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Potential Output and Output Gap in Central America, Panama and Dominican Republic

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Abstract

Potential Output is a key factor for debt sustaintability analysis and for developing strategies for growth, but unfortunately it is an unobservable variable. Using three methodologies (production function, switching, and state-space), this paper computes potential output for CAPDR countries using annual data. Main findings are: i) CAPDR potential growth is about 4.4 percent while output gap volatility is about 1.9 percent; ii) The highest-potential growth country is Panama (6.5 percent) while the lowest-growth country is El Salvador (2.6 percent); iii) CAPDR business cycle is about eigth years.

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Contents Page

Abstract....2

I. Introduction....3

II. The Production Function Approach....4
A. Empirical Results for the TFP Approach....4
B. Computing the Output Gap....8
C. Empirical Results: TFP Vis-à-vis HP....8

III. Switching Model....12
A. Application to CAPDR: Identifying Potential Growth and Output Gap Volatility....14
B. Identifying Conditional Probabilities....18
C. Conditional Probabilities and Cycle Indicator Function....19

IV. State-Space Models....22
A. Model I: Deterministic Drift....23
B. Model II: Drift with Mean Reversion....24
C. Estimation and Empirical Results....25

V. Summary of the Models....28

VI. Conclusions....30

Appendix....31
A. Switching Model: Gauss Code....31
B. State-Space Model: Gauss Code....32
C. Optimal Lambda using the Pedersen (2001, 2002) Method: Nicaragua....38

References....39

Tables and Figures
Table 1. CAPDR Growth Decomposition: Contributions and Stylized Facts....6
Table 2. Switching Model: Three States for the Economy....15
Table 3. Relation between Growth and Volatility: State-Space Models....26
Table 4. Three Approaches: Growth and Output Gap Volatilities in CAPDR....28

Figure 1. CAPDR: Capital, Labor and Productivity Contribution to Growth....7
Figure 2. CAPDR and US Output Gaps: Cycles and Correlation....9
Figure 3. CAPDR: Potential Output and Output Gap–Production Function Approach....10
Figure 4. CAPDR: Potential Output and Output Gap–HP Filter....11
Figure 5. Mixture and Density Functions for each State of the Economy: CAPDR....17
Figure 6. CAPDR: Probabilities for each Scenario and Heat Map–Switching Model....20
Figure 7. CAPDR: Potential Output and Output Gap–Switching Model....21
Figure 8. CAPDR: Potential Output and Output Gap–State-Space Model....27
Figure 9. Potential Growth: average of the three methodologies....28
Figure 10. CAPDR: Potential Growth across Models....29
Figure 11. CAPDR: Output Gap Volatilities across Models....29
Figure 12. Q Loss Function and Optimal Lambda....38

I. INTRODUCTION

This paper studies alternative methodologies to assess growth in Central America, Panama and the Dominican Republic (CAPDR). These small economies are open to international trade and financial sector and consequently exposed to international shocks and cycles. It is remarkable how diverse economic policies could bring some much similarity in terms of growth. For instance, Panama and El Salvador have economies integrated to the international financial markets with no domestic currency or active monetary policy (at least in the traditional definition of monetary policy). With a different approach we have Nicaragua, a highly dollarized economy with a crawling peg exchange rate regime.[2] Then we have the Dominican Republic, with a crawl-like arrangement for the exchange rate (IMF, 2011) and an inflation targeting regime (IT). And finally, Costa Rica, which is in the process of implementing an IT regime and currently has a managed arrangement for the exchange rate (IMF, 2011).

Differences in economic growth have been attributed to a variety of ideas, most of them affecting total factor productivity. Accountability, quality of institutions, policy implementation (Swiston and Barrot, 2011), resource misallocation and selection (Hsieh and Klenow, 2009; Bartelsman et al., 2013), slow technology diffusion (Howitt, 2000), and radical institutional reforms (Acemoglu et al., 2011) are among the core concepts used to give explanation of growth performance.

