WP/13/86

IMF Working Paper

"Leaning Against the Wind" and the Timing of Monetary Policy

Itai Agur and Maria Demertzis

INTERNATIONAL MONETARY FUND

 WP/13/86

IMF Working Paper

IMF - Singapore Regional Training Institute

"Leaning Against the Wind" and the Timing of Monetary Policy[1]

Prepared by Itai Agur and Maria Demertzis

Authorized for distribution by Sunil Sharma

April 2013

Abstract

If monetary policy is to aim also at financial stability, how would it change? To analyze this question, this paper develops a general-form framework. Financial stability objectives are shown to make monetary policy more aggressive: in reaction to negative shocks, cuts are deeper but shorter-lived than otherwise. By keeping cuts brief, monetary policy tightens as soon as bank risk appetite heats up. Within this shorter time span, cuts must then be deeper than otherwise to also achieve standard objectives. Finally, we analyze how robust this result is to the presence of a bank regulatory tool, and provide a parameterized example.

JEL Classification Numbers: E52, G01, G21

Keywords: Monetary policy, financial stability, bank risk, regulation

Author's E-Mail Address:iagur@imf.org, m.demertzis@dnb.nl

[1] This paper is forthcoming in the Journal of International Money and Finance. It has benefited from the authors' discussions with Gabriele Galati, Nicola Viegi, Refet Gurkaynak, Markus Brunnermeier, Viral Acharya, Hans Degryse, Wolf Wagner, Andrew Hughes Hallett, Philipp Hartmann, Claudio Borio, Graciela Kaminsky, and Olivier Pierrard, written comments by Selim Elekdag, and comments from seminar participants at the IMF, the BIS, the CEPR-EBC conference in Tilburg, and at the Dutch Central Bank. All remaining errors are our own.

Contents

I. Introduction

Various authors have argued that central banks' prolonged accommodative policies spurred risk taking incentives among the financial intermediaries that were at the heart of the recent financial crisis.[1] This had led to calls for a monetary policy that explicitly considers bank risk taking and financial stability,[2] or as it is known in this literature, to "lean against the wind". The main mechanisms through which the so-called risk taking channel of monetary policy is thought to work are: valuation effects, such as collateral which gains value from expansive policy, encouraging riskier profiles (Borio and Zhu, 2012); a search-for-yield that is driven by institutional factors leading some fund managers to seek higher risk to maintain yields after rates on safer assets decline (Rajan, 2006); and cheaper short-term debt, which raises levering incentives, and through interaction with banks' limited liability consequently also asset risk incentives (Agur and Demertzis, 2012; Dell'Ariccia, Laeven and Marquez, 2013).

Empirically, Delis, Hasan and Mylonidis (2011) use micro-level datasets from the US banking sector to examine the relationship between policy rates and risk taking. They find that low interest rates significantly strengthen banks' incentives to take on risky assets, and this is especially true for prolonged rate cuts. Maddaloni and Peydro (2011) use data from the Euro Area Bank Lending Survey to show that lower overnight rates soften lending standards. They also find evidence that keeping rates "low for too long" reduces credit standards even further. Similarly, Altunbas, Gambacorta and Marquez-Ibanez (2010) find that keeping rates low for an extended period of time significantly raises banks' risk profiles. They obtain this result from a data set that includes quarterly balance sheet information on listed banks in the EU and the US.[3]

In this paper we give an analytical interpretation for the meaning of keeping rates "low for too long", on the basis of the persistence of risk on banks' balance sheets, which relates to the long maturity of their assets. In a general form approach, in which we take the objectives of the monetary authority as given, we show that there are two main effects on optimal policy rates following a shock: the first is upon impact, and the second refers to the dynamic path of interest rates.

Faced with a negative shock, the authority that "leans against the wind" would cut interest rates deeper upon impact, than absent of a financial objective. However, its dynamic response will be to return to the equilibrium level quicker. Intuitively, the monetary authority cuts extra deep at the bottom of the recession, when banks are in the process of building down

[1] See Dell'Ariccia, Igan and Laeven (2008), Calomiris (2009), Brunnermeier (2009), Brunnermeier and others (2009), Allen, Babus and Carletti (2009), Diamond and Rajan (2009) and Kannan, Rabanal and Scott (2009), Adrian and Shin (2010), and Borio and Zhu (2012).

[2] Borio and White (2004), Borio and Zhu (2012), Adrian and Shin (2008, 2009, 2010) and Disyatat (2010).

[3] Other empirical papers that focus on the relationship between monetary policy and bank risk taking are Jiménez and others (2009), Ioannidou, Ongena and Peydro (2009), Buch, Eickmeier and Prieto (2010), Delis and Brissimis (2010), Delis and Kouretas (2011), Paligorova and Santos (2012) and Dell'Ariccia, Laeven and Suarez (2013). Unlike the three papers cited in the text, however, these papers do not analyze the relation to the duration of a rate change.

risks. This extra depth of the cut is intended to compensate for its short duration. The reason the cut is short-lived is that as soon as the economy is past its lowest point, the monetary authority wants to raise back rates quickly in order to tame the banks' renewed appetite for risk. If this financial stability objective is indeed representative of a welfare gain, then by comparison to it, the interest rate path of a standard-objectives authority is "low for too long".

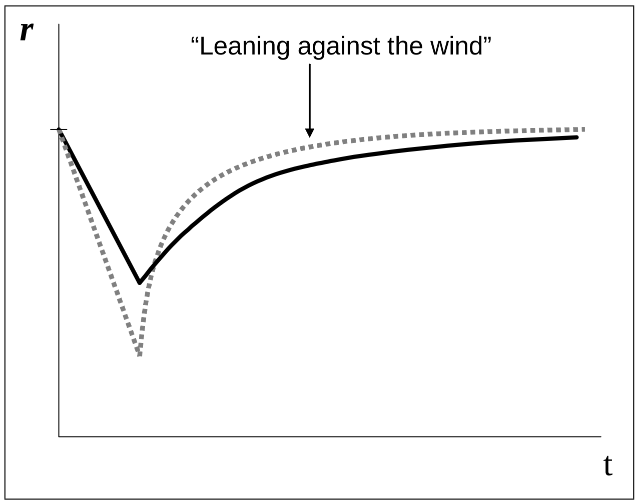

Figure 1: The timing of monetary policy

The result is summarized in figure 1, which represents the response of the monetary authority to a negative economic shock. The dotted line graphs the policy of an authority whose objectives include financial stability, while the solid line is that of an authority with standard-objectives. While we derive our result in a general-form model, we also provide a parameterized example of the model to highlight its mechanisms (Appendix D).

In an extension we show that a different interpretation can be given as well for the relationship between bank risk and the duration of a rate cut: since banks know that unwinding risk takes time, they only adjust their portfolio towards greater risk when they foresee that cuts last long.

We investigate the robustness of our result to the presence of bank regulation, which is assigned to the same authority that also controls monetary policy. We show that in the presence of a perfect regulatory tool, perfect in the sense of being both capable of exactly targeting bank risk as well as of full dynamic adjustment, our monetary policy result vanishes. That is, the regulatory tool targets bank risk whereas the interest rate targets only standard objectives, regardless of the weight that the central bank places on financial stability. However, when the regulatory tool is imperfect in terms of precision and dynamic adjustability, as arguably the most common tools such as capital requirements indeed are, the difference between a "leaning against the wind" and standard objectives monetary policy in the sense that we have described re-emerges.

The paper is organized as follows. Section 2 discusses related literature. Section 3 presents the basic model, and section 4 derives the main result. Section 5 then extends to a regulatory tool. Additional extensions to bank optimization, generalized central bank objectives and a parameterized example can be found in the Appendices B-D. All proofs of the propositions in the text are presented in Appendix A.

II. Literature

Although we take the monetary authority's financial stability objective as given, several recent papers model a reason for it to exist. Agur and Demertzis (2012) use a banking model to show how exogenous changes in monetary policy affect bank risk taking, and how an optimizing regulator is not in the position to neutralize this effect. The reason is that monetary policy affects both sides of the regulator's trade-off, namely financial stability and credit provision, so that a rate change essentially tilts the regulatory possibilities frontier. With a regulator that is unable to neutralize the risk-taking channel of monetary policy, there is justification for a coordinated regulatory-monetary policy decision. Acharya and Naqvi (2012) introduce an agency consideration into the analysis of monetary transmission: bank loan officers are compensated on the basis of generated loan volume. This causes an asset bubble, which a monetary authority can prevent by "leaning against liquidity". Loisel, Pommeret and Portier (2012) construct a model in which it is optimal for the monetary authority to lean against asset bubbles by affecting entrepreneurs' cost of resources in order to prevent herd behavior.[4]

A different approach is taken within the DSGE macro literature. Rather than providing a qualitative analysis, the models of Angeloni and Faia (2013), Angeloni, Faia and Lo Duca (2013) and Gertler and Karadi (2011) make a quantitative comparison of welfare under different central bank objectives, showing numerically that financial objectives can be valid.[5] Compared to these papers, the persistence of bank risk that we model is what gives rise to our main result. This type of issue would be hard to analyze within the DGSE framework because it induces an asymmetric payoff structure that cannot be linearized. But in comparison to these micro-founded models our work obviously lacks the rich endogenous reactions that they can produce.

