $l_i = 1$, but differ in terms of their ownership of the aggregate capital stock $\bar{K} > 0$. Agent i's ownership share is denoted by $\theta_i \in \Theta = [0, 1]$. These shares are continuously distributed across the population according to the cumulative distribution function $G : \Theta \to [0, 1]$. Inverting the cumulative distribution function, we obtain the quantile function $Q : \mathcal{N} \to [0, 1]$ and associated quantile density function $q : \mathcal{N} \to [0, 1]$. Without loss of generality, we order agents by their ownership shares such that the index of agent i also denotes the measure of the set $[0, i]$. This implies that we can write $\theta_i = q(i)$, where by definition $\int_0^1 q(i)di = Q(1) = 1$ since the sum of shares must equal one. In order to measure the degree of wealth inequality, we define a scaling parameter $\beta > 0$ that determines the statistical dispersion of the probability distribution $G(\theta; \beta)$. As β gets large, the distribution of shares becomes increasingly unequal; as β goes to zero, the distribution of shares becomes increasingly uniform.

Assumption 1. *The distribution function* $G : \Theta \to [0,1]$ *is such that the quantile density function* $q(\cdot)$ *is continuously differentiable and monotonically increasing.*

3.2 Income Distribution

Taking the price of the secondary good as the numeraire, we can write the budget constraint of agent i with income y_i as follows

$$p_1 c_{i,1} + c_{i,2} \leq y_i \equiv w l_i + \theta_i r \bar{K}, \quad \forall i \in N \tag{1}$$

where $w > 0$ and $r > 0$ denote the wage and interest rate, respectively. Given these budget sets, we can easily derive the implied income distribution for this economy. We use the Gini coefficient to measure the degree of income inequality. Using agents' budget constraints and the fact that $\theta_i \sim G(\theta)$, the distribution of income can then be written as

$$y_i \sim H(y) \equiv G\left(\frac{y - w l_i}{r \bar{K}}\right)$$

where $y \in [y_l, y_h]$ with $y_l = w l_i$ and $y_h = w l_i + r \bar{K}$.

Definition 1. *Given a piecewise differentiable distribution function* $H(y) : [y_l, y_h] \to [0,1]$ *with associated density function* $h(y) : [y_l, y_h] \to [0,1]$*, the Gini coefficient* Γ *is defined as*

$$\Gamma = \frac{\int_{y_l}^{y_h} H(y)(1 - H(y))dy}{\int_{y_l}^{y_h} y h(y) dy}$$

4 Frictionless Economy

We first consider the case of a frictionless labor market in order to analytically characterize the properties of the Gini coefficient for income. This allows us to explicitly identify the price channels contributing to the counter-cyclicality of income inequality. We address the issue of equilibrium unemployment in Section 5 below.

Production takes place using two factors of production: capital (K) and labor (L). Since agents incur no disutility from labor, it must be that $l_i = 1$ for all

$i \in N$ in equilibrium. It follows that aggregate labor supply is constant and equal to $\bar{L} \equiv \int_{i \in N} l_i di = 1$. In line with the stylized facts presented above, the secondary good sector is relatively labor intensive, while the basic good sector is relatively capital intensive. To simplify the analysis, we assume that the production of the basic good requires only capital as an input, while the production of the secondary good combines both factors of production. Formally, the production technology in the basic good sector is

$$Y_1(K_1) = AK_1$$

while the secondary good is produced using a using a Cobb-Douglas production technology, such that

$$Y_2(K_2, L_2) = AK_2^{\alpha} L_2^{1-\alpha}$$

where $\alpha \in (0,1)$ and $A > 0$ denotes a Hicks-neutral productivity parameter[7] Given these production technologies, profit maximization in the secondary good sector implies that the equilibrium interest rate must satisfy

$$r(K_2, L_2) = \frac{\partial Y_2}{\partial K_2} = \alpha A \left(\frac{L_2}{K_2} \right)^{1-\alpha} \tag{2}$$

and the equilibrium wage rate will be

$$w(K_2, L_2) = \frac{\partial Y_2}{\partial L_2} = (1-\alpha) A \left(\frac{K_2}{L_2} \right)^{\alpha} \tag{3}$$

Definition 2. *A competitive equilibrium consists of prices* (r^*, w^*, p_1^*, p_2^*) *and quantities* $(c_1^*, c_2^*, K_1^*, K_2^*, L_2^*)$ *such that*

1. *All agents* $i \in N$ *choose consumption bundles* $(c_{i,1}, c_{i,2})$ *in order to maximize their utility subject to their budget constraints, taking prices as given.*

[7]The assumption that labor does not enter the production of basic goods is without loss of generality in the sense that all results would hold even in the case of a Cobb-Douglas production function

$$Y_1(K_1, L_1) = K_1^{\phi} L_1^{1-\phi}$$

providing that $\phi > \alpha$.

2. *Firms in both sectors $s \in \{1, 2\}$ choose factor inputs (K_s, L_s) in order to maximize their profits, taking prices as given.*

3. *Labor, capital and goods markets clear.*

4.1 Market Equilibrium

Given the structure of agents' preferences and their budget constraints (1), utility maximization implies that agent i consumes a positive quantity of the secondary good if and only if the following condition is satisfied

$$\frac{w + \theta_i r \bar{K}}{p_1} > \bar{c}_1 \tag{4}$$

Since an agent's income is strictly increasing in the value of his ownership share θ_i, it follows that any equilibrium must have a threshold structure: i.e. only agents with an ownership share greater than some (endogenous) threshold $\theta_i > \hat{\theta}$ will consume a positive quantity of the secondary good. Using condition (4), we can derive an expression for this threshold share as follows

$$\hat{\theta} = \frac{p_1 \bar{c}_1 - w}{r \bar{K}} \tag{5}$$

Henceforth, we denote by $\hat{\imath}$ the marginal agent such that $\theta_{\hat{\imath}} = \hat{\theta}$. Market clearing in the basic good sector requires that

$$\int_0^{\hat{\imath}} \left(\frac{w + q(i) r \bar{K}}{p_1} \right) di + (1 - \hat{\imath}) \bar{c}_1 = A K_1 \tag{6}$$

where $\hat{\imath} \in (0, 1)$ denotes the measure of constrained agents: i.e. agents too poor to demand a positive quantity of the secondary good. This market clearing condition, together with threshold condition (5), define a system of two non-linear equations in two unknowns: the measure of constrained agents $\hat{\imath} \in (0, 1)$ and the capital supplied to the secondary good sector K_2. Its solution fully characterizes the equilibrium prices and quantities for this economy.

Assumption 2. *The distribution of ownership shares is such that*

$$(1-\alpha(1-q(0)))A\bar{K} < \bar{c}_1 < q(1)A\bar{K}$$

Proposition 1. *If Assumptions 1 and 2 are satisfied, there exists a unique interior competitive equilibrium with $\hat{\theta}^* \in (0,1)$.*

Proof. See Appendix B. □

4.2 Capital Reallocation, Factor Prices and Inequality

Before analyzing the properties of the Gini coefficient, we examine how equilibrium prices and quantities - especially the equilibrium allocation of capital across sectors - react to productivity shocks in this economy.