This paper measures potential growth, output gap and output gap volatility for CAPDR countries using three different techniques. First, we have the production function approach, which decomposes GDP using employment and capital stock data, assuming certain technology. Next, we estimate and calibrate a switching regime model to associate growth and growth volatility for each state of the economy (recession, sustainable growth and overheating economy).[3] Finally, a state-space approach is used to decompose observed growth in potential and output gap (both unobserved variables). Main findings are that in CAPDR, potential growth is about 4.6 percent with an output gap volatility of about 1.8 percent. Second, the country with the highest potential growth is Panama (6.7 percent), while El Salvador has only 2.7 percent. Next, CAPDR business cycle is about eight years. Finally, it is well documented that there is a statistically negative correlation between potential growth and output gap volatility.

The rest of the article is structured as follows. Section II describes the production function approach and its results. Section III describes the switching model while section IV presents the state-space approach. Section V summarizes and elaborates a comparative analysis for the region. Section VI concludes, while GAUSS program codes and other methodological details are contained in the Appendix.

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[2] Nicaragua is one of the three economies classified under this exchange rate arrangement. The other two are Botswana and Uzbekistan (IMF, 2011).

[3] Our definition of sustainable growth will define the pattern for potential growth and output gap.

II. THE PRODUCTION FUNCTION APPROACH

The growth accounting exercise was performed over the 1994-2011 period. As it is standard in the literature, these economies were characterized by a Cobb-Douglas production function assuming constant returns to scale (CRS) technology:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where Y_t is output, K_t and L_t are capital and labor services (for simplicity, labor input is defined as the number of employees in the economy[4]), while A_t is the contribution of technology or total factor productivity (TFP); and where the output elasticities (α denotes capital-output elasticity) sum up to one reflecting CRS.

Because capital input is not available, it is generated using the usual perpetual inventory model (Epstein and Macchiarelli, 2010, Teixeira de Silva, 2001) as in:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{2}$$

where the depreciation rate δ was parameterized as 0.05 consistent with a vast empirical literature, while the initial capital stock is computed as $K_0 = I^*/(g+\delta)$. I^* is the benchmark investment (calculated as the average proportion of investment in the total GDP) while g is the average growth of the economy during the sample period 1994-2011. Hence, based on these parameters, the initial capital stock is derived by: $K_0 = \left(I/Y\right) \cdot Y_{1994}/(g+\delta)$. The procedure was implemented across CAPDR.

Since TFP is not observable, the usual procedure applies and is computed inverting the technological process from equation (1) as follows:

$$A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}} \tag{3}$$

Now with the TFP series and using the other inputs in (1), it is possible to decompose GDP growth (Figure 1).

A. Empirical Results for the TFP Approach

This section presents the output growth decomposition and the factor's contribution to growth. Next section, output gaps are generated using the production function approach. All

[4] The growth accounting exercise could benefit from adjusting the labor force by human capital (see, for instance, Sosa, Tsounta, and Kim, 2013). Otherwise, changes in the quality of the labor force are automatically imputed to the estimated TFP measure. A caveat on the measure of TFP: changes in the use of land (not considered here) would contaminate our TFP measure.

the variables (labor, capital and output measured by GDP) have been logged, and as we said, sample period is 1994–2011.[5]

Average regional growth is about 4.3 percent with Panama and Dominican Republic leading the region in terms of growth and volatility, while El Salvador and Nicaragua present the worst performance (Table 1 and Figure 1). In terms of decomposition, capital dynamic explains most of the growth in each country with about 2.1 percent on average for the region, while labor explains 1.5 percent of the regional growth. One interesting and robust result is that TFPs explain about 0.8 of the total regional growth presenting the highest standard deviation (2.3 percent). Dominican Republic and Panama present a TFP of about 2 percent while Honduras and Nicaragua report negative TFPs (-0.4 and -0.2 percent, respectively).

Given the relatively stable contribution to growth of the capital stock (about 2.1 percent with a standard deviation of 0.4 percent; see Table 1), reflected in its low volatility across countries (0.7 percent in average), the exercise reveals the negative correlation between labor and productivity growth. It is common feature that when employment increases in episodes of low GDP growth (below trend or in recessionary cycles), the residual TFP reports negative contribution. This was the case for instance for Costa Rica 2000–2001, El Salvador 2009, Nicaragua 2000, and Panama 2001–2002.