All of the above literature focuses on how monetary policy affects the buildup of financial

[4] Other papers that model the transmission from monetary policy to bank risk, but without focusing on an argument for why this would affect monetary policy strategy, are Dell'Ariccia and Marquez (2006), Dubecq, Mojon and Ragot (2009), Drees, Eckwert and Várdy (2010), Valencia (2011), and Dell'Ariccia, Laeven and Marquez (2013).

[5] There have been many other papers that build on the framework of Bernanke, Gertler and Gilchrist (1999) by incorporating financial frictions into DSGE models. These are reviewed in Gertler and Kyotaki (2010). However, banks are usually a passive friction in this literature, with the exception of the papers cited in the text, and Cociuba, Shukayev and Ueberfeldt (2012), who numerically analyze monetary transmission on banks' incentives to "search for yield", but do not focus on the optimality of "leaning against the wind". See also Goodhart, Osorio and Tsomocos (2009) who have bank default in their model and their simulations show how it is affected by the policy rate. Their model does not have an optimizing central bank, however.

sector risks, and in all there is a negative relationship between the interest rate and risk taking. However, there is much reason to also consider how monetary policy is optimally set in the aftermath of a financial crisis, when lower rates reduce imminent default risk. This is done by Diamond and Rajan (2012) and Farhi and Tirole (2012), who proceed to investigate how such ex-post interest rate bailouts affect ex-ante risk taking incentives.

III. Model

We describe the economy by the general aggregate demand function:

$$y_t\left(\alpha_t, \varepsilon_t, r_t^f, r_{t-1}^f, ..., r_0^f\right), \tag{1}$$

where $y_t(\cdot)$ is the output gap; $r_t^f, r_{t-1}^f, ..., r_0^f$ are the current and all past interest rates. The standard arguments of the IS equation imply that:

$$\frac{\partial y_t\left(\alpha_t, \varepsilon_t, r_t^f, r_{t-1}^f, ..., r_0^f\right)}{\partial r_{t-s}^f} < 0 \qquad \forall s \leq t. \tag{2}$$

Variable ε_t represents a persistent demand shock:

$$\varepsilon_t = \phi\varepsilon_{t-1} + \nu_t, \tag{3}$$

with $\phi \in (0,1)$ the persistence parameter, and ν_t an iid shock. The impact on the business cycle is such that:

$$\frac{\partial y_t(\cdot)}{\partial \varepsilon_t} > 0. \tag{4}$$

Finally, α_t represents the bank risk profile, taken by financial institutions. Although we do not attempt to model risk explicitly here, the types of concepts that we have in mind for risk are, for example, the share of risky loans in a bank's portfolio, or the extent of financial innovation, which may bestow both benefits and costs on society (Tufano, 2003; Lerner and Tufano, 2011). This would suggest that there is an optimal level of risk taking in as far as welfare is concerned. We denote this as α^w. Any negative deviations from it would imply missing out on welfare enhancing opportunities; any positive deviations would be identified with "excessive risk taking".

$$\begin{aligned} \frac{\partial y_t(\cdot)}{\partial \alpha_t} &> 0, \quad \forall\, \alpha_t \in [0, \alpha^w), \\ \frac{\partial y_t(\cdot)}{\partial \alpha_t} &< 0, \quad \forall\, \alpha_t \in (\alpha^w, 1]. \end{aligned}$$

The monetary authority combines its two objectives in the inter-temporal function, with discount rate $\delta < 1$, like in Disyatat (2010):

$$\begin{aligned} \min_{r_t^f, \quad t \geq 0} \mathbb{E}[L] &= \min_{r_t^f, \quad t \geq 0} \left\{ E \sum_{t=0}^{\infty} \delta^t \left[(1-\rho) f(y_t(\cdot)) + \rho g(\alpha_t - \alpha^w) \right] \right\} \\ \text{s.t.:} \quad & \quad y_t(\cdot) . \end{aligned} \tag{5}$$

Here, $(\alpha_t - \alpha^w)$ is the distance between bank risk and socially optimal risk. The monetary authority places a weight of ρ on preventing the costs arising from excessive risk, captured by the function $g(\alpha_t - \alpha^w)$. And it places a weight of $(1-\rho)$ on the "standard" objective of minimizing output gap fluctuations, represented by the function $f(y_t(\cdot))$.[6] We assume that both $f(y_t(\cdot))$ and $g(\alpha_t - \alpha^w)$ are continuous and twice differentiable functions with $f(0) = 0$, $f''(y_t(\cdot)) = c_1 > 0$, $g(0) = 0$, and $g''(\cdot) = c_2 > 0$. This means that losses are minimized when, respectively, the output gap and the deviation of risk taking from the social optimum are zero. Moreover, the strictly positive second derivatives also imply that when the distance away from zero increases, losses rise more than linearly (i.e. a large output gap, or a large difference between realized and socially optimal bank risk taking are damaging to welfare).

Furthermore, the fact that both second derivatives are equal to a (positive) constant means that the functions are symmetric around the minimum. This symmetry property is not necessary for our results in section 4, but it simplifies the analysis with regulation in section 5. Note that for the output gap these assumptions nest the standard specification of a quadratic loss function.

Finally, assuming a linear form of the central bank objective (in terms of ρ) is not necessary for our results, but rather used for expositional convenience. In Appendix B we give a general form objective function and the required restrictions on it that are sufficient for our purposes.

Within the economy described above we introduce a banking sector that is modelled based on the following three axioms:

Axiom 1 Bank optimal risk taking is larger than the social optimum.

Axiom 2 Risk taking is procyclical.

Axiom 3 Risk is persistent.

Each of these can be obtained from various specific functional forms. In particular, the first axiom relates to bank moral hazard, which is quite a standard feature of the banking literature in general (Freixas and Rochet, 1997). The bank does not fully internalize the social costs of its risky loans. Part of the cost of its potential insolvency is borne by society

[6] We ignore inflation without any loss of generality. As we will only be looking at demand shocks, a policy effort to close the output gap will at the same time close the inflation gap.

rather than by bank shareholders, through limited liability, bailouts, deposit insurance or lost bank-specific relations to its customers. Therefore, the bank takes more risk than is socially optimal. The second axiom relates to the returns on risky projects being positively influenced by the state of the business cycle. Procyclicality is a well-established feature of banking empirical studies. The literature survey of Drumond (2009) discusses various mechanism through which procyclicality arises. Finally, the third axiom holds whenever risky projects are of relatively long maturity. Maturity mismatch between assets and liabilities has always been a key feature of banking, and gained particular prominence in the buildup to the previous crisis (Brunnermeier, 2009; Adrian and Shin, 2010), as even 30-year mortgages were often financed using very short-term instruments. Maturity mismatch implies that building down risk on the asset side is a time consuming process.

We model one bank, whose management is risk neutral. This bank can be seen as representing the banking sector's aggregate balance sheet. The bank chooses a risk profile α_t to maximize its profit, $P_t(\alpha_t, y_t(\cdot))$. We call the bank's profit maximizing risk profile, α_t^b, and operationalize the first axiom as

$$\alpha_t^b > \alpha^w. \tag{6}$$

The second axiom, on the procyclicality of risk taking, is given by:

$$\frac{\partial \alpha_t^b}{\partial y_t(\cdot)} > 0. \tag{7}$$

Finally, the third axiom is operationalized with the constraint

$$\alpha_t \geq \beta \alpha_{t-1}. \tag{8}$$

Here, the bank can only gradually shed risk from its balance sheet.[7] We let $\beta \in (0,1)$, but later (Proposition 1) impose an additional restriction on the size of β, namely that it be small enough to have some impact, that is, the constraint in (8) should bind in at least one period.

IV. A brief but deep cut

We examine next the effects of a persistent shock on the dynamic path of the interest rate (r_t^f, $\forall t$) and bank risk taking (α_t, $\forall t$). At time $t = 1$ a random shock ν_1 occurs, which determines the path of ε_t through the persistence parameter ϕ. Since we consider a one period shock only, the dynamic aspect of our exercise relates to how an authority chooses to 'spread' a given policy across time. When a negative shock hits, will it choose a short, deep cut or a longer, smoother response?

