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Systemic Contingent Claims Analysis – Estimating Market-Implied Systemic Risk

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Systemic Contingent Claims Analysis – Estimating Market-Implied Systemic Risk

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Abstract

The recent global financial crisis has forced a re-examination of risk transmission in the financial sector and how it affects financial stability. Current macroprudential policy and surveillance (MPS) efforts are aimed establishing a regulatory framework that helps mitigate the risk from systemic linkages with a view towards enhancing the resilience of the financial sector. This paper presents a forward-looking framework ("Systemic CCA") to measure systemic solvency risk based on market-implied expected losses of financial institutions with practical applications for the financial sector risk management and the system-wide capital assessment in top-down stress testing. The suggested approach uses advanced contingent claims analysis (CCA) to generate aggregate estimates of the joint default risk of multiple institutions as a conditional tail expectation using multivariate extreme value theory (EVT). In addition, the framework also helps quantify the individual contributions to systemic risk and contingent liabilities of the financial sector during times of stress.

JEL Classification Numbers: C46, C51, G01, G13, G21, G28.

Keywords: macroprudential policy and surveillance, contingent claims analysis (CCA), systemic CCA, systemic risk, conditional tail expectation (CTE), contingent liabilities, extreme value theory (EVT), risk-adjusted balance sheets, stress testing.

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I. INTRODUCTION[2]

In the wake of the global financial crisis, there has been increased focus on systemic risk as a key aspect of macroprudential policy and surveillance (MPS) with a view towards enhancing the resilience of the financial sector.[3] MPS is predicated on (i) the assessment of system-wide vulnerabilities and the accurate identification of threats arising from the build-up and unwinding of financial imbalances, (ii) shared exposures to macro-financial shocks, and (iii) possible contagion/spillover effects from individual institutions and markets due to direct or indirect connectedness. Thus, it aims to limit, mitigate or reduce systemic risk, thereby minimizing the incidence and impact of disruptions in the provision of key financial services that can have adverse consequences for the real economy (and broader implications for economic growth).[4,5] Systemic risk refers to individual or collective financial

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[2] Technical elements of this model have been applied within the stress testing exercise of the *Financial Sector Assessment Program* (FSAP) for Germany, Spain, Sweden, the United Kingdom, and the United States between 2010 and 2012, the Global Financial Stability Report (IMF, 2009a and 2009b), as well as the financial stability analyses in the context of bilateral surveillance. An earlier and abridged version of this paper was published as "New Directions in Financial Sector and Sovereign Risk Management" in the *Journal of Investment Management* (Gray and Jobst, 2010a). The authors are grateful to Laura Kodres, Chris Towe, Robert C. Merton, Robert Engle, Zvi Bodie, Samuel Malone, Mikhail Oet, and Andrea Maechler for useful comments and helpful suggestions. We also thank participants at the conference on "Beyond the Financial Crisis: Systemic Risk, Spillovers and Regulation" (28-29 October 2010, Technische Universität Dresden, Germany), the "Banking Law Symposium on Managing Systemic Risk" (7-9 April 2010, University of Warwick, U.K.), and at seminar presentations at the U.S. Federal Board of Governors, the U.K. Financial Services Authority (FSA), and the Deutsche Bundesbank for their feedback.

[3] In a recent progress report to the G-20 (FSB/IMF/BIS, 2011b), which followed an earlier update on macroprudential policies (FSB/IMF/BIS, 2011a), the FSB takes stock of the development of governance structures that facilitate the identification and monitoring of systemic financial risk as well as the designation and calibration of instruments for macroprudential purposes aimed at limiting systemic risk. While the report acknowledges considerable progress in the conduct of macroprudential policy, the report finds that there is still much scope for systemic risk regulation and institutional arrangements for the conduct of policy. Note that similar efforts in the banking sector are more advanced. The CGFS (2012) recently published a report on operationalizing the selection and application of macroprudential policies, which provides guidance on the effectiveness and timing of banking sector-related instruments (affecting the treatment of capital, liquidity and assets for the purposes of mitigating the cyclical impact of shocks and enhancing system-wide resilience to joint distress events). For a brief summary of the scope of MPS, see Nier and others (2011) as well as Jácome and Nier (2012). See Acharya and others (2010a and 2010b) for the implications of systemic risk in MPS in the U.S. context.

[4] Such risk to financial stability arises from fault lines in the architecture of the financial system, for instance between banking and non-banking financial sector activities, and the collective impact of common shocks on a material number of financial institutions, possibly amplified by market failures.

[5] The traditional approach to financial stability analysis concentrates analytical efforts on the identification of vulnerabilities prior to stress from individual failures, assuming that the financial system is in equilibrium and adjusts when it experiences a shock. As opposed to this conventional approach, the tenet of MPS centers on monitoring the build-up of systemic vulnerabilities in areas where the impact of disruptions to financial stability is deemed most severe and wide-spread – and especially in areas of economic significance to both the financial sector and the real economy.

arrangements—both institutional and market-based—that could either lead directly to system-wide distress in the financial sector and/or significantly amplify its consequences (with adverse effects on other sectors, in particular capital formation in the real economy). Typically, such distress manifests itself in disruptions to the flow of financial services due to an impairment of all or parts of the financial system that are deemed material to the functioning of the real economy.[6,7]

Current policy efforts are geared towards establishing a regulatory framework that includes market-wide perspective of supervision rather than being concerned with the viability of individual institutions only. The comprehensive assessment of systemic risk has resulted in a multi-faceted array of complementary measures in areas of regulatory policies, supervisory scope, and resolution arrangements aimed at mitigating system-wide vulnerabilities while avoiding impairment to efficient activities that do not cause and/or amplify stress in any meaningful manner. Its successful implementation depends on the quality of surveillance activities/analytical tools, institutional strength of the supervisory measures, effectiveness of policy instruments (including the persuasiveness of recommendations). Some measures include more stringent prudential standards,[8] such as limits on leverage and higher capital requirements as a way to limit the scale and scope of banking activities, a broader adoption of contingent capital initiatives (including the adoption of mandatory debt-to-equity swaps as part of "bail-in" provisions), designing "living wills", strengthening resolution processes for large complex financial institutions (LCFIs), in combination with the establishment of a specialized macro-prudential supervisor of systemically important entities, such as the Financial Stability Oversight Council (FSOC) in the United States, the European Systemic Risk Board (ESRB), and the Financial Policy Committee (FPC) in the United Kingdom.[9]

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[6] Impairment to the flow of financial services occurs where certain financial services are temporarily unavailable, as well as situations where the cost of obtaining the financial services is sharply increased. It would include disruptions due to shocks originating outside the financial system that have an impact on it, as well as shocks originating from within the financial system. These disruptions due to shocks originating outside and within the financial system generate externalities on economic activity or processes that affect those that are not directly involved.

[7] For a discussion of ways to assess the systemic importance of financial institutions and markets see FSB/IMF/BIS (2009).

[8] This approach involves reducing the size and business activities of large financial institutions and providing incentives for downsizing via capital and liquidity requirements in order to lessen potentially systemic linkages.

[9] These efforts have also been accompanied by the development of criteria for the identification of systemically important jurisdictions for purposes of prioritization in policy discussions on global financial stability. In this regard, the FSB, IMF and BIS (2011a and 2011b) have identified data gaps (IMF/FSB, 2009 and 2010) and proposed—as possible measures guiding MPS—several macroeconomic/financial sector indicators and the degree of adherence to international cooperation and information exchange standards, such as the compliance grades from IMF-World Bank detailed assessments of observance of relevant principles within the Basel

(continued)

Table 1. General Systemic Risk Measurement Approaches.

	Contribution approach ("Risk Agitation")	**Participation approach** ("Risk Amplification")
Concept	systemic resilience to individual failure	individual relience to common shock
Description	a contribution to systemic risk conditional on individual failure due to knock-on effect	expected loss from systemic event due to common exposure and risk concentration
Risk transmission	"institution-to-institution"	"institution-to-aggregate"
Risk indicators	economic significance of asset holdings ("size")	claims on other financial sector participants (credit exposure)
	intra- and inter-system liabilities ("connectedness")	market risk exposure (interest rates, credit spreads, currencies)
	degree of transparency and resolvability ("complexity")	risk-bearing capacity (solvency and liquidity buffers, leverage)
	participation in system-critical function/service, e.g., payment and settlement system ("substitutability")	economic significance of asset holdings, maturity mismatches debt pressure ("asset liquidation")
Policy objectives	avoid/mitigate contagion effect (by containing systemic impact upon failure)	maintain overall functioning of system and maximize survivorship
	avoid moral hazard	preserve mechanisms of collective burden sharing

Sources: Tarashev and others (2009), Drehmann and Tarashev (2011), FSB (2011), Weistroffer (2011), and Jobst (2012).
Note: The policy objectives and different indicators to measure systemic risk under both contribution and participation approaches are not exclusive to each concept. Moreover, the availability of certain types of balance sheet information and/or market data underpinning the various risk indicators varies between different groups of financial institutions, which requires a certain degree of customization of the measurement approach to the distinct characteristics of a particular group of financial institutions.

While there is still no comprehensive theory of MPS related to the measurement of systemic risk, existing approaches can be broadly distinguished based on their conceptual underpinnings regarding several core principles. There are two general approaches: (i) a particular activity causes a firm to fail, whose importance to the system imposes marginal distress on the system due to the nature, scope, size, scale, concentration, or connectedness of its activities with other financial institutions (“contribution approach”), or (ii) a firm experiences losses from a single (or multiple) large shock(s) due a significant exposure to the commonly affected sector, country and/or currency (“participation

Committee on Banking Supervision (BCBS), International Association of Insurance Supervisors (IAIS), and the International Organization of Securities Commissions (IOSCO) standards.

approach").[10] In the case of the former, the contribution to systemic risk arises from the initial effect of direct exposures to the failing institution (e.g., defaults on liabilities to counterparties, investors, or other market participants), which could also spillover to previously unrelated institutions and markets as a result of greater uncertainty or the reassessment of financial risk (i.e., changes in risk appetite and/or the market price of risk). For instance, leverage and maturity mismatches amplify the potential of material financial distress, if the sudden disposal of large asset positions of an institution significantly disrupts trading and/or causes significant losses for other firms with similar holdings due to increases in asset and funding liquidity risk. In this case, the participation in systemic risk occurs via an institution's common exposures to certain asset classes, industry sectors, and markets. Such indirect linkages can affect systemic risk if these exposures are significant enough to cause either material impairment of other financial institutions (by threatening their financial condition and/or competitive position) or disruptions to critical functions of the sector and/or financial system. Table 1 above summarizes the distinguishing features of both approaches.

The growing literature on systemic risk measurement has responded to greater demand placed on the ability to develop a better understanding of the interlinkages between firms and their implications for financial stability (see Table 2 and Appendix 5). The specification of dependence between firms helps identify common vulnerabilities to risks as a result of the (assumed) collective behavior under common distress. Most of the prominent institution-level measurement approaches, such as CoVaR (Adrian and Brunnermeier, 2008), CoRisk (Chan-Lau, 2010), Systemic Expected Shortfall (SES) (Acharya and others, 2009, 2010, and 2012) (as well as extensions thereof, such as the Distress Insurance Premium (DIP) by Huang and others (2009 and 2010)), Granger Causality (Billio and others, 2010), SRISK (Brownlees and Engle, 2011), and the Joint Probability of Distress (Segoviano and Goodhart, 2009), have focused on the "contribution approach" by including an implicit or explicit treatment of statistical dependence in determining joint default risk or expected losses.[11]

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[10] Drehmann and Tarashev (2011) refer to this as a "bottom-up approach", whereas as a "top-down approach" would be predicated on the quantification of expected losses of the system, with and without a particular institution being part of it, which determines the institution's marginal contribution to systemic risk.

[11] There are also several approaches that specifically address dependence based on changes in statistical properties of time series data. For instance, Kritzman and others (2010) define the absorption ratio (AR) as a measure of systemic risk based on eigenvalue decomposition of the covariance matrix for time series data using principal components analysis (PCA). Since eigenvalues represent the share of the total variance that is taken up by each eigenvector, a relatively small number of eigenvalues that are disproportionally large indicates that the time series are closely aligned and move together. The AR is defined as the fraction of the total variance of a set of time series explained or "absorbed" by a fixed number of eigenvectors. Other examples include Kritzman and Yaunzhen (2010) as well as Billio and others (2010). See IMF (2009a and 2009b), Bisias and others (2012), and Markeloff and others (2012) for an overview of different systemic risk measurement approaches. See De Brandt and Hartmann (2000) for an early account of systemic risk measures prior to the start of the financial crisis in 2007. For an overview of empirical approaches to measuring systemic liquidity risk, see IMF (2010a and 2011a).

Table 2. Selected Institution-Level Systemic Risk Models.

	Conditional Value-at-Risk (CoVaR)	Conditional Risk (CoRisk)	SRISK	Systemic Expected Shortfall (SES)	Distress Insurance Premium (DIP)	Joint Probability of Default (JPoD)	Systemic Contingent Claims Analysis (CCA)
Systemic Risk Measure	Value-at-Risk	Value-at-Risk	Expected Shortfall	Expected Shortfall	Expected Shortfall	conditional probabilities	Expected Shortfall
Conditionality	percentile of individual return	percentile of joint default risk	threshold of capital adequacy	threshold of capital adequacy	percentage threshold of system return	various (individual or joint expected losses)	various (individual or joint expected losses)
Dimensionality	multivariate	bivariate	bivariate	bivariate	bivariate	multivariate	multivariate
Dependence Measure	linear, parametric	linear, parametric	parametric	empirical	parametric	non-linear, non-parametric	non-linear, non-parametric
Method	panel quantile regression	bivariate quantile regression	dynamic conditional correlation (DCC GARCH) and Monte Carlo simulation	empirical sampling and scaling; Gaussian and power law	dynamic conditional correlation (DCC GARCH) and Monte Carlo simulation	empirical copula	empirical copula
Data Source	equity prices and balance sheet information	CDS spreads	equity prices and balance sheet information	equity prices and balance sheet information	equity prices and CDS spreads	CDS spreads	equity prices and balance sheet information
Data Input	quasi-asset returns	CDS-implied default probabilities	quasi-asset returns	quasi-asset returns	equity returns and CDS-implied default probabilities	CDS-implied default probabilities	expected losses ("implicit put option")
Reference	Adrian and Brunnermeier (2008)	Chan-Lau (2010)	Brownlees and Engle (2011)	Acharya and others (2009, 2010 and 2012)	Huang and others (2009 and 2010)	Segoviano and Goodhart (2009)	Gray and Jobst (2010 and 2011)

A chapter of a recent issue of the Global Financial Stability Report (GFSR) provides comparative analysis of several of these systemic risk measures in the context of designing early warning indicators for MPS (IMF, 2011g).[12] There are also several studies on network analysis and agent-based models that are more closely related to the "participation approach" by modeling how inter-linked asset holdings matter in the generation and propagation of systemic risk (Allen and others 2011; Espinosa-Vega and Solé, 2011; OECD, 2012). Haldane and Nelson (2012) underscore this observation by arguing that networks can produce non-linearity and unpredictability with the attendant extreme (or fat-tailed) events.[13]

However, only a few of these models estimate multivariate (firm-by-firm) dependence through either a closed-form specification or the simulation of joint probabilities using historically informed measures of association, and none of those include a structural definition of default risk. Against this background, we propose a forward-looking framework for the quantification of systemic risk from market-implied interlinkages between financial institutions in order to fill this gap in the existing literature. The suggested approach ("Systemic Contingent Claims Analysis," or in short "Systemic CCA") extends the risk-adjusted (or economic) balance sheet approach to generate estimates of the joint default risk of multiple institutions as a measure of systemic risk. Under this approach, the magnitude of systemic risk depends on the firms' size and interconnectedness and is defined by the multivariate density of combined expected losses within a given system of financial institutions.

Systemic CCA identifies endogenous linkages affecting joint expected losses during times of stress, which can deliver important insights about the joint tail risk of multiple entities. A sample of firms (as proxy for an entire financial system, or parts thereof) is viewed as a portfolio of individual expected losses (with individual risk parameters) whose sensitivity to common risk factors is accounted for by including the statistical dependence of their individual expected losses ("dependence structure").[14] Like other academic proposals,

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[12] A comparison of three measures of institution-level systemic risk exposure (Sedunov, 2012) shows that CoVaR shows good forecasting power of the within-crisis performance of financial institutions during two systemic crisis periods marked by the collapse of large firms (LTCM in 1998 and Lehman Brothers in 2008). In contrast, his findings suggest that SES and Granger Causality do not seem to forecast the performance of firm performance reliably during crises.

[13] One empirical example of the network literature is based on a Federal Reserve (Fed) data set, which allowed for the mapping of bilateral exposures of 22 global banks that accessed Fed emergency loans in the period 2008 to 2010 (Battiston and others, 2012). The authors find that size is not a relevant factor to determine systemic importance.

[14] In this context, risk parameters refer to components of the default risk model, which comprise the implied asset value, the volatility of asset returns, and the debt service obligation over a specified time horizon.

this approach helps assess individual firms' contributions to systemic solvency risk (at different levels of statistical confidence). However, by accounting for the time-varying dependence structure, this method links the market-based assessment of each firm's risk profile with the risk characteristics of other firms that are subject to common changes in market conditions. Based on the expected losses arising from the variation of each individual firm's expected losses, the joint probability of all firms experiencing distress simultaneously can be estimated.

In addition, the Systemic CCA framework can generate close-form solutions for market-implied estimates of capital adequacy under various stress test scenarios. By modeling how macroeconomic conditions and bank-specific income and loss elements (net interest income, fee income, trading income, operating expenses, and credit losses) have influenced the changes in market-implied expected losses (as measured by implicit put option values), it is possible to link a particular macroeconomic path to financial sector performance in the future and derive estimates of joint capital need to maintain current capital adequacy. As a result, the market-implied joint capital need arises from the amount of expected losses relative to current (core) capital levels as well as its escalation during extreme market stresses at a very high statistical confidence level (expressed as "tail risk").[15]

Since its measure of systemic risk is derived from the joint distribution of individual risk profiles, Systemic CCA addresses the general identification problem of existing approaches. It does not rely on stylized assumptions affecting asset valuation, such as linearity and normally distributed prediction errors, and generates point estimates of systemic risk without the need of re-estimation for different levels of desired statistical confidence. None of the recently published systemic risk models, such as Adrian and Brunnermeier (2008), Acharya and others (2009, 2010, and 2012), and Huang and others (2009 and 2010), applies multivariate density estimation, which allows the determination of the marginal contribution of an individual institution to concurrent changes of both the severity of systemic risk and the dependence structure across any combination of sample institutions for *any* level of statistical confidence.

The paper is structured as follows. The next section (Section II) introduces the general concept of the Systemic CCA framework and provides a detailed description of how joint default risk is modeled via a portfolio-based approach using Contingent Claims Analysis (CAA). Section III introduces various extensions to the Systemic CCA framework, including

[15] See also Gray and Jobst (2010b, 2011a, 2011b, and 2011c) as well as Gray and others (2010) for a more general application of this approach to the integrated balance sheets of an entire economy. For first versions of the model, see Gray and Jobst (2009 and 2010a), Gray and others (2010), as well as González-Hermosillo and others (2009). An application of the Systemic CCA approach in the context of FSAP stress tests and spillover analysis are published in IMF (2010b, 2010c, 2011b, 2011c, 2011d, 2011e, 2011f, and 2012). For an earlier application of the methodology underpinning this framework, see also IMF (2009a and 2009b).

the estimation of contingent claims to the financial sector, the design of macroprudential policy measures to mitigate systemic risk, and the assessment of capital adequacy using market-implied measures of system-wide solvency in the context of macroprudential stress testing. Sections IVand V present estimation results of the Systemic CCA framework for the U.S. financial system and the U.K. banking sector as results of the recent *Financial Sector Assessment Program* (FSAP) reviews completed by Fund staff in these countries. The paper concludes with possible extensions to the presented methodology (and its scope of application) and provides some caveats on systemic risk measurement.

II. Methodology

A. Overview

Systemic CCA is a forward-looking, market data-based analytical framework for measuring systemic solvency risk by means of a multivariate extension to contingent claims analysis (CCA) paired with the concept of extreme value theory (EVT).[16] As a logical extension to the individual firm-level analysis using CCA, the magnitude of systemic risk jointly posed by multiple institutions falling into distress is modeled as a portfolio of individual expected losses (with individual risk parameters) calculated from equity market and balance sheet information. The model combines expected losses of individual firms and the dependence between them in order to generate a multivariate distribution that formally captures the potential of extreme realizations of joint expected losses as a combined measure of default risk.

The Systemic CCA framework can be decomposed into two sequential estimation steps in order to measure joint market-implied expected losses as a result of system-wide effects on solvency conditions. First, each firm's expected losses (and associated change in existing capital levels) are estimated using an enhanced form of CCA, which has been widely applied to measure and evaluate credit risk at financial institutions (see Appendix 1).[17] Second, these expected losses are assumed to follow an Generalized Extreme Value (GEV) distribution, which are combined to a multivariate solution by utilizing a novel application of a non-parametric dependence measure in order to derive the amount of joint expected losses

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[16] EVT is a useful statistical concept to study the tail behavior of heavily skewed data, which specifies residual risk at high percentile levels through a generalized parametric estimation of order statistics.

[17] The CCA is a generalization of option pricing theory pioneered by Black and Scholes (1973) and Merton (1973 and 1974). It is based on three principles that are applied in this chapter: (i) the values of liabilities are derived from assets; (ii) assets follow a stochastic process; and (iii) liabilities have different priorities (senior and junior claims). Equity can be modeled as an implicit call option, while risky debt can be modeled as the default-free value of debt less an implicit put option that captures expected losses. In the Systemic CCA model, advance option pricing is applied to account for biases in the Black-Scholes-Merton (BSM) specification.

(and changes in corresponding capital levels) as the multivariate conditional tail expectation (CTE).

Table 3. Main Features of the Systemic CCA Model.

Model Description	The model extends the traditional risk-adjusted balance sheet model (based on *contingent claims analysis* (CCA)) to determine the magnitude of systemic risk from the interlinkages between institutions based on the time-varying likelihood of a joint decline of implied asset values below the debt-driven "default barrier." This approach also helps quantify the contribution of individual institutions to systemic (solvency) risk and assess spillover risks from the financial sector to the public sector (and vice-versa) in the form of contingent liabilities (see Section III.B below).
Treatment of solvency risk	Stochastic capital assessment based on expected losses embedded in equity prices and implied asset value, asset volatility, and debt service obligations within risk horizon; relies on option-pricing models to assess the impact of maturity mismatches and leverage on the assessment of capital adequacy.
Treatment of channels of systemic risk	Estimates the non-linear, non-parametric dependence structure between sample firms so linkages are endogenous to the model and change dynamically.
Application/Data requirements	Model is appropriate in situations where access to prudential data is limited but market information is readily available; minimal use of supervisory data: accounting information (amounts/maturities) of outstanding liabilities; market data on equity and equity option prices; various market rates and macro data for satellite model underpinning the stress test.
Strengths	The model integrates market-implied expected losses (and endogenizes loss-given default (LGD)) in a multivariate specification of joint default risk; approach is highly flexible and can be used (i) to quantify an individual institution's time-varying contribution to systemic solvency risk under normal and stressed conditions, and (ii) serve as a macroprudential tool to price a commensurate systemic risk charge.
Weaknesses	Assumptions are required regarding the specification of the option pricing model (for the determination of implied asset and asset volatility of firms); technique is complex and resource-intensive.

In order to understand individual risk exposures, first, CCA is applied to construct risk-adjusted (economic) balance sheets of financial institutions and estimate their expected losses. In its basic concept, CCA quantifies default risk on the assumption that owners of corporate equity in leveraged firms hold a call option on the firm value after outstanding liabilities have been paid off. So, corporate bond holders effectively write a European put option to equity owners, who hold a residual claim (i.e., call option) on the firm's asset value in non-default states of the world. The pricing of these state-contingent contracts is predicated on the risk-adjusted valuation of the balance sheet of firms whose assets are stochastic and may be above or below promised payments on debt over a specified period of time. When there is a chance of default, the repayment of debt is considered "risky"—to the extent that it is not guaranteed in the event of default—and, thus, generates expected losses conditional on the probability and the degree to which the future asset value could drop below the contemporaneous debt payment. Higher uncertainty about changes in future asset value, relative to the default barrier, increases default risk which occurs when assets decline below the barrier (see Appendix 1).