It is symptomatic that low TFPs explain most of the low level of growth for nearly all of the countries in the region. Policies conducted to increase productivity would help to increase potential growth in cases such as Costa Rica, El Salvador, Honduras and Nicaragua. Also policies to increase labor participation would enhance growth. This is the case of countries such as Dominican Republic and El Salvador, where the labor contribution of about 0.7-0.9 percent seems very low.

[5] Guatemala was not considered in the production function approach as employment data was not available. Data source: WEO. Some methodologies (HP Filter) use data up to 2017 to avoid end-of-sample bias.

Table 1. CAPDR Growth Decomposition: Contributions and Stylized Facts

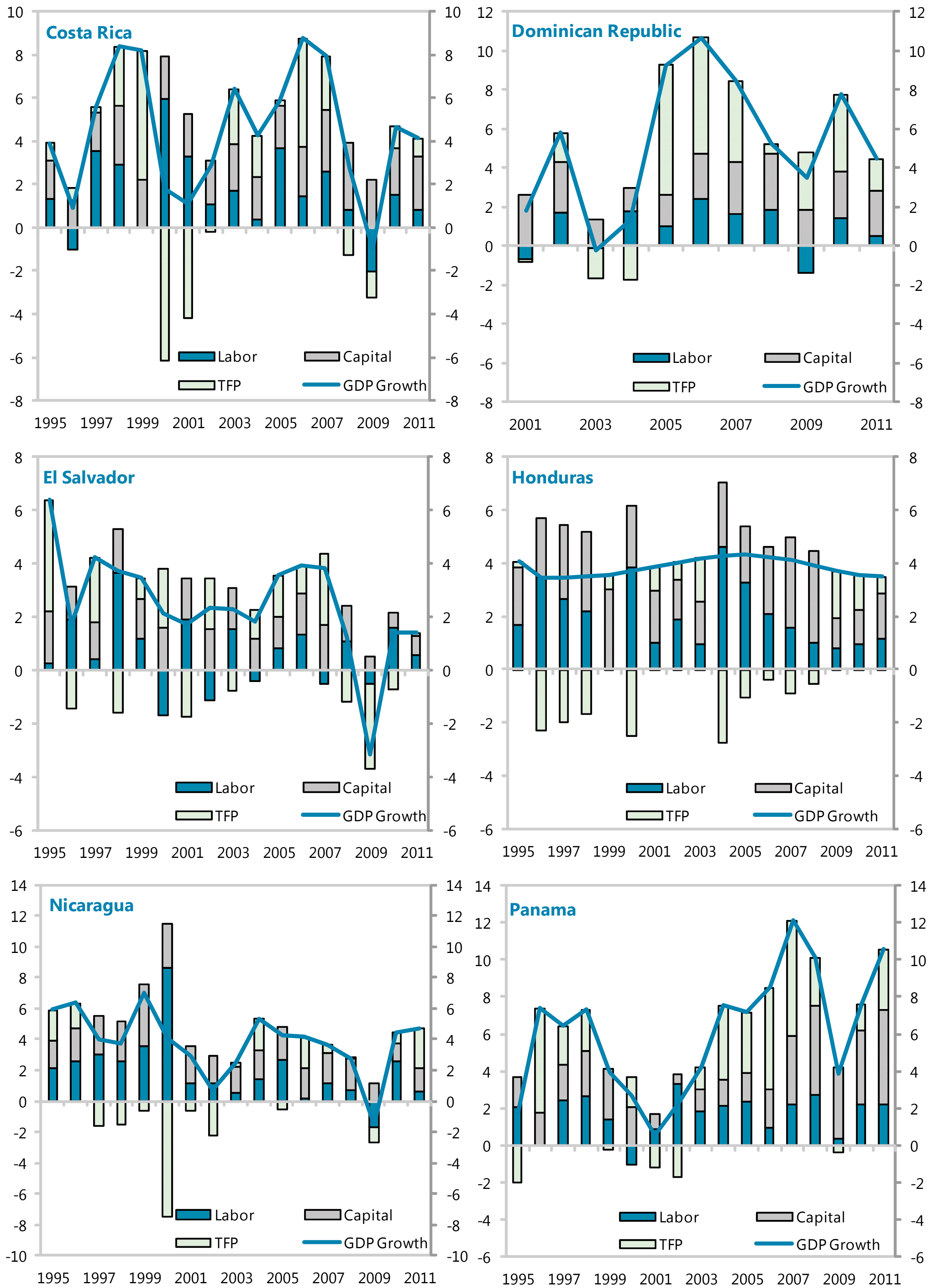
		Costa Rica	Dominican Republic[1/]	El Salvador	Honduras	Nicaragua	Panama	CAPDR[2/]	CAPDR[3/]
GDP Growth									
	Average	4.5	5.3	2.5	3.8	3.8	6.1	**4.3**	**1.3**
	Min	-1.0	-0.3	-3.1	-2.1	-1.5	0.6	**-1.2**	**1.3**
	Max	8.8	10.7	6.4	6.6	7.0	12.1	**8.6**	**2.4**
	Std. Dev.	2.9	3.5	2.0	2.5	2.0	3.3	**2.7**	**0.6**
Labor									
	Average	1.7	0.9	0.7	2.0	2.0	1.7	**1.5**	**0.5**
	Min	-2.0	-1.4	-1.7	0.0	-1.7	-1.0	**-1.3**	**0.7**
	Max	6.0	2.4	3.6	4.6	8.6	3.3	**4.8**	**2.2**
	Std. Dev.	1.9	1.2	1.3	1.3	2.1	1.1	**1.5**	**0.4**
Capital									
	Average	2.2	2.2	1.3	2.3	2.1	2.4	**2.1**	**0.4**
	Min	1.3	1.2	0.5	1.1	1.2	0.6	**1.0**	**0.3**
	Max	3.1	2.9	1.9	3.4	4.1	5.1	**3.4**	**1.1**
	Std. Dev.	0.4	0.6	0.4	0.7	0.7	1.4	**0.7**	**0.3**
TFP									
	Average	0.7	2.2	0.4	-0.4	-0.2	2.0	**0.8**	**1.1**
	Min	-6.1	-1.7	-3.2	-2.7	-7.4	-2.0	**-3.9**	**2.4**
	Max	6.0	6.6	4.2	1.8	2.5	6.2	**4.6**	**2.0**
	Std. Dev.	2.9	2.8	2.0	1.5	2.3	2.5	**2.3**	**0.5**