Corollary 1. *Following a positive/negative Hicks-neutral productivity shock, capital is reallocated from the basic/secondary good sector to the secondary/basic good sector.*

Proof. See Appendix B. □

What is the mechanism driving the reallocation of capital across sectors? For illustrative purposes, consider the case of a negative Hicks-neutral shock. The productivity shock obviously has as an immediate consequence a reduction of income for all agents. However, the non-homothetic preferences of consumers results in this productivity shock also engendering a recomposition of demand away from secondary goods and towards basic goods. In other words, a greater share of aggregate income is now spent on the basic good. Because of this *demand composition effect*, a greater share of capital (which is in fixed supply) is reallocated from the secondary goods sector to the basic goods sector.

Since capital and labor are complements in production of the secondary good, the reallocation of capital leads to a lowering of the marginal product of labor in the secondary goods sector. As labor is inelastically supplied, this results in a lowering of the wage rate, and thereby a further decrease in the income of workers over and above the magnitude of the initial productivity shock. To see this, differentiate the wage condition (3) to obtain

$$\frac{dw^*}{dA} = \underbrace{(1-\alpha)K_2^{*\alpha}}_{\text{direct effect}} + \underbrace{\alpha(1-\alpha)AK_2^{*\alpha-1}\frac{dK_2^*}{dA}}_{\text{reallocation effect}} > 0$$

The first term of this derivative corresponds to the direct effect of a productivity shock on the marginal product of labor, for a given supply of capital to the secondary good sector. The second term corresponds to the indirect effect of a productivity shock on the marginal product of labor engendered by the reallocation of capital to or from the secondary good sector. Turning now to the interest rate, differentiating condition (2) yields

$$\frac{dr^*}{dA} = \underbrace{\alpha K_2^{*\alpha-1}}_{\text{direct effect}} - \underbrace{\alpha(1-\alpha)AK_2^{*\alpha-1}\frac{\frac{dK_2^*}{dA}}{K_2^*}}_{\text{scarcity effect}} \lessgtr 0$$

The change in the interest rate following a Hick-neutral productivity shock again consists of a direct (productivity) component and an indirect (reallocation) component. Why is the interest rate, contrary to the wage rate, not always increasing in A? The reason lies in the fact that even though capital becomes more/less productive following a positive/negative productivity shock, it also becomes relatively less/more scarce (i.e. the demand for the capital-intensive good increases/decreases in relative terms). This (negative) scarcity effect counterbalances the (positive) productivity effect. It can be shown that for sufficiently small values of $\bar{K}$, the scarcity effect can in fact dominate the productivity effect, so that the interest rate will be decreasing in A. However, regardless of whether the interest rate increases or decreases, the ratio of labor to capital income will always be increasing in the productivity parameter A. This is the *factor demand effect.* Let $\rho = w/r$ denote the wage-interest rate ratio. It can be easily verified that

$$\frac{d\rho(K_2^*)}{dA} = \frac{1-\alpha}{\alpha}\frac{dK_2^*}{dA} > 0$$

This last result is key to understand the counter-cyclical properties of the Gini coefficient. Indeed, in this economy without frictions, the counter-cyclicality of the Gini coefficient is a direct consequence of changes in the wage-interest rate ratio.

Proposition 2. *If Assumption 1 is satisfied, the Gini coefficient* Γ *for income is decreasing in the productivity parameter* A*.*

Proof. See Appendix B. □

To understand the intuition behind this result further, notice that because the Gini coefficient is scale invariant, changes in the wage and interest rate *pari passu* do not affect the degree of income inequality. However, since the wage-interest rate ratio is increasing in A, the factor by which an agent's income changes following a productivity shock is decreasing in the level of his *ex ante* wealth. This can be seen formally by noticing that the relative change in agents' income after a shock varies as a function of agents' capital ownership position

$$\frac{d\rho(K)}{dA} > 0 \quad \Rightarrow \quad \frac{d}{d\theta_i}\left(\frac{y_i + \frac{dy_i}{dA}}{y_i}\right) < 0$$

Alternatively, a simple way to interpret the cyclical dynamics of the income distribution is to notice that labor income is uniformly distributed across the population, while capital income is not. Therefore, whenever the wage increases/decreases relatively more than than the interest rate, the share of aggregate income that is uniformly distributed increases/decreases relative to the share that is unequally distributed.

5 Economy with Unemployment

As pointed out in the introduction, a large part of the variation in income inequality over the business cycle appears to be due to changes in labor income, and more specifically variation in the level of employment rate. Obviously, the frictionless economy analyzed above cannot account for changes in the level of employment. To address this shortcoming, we extend the model to account for labor market frictions so that some agents remain unemployed in equilibrium. When frictions are introduced, changes in the composition of demand (insofar as they change the matching rate on the labor market) directly affect the level of employment and, by extension, the level of aggregate output. As we shall see below, these fluctuations in employment are key to understand the cyclical properties of income inequality.

5.1 Labor Market Frictions

There are many ways in which labor market frictions can be modeled. A commonly used framework is the canonical random search model with Nash bargaining *à la* Pissarides (2000). We model frictions following the competitive search literature, as developed by Montgomery (1991) and Moen (Moen 1997). The approach has the advantage of endogenously determining the equilibrium wage schedule without relying on *ad hoc* assumptions about the distribution of bargaining power between workers and firms. That said, our modeling choice is mostly made for the sake of analytical convenience, and the qualitative nature of our results do not fundamentally depend on the details of the wage-setting process.

We begin by solving for the (partial) equilibrium in the labor market, treating the matching frictions as an exogenous technological constraint. Interested readers are referred to Appendix A in which the micro-foundations of the matching frictions are derived in full. More specifically, and contrary to the frictionless economy studied above, the secondary good is no longer produced by a representative firm using a Cobb-Douglas production technology. Instead, we assume the secondary good sector to consist of a continuum of homogeneous firms, each employing at most one worker.

We index active firms in the secondary goods sector by $j \in F$, with measure equal to $\mathcal{F} \in \mathbb{R}_+$. Each firm needs one unit of capital to produce, which it rents on a competitive credit market at the interest rate $r > 0$. Production takes place using a constant returns-to-scale technology, so that each firm employing a worker produces $A > 0$ unit of the secondary good. Aggregate capital demanded by the secondary goods sector is thus given by

$$K_2 = \int_0^{\mathcal{F}} dj = \mathcal{F}$$

Matching frictions imply that not every active firm succeeds in hiring a worker, and hence not every active firm produces output in equilibrium. Formally, the probability that firm j successfully hires a worker is equal to

$$\mu(K_2) = (1 - e^{-\frac{1}{K_2}})$$

which is strictly decreasing in the quantity of capital supplied to the secondary goods sector.[8] It follows that the level of employment in this economy equals the measure of active firms successfully hiring a worker, so that

$$L_2(K_2) = \mu(K_2)K_2 \tag{7}$$

Lemma 1. *The level of employment is strictly increasing in the quantity of capital allocated to the secondary good sector.*

Proof. Omitted. □

Factor prices in equilibrium are determined by firms' profit maximization problem. The expected profits of firm j posting wage w_j are

$$\mathbf{E}[\pi_j] = \mu(K_2)(A - w_j) - r, \quad \forall j \in \mathcal{F} \tag{8}$$

[8]See Appendix A for a detailed explanation of how this matching function is derived.