In this framework, the expected loss of a financial institution can be valued as an implicit put option in the form of a credit spread that compensates investors for holding risky debt. The put option value associated with outstanding liabilities is determined by the duration of the total debt claim, the leverage of the firm, and the volatility of its asset value. In this paper, this option pricing concept based on the standard Black-Scholes-Merton (BSM) (Black and Scholes, 1973; Merton, 1973 and 1974) valuation model is enhanced (but remains within a closed-form solution) using more advanced valuation methods, such as the Gram-Charlier expansion of the density function of asset changes (Backus and others, 2004) or a jump diffusion process of asset value changes in order to achieve more robust and reliable estimation results.[18] These approaches redress some of the most salient empirical shortcomings of the BSM model. Other suggested extensions are aimed at imposing more realistic assumptions, such as the introduction of stationary leverage ratios (Collin-Dufresne and Goldstein, 2001) and stochastic interest rates (Longstaff and Schwartz, 1995).[19]

A suitable assessment of the systemic risk stemming from the joint impact of expected losses of multiple institutions, however, entails a non-trivial aggregation problem. Thus, measuring the magnitude of risk stemming from the distress of multiple institutions warrants measuring aggregate default risk from individually estimated put options. Since the simple summation of implicit put option values would presuppose perfect correlation, i.e., a coincidence of defaults, the correct estimation of aggregate risk requires knowledge about the dependence structure of individual balance sheets and associated expected losses. While it is necessary to move beyond "singular CCA" by accounting for the dependence structure of individual balance sheets, the estimation of systemic risk through correlation becomes exceedingly unreliable in the presence of "fat tails."

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[18] The suggested approach takes into account the non-linear characteristics of asset price changes, which deviate from normality and are frequently influenced by stochastic volatility. The BSM model has shown to consistently understate spreads (Jones and others, 1984; Ogden, 1987; Lyden and Saranti, 2000), with more recent studies pointing to considerable pricing errors due to its simplistic nature.

[19] Incorporating early default (Black and Cox, 1976) does not represent a useful extension in this context given the short estimation and forecasting time window used for the CCA analysis.

As traditional (pair-wise) correlation measures are ill-suited for systemic risk analysis when extreme events occur jointly (and in a non-linear fashion), one way forward is to view the financial sector as a portfolio of individual expected losses (with individual risk parameters). Correlation describes the complete dependence structure between two variables correctly only if the joint (bivariate) probability distribution is elliptical – an ideal assumption rarely encountered in practice.[20] This is especially true in times of stress, when default risk is highly skewed, and higher volatility inflates conventional correlation measures automatically (as covariance increases disproportionately to the standard deviation), so that large extremes may even cause the mean of the distribution to become undefined. In these instances, default risk becomes more frequent and severe than suggested by the standard (distributional) assumption of normality, i.e., there is a higher probability of large losses (and more extreme outcomes) due to a considerable shift of the average away from the median ("excess skewness"), a narrower peak ("excess kurtosis"), and/or a non-linear increase of default risk in response to a declining market value of assets. Thus, widening the scope of measuring dependence to include these situations can deliver important insights about the joint tail risk of multiple (rather than only two) entities, given that large shocks are transmitted across entities differently than small shocks.

Thus, the marginal distributions of individual expected losses are combined with their time-varying dependence structure to generate the multivariate distribution of joint expected losses of all sample firms. This approach of analyzing extreme linkages of multiple entities links the univariate marginal distributions in a way that formally captures both linear and non-linear dependence and its impact on joint tail risk behavior over time. The expected losses of each firm are assumed to be "fat-tailed" and fall within the domain of the GEV distribution, which identifies asymptotic tail behavior of normalized extremes.[21] The joint distribution is estimated iteratively over a pre-specified time window with the frequency of updating determined by the periodicity of available data. The specification of a closed-form solution can then be used to (i) determine point estimates of joint expected losses as a measure multivariate *conditional tail expectation* (CTE) and (ii) quantify the contribution of specific institutions to the dynamics of systemic risk (at different levels of statistical confidence, especially at higher percentile levels).[22]

[20] Note that an elliptical joint distribution implies normal marginal distributions but not vice-versa.

[21] The choice of the empirical distribution function of the underlying data to model the marginal distributions avoids problems associated with using specific parameters that may or may not fit these distributions well (Gray and Jobst, 2009).

[22] The contribution to systemic (joint tail risk) is derived as the partial derivative of the multivariate density relative to changes in the relative weight of the univariate marginal distribution of each bank at the specified percentile.

While alternative approaches have also placed the emphasis squarely on modeling dependence in response to extreme changes in market conditions, they do so without controlling for firm-to-firm relationships and/or considering the time-variation of point estimates of associated risk measures. For instance, Acharya and others (2012) estimate potential losses as Marginal Expected Shortfall (MES) of individual banks in the event of a systemic crisis, which is defined as the situation when the aggregate equity capital of sample banks falls below some fraction of aggregate assets. The MES specifies historical expected losses, conditional on a firm having breached some high systemic risk threshold based on its historical equity returns. Adjusting MES by the degree of firm-specific leverage and capitalization yields the Systemic Expected Shortfall (SES). This method, however, generates a purely empirical measure of linear and bivariate dependence rather than a closed-form solution. Brownlees and Engle (2011) apply the same definition for a systemic crisis and formulate a capital shortfall measure ("SRISK index") that is similar to SES; however, they provide a close-form specification of extreme value dependence underpinning MES by modeling the correlations between the firm and market returns using the Dynamic Conditional Correlation (DCC)-GARCH (Engle, 2001 and 2002). Also Huang and others (2009 and 2010) derive correlation of equity returns via DCC-GARCH as statistical support to motivate the specification of a dependence structure. Both CoVaR and CoRisk follow the same logic of deriving a bivariate measure of dependence between a firm's financial performance and an extreme deterioration of market conditions (or that of its peers). None of these approaches, however, applies multivariate density estimation like Systemic CCA, in order to determine the marginal contribution of an individual institution to concurrent changes of both the severity of systemic risk and the dependence structure across any combination of sample institutions for *any* level of statistical confidence and at *any* given point in time (see Table 2 and Appendix 5).

The Systemic CCA model accounts for changes in firm-specific and common factors determining individual default risk, their implications for the market-implied linkages between firms, and the resulting impact on the overall assessment of systemic risk. In particular, it accomplishes two essential goals of risk measures in this area: (i) measuring the extent to which an institution contributes to systemic risk (in keeping with the "contribution approach" as shown in Table 1), and (ii) using such a measure to price the potential public sector cost of assisting an institution if it were to face capital need. The systemic dimension of the model is captured by three properties (see Table 3):

(i) *drawing on the market's evaluation of a firm's risk profile.* The evaluation of default risk is inherently linked to investor perception as implied by the institution's equity and equity options (which determine the implied asset value of the firm conditional on its leverage and debt level);

(ii) *controlling for common factors affecting the firm's solvency.* The framework combines market prices and balance sheet information to inform a risk-adjusted measure of systemic solvency risk. The implied asset value of a firm is modeled as being sensitive to the same markets as the implied asset value of every other institution but by varying degrees due to a particular capital structure (and its implications for the market's perception). Changes in market conditions (and their impact on the perceived risk profile of each firm via its equity price and volatility) establish market-induced linkages among sample firms. Thus, measuring the joint expected losses via option prices links institutions implicitly to the markets in which they obtain equity capital and funding; and

(iii) *quantifying the chance of simultaneous default via joint probability distributions.* The probability that multiple firms experience a realization of expected losses simultaneously is made explicit by computing joint probability distributions (which also account for differences in the magnitude of individual expected losses). Hence, the default risk—i.e., the likelihood that the implied asset value (including available cash flows from operations and asset sales) falls below the amount of required funding to satisfy debt payment obligations—is assessed not only for individual institutions but for all firms within a system in order to generate estimates of systemic risk.

B. Model Specification and Estimation Steps

The Systemic CCA framework follows a two-step estimation process. First, the changes to default risk causing a bank to fail (i.e., future debt payments exceeding the asset value of the firm) are modeled as a put option in order to estimate the market-implied expected losses over a certain time horizon (consistent with the application of CCA). Second, these individually estimated expected losses are combined into a multivariate distribution that determines the probabilistic measure of joint expected losses at a system-wide level for a certain level of statistical confidence. Finally, the sensitivity of systemic solvency risk to changes in individual default risk of a single firm (and its implications for the dependence structure of expected losses across all firms) quantifies the degree to which a particular firm contributes to total default risk.

1. Step 1 – Calculating expected losses from contingent claims analysis (CCA) using option pricing

CCA is a generalization of the option pricing theory (OPT) pioneered by Black and Scholes (1973) as well as Merton (1973 and 1974) to the corporate capital structure context. When applied to the analysis and measurement of credit risk, it is commonly called the *Black-Scholes-Merton* (BSM) (or in short, *Merton*) model.[23] In its basic concept, it

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[23] The *Merton* approach assumes that the firm's debt consists of a zero-coupon bond B with a notional value F and a maturity term of T periods. The firm's outstanding liabilities constitute the bankruptcy level, whose standard normal density defines the "distance to default" relative to the firm value. This capital structure-based evaluation of contingent claims on firm performance implies that a firm defaults if its asset value is insufficient

(continued)

quantifies default risk on the assumption that owners of equity in leveraged firms hold a call option on the firm value after outstanding liabilities have been paid off. The concept of a risk-adjusted balance sheet is instrumental in understanding default risk. The total market value of firm assets, A, at any time, t, is equal to the sum of its equity market value, E, and its risky debt, D, maturing at time T. The asset value follows a random, continuous process and may fall below the bankruptcy level ('default threshold' or 'distress barrier'), which is defined as the present value of promised payments on risky debt B, discounted at the risk free rate. The variability of the future asset value relative to promised debt payments is the driver of credit and default risk. Default happens when assets are insufficient to meet the amount of debt owed to creditors at maturity. Thus, the market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option (and its cost reflected in a credit spread above the risk-free rate that compensates investors for holding risky debt). The put option value is influence by the duration of the total debt claim, the leverage of the firm, and the volatility of its asset value (see Appendix 1).

Table 4. Traditional Accounting Bank Balance Sheet.

Assets	**Liabilities**
Accounting Assets	Debt and Deposits
(i.e., cash, reserves, loans, credits, and other exposures)	Book Equity

In the traditional definition of bank balance sheets, a change in accounting assets entails a one for one change in book equity. Assets comprise cash and cash-equivalents, investments, loans, mortgages, other cash claims on counterparties as well as non-cash claims (derivatives and contingent assets), and liabilities consist of book equity and book value of debt and deposits. When assets change, the full change affects book equity (see Table 4). In the conventional definition of credit risk, the concept of "expected losses" refers to exposures on the asset side of the bank's balance sheet. In this context, expected loss is calculated as the probability of default (PD) times a loss given default (LGD) times the exposure at default (EAD). The expected loss of different exposures are aggregated (using certain assumptions regarding correlation, etc.) and used as an input into loss distribution calculations, which are in turn used for the estimation of regulatory capital.

to meet the amount of debt owed to bondholders at maturity. Conversely, if the "distance to default" is positive, and the asset value of the firm exceeds the bankruptcy level, the call option held by equity holders on firm value has intrinsic value (in addition to its time value until the maturity of debt).

Table 5. Risk-adjusted (CCA) Bank Balance Sheet.

Assets	Liabilities
(Implied) Market Value of Assets (A)	"Risky" Debt (D)
(i.e., cash, reserves, and implied market value of "risky" assets)	*(= default-free value of debt and deposits minus expected losses to bank creditors)*
	(Observable) Market Value of Equity (E)
	(= market capitalization)

CCA constructs a risk-adjusted (economic) balance sheet by transforming accounting identities into exposures so that changes in the value of assets are directly linked to changes in the market value of equity and expected losses in an integrated framework. A decline in the value of assets increases expected losses to creditors and leads to less than one-to-one decline in the market value of equity; the amount of change in equity depends on the severity of financial distress (i.e., the perceived impact of anticipated operating losses on equity returns), the degree of leverage, and the volatility of asset returns. While expected loss in this case also relates to the total debt and deposits on the full bank balance sheet, the underlying "exposure" represented by the default-free value of the bank's total debt and deposits after accounting for the (now) higher probability of assets creating insufficient cash flows to meet debt payments (without compromising the integrity of the deposit base). The expected loss to creditors is a "risk exposure" in the risk-adjusted balance sheet (see Table 5).[24]

Thus, the integrated modeling of default risk via CCA can quantify the impact on bank borrowing costs of higher (or lower) levels of equity, the impact of changes in risk aversion, and the implications of public sector support for the market-implied capital assessment. A lower market value of equity would increase the probability of a bank to generate capital losses, which is directly related to higher bank funding costs.[25] The impact of changes in investor risk appetite on the valuation of banks can also be measured in this

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[24] Note that the risk-adjusted bank balance sheet and the traditional accounting bank balance sheet can be reconciled if uncertainty about the default risk is ignored. The risk-adjusted balance accords with the accounting balance when uncertainty is eliminated (i.e. bank's assets have no volatility). Without volatility affecting the asset values on the balance sheet, the expected loss to bank creditors declines to zero, and equity becomes book equity. The "risk exposure" becomes zero (Gray and Malone, 2008).

[25] For example, Haldane (2011) states that "market-based metrics of bank solvency could be based around the market rather than book value of capital ... e.g., [the] ratio of a bank's market capitalization to its total assets. … Market-based measures of capital offered clear advance signals of impending distress beginning April 2007 … , replacing the book value of capital by the market value lowers errors by half. Market measures provide both fewer false positives and more reliable advance warnings of future banking distress."

framework. The model implicitly captures the fact that greater risk aversion raises the cost of capital, and, by extension, increases expected losses (all else equal). For instance, during the crisis, implicit and explicit government guarantees had an important impact on reducing bank borrowing costs (and shifting risk to the public sector), which reduced expected losses based on rising equity valuation.

First, the amount of expected loss is modeled as a put option based on the individual risk-adjusted balance sheet (see Appendix 1). The expected loss can then be quantified by viewing default risk as if it were a put option written on the amount of outstanding liabilities, where the present value of debt (i.e., the default barrier) represents the "strike price," with the value and volatility of assets determined by changes in the equity and equity options prices of the firm. The value of the put option increases the higher the probability of the implied asset value falling below the default barrier over a pre-defined horizon. Such probability is influenced by changes in the level and the volatility of their implied asset value reflected in the institution's equity and equity option prices conditional on its capital structure.[26] Thus, the present value of market-implied expected losses associated with outstanding liabilities in keeping with the traditional BSM model can be valued as an implicit (European) put option value

$$P_E(t) = Be^{-r(T-t)}\Phi\left(-d-\sigma_A\sqrt{T-t}\right) - A(t)\Phi(-d), \tag{1}$$

with the present value of debt B as strike price on the asset value $A(t)$ and asset return volatility[27]

$$\sigma_A = \frac{E(t)\sigma_E}{A(t)\Phi(d)} = \left(1 - \frac{Be^{-r(T-t)}\Phi\left(d-\sigma_A\sqrt{T-t}\right)}{A(t)\Phi(d)}\right)\sigma_E \tag{2}$$

[26] Note that the absence of market data for non-listed companies does not necessarily rule out the application of CCA. For instance, in the case of the IMF's FSAP Update of the United Kingdom (IMF, 2011c), additional calculations were necessary for the estimation of market-implied expected losses of two firms included in the stress test of the largest banks. Since Santander U.K. and Nationwide are not listed companies, there are no observable equity prices and volatilities of equity returns. Instead, the implied asset values and asset volatilities underpinning the CCA model were derived via peer group analysis and historical balance sheet data. For instance, in the case of Nationwide, the implied assets were derived from quarterly reported total assets, scaled by the median ratio between the individual option-derived implied asset value and quarterly reported total assets for all other sample firms with available equity prices. Its historical asset volatility was estimated at quarterly frequency (via a simple GARCH(1,1) specification using total assets) and interpolated for daily values by using the dynamics from the median asset volatility of sample firms. Balance sheet information on total liabilities was available from public accounts, and did not require similar adjustments.

[27] Moody's KMV defines this barrier equal to total short-term debt plus one-half of long-term debt.

over time horizon $T–t$, where σ_E is the (observable) equity volatility, $E(t)$ is the equity price, r is the risk-free discount rate, and $\Phi(\bullet)$ is the cumulative probability of the standard normal density function, subject to the duration of debt claims, the asset volatility, and the sensitivity of the option price to changes in $A(t)$, i.e., the leverage

$$d = \left(\ln\left(\frac{A(t)}{B} \right) + \left(r + \frac{\sigma_A^2}{2} \right)(T-t) \right) \Big/ \sigma_A \sqrt{T-t}. \tag{3}$$

This specification of option price-based expected losses, however, does not incorporate skewness and kurtosis, and stochastic volatility, which can account for implied volatility smiles of equity prices.[28] These shortcomings of the conventional Merton model can be addressed with more advanced valuation models, which have been applied in the empirical use of the Systemic CCA framework here (see Box 1).[29]

Since the asset value is influenced by the empirical irregularities contained in the Merton model (which also affects the model-based calibration of implied asset volatility), it is estimated directly from observable equity option prices.[30] The state-price density (SPD) of the implied asset value is estimated from the risk-neutral probability distribution of the underlying asset price at the maturity date of equity options with different strike prices, without any assumptions on the underlying asset diffusion process (which is assumed to be lognormal in the Merton model). The implied asset value is defined as the expectation over the empirical SPD by adapting the Breeden and Litzenberger (1978) method (see Appendix 2), together with a semi-parametric specification of the Black-Scholes option pricing formula (Aït-Sahalia and Lo, 1998).[31] More specifically, this approach uses the

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[28] Since the Merton model contains simplifying assumptions, such as constant volatility as well as a lognormal and continuous asset process, its empirical irregularities are more pronounced the higher the intrinsic value of the put option (and the closer the asset values are from the default barrier). In other words, alternative (and more accurate) option pricing methods would generate expected losses *similar* to the ones under the Merton model as distress abates.

[29] Note that the asset volatility under risk-neutral valuation can also be derived explicitly based on several inputs to the Moody's KMV model in absence of market information about equity volatility (see Appendix 4).

[30] Given that the implied asset value is derived separately, this approach avoids the traditional "two-equations-two-unknowns" approach to derive implied assets and asset volatility based on Jones and others (1984), which was subsequently extended by Ronn and Verma (1986) to a single equation to solve two simultaneous equations for asset value and volatility as two unknowns. Duan (1994) shows that the volatility relation between implied assets and equity could become redundant if the equity volatility is stochastic. An alternative estimation technique for asset volatility introduces a maximum likelihood approach (Ericsson and Reneby, 2004 and 2005) which generates good prediction results.

[31] The implied asset value is also adjusted for dividend payments as specified in Appendix 1.

second derivative of the call pricing function (on European options) with respect to the strike price (rather than option prices). Estimates are based on option contracts with identical time to maturity, assuming a continuum of strike prices.[32]

Note, however, that the presented valuation model is subject to varying degrees of estimation uncertainty and parametric assumptions, which need to be taken into account when drawing policy conclusions. The option pricing model (given its specific distributional assumptions, the derivation of both implied assets and asset volatility, and assumptions about the default barrier) could fail to capture some relevant economics that are needed to fully understand default risk, and, thus, could generate biased estimators of expected losses. Moreover, equity prices might not only reflect fundamental values due to both shareholder dilution and trading behavior that obfuscate proper economic interpretation. For instance, during the credit crisis rapid declines in market capitalization of firms were not only a signal about future solvency risk, but also reflected a "flight to quality" motive that was largely unrelated to expectations about future firm earnings or profitability. The decline of equity prices could also have been exacerbated by the shareholder dilution effected after capital injections from the government.

Box 1. Extension of BSM Model Using the Gram-Charlier (GC) Specification or Jump Diffusion.[33]

The BSM model above can also enhanced, without altering the analytical form, by means of closed-form extensions that allow for kurtosis and skewness in returns based on the same diffusion process for asset prices (without the use of option prices in its calibration).

One possibility is the application of the Gram-Charlier (GC) expansion of the density function of asset changes defined by a standard normal random variable in the BSM model (Backus and others, 2004). For default threshold *B* as strike price on the asset value $A(t)$ of each institution, the price of the put option can be written as

$$P_{E_{GC}}(t) = Be^{-r(T-t)}\Phi\left(-d-\sigma_A\sqrt{T-t}\right) - A(t)\Phi(-d)$$
$$-A(t)\Phi(d)\sigma_A\left[\frac{\gamma_1(t)}{3!}(2\sigma_A-d) - \frac{\gamma_2(t)}{4!}\left(1-d^2+3d\sigma_A-3\sigma_A^2\right)\right]$$

[32] Since available strike prices are always discretely spaced on a finite range around the actual asset value, interpolation of the call pricing function inside this range and extrapolation outside this range are performed by means nonparametric (local polynomial) regression of the implied volatility surface (Rookley, 1997).

[33] These enhancements to the BSM model were applied within the Systemic CCA framework—as part of the IMF's FSAP stress tests—with the Gram-Charlier (GC) extension in the case of the United States, Sweden, and Germany (IMF, 2010b, 2011e, and 2011f), and with a jump diffusion process for the United Kingdom and Spain (IMF, 2011c and 2012).

over time horizon $T–t$ at risk-free rate r, subject to the duration of debt claims, the leverage of the firm, and asset volatility σ_A, with the correction terms for t-period skewness γ_1 and kurtosis γ_2 in returns based on the same diffusion process for asset prices.[34]

Alternatively, the model can constructed around a jump diffusion that follows a standard Poisson process

$$\Pr\left(N(t)=k\right)=\frac{(\lambda t)^k}{k!\,e^{-\lambda t}},$$

where λ is the average number of expected jumps per unit time (i.e., the number of jump events up to time t). The jump size follows a log-normal distribution

$$J \sim \varphi \exp\left(-\nu^2/2+\nu\Phi(0,1)\right)$$

with average jump size φ and volatility ν of the jump size calibrated over an estimation time period with τ-number of observations (e.g., τ = 120 days).[35] The k^{th} term in this series corresponds to k jumps to occur over the specified observation time horizon. By conditioning the asset value on the expected jump process over a specific time period (such as a rolling time window with periodic updating), the put option value at time T-t to maturity can be written as

$$P_{E_k}(t)=\sum_{k=0}^{\infty}\frac{\exp(-\varphi\lambda t)(\varphi\lambda t)^k}{k!}Be^{-r(T-t)}\Phi\left(-d_k-\sigma_{A_k}\sqrt{T-t}\right)-A(t)\Phi\left(-d_k\right).$$

The asset volatility—consistent with the above specification of a jump diffusion process—is defined as

$$\sigma_{A_k}=\sqrt{\sigma_A^2+k\nu^2/(T-t)},$$

which is derived by combining

[34] The GC model is only slightly more complicated to implement than the Merton model because only two additional parameters—skewness and kurtosis—need to be estimated.

[35] Further refinements of this option pricing model are possible, including various simulation approaches, which might come at the expense of losing analytical tractability. The ad hoc model by Dumas and others (1998) is designed to accommodate the implied volatility smile and is easy to implement, but requires a large number of market option prices. The pricing models by Heston (1993) and Heston and Nandi (2000) allow for stochastic volatility, but the parameters driving these models can be difficult to estimate. Many other models have been proposed, to incorporate stochastic volatility, jumps, and stochastic interest rates. Bakshi and others (1997), however, suggest that most of the improvement in pricing comes from introducing stochastic volatility. Introducing jumps in asset prices leads to small improvements in the accuracy of option prices. Other option pricing models include those based on copulas, Levy processes, neural networks, GARCH models, and non-parametric methods. Finally, the binomial tree proposed by Cox and others (1979) spurred the development of lattices, which are discrete-time models that can be used to price any type of option—European or American, plain-vanilla or exotic.

$$\sigma_{A_k} = \sqrt{\left(\frac{E(t)}{A(t)\Phi(d_k)}\sigma_E\right)^2 + \frac{k\nu^2}{T-t}},$$

and the updated risk-free interest rate

$$r_k = r - \lambda(\varphi - 1) + k\ln(\varphi)/(T-t)$$ [36]

to arrive at the leverage parameter which, subject to the assumed jump process can be re-written as

$$d_k = \left(\ln\left(A(t)/B\right) + \left(r_k + \sigma_{A_k}^2/2\right)(T-t)\right)\Big/\sigma_{A_k}\sqrt{T-t}.$$

2. Step 2 – Estimating the joint expected losses from default risk

Second, the individually estimated expected losses are combined to determine the magnitude of default risk on a system-wide level. The distribution of expected losses is assumed to be "fat-tailed" in keeping with extreme value theory (EVT). This specification of individual loss estimates then informs the estimation of the dependence function. As part of a four-step sub-process, we define the univariate marginal density functions of all firms, which are then combined with their dependence function in order to generate an aggregate measure of default risk.[37] This multivariate set-up underpinning the Systemic CCA framework formally captures the realizations of joint expected losses. We can then use tail risk estimates, such as the *conditional Value-at-Risk* (VaR) (or *expected shortfall* (ES)), in order to gauge systemic solvency risk in times of stress at a statistical confidence level of choice.

(i) Estimating the marginal distributions of individual expected losses

We first specify the statistical distribution of individual expected losses (based the series of put option values obtained for each firm) in accordance with extreme value theory (EVT), which is a general statistical concept of deriving a limit law for sample maxima. Let the vector-valued series

$$\mathbf{X}_j^n = P_{E,1}^n(t), \ldots, P_{E,m}^n(t) \tag{4}$$

[36] The value of a derivative with a convex payoff (which includes this put option specification) increases when jumps are present (i.e., when λ>0)—regardless of the average jump direction.