[1/]Dominican Republic: 2001-2011.

[2/]Simple average.

[3/]Standard deviation.

Figure 1. CAPDR: Capital, Labor and Productivity Contribution to Growth



B. Computing the Output Gap

In the production function approach, output gap is computed using the TFP generated from (3), but rewriting the production function (1) now using trends for all the variables. The standard Hodrick-Prescott (HP) filter[6] is used to generate those trends assuming a smoothness parameter lambda of 100 (the full-capacity stock of capital is approximated by the actual so the HP filter was not applied to the capital stock).[7] The assumption of constant returns to scale was maintained in all countries, calibrating in 0.5 the elasticity of labor to output $(1-\alpha)$.[8] So, starting from the following expression:

$$Y_t^* = A_t^* K_t^{\alpha} (L_t^*)^{1-\alpha} \tag{4}$$

output gap is calculated as a percentage of the potential output as follows:

$$gap_t \equiv \frac{Y_t - Y_t^*}{Y_t^*} \cdot 100 \tag{5}$$

However in our analytical implementation, we used the logarithmic approximation as in:

$$gap_t \equiv (lnY_t - lnY_t^*) \cdot 100 \tag{6}$$

We now turn to compute the output gaps for CAPDR.

C. Empirical Results: TFP Vis-à-vis HP

Using the standard HP filter as a benchmark, the computed potential outputs show not much difference in terms of the output gaps for all CAPDR countries (Figures 3 and 4). Figure 3 provides measures of potential output (all in logs) and output gaps for each country on the region using the production function approach while Figure 4 provides the same analysis using the HP de-trending method.