[7] In fact, given that riskier projects generally involve longer maturities, we could write in more general notation: $\beta(\alpha_t)$, with $\beta'(\alpha_t) > 0$. That is, the riskier a bank's profile, the longer the maturities of its loans, the fewer loans terminate each period and, therefore, the more persistent its balance sheet becomes. However, this complicates notation, while not making a qualitative difference to the proofs.

Before answering this question we must first make precise what we mean by a "short, deep" cut. Let us define λ as the profile of the monetary authority's policy response, where we assign $\lambda = 0$ to the optimal policy of the monetary authority with $\rho = 0$. This is the baseline case of an authority that does not lean against the wind. We then define a higher λ as:

Definition 1: *Policy profile i has a higher λ than policy profile j if:*

$$\exists \widehat{t}: \quad \left(\left|r^f_{t,i} - \overline{r}^f\right| \geq \left|r^f_{t,j} - \overline{r}^f\right| \quad \forall t < \widehat{t}\right) \wedge \left(\left|r^f_{t,i} - \overline{r}^f\right| \leq \left|r^f_{t,j} - \overline{r}^f\right| \quad \forall t > \widehat{t}\right),$$

and for some $t < \widehat{t}$ and some $t > \widehat{t}$ the respective conditions are strictly binding. Here, $\overline{r}^f$ is the steady state interest rate and policy is thus defined in deviations from that steady state.

Thus, a higher λ means a deeper but shorter-lived policy. We can now state this section's main result:

Proposition 1 *Following a negative shock ($\nu_1 < 0$), and assuming that β is sufficiently small such that the constraint in (8) is binding in at least one period: a monetary authority that "leans against the wind" ($\rho > 0$) chooses a profile $\lambda > 0$ for its interest rates. It thus opts for a deeper but shorter response, compared to an authority, which only has standard objectives ($\rho = 0$): $\frac{d\lambda}{d\rho} > 0$.*

Intuitively, banks build up risk when the economy picks up again, while rates are still low. This is the pattern that some argue was observed in the aftermath of the 2001-2003 recession, and contributed to the current crisis (see footnote 1). An authority that "leans against the wind" wants to prevent this type of pattern. By raising rates quickly after an initial cut, the monetary authority mitigates the incentives to buildup risk later. This comes at a cost in terms of the optimal output gap stabilization. The more an authority cares about preventing excessive risk, the more it is willing to bear such costs. Thus, the larger the weight on the financial stability objective, the shorter are its rate cuts. Given the short window of time in which rates are lowered, the authority then chooses a relatively deep cut, in order to sufficiently stimulate the economy. Overall, this yields figure 1 in the introduction, where the dotted line represents $\rho > 0$ and the solid line $\rho = 0$.

Note that throughout the remainder of the paper we assume that β is indeed small enough to make the third axiom of relevance (i.e., constraint (8) binding) in at least one period. When this is not the case it is quite obvious that the "leaning against the wind" effect vanishes: it is the persistence of bank risk that drives the result in Proposition 1.

In Appendix D we present a parametric example of our model for a two period optimization, the minimum required to demonstrate our result. Assuming that the constraint on risk persistence, from (8), binds in the first period and is released in the second, we show that a monetary authority that has a financial objective in addition to its traditional objective, will indeed cut interest rates by more in the first period before it returns to the steady state, in line with our Proposition 1.

Corollary 1 *Proposition 1 does not extend to an upturn ($\nu_1 > 0$). No unambiguous statement can be made about the effect of a higher ρ on the dynamics of monetary policy response to a positive shock.*

Intuitively, moving the asset portfolio from shorter to longer maturities is not time consuming. But the converse is: building down risk takes time, as risky loans involve long-term commitments. This is implicit in the formalization of the third axiom (equation (8)), which drives the asymmetry between positive and negative shocks.

V. Regulation

We examine next the interaction with bank regulation: if there are prudential tools that can specifically target bank risk, how does this affect the dynamic path of monetary policy? To what extent and - perhaps more precisely in the case of a qualitative model like our own - under what conditions does a financial stability concern still induce a deviation from the standard policy rate path?

One challenge we face is that incorporating an explicit regulatory tool, such as risk-weighted capital requirements, requires modelling a bank liability side, which our reduced form approach does not include. Agur and Demertzis (2012) do provide a microfounded banking model in which regulation affects bank liabilities and, through it, also bank asset side optimization. That model has no monetary policy dynamics in it, however, which is the focus here.

In line with our approach of the previous sections, we introduce bank regulation in general form. We assume that there is a tool available that allows for the implementation of a risk cap on banks, $\overline{\alpha}$. This can be interpreted as a risk-weighted capital requirement, which assuming banks come in with a given amount of capital, places an upper bound on the amount of risk they are allowed to take. As a benchmark case we initially assume that the tool is perfect, in the sense that it can be adjusted to always obtain the desired $\overline{\alpha}$. Moreover, we first assume that the tool is also dynamically adjustable over the cycle, $\overline{\alpha}_t$.

Subsequently, we relax these assumption and investigate the implications of an imperfect correspondence between the regulatory tool and $\overline{\alpha}_t$ (the authority faces uncertainty about how bank risk is affected by its policies), as well as the inability to dynamically adjust the tool. The latter is based on the observation that capital requirements are cumbersome to adjust in reality, and tend to remain fixed at internationally (e.g. Basel) negotiated levels over long periods of time.

A. Benchmark

We assume that monetary policy and bank regulation are jointly determined by one central bank, which has two tools, the interest rate r_t^f and a perfect regulatory tool $\overline{\alpha}_t$. Although issues of policy coordination between the monetary authority and a prudential regulator, and questions on the optimal institutional arrangement (joint or separate) of the tasks, are of great interest, they lie beyond the scope of this paper. We write the optimization problem as follows:

$$\begin{aligned} \min_{r_t^f,\overline{\alpha}_t \quad t\geq 0} \mathbb{E}\left[L\right] &= \min_{r_t^f,\overline{\alpha}_t \quad t\geq 0} \left\{ E\sum_{t=0}^{\infty} \delta^t \left[(1-\rho) f\left(y_t\left(\cdot\right)\right) + \rho g\left(\alpha_t - \alpha^w\right)\right] \right\} \\ \text{s.t.:} & \quad y_t\left(\cdot\right), \alpha_t \leq \overline{\alpha}_t, \end{aligned} \tag{9}$$

and where the assumptions embodied in equations (1)-(4), and (6)-(8) apply as before.

Proposition 2 r_t^f *is the same for any* ρ. *The optimal policy path is now always equal to that of the standard objectives* ($\rho = 0$) *authority in Proposition 1.*

The availability of a perfect regulatory tool thus eliminates the need to "lean against the wind". This tool already manages to get bank risk at its optimum, so that monetary policy is freed up to focus purely on its standard objectives.

B. Imperfect regulatory tool

We now consider that the regulatory tool suffers from two types of imperfections, namely lack of precision and dynamic inflexibility. We assume that the central bank can only set its regulatory policy, such as a capital requirements, once namely at the beginning of time. Moreover, the central bank cannot directly determine the risk cap $\overline{\alpha}$. Rather it adjusts its tool so as to target a desired risk cap, which we call $\widehat{\alpha}$, but it knows that the realization of $\overline{\alpha}$ can differ from its aim.[8] To capture this we assume that:

$$\overline{\alpha} = \widehat{\alpha} + \xi, \tag{10}$$

where ξ is a symmetric, random shock with zero mean, which implies that $E\left[\overline{\alpha}\right] = \widehat{\alpha}$ and the central bank hits its targeted risk cap on average.

The reason that we consider both these imperfections simultaneously is that solving the problem with an imprecise tool is very complicated if that tool is dynamically adjustable, because the problem then interacts with the dynamic constraint in equation (8) in intricate ways. Conversely, having only dynamic inflexibility but with a precise tool would currently be meaningless, since, as seen in Proposition 2, the optimal solution of the central bank would be to just set a constant $\overline{\alpha} = \alpha^w$. Thus, we consider this combination of limitations on the regulatory tool for the time being. However, in section 5C we extend to a dynamic α_t^w,

[8] This is due to the fact that the central bank is not able to precisely influence bank behavior. Additionally, of course, there may be political economy considerations why a central bank would not want to act too fiercely towards the financial sector with macro-prudential regulation (Agur and Sharma, 2013), but this lies beyond the scope of our analysis here.

in which case we can focus on only the limitation of a dynamically inflexible tool, without the need to simultaneously invoke imprecision.