We show in Appendix A that the equilibrium wage function that maximizes firms' profits satisfies

$$w_j = \frac{A}{K_2(e^{\frac{1}{K_2}} - 1)}, \quad \forall j \in F \tag{9}$$

Free-entry into the secondary good sector implies that firms' expected profits must be equal to zero in equilibrium. This allows us to pin down the equilibrium measure of active firms as a function of the interest rate. Substituting the equilibrium wage into the firms' profit function (8) and solving for r yields an implicit condition pinning down the capital demanded by the secondary good sector, implying that

$$r = A\left(1 - \left(1 + \frac{1}{K_2(r;A)}\right) e^{-\frac{1}{K_2(r;A)}}\right) \tag{10}$$

Lemma 2. *Given any interest rate $r \in (0, A]$, there exists a unique partial equilibrium in the labor market. Moreover, the equilibrium measure of active firms in the secondary good sector is decreasing in r and increasing in A.*

Proof. See Appendix A. □

5.2 Equilibrium Income Distribution

We now return to the general equilibrium model and introduce the matching frictions outlined above. Contrary to the frictionless economy, agents now differ both in terms of their initial ownership of the aggregate capital stock and their employment status (i.e. whether they are employed or unemployed). Importantly, an individual agents' employment status is independent of his capital ownership position. Given this, the market clearing condition (6) in the basic good sector becomes

$$(1-L_2)\left(\int_0^{\hat{i}^U} \frac{q(i)r\bar{K}}{p_1} di + (1-\hat{i}^U)\bar{c}_1\right) + L_2\left(\int_0^{\hat{i}^E} \left(\frac{w + q(i)r\bar{K}}{p_1}\right) di + (1-\hat{i}^E)\bar{c}_1\right) = AK_1$$

where L_2 denotes the employment rate as defined by condition (7), $\hat{i}^U \in (0,1)$ denotes the marginal unemployed agent, and $\hat{i}^E \in [0,1)$ denotes the marginal employed agent. Using the threshold condition (5), we can derive explicit expressions for the marginal unemployed and employed agent. Formally, these threshold conditions are given by

$$\hat{\imath}^U = q^{-1}\left(\frac{\bar{c}_1}{A\bar{K}}\right) \quad \text{and} \quad \hat{\imath}^E = \max\left\{0, q^{-1}\left(\frac{\bar{c}_1}{A\bar{K}} - \frac{\rho(K_2)}{\bar{K}}\right)\right\}$$

Notice that the condition pinning down the measure of constrained unemployed agents does not depend on the allocation of capital across sectors. Hence, even though the measure of unemployed agents varies as a function of the quantity of capital allocated to the secondary good sector, the quantity of basic good demanded by each unemployed agent will be constant. Intuitively, this is because unemployed agents by definition do not earn a wage, and their income is thus unaffected by changes in the wage-interest rate ratio.

Assumption 3. *The distribution of ownership shares is such that*

$$q(0)A\bar{K} < \bar{c}_1 < q(1)A\bar{K}$$

Proposition 3. *If Assumptions 1 and 3 are satisfied, there exists a unique interior equilibrium in the model with frictions.*

Proof. See Appendix B. □

It can be easily verified that the capital reallocation effect remains once labor market frictions are introduced. Moreover, from Lemma 1, this implies that the level of employment varies as a function of aggregate productivity.

Corollary 2. *Following a positive/negative Hicks-neutral productivity shock, capital is reallocated from the basic/secondary good sector to the secondary/basic good sector. Moreover, the level of employment is increasing in the productivity parameter A.*

Proof. See Appendix B. □

Broadly speaking, this result stems from the fact that productivity shocks, insofar as they change the composition of demand due to the non-homotheticity of consumer preferences, change the measure of firms active in the secondary good sector. As total employment is proportional to the measure of active firms in the secondary good sector, productivity shocks will directly affect the level of equilibrium employment. Contrary to the frictionless case in which productivity shocks only affected relative prices, the model with frictions is also able to capture variation along the extensive margin. This, in turn, implies that changes in the distribution of income will be determined both by changes in the wage-interest rate ratio and by changes in the level of employment.

Deriving the income distribution in the model with frictions is more involved than in the frictionless case, since the set of agents is now partitioned into employed and unemployed workers. However, the task is simplified by the fact that an individual agent's employment status is independent of his wealth. Partitioning agents based on their employment status, we have that

$$y_i^E = w + \theta_i r\bar{K} \quad \text{and} \quad y_i^U = \theta_i r\bar{K}$$

where y_i^E and y_i^U denotes the income of employed and unemployed agents, respectively. Again, since $\theta_i \sim G(\theta)$, this implies

$$y_i^E \sim G\left(\frac{y^E - w}{r\bar{K}}\right) \quad \text{and} \quad y_i^U \sim G\left(\frac{y^U}{r\bar{K}}\right)$$

where $y^E \in [w, w + r\bar{K}]$ and $y^U \in [0, r\bar{K}]$. It follows that the distribution of income is given by the following piecewise continuous function

$$y_i \sim H(y) \equiv \mathbf{1}_{y \leq w} G\left(\frac{y}{r\bar{K}}\right)(1 - L_2) +$$
$$\mathbf{1}_{w<y<r\bar{K}} \left(G\left(\frac{y}{r\bar{K}}\right)(1 - L_2) + G\left(\frac{y-w}{r\bar{K}}\right) L_2\right) + \mathbf{1}_{y \geq r\bar{K}} \left((1 - L_2) + G\left(\frac{y-w}{r\bar{K}}\right) L_2\right)$$

Although well defined, deriving an analytical expression for the Gini coefficient using this income distribution function is quite tedious. Consequently, we turn to some simple numerical simulations in order to analyze how the distribution of income is affected by aggregate productivity shocks.

We begin by parameterizing the model with labor market frictions in order to obtain an empirically sensible value for the level of income (and wealth) inequality, taking the US as a benchmark. Using this baseline parametrization, we calculate the semi-elasticity of the Gini coefficient with respect to productivity shocks of plausible magnitudes, and analyze the extent to which variations in the level of employment on the one hand, and changes in factor prices on the other, affect income inequality over the business cycle. We then study how modifying the model's key parameters around this calibrated benchmark affect the level and cyclical properties of the income distribution. *Inter alia*, we find that, notwithstanding the highly stylized nature of the model, the simulated reaction of income inequality to productivity shocks is surprisingly close to the estimated value using US data. Second, and consistent with empirical evidence, we find that both unemployment and wage channels drive the counter-cyclicality of income inequality, even though the unemployment channel is significantly stronger. Finally, we find that while the level of income inequality is largely unaffected by changes in the concentration of capital ownership, the level of employment can either decrease or increase following a progressive redistribution of wealth. These comparative static results have interesting implications for the design of redistributive policies.