A regional comparison of the output gaps are represented in the bottom two charts of Figure 3. The left chart at the bottom provides some evidence of the high correlation among regional

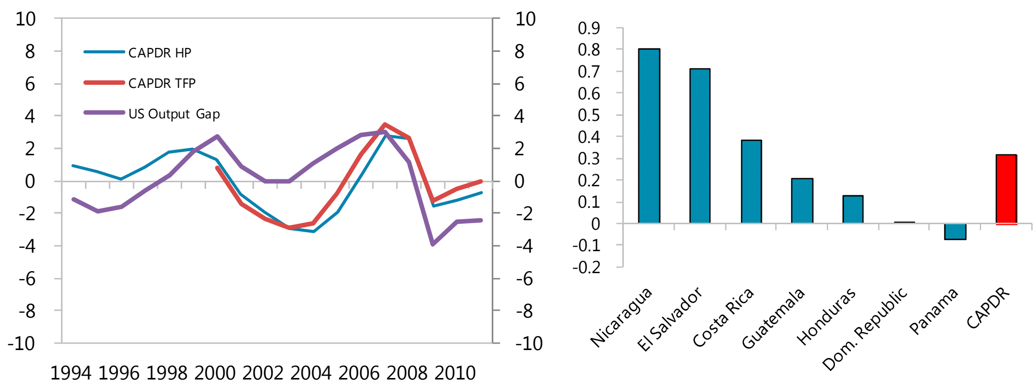
[6] To avoid end-of-sample bias data included projections up to 2017.

[7] Exploratory analysis done using the method of optimal filtering (Pedersen, 2001, 2002) didn't make any significance difference. For instance, for Nicaragua the optimal lambda was 181, however the loss function of the method is very flat between lambdas 100 and 300, implying very similar potential output dynamic for lambdas belonging to this interval. See Appendix B for details.

[8] A cointegration approach was explored to generate estimates for labor and capital-output elasticities. Initial estimations show that all the series are integrated of order 1 (analysis with panel and individual unit root tests), while the Johansen Cointegration test indicates one cointegrating equation. However results from the VECM (DOLS) were not reliable in terms of the value of the estimated coefficients. This extension is left for future research.

output gaps. Magnitudes, frequencies and synchronization of the cycles look similar, aiming the hypothesis of a common factor driving regional growth. The international trade linked to the US economy could be part of the explanation of this common cycle. The following two charts (Figure 2) present some evidence in this direction. US Output gap presents a correlation of about 0.31 with CAPRD Output gap, but partial correlations between countries and the US Output gap are rather heterogeneous. A number of results are worth emphasizing here. First, Nicaragua presents the highest correlation (0.81) followed by El Salvador with 0.72; next, we have Costa Rica, Guatemala and Honduras with correlations in the range of 0.39–0.13, and finally, Dominican Republic and Panama present the lowest correlations with the US Output gap (a remarkable result for Panama given the monetary policy regime).

Figure 2. CAPDR and US Output Gaps: Cycles and Correlation



Note: Output Gap: in percent of Potential GDP

As we can see in Figure 2, TFP and HP approaches give very similar results for output gaps in 2011. The last charts on the right in Figures 3 and 4 provide a closer look of this analysis presenting a 95 percent confidence interval for CAPDR output gap (simple regional average). Almost all the countries present a statistically zero output gap for 2011, with the exception of Panama, which presents a minor “overheating” sign[9] (as we will see, this result is consistent across methodologies).

A number of conclusions are worth emphasizing here. First, the production function and the HP de-trending methods provide indistinguishing results in terms of output gaps for CAPDR countries. Second, there exists a considerable synchronization among the CAPDR output cycles, which could be partially explained by the US output dynamic.[10] Third, all the countries, with the exception of Panama, seem to present an output gap by about zero at the end of the sample (or statistically zero if we consider a 95 percent confidence interval for the CAPDR output gap). Finally, the average business cycle seems to last about eight years.

[9] By overheating we mean growing above potential or trend, implying a positive output gap and consequently inflationary pressures. The usual pass-through to inflation is invoked but not tested here.