The optimization problem of the central bank now becomes:

$$\begin{aligned} \min_{r_t^f, \widehat{\alpha} \quad t\geq 0} \mathbb{E}[L] &= \min_{r_t^f, \widehat{\alpha} \quad t\geq 0} \left\{ E \sum_{t=0}^{\infty} \delta^t \left[(1-\rho) f(y_t(\cdot)) + \rho g(\alpha_t - \alpha^w) \right] \right\} \\ \text{s.t.:} \quad & y_t(\cdot), \alpha_t \leq \widehat{\alpha} + \xi. \end{aligned} \tag{11}$$

Proposition 3 *The optimal policy of the central bank is to set $\widehat{\alpha} > \alpha^w$, and therefore to allow bank risk to exceed the social optimum on average. Its interest rate policy is then qualitatively the same as in Proposition 1: the $\rho > 0$ authority chooses a profile with deeper and shorter-lived response ($\lambda > 0$).*

The intuition for this result is that imprecision of the regulatory tool is less costly, in expected terms, when it is set above the socially optimal risk level, α^w, than when it is set below it. This is due to the fact that when the risk cap materializes well above α^w this has only limited costs, since banks will not choose to take more risk than a certain level (α_t^b), and instead if the risk constraint is much below α^w the central bank experiences the full loss. This asymmetry derives from the inequality in the first axiom (equation (6)) where banks' optimal risk taking is always greater than the social optimum.

Due to the fact that the central bank now relaxes regulatory constraints and expects banks to take on excessive risk, the same qualitative arguments apply as in section 4, and the optimal policy rate path will be shorter and deeper when the central bank considers financial stability objectives than when it cares only for its standard objectives. This, of course, says nothing of the quantitative change. We argue - but do not prove - that the v-shape dampens in comparison to section 4: for $\rho > 0$ the availability of regulatory policy relieves monetary policy from some, though not all, of its financial stability burden.

C. Dynamic socially optimal risk

So far we have assumed that the socially optimal level of risk taking, α^w, is constant. This assumption is actually more restrictive than is necessary for Proposition 1, which would carry through as long as (proof available upon request):

$$\frac{\partial \left(\alpha_t^b - \alpha_t^w \right)}{\partial y_t(\cdot)} > 0. \tag{12}$$

That is, what is required is that excessive risk taking is procyclical, in the sense that the gap between the risk that banks want to take, and the risk that society would like them to take, expands during booms. We replace equation (7) with (12). We do this with a specific purpose in mind, namely to be able to analyze the impact of a regulatory tool that displays only one imperfection: dynamic inflexibility. Thus, like in the benchmark case, the regulator is able to directly determine the risk cap, $\overline{\alpha}$. However, as in section 5B that risk cap is set at the beginning of time only and cannot be adjusted over the cycle. As we show, this one limitation of the regulatory tool is enough to (qualitatively) retain the v-shape result of Proposition 1. The optimization problem of the central bank now is:

$$\begin{aligned} \min_{r_t^f, \overline{\alpha} \quad t \geq 0} \mathbb{E}[L] &= \min_{r_t^f, \overline{\alpha} \quad t \geq 0} \left\{ E \sum_{t=0}^{\infty} \delta^t \left[(1-\rho) f(y_t(\cdot)) + \rho g(\alpha_t - \alpha_t^w) \right] \right\} \\ \text{s.t.:} & \quad y_t(\cdot), \alpha_t \leq \overline{\alpha}. \end{aligned} \tag{13}$$

Proposition 4 *The central banks sets $\overline{\alpha} > \min_t \alpha_t^w$. Hence, there are periods when banks take excessive risk, and in those periods the arguments of Proposition 1 apply.*

As socially optimal risk taking itself is now time-variant, the central bank must take care not to be overly restrictive on its regulatory policy, which it can set only at the beginning of time. But by allowing bank risk to be larger than socially optimal in some periods, there is still scope for monetary policy to play a role in improving financial stability. Simply put, the reason for monetary policy to "lean against the wind" here is the fact that it can be adjusted more frequently than regulation. And the way in which monetary policy does so, is in the manner described before, even if, as discussed earlier, the difference between the policy paths of a standard-objectives and a financial-stability oriented authority should be quantitatively smaller compared to a world without bank regulation.

VI. Conclusions

In this paper we examine how monetary policy would be altered if it were to account for financial imbalances. We allow for the economy to affect risk in a procyclical way, taking into account the persistent nature of long term loans on banks' balance sheets, and then examine how the interaction between bank risk taking and monetary policy would affect the path of interest rates. We find that, when faced with negative shocks, monetary authorities that have a financial stability objective are better off keeping rate cuts brief. But wishing to close the output gap as well then implies that the cut needs to be bigger than otherwise. The monetary authority essentially "frontloads" the cut to occur at the trough of a recession, when banks are building down risk, so that by the time banks want to start building up risk again, rates increase quickly. In an extension we show that there is also different way to look at this, namely that when banks foresee a long period of low rates they are keener to pursue a risky strategy, than when cuts are brief. We acknowledge that while accounting for financial imbalances has a very clear implication for the path of interest rates, the definition and measurement of risk remains a considerable challenge in its implementation.

Policy makers tend to dislike interest rate variability and prefer slow, smooth movements in the policy rate. In a sense, this paper provides an argument against relatively slow dynamics in monetary policy. If financial stability targets do become a part of monetary authorities' task, this may necessitate a more fluid, and perhaps more aggressive conduct of monetary policy in the future.

Appendix A: Proofs

Proof of Proposition 1. We outline our proof in figure 2 where we plot, on the left and right panes respectively, the interest rates and the associated levels of risk taking for different λ. In the right pane the dashed (red) line represents how the constraint on risk ($\alpha_t \geq \beta\alpha_{t-1}$) prevents the reduction of risk from one period to the next. In other words, up to point t' even though the interest rate drop is different for the two policies (as shown in the left pane), the risk reduction is identical and captured by this line. After that point, the two policies differ with the risk being higher (quicker to return to the initial state) for the policy for which $\lambda = 0$. To demonstrate this, consider first $\beta = 0$, i.e. no dynamic constraint on risk taking. First, by $\frac{\partial\alpha_t^b}{\partial y_t(\cdot)}\frac{\partial y_t(\cdot)}{\partial\varepsilon_t} = (+)(+) > 0$ a negative shock, $\nu_1 < 0$, implies that α_t^b decreases and then, as $\varepsilon_t \to 0$, gradually returns to $\overline{\alpha}^b$, the bank's steady state optimal risk taking. This is true for any policy irrespective of λ. Then, for $\beta > 0$, the constraint $\alpha_t \geq \beta\alpha_{t-1}$ will be binding from $t = 0$ up to a t', at which point $\alpha_{t'}^b\big|_{\beta=0} = \beta\alpha_{t'-1}^b$(or $= \beta^{t'}\overline{\alpha}^b$).

For a sufficient proof set $\widehat{t} = t'$. We observe that for $t < t'$ policy cuts $\left|r_t^f - \overline{r}^f\right|$ are less deep for $\lambda = 0$, generating risk taking that is closer to society's optimal. For $t > t'$, policy cuts $\left|r_t^f - \overline{r}^f\right|$ implied by $\lambda > 0$ however, generate risk taking that is closer to society's optimal. Then up to t' the constrained paths of $\lambda = 0$ and $\lambda > 0$ are equivalent. But, subsequently, $\lambda > 0$ has lower risk taking. In terms of financial stability, the $\lambda > 0$ thus offers an unambiguous gain on the financial stability objective, i.e.: $\frac{d}{d\lambda}\sum_{t=0}^{T}\delta^t\left[g\left(\alpha_t - \alpha^w\right)\right] < 0$. However, it is also an unambiguous loss on $\sum_{t=0}^{T}\delta^t\left[f\left(y_t(\cdot)\right)\right]$ by the definition that $\lambda = 0$ is the path of the $\rho = 0$ authority, which minimizes $f\left(y_t(\cdot)\right)$. It follows that the more weight the authority puts on preventing financial imbalances (higher ρ), the more it is willing to give up on minimizing $f\left(y_t(\cdot)\right)$ to achieve a lower $g\left(\alpha_t - \alpha^w\right)$, which implies that $\frac{d\lambda}{d\rho} > 0$. ■