5.3 Simulation and Decomposition

The model has four free parameters: the degree of wealth inequality β, total factor productivity A, the consumption satiation point $\bar{c}$ and the aggregate capital stock $\bar{K}$. To begin, we impose a functional form for the distribution of wealth and assume the ownership shares are Pareto distributed across the population. Formally, the cumulative distribution function of the truncated Pareto distribution over the interval $[\underline{l}, 1]$ is given by:

$$\theta_i \sim Pa(\theta; \beta, \underline{l}) = \frac{1 - \underline{l}^{\beta}\theta^{-\beta}}{1 - \underline{l}^{\beta}}$$

where $\underline{l} \in (0, 1)$ denotes the lower bound of the distribution and $\beta > 0$ is the scaling parameter. Together, these two parameters determine the degree of wealth inequality. We set these parameters such that the Gini coefficient for wealth equals 0.73, the recorded value for the US in the late 2000s (see Piketty (2014)). We do this by fixing the value of β, and numerically solving for the value of the lower bound of the distribution such that the Gini coefficient for wealth takes on the desired value. The implied value is $\underline{l} = 0.001$ when $\beta = 0.01$.

Target Variable	Model	US Data	Parameter Value
Wealth Gini	0.72	0.73	$(\beta, \bar{l}) = (0.01, 0.001)$
Income Gini	0.35	0.35	$\bar{c} = 6$
Employment-to-Population Ratio	0.68	0.71	$\bar{K} = 5$
Productivity	-	-	$A = 1$

Table 3: Baseline Parametrization

We normalize the technology parameter $A = 1$ so that productivity shocks can be easily expressed in terms of percent deviations from the benchmark value. The consumption satiation point $\bar{c}$ and the capital stock $\bar{K}$ are then chosen in order to simultaneously match the observed degree of income inequality in the US and to obtain a plausible value for the level of equilibrium employment. We use the Gini coefficient for income before taxes and transfers reported by the BEA for the US in 2004, with a value of 0.35. The benchmark model then sets values of $\bar{c}$ and $\bar{K}$ equal to 6 and 5, respectively. This parametrization implies an employment-to-population ratio equal to 0.68, which is close to the value of 0.71 for the US in 2004 reported by the OECD.[9] The baseline parametrization and the associated targeted values are summarized in Table 3.

[9]The data can be accessed from http://stats.oecd.org/.

Figure 4: Decomposition of Simulated Gini Coefficient.

Using this baseline model, we simulate the cyclical behavior of the income distribution. We do so by measuring the contemporaneous response of the simulated Gini coefficient for income to shocks to the productivity parameter A. In order to use sensible values for the evolution of A, we parametrize the magnitude of shocks in order to match the observed pattern of (de-trended) TFP growth in the US between 1979 and 2004, as measured by the San Francisco Federal Reserve.[10] In addition, we examine how changes in the level of employment on the one hand, and changes in factor prices on the other, contribute to the counter-cyclical movements of income inequality. We do this by calculating the Gini coefficient anew using the simulated

[10]The data can be accessed from http://www.frbsf.org/economic-research/total-factor-productivity-tfp/.

equilibrium values, but fixing the employment rate at its benchmark value. In this case, movements in the Gini coefficient will only reflect changes in the wage-interest rate ratio. The output of these simulations are presented graphically in Figure 4.

We find that the Gini coefficient for income remains counter-cyclical in the model with labor market frictions. This should not come as a surprise, given that the employment rate is itself pro-cyclical. In terms of magnitude, the model predicts that a shock that increases (decreases) TFP by 1% is associated with a fall (rise) in the Gini coefficient of 0.003 units. This elasticity is also found to be linear, with a 2% shock associated with a rise (fall) in the Gini of 0.006 units and a 5% shock associated with a rise (fall) in the Gini of 0.015 units. Also, the decomposition exercise clearly shows that, consistent with empirical evidence, variations in the level of employment is the key channel explaining the behavior of the Gini coefficient: according to this baseline parametrization, only one third of the variation in the Gini coefficient is caused by changes in relative prices, implying that the remaining two thirds result from changes in the employment rate.

Interestingly, the simulated elasticity of the income distribution is surprisingly close to the actual elasticity estimated using US data between 1979-2005: regressing the cyclical component of the Gini data from the BEA on contemporaneous (detrended) TFP growth data from the FRSF yields an estimated coefficient equal to -0.0031 with a standard error equal to 0.002.[11] Given the uncertainty surrounding empirical TFP data series, this coefficient should naturally be interpreted with caution. Still, it suggests that in terms of order of magnitude, the channels in the model are important factors driving variations in the Gini coefficient over the business cycle.

5.4 Comparative Statics and Policy Implications

Turning now to the comparative statics, this section examines how the simulated economy reacts to changes in the model's key parameters. In particular, we are

[11]This coefficient goes up to -0.0038 with a standard error of 0.0019 when using a one year lag for the TFP data. See Appendix C for more details on the data and methodology.

interested in understanding how both the level of income inequality and the semi-elasticity of the Gini coefficient for income are affected by changes in the level of employment and the degree of wealth inequality.[12] To answer this question, we examine the effect of changes in the value of the scaling parameter β, which measures the dispersion of capital ownership across households.

β	Gini Income	Semi-Elasticity	L^*	Gini Wealth
0.01	0.35	0.0030	0.68	0.72
0.02	0.35	0.0029	0.68	0.73
0.04	0.34	0.0027	0.68	0.73
0.06	0.34	0.0026	0.68	0.74
0.08	0.34	0.0025	0.69	0.75
0.10	0.34	0.0024	0.69	0.76

Table 4: Comparative statics for β around benchmark parametrization

The results reported in Table 4 first indicate that the level of income inequality is largely unaffected by changes in the distribution of capital ownership. Standard general equilibrium effects are the cause of this seemingly paradoxical result. As an illustration, consider what would happen following an exogenous redistributive shock that leads to a reduction in the degree of wealth inequality. As the revenue accruing to owners of the capital stock will now be more equitably distributed, the direct effect of this redistribution will be a reduction in inequality. However, the implied income effects will also engender a recomposition of aggregate demand away from secondary (labor-intensive) goods towards basic (capital-intensive) goods. This translates into a greater share of aggregate income accruing to capital as it becomes the relatively more scare factor of production. In equilibrium, these two effects almost perfectly cancel each other out: the decrease in inequality caused by the initial redistribution is neutralized by a fall in the wage-interest rate ratio and employment rate, which itself leads to an increase in the returns to capital compared to labor.

Table 4 also shows that the counter-cyclicality of the Gini coefficient for income

[12]For robustness, we have also calculated comparative statics with respect to the aggregate capital stock $\bar{K}$ and the consumption satiation point $\bar{c}$. These results are found in Appendix C.

is somewhat attenuated for higher values of wealth inequality. Broadly speaking, this result stems from the fact that the degree to which income inequality reacts to productivity shocks depends on the level of employment. Since the degree of demand recomposition (and thus capital reallocation) across sectors varies inversely with the level of employment, economies characterized by relatively high employment levels should *ceteris paribus* exhibit less pronounced counter-cyclical variations in income inequality.