[37] The aggregation of individual expected losses (rather than their underlying assets and liabilities of each firm's risk-adjusted balance sheet) is crucial to preserve individual balance sheet risk in the estimation of aggregate default risk. Calculating joint expected losses based on a single put option specification (using an aggregate default barrier) would result in a misrepresentation of overall default risk (unless default risk is uniformly distributed within the selected sample of firms).

denote i.i.d. random observations of expected losses (i.e., a total of n-number of daily put option values $P^n_{E,m}(t)$ up to time t), each estimated according to equation (1) above over a rolling window of τ observations with periodic updating (e.g., a daily sliding window of 120 days) for $j \in m$ firms in the sample. The individual asymptotic tail behavior is modeled in accordance with the Fisher-Tippett-Gnedenko theorem (Fisher and Tippett, 1928; Gnedenko, 1943), which defines the attribution of a given distribution of normalized maxima (or minima) to be of extremal type (assuming that the underlying function is continuous on a closed interval). Given

$$\mathbf{X} = \max\left(P^n_{E,j}(t),...,P^n_{E,m}(t)\right), \tag{5}$$

there exists a choice of normalizing constants $\beta^n_j > 0$ and α^n_j, such that the probability of each ordered n-sequence of normalized sample maxima $\left(\mathbf{X}-\alpha^n\right)/\beta^n$ converges to the non-degenerate limit distribution $H(x)$ as $n \to \infty$ and $x \in \mathbb{R}$, so that

$$F_n^{\left[\beta^n x+\alpha^n\right]}(x) = \lim_{n\to\infty} \Pr\left(\left(X-\alpha^n\right)/\beta^n \leq y\right) = \left[F\left(\beta^n y+\alpha^n\right)\right]^n \to H(x), \tag{6}$$

falls within the maximum domain of attraction (MDA) of the GEV distribution and conforms to one of three distinct types of extremal behavior—Gumbel (EV0), Fréchet (EV1) or negative Weibull (EV2)—as limiting distributions of maxima of dependent random variables (Coles and others, 1999; Poon and others, 2003; Stephenson, 2003; Jobst, 2007):

$$\text{EV0: } H_0(x) = \exp\left(-\exp(-x)\right) \quad \textit{if } x \geq 0, \xi = 0, \tag{7}$$

$$\text{EV1: } H_{1,\xi}(x) = \exp\left(-x^{-1/\xi}\right) \quad \textit{if } x \in \left[\mu-\sigma/\xi, \infty\right), \xi > 0, \text{ and} \tag{8}$$

$$\text{EV2: } H_{2,\xi}(x) = \exp\left(-(-x)^{-1/\xi}\right) \quad \textit{if } x \in \left(-\infty, \mu-\sigma/\xi\right], \xi < 0.\text{[38]} \tag{9}$$

The limit distributions above are combined into a *unified* parametric family of the GEV probability distribution function

$$h_{\mu,\sigma,\xi}(x) = \frac{1}{\sigma}\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{(-1/\xi)-1} \exp\left(-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)_+^{-1/\xi}\right), \tag{10}$$

[38] See Embrechts and others (1997) as well as Vandewalle and others (2004) for additional information on the definition of extreme value theory (EVT).

and the cumulative distribution function

$$H_{\mu,\sigma,\xi}(x) = \begin{cases} \exp\left(-\left(1+\xi\dfrac{(x-\mu)}{\sigma}\right)^{-1/\xi}\right) & \text{if } 1+\dfrac{\xi(x-\mu)}{\sigma} \geq 0 \\ \exp\left(-\exp\left(-\dfrac{(x-\mu)}{\sigma}\right)\right) & \text{if } x \in \mathbb{R}, \xi = 0. \end{cases} \tag{11}$$

Thus, the j^{th} univariate marginal density function of each expected loss series converging to GEV in the limit is defined as

$$y_j(x) = \left(1+\hat{\xi}_j \frac{\left(x-\hat{\mu}_j\right)}{\hat{\sigma}_j}\right)_+^{-1/\hat{\xi}_j} \quad \text{(for } j = 1,...,m\text{)} \tag{12}$$

where $1+\xi_j\left(x-\mu_j\right)/\sigma_j > 0$, scale parameter $\sigma_j > 0$, location parameter μ_j, and shape parameter $\xi_j \neq 0$.[39]

The moments of the univariate density function in equation (12) above are defined as

$$\textit{mean}: \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi} h_1 \text{ given } \begin{cases} \mu + \sigma\left(\Gamma(1-\xi)-1\right)/\xi & \text{if } \xi \neq 0, \xi < 1 \\ \mu + \sigma\gamma & \text{if } \xi = 0 \\ \infty & \text{if } \xi \geq 1 \end{cases}, \tag{13}$$

$$\textit{variance}: \frac{\sigma^2\left(h_2 - h_1^2\right)}{\xi^2} \text{ given } \begin{cases} \sigma^2\left(h_1 - h_1^2\right)/\xi^2 & \text{if } \xi \neq 0, \xi < 1/2 \\ \sigma^2 \pi^2/6 & \text{if } \xi = 0 \\ \infty & \text{if } \xi \geq 1/2 \end{cases}, \tag{14}$$

$$\textit{skewness}: \frac{h_3 - 3h_1h_2 + 2h_1^3}{\left(h_2 - h_1^2\right)^{3/2}} \text{ given } \begin{cases} \dfrac{h_3 - 3h_1h_2 + 2h_1^3}{\left(h_2 - h_1^2\right)^{3/2}} & \text{if } \xi \neq 0 \\ \dfrac{12\sqrt{6}\zeta(3)}{\pi^3} & \text{if } \xi = 0 \end{cases}, \text{ and} \tag{15}$$

[39] The upper tails of most (conventional) limit distributions (weakly) converge to this parametric specification of asymptotic behavior, irrespective of the original distribution of observed maxima (unlike parametric VaR models). The higher the absolute value of shape parameter, the larger the weight of the tail and the slower the speed at which the tail approaches its limit.

$$kurtosis\text{: } \frac{h_4 - 4h_1h_3 + 6h_2h_1^2 - 3h_1^4}{\left(h_2 - h_1^2\right)^2} - 3 \text{ given } \begin{cases} \dfrac{h_4 - 4h_1h_3 + 6h_2h_1^2 - 2h_1^4}{\left(h_2 - h_1^2\right)^2} & if\, \xi \neq 0 \\ \dfrac{12}{5} & if\, \xi = 0 \end{cases}, \tag{16}$$

with $h_p = \Gamma(1 - p\xi)$ for $p \in \{1,...,4\}$, Euler's constant γ (Sondow, 1998) and Riemann's zeta function $\zeta(t)$ (Borwein and others, 2000) and gamma probability density function $\Gamma(t)$. These moments are estimated concurrently by means of numerical iteration via maximum likelihood (ML), which identifies possible limiting laws of asymptotic tail behavior, i.e., the likelihood of even larger extremes as the level of statistical confidence approaches certainty (Coles and others, 1999; Poon and others, 2003; Jobst, 2007). The ML estimator in the GEV model is evaluated numerically by using an iteration procedure (e.g., over a rolling window of τ observations with periodic updating) to maximize the likelihood $\prod_{i=1}^{n} h_{\mu,\sigma,\xi}\left(x|\theta\right)$ over all three parameters $\theta = (\mu, \sigma, \xi)$ simultaneously, where the *linear combinations of ratios of spacings* (LRS) estimator serves as an initial value (see Annex 3 and Jobst, 2012).[40]

(ii) Estimating the dependence structure of individual expected losses

Second, we define a non-parametric, multivariate dependence function between the marginal distributions of expected losses by expanding the bivariate logistic method proposed by Pickands (1981) to the multivariate case and adjusting the margins according to Hall and Tajvidi (2000) so that

$$\Upsilon(\omega) = \min\left\{1, \max\left\{n\left(\sum_{i=1}^{n} \bigwedge\nolimits_{j=1}^{m} \frac{y_{i,j}/\hat{y}_{\bullet j}}{\omega_j}\right)^{-1}, \omega, 1-\omega\right\}\right\}, \tag{17}$$

where $\hat{y}_{\bullet j} = \sum_{i=1}^{n} y_{i,j}/n$ reflects the average marginal density of all $i \in n$ put option values and $0 \leq \max(\omega_1,...,\omega_{m-1}) \leq \Upsilon(\omega_j) \leq 1$ for all $0 \leq \omega_j \leq 1$.[41] $\Upsilon(\bullet)$ represents a convex function

[40] Note that the ML estimator fails for $\xi \leq -1$ since the likelihood function does not have a global maximum in this case. However, a local maximum close to the initial value can be attained.

[41] Note that the marginal density of a given extreme relative to the average marginal density of all extremes is minimized ("$\bigwedge$") across all firms $j \in m$, subject to the choice of factor ω_j. A bivariate version of this approach has been implemented in Jobst and Kamil (2008).

on $[0,1]$ with $\Upsilon(0)=\Upsilon(1)=1$, i.e., the upper and lower limits of $\Upsilon(\bullet)$ are obtained under complete dependence and mutual independence, respectively. It is estimated iteratively (and over a rolling window of τ observations with periodic updating (e.g., a daily sliding window of 120 days) subject to the optimization of the (m-1)-dimensional unit simplex

$$S_m=\left\{(\omega_1,...,\omega_{m-1})\in\mathbb{R}_+^m:\ \omega_j\geq 0, 1\leq j\leq m-1; \sum_{j=1}^{m-1}\omega_j\leq 1 \text{ and } \omega_m=1-\sum_{j=1}^{m-1}\omega_j\right\}, \tag{18}$$

which establishes the degree of coincidence of multiple series of cross-classified random variables similar to a χ-statistic that measures the statistical likelihood that observed values differ from their expected distribution. This specification stands in contrast to a general copula function that links the marginal distributions using only a single (and time-invariant) dependence parameter.

(iii) Estimating the joint distribution of expected losses

We then combine the marginal distributions of these individual expected losses with their dependence structure to generate a *multivariate extreme value distribution* (MGEV) over the same estimation period as above.[42] Analogous to equations (10)-(12), the resultant cumulative distribution function is specified as

$$G_{t,m}(x)=\exp\left\{-\left(\sum_{j=1}^{m}y_{t,j}\right)\Upsilon_t(\omega)\right\} \tag{19}$$

with corresponding probability density function

$$g_{t,m}(x)=\hat{\sigma}_{t,m}^{-1}\left(\left(\sum_{j=1}^{m}y_{t,j}\right)\Upsilon_t(\omega)\right)^{\hat{\xi}_{t,m}+1}\exp\left\{-\left(\sum_{j=1}^{m}y_{t,j}\right)\Upsilon_t(\omega)\right\} \tag{20}$$

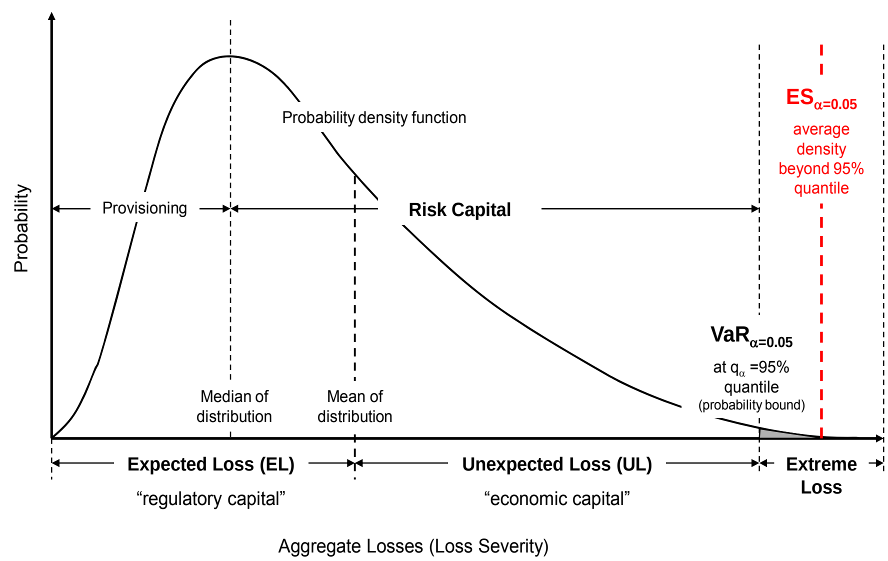
---

[42] The analysis of dependence in this presented approach is completed separately from the analysis of marginal distributions, and, thus, differs from the classical approach, where multivariate analysis is performed jointly for marginal distributions and their dependence structure by considering the complete variance-covariance matrix, such as the MGARCH approach. To obtain a multivariate distribution function, the dependence function above combines several marginal distribution functions in accordance with Sklar's (1959) theorem on constructing joint distributions with arbitrary marginal distributions via copula functions (which completely describes the dependence structure and contains all the information to link the marginal distributions). For the formal treatment of copulas and their properties, see Hutchinson and Lai (1990), Dall'Aglio and others (1991), and Joe (1997).

at time $t=\tau+1$ by maximizing the likelihood $\prod_{j=1}^{m} g_{t,m}\left(x\middle|\theta\right)$ over all three parameters $\theta=\left(\mu,\sigma,\xi\right)$ simultaneously. Equation (20) above represents the functional form of the joint expected losses of all sample firms based on the empirical observations in equation (4) above. Since the logarithm is a continuously increasing function over the range of likelihood, the parameter values that maximize the likelihood will also maximize the logarithm as global maximum. Thus, we can write the lognormal likelihood function as $\sum_{j=1}^{m} \ln g\left(x\middle|\theta\right)$ so that the maximum likelihood estimate (MLE) of the true values θ_0 is

$$\hat{\theta}_{MLE} = \underset{\theta\in\Theta}{\arg\max}\, \hat{\ell}\left(\theta\middle|x\right) \rightarrow \theta_0. \tag{21}$$

Figure 1. The Location of Expected Shortfall (ES) in a Stylized Loss Distribution.



(iv) <u>Estimating a tail risk measure of joint expected losses</u>

Finally, we obtain the joint *expected shortfall* (ES) (or conditional *Value-at-Risk* (VaR)) as a measure conditional tail expectation (CTE). ES defines the probability-weighted residual density (i.e., the average aggregate expected losses) beyond a pre-specified statistical confidence level ("severity threshold") over a given estimation time period. Given the dependence structure defined in equation (17) above, and in accordance with the general definition of ES (Artzner and others, 1999) as

$$
\begin{aligned}
ES_a &= \mathbf{E}\left[X \middle| X > VaR_a\right] \\
&= \frac{1}{1-F\left(VaR_a\right)}\int_{VaR_a}^{\infty}\left(1-F(x)\right)dx = \frac{1}{1-a}\int_a^1 VaR_a da,
\end{aligned}
\tag{22}
$$

for a continuous random variable *X*, where $VaR_a \equiv \inf\left\{x : F(x) \geq a\right\}$ is the quantile of order $0 < a < 1$ (say, *a*=0.95) pertaining to the distribution function *F*, we can calculate ES in a multivariate context as

$$
\begin{aligned}
ES_{t,\tau,m,a} &= -\mathbf{E}\left[z_\tau \middle| z_\tau \geq G^{-1}_{t,\tau,m}(a) = VaR_{t,a}\right] \\
&= -\frac{1}{a}\int_0^a G^{\leftarrow}_{t,\tau,m}(x)\,dx = -\frac{1}{a}\int_0^a G^{-1}_{t,\tau,m}(x)\,dx,
\end{aligned}
\tag{23}
$$

where $G^{-1}(a)$ is the quantile corresponding to probability, $z \in \mathbb{R}$ and $G^{-1}(a) = G^{\leftarrow}(a)$ with $G^{\leftarrow}(a) \equiv \inf\left(x \middle| G(x \geq a)\right)$ so that

$$
VaR_{t,a} = \sup\left\{G^{-1}_{t,\tau,m}(a) \middle| \Pr\left[z_\tau > G^{-1}_{t,\tau,m}(a)\right] \geq a = 0.95\right\},
\tag{24}
$$

with the point estimate of joint potential losses of *m* firms at time *t* defined as

$$
G^{-1}_{t,\tau,m}(a) = \hat{\mu}_{t,m} + \hat{\sigma}_{t,m} \big/ \hat{\xi}_{t,m}\left(\left(-\frac{\ln(a)}{\Upsilon_t(\omega)}\right)^{-\hat{\xi}_{t,m}} - 1\right).^{43}
\tag{25}
$$

ES is a coherent risk measure as a real functional ϑ defined on a space of random variables that satisfy the following axioms:

H1. *Subadditivity:* for all variables X and Y, $\vartheta(X+Y) \leq \vartheta(X) + \vartheta(Y)$.

H2. *Monotonicity:* if $X \leq Y$ for each outcome, then $\vartheta(X) \leq \vartheta(Y)$.

H3. *Positive Homogeneity:* for positive constant λ, $\vartheta(\lambda X) = \lambda\vartheta(X)$

[43] Expected shortfall (ES) is an improvement over Value-at-Risk (VaR), which, in addition to being a pure frequency measure, is "incoherent," i.e., it violates several axioms of convexity, homogeneity, and sub-additivity found in coherent risk measures. For the application of this kind of coherent risk measures we refer to Artzner and others (1997 and 1999) as well as Wirch and Hardy (1999). For example, sub-additivity, which is a mathematical way to say that diversification leads to less risk, is not satisfied by VaR.

H4. *Translation Invariance:* for constant c, $\vartheta(X+c)=\vartheta(X)+c$.

(v) Estimating the individual contribution to joint expected losses

The contribution of each firm is determined by calculating the cross-partial derivative of the joint distribution of expected losses.[44] The joint ES can also be written as a linear combination of individual ES values, $ES_{t,\tau,j,a}$, where the relative weights $\psi_{\tau,m,a}$ (in the weighted sum) are given by the second order cross-partial derivative of the inverse of the joint probability density function $G_{t,\tau,m}^{-1}(\bullet)$ to changes in both the dependence function $\Upsilon_t(\bullet)$ and the individual marginal severity $y_{t,j}$ of expected losses. Thus, the contribution can be derived as the partial derivative of the multivariate density to changes in the relative weight of the univariate marginal distribution of expected losses and its impact on the dependence function (of all expected losses of sample firms) at the specified percentile. By re-writing $ES_{t,\tau,m,a}$ in equation (23) above, we obtain

$$ES_{t,\tau,m,a}=-\sum_{j}^{m}\psi_{\tau,m,a}\mathbf{E}\left[z_{\tau,j}\,\middle|\,z_{\tau,m}\geq G_{t,\tau,m}^{-1}(a)=VaR_{t,a}\right], \tag{26}$$

where the relative weight of institution *j* at statistical confidence level *a* is defined as the marginal contribution

$$\psi_{\tau,m,a}=\frac{\partial^{2}G_{t,\tau,m}^{-1}(a)}{\partial y_{t,j}\partial\Upsilon_{t}(\omega)}\quad \text{s.t.}\ \sum_{j}^{m}\psi_{\tau,j,a}=1\ \text{ and }\ \psi_{\tau,j,a}z_{\tau,j}\leq z_{\tau,m}, \tag{27}$$

attributable to the joint effect of both the marginal density and the change of the dependence function due to the presence of institution $j\in m$ in the sample.

[44] Note that this approach could also be used to identify the effectiveness of closer supervisory monitoring of identified liquidity problems of a particular bank. If remedial actions are effective, they would decrease the bank's contribution to overall systemic liquidity risk to a level that closely matches the individual liquidity risk.

III. Extensions of the Systemic CCA Framework

A. Price-based Macroprudential Measure for Systemic Risk

The joint expected losses estimated via Systemic CCA could support the design of MPS instruments that mitigate the impact of systemically important financial institutions (SIFIs) on financial stability. SIFIs, which are also referred to as large complex financial institutions (LCFIs) or "too-big-to-fail" (TBTF), are defined as "financial institutions whose disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the financial system and economic activity (FSB, 2009 and 2010)." Their viability is deemed crucial for the functioning of the financial system, with the complexity of their operations complicating timely resolution in distress due to multiple linkages to institutions and markets. A material financial distress at such a firm, related to its nature, scope, size, scale, concentration, interconnectedness, or the mix of the activities of the firm (FSOC, 2011), can adversely affect financial stability due to the spillover effects of their actions on the financial system and the wider economy. Thus, the most recent examples of financial sector reform include capital charges on SIFIs, such as the proposed "loss absorbency requirement" for "global systemically important banks (G-SIBs)" (BCBS, 2011a).[45,46]

The marginal (time-varying) contribution of an institution to joint expected losses could motivate an insurance policy that covers the impact of individual default risk on the likelihood on system-wide distress (see equation (22) above). An insurance policy that indemnifies a single firm's contribution to systemic risk would be based on the actuarial assessment of the conditional probability that a bank, in concert with other institutions, fails to meet minimum solvency standards after the realization of expected losses.[47]

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[45] However, such surcharges entail important policy considerations as they could create additional burden on the financial sector at a time when capital is scarce and credit supply is so much needed to sustain the recovery. Thus, a careful calibration and possible implementation is essential to ensure that availability of adequate credit to support the ongoing recovery is not impeded. The long implementation periods are meant to accommodate this issue.

[46] The selected indicators for the designation of G-SIBs were chosen by the BCBS to reflect the different aspects of what generates negative externalities and makes a bank critical for the financial system. They include the *size* of banks, their *interconnectedness*, the lack of *substitutability* for the services they provide (all of which were identified by the FSB (2009, 2010, and 2011) with regard to systemically important financial institutions, markets, and instruments), their *global activity*, and their *complexity*. The intention is for the quantitative approach to be supplemented with qualitative information that is incorporated through a framework for supervisory judgment in exceptional, egregious cases, subject to international peer review to ensure consistency in its application.

[47] This approach could also be refined to cover only the degree of contingent capital support that a systemically important bank would expect to receive in times of distress, and, thus, would represent the potential public sector cost (i.e., contingent liability).

The insurance premium would promote the internalization of own default risks and/or the impact of own business activities on system-wide solvency risk. As with any insurance premium, the primary objective would not be the collection of fees for the *ex post* funding of any potential public sector cost associated the negative impact of individual failure on financial stability but the change in firm behavior.[48]

The fair value insurance premium would be commensurate to a risk-based surcharge that is calculated based on the individual contribution of each institution to the joint expected losses relative the aggregate amount of outstanding liabilities of all sample firms. In the context of the Systemic CCA framework, it represents the actuarial value of two or more firms experiencing expected loss when debt payments exceed their implied asset value during times of stress over a pre-defined risk horizon. The estimated joint expected losses are combined with the sum of individual default barriers for all sample firms to inform a subjective default probability in a standard hazard rate model. More specifically, the insurance premium can be obtained as the natural logarithm of one minus the hazard rate,

$$\varphi \approx \frac{\frac{1}{4}\sum_{t=-4}^{0}\left(\overline{\psi}_{t,\tau,j,a} \times \overline{ES}_{t,\tau,j,a}\right)}{\frac{1}{4}\sum_{j}^{m}\sum_{t=-4}^{0} B_{t,j}e^{-r(T-t)}}, \tag{28}$$

which is defined as the ratio between the average marginal contribution of each firm to the average expected shortfall $\overline{ES}_{t,\tau,j,a}$ (with statistical significance *a*) over multiple time periods of observations (say, the last four quarters if expected losses are estimated daily) and the average of the discounted present value of total liabilities of all sample firms as aggregate default barrier, $\sum_{j}^{m} B_{t,j}e^{-r(T-t)}$, over the same time periods.[49] This assumes that the conditional probability of each firm contributing to joint default risk under the risk-neutral measure is constant over the risk horizon *t* and can be expressed as an exponential function, given the survival probability

[48] Note that if the pricing of insurance coverage were based on the aggregate incidence of expected losses (and associated potential capital shortfall), the risk is "pooled" with others in the sample that made up the joint probability—making it less expensive.