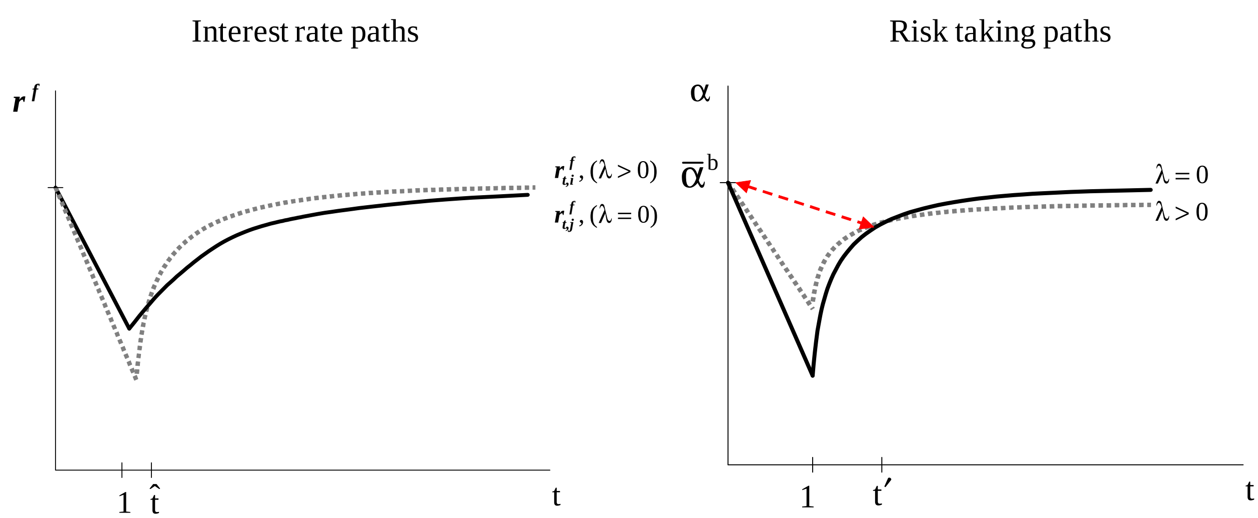

Figure 2: Interest rate and risk taking paths

Proof of Corollary 1. The proof of Proposition 1 implies that $\int_t \alpha_t^b dt$ is unambiguously smaller under a higher λ, as α_t^b is the same till $\widehat{t}$, and less afterwards. This does not extend to a positive shock, however. A higher λ, which here implies steeper initial rate hike, does translate into a smaller α_0^b. But for $t > \widehat{t}$: $\alpha_t^b\big|_{\lambda>0} > \alpha_t^b\big|_{\lambda=0}$. Thus, there is a parameter-dependent trade-off, and no general proof can be derived. ■

Proof of Proposition 2. As discussed in the proof of Proposition 1, the policy rate path of the $\rho = 0$ authority is the one that minimizes $y_t(\cdot)$ and therefore, by $f(0) = 0$, and $f''(y_t(\cdot)) > 0$ minimizes $f(y_t(\cdot))$. Moreover, by $g(0) = 0$, and $g''(\cdot) > 0$ we have that $\overline{\alpha}_t = \alpha^w$ minimizes $g(\alpha_t - \alpha^w)$. Therefore, for any ρ, the central bank cannot do better than to set $\overline{\alpha}_t = \alpha^w$ and r_t^f equal to the standard-objectives authority.

Note that the dynamic constraint of equation (8) does not affect the optimization of the central bank here: by $\alpha_t^b > \alpha^w$ we can be certain that $\overline{\alpha}_t = \alpha^w \Rightarrow \alpha_t = \overline{\alpha}_t \forall t$. ■

Proof of Proposition 3. The proof relies on an asymmetry in the costs of "getting it wrong" in terms of regulatory policy. By $g(0) = 0$ and $g''(\cdot) = c_2 > 0$, we have that g is a function that reaches its minimum at $\alpha_t = \alpha^w$ and is convex and symmetric around that minimum. From (6) we have that whenever $\xi < 0$, $g(\alpha_t - \alpha^w) = g((\widehat{\alpha} + \xi) - \alpha^w)$ whereas instead whenever $\xi > 0$, $g(\alpha_t - \alpha^w) = g((\widehat{\alpha} + \xi) - \alpha^w)$ only when $\widehat{\alpha} + \xi < \alpha_t^b$ and instead when $\xi > 0$ is large enough that $\widehat{\alpha} + \xi \geq \alpha_t^b$, the regulatory imprecision no longer "bites" and $g(\alpha_t - \alpha^w)$ is capped at $g\left(\alpha_t^b - \alpha^w\right)$. Thus, imprecision ($\xi$) is less costly (in terms of g) on average when $\widehat{\alpha}$ is located closer to α_t^b. Therefore, the optimal regulatory policy involves $\widehat{\alpha} > \alpha^w$ and $\widehat{\alpha} < \alpha_t^b$ ($\forall t$). When $\widehat{\alpha} > \alpha^w$ all arguments of Proposition 1 go through unaltered, and hence $\frac{d\lambda}{d\rho} > 0$. ■

Proof of Proposition 4. Suppose $\overline{\alpha} = \min_t \alpha_t^w$. Then, at $t' = \arg\min_t \alpha_t^w$ we have $g(\alpha_{t'} - \alpha_{t'}^w) = g(0) = 0$, and for every other period $g(\alpha_t - \alpha_t^w) > 0$. Moreover, by the definition of a global minimum (the existence of which follows from $g''(\cdot) > 0$): $g'(0) = 0$. Hence, marginally increasing $\overline{\alpha}$ above $\alpha_{t'}^w$ generates near zero welfare loss in t' but (by the convexity of $g(\cdot)$) larger welfare gains in every other period. Hence $\overline{\alpha} > \min_t \alpha_t^w$ must be optimal. ■

Appendix B: Central bank objective in general form

The formulation of the monetary authority's optimization problem in (5) can instead be written to the general form:

$$\begin{aligned} \min_{r_t^f,\quad t\geq 0} \mathbb{E}\left[L\right] &= \min_{r_t^f,\quad t\geq 0} \left\{ E \sum_{t=0}^{\infty} \delta^t H^\rho \left(f\left(y_t\left(\cdot\right)\right), g\left(\alpha_t, \alpha^w\right)\right)\right\} \\ \text{s.t.:} \quad & y_t\left(\cdot\right), \end{aligned} \tag{14}$$

where H^ρ is a set of functions such that when $f\left(y_t\left(\cdot\right)\right) > g\left(\alpha_t, \alpha^w\right)$ the value of $H^\rho\left(f\left(y_t\left(\cdot\right)\right), g\left(\alpha_t, \alpha^w\right)\right)$ decreases in ρ, and when $f\left(y_t\left(\cdot\right)\right) < g\left(\alpha_t, \alpha^w\right)$ the value of $H^\rho\left(f\left(y_t\left(\cdot\right)\right), g\left(\alpha_t, \alpha^w\right)\right)$ increases in ρ.

Moreover, $g\left(\alpha_t, \alpha^w\right)$ satisfies

$$\begin{aligned} \left.\frac{\partial g\left(\alpha_t, \alpha^w\right)}{\partial \alpha_t}\right|_{\alpha_t > \alpha^w} &> 0 \\ \left.\frac{\partial g\left(\alpha_t, \alpha^w\right)}{\partial \alpha_t}\right|_{\alpha_t < \alpha^w} &< 0 \\ \left.\frac{\partial g\left(\alpha_t, \alpha^w\right)}{\partial \alpha^w}\right|_{\alpha_t > \alpha^w} &< 0 \\ \left.\frac{\partial g\left(\alpha_t, \alpha^w\right)}{\partial \alpha^w}\right|_{\alpha_t < \alpha^w} &> 0. \end{aligned}$$

And, furthermore, the same properties as before for $f\left(y_t\left(\cdot\right)\right)$ and $g\left(\alpha_t, \alpha^w\right)$ apply, namely: they are continuous and twice differentiable functions with $f\left(0\right) = 0$, $f''\left(y_t\left(\cdot\right)\right) = c_1 > 0$, $g\left(0\right) = 0$, and $g''\left(\cdot\right) = c_2 > 0$.

Appendix C: Bank optimization

In this extension we consider a setup with banks that optimize their risk profile, taking into account the path of interest rates. To keep the problem tractable, we make the simplifying assumption that there is a continuum of atomistic banks. This implies that each bank sets its risk profile taking the state of the economy as given, that is, without considering how its own risk taking decision will affect the overall business cycle.