This begs the question, under what conditions will the level of employment rise or fall following a progressive redistribution of wealth? According to the baseline parametrization results presented in Table 4, making the wealth distribution more unequal leads to an increase in the level of employment (and thereby a decrease in the level of income inequality). Although this effect may be small in magnitude, it suggests that policy makers may sometimes face a trade-off between reducing wealth inequality on the one hand, and increasing employment and output on the other hand. That being said, we can show that the existence of such a trade-off fundamentally depends on the way wealth is redistributed across households. To see this formally, rearrange condition (5) to obtain

$$q(\hat{\imath};\beta) = \frac{\bar{c}_1}{A\bar{K}} - \frac{\rho(K_2)}{\bar{K}}$$

where we have used the fact that $q(\hat{\imath}) = \hat{\theta}$. Differentiating this condition with respect to β implies

$$\frac{dL_2}{d\beta} \propto -\frac{\bar{K}}{\rho'(K_2)}\left(\frac{\partial q}{\partial \hat{\imath}}\frac{d\hat{\imath}}{d\beta} + \frac{\partial q}{\partial \beta}\right) \lesseqgtr 0$$

The last condition shows that the existence of a trade-off is determined by whether the cumulative wealth of unconstrained agents (i.e. those with enough income to consume the secondary good) increases or decreases following a progressive redistribution of wealth. In the simulated economy studied above, the measure of constrained agents $\hat{\imath}$ is *increasing* in the degree of inequality. This effect, by itself, implies that employment should increase following a progressive redistribution of wealth. However, it is counteracted by the fact that the wealth of the marginal agent $q(\hat{i};\beta)$ is *decreasing* in β; or, equivalently, that the cumulative wealth of unconstrained agents decreases following a progressive redistribution of wealth. This second effect is so strong that it in fact dominates the fall in the number of constrained agents after redistribution. Interestingly, the model thus suggests that redistribution policies are not all equivalent, and that the way inequality is reduced is crucial in order to understand its consequences for employment. Policies designed to reduce wealth inequity should take into account their effects on the aggregate demand for labor-intensive goods and services in order to avoid unnecessary trade-offs between redistribution and output.

6 Conclusion

This paper proposes a new theory explaining the counter-cyclical property of the income distribution. After motivating empirically the extent of demand recomposition over the business cycle, we developed a model to study how such demand composition effects influence the distribution of income in the short run. To this end, we designed a two-sector general equilibrium model with labor market frictions in which (i) the ownership of capital is unequally distributed among the population, (ii) consumers have non-homothetic preferences and (iii) sectors differ in terms of their relative labor- and capital-intensity. Using this framework, we first show that changes in the composition of demand are an important channel through which productivity shocks are propagated through the economy. Second, and more importantly, we cast a new light on the specific channels driving short-run changes in the distribution of income. Income inequality (as measured by the Gini coefficient) is found to be counter-cyclical, and this effect is driven by changes in the level of employment and, to a lesser degree, by changes in relative factor prices. Interestingly, these theoretical results go a long way in rationalizing recent empirical findings that inequality rises during recessions because high unemployment and lower wages worsen the relative position of low-income groups. The semi-elasticity of the Gini coefficient for income implied by the model is also surprisingly close to that observed in US data between 1979 and 2005. While stylized, our framework thus seems able to capture key aspects of the observed cyclical properties of the US income distribution. The model's comparative statics also suggest that changes in the concentration of capital ownership have ambiguous effects on the level of employment, and that the presence (or absence) of a trade-off between equity and employment depends on how redistributing wealth affects the composition of demand across sectors.

More generally, we believe this paper calls for additional research on the short-run consequences of changes in the composition of aggregate demand. To date, most studies have examined variations in spending over the business cycle using characteristics of the products, such as their tradability or durability. However, as stated above, sorting goods and services by factor-intensity of inputs (rather than end-use) suggests that there are significant differences in the way sectors respond to business cycle shocks, with important consequences for factor prices and utilization rates. *Inter alia*, explicitly modeling such demand composition effects might help in addressing some of the shortcomings of standard representative-agent models, which fail to account for the distributional consequences of business-cycle shocks. This would allow, in turn, for a more detailed examination of the welfare costs of business cycles. Introducing and exploring the consequences of these largely-ignored aspects of economic fluctuations constitutes an important avenue for further research.

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Appendix A: Competitive Search Equilibrium

Under competitive search, firms post wage announcements $w_j \geq 0$. After having observed the distribution of wage announcements, each worker chooses a (symmetric) application strategy, denoted by $\sigma_j \in [0,1]$ for all $j \in F$ such that $\int_0^{\mathcal{F}} \sigma_j dj = 1$. The workers' application strategies induce queues at each firm, with expected length denoted by $\lambda_j \geq 0$. This corresponds to the expected number of job applicants at a firm posting wage w_j. Given the assumption that application strategies are symmetric and independent across workers, the actual number of applicants at a firm posting wage w_j is a Poisson random variable with mean λ_j. That is, each firm posting wage w_j receives $z \in \{0, 1, 2, ...\}$ applicants with probability $\frac{\lambda_j^z e^{-\lambda_j}}{z!}$. It follows that the probability that a worker applying to a firm posting a wage w_j is hired is equal to

$$\nu(\lambda_j) = \lim_{\bar{z} \to \infty} \sum_{z=0}^{\bar{z}} \frac{1}{(z+1)} \frac{\lambda_j^z e^{-\lambda_j}}{z!} = \frac{1 - e^{-\lambda_j}}{\lambda_j}$$

Thus, the probability that firm j successfully hires a worker must equal

$$\mu(\lambda_j) \equiv \lambda_j v(\lambda_j) = (1 - e^{-\lambda_j})$$

The equilibrium queue lengths are determined such that each worker obtains an expected utility of at least $V > 0$ from applying to any active firm. Since a worker facing a queue of length λ_j is hired with probability $\nu(\lambda_j)$, the following indifference condition must hold in equilibrium

$$\nu(\lambda_j) w_j = V \tag{11}$$

Labor market clearing requires that the total number of workers searching for a job must equal the aggregate labor supply. Formally, this implies

$$\int_0^{\mathcal{F}} \lambda_j dj = 1 \tag{12}$$

Definition 3. *A competitive search equilibrium is defined as a tuple* $\langle w, \lambda, V, \mathcal{F} \rangle$ *such that*

- *Firms choose wages* w *to maximize expected profits, taking as given workers' expected utility* V *and queue lengths* λ*.*
- *Each worker applies to exactly one firm thereby inducing queue lengths* λ*, taking the profile of wages* w *as given.*
- *Queue lengths* λ *and the measure of firms entering the market* $\mathcal{F}$ *are such that the labor market clears.*

Proof of Lemma 2.