[49] Averaging over several periodic observations helps mitigate statistical distortions from one-off events during a particular time period. This approach also follows the rationale of the current bank regulatory guidelines for market risk regarding the choice of time horizon. The BCBS (2011b) defines the capital requirement based on Value-at-Risk (VaR) that is calculated each day and compared to three times the average quarterly VaRs over the last four quarters. The maximum of these two numbers becomes the required amount of regulatory capital for market risk (Jobst, 2012).

$$\exp\left(-\int_0^t \varphi(u)\,du\right) = \exp(-\varphi t)\,. \tag{29}$$

Thus, the cost of insuring the downside risk of available funds being insufficient to satisfy contemporaneous debt service obligations of the firm during times of system-wide stress can be calculated by converting the insurance premium (in basis points), based on the constant hazard rate defined in equation (28) above, into decimal form and multiplying the product by the average value of the firm's short-term liabilities ("default barrier") so that

$$\underbrace{-\frac{1}{T}\ln(1-\varphi)}_{\text{insurance premium}} \times 10{,}000 \times \frac{1}{4}\sum\nolimits_{t=-4}^{0} \overline{B}_{t,j}^{ST}\,. \tag{30}$$

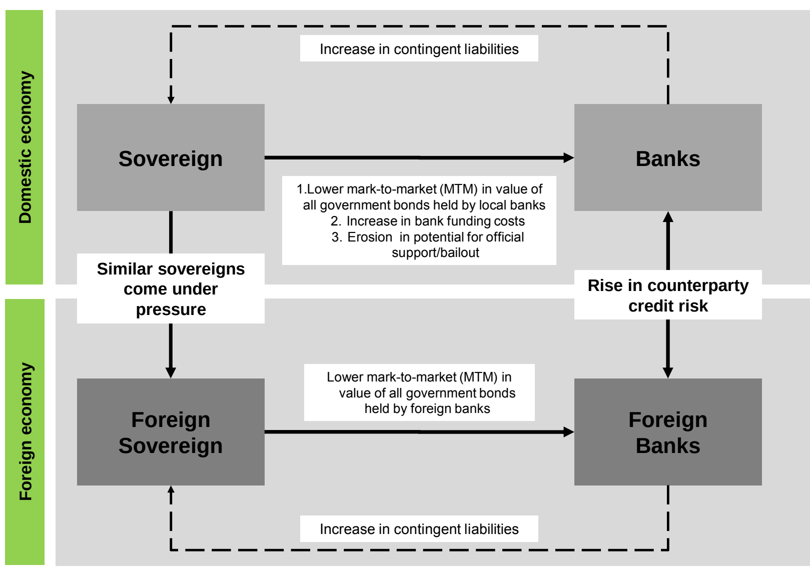
While the actuarial motivation of this risk-based levy has considerable conceptual appeal it is not without risks. A systemic risk surcharge creates a sense of awareness of the negative externalities of individual default risk firms pose for financial stability while mitigating future burdens on taxpayers. However, the institutionalization of systemic importance could aggravate the underlying problem of moral hazard by breeding a sense of entitlement to government support. There is also the question of whether such surcharges should be charged *ex post* or *ex ante*, and whether proceeds would go to special funds or to general government revenue.

B. Estimating the Contingent Liabilities to the Public Sector

Large implicit government financial guarantees (valued as contingent liabilities) create significant valuation linkages between sovereigns and financial sector risks. These linkages could give rise to a destabilization process that increases the susceptibility of public finances to the potential impact of distress in an outsized financial sector (see Figure 2 and Box 2). The financial crisis that started in 2007 is a stark reminder of how public sector support measures provided to large financial institutions can result in considerable risk transfer to the government, which places even greater emphasis on long-term fiscal sustainability. A negative feedback loop between the financial sector risks and fiscal policy arises because a decline in the market value of sovereign debt does not only weaken the implicit sovereign guarantees to systemically-importance financial institutions (and large banks in particular), but also raises default risk in the financial sector overall (which increases the cost of borrowing). There is also a higher likelihood of downgrades to financial institutions following downgrades of sovereign credit ratings, which establish an upper ceiling to their unsecured funding ratings. This linkage between the creditworthiness of the sovereign and the financial system (in particular, the banking sector) is prone to perpetuate a negative feedback effect that is accentuated by the lender-borrower channel in situations where financial institutions hold large exposures to public sector entities.

Based on CCA, the market-implied expected losses calculated from equity market and balance sheet information of financial institutions can be combined with information from their CDS contracts to estimate the government's contingent liabilities.[50] Since systemically important institutions enjoy an implicit government guarantee, their creditors have an expectation to be bailed out by the government in the case of failure.[51] Conversely, the cost of insuring the non-guaranteed debt (as reflected in the senior credit default swap (CDS) spread) should capture only the expected loss *retained* by such financial institutions (and borne by unsecured senior creditors), after accounting for any implicit public sector support due to their systemic significance. Thus, the market-implied government guarantee for large banks (as prime candidates for SIFI status) can be derived heuristically as difference between the total expected loss (i.e., the value of a put option $P_E(t)$ derived from the bank's equity price) and the value of an implicit put option $P_{CDS}(t)$ derived from the bank's CDS spread—assuming that equity values remain unaffected (which is very likely when banks are close to the default barrier, as the call option value from the residual claim of equity to future profits after debt payment converges to zero).

Figure 2. Valuation Linkages between the Sovereign and Banking Sector.



[50] See Gray and Jobst (2009b). See also Gapen (2009) for an application on CCA to the measurement of contingent liabilities from government-sponsored enterprises (GSEs) in the United States.

[51] As a result, the cost of funding for these institutions remains below the level commensurate to their actual default risk.

By replacing individual expected losses with this measure of contingent liabilities, the Systemic CCA framework can be used to derive an estimate of systemic risk from joint contingent liabilities. Since the put option value $P_{CDS}(t) \leq P_E(t)$, it reflects the expected losses associated with default net of any financial guarantees, i.e., residual default risk on unsecured senior debt. $P_{CDS}(t)$ can be written as

$$P_{CDS}(t) = \left(1 - \exp\left(-\left(\frac{s_{CDS}(t)}{10{,}000}\right)\left(\frac{B}{D(t)} - 1\right)(T-t)\right)\right)Be^{-r(T-t)} \tag{31}$$

after rearranging the specification of the CDS spread (in basis points), $s_{CDS}(t)$, under the risk-neutral measure as

$$\begin{aligned} s_{CDS}(t) &= -\frac{1}{T-t}\ln\left(1 - \frac{P_{CDS}(t)}{Be^{-r(T-t)}}\right) \times \left(\frac{B}{D(t)} - 1\right) \times 10{,}000 \\ &= -\frac{10{,}000}{T-t}\ln\left(1 - \frac{P_{CDS}(t)}{Be^{-r(T-t)}}\right) \times \left(\frac{B}{Be^{-r(T-t)} - P_E(t)} - 1\right), \end{aligned} \tag{32}$$

assuming a default probability

$$1 - \exp\left(-\int_0^t \varphi(u)\,du\right) = 1 - \exp(-\varphi t) \equiv \frac{P_{CDS}(t)}{Be^{-r(T-t)}} \tag{33}$$

at time *t*, constant hazard rate $s_{CDS}(t) \approx \varphi$, and the implied yield to maturity, $y = \varphi - r$, on the risky debt, which is defined by

$$D(t) = Be^{-y(T-t)} \Leftrightarrow e^{-y(T-t)} = \frac{D(t)}{B} = \frac{Be^{-r(T-t)} - P_E(t)}{B} \tag{34}$$

so that $\left(1 - s_{CDS}(t)\right)Be^{-r(T-t)} = PD \times LGD$ over one period $T - t = 1$. A linear adjustment of $B/D(t) - 1$ is needed in order to control for the impact of any difference between the market price and the repayable face value of outstanding debt on the fair pricing of credit protection. If outstanding debt trades above par, $D(t) > B$, the recovery of the nominal value under the CDS contract ("recovery at face value" or RFV) drops below the recovery rate implied by the market price of debt ("recovery at market value" or RMV). As a result, the CDS spread,

$s_{CDS}(t)$, becomes drops below the bond spread.[52] Conversely, a below-par bond price pushes up the implied recovery rate of CDS, causing a mark-up of CDS spreads over comparable bond spreads for the same estimated default risk.

Estimated expected losses derived from both equity prices and credit spreads can be combined to derive the market-implied degree to which potential public sector support depresses the cost of credit. Given $P_{CDS}(t)$, the ratio

$$\alpha(t) = 1 - \frac{P_{CDS}(t)}{P_E(t)} \tag{35}$$

defines the share of expected loss covered by implicit (or explicit) government guarantees that depress the CDS spread below the level that would be warranted by the equity-implied default risk.[53]

$$\alpha(t) P_E(t) = \left(1 - \frac{P_{CDS}(t)}{P_E(t)}\right) P_E(t) = P_E(t) - P_{CDS}(t) \tag{36}$$

represents the fraction of default risk covered by the government (and no longer reflected in the marked-implied default risk), and $(1-\alpha(t))P_E(t)$ is the risk retained by an institution and reflected in the senior CDS spread. Thus, the time pattern of the government's contingent liabilities and the retained risk in the financial sector can be measured.

While this definition of market-implied contingent liabilities provides a useful indication of possible sovereign risk transfer, the presented estimation method depends on a variety of assumptions that influence the assessment of the likelihood of public sector support, especially at times of extreme stress. The extent to which the equity put option values differ from the one implied by CDS spreads might reflect distortions stemming

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[52] The difference in recovery values (often referred to as "basis") implied by a divergence of CDS and bond spreads is approximated by using the fair value CDS (FVCDS) spread and the fair value option adjusted spread (FVOAS) reported by Moody's *KMV CreditEdge* (MKMV). Both FVOAS and FVCDS represent credit spreads (in basis points) for the bond and CDS contract of a particular firm based on risk horizon of t years (where t=1 to 10 years). Both spreads imply a loss given default (LGD) determined by the industry category. In practice, this adjustment factor is very close to unity for most of the cases, with a few cases where the factor falls within a 20 percentage point range (0.9 to 1.1).

[53] Note that the estimation assumes a European put option, which does not recognize the possibility of premature execution. This might overstate the actual expected losses inferred from put option values in comparison with the put option derived from CDS spreads.

from the modeling choice (and the breakdown of efficient asset pricing in situations of illiquidity), changes in market conditions, and the capital structure impact of government interventions, such as equity dilution in the wake of capital injections, beyond the influence of explicit or implicit guarantees.

Even though the equality condition of default probabilities derived from equity prices and CDS spreads implies that positive $\alpha(t)$ values cannot exist in absence of arbitrage, empirical evidence during times of stress suggest otherwise.[54] Carr and Wu (2007) show that for many firms the put option values from equity options and CDS are indeed closely related.[55] In stress situations, however, the implicit put options from equity markets and CDS spreads can differ in their capital structure impact, and, thus, should be priced differently. Besides guarantees, there are several distortions that could set apart put option values derived from CDS and equity prices, even if the risk-neutral default probability (RNDP) implied by the CDS spread (based on an exponential hazard rate) were the same as the RNDP component of the equity put option value. Some of these factors include (i) the recovery-at-face value assumption underlying CDS spreads, and (ii) different risk horizons of put option values derived from CDS and equity prices. We address these two potential sources of distortion via an adjustment for "basis risk" in equation (30) above.

Box 2. Interaction and Feedback between the Sovereign and Financial Sector Balance Sheets Using the Systemic CCA Framework.

CCA can be used to derive a risk-adjusted measure of expected losses implied by the sovereign and financial sector balance sheets in order to illustrate the interaction and potential destabilization of credit spreads in both the sovereign and banking sectors (Gray and Jobst, 2010b and 2011a). In the absence of measureable equity and equity volatility for sovereign debtors, the term structure of sovereign spreads can be used to estimate implied sovereign assets and asset volatility and calibrate market-implied sovereign risk-adjusted balance sheets.

The sovereign credit default swap (CDS) spread

$$s_{CDS_{sov}}(t) = -\frac{1}{T-t}\ln\left(1-\frac{P_{sov}(t)}{B_{sov}e^{-r(T-t)}}\right)\times 10{,}000$$

is defined by the sovereign implicit put option

[54] Arbitrage trading between both prices shows that synthetic replication of credit protection on guaranteed bonds using equity can be obtained from combining a long position in an equity option "straddle" with a short CDS position.

[55] Carr and Wu (2007) find that equity options used in a modified CCA seem to produce risk-neutral default probabilities (RNDP) that closely match RNDPs derived from CDS (sometimes higher, sometimes lower, and differences seem to predict future movements in both markets). Yu (2006) uses a less sophisticated model based on *CreditGrades*, which contains some simplifying assumptions.

$$P_{sov}(t) = B_{sov} e^{-r(T-t)} \Phi\left(-d - \sigma_{A_{sov}} \sqrt{T-t}\right) - A_{sov}(t) \Phi(-d),$$

with default barrier, B_{sov} (or threshold that debt restructuring is triggered), implied sovereign assets $A_{sov}(t)$, asset volatility $\sigma_{A_{sov}}$, over time horizon *T–t* at the risk-free discount rate *r*, subject to the duration of debt claims and leverage $A_{sov}(t)/B_{sov}$ of the sovereign. Rearranging the equation above for the implicit sovereign put option gives

$$\frac{P_{sov}(t)}{B_{sov} e^{-r(T-t)}} = \Phi\left(-d - \sigma_{A_{sov}} \sqrt{T-t}\right) - \frac{A_{sov}(t)}{B_{sov} e^{-r(T-t)}} \Phi(-d),$$

which can be inserted in the first equation for sovereign spreads so that

$$\hat{s}_{CDS_{sov}}(t) = -\frac{1}{T-t} \ln\left(1 - \left(\Phi\left(-d - \sigma_{A_{sov}} \sqrt{T-t}\right) - \frac{A(t)_{sov}}{B_{sov} e^{-r(T-t)}} \Phi(-d)\right)\right) \times 10{,}000.$$

Thus, the sovereign default barrier (based on available information on the periodic debt service) and the full term structure of the sovereign credit curve (e.g., the sovereign CDS spreads at maturity tenors of 1, 3, 5, 7, and 10 years) can be used to determine what combination of both sovereign assets and asset volatility generates a modeled credit spread that most closely matches the observed credit spread at the chosen maturity term.

This specification also helps assess the effect of changes in any constituent component of the sovereign asset value —reserves, the primary fiscal balance, and the implicit banking sector guarantee—on sovereign default risk (and corresponding sovereign credit spreads) for the purposes of sensitivity analysis. The sovereign asset value is defined as

$$A(t)_{sov} = R(t) + PS(t) e^{-r(T-t)} - \alpha P(t)_{bank} + Other,$$

with foreign currency reserves, R, net fiscal assets (i.e., the present value of the primary fiscal surplus (PS)), the implicit and explicit contingent liabilities $\alpha P(t)_{bank}$ to the banking sector, and remainder items ("Other"). The value of reserves can be observed, and the contingent liabilities to the banking sector are defined as the share α of expected losses in the banking sector (see equation (35)), denoted by the put option

$$P_{bank}(t) = B_{bank} e^{-r(T-t)} \Phi\left(-d - \sigma_{A_{bank}} \sqrt{T-t}\right) - A_{bank}(t) \Phi(-d),$$

which can be estimated jointly for all banks using the Systemic CCA framework.[56] Thus, the full specification of the sovereign CDS spread would be

[56] This residual includes a number of government assets and various unrealized liabilities, such as pension and healthcare obligations as well as contingent financial support to non-bank financial institutions, guarantees from other governments or multilaterals, and/or backstop assets (e.g., land or other public sector assets of value).

$$\hat{s}_{CDS_{sov}}(t) = -\frac{1}{T-t}\ln\left(1-\left(\Phi\left(-d-\sigma_{A_{sov}}\sqrt{T-t}\right)-\frac{R(t)+PS(t)e^{-r(T-t)}-\alpha P(t)_{bank}+Other}{B_{sov}e^{-r(T-t)}}\Phi(-d)\right)\right)\times 10,000.$$

Conversely, the effect of contingent liabilities on the credit spreads of banks can be seen as the fraction of default risk $(1-\alpha)P(t)_{bank}$ retained by banks (see equation (34)) plus the constant, δ, which reflects an add-on to the CDS premium and solves the equation below if high sovereign spreads were to increase expected losses in the banking sector such that

$$\hat{s}_{CDS}(t)_{bank} = \left(-\frac{1}{T-t}\ln\left(1-\frac{(1-\alpha)P(t)_{bank}}{B_{bank}e^{-r(T-t)}}\right)+\delta^{*}\right)\times 10,000\,.$$

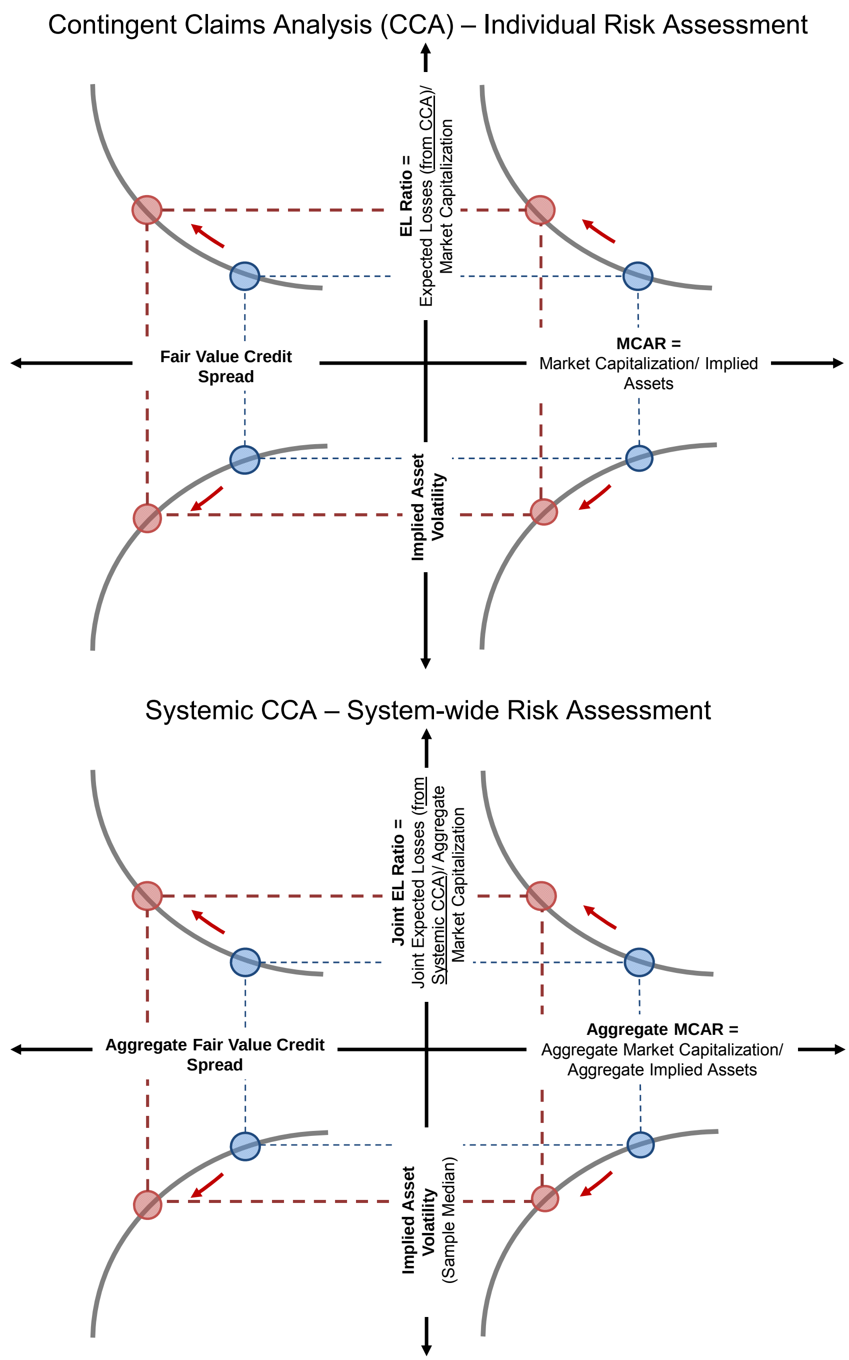
falls below the actual (i.e., observed) bank CDS spread, $s_{CDS_{bank}}(t)$.

This simple model shows various ways in which sovereign and bank spreads can interact and potentially lead to a destabilization process. If sovereign spreads increase this can lead to an increase in bank spreads as the potential for sovereign guarantees decreases (i.e., the value of α decreases), (ii) the value of the implicit bank put option increases as the value of the bank's holdings of government debt decreases, and (iii) the bank default barrier may increase due to higher borrowing costs as the premium (δ) increases.

C. Integrated Market-implied Capital Assessment Using CCA and Systemic CCA

Since the evaluation of default risk is inherently linked to perceived riskiness as implied by the changes in equity and equity option prices (see Section II.B above), the risk-adjusted (economic) balance sheet approach—and its multivariate extension in the form of Systemic CCA—provide an integrated analytical framework for a market-based assessment of individual and system-wide solvency. Higher expected losses to creditors lead to less than a one-to-one decline in the market value of equity, depending on the severity of the perceived (and actual) decline in the value of assets, the degree of leverage, and the volatility of assets. Thus, the interaction between these constituent elements of the risk-adjusted (economic) balance sheet gives rise to a market-based capital assessment, which links changes in the value of assets directly to the market value of equity.

Figure 3. Integrated Market-based Capital Assessment Using CCA and Systemic CCA (based on non-linear relation between CCA-based CAR, EL Ratio, Fair Value Credit Spread, and Implied Asset Volatility).



As opposed to the regulatory definition of capitalization (based on discrete nature of accounting identities), the CCA-based capital assessment hinges on the historical dynamics of implied asset values and their effect on the magnitude of expected losses.

Two broad indicators serve to illustrate this point: (i) the *market-implied capital adequacy ratio* (MCAR), which is defined as the ratio between the amount of market capitalization divided by the implied asset value of a firm, and (ii) the *expected loss ratio* ("EL Ratio") between individual expected losses (at a defined percentile of statistical confidence) and the amount of market capitalization (see Figure 3, upper chart). Both ratios can also be generated on a system-wide basis, with inputs defined as sample aggregates of input variables to the multivariate specification of the Systemic CCA. In this case, the EL ratio is defined by magnitude of joint expected losses (at the 95th percentile, for instance) relative to aggregate market capitalization of all sample firms (see Figure 3, lower chart).

The market-implied capital assessment can also help identify a capital shortfall. The MCAR represents an important analogue to the regulatory definition of capital adequacy, which reflects the market's perception of solvency (based on the implied value of assets relative to the prevailing default risk) and is completely removed from prudential determinants of default risk affecting a bank's capital assessment, such as risk-weighted assets (RWAs). In the empirical section of the paper the relation between the MCAR and regulatory capital adequacy is investigated further, which establishes the possibility of defining a systemically-based capital shortfall using CCA-based measures of solvency (see Section V).

In addition, Figure 3 shows the stylized sensitivity of both measures at the individual and system-wide level to the fair value credit spread and the asset volatility to illustrate the non-linear dynamics between the different components of the valuation model to changes in market-implied default risk. For instance, as higher expected losses threaten to erode market capitalization (as a lower implied asset value and higher implied asset volatility increase the risk premium for equity), the EL ratio rises, and the MCAR decreases, which also results in a higher fair value credit spread. Similar to the specification of the CDS spread under the risk-neutral measure in equation (32), the fair value credit spread (in basis points), $s_{FV_j}(t)$, based on individual and joint expected losses of $j \in m$ can be defined as a constant hazard rate and written as

$$s_{FV_j}(t) = -\frac{1}{T-t}\ln\left(1-\frac{P_E(t)}{B_j e^{-r(T-t)}}\right)\times 10{,}000, \tag{37}$$

and, using the specification of the quantile function of joint expected losses in equation (25),[57]

[57] Note that for simplicity, the notation for the estimation time horizon and sample size has been suppressed in this section.

$$s_{FV_m}(t) = -\frac{1}{T-t}\ln\left(1-\frac{G^{-1}(a)}{\sum_j^m B_j e^{-r(T-t)}}\right)\times 10{,}000, \tag{38}$$

respectively, which implies an individual and joint periodic default probabilities

$$1-\exp\left(-s_{FV_j}(t)\times t\right) \equiv \frac{P_E(t)}{B_j e^{-r(T-t)}} \tag{39}$$

and

$$1-\exp\left(-s_{FV_m}(t)\times t\right) \equiv \frac{G^{-1}(a)}{\sum_j^m B_j e^{-r(T-t)}} \tag{40}$$

so that $\left(1-s_{FV_j}(t)\right)B_j e^{-r(T-t)} = PD_j \times LGD_j$ and $\left(1-s_{FV_m}(t)\right)\sum_j^m B_j e^{-r(T-t)} = PD_m \times LGD_m$ at time to maturity $T-t=1$.

These market-based measures of assessing capital adequacy can be transposed into estimates of capital shortfalls in order to compare them to those obtained from balance sheet-based analysis. In the context of CCA, the market-implied capital shortfall can be defined as the amount of capital required to maintain solvency conditions under stress irrespective of changes in expected losses in excess of any existing capital buffer of common equity. In this way, the risk-based assessment of capital adequacy can be reconciled with prudential solvency standards.[58]

D. Systemic CCA and Stress Testing

The Systemic CCA framework can also be used to project the dynamics of systemic solvency risk for the purposes of assessing capital adequacy under stress scenarios. Based on the historical distribution of firm-specific market-implied expected losses (or the associated contingent liabilities) (which can be generated using conventional or more advanced option pricing techniques (see Box 1)), future changes in market-implied default risk of each firm can be estimated over the selected forecast horizon (based on their macro-financial linkages under different stress scenarios) and finally combined to define system-wide solvency risk within the Systemic CCA framework.