Moreover, in this setup we take monetary policy as given. We thus abstract from optimizing monetary policy here, and aim only to show how bank risk is affected by different policy paths. In spite of these simplifications, the bank optimization problem will still prove quite intricate. It can be written as

$$\max_{\alpha_{it}} \sum_{t=0}^{\infty} \delta^t P_{it}\left(\alpha_{it}, y_t\left(\alpha_t, r_t^f, r_{t-1}^f, ..., r_0^f\right)\right) \tag{15}$$
$$\text{s.t. } \alpha_{it} \geq \beta\alpha_{it-1}$$

where α_{it} is bank i's risk profile, and α_t is the average risk of the banking sector. Here the properties of $y_t\left(\alpha_t, r_t^f, r_{t-1}^f, ..., r_0^f\right)$ with respect to risk and interest rates are as before, and $P_{it}\left(\alpha_{it}, y_t\left(\alpha_t, r_t^f, r_{t-1}^f, ..., r_0^f\right)\right)$ is assumed to satisfy (7), meaning that banks would like to take more risk when the output gap is larger: $\frac{\partial\alpha_{it}^b}{\partial y_t(\cdot)} > 0$ (with α_{it}^b being bank i's optimal risk taking in period t). Given identical banks, it is clear that $\alpha_t = \alpha_{it}$: each bank optimizes taking into account that all other banks face the same optimization problem, and will make the same choice. Importantly, even though each other bank will do the same, this is not a consequence of bank i's action, but just a result of facing the identical optimization problem. Since bank i is atomistic, it therefore does not consider its effect on α_t and hence on y_t. Furthermore, the policy rate path $r_t^f, r_{t-1}^f, ..., r_0^f$ is also given, meaning that bank i decides about the path for α_{it} for a given path of $y_t\left(\alpha_t, r_t^f, r_{t-1}^f, ..., r_0^f\right)$. What makes this problem interesting is the risk constraint, $\alpha_{it} \geq \beta\alpha_{it-1}$.

Proposition 5 $\exists\widehat{\lambda} > 0 : \alpha_{it}|_{\lambda=\widehat{\lambda}} < \alpha_{it}|_{\lambda=0}\,\forall t$: *interest rate paths can be found which involve shorter and deeper cuts than the $\lambda = 0$ path, and bring about an unambiguous (i.e. each period) reduction in risk taking.*

Proof. From the definition of λ it is obvious that in any period $t > \widehat{t}$ we have $\alpha_{it}^b\big|_{\lambda=\widehat{\lambda}} < \alpha_{it}^b\big|_{\lambda=0}$. Instead, in periods $t < \widehat{t}$ there is a trade-off: on the one hand, $r_t^f\Big|_{\lambda=\widehat{\lambda}} < r_t^f\Big|_{\lambda=0}$ in such periods, which works to raise $\alpha_{it}^b\big|_{\lambda=\widehat{\lambda}}$ as compared to $\alpha_{it}^b\big|_{\lambda=0}$. On the other hand, the fact that a bank wants to have lower risk in the future ($t > \widehat{t}$) when faced with $\lambda = \widehat{\lambda}$ than with the $\lambda = 0$ path, makes it prefer lower risk in the current period as well, due to the constraint $\alpha_{it} \geq \beta\alpha_{it-1}$. In periods that this constraint is binding, the aim to have lower future risk necessitates lower current risk.

It is possible to choose $\widehat{\lambda} > 0$ such that the trade-off is unambiguously dominated by the latter argument. We reiterate that this says nothing of the optimality of such paths from the perspective of monetary policy, since we are not deriving the optimal response here.

A proof of existence thus suffices. Take a $\widehat{\lambda}$ such that at $t = 1$, $r_1^f\Big|_{\lambda=\widehat{\lambda}} = r_t^f\Big|_{\lambda=0} - \epsilon$ and subsequently $r_t^f\Big|_{\lambda=\widehat{\lambda}} = \overline{r}_t^f$ from $t = 2$ onwards. One can take ϵ arbitrarily small such that instantaneous (period 1) gain in $P_{i1}(\cdot)$ from a higher α_{it} is offset by the losses in $P_{i1}(\cdot)$ for $t \in (2, t')$ (even just at $t = 2$ suffices). Hence $\widehat{\lambda} > 0$ can be defined such that $\alpha_{it}|_{\lambda=\widehat{\lambda}} < \alpha_{it}|_{\lambda=0}\, \forall t$. ■

Appendix D: A parametric example

The point we make in this paper is about the dynamic response on the way to equilibrium. In order to demonstrate this in a parameterized example, therefore, we require a minimum of 2 periods in order to show the inter-temporal responses of the Central Bank. In this extension we thus solve analytically for the optimal interest rate paths of the $\rho = 0$ (traditional objectives) and $\rho > 0$ (financial stability concerned) Central Banks, and compare these, showing that it matches our main result. Using a parametric example helps to highlight the mechanisms behind our model, and Proposition 1 in particular.

The IS curve for the two periods is:

$$\begin{aligned} y_1 &= -\gamma r_1^f + \phi\varepsilon_1 \\ y_2 &= -\gamma\left(r_2^f + \theta r_1^f\right) + \phi\varepsilon_2, \end{aligned}$$

with $\theta, \gamma, \phi \in (0,1)$. That is, period 1 output gap, y_1, is affected by the contemporaneous interest rate, while period 2's output gap is also affected by the lagged interest rate of period 1. Moreover, in our model we investigate the effects of a single shock. Starting from equilibrium, the shock occurs in $t = 1$, which carries over to period 2 through persistence:

$$\varepsilon_2 = \phi\varepsilon_1,$$

and in turn

$$y_1 = -\gamma r_1^f + \phi\varepsilon_1 \tag{16}$$

$$y_2 = -\gamma\left(r_2^f + \theta r_1^f\right) + \phi^2\varepsilon_1. \tag{17}$$

The evolution of bank risk taking is given by

$$\alpha_1 = \max\left\{A + \tau y_1, \beta\alpha_0\right\} \tag{18}$$

$$\alpha_2 = \max\left\{A + \tau y_2, \beta\alpha_1\right\}, \tag{19}$$

where $\tau \in (0,1)$. Here τy_2 captures axiom 2 on the procyclicality of risk. Moreover, the constant term A relates to axiom 1, which says that bank risk taking α_t is always larger than the social optimum, α^w. Without a constant term, we would have that at steady state $(y_t = 0)$ α_t must be zero since the constraint $\beta\alpha_{t-1}$ ceases to bind over time.

The optimization problem of the Central Bank is:

$$\min_{r_1^f, r_2^f}\{L\} = \min_{r_1^f, r_2^f}\left\{(1-\rho)\left(y_1^2 + y_2^2\right) + \rho\left(\alpha_1 + \alpha_2\right)\right\}. \tag{20}$$

The $\rho = 0$ authority

For the $\rho = 0$ authority, the optimization yields the following FOCs for r_1^f and r_2^f:

$$\begin{aligned} \frac{\partial\left(y_1^2+y_2^2\right)}{\partial r_1^f} &= 0 \\ \frac{\partial\left(y_1^2+y_2^2\right)}{\partial r_2^f} &= 0 \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} \partial\frac{\left(-\gamma r_1^f+\phi\varepsilon_1\right)^2+\left(-\gamma\left(r_2^f+\theta r_1^f\right)+\phi^2\varepsilon_1\right)^2}{\partial r_1^f} &= 0 \\ \partial\frac{\left(-\gamma r_1^f+\phi\varepsilon_1\right)^2+\left(-\gamma\left(r_2^f+\theta r_1^f\right)+\phi^2\varepsilon_1\right)^2}{\partial r_2^f} &= 0 \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} r_1^f &= \frac{\phi\varepsilon_1+\theta\phi^2\varepsilon_1}{\gamma\left(1+\theta^2\right)}-\frac{\theta}{1+\theta^2}r_2^f \\ r_2^f &= \frac{\phi^2}{\gamma}\varepsilon_1-\theta r_1^f, \end{aligned}$$

and solving for these two equations in the two variables gives:

$$\left(r_1^f\right)^* = \frac{\phi}{\gamma}\varepsilon_1 \tag{21}$$

$$\left(r_2^f\right)^* = \frac{\phi^2}{\gamma}\varepsilon_1-\frac{\theta\phi}{\gamma}\varepsilon_1=\frac{\phi\left(\phi-\theta\right)}{\gamma}\varepsilon_1. \tag{22}$$

Replacing the optimal interest rates into equations (16) and (17) gives

$$\begin{aligned} y_1^* &= 0 \\ y_2^* &= 0, \end{aligned}$$

which is not surprising since output gap stabilization is the only objective, and the Central Bank has full controllability.