[10] This hypothesis, causality tests and the inclusion of the US growth into the model were left for future research.

Figure 3. CAPDR: Potential Output and Output Gap–Production Function Approach

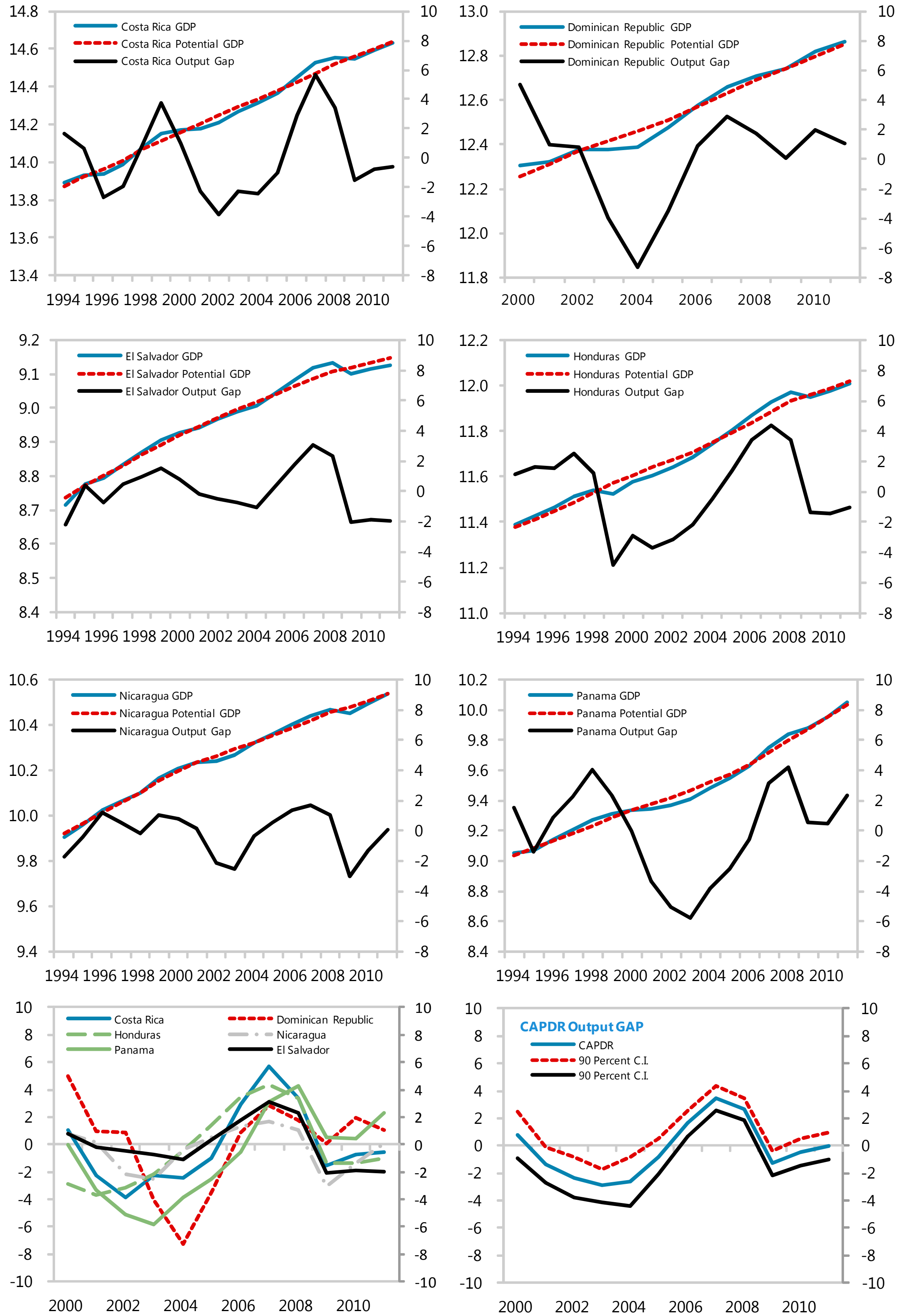