When solving for the partial equilibrium in the labor market, we take the interest rate $r > 0$ as exogenous. Substituting workers' indifference condition into firms' profit function as given by condition (8), we obtain

$$\mathbf{E}[\pi_j] = A(1 - e^{-\lambda_j}) - \lambda_j V - r$$

Differentiating with respect to λ_j yields the following first-order condition

$$\lambda_j = \log\left(\frac{A}{V}\right), \quad \forall j \in \mathcal{F}$$

Since the RHS of this condition does not depend on j, it must be that the equilibrium queue lengths (and thus the equilibrium wage announcements) are the same for all active firms. Given this, the labor market clearing condition (12) implies

$$\lambda(\mathcal{F}) = \frac{1}{\mathcal{F}}$$

Combining these last two equations allows us to solve for the expected utility of workers in equilibrium

$$V(\mathcal{F}; A) = Ae^{-\frac{1}{\mathcal{F}}}$$

and plugging this condition into workers' indifference condition (11) yields the wage posted by firms in equilibrium

$$w = \frac{A}{\mathcal{F}(e^{\frac{1}{\mathcal{F}}} - 1)}$$

Substituting this into the expected profit condition (8) and simplifying, we obtain

$$\mathbf{E}[\pi] = A\left(1 - \left(1 + \frac{1}{\mathcal{F}}\right)e^{-\frac{1}{\mathcal{F}}}\right) - r$$

Free-entry into the secondary good sector implies that firms' expected profits must equal to zero in equilibrium. This pins down the equilibrium measure of active firms as a function of the interest rate. Setting the last condition equal to zero and solving for r yields

$$r = A\left(1 - \left(1 + \frac{1}{\mathcal{F}(r;A)}\right)e^{-\frac{1}{\mathcal{F}(r;A)}}\right)$$

Finally, notice that firms' expected gross revenue is a continuous and monotonically decreasing function of the measure of active firms, beginning at A when $\mathcal{F} = 0$ and converging to 0 as $\mathcal{F} \to \infty$. Formally,

$$\frac{d}{d\mathcal{F}}\left(A\left(1 - \left(1 + \frac{1}{\mathcal{F}}\right)e^{-\frac{1}{\mathcal{F}}}\right)\right) = -\frac{e^{-\frac{1}{\mathcal{F}}}}{\mathcal{F}^3} < 0$$

It follows that given any (exogenous) interest rate $r \in (0, A]$, there exists a unique and finite equilibrium measure of active firms $\mathcal{F}^*$. It can be easily verified that the equilibrium measure of active firms in the secondary good sector $\mathcal{F}^*$ is decreasing in r and increasing in A. $\square$

Appendix B: Proofs

Proof of Proposition 1.

Free-entry in the basic good sector pins down the price of the basic good as a function of the interest rate

$$p_1 = \frac{r}{A} \tag{13}$$

In order to guarantee the existence of an interior equilibrium whereby some, but not all, agents consume a positive quantity of the secondary good, we must impose some parametric restrictions so that agents are neither too rich nor too poor. Formally, the existence of an interior equilibrium requires that

$$\frac{w + \theta_0 r \bar{K}}{p_1} < \bar{c}_1 < \frac{w + \theta_1 r \bar{K}}{p_1}$$

Using the factor price equations (2) and (3) derived above, and recalling that $\theta_i = q(i)$, we are led to the inequality outlined in Assumption 2.

Using the free-entry condition (13) and the feasibility constraint $K_1 + K_2 = \bar{K}$, the market clearing condition 6 simplifies to

$$A\left(\hat{i}\rho(K_2) + Q(\hat{i})\bar{K}\right) + (1 - \hat{i})\bar{c}_1 = A(\bar{K} - K_2) \tag{14}$$

where $\rho(K_2) > 0$ denotes the wage-interest rate ratio. Moreover, using the factor price equations (2) and (3) and the fact that $\hat{\theta} = q(\hat{i})$, we can rewrite the threshold condition (5) as follows

$$q(\hat{i}) = \frac{\bar{c}_1}{A\bar{K}} - \frac{\rho(K_2)}{\bar{K}} \tag{15}$$

Recall that the LHS of condition (14) corresponds to the aggregate demand for the basic good, while the RHS equals the aggregate supply of the basic good. It is easy to verify that the RHS is monotonically decreasing in K_2 from $[A\bar{K}, 0]$ on the interval $K_2 \in [0, \bar{K}]$. Differentiating the LHS with respect to K_2, we obtain

$$\frac{d\hat{i}}{dK_2} \underbrace{\left(A \left(\rho(K_2) + q(\hat{i})\bar{K} \right) - \bar{c}_1 \right)}_{=0} + A \left(\frac{1-\alpha}{\alpha} \right) \hat{i} > 0$$

where the inequality follows from condition (15). It follows that aggregate demand for the basic good is monotonically increasing in K_2. Evaluating the LHS of the market clearing condition at $K_2 = 0$, we have that

$$Q(\hat{i})A\bar{K} + (1-\hat{i})\bar{c}_1 < A\bar{K}$$

since $\rho(0) = 0$. Rearranging, we obtain

$$(1 - Q(\hat{i}))A\bar{K} > (1-\hat{i})\bar{c}_1$$

where the inequality follows from Assumption 1 since $Q(i) < i$ for all $i \in (0, 1)$, and Assumption 2 since $A\bar{K} > \bar{c}_1$. □

Proof of Corollary 1.

Rewriting the market clearing condition (14) and differentiating with respect to A yields

$$\frac{dK_2^*}{dA} = (1-\hat{i}^*)\frac{\bar{c}_1}{A^2} - \hat{i}^*\rho'(K_2^*)\frac{dK_2^*}{dA} + \frac{d\hat{i}^*}{dA} \underbrace{\left(\frac{\bar{c}_1}{A} - \rho(K_2^*) - q(\hat{i}^*)\bar{K} \right)}_{=0}$$

Solving for dK_2^*/dA, we obtain

$$\frac{dK_2^*}{dA} = \omega(\hat{i}^*; \alpha)\frac{\bar{c}_1}{A^2} > 0$$

where we have that

$$\omega(\hat{i};\alpha) = \frac{\alpha(1-\hat{i})}{\alpha+(1-\alpha)\hat{i}} \in (0,1) \quad \square$$

Proof of Proposition 2.