Note that the steady state interest rate here is simply $\overline{r}^f = 0$ which can be seen from implementing $\varepsilon_1 = 0$ in the optimal interest rate equation at the last period.[9]

[9] Note also that if $\phi < \theta$ then equations (21) and (22) imply that in response to a negative shock ($\varepsilon_1 < 0$) the Central Bank first cuts rates and then in the second period actually raises them above the steady state rate. This too is not surprising, however, because $\phi < \theta$ means that the effect on the output gap of the shocks is less persistent than the effect of the lagged interest rate, so that the Central Bank compensates for the lagged interest rate effect more than it compensates for the lagged shock effect.

The $\rho > 0$ authority

Our aim is to match the parametric example to our general-form model. In Proposition 1 of our paper we consider that the risk persistence constraint is binding for at least one period. Here, in our two period example, we then need to have that the risk constraint $\alpha_t \geq \beta\alpha_{t-1}$ binds in the first period. In other words we need to assume that:

$$\beta\alpha_0 > A, \tag{23}$$

while

$$\beta^2\alpha_0 = \beta\alpha_1 < A, \tag{24}$$

where α_0 is the given initial level of bank risk exposure.

Admittedly conditions (23) and (24) are a bit of a knife-edge in a two-period case, but one can see how they would apply more generally in a setting with more periods: initially the constraint would bind, while it would cease to at some later point as risk is unwound (see the discussion on axiom 3).

The optimization of the $\rho > 0$ authority is then:

$$\min_{r_1^f, r_2^f} \{L\} = \min_{r_1^f, r_2^f} \left\{ \begin{array}{c} (1-\rho)\left(\left(-\gamma r_1^f + \phi\varepsilon_1\right)^2 + \left(-\gamma\left(r_2^f + \theta r_1^f\right) + \phi^2\varepsilon_1\right)^2\right) \\ +\rho\left((\beta\alpha_0) + \left(A + \tau\left[-\gamma\left(r_2^f + \theta r_1^f\right) + \phi^2\varepsilon_1\right]\right)\right), \end{array} \right\} \tag{25}$$

which gives FOCs

$$\begin{aligned} -2\gamma(1-\rho)\left(-\gamma r_1^f + \phi\varepsilon_1\right) - 2\gamma\theta(1-\rho)\left(-\gamma\left(r_2^f + \theta r_1^f\right) + \phi^2\varepsilon_1\right) - \tau\rho\gamma\theta &= 0 \\ -2\gamma(1-\rho)\left(-\gamma\left(r_2^f + \theta r_1^f\right) + \phi^2\varepsilon_1\right) - \tau\rho\gamma &= 0 \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} r_1^f &= \frac{\tau\rho\theta}{2\gamma\left(1+\theta^2\right)(1-\rho)} + \frac{\phi(1+\theta\phi)}{\gamma\left(1+\theta^2\right)}\varepsilon_1 - \frac{\theta}{\left(1+\theta^2\right)}r_2^f \\ r_2^f &= \frac{\tau\rho}{2\gamma(1-\rho)} + \frac{\phi^2}{\gamma}\varepsilon_1 - \theta r_1^f, \end{aligned}$$

and replacing between these equations gives

$$\left(r_1^f\right)^* = \frac{\phi}{\gamma}\varepsilon_1 \tag{26}$$

$$\left(r_2^f\right)^* = \frac{\rho\tau}{2\gamma(1-\rho)} + \frac{\phi(\phi-\theta)}{\gamma}\varepsilon_1. \tag{27}$$

In equilibrium there are no shocks and therefore the steady-state interest rate is

$$\overline{r}^f = \frac{\rho\tau}{2\gamma(1-\rho)}.$$

Comparison

We now compare the dynamic response for the two different Central Banks. Take any $\varepsilon_1 < 0$ (negative shock), then

$$\begin{aligned} \left.\left(\left(r_1^f\right)^* - \overline{r}^f\right)\right|_{\rho=0} &= \frac{\phi}{\gamma}\varepsilon_1 - 0 < 0 \text{ (because } \varepsilon_1 < 0\text{)} \\ \left.\left(\left(r_1^f\right)^* - \overline{r}^f\right)\right|_{\rho>0} &= \frac{\phi}{\gamma}\varepsilon_1 - \frac{\rho\tau}{2\gamma(1-\rho)} < 0, \end{aligned}$$

and, importantly,

$$\left.\left|\left(r_1^f\right)^* - \overline{r}^f\right|\right|_{\rho>0} - \left.\left|\left(r_1^f\right)^* - \overline{r}^f\right|\right|_{\rho=0} = \frac{\rho\tau}{2\gamma(1-\rho)}, \tag{28}$$

so that compared to the steady state rate, the initial rate cut is deeper for the $\rho > 0$ authority than for the $\rho = 0$ authority.

We can also depict this graphically. Let us say, for visual simplicity, that $\varepsilon_3 = 0$ (the shock lasts only two periods - then return to steady state). And let us take $\varepsilon_1 = -1$, $\phi = 0.5$, $\gamma = 0.5$, $\rho = 0.5$, $\tau = 0.5$, $\theta = 0.25$ (this is just an illustrative example which can be generalized for any parameterization through (28)). The two interest rate paths are now:

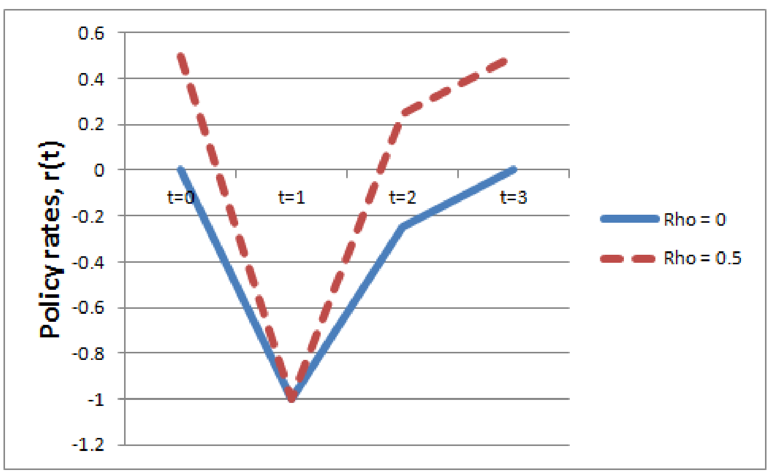

Figure 3: Parameterized example: optimal rates

And in terms of deviations from their steady state:

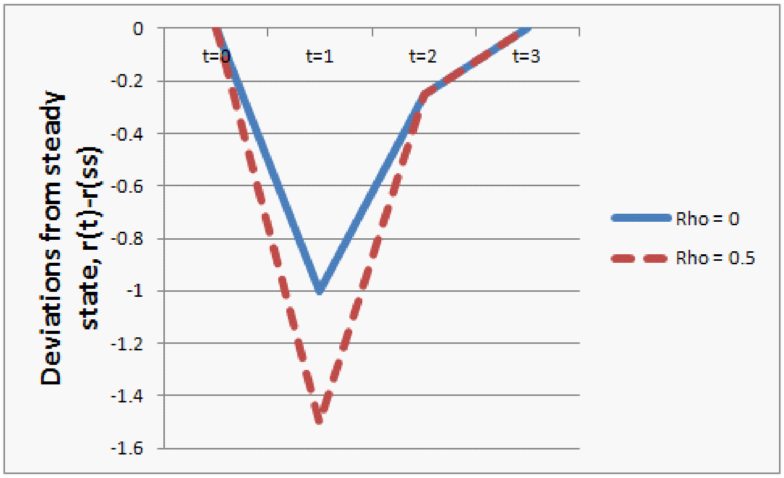

Figure 4: Parameterized example: deviations from the steady state

Thus, in this parametric example optimal interest rates behave as described in our general-form model: the authority that cares about bank risk will implement a deeper cut and then raise rates back quickly.

References

Acharya, Viral V., and Hassan Naqvi, 2012, "The Seeds of a Crisis: A Theory of Bank Liquidity and Risk-Taking over the Business Cycle," *Journal of Financial Economics*, Vol. 106(2), pp. 349-366.

Adrian, Tobias, and Hyun Song Shin, 2008, "Financial Intermediaries, Financial Stability, and Monetary Policy," Federal Reserve Bank of New York Staff Report No. 346 (New York: Federal Reserve Bank).

Adrian, Tobias, and Hyun Song Shin, 2009, "Money, Liquidity and Monetary Policy," *American Economic Review: Papers and Proceedings*, pp. 600-605.

Adrian, Tobias, and Hyun Song Shin, 2010, "Financial Intermediaries and Monetary Economics," in the *Handbook of Monetary Economics* 3A, Ch. 12, eds. Benjamin Friedman and Michael Woodford, pp. 601-650 (Amsterdam: Elsevier).