First, a suitable univariate input is defined. Individual CCA-based estimates of market-

[58] Note that this aspect will be further explored in the next section.

implied expected losses can generate the following measures that support a firm-specific assessment of capital adequacy (and which have been applied empirically already):

(i) *contingent liabilities as the share of expected losses that are potentially transferred to the public sector* if a systemically relevant firm were to fail (see Section III.B);

(ii) *capital shortfall based on the amount of expected losses in excess of existing common (core) equity Tier 1 capital above the regulatory minimum* (see Section III.C); [59] and

(iii) *capital shortfall based on the market-implied capital adequacy ratio (MCAR)* generated from the change in market capitalization relative to the asset value under the impact of expected losses (see Section III.C);

The first two measures are presented in the empirical sections of this paper (see Sections IV and V), which illustrate the practical application of the Systemic CCA framework for macroprudential stress testing as part of the IMF FSAP exercise.

Second, empirical and theoretical (endogenous) models can be used to specify the macro-financial linkages of expected losses under certain stress scenarios. By modeling how the impact of macroeconomic conditions on income components and asset impairment (such as net interest income, fee income, trading income, operating expenses, and credit losses) has influenced a bank's market-implied expected losses in the past, it is possible to link individual estimates of expected losses (and their implications for contingent liabilities and potential capital shortfall) to a particular macroeconomic path (and associated financial sector performance) in the future. Alternatively, the historical sensitivity of the main components of the option pricing formula themselves (i.e., the implied asset value and asset volatility) can be determined by applying stress conditions directly to the theoretical specification of market-implied expected losses. Finally, the implied asset value underpinning the put option value of each sample bank could be adjusted by the projected profitability under different stress scenarios in order to determine the corresponding change in the level of market-implied expected losses.

[59] The capital shortfall of each firm is defined as the amount of capital required to preserve pre-stress solvency conditions, i.e., the existing capital level above the regulatory minimum. It is measured as the marginal change of expected losses over the forecast horizon relative to the expected losses measured at the end point of the historical sample of observations. Note that the market value of equity is considered equivalent to common equity Tier 1 capital since CCA does not specify reported capital tiers but implicitly assumes that any potential loss first affects the most junior claims on firm assets. However, cross-sectional differences in the quality of capital held by firms will affect changes in valuation, and, thus, individual estimates of market-implied capital shortfall. In the empirical application of Systemic CCA for capital assessment, Tier 1 capital was used for consistency purposes (due to divergent regulatory treatments of more junior capital tiers).

The following methods have been used thus far to model the macro-financial linkages affecting the change in expected losses (and its impact on contingent liabilities and potential capital shortfall):

(i) *satellite model*—The historical sensitivity of the market-implied expected losses (or the underlying option pricing elements themselves) is estimated based on several macroeconomic variables (such as short-term and long-term interest rates, real GDP, and unemployment) and bank-specific variables (net interest income, operating profit before taxes, credit losses, leverage, and funding gap) using some econometric approach, such as a dynamic panel regression specification (IMF, 2011b, 2011c, 2011e, 2011f, and 2012);

(ii) *structural model*—The value of implied assets is adjusted by forecasts of net operating income in order to derive a revised put option value (after re-estimating implied asset volatility), which determines changes in market-implied expected losses relative to the change of the payment obligations arising from maturing debt claims in each year over the forecast horizon (IMF, 2011c and 2011e).[60]

Third, the joint contingent liabilities (and potential capital shortfall) are derived from estimating the multivariate density of each bank's individual marginal distribution of forecasted expected losses (if any) and their dependence structure. If the number of forecasted expected losses is insufficient due to the low frequency of macroeconomic variables chosen for the specification of macro-financial linkages (e.g., quarterly values over a five-year forecast horizon), the time series of expected losses could be supplemented by historical expected losses to a point when the number of observations are sufficient for the estimation of joint contingent liabilities or capital shortfall.[61]

Box 3. The Importance of Distributions and Dependence in Stress Testing.

For stress testing to inform a realistic measure of capital, the Systemic CCA framework satisfies two essential objectives—the variability of risk factors and their dependence under all plausible scenarios (in addition to their likelihood and severity). While it is generally straightforward to generate stress test results based on the effect of a *single* risk, combining multiple risk factors (and the extent to which firms might affect each other in terms of default risk) under different scenarios tends to complicate a reliable capital assessment. In particular, most conventional balance sheet-based stress tests (similar to recent system-wide capital assessments of banking sectors) do not account for default dependencies across institutions, omitting the potential role of

[60] This approach also generates estimates of funding costs and the CCA-based capital ratio (i.e., market value of equity to market value of assets).

[61] A sufficient number of observations are required for a reliable and robust estimation of the univariate distribution functions and the dependence structure.

non-linearities, and, hence, possibly underestimating extreme outcomes.[62] Many of these approaches also tend to imply sequential rather than concurrent changes of prudential indicators whose realization is assumed to be fully reflected in their accounting treatment.

Thus, consideration needs to be given to the possibility that one risk factor might increase the likelihood of a realization in other risk factors (with several factors affecting one firm or common shocks affecting multiple firms at the same time), especially under stress conditions. Assuming that risk factors are not fully correlated, it is reasonable to account for their dependence structure and combinations of stress testing parameters in which the individual impact of each risk is lower than the appropriate percentile for that risk in isolation. However, risk factors may be weakly correlated under normal economic circumstances but highly correlated in times of distress (as correlations become very high in downturn conditions).[63] Similarly, the joint default risk within a system of firms varies over time and depends on the individual firm's likelihood to cause and/or propagate shocks arising from the adverse change in one or more risk factors. Given that large shocks are transmitted across entities differently than small shocks, measuring non-linear dependence in stress testing can deliver important insights about the joint tail risks that arise in extreme loss scenarios (Gray and Jobst, 2009a; Jobst, *forthcoming*). This would also include measuring the different impact of combinations of risk factors on estimates of uncertainty about the realization of joint outcomes, which affects system-wide capital adequacy.

Another critical area is the sensitivity of stress test results to the historical volatility of risk factors. Ideally, this would be done by assigning a probability estimate to the capital assessment using a risk-adjusted measure of solvency, recognizing that prudential information at a certain point in time reflects the outcome of a *stochastic process rather than a discrete value*. Figure 4 compares the conceptual differences of loss measurement under both balance sheet- and distribution-based approaches affecting the capital assessment. Instead of relying only on accounting values, a risk-based measure of solvency, such as market-implied expected losses (and associated capital shortfall) within the Systemic CCA framework considers the historical dynamics of default risk that affect the capital assessment under stress conditions based on VaR, or, as a more coherent risk metric (Artzner and others, 1997), the *Expected Shortfall* (ES), i.e., the average density of extreme losses beyond VaR at a selected percentile level.

A risk-based approach is likely to involve valuation methods, which entails distinct benefits but also significant drawbacks. On one hand, valuation methods offer an expedient way to characterize scenarios, and accommodate different combinations of risk factors, for the purpose of examining capital at different levels of statistical confidence. They allow stress testers to carry out sensitivity analysis without re-estimating "satellite models" (i.e., translating key risk parameters' response to adverse shocks into an economic assessment of solvency) or re-calculating the effect of shocks to income generation (such as in balance sheet-based tests). On the other hand, the validity of valuation models could be undermined by distress events. Economic models are constructed as general representations of reality that, at best, capture the broad outlines of economic phenomena in a steady-state. Thus, the statistical apparatus underlying conventional asset pricing theory fails to capture

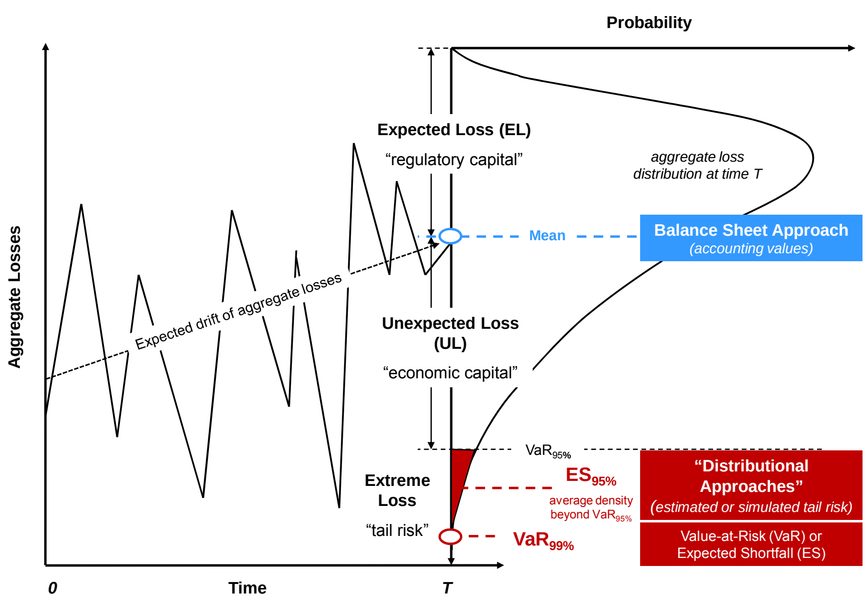
[62] The most important stress test exercises were the *Supervisory Capital Assistance Program* (SCAP) in 2009 and the *Comprehensive Capital Analysis and Review* (CCAR) in 2012 by the U.S. Federal Reserve as well as the stress tests administered by the European banking supervisors (CEBS and EBA) in 2010 and 2011, respectively.

[63] In these instances, there is a higher probability of larger joint outcomes due to a considerable shift of the average away from the median ("excess skewness") and a narrower peak ("excess kurtosis") of the probability distribution. If distributions become highly skewed, large extremes may even cause the mean to become undefined, which is an important consideration when applying stress tests.

sudden and unexpected realizations beyond historical precedent, such as the rare and non-recurring events during the recent financial crisis.

A comprehensive capital assessment for financial stability analysis would require the system-wide characterization of dependence and historical dynamics of risk factors. Balance sheet-based approaches capture mostly the firm-specific impact of institutional distress in response to exogenous shocks *(microprudential perspective)*; in contrast, considering the dependence of risk factors and their stochastic impact on the solvency of multiple firms helps identify common vulnerabilities to risks (and their changes over time) as a result of the (assumed) collective behavior under system-wide distress *(macroprudential perspective)*. As a result, in recent FSAPs—most notably, in the cases of Germany, Spain, Sweden, the United Kingdom, and the United States (IMF, 2010c, 2011c, 2011e, 2011f, and 2012)—institution-level stress tests were combined with the portfolio-based Systemic CCA framework to derive capital assessments from a system-wide perspective—after controlling for the dependence structure of risk factors and the stochastic nature input parameters—in support of a more comprehensive assessment of the strengths and vulnerabilities of the financial system.

Figure 4. Key Conceptual Differences in Loss Measurements.



IV. Empirical Application: Systemic Risk from Expected Losses and Contingent Liabilities in the U.S. Financial System

This section illustrates some of the results from applying the Systemic CCA framework as a market-based, top-down (TD) capital assessment methodology supporting the financial stability analysis of the U.S. financial system in the context of the IMF's recent FSAP (IMF, 2010c). The model was estimated based on market and balance sheet information of 33 large commercial banks, investment banks, insurance companies, and special purpose financial institutions using daily data between January 1, 2007 and end-January 2010.[64,65] By treating these sample firms as a portfolio of risk-adjusted asset values, the associated individual and joint market-implied expected losses (and contingent liabilities) of the financial sector were estimated in order to quantify the individual firms' contributions to systemic risk in the event of distress ("tail risk"). The following exhibits (see Figures 5-7 and Tables 6-7) illustrate—in broad strokes—the analytical progression from the summation of individual contingent liabilities derived from implicit put option values to their multivariate distribution. Based on these results, we also present the individual contributions of sample firms to the change of joint contingent liabilities (see Figure 8) and calculate their institution-specific systemic risk surcharge in accordance with the specification in equation (30) above.

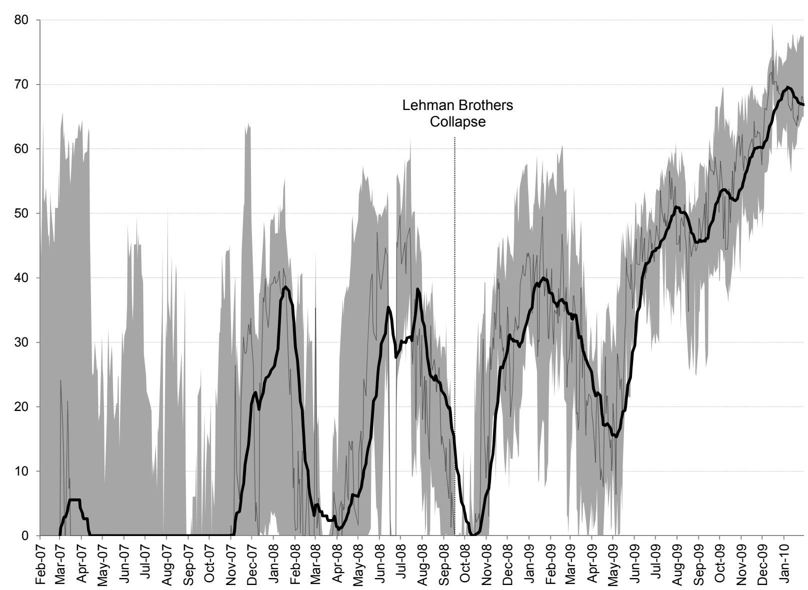
[64] The sample comprised the following firms: Bank of America, J.P. Morgan Chase, Citigroup, Wells Fargo ("Top 4 Banks"); Goldman Sachs, Morgan Stanley, Bear Stearns, Lehman Brothers ("Investment Banks"); PNC Financial Services Group, U.S. Bancorp, SunTrust Banks, BB&T Corp., Regions Financial Corporation, Fifth Third Bancorp, KeyCorp ("Regional Banks"); Bank of New York Mellon Corp., State Street Corp., Northern Trust Corp. ("Processing Banks"); CIT, Ameriprise ("Consumer Banks"), Fannie Mae, Freddie Mac, Sallie Mae ("GSEs"); AIG, Metlife, Prudential, Hartford, Allstate, Principal, Travelers, Genworth, Aflac, Lincoln ("Insurance").

[65] Key inputs used were the daily implied asset values derived as the expectation over the state price density (SPD) using equity option data (from Bloomberg), the default barrier estimated for each firm based on quarterly financial accounts (from Moody's *KMV CreditEdge*), the risk-free rate of interest (r=3.0 percent), a one-year time horizon T=1, and one-year credit default swap (CDS) spreads on senior debt (from *MarkIt)*. The default barrier was set equal to total short-term debt plus one-half of long-term debt, consistent with the approach taken by Moody's *KMV CreditEdge*. More specifically, the SPD is calculated using daily European-style equity call option price data over the sample time period with a time-to-maturity of three months. Besides the maturity tenor, option price data also included information on strike prices as well as actual and corrected spot prices (with implied dividends taken into account). Outputs were the expected losses (i.e., the implicit put option value over a one-year horizon) and the associated contingent liabilities from each bank (i.e., alpha-value times the implicit put option derived from equity). Their multivariate distribution across all sample firms was estimated over a rolling window of 60 working days (i.e., three months) with daily updating, consistent with the maturity tenor of equity option prices used for the estimation of the implied asset value of each firm.

A. Estimating Individual Contingent Liabilities

Based on the market-implied expected losses from market price data and firm-specific information on leverage as well as the maturity and amount of debt service, the fraction of those losses that might become contingent liabilities to the public sector in cases of individual distress can be calculated. Figure 5 below shows the changes of the median and the inter-quartile range of the distribution of alpha-values (as defined in equation (35) above) across all sample institutions that generated significant contingent liabilities over time. The secular increase (and decreasing dispersion) of alpha-values indicates growing financial sector support to the financial sector as a result of a widening gap between default risk implied by equity and CDS put option values. Sudden declines in alpha-values around April 2008, October 2008, and May 2009 are attributable to little or no market confidence in any explicit or implicit public sector support to the sample firms amid a greater alignment of a decline in market capitalization and a commensurate rise in CDS-implied default risk of distressed firms.[66]

Figure 5. United States: Financial Sector – Time Pattern of the Alpha-Value.



Sample period: 01/03/2007-01/29/2010. *Note:* The chart shows the individual alpha-values of 29 sample firms (commercial banks, investment banks, insurance companies, and special purpose financial institutions) with significant results. The solid black line shows the median alpha-value over a 20-(working) day rolling window, which represents the trend line of daily observations (light grey line). The shaded area shows the inter-quartile range (IQR) between the 25th and the 75th percentile of the distribution of alpha-values.

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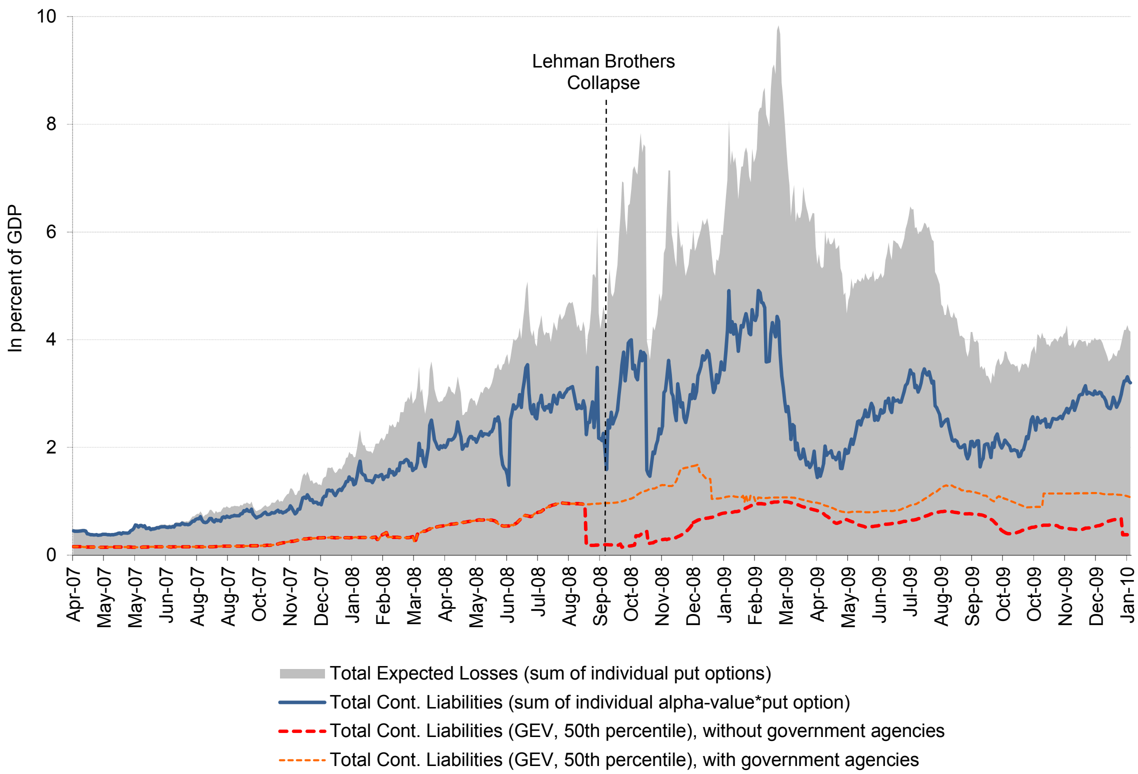
[66] This means that potential contingent liabilities to the financial sector have not disappeared but are persistent (outside the current specification of the alpha-value).

Figure 6. United States: Systemic CCA Estimates of Market-Implied Total Contingent Liabilities and Multivariate Density of Contingent Liabilities.

(In percent of real GDP)



Sample period: 01/03/2007-01/29/2010, daily observations of individual put option values (i.e., expected losses) conditional on the endogenous alpha-value of implicit guarantees of 36 sample banks, insurance companies, and other financial institutions. *Note:* The multivariate density is generated from univariate marginals that conform to the Generalized Extreme Value Distribution (GEV) and a non-parametrically identified time-varying dependence structure. The marginal severity and dependence are estimated over a window of 60 working days (with daily updating) (see Appendix 3). There are two 50th percentile lines in this figure. The solid line shows results for the case where government-sponsored financing agencies were *de facto* nationalized (which warrants their exclusion from the sample on September 8, 2008).The second, dashed, 50th percentile line shows the case where these government-sponsored financing agencies are left in the sample. Note that daily equity prices were still available after the exclusion date but it can be argued these data may be much less informative.

Figure 6 shows that total expected losses (area) and contingent liabilities (lines) had begun to increasingly diverge during the beginning of the financial crisis, with each having reached their highest values between the periods just after the collapse of Lehman Brothers in September 2008 and end-July 2009. The persistent difference between expected losses and contingent liabilities (as indicated by the gap between the contours of the shaded area and the blue line) suggests that markets expected that on average more than 50 percent of total expected losses could have been transferred to the government in the event of default. The sum of individual contingent liabilities peaks at more than 4 percent of GDP at the end of February 2009, averaging 2.5 percent of GDP over the entire sample period.

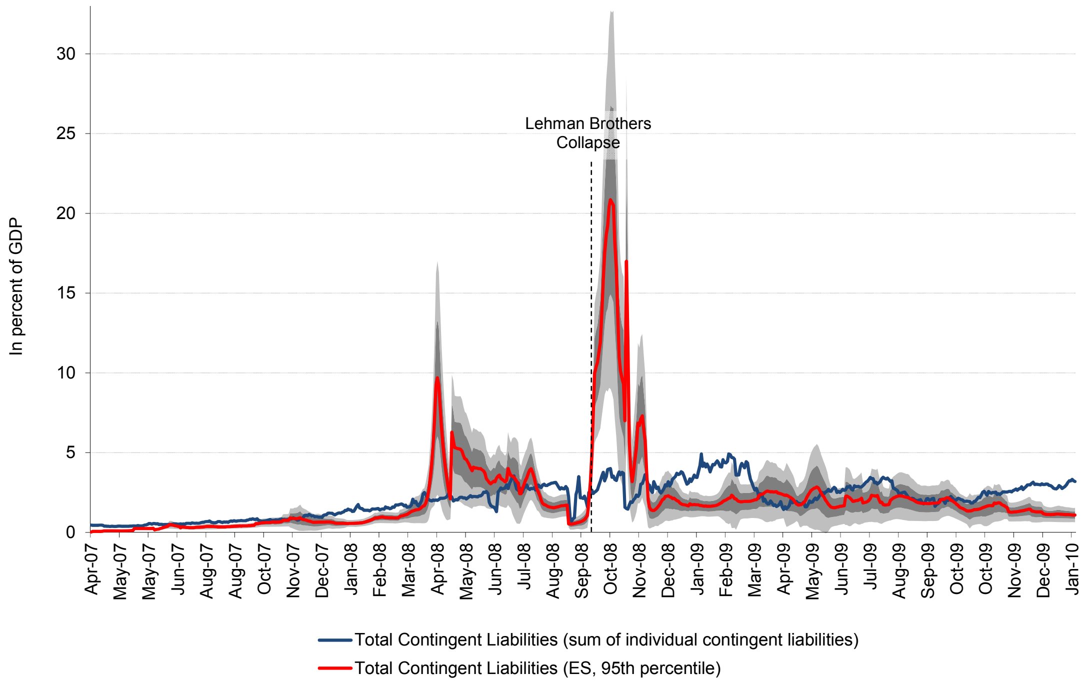
While the summation of expected losses and contingent liabilities represents a correct measure of aggregate systemic risk, it would reflect the concurrent realization of individual distress at an average degree of severity without consideration of their conditional probability of default. Thus, it is critical to take into account the intertemporal changes in the dependence structure of risk-adjusted default risk within this "portfolio" of sample institutions for the estimation of systemic risk arising from the *joint* default probability. After controlling for the dependence structure using the aggregation technique underpinning the Systemic CCA framework (see equations (19) and (20) above), aggregate measure of contingent liabilities, i.e., the median value (50^{th} percentile) of the multivariate distribution of contingent liabilities, dropped significantly (see orange and red dotted lines in Figure 6).

B. Estimating the Systemic Risk from Joint Contingent Liabilities

After controlling for the market perception (via CDS prices) about the residual risk retained in the financial sector, estimates of systemic risk were obtained from the multivariate distribution of contingent liabilities, which was derived on a daily basis over 60-day rolling window over the historical sample time period. The systemic risk from contingent liabilities was considerable during the financial crisis, and increased sharply after the collapse of Lehman Brothers (see Figure 7). However, the time pattern of joint contingent liabilities showed spikes as early as April 2008, indicating already high potential government exposure to joint financial sector distress in the wake of the Bear Stearns bailout. In fact, extreme tail risk (i.e., the 95^{th} percentile ES (see equation (26) above)) of expected losses transferred to the government (red line) exceeded nine percent of GDP already in April and almost reached 20 percent of GDP in October 2008. In other words, during this period of exceptional systemic distress, market prices of sample institutions implied joint contingent liabilities of 20 percent of end-2009 GDP with a probability of less than five percent over a one-year time horizon. The magnitude of such tail risk, however, dropped to under two percent of end-2009 GDP during the remainder of 2008 before rising again in May the following year.