Begin by rewriting the Gini coefficient in terms of the quantile function. Formally,

$$\Gamma = 1 - 2\int_0^1 L(x)dx$$

where

$$L(x) = \frac{\int_0^x H^{-1}(p)dp}{\int_0^1 H^{-1}(p)dp}$$

is the Lorenz curve and $H^{-1}(p) = w^* + Q(p)r^*\bar{K}$ is the income quantile function. It follows that the Gini coefficient will be decreasing in A if and only if the Lorenz curve is increasing in A. Formally,

$$\frac{d}{dA}\frac{\int_0^x w^* + Q(p)r^*\bar{K}dp}{\int_0^1 w^* + Q(p)r^*\bar{K}dp} > 0$$

Multiplying and dividing by r^*, we have

$$\frac{d}{dA}\frac{\int_0^x \rho^*(A) + Q(p)\bar{K}dp}{\int_0^1 \rho^*(A) + Q(p)\bar{K}dp} > 0$$

which implies

$$\left(\int_0^x \frac{d}{dA}\rho^*(A)dp\right)\int_0^1 H^{-1}(p)dp - \left(\int_0^1 \frac{d}{dA}\rho^*(A)dp\right)\int_0^x H^{-1}(p)dp > 0$$

Simplifying, we obtain

$$\left(x\int_0^1 H^{-1}(p)dp - \int_0^x H^{-1}(p)dp\right)\frac{d}{dA}\rho^*(A) > 0$$

From Assumption 1, we must have

$$x > L(x) = \frac{\int_0^x H^{-1}(p)dp}{\int_0^1 H^{-1}(p)dp}$$

As $\rho^*(A)$ is always increasing in A, this completes the proof. $\square$

Proof of Proposition 3.

Again, we restrict attention to interior equilibria, implying that some (but not all) unemployed agents consume the secondary good. Since unemployed agents receive no wage income, Assumption (2) simplifies to Assumption (3).

Using the free-entry condition $p_1 = r/A$ and the feasibility condition $K_1 + K_2 = \bar{K}$, we can rewrite the market clearing condition as follows

$$A(\bar{K}-K_2) = (1-L_2)\left((1-\hat{i}^U)\bar{c}_1 + Q(\hat{i}^U)A\bar{K}\right) + L_2\left((1-\hat{i}^E)\bar{c}_1 + Q(\hat{i}^E)A\bar{K} + \hat{i}^E A\rho(K_2)\right)$$

As before, these conditions constitute a system of two non-linear equations in two unknowns: the capital supplied to the secondary good sector $K_2 \in \mathbb{R}_{++}$ and the measure of constrained employed agents $\hat{i}^E \in (0,1)$. Differentiating the RHS of the marketed clearing condition with respect to K_2 yields

$$\underbrace{\frac{dL_2}{dK_2}}_{(+)}\left(D_1^E(K_2) - \bar{D}_1^U\right) + L_2\left(\underbrace{\frac{d\hat{i}^E}{dK_2}\left(A\rho(K_2) + q(\hat{i}^E)A\bar{K} - \bar{c}_1\right)}_{=0} + \hat{i}^E A \underbrace{\rho'(K_2)}_{(+)}\right)$$

where $D_1^E(K_2)$ and $\bar{D}_1^U$ denotes the quantity of basic good demanded by employed and unemployed agents, respectively. Notice that, contrary to the frictionless case, all employed agents can be unconstrained in equilibrium. That is, we can have

$$\frac{w + q(\hat{i}^E)r\bar{K}}{p_1} > \bar{c}_1$$

implying that $\hat{i}^E = 0$. By definition, in such a case we will have $d\hat{i}^E/dK_2 = 0$. Using the capital demand, wage and free-entry conditions (9)-(10) we have that

$$\rho'(K_2) = \rho(K_2)^2 \left(2 - \left(1 - \frac{1}{K_2}\right) e^{\frac{1}{K_2}} - \left(1 + \frac{1}{K_2} + \frac{1}{{K_2}^2}\right) e^{-\frac{1}{K_2}}\right) > 0$$

It follows that the aggregate demand of basic good will be monotonically increasing in K_2 if and only if employed agents demand strictly more basic good than unemployed agents. Formally,

$$D_1^E(K_2) - \bar{D}_1^U = (\hat{i}^U - \hat{i}^E)\bar{c}_1 + \hat{i}^E A\rho(K_2) - Q(\hat{i}^U - \hat{i}^E)A\bar{K} > 0$$

Dividing by $A\bar{K}$ and noticing that $q(\hat{i}^U) = \bar{c}_1/A\bar{K}$, we obtain

$$(\hat{i}^U - \hat{i}^E)q(\hat{i}^U) + \hat{i}^E \frac{\rho(K_2)}{\bar{K}} - Q(\hat{i}^U - \hat{i}^E) > 0$$

where the inequality follows from Assumption 1 as long as $\hat{i}^U \neq \hat{i}^E$. From Assumption 3, we have that $\hat{i}^U > \hat{i}^E$ since $\hat{i}^U > 0$ and $\hat{i}^E < 1$. Evaluating aggregate demand for the basic good at $K_2 = 0$, and noticing that $L_2 = 0$ when $K_2 = 0$, we must have

$$Q(\hat{i}^U)A\bar{K} + (1 - \hat{i}^U)\bar{c}_1 < A\bar{K}$$

Rearranging, we obtain

$$(1 - Q(\hat{i}^U))A\bar{K} > (1 - \hat{i}^U)\bar{c}_1$$

which is always the case as long as $\hat{i}^U < 1$ since $Q(\hat{i}^U) < \hat{i}^U$ and $A\bar{K} > \bar{c}_1$ by assumption. As aggregate supply of the basic good is monotonically decreasing in K_2 starting at $A\bar{K}$ when $K_2 = 0$, it follows that there exists a unique competitive equilibrium. $\square$

Proof of Corollary 2.

Rearranging the market clearing condition, we obtain

$$K_2^* = \bar{K} - (1-L_2)\left((1-\hat{i}^U)\frac{\bar{c}_1}{A} + Q(\hat{i}^U)\bar{K}\right) - L_2\left((1-\hat{i}^{E*})\frac{\bar{c}_1}{A} + Q(\hat{i}^{E*})\bar{K} + \hat{i}^{E*}\rho(K_2^*)\right)$$

Differentiating this condition with respect to A yields

$$\frac{dK_2^*}{dA} = \frac{(1-L_2)(1-\hat{i}^U)\bar{c}_1 + L_2(1-\hat{i}^{E*})\bar{c}_1}{A^2} -$$
$$(D_1^E(K_2^*) - \bar{D}_2^U)\frac{\partial L_2}{\partial K_2}\frac{dK_2^*}{dA} - L_2\left(\hat{i}^{E*}\rho'(K_2^*)\frac{dK_2^*}{dA} + \underbrace{\frac{d\hat{i}^{E*}}{dA}\left(\rho(K_2^*) + q(\hat{i}^{E*})\bar{K} - \frac{\bar{c}_1}{A}\right)}_{=0}\right)$$

where again we have that whenever $\hat{i}^E = 0$ we will have $d\hat{i}^E/dA = 0$. Rearranging yields the following comparative static condition

$$\frac{dK_2^*}{dA} = \frac{(1-L_2)(1-\hat{i}^U)\bar{c}_1 + L_2(1-\hat{i}^{E*})\bar{c}_1}{A^2}\left(1 + \hat{i}^{E*}\rho'(K_2^*)L_2 + (D_1^E(K_2^*) - D_2^U)\frac{dL_2}{dK_2}\right)^{-1} > 0 \quad \square$$