Agur, Itai and Maria Demertzis, 2012, "Excessive Bank Risk-Taking and Monetary Policy," ECB Working Paper No. 1457 (Frankfurt: European Central Bank).

Agur, Itai and Sunil Sharma, 2013, "Rules, Discretion, and Macro-Prudential Policy," IMF Working Paper No. 13/65 (Washington: International Monetary Fund).

Allen, Franklin, Ana Babus, and Elena Carletti, 2009, "Financial Crises: Theory and Evidence," *Annual Review of Financial Economics,* Vol. 1, pp. 97-116.

Altunbas, Yener, Leonardo Gambacorta, and David Marquez-Ibanez, 2010, "Bank Risk and Monetary Policy," *Journal of Financial Stability,* Vol. 6, pp. 121-129.

Angeloni, Ignazio and Ester Faia, 2013, "Capital Regulation and Monetary Policy with Fragile Banks," *Journal of Monetary Economics*, forthcoming.

Angeloni, Ignazio, Ester Faia and Marco Lo Duca, 2013, "Monetary Policy and Risk Taking, mimeo (Frankfurt: European Central Bank and Goethe University).

Bernanke, Ben S., Mark Gertler and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. B. Taylor and M. Woodford (eds.) *Handbook of Macroeconomics*, vol. 1., (Amsterdam: Elsevier).

Borio, Claudio and William White, 2004, "Whither Monetary Policy and Financial Stability? The Implications of Evolving Policy Regimes," BIS Working Paper No. 147 (Basel: Bank for International Settlements).

Borio, Claudio and Haibin Zhu, 2012, "Capital Regulation, Risk-Taking and Monetary Policy: a Missing Link in the Transmission Mechanism," *Journal of Financial Stability,* Vol. 8, pp. 236-251.

Brunnermeier, Markus K., 2009, "Deciphering the Liquidity and Credit Crunch 2007-2008," *Journal of Economic Perspectives,* Vol. 23, pp. 77-100.

Brunnermeier, Markus K., Andrew Crockett, Charles Goodhart, Avinash D. Persaud and Hyun Song Shin, 2009, *The Fundamental Principles of Financial Regulation,* Geneva Reports on the World Economy 11 (Geneva).

Buch, Claudia M., Sandra Eickmeier and Esteban Prieto, 2010, "Macroeconomic Factors and Micro-Level Bank Risk," CESifo Working Paper No. 3194 (Munich: Ifo Center for Economic Studies).

Calomiris, Charles W., 2009, "The Subprime Crisis: What's Old, What's New, and What's Next?," mimeo (New York: Columbia University).

Cociuba, Simona E., Malik Shukayev and Alexander Ueberfeldt, 2012, "Collateralized Borrowing and Risk Taking at Low Interest Rates," mimeo (Bank of Canada and University of Western Ontario).

Delis, Manthos D., and Sophocles N. Brissimis, 2010, "Bank Heterogeneity and Monetary Policy Transmission," ECB Working Paper No. 1233 (Frankfurt: European Central Bank).

Delis, Manthos D., and Georgios P. Kouretas, 2011, "Interest Rates and Bank Risk Taking," *Journal of Banking and Finance*, Vol. 35(4), pp. 840-855.

Delis, Manthos D., Iftekhar Hasan, Nikolaos Mylonidis, 2011, "The Risk-Taking Channel of Monetary Policy in the USA: Evidence from Micro-Level Data," MPRA Paper No. 34084 (Munich: University Library of Munich).

Dell'Ariccia, Giovanni and Robert Marquez, 2006, "Lending Booms and Lending Standards," *Journal of Finance,* Vol. 61(5), pp. 2511-2546.

Dell'Ariccia, Giovanni, Deniz Igan and Luc Laeven, 2008, "Credit Booms and Lending Standards: Evidence From the Subprime Mortgage Market," IMF Working Paper No. 08/106 (Washington: International Monetary Fund).

Dell'Ariccia, Giovanni, Luc Laeven, and Robert Marquez, 2013, "Monetary Policy, Leverage, and Bank Risk-Taking," *Journal of Economic Theory,* forthcoming.

Dell'Ariccia, Giovanni, Luc Laeven, and Gustavo Suarez, 2013, "Bank Leverage and Monetary Policy's Risk-Taking Channel: Evidence from the United States," mimeo.

Diamond, Douglas W., and Raghuram G. Rajan, 2009, "The Credit Crisis: Conjectures about Causes and Remedies," *American Economic Review, Papers and Proceedings,* pp. 606-610.

Diamond, Douglas W., and Raghuram G. Rajan, 2012, "Illiquid banks, Financial Stability, and Interest Rate Policy," *Journal of Political Economy,* Vol. 120(3), pp. 552-591.

Disyatat, Piti, 2010, "Inflation Targeting, Asset Prices and Financial Imbalances: Contextualizing the Debate," *Journal of Financial Stability,* Vol. 6(3), pp. 145-155.

Drees, Burkhard, Bernhard Eckwert and Felix Várdy, 2010, "Cheap Money and Risk Taking: Opacity versus Underlying Risk," mimeo.

Drumond, Ines, 2009, "Bank Capital Requirements, Business Cycle Fluctuations and the Basel Accords: A Synthesis," *Journal of Economic Surveys,* Vol. 23(5), 798-830.

Dubecq, Simon, Benoit Mojon and Xavier Ragot, 2009, "Fuzzy Capital Requirements, Risk-Shifting and the Risk-Taking Channel of Monetary Policy," Banque de France Working Paper No. 254 (Paris: Banque de France).

Farhi, Emmanuel and Jean Tirole, 2012, "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, Vol. 102(1), pp. 60-93.

Freixas, Xavier, and Jean-Charles Rochet, 1997, *The Microeconomics of Banking*, (Boston: MIT Press).

Gertler, Mark and Nobuhiro Kiyotaki, 2010, "Financial Intermediation and Credit Policy in Business Cycle Dynamics," in the *Handbook of Monetary Economics* 3, eds. Benjamin Friedman and Michael Woodford (Amsterdam: Elsevier).

Gertler, Mark and Peter Karadi, 2011, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, Vol. 58(1), pp. 17-34.

Goodhart, Charles A.E., Carolina Osorio and Dimitrios P. Tsomocos, 2009, "Analysis of Monetary Policy and Financial Stability: A New Paradigm," CESifo Working Paper No. 2885 (Munich: Ifo Center for Economic Studies).

Ioannidou, Vasso, Steven Ongena and José-Luis Peydro, 2009, "Monetary Policy, Risk Taking and Pricing: Evidence From a Quasi-Natural Experiment," European Banking Center Discussion Paper 2009-04S (Tilburg: European Banking Center).

Jiménez, Gabriel, Steven Ongena, José-Luis Peydro and Jesús Saurina, 2009, "Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say about the Effects of Monetary Policy on Credit Risk-Taking?" Banco de Espana Working Paper No. 833 (Madrid: Banco de Espana).

Kannan, Prakash, Pau Rabanal, and Alasdair Scott, 2009, "Macroeconomic Patterns and Monetary Policy in the Run-Up to Asset Price Busts," IMF Working Paper No. 09/252 (Washington: International Monetary Fund).

Lerner, Josh and Peter Tufano, 2011, "The Consequences of Financial Innovation: A Counterfactual Research Agenda," NBER Working Paper No. 16780 (Cambridge, MA: National Bureau of Economic Research).

Loisel, Olivier, Aude Pommeret and Franck Portier, 2012, "Monetary Policy and Herd Behavior: Leaning Against Bubbles," Banque de France Working Paper No. 412 (Paris: Banque de France).

Maddaloni, Angela and José-Luis Peydro, 2011, "Bank Risk-Taking, Securitization, Supervision, and Low Interest-Rates: Evidence from the Euro Area and US Lending Standards," *Review of Financial Studies*, Vol. 24, pp. 2121-65.

Paligorova, Teodora and João Santos, 2012, "When is it Less Costly for Risky Firms to Borrow? Evidence from the Bank Risk-Taking Channel of Monetary Policy," Bank of Canada Working Paper No. 2012-10 (Ottawa: Bank of Canada).

Rajan, Raghuram G., 2006, "Has Finance Made the World Riskier?" *European Financial Management*, Vol. 12(4), pp. 313-369.

Tufano, Peter, 2003, "Financial Innovation," Chapter 6 of the *Handbook of the Economics of Finance*, Vol. 1, Part 1, pp. 307-335 (Amsterdam: Elsevier).

Valencia, Fabian, 2011, "Monetary Policy, Bank Leverage, and Financial Stability," IMF Working Paper No. 11/244 (Washington: International Monetary Fund).