Figure 7. United States: Systemic CCA Estimates of Market-Implied Average Daily Expected Shortfall (ES) [95th Percentile] based on Multivariate Density of Contingent Liabilities.

(In percent of real GDP)



Sample period: 01/03/2007-01/29/2010, daily observations of individual put option values (i.e., expected losses) conditional on the endogenous alpha-value of implicit guarantees of 36 banks, insurance companies, and other financial institutions. *Note:* The red line shows the expected shortfall (ES) for the entire sample at a 95th percentile threshold within a confidence band of one and two standard deviations (grey areas). The multivariate density is generated from univariate marginals that conform to the Generalized Extreme Value Distribution (GEV) and a non-parametrically identified time-varying dependence structure. The marginal severity and dependence are estimated over a window of 60 working days (with daily updating).

C. Estimating the Individual Contributions to Systemic Risk from Contingent Liabilities

Failed and bailed-out firms were on average the largest contributors to systemic risk from joint contingent liabilities, especially prior to the collapse of Lehman Brothers. An examination of the percentage share of systemic risk in different periods (see Table 6 and Figure 8) by four major groups—commercial/investment banks, insurance companies, failed/bailed-out firms, and other financial institutions, suggests that pre-crisis (up to September 14, 2008), the nine financial institutions that either failed (6) or were rescued (3) later on were the biggest contributors at 69.5 percent. As the six failed institutions gradually dropped out of the sample after September 15, 2008, their systemic risk contribution declined and large banks increased their share to about one-third of total systemic risk. Interestingly, the contribution of insurance companies remained low during the height of the credit crisis before rising to nearly 24 percent on average during the second half of 2009.

Table 6. United States: Systemic CCA Estimates of Market-Implied Average Individual Contribution to Systemic Risk from Contingent Liabilities.

(In percent of average expected shortfall at the 95th percentile during each time period)

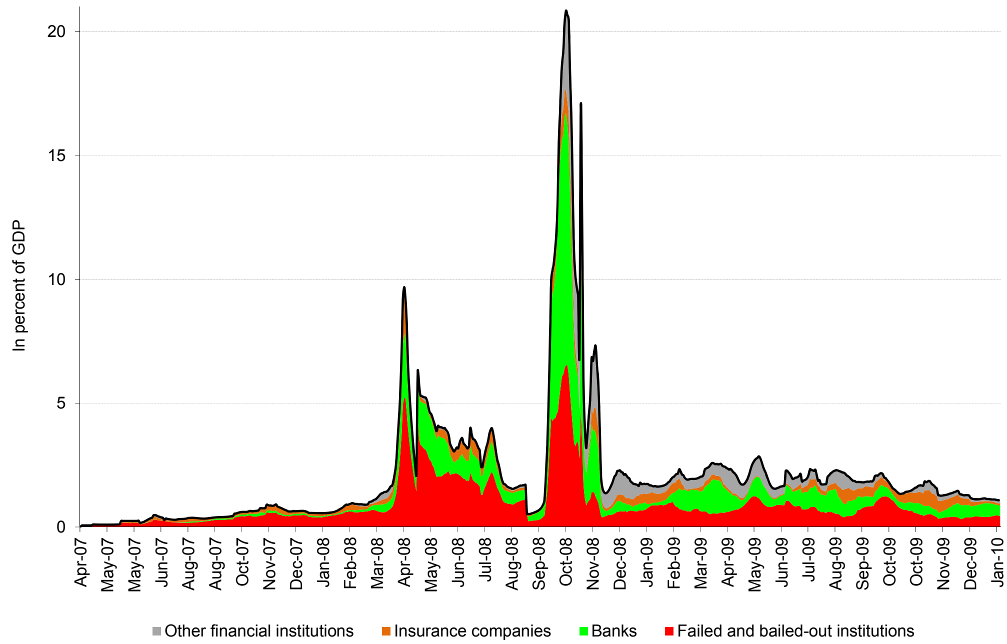
	Pre-Crisis	Crisis Main Periods			Total Period		
Sample Group	July 1, 2007—Sept. 14, 2008	Sept. 15—Dec. 31, 2008	Jan. 1—May 8, 2009	May 11—Dec. 31, 2009	April 1, 2007—Jan. 29, 2010	*Share of total liabilities*	*Share of total contingent liabilities*
Banks	15.6	36.1	34.6	37.7	27.2	*37.2*	*7.7*
Insurance companies	13.9	9.8	14.1	23.7	15.0	*16.3*	*4.8*
Other, non-bank financial institutions	3.0	31.8	32.2	21.0	14.7	*2.8*	*0.9*
Failed (or bailed-out) financial institutions	69.5	22.3	19.1	17.6	43.1	*43.7*	*86.6*

Note: Each group's percentage share aggregates each constituent institution's time-varying contribution to the expected shortfall at the 95th percentile of the multivariate density of total contingent liabilities during different time periods. The share of failed or bailed-out financial institutions declines over time as they drop out of the original sample. The two additional columns to the far right compare the groupwise contribution to systemic risk from joint contingent liabilities to other relative measures (which would inform a naïve attribution of systemic relevance), such as the share of total contingent liabilities (i.e., the sum of individual equity put option values multiplied by the corresponding alpha-values) and the share of total reported liabilities.

The results also show that certain segments of the financial sector contributed more to systemic risk than the sum of contingent liabilities would otherwise suggest. For instance, ignoring the dependence structure of market-implied default risk using risk-adjusted balance sheets by simply summing up contingent liabilities would have *overstated* the tail risk contribution of failed/rescued financial institutions (by 43.5 percentage points on average; see fifth and last columns of Table 6) over the total time period (between April 2007 and end-January 2010) while severely *understating* the contribution of all other, individually more resilient financial institutions, including other banks and insurance companies (by 19.5 percentage points and 10.2 percentage points, respectively). That being said, using the share of reported total liabilities of each group in the financial sector as a naïve indicator seems more consistent with the groupwise contributions to systemic risk derived from the Systemic CCA approach, especially for failed/rescued financial institutions and insurance companies.

Figure 8. United States: Decomposition of Systemic CCA Estimates of Market-Implied Average Daily Expected Shortfall (ES) [95th Percentile] based on Multivariate Density of Contingent Liabilities.

(In percent of real GDP)



Sample period: 01/03/2007-01/29/2010, daily observations of individual put option values (i.e., expected losses) conditional on the endogenous alpha-value of 36 banks, insurance companies, and other financial institutions. *Note:* The chart shows the group-wise contribution to expected shortfall (ES) at a 95th percentile threshold. The multivariate density is generated from univariate marginals that conform to the Generalized Extreme Value Distribution (GEV) and a non-parametrically identified time-varying dependence structure. The marginal severity and dependence are estimated over a window of 60 working days (with daily updating).

D. Calculating a Systemic Risk Surcharge for Contingent Liabilities

The amount of individual contributions to the estimated joint contingent liabilities also helps assess the fair value price of a possible systemic risk surcharge, which was modeled as an insurance contract to cover expected cost of public sector interventions. The premium rate was calculated based on the systemically-based dollar losses of each institution as if a five percent tail event had occurred relative to the aggregate amount of outstanding liabilities of all sample firms (see equation (30)). More specifically, the average marginal contribution of each firm to joint contingent liabilities over the specified time period was divided by the average of the discounted present value of total liabilities of all firms over the same time period in order to derive the hazard rate.

Findings from this analysis for different time periods suggest an annual systemic surcharge for systemically important financial institutions of about 50 basis points on average. A reasonable average systemic surcharge for each sample firm during the pre-crisis period would have been as low as 39 basis points per year between July 2007 and September 2008 (before the Lehman collapse) (see Table 7). At a higher statistical confidence level (such as 95 percent), this value would have reached 60 (and 317) basis points shortly before (after) the collapse of Lehman Brothers as most tail events would be priced into the systemic risk charge. Since controlling for the share of insured deposits reduces the value of the total default barrier *B* by 15 percent on average, the estimated systemic risk surcharge would have decreased accordingly.[67]

Table 7. United States: Systemic CCA Estimates of Market-Implied Fair Value Surcharge for Systemic Risk based on Total Contingent Liabilities.

(In average values of point estimates at the 50th and 95th percentiles)

	50th percentile (median)		**95th percentile**	
Time Period	Amount *(In billion U.S. dollars)*	Annual fee/surcharge *(In basis points)*	Amount *(In billion U.S. dollars)*	Annual fee/surcharge *(In basis points)*
Pre-Crisis				
July 1, 2007—Sept. 14, 2008	59	39	91	60
Crisis Main Periods				
Sept. 15—Dec. 31, 2008	43	28	479	317
Jan. 1—May 8, 2009	119	76	359	232
May 11—Dec. 31, 2009	88	58	227	150
Total Period				
April 1, 2007—Jan. 29, 2010	74	49	214	142

V. Empirical Application: Stress Testing Systemic Risk from Expected Losses in the U.K. Banking Sector

This section summarizes the findings from applying the Systemic CCA framework as a market-based top-down (TD) solvency stress testing model as part of the IMF's FSAP stress test of the U.K. banking sector (IMF, 2011c).[68] The model was estimated based on market and balance sheet information of the five largest commercial banks, the largest building society, and the largest foreign retail bank using daily data between January 3, 2005

[67] Given the time-variation of contingent liabilities (and their distributional behavior), this surcharge could be augmented with counter-cyclical properties by combining estimates over different estimation time periods and at different percentile levels of statistical confidence.

[68] The Systemic CCA framework also served as a means of cross-validating the stress testing results from bottom-up exercise conducted jointly with FSA.

and end-March 2011.[69,70] Like in the case of the United States (see Section IV above), the individual market-implied expected losses were treated as a portfolio, whose combined default risk over a pre-specified time horizon generated estimates of individual firm' contributions to systemic risk in the event of distress ("tail risk").

A. Illustrating the Non-linearity of Historical Estimates of Individual and Joint Expected Losses

Based on the historical estimates of expected losses, the risk-based valuation of default risk within the Systemic CCA framework supports an integrated market-based capital assessment (see Section III.C above). Figure 9 illustrates the non-linear relation between the daily estimates of the market-implied capital adequacy ratio (MCAR), the EL ratio, the fair value credit spread, and the implied asset volatility of one selected sample firm in accordance with the stylized example shown in Figure 3. As expected, the rising default risk during the financial crisis establishes a high sensitivity of MCAR to a precipitous increase of expected losses at a level consistent with the regulatory measure of capital adequacy (see Figure 9, bottom chart). However, the growing gap between MCAR and the Core Tier 1 ratio towards the end point of the estimation time period implies that capitalization of the selected bank was perceived by investors as much lower than what regulatory standards would otherwise suggest.

Similarly, the interaction between these variables can also be shown on a system-wide basis (see Figure 10). For all firms in the sample, the firm-specific variables underpinning the presented measure were replaced with system-wide measures, such as the joint EL ratio (defined as the sum of 95 percent VaR values of individual expected losses or the joint 95 percent ES divided by the aggregate market capitalization) and the aggregate MCAR. The empirical results suggested an even higher sensitivity of MCAR to changes in system-wide expected losses than in the case of the single-firm example shown in Figure 9. In addition, the comparison of MCAR to the regulatory benchmark (Core Tier 1) indicates that the improvement of solvency conditions after the peak of the financial crisis in March 2009 appeared more protracted across all sample firms (see Figure 10, bottom chart).

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[69] The sample comprised the following firms: HSBC, Barclays, RBS, Lloyds's Banking Group, Standard Chartered Bank, Nationwide, and Santander UK.

[70] Key inputs used were the same as the ones obtained for the estimation of the Systemic CCA in the context of the FSAP for the United States (IMF, 2010c; see Section IV above). However, in absence of any estimation of contingent liabilities (which were not included in the stress testing exercise), CDS data were not required. Thus, the output was expected losses (i.e., the implicit put option value over a one-year horizon). Their multivariate distribution across all sample firms was estimated over a rolling window of 120 working days (i.e., six months) with daily updating in order to reduce model error for the calibration of individual expected loss series under extreme value theory (EVT).

Figure 9. United Kingdom: Integrated Individual Market-based Capital Assessment Based on CCA-derived Estimates of Expected Losses.

(based on a large commercial bank, one-year risk horizon)

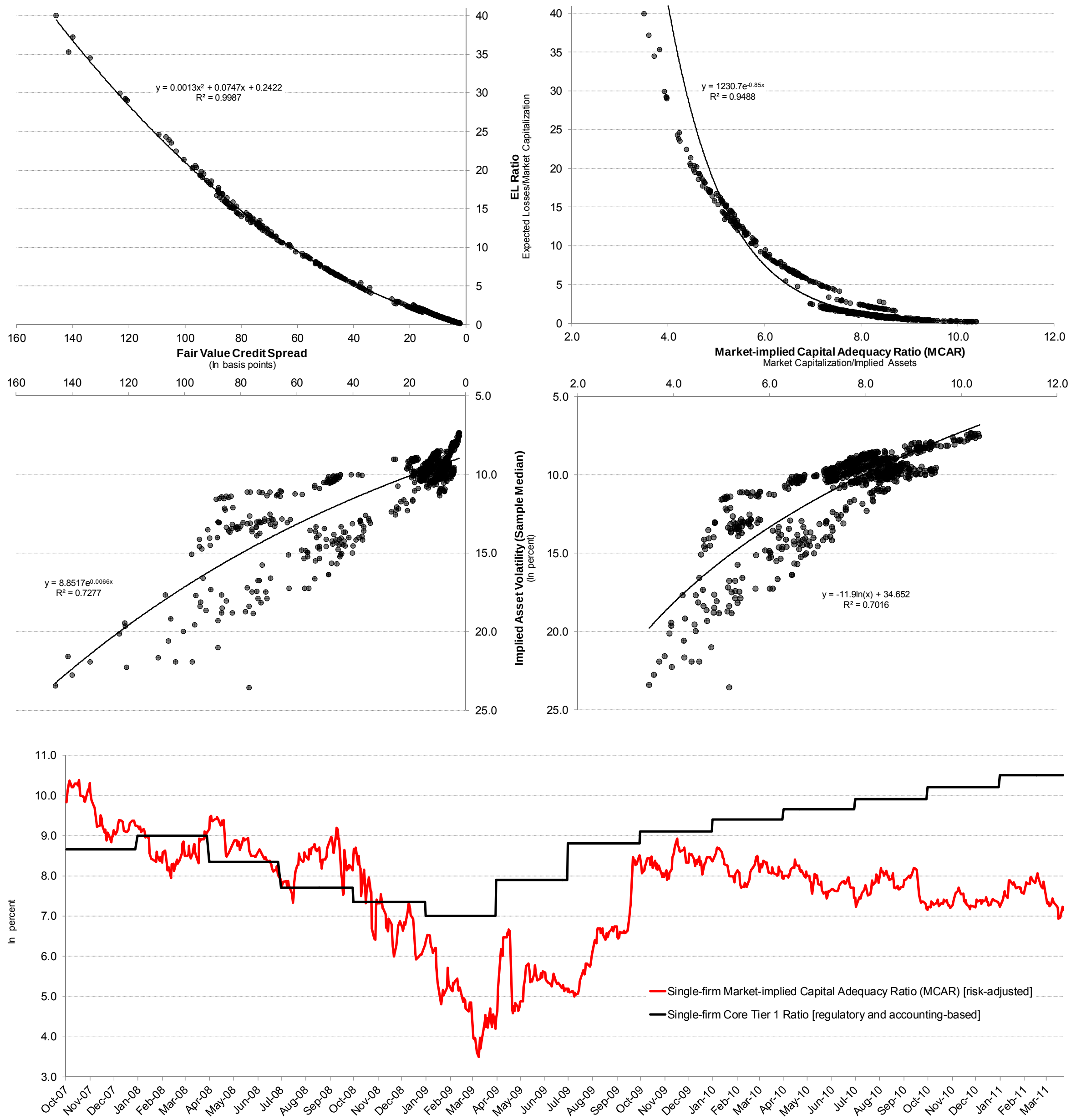
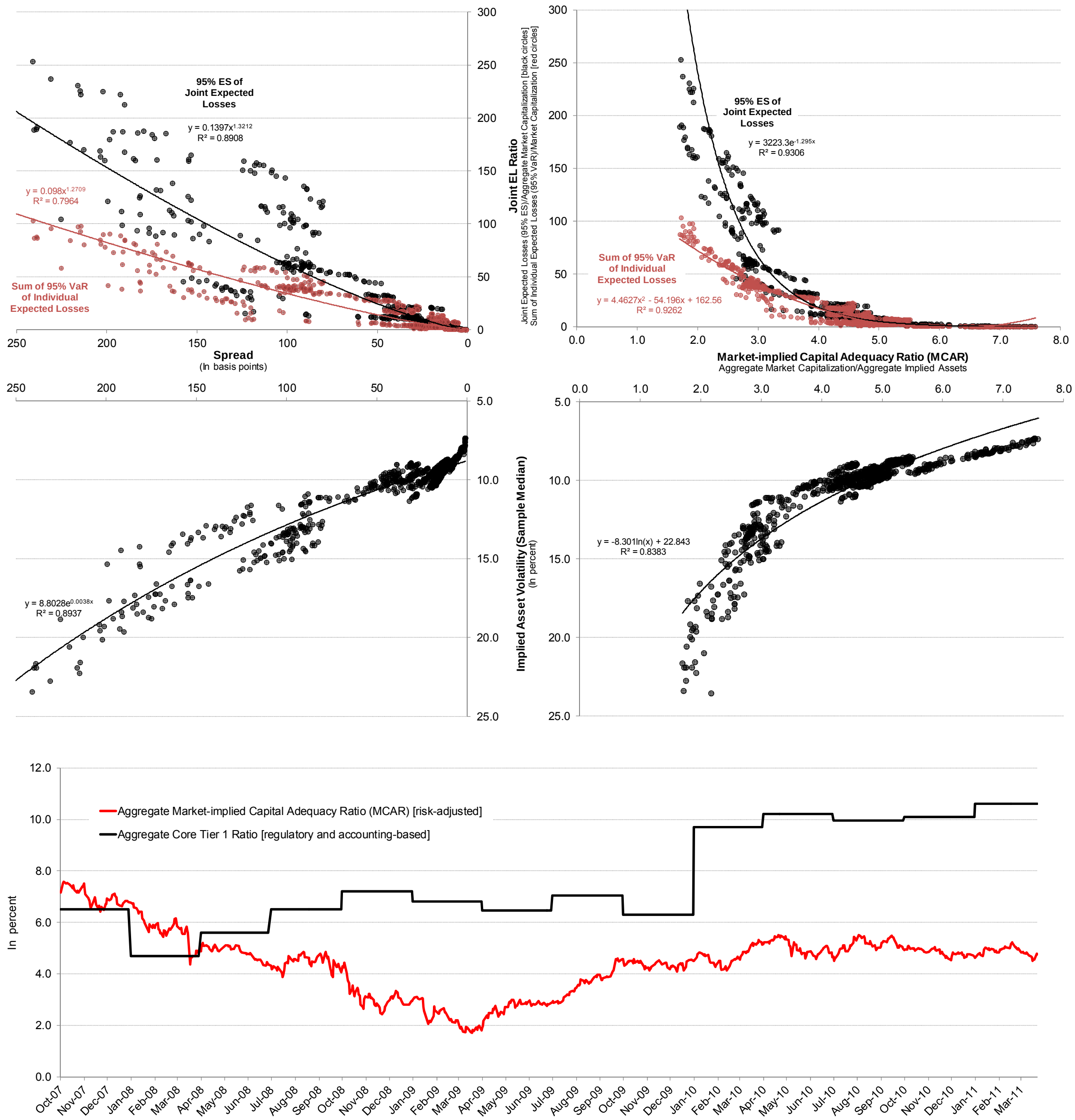


Figure 10. United Kingdom: Integrated Aggregate Market-based Capital Assessment Based on Systemic CCA-derived Estimates of Joint Expected Losses.

(based on all sample firms, one-year risk horizon)



B. Estimating Expected Losses Under Stress Conditions Based on Macro-Financial Linkages

Individual expected losses under stress conditions were derived from their historically implied variability. Given the sensitivity of market-implied individual expected losses to past macroeconomic conditions, expected losses under different stress scenarios were forecasted under two different methods for the specification of the macro-financial linkages of expected losses as measured by the implicit put option values (see Section III.B above):[71]

- In the first model ("IMF satellite model"), expected losses were estimated using a dynamic panel regression specification with several macroeconomic variables (short-term interest rate [+], long-term interest rate [-], real GDP [-], and unemployment [+]) and projected individual bank performance as RAMSI model output (see Figure 8 below) in the form of several bank-specific output variables (net interest income [+], operating profit before taxes [-], credit losses [+], leverage [+], and funding gap [+]).[72]

- In the second model ("structural model"), the value of implied assets of each bank as at end-2010 is adjusted by forecasts of operating profit and credit losses generated in the Bank of England's *Risk Assessment Model for Systemic Institutions* (RAMSI) model in order to derive a revised put option value (after re-estimating implied asset volatility), which determines the market-implied capital loss.[73]

The individually estimated expected losses were then transposed into a capital shortfall. The potential capital loss was determined on an individual basis as the marginal change of expected losses over the forecast horizon (2011-2015) and evaluated for each macro-financial approach and under each of the specified adverse scenarios. As an alternative, the market-implied CAR (see Figures 9 and 10 above) could have been used as a basis for

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[71] The different macroeconomic scenarios of the stress test include a baseline trend, which is specified as the IMF's WEO baseline projections, and three adverse ones based on different magnitudes of deviation of GDP from the baseline: (i) a one standard deviation shock to real GDP growth (based on the volatility of the two-year growth rate between 1980 and 2010) from the baseline growth trend over a five-year horizon with positive adjustment dynamics during the subsequent three years in which a shock to economic growth results in a sharp decline in output and rising employment over two years ("mild double-dip recession" or "DD mild"); (ii) a two standard deviations of the same, consistent with the FSA's 2011 anchor scenario ("severe double-dip recession" or "DD severe"); and (iii) a prolonged slow growth—i.e., severe and long-term—scenario with a cumulative negative deviation of about 7.5 percentage points from baseline growth, or an average annual growth rate of about 0.9 percent over a five-year horizon, as a result of a permanent shock to productive capacity amid rising inflation expectations ("prolonged slow growth" or "SG").

[72] The "+/-" signs indicate whether the selected variable exhibit a positive/negative regression coefficient. To be included in the model the variables needed to be statistically significant at least at the 10 percent level.

[73] This approach also assumed a graduated increase of the default barrier consistent with the transition period to higher capital requirements for the most junior levels of equity under Basel III standards (BCBS, 2012).

assessing capital adequacy and the extent to which expected losses results in potential capital shortfall. However, in the context of this exercise, the CCA-based results were transposed into a conventional (regulatory) measure of solvency, assuming that any increase in expected losses in excess of existing capital buffers constitutes a shortfall of Tier 1 capital (which was chosen as the closest approximation of common equity in CCA).

Finally, individual capital losses were combined using the Systemic CCA framework in order to determine the joint capital shortfall relative to the current solvency level over the forecast horizon. The univariate density functions of individual capital shortfalls and their dependence structure were combined to a multivariate probability distribution of joint capital shortfall (with a five-year sliding window and monthly updates). The estimated distribution allowed for an assessment of *joint* capital adequacy under stress conditions at different levels of statistical confidence (e.g., 50^{th} and 95^{th} percentiles) as opposed to the summation of individual institutions' capital adequacy (see Section II.B.2 (iii) above).[74] Any capital shortfall arose when the level of Tier 1 capital after accounting for joint capital losses fell below the hurdle rate of regulatory capital defined by the transition period to Basel III (and compared against the FSA's interim capital regime requirements).