Appendix C: Supplementary Material

Labor and Capital Shares of Different Industries

Rank	Industry Title	Labor Share	Capital Share
1	Educational services	91,8%	8,2%
2	Hospitals and nursing and residential care facilities	91,5%	8,5%
3	Securities, commodity contracts, and investments	90,7%	9,3%
4	Computer systems design and related services	90,4%	9,6%
5	Management of companies and enterprises	90,0%	10,0%
6	Printing and related support activities	87,6%	12,4%
7	Social assistance	82,4%	17,6%
8	Motor vehicles, bodies and trailers, and parts	81,6%	18,4%
9	Warehousing and storage	80,2%	19,8%
10	Air transportation	79,0%	21,0%
11	Other transportation equipment	78,0%	22,0%
12	Computer and electronic products	76,9%	23,1%
13	Administrative and support services	76,3%	23,7%
14	Wood products	75,7%	24,3%
15	Apparel and leather and allied products	75,3%	24,7%
16	Ambulatory health care services	75,3%	24,7%
17	Textile mills and textile product mills	75,0%	25,0%
18	Food services and drinking places	74,6%	25,4%
19	Furniture and related products	74,3%	25,7%
20	Retail trade	72,4%	27,6%
21	Primary metals	72,3%	27,7%
22	Machinery	71,7%	28,3%
23	Rail transportation	70,7%	29,3%
24	Other transportation and support activities	70,6%	29,4%
25	Fabricated metal products	70,1%	29,9%
26	Wholesale trade	69,0%	31,0%
27	Support activities for mining	68,8%	31,2%
28	Construction	67,7%	32,3%
29	Amusements, gambling, and recreation industries	67,7%	32,3%
30	Information and data processing services	67,3%	32,7%
31	Other services, except government	66,8%	33,2%
32	Accommodation	66,6%	33,4%
33	Nonmetallic mineral products	66,5%	33,5%
34	Truck transportation	64,3%	35,7%
35	Plastics and rubber products	64,1%	35,9%
36	Electrical equipment, appliances, and components	63,7%	36,3%
37	Miscellaneous professional, scientific, and technical services	63,3%	36,7%
38	Transit and ground passenger transportation	62,7%	37,3%
39	Miscellaneous manufacturing	61,7%	38,3%
40	Publishing industries (includes software)	61,2%	38,8%
41	Insurance carriers and related activities	60,2%	39,8%
42	Waste management and remediation services	59,9%	40,1%
43	Paper products	58,3%	41,7%
44	Legal services	57,4%	42,6%
45	Performing arts, spectator sports, museums, and related activities	56,5%	43,5%
46	Funds, trusts, and other financial vehicles	55,7%	44,3%
47	Mining, except oil and gas	53,0%	47,0%
48	Water transportation	52,6%	47,4%
49	Food and beverage and tobacco products	51,8%	48,2%
50	Motion picture and sound recording industries	51,8%	48,2%
51	Forestry, fishing, and related activities	50,4%	49,6%
52	Pipeline transportation	47,6%	52,4%
53	Chemical products	45,9%	54,1%
54	Federal Reserve banks, credit intermediation, and related activities	42,6%	57,4%
55	Broadcasting and telecommunications	40,5%	59,5%
56	Utilities	31,2%	68,8%
57	Oil and gas extraction	26,7%	73,3%
58	Petroleum and coal products	23,6%	76,4%
59	Farms	19,4%	80,6%
60	Rental and leasing services and lessors of intangible assets	18,5%	81,5%
61	Real estate	5,6%	94,4%
	Min	5,6%	8,2%
	Max	91,8%	94,4%
	Mean	63,4%	36,6%
	Median	66,8%	33,2%

Estimated Semi-Elasticity

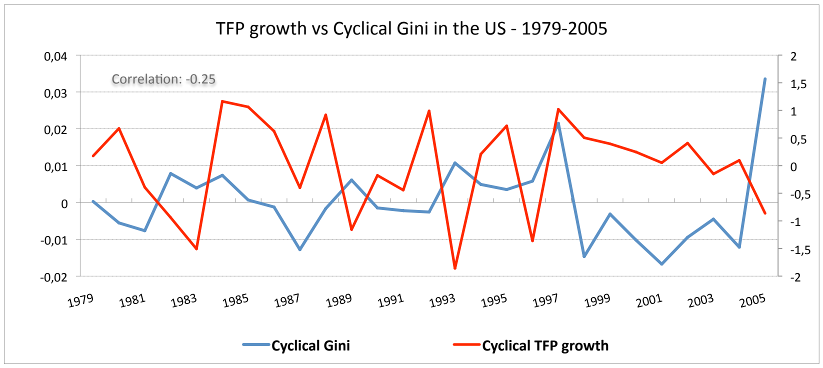

Figure 5: Cyclical Component of Gini Coefficient and De-trended TFP Growth, US 1979-2005. *Source: BEA and San Francisco Federal Reserve*

In order to obtain an estimate for the semi-elasticity of the Gini coefficient for income, we simply regress the cyclical component of the Gini coefficient on de-trended total factor productivity growth. To do so, we gathered annual data on the Gini coefficient for income before taxes and transfers from the BEA from 1979 to 2005. De-trending this series, we isolated the cyclical component of income inequality as measured by the Gini coefficient. We then calculated the simulated cyclical component of the Gini coefficient using the same data on de-trended total factor productivity growth described in the main text.

Variable	**Coefficient** (Std. Err)	**Coefficient** (Std. Err.)
TFP_t	-0.0031	-
	(0.0023)	
TFP_{t-1}	-	$\mathbf{-0.0038}^*$
		(0.0019)

Table 5: Estimated Semi-Elasticity of Gini Coefficient for Income, US 1979-2005.

Table 5 reports results for two specifications: one in which we use the contemporaneous value of TFP growth and one in which TFP growth enters with a one year lag. Since the data is de-trended, we do not include a constant in the regression.

Additional Comparative Statics

Changes to the aggregate capital stock $\bar{K}$ and the consumption satiation point $\bar{c}$ have opposite, but otherwise similar effects on both the level of income inequality and its cyclical properties. For example, a lower aggregate capital stock for a given satiation point, or a higher satiation point for a given size of the capital stock, leads to higher level of income inequality. This is because such changes lead to a sizeable decrease in the employment rate as less capital is supplied to the labor-intensive sector. We also find that at higher levels of income inequality, the Gini coefficient for income is more sensitive to productivity shocks. Again, this is because for a productivity shock of a given magnitude, the degree of capital reallocation across sectors will be increasing in the level of inequality.

$\bar{K}$	Gini Coefficient	Semi-Elasticity	L^*
5.0	0.35	0.0030	0.68
4.5	0.37	0.0031	0.65
4.0	0.40	0.0032	0.61
3.5	0.43	0.0034	0.57
3.0	0.47	0.0038	0.52

Table 6: Comparative statics for $\bar{K}$ around benchmark parametrization

$\bar{c}$	Gini Coefficient	Semi-Elasticity	L^*
6	0.35	0.0030	0.68
8	0.39	0.0036	0.64
10	0.42	0.0043	0.61
12	0.46	0.0050	0.57
14	0.50	0.0057	0.53

Table 7: Comparative statics for $\bar{c}$ around benchmark parametrization