Estimation results of the joint capital shortfall suggested that any impact from the realization of systemic solvency risk would have been limited even in a severe recession. The joint capital loss at the 50^{th} percentile would have been contained under all adverse scenarios. Existing capital buffers were sufficient to absorb the realization of central case (median) joint solvency risks. The severe double-dip recession scenario had the biggest impact on the banking system, but there would have been no capital shortfall (see Figure 12 and Table 8):

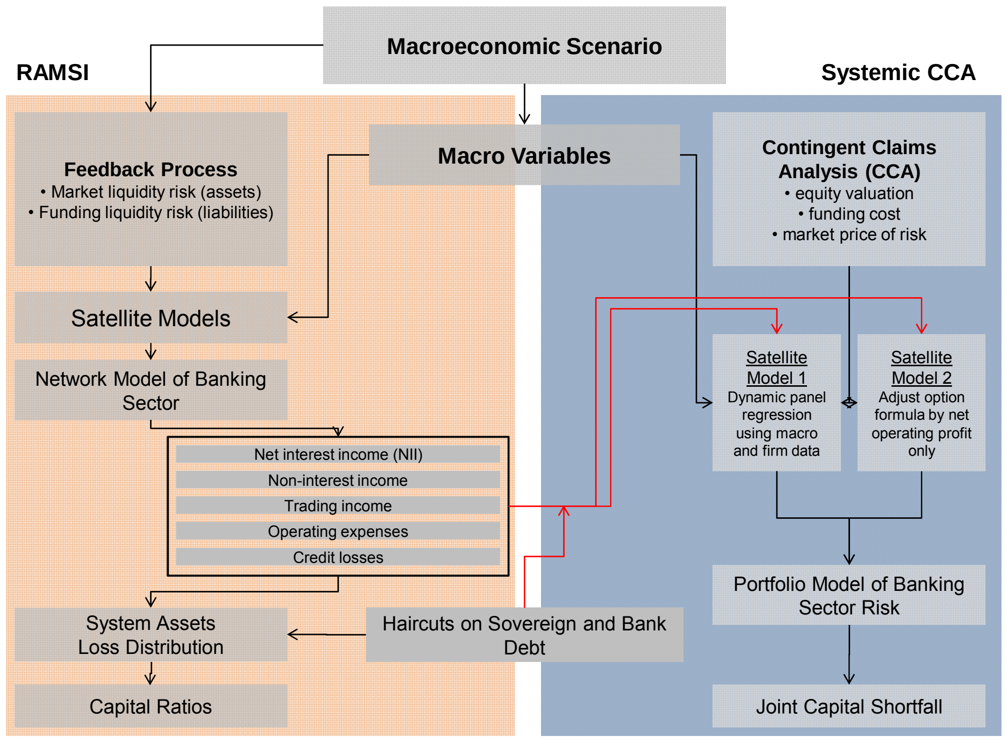
- *Overall, both satellite models, i.e., the IMF satellite model and the structural model, yielded similar and consistent results despite their rather different specifications.* They suggested robustness of estimates under either a panel regression approach or a structural approach via updates of model parameters using forecast changes in profitability (see Table 8).

- *The severe double-dip recession scenario had the biggest impact on the banking system.* Under baseline conditions, potential joint solvency pressures from the realization of slowing profitability, moderate credit losses, and risks to sovereign bank debt holdings would have been relatively benign, resulting in joint potential

[74] It should be noted that the application of the Systemic CCA framework for the TD stress test of the UK FSAP also took into account further declines in the market value of sovereign and bank debt (in the form of valuation haircuts applied to banks' debt holdings of all non-"AAA" rated sovereigns and banks in both the trading and banking books), which was added to market-implied expected losses (see Figure 11). The severity and dynamics of these haircuts was informed by the forward term structure of five-year sovereign credit default swap (CDS) spreads as at end-2010 (Jobst and others, *forthcoming*).

capital losses averaging £0.4 billion (equivalent to 0.03 percent of end-2010 GDP) over 2011–15. In the event that the severe double-dip scenario had occurred, capital losses could have amounted up to an average 0.12 percent of end-2010 GDP (£1.8 billion) over the forecast horizon. The resulting capital balance would have remained comfortably above the "distress barrier" under the Basel III capital hurdle rates, and there would have been no resulting shortfall.[75]

Figure 11. United Kingdom: Integration of the RAMSI and Systemic CCA Models based on Common Specification of Macro-Financial Linkages.



Source: IMF (2011c).

However, the Systemic CCA framework also allowed for the examination of more extreme outcomes amid a rapidly deteriorating macroeconomic environment. The estimation results above, which show relatively benign outcomes, pointed to the existing challenges of incorporating the "extreme tail risk" of such a situation. Since the Systemic CCA framework considers the stochastic nature of risk factors that determine expected losses and the associated degree of potential capital shortfall (see Box 3), we examined the realization of a five percent "tail of the tail" risk event (as the average density beyond the 95th percentile statistical confidence level) for multiple firms experiencing a dramatic escalation

[75] There would have also been no shortfall under the FSA's interim capital regime requirements.

of losses. Under the adverse scenarios, the market-implied capital shortfall would have been significantly increased, likely as a result of significantly lower profitability in conjunction with a sharp deterioration in asset quality and weaker fee-based income reducing the implied asset value of sample banks:

- *A historically very negative response of sample banks to assumed macro shocks could have eroded reported capital levels.*[76] The severe adverse scenario could have resulted in average joint capital losses of up to 3.4 percent of 2010 GDP (£50 billion), albeit still well below the peaks seen during the crisis (Figure 12). Under this scenario, the estimated capital loss would have caused an average capital shortfall of between 1.3–1.6 percent of end-2010 GDP relative to the Basel III Tier 1 hurdle rates (and 1.6–1.8 percent of end-2010 GDP relative to the FSA interim capital regime Tier 1 requirements), depending on the choice of satellite model; see Table 8). The prolonged slow growth scenario could have generated joint capital losses of up to 2 percent of end-2010 GDP (£29 billion), which would have translated to an average capital shortfall of up to 0.4 percent of end-2010 GDP (£6.3 billion).

- *Depending on the timing and adversity of macroeconomic conditions as well as the evolution of sovereign risk affecting banks' government and bank debt holdings, capital losses could range widely over the five-year horizon.* Estimates suggest that potential losses could range between zero (baseline, mild double-dip, and severe double-dip without debt haircuts, from 2013 onwards) to an equivalent of between 6.4–7.1 percent of end-2010 GDP, or £94–104 billion (under the severe double-dip scenario with sovereign and bank debt haircuts in 2011 and depending on the use of either the structural or the IMF satellite model; see Table 8). These losses would potentially result in an estimated capital shortfall of between 4.4–5.0 percent of end-2010 GDP, or £63–73 billion, relative to Basel III Tier 1 capital hurdles (and 4.9–5.6 percent of end-2010 GDP relative to FSA interim capital regime requirements).

The dispersion of individual contributions to the realization of the central case and extreme tail systemic risk suggest that a few banks contributed disproportionately to joint solvency risk under stress. Table 9 shows the percentage share of the minimum, maximum, and inter-quartile range (IQR) of the individual banks' time-varying contribution to the multivariate density of potential losses at the 50th (median) and 95th percentiles. With the exception of the first phase of the European sovereign debt crisis (during most of 2010 and the beginning of 2011), one bank, represented by the maximum of the distribution of all banks, seemed to have accounted for more than half of the solvency risk in the sample.

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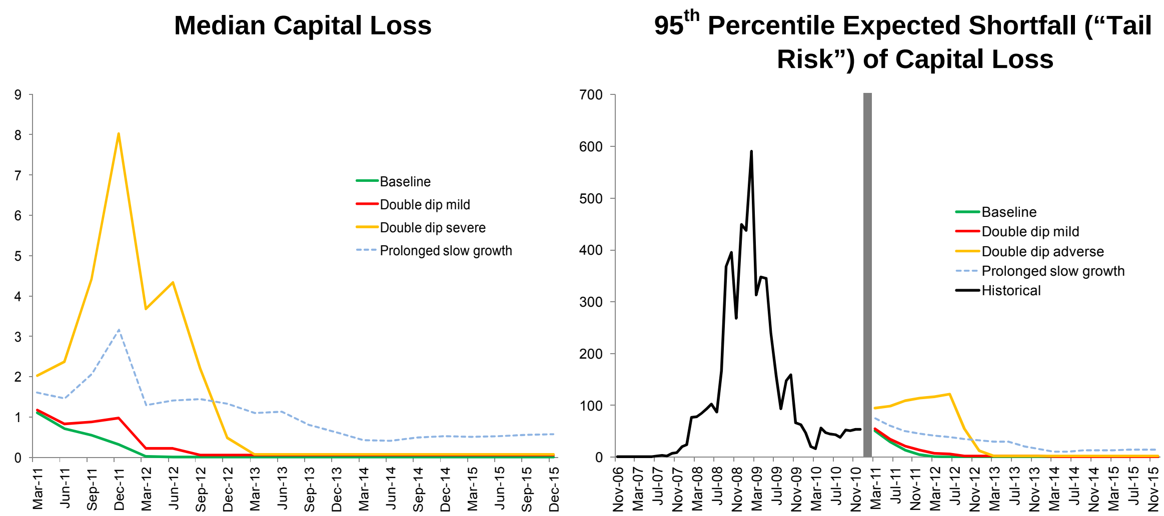
[76] Given that a severe double-dip recession scenario would have occurred with a two percent probability, the realization of joint capital losses beyond the 95th percentile of the historical distribution (as the 95th percent ES, or conditional VaR) constituted a less than 0.1 percent probability event (given that [0.02(1-0.95)]=0.001).

Figure 12. United Kingdom: Systemic CCA Estimates of Market-Implied Joint Capital Losses from the U.K. FSAP Update Top-Down Stress Tests *(with IMF Satellite Model)*.

(In billions of Pound Sterling)



Sample period: 01/03/2005-03/29/2010, monthly observations of historical and forecasted joint capital losses of seven sample firms. *Note:* The multivariate density is generated from univariate marginals that conform to the Generalized Extreme Value Distribution (GEV) and a non-parametrically identified time-varying dependence structure. The marginal severity and dependence measure are estimated (i) over a rolling window of 120 working days (with daily updating) for the historical measure (up to a sample cut-off at end-2010) and (ii) over a rolling window of 60 months (with monthly updating) for the generation of forecasted values (using he historical dynamics of capital losses as statistical support).

Table 8. United Kingdom: Systemic CCA Estimates of Market-Implied Joint Potential Capital Loss.

(Average over time period, in billions of Pound Sterling unless stated otherwise)

	IMF Satellite Model 1/								**Structural Model 2/**							
	without sovereign/bank debt haircuts				*with sovereign/bank debt haircuts*				*without sovereign/bank debt haircuts*				*with sovereign/bank debt haircuts*			
	Baseline	*Adverse Scenarios*			Baseline	*Adverse Scenarios*			Baseline	*Adverse Scenarios*			Baseline	*Adverse Scenarios*		
		DD mild	DD severe	SG		DD mild	DD severe	SG		DD mild	DD severe	SG		DD mild	DD severe	SG
	Market-implied Joint Potential Capital Loss															
	Median															
2011-2015	0.13	0.20	1.35	0.67	0.14	0.26	1.43	1.07	0.43	0.62	0.98	0.71	0.48	1.04	1.77	0.98
In percent of GDP	*0.01*	*0.02*	*0.11*	*0.05*	*0.01*	*0.02*	*0.11*	*0.08*	*0.03*	*0.05*	*0.08*	*0.06*	*0.04*	*0.08*	*0.14*	*0.08*
2011	0.66	0.91	4.13	1.65	0.67	0.97	4.21	2.08	0.90	1.35	1.94	1.20	1.00	1.92	2.96	1.53
2012	0.00	0.08	2.60	0.96	0.02	0.15	2.69	1.37	0.38	0.63	1.41	0.65	0.43	1.12	2.40	0.93
2013	0.00	0.00	0.00	0.52	0.01	0.06	0.08	0.92	0.27	0.37	0.61	0.50	0.30	0.74	1.33	0.73
2014	0.00	0.00	0.00	0.08	0.01	0.06	0.08	0.46	0.30	0.42	0.54	0.61	0.33	0.80	1.20	0.86
2015	0.00	0.00	0.00	0.16	0.01	0.06	0.08	0.54	0.30	0.34	0.41	0.59	0.33	0.64	0.94	0.83
	95 Percent (Tail Risk) 3/															
2011-2015	4.9	6.4	35.1	18.5	5.18	8.01	37.21	29.02	12.3	17.8	28.0	20.0	13.7	29.6	49.9	27.3
In percent of GDP	*0.38*	*0.50*	*2.75*	*1.45*	*0.41*	*0.63*	*2.91*	*2.27*	*0.96*	*1.40*	*2.20*	*1.57*	*1.07*	*2.32*	*3.90*	*2.14*
2011	24.4	29.6	101.5	46.9	24.69	31.16	103.60	57.80	28.6	42.7	61.5	38.0	31.7	60.8	93.7	48.4
2012	0.1	2.5	73.9	26.2	0.43	4.18	76.15	37.07	10.3	17.0	38.3	17.7	11.8	30.6	65.1	25.2
2013	0.0	0.0	0.0	13.6	0.27	1.63	2.19	24.10	7.0	9.7	15.9	13.1	7.9	19.4	34.9	19.0
2014	0.0	0.0	0.0	1.9	0.26	1.57	2.10	12.04	7.9	11.0	14.0	15.9	8.7	20.7	31.2	22.4
2015	0.0	0.0	0.0	4.1	0.25	1.52	2.01	14.11	7.8	8.8	10.6	15.4	8.6	16.6	24.4	21.6

Sample period: 01/03/2005-03/29/2010. *Note:* The estimations show the joint capital requirements for maintaining the market value of Tier 1 capital, with a gradual increase of the hurdle rate from 2013 onwards consistent with the Basel III proposal as at December 2010; 1/ The IMF satellite model uses a set of macroeconomic variables (short-term interest rates, long-term interest rates, real GDP growth, and unemployment) as well as income elements specific to each bank (operating profit, net interest income) to project potential losses generated by the CCA methodology; 2/ As an alternative, projected operating profit based on RAMSI model results is integrated in the CCA framework by adjusting implied firm assets, which increase potential losses via an option pricing approach. The treatment of losses from haircuts on holdings of sovereign and bank debt differs between both satellite model approaches. In the case of the former, these losses are calculated each year and added to the estimated overall potential losses. In contrast, for the alternative satellite model, losses from these debt holdings are subtracted from the RAMSI-model projected operating profit each quarter; 3/ The tail risk at the 95th percentile represents the average probability density beyond the 95th percentile as a threshold level.

Table 9. United Kingdom: Systemic CCA Estimates of Market-Implied Individual Contributions of Sample Banks to Systemic Risk—Market-Implied Joint Capital Loss.

(Average per time period, in percent of joint capital shortfall)

	Pre-Crisis: End-June 2005– End-June 2007	Subprime Crisis: Jul 1, 2007– Sep 14, 2008	Crisis Period 1: Sep 15–Dec 31, 2008	Crisis Period 2: Jan 1–Sep 30, 2009	Crisis Period 3: Oct 1, 2009– Feb 28, 2010	Sovereign Crisis: Mar 1, 2010– Mar 2011	Average
	Expected Joint Capital Shortfall (Median) 1/						
Minimum	0.1	0.3	0.8	0.6	0.2	1.0	0.4
25th percentile	2.4	1.5	1.8	1.5	2.1	2.6	2.0
Median	5.8	4.9	3.1	4.5	6.8	8.1	5.7
75th percentile	15.3	16.0	6.8	20.0	21.8	22.1	17.2
Maximum	58.9	59.9	79.0	51.9	45.2	41.4	55.3
	Extreme Joint Capital Shortfall (95th Percentile) 1/						
Minimum	0.1	0.4	1.4	1.0	0.6	1.1	0.6
25th percentile	3.6	2.9	4.3	3.2	2.9	4.8	3.6
Median	7.4	7.3	6.2	6.5	4.9	9.5	7.5
75th percentile	14.0	19.1	13.4	14.0	16.0	20.3	16.2
Maximum	57.2	48.4	56.9	58.0	56.7	39.3	52.2

Sample period: 01/03/2005-03/29/2010. *Note:* Each bank's percentage share is based on its time-varying contribution to the multivariate density of capital losses at the 50th (median) and 95th percentile.

VI. CONCLUSION

The identification of systemic risk is an integral element in the design and implementation of MPS with a view towards enhancing the resilience of the financial sector. However, assessing the magnitude of systemic risk is not a straightforward exercise and has many conceptual challenges. In particular, there is danger of underestimating practical problems of applying systemic risk management approaches to a real-life situation as complex modeling risks loses transparency, with conclusions being highly dependent on the assumptions, the differences in business models of financial institutions, and differences in regulatory and supervisory approaches across countries.

This paper presented a new modelling framework ("Systemic CCA"), which can help the measurement and analysis of systemic solvency risk by estimating the joint expected losses and associated contingent liabilities of financial institutions based on multivariate extreme value theory (EVT)—with practical applications to the analysis of financial sector risk management and stress testing. Based on two recent applications of the Systemic CCA framework in the context of the IMF's FSAP exercises for the United States (2010) and the United Kingdom (2011), the paper demonstrates how the individual contributions to systemic risk can be quantified under different stress scenarios and how such systemic relevance of financial institutions can be used to design macroprudential instruments, such as systemic risk surcharges. The aggregation model underlying the Systemic CCA framework was also used to illustrate the systemic risk of the financial sector implicitly taken on by governments and the potential (non-linear) destabilizing feedback processes between the financial sector and the sovereign balance sheet.

Some limitations of the Systemic CCA framework need to be acknowledged and have already been reflected as caveats in the interpretation of the findings. For instance, the marked-based determination of default risk might be influenced by the validity of valuation models. Economic models are constructed as general representations of reality that, at best, capture the broad outlines of economic phenomena in a steady-state. However, financial market behavior might defy statistical assumptions of valuation models, such as the option pricing theory, extreme value measurement, and non-parametric specification of dependence between individual default probabilities in the Systemic CCA approach. For instance, during the credit crisis financial market behavior was characterized by rare and non-recurring events, and *not* by repeated realizations of predictable outcomes generated by a process of random events that exhibits stochastic stability. Thus, the statistical apparatus underlying conventional asset pricing theory fails to fully capture sudden and unexpected realizations beyond historical precedent.[77,78]

[77] One also needs to acknowledge the irreducible core of unpredictable outcomes in the application of valuation models during distress events.

Going forward, the Systemic CCA framework could be expanded in scale and scope to include other types of risks. For instance, most recently, the Systemic CCA framework has also been adapted to measure systemic liquidity risk by transforming the Net Stable Funding Ratio (NSFR) as standard liquidity measure under Basel III into a stochastic measure of aggregate funding risk.[79] In addition, given the flexibility of this framework, the financial sector and sovereign risk analysis could be integrated with macro-financial feedbacks in order to design monetary and fiscal policies. Such an approach could inform the calibration of stress scenarios of banking and sovereign balance sheets and the appropriate use of macroprudential regulation. Using an economy-wide Systemic CCA for all sectors, including the financial sector, non-financial corporates, households, and the government, can also provide new measures of economic output – the present value of risk-adjusted GDP (see Gray and others, 2010).

[78] Parameter sensitivity and estimation risk warrants the use of different valuation techniques in order to substantiate a comprehensive risk assessment. For instance, exploring different option pricing models can help determine any pricing distortions created by aberrant price dynamics.

[79] The Systemic Risk-adjusted Liquidity (SRL) model (IMF, 2011a; Jobst, 2012) combines option pricing with market information and balance sheet data to generate a probabilistic measure of multiple entities experiencing a joint liquidity event. It links a firm's maturity mismatch between assets and liabilities, its overall risk profile, and the stability of its funding with those characteristics of other firms that are subject to common changes in market conditions.

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APPENDICES

Appendix 1. Standard Definition of Contingent Claims Analysis (CCA)

CCA is used to construct risk-adjusted balance sheets, based on three principles: (i) the values of liabilities (equity and debt) are derived from assets; (ii) liabilities have different priority (i.e., senior and junior claims); and (iii) assets follow a stochastic process. The asset price of a firm (such as the present value of income flows and proceeds from asset sales) changes over time and may be above or below promised payments on debt which constitute a default barrier. Uncertain changes in future asset value, relative to the default barrier, determine the probability of default risk, where default occurs when assets decline below the barrier. When there is a chance of default, the repayment of debt is considered "risky," unless it is guaranteed in the event of default. The guarantee can be held by the debt holder, in which case it can be thought of as the expected loss from possible default or by a third-party guarantor, such as the government (Merton and Bodie, 1992).

In the first structural specification, commonly referred to as the *Black-Scholes-Merton* (BSM) framework (or short "Merton model") of capital structure-based *option pricing theory* (OPT), the total value of firm assets follows a stochastic process and may fall below the value of outstanding liabilities. In its basic concept, the BSM model assumes that owners of corporate equity in leveraged firms hold a call option as a residual asset claim on the firm value after outstanding liabilities have been paid off. They also have the option to default if their firm's asset value ("reference asset") falls below the present value of the notional amount of outstanding debt ("strike price") owed to bondholders at maturity. So, corporate bond holders effectively write a European put option to equity owners, who hold a residual claim on the firm's asset value in non-default states of the world. Bond holders receive a put option premium in the form of a credit spread above the risk-free rate in return for holding risky corporate debt (and bearing the potential loss) due to the limited liability of equity owners. The value of the put option is determined by the duration of debt claim, the leverage of the firm, and asset price volatility.

More specifically, the value of firm assets is stochastic and may fall below the value of outstanding liabilities, which would results in default. The asset value *A(t)* at time *t* is assumed to evolve (under the risk-neutral probability measure *Q*) following the stochastic differential equation

$$dA(t) = A(t) r_A dt + A(t) \sigma_A dW^Q(t) \quad \text{(A1.1)}$$

with drift r_A, implied asset volatility σ_A, and diffusion defined by a standard *geometric Brownian motion* (GBM) $\Delta W^Q(t) \sim \varphi(0, \Delta t)$ with Wiener process $z \sim \varphi(0, \sigma)$ of instantaneous asset value change. After application of the Itô-Döblin theorem (Itô, 1944;

Döblin, 2000), the discrete form analog for initial value $A(0)$ can be written as a lognormal asset process

$$\ln A(t) - \ln A(0) \sim \phi\left[\ln A(0) + \left(r_A - \frac{\sigma_A^2}{2}\right)t; \sigma_A^2\sqrt{t}\right], \tag{A1.2}$$

where $\phi(\bullet)$ is the standard normal probability density function, and with drift r_A set equal to the general risk-free rate *r*. Thus, the asset value $A(t)$ at time *t* describes a continuous asset process so that the physical probability distribution of the end-of-period value is

$$A(T-t) \sim A(t)\exp\left[\left(r_A + \frac{\sigma_A^2}{2}\right)(T-t) + \sigma_A\sqrt{T-t}z\right], \tag{A1.3}$$

for time to maturity *T-t*. More specifically, $A(t)$ is equal to the sum of the equity market value, $E(t)$, and risky debt, $D(t)$, so that $A(t) = E(t) + D(t)$.

Default occurs if the asset value is insufficient to satisfy the amount of debt that is owed to creditors at maturity, i.e., the bankruptcy level ("default threshold" or "distress barrier"). Given firm leverage $B/A(t)$ as the ratio of the repayable face value of outstanding debt *B* and the asset value, the expected *(physical)* probability of default

$$\Pr\left(A(t) \le B\right) \approx \Pr\left(\ln A(t) \le \ln B\right) \tag{A1.4}$$

at time *t* is defined as

$$\begin{aligned}
&\Phi\left(\ln B - \left(\ln A(t) - \left(r_A + \frac{\sigma_A^2}{2}\right)(T-t)\right)\frac{1}{\sigma_A\sqrt{T-t}}\right) \\
&= \Phi\left(\ln\left(\frac{B}{A(t)}\right) + \left(r_A + \frac{\sigma_A^2}{2}\right)(T-t)\frac{1}{\sigma_A\sqrt{T-t}}\right) \\
&= \Phi(-d) = 1 - \Phi(d),
\end{aligned} \tag{A1.5}$$

with the cumulative probability $\Phi(\bullet)$ of the standard normal distribution function. For simplicity, it is assumed that the asset drift r_A is equivalent to the general risk-free rate *r*, so that the *distance to default* (DD) measure is

$$d = \frac{\ln\left(\frac{A(t)}{B}\right) + \left(r + \frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A \sqrt{T-t}}, \tag{A1.6}$$

whose probability density defines the "survival probability"

$$\Phi(d) = 1 - P(t) = \Pr\left(A(t) > B\right) \approx \Pr\left(\ln A(t) > \ln B\right). \tag{A1.7}$$

Given the assumed capital structure, the equity value $E(t)$ at any moment in time is defined as the residual claim on assets once creditors are paid the full amount $D(t)$. Thus, the equity value is an implicit call option on assets as the underlying, with an exercise price (or strike) equal to default barrier, based on the conditional expectation (under the risk-neutral probability measure *Q*) of termination payoff

$$E(t) = \mathbf{E}_t^Q\left[e^{-r(T-t)} \max\left(A(t) - D(t), 0\right)\right], \tag{A1.8}$$

which can be computed as the value of a call option

$$E(t) = A(t)\Phi(d) - Be^{-r(T-t)}\Phi\left(d - \sigma_A\sqrt{T-t}\right), \tag{A1.9}$$

assuming that the asset value follows a lognormal distribution as specified in equation (A1.2) above.

Both the asset and asset volatility are valued *after* the dividend payouts. As in Gray and Malone (2008), and consistent with the discussion in Hull (2006), accounting for dividend payouts included in the asset value requires subtracting the present value of dividends S, discounted at the risk-free rate *r*. Dividend payments, *S*, paid out from the asset value are modeled as "lumpy dividend" payments.[80] The dividend *S* is the discounted dividends up to time horizon *T*, i.e.,

$$S = e^{-r(t_1 - t_0)}S_1 + e^{-r(t_2 - t_0)}S_2 + \ldots + e^{-r(t_N - t_0)}S_T\,. \tag{A1.10}$$

Given asset value $\hat{A}(t)$ before dividend payout, the asset value after dividend payout is

[80] Such payments are not easily modeled with a continuous dividend payment from the asset. It is more flexible to model it as a "lumpy dividend" payment.

$$A(t) \equiv \hat{A}(t) - S. \tag{A1.11}$$

Since the relation between assets and asset volatility with "lumpy dividends" (Chriss, 1997) can be written

$$\sigma_A = \frac{\hat{A}(t)}{A(t)} \sigma_{\hat{A}}, \tag{A1.12}$$

this concept can be generalized to applied to many different factors that impact the financial institution's asset. It applies to inflows and outflows from the asset and to asset value changes from valuation losses. Profits, loan losses, dividends, taxes and other factors can drive changes in asset values, which amount to δA so that

$$A(t) \equiv \hat{A}(t) - \delta A. \tag{A1.13}$$

In this paper, the following more general relation is used to adjust the estimated volatility

$$\sigma_A \equiv \left(\hat{A}(t) / A(t) + \delta A \right)^{\gamma} \sigma_{\hat{A}} = \sigma_{A+\delta A}. \tag{A1.14}$$

The parameter, γ, is calculated empirically from the observed relation between implied asset values (after application of Appendix 2) and implied asset volatility over an estimation period consistent with the calibration of expected losses in the univariate CCA model (see Section II).

The value of risky debt is equal to default-free debt minus the present value of expected loss due to default,

$$D(t) = Be^{-r(T-t)} - P_E(t). \tag{A1.15}$$

Thus, the present value of market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option, which is calculated with the default threshold *B* as strike price on the asset value *A(t)* of each institution. Thus, the present value of market-implied expected loss can be computed as

$$P_E(t) = Be^{-r(T-t)} \Phi\left(-d - \sigma_A \sqrt{T-t}\right) - A(t) \Phi(-d), \tag{A1.16}$$

over time horizon *T–t* at risk-free rate *r*, subject to the duration of debt claims, and the leverage of the firm. Both the risk-free rate and the asset volatility are assumed to be time-invariant in this specification. Since equity is a function of assets, we can use the Itô-Döblin theorem to derive an expression for the diffusion process of $E(t)$ based on equation (A1.9). Given the GBM-based asset diffusion process in equation (A1.1), we have that

$$
\begin{aligned}
dE(t) &= \frac{\partial E}{\partial A} dA(t) + \frac{1}{2}\frac{\partial^2 E}{\partial A^2} dA^2(t)\sigma_A^2 dt \\
&= \frac{\partial E}{\partial A}\left(A(t) r_A dt + A(t)\sigma_A dW^Q(t)\right) + \frac{1}{2}\frac{\partial^2 E}{\partial A^2} dA^2(t)\sigma_A^2 dt \\
&= \left(\frac{\partial E}{\partial A} A(t) r_A + \frac{1}{2}\frac{\partial^2 E}{\partial A^2} A^2(t)\sigma_A^2\right) dt + \frac{\partial E}{\partial A} A(t)\sigma_A dW^Q(t),
\end{aligned}
\tag{A1.17}
$$

Assuming that equity also follows a lognormal process implies that

$$dE(t) = E(t) r_A dt + E(t)\sigma_E dW^Q(t), \tag{A1.18}$$

Matching both terms in equations (A1.17) and (A1.18) above implies that

$$E(t) r_A = \frac{\partial E}{\partial A} A(t) r_A + \frac{1}{2}\frac{\partial^2 E}{\partial A^2} A^2(t)\sigma_A^2, \tag{A1.19}$$

and

$$E(t)\sigma_E = \frac{\partial E}{\partial A} A(t)\sigma_A, \tag{A1.20}$$

so that we can solve for asset volatility

$$\sigma_A = \frac{E(t)\sigma_E}{A(t)\Phi(d)} = \left(1 - \frac{Be^{-r(T-t)}\Phi\left(d - \sigma_A\sqrt{T-t}\right)}{A(t)\Phi(d)}\right)\sigma_E, \tag{A1.21}$$

which is “embedded” in the equity option formula. Since the implicit put option $P_E(t)$ can be decomposed into PD and LGD by re-arranging equation (A1.16) so that

$$P_E(t) = \underbrace{\Phi\left(-d-\sigma_A\sqrt{T-t}\right)}_{PD}\underbrace{\left(1-\frac{\Phi(-d)}{\Phi\left(-d-\sigma_A\sqrt{T-t}\right)}\frac{A(t)}{Be^{-r(T-t)}}\right)Be^{-r(T-t)}}_{LGD}. \qquad \text{(A1.22)}$$

Note that there is no need to introduce potential inaccuracy of assuming a certain LGD. Given the underlying asset dynamics according to equation (A1.3), the risk-neutral probability distribution (or *state price density*, SPD) of $A(t)$ is a log-normal density

$$f_t^*\left(A(T-t)\right) = e^{-r_{A_t,T-t}T-t}\frac{\partial^2 E}{\partial B^2}\Big|_{B=A(T-t)}$$
$$= \frac{1}{A(T-t)\sqrt{2\pi\sigma_A^2(T-t)}}\exp\left[-\frac{\left[\ln\left(A(T-t)/A(t)\right)-\left(r_{A_t,T-t}-\sigma_A^2/2\right)(T-t)\right]^2}{2\sigma_A^2(T-t)}\right] \qquad \text{(A1.23)}$$

with mean $\left(r_{A_t,T-t}-\sigma_A^2/2\right)(T-t)$ and variance $\sigma_A^2(T-t)$ for $\ln\left(A(T-t)/A(t)\right)$, where $r_{A_t,T-t}$ and $f^*(\bullet)$ denote the risk-free interest rate and the risk-neutral probability density function (or SPD) at time *t*, with risk measures

$$\Delta \overset{def}{=} \frac{\partial E}{\partial A} = \Phi(d) \qquad \text{(A1.24)}$$

and

$$\Gamma \overset{def}{=} \frac{\partial^2 E}{\partial A^2} = \frac{\Phi(d)}{A(t)\sigma_A\sqrt{T-t}}. \qquad \text{(A1.25)}$$

Since both the market value of equity and the volatility of equity returns are observable, equations (A1.9) and (A1.20) can be used to solve for the implied value of assets and asset return volatility. However, solving this system of equations within the BSM model has several empirical shortcomings that have been addressed in a more advanced version of CCA presented in the main text.

Appendix 2. Estimation of the Empirical State Price Density (SPD)

Breeden and Litzenberger (1978) show Arrow-Debreu prices can be replicated via the concept of the *butterfly spread* on European call options as basis for extracting the *state price density* (SPD) from observable derivatives prices. This spread entails selling two call options at strike price *K* and buying two call options with adjacent strike prices $K^- = K - \Delta K$ and $K^+ = K + \Delta K$, respectively, with the stepsize ΔK between the two call strikes. If the terminal underlying asset value $A(T) = K$, then the payoff $Z(\bullet)$ of $1/\Delta K$ of such butterfly spreads at time t (with time to maturity $T-t$) is defined as

$$Z\big(A(T),K;\Delta K\big) = Price\big(A(t),T-t,K;\Delta K\big)\Big|_{T-t=0} = \frac{u_1 - u_2}{\Delta K}\Big|_{A(T)=K,T-t=0} = 1, \quad \text{(A2.1)}$$

with

$$u_1 = C\big(A(t),T-t,K+\Delta K\big) - C\big(A(t),T-t,K\big) \quad \text{(A2.2)}$$

and

$$u_2 = C\big(A(t),T-t,K\big) - C\big(A(t),T-t,K-\Delta K\big), \quad \text{(A2.3)}$$

where $C\big(A(t),T-t,K\big)$ denotes the price of a European call option with an underlying asset price *A* and strike price *K*. As $\Delta K \to 0$, $Price\big(A(t),T-t,K;\Delta K\big)$ of the position value of the butterfly spread becomes an Arrow-Debreu security paying 1 if $A(T) = K$ and 0 in other states. If $A(T) \in \mathbb{R}^+$ is continuous, we can obtain a security price

$$\lim_{\Delta K \to 0}\left(\frac{Price\big(A(t),T-t,K;\Delta K\big)}{\Delta K}\right)\Bigg|_{K=A(T)} = f_t^*\big(A(T)\big)e^{-r_t(T-t)}, \quad \text{(A2.4)}$$

where r_t and $f^*(\bullet)$ denote the risk-free rate and the risk-neutral probability density function (or SPD) at time *t*, respectively. On a continuum of strike prices *K* at infinitely small ΔK, a complete state pricing function can be defined. As $\Delta K \to 0$, the price

$$\lim_{\Delta K \to 0}\left(\frac{Price\big(A(t),T-t,K;\Delta K\big)}{\Delta K}\right) = \lim_{\Delta K \to 0}\frac{u_1 - u_2}{(\Delta K)^2} = \frac{\partial^2 C_t(\bullet)}{\partial K^2} \quad \text{(A2.5)}$$

will tend to the second derivative of the call pricing function with respect to the strike price evaluated at *K*, provided that $C(\bullet)$ is twice differentiable. Thus, by combining equations (A2.4) and (A2.5) we can write

$$\left.\frac{\partial^2 C_t(\bullet)}{\partial K^2}\right|_{K=A(T)} = f_t^*\left(A(T)\right)e^{-r_t(T-t)} \tag{A2.6}$$

across all states, which yields the SPD

$$f_t^*\left(A(T)\right) = e^{-r_t(T-t)}\left.\frac{\partial^2 C_t(\bullet)}{\partial K^2}\right|_{K=A(T)} \tag{A2.7}$$

under no-arbitrage conditions and without assumptions on the underlying asset dynamics. Preferences are not restricted since no-arbitrage conditions only assume risk-neutrality with respect to the underlying asset. The only requirements for this method are that markets are perfect, i.e., there are no transactions costs or restrictions on sales, and agents are able to borrow and lend at the risk-free interest rate.

Appendix 3. Estimation of the Shape Parameter Using the Linear Combination of Ratios of Spacings (LRS) Method As Initial Value for the Maximum Likelihood (ML) Function

Since all raw moments of GEV cumulative distribution function $H(\bullet)$ are defined contingent on the asymptotic tail behavior (see equation (12) in the main text), the natural estimator of the shape parameter $\hat{\xi}$ is derived by means of the *Linear Combination of Ratios of Spacings* (LRS) method, which specifies the marginal increase of extreme values (relative to the rest of the distribution) based on

$$\hat{\xi} = \frac{4}{n} \sum_{i=1}^{(n/4)} \frac{\ln(\hat{v}_i)}{-\ln(c)} \tag{A3.1}$$

for all $i \in n$ observations (with the n^{th} order statistics $x_{1:n}$ and $x_{n:n}$ of the empirical data series) and quantiles $a = i/n$, where

$$\hat{v}_i = \frac{x_{n(1-a):n} - x_{na^c:n}}{x_{na^c:n} - x_{na:n}} \tag{A3.2}$$

and

$$c = \sqrt{\frac{\ln(1-a)}{\ln(a)}}. \tag{A3.3}$$

Since $x_{na:n} = H^{-1}_{\hat{\mu},\hat{\sigma},\hat{\xi}}(a)$ at statistical significance *a*, the approximation

$$\hat{v}_i \approx \frac{H^{-1}_{\hat{\mu},\hat{\sigma},\hat{\xi}}(1-a) - H^{-1}_{\hat{\mu},\hat{\sigma},\hat{\xi}}(a^c)}{H^{-1}_{\hat{\mu},\hat{\sigma},\hat{\xi}}(a^c) - H^{-1}_{\hat{\mu},\hat{\sigma},\hat{\xi}}(a)} = c^{-1+\xi} \tag{A3.4}$$

holds. For more information on the LRS method, see Jobst (2007).

Appendix 4. Derivation of the Implied Asset Volatility Using the Moody's KMV Model

This appendix illustrates how the estimated default frequency (EDF) in Moody's global KMV model (Crosbie and Bohn, 2003) can be used to arrive at the implied asset volatility for the Systemic CCA framework under risk neutrality. The default risk in the KMV model is defined—under the *physical* measure (see Appendix 1)—as

$$EDF_{MKMV} = \Phi\left(d_{MKMV} - \sigma_{A_{MKMV}}\sqrt{T-t}\right) \tag{A4.1}$$

using a Merton-style option pricing definition. This specification is inappropriate for estimation of expected losses within the Systemic CCA model, because it does not account for risk aversion, i.e., the market's assessment of risk ("market price of risk"). Hence, under the risk-neutral measure, we can re-write equation (A4.1) as

$$EDF_{MKMV} = \Phi\left(\frac{\ln\left(\frac{A(t)_{MKMV}}{B_{MKMV}}\right) + \left(r_{A_{MKMV}} + \rho_{A_{MKMV},M}\sigma_A^* \frac{\mu_M - r}{\sigma_M} - \frac{\left(\sigma_A^*\right)^2}{2}\right)(T-t)}{\sigma_A^*\sqrt{T-t}}\right) \tag{A4.2}$$

with the correlation between the asset and market return $\rho_{A_{MKMV},M}$, asset return $r_{A_{MKMV}}$, the "Market Sharpe Ratio" $(\mu_M - r)/\sigma_M$ (which changes daily and is assumed to be the same for all firms and financial institutions, consisting of the general risk-free rate r, average market return μ_M, and market volatility σ_M). For $T-t=1$, re-arranging equation (A4.2) above gives

$$\begin{aligned} &\ln\left(\frac{A(t)_{MKMV}}{B_{MKMV}}\right) + r_{A_{MKMV}} + \rho_{A_{MKMV},M}\sigma_A^* \frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{MKMV}\right)\sigma_A^* - \frac{\left(\sigma_A^*\right)^2}{2} = 0 \\ &\Leftrightarrow \underbrace{\ln\left(\frac{A(t)_{MKMV}}{B_{MKMV}}\right) + r_{A_{MKMV}}}_{a} + \underbrace{\left(\rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{MKMV}\right)\right)}_{b}\sigma_A^* \underbrace{-\frac{1}{2}}_{c}\left(\sigma_A^*\right)^2 = 0 \end{aligned} \tag{A4.3}$$

so the quadratic formula $\sigma_A^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to recover a measure of the *adjusted* asset volatility

$$\sigma_A^* = \frac{-\left(\rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{MKMV}\right)\right) \pm \sqrt{\left(\rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{MKMV}\right)\right)^2 + 2\left(\ln\left(\frac{A_{MKMV}(t)}{B_{MKMV}}\right) + r_{A_{MKMV}}\right)}}{2\left(\ln\left(\frac{A_{MKMV}(t)}{B_{MKMV}}\right) + r_{A_{MKMV}}\right)}. \tag{A4.4}$$

Given equation (A1.20), the actual asset volatility, σ_A, as a risk-neutral model input, is derived by solving the optimization constraint

$$\frac{1}{\tau}\sum_{t=1}^{\tau}\left(\sigma_A^*\sqrt{\frac{A(t)}{E(t)}}\frac{\gamma(t)}{\eta^*}\right) = \frac{1}{\tau}\sum_{t=1}^{\tau}\sigma_A^* \tag{A4.5}$$

over a rolling time window of τ number of estimation days (consistent with the maximum-likelihood estimation of the multivariate distribution in Section II.A.2(i)) on η^*, so that

$$\sigma_A = \sigma_A^*\frac{\gamma(t)}{\eta^*}\sqrt{\frac{A(t)}{E(t)}}, \tag{A4.6}$$

where

$$\gamma(t) = \frac{\sigma_{A(t)}\sigma_{A(t+\tau)_{MKMV}}}{\sigma_{A(t+\tau)}\sigma_{A(t)_{MKMV}}} \tag{A4.7}$$

is an intertemporal error correction over the same estimation window (with $\sigma_{A(t)_{MKMV}}$ and $\sigma_{A(t+\tau)_{MKMV}}$ denoting the asset volatility at time *t* and $t+\tau$ days reported by Moody's KMV for an individual entity).[81]

[81] The general asset volatility under risk-neutrality would need to be further refined depending on the relevant option pricing model (and its implications for asset volatility).

Appendix 5. Comparison of Different Systemic Risk Measures vis-à-vis Systemic CCA

One goal of financial systemic risk measures is to determine the contribution of individual financial institutions to systemic risk, including capturing spillover and contagion effects between institutions within and across different sectors and national boundaries. The ultimate objective is to assess how the adverse impact of this contribution on financial stability (i.e., the "negative externality of individual activities on the system) could be internalized through special taxes, risk-based surcharges, and/or insurance premiums that mitigate excessive risk-taking. This section presents three main systemic risk models that have been proposed and compares them to the Systemic CCA framework—CoVaR, Systemic Expected Shortfall (SES), and the Distress Insurance Premium (DIP). See also Table 2 for a more succinct comparison, which also incorporates other institution-level systemic risk models.

CoVaR (Adrian and Brunnermeier, 2008)—The CoVaR quantifies how financial difficulties of one institution can increase the tail risk of others. CoVaR for a certain institution is defined as the VaR (as a measure of extreme default risk) of the whole sector conditional on a particular institution being in distress. Bank *X*'s marginal contribution to systemic risk is then computed as the difference between its CoVaR value and the financial system's VaR. The methodology uses quantile regression analysis to predict future CoVaR on a quarterly basis, which are then related to particular characteristics (e.g., leverage) and observed market risk factors (e.g., CDS spreads). The model is not structural and is motivated by the use of bank-specific VaR estimates that measure tail risks.

Systemic Expected Shortfall (SES) (Acharya and others, 2009, 2010 and 2012) — The Marginal Expected Shortfall (MES) specifies historical expected losses, conditional on a firm having breached some high systemic risk threshold. Adjusting MES by the degree of firm-specific leverage and capitalization yields the Systemic Expected Shortfall (SES). MES measures only the average, linear, bivariate dependence. It does not consider interaction between subsets of banks and implicitly assumes that the *entire* banking sector is undercapitalized.

Distress Insurance Premium (DIP) (Huang and others, 2009 and 2010) — This approach to measuring and stress-testing systemic risk identifies the contribution of individual financial institutions to systemic risk based on a hybrid approach using information from both equity and CDS prices. The systemic risk measure, the so-called the Distress Insurance Premium (DIP), is defined as the insurance cost to protect against distressed losses in a banking system and serves as a summary indicator of market perceived risk that reflects expected default risk of individual banks and correlation of defaults. It combines estimates of default risk backed out of CDS spreads with correlations backed out of bank equity returns.

Systemic Contingent Claims Approach (Systemic CCA) — This approach uses CCA for individual institutions and estimates a multivariate distribution allowing for a nonlinear dependence structure across institutions to derive a market-implied measure of systemic risk based on estimates of joint expected losses (and associated contingent liabilities). Market data (equity and equity option prices) as well as capital structure information are used to generate a non-linear metric of system-wide default risk based on Expected Shortfall (ES). This multivariate conditional tail expectation (CTE) measure provides two distinct benefits: (i) by applying a multivariate density estimation, it quantifies the marginal contribution of an individual firm, while accounting for rapidly changing market valuations of balance sheet structures; and (ii) it can be used to value systemic risk charges within a consistent framework for estimating potential losses based on current market conditions rather than on historical experience (Khandani and others, 2009).

In comparing all four approaches important methodological differences and their implications for the measurement and interpretation of systemic risk should be recognized:

- *Risk measures should adequately reflect tail risks.* Expected shortfall (ES) is an improvement over VaR but needs to be qualified. While ES is a coherent risk measure (Artzner and others, 1997 and 1999), conditioning ES on the most severe outcomes for the entire sample of banks ignores the possibility that the incidence of joint expected losses below the ES threshold influences the probability of tail events if this tail risk measure were applied in empirical models (such as in the SES/MES approach). Moreover, like CoVaR, the parametric specification of SES/MES conditional on quarterly estimated data (e.g., the necessity to estimate a leverage ratio from quarterly available data) is insensitive to rapidly changing market valuations of balance sheet structures, and requires re-estimation with the potential of parameter uncertainty;

- *The complete specification of different risk components facilitates model robustness.* For instance, the measures of tail risk generated by the SES approach and Systemic CCA are adjusted for leverage as an important determinant of default risk. However, in absence of a structural definition in the SES approach, leverage is commingled with other (non-specified) components of default risk, such as the maturity of debt payments and asset volatility, which complicates the identification of risk drivers (and the possible design of mitigants of systemic risk).

- *The specification of the dependence structure matters.* In the DIP approach, a correlation measure is derived from equity market returns; however, default probabilities are backed out of the dynamics of CDS prices, which reflect the residual default risk of an institution in the presence of explicit and/or implicit government guarantees (see Section III.C above). As a result, the dependence structure does not isolate market-implied linkages (which are primarily influenced by risk factors that are not covered by any form of guarantee). Moreover, correlation represents only average, linear, bivariate dependence, which does not account for changes in dependence relative to the magnitude of shocks affecting individual default risk.

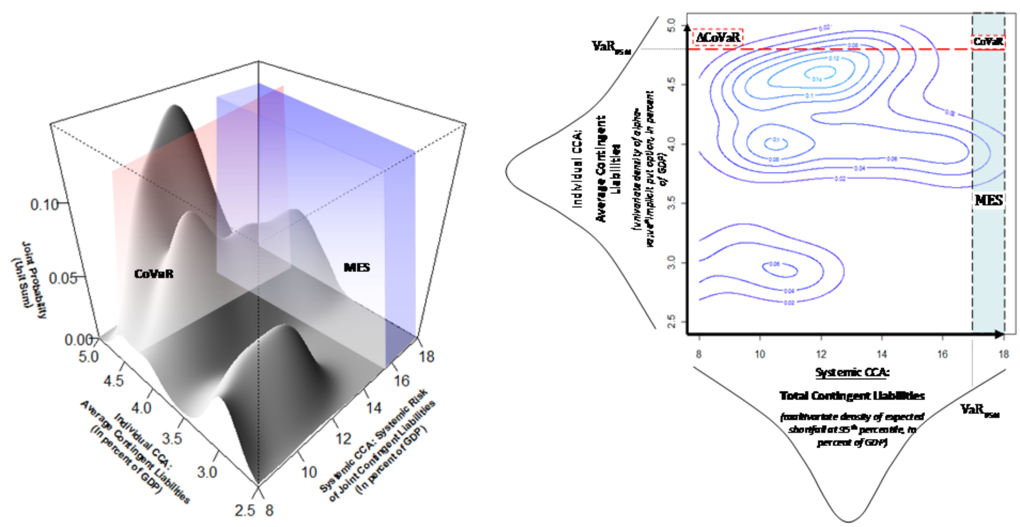
Overall, the Systemic CCA framework seems to be more comprehensive and flexible than CoVaR or MES in the definition and measurement of systemic risk, which can be seen by examining important features of the Systemic CCA method:

- By combining individual expected losses (or contingent liabilities based on additional calculations as presented in Section III.C) to a measure of joint expected losses (or contingent liabilities) the Systemic CCA generates a conditional, non-linear metric of systemic risk sensitivity to individual firms. Figure A1 illustrates the marginal rate of substitution (MRS) between the individual contributions to systemic risk and the joint impact of default risk on systemic risk as the bivariate density of two vectors—the individual expected losses (or individual contingent liabilities) and the joint expected losses (or joint contingent liabilities) over the entire distribution of observable values. Hence, the model uses these elements in a non-deterministic and flexible fashion.

- Both SES and CoVaR are empirically motivated and depend on a pre-specified percentile choice, which covers only a fraction of available data that could be usefully integrated into a system-wide assessment. Figure A1 shows an example kernel density function (Chart 1) and a contour plot (Chart 2) of individual contingent liabilities and systemic risk from total contingent liabilities derived from the Systemic CCA framework. CoVaR represents the average systemic risk at a specific level of statistical confidence of individual contingent liabilities only (perpendicular red plane (Chart 1) and red dashed line (Chart 2)). In contrast, MES, as key element of SES, covers the density contained in the blue rectangle on the right, whose left side defines the threshold of the expected shortfall measure (Chart 2, Figure A1). Neither approach applies multivariate density estimation like Systemic CCA, which would allow the determination of the marginal contribution of an individual institution to changes of both the severity of systemic risk and the dependence structure across institutions of contingent liabilities for any level of statistical confidence and at any given point in time (which would imply coverage of the entire surface of the bivariate kernel density in Chart 1 and the complete contour plot in Chart 2 of Figure A1. Since Systemic CCA also considers the time-variation of point estimates of systemic risk (without re-estimation), it is more comprehensive than CoVaR and SES if these methods were applied to measure systemic risk from contingent liabilities across different levels of statistical confidence at daily frequencies.

Figure A1. Stylized Illustration of the Marginal Rate of Substitution Between Individual and Systemic Risk: Bivariate Kernel Density Function and Contour Plot of Individual Contingent Liabilities and Systemic Risk from Joint Contingent Liabilities (Systemic CCA).

Chart 1 Chart 2



Sample period: 482 observations of individual put option values of sample banks in a small European country with an outsized financial sector relative to GDP. Note: The chart show the bivariate probability distribution of (i) the average contingent liabilities (*alpha*-value*implicit put option) in percent of GDP (as of end-2008) (y-axis) and (ii) total systemic risk from contingent liabilities in percent of GDP (x-axis). Kernel density estimation with Epanechnikov (1969) kernel function and linear binning, using an empirically derived bandwidth.