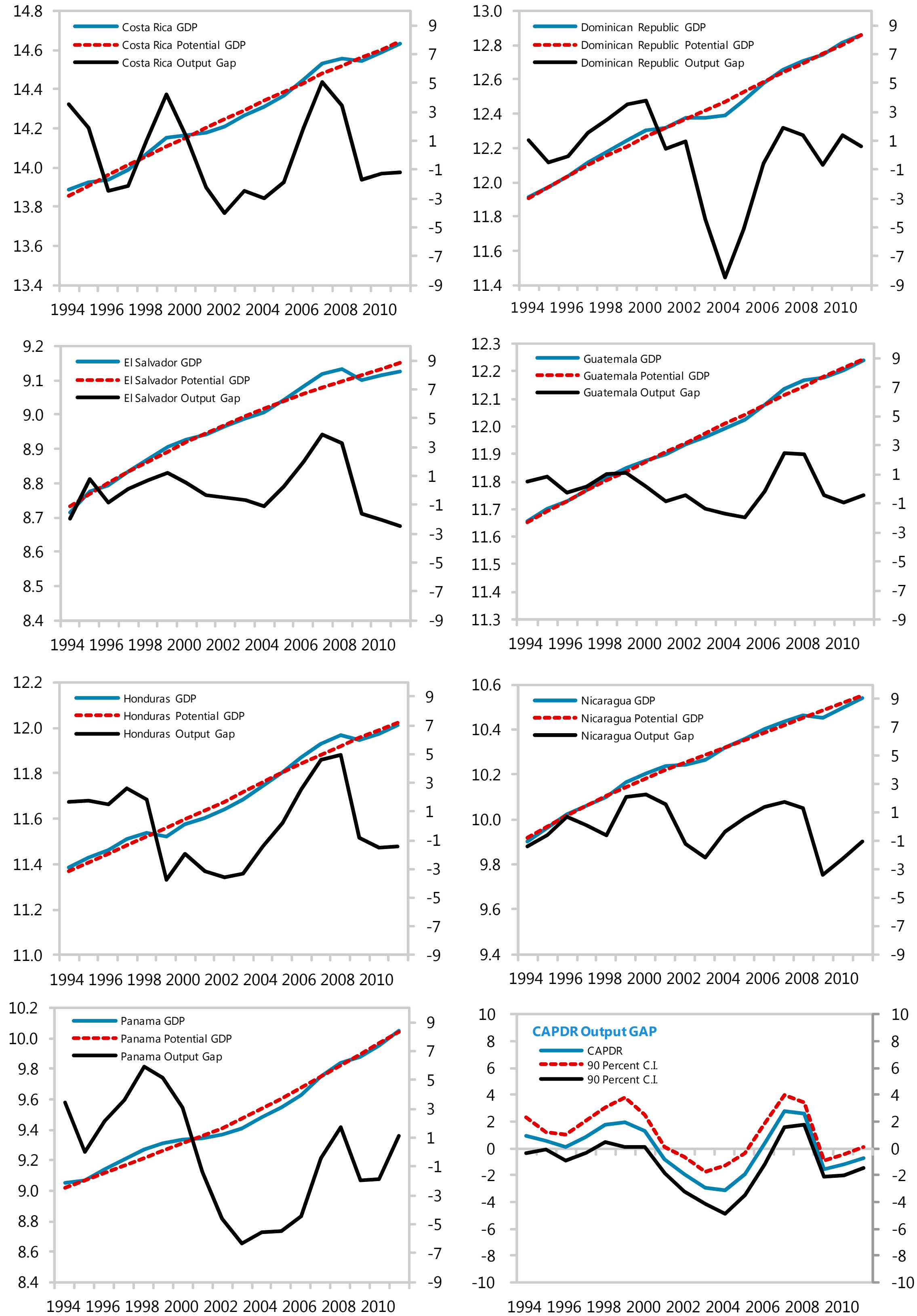


Figure 4. CAPDR: Potential Output and Output Gap–HP Filter



III. Switching Model

This section applies the switching model to compute steady-state growth and output gap volatilities for CAPDR countries.

Regime Switching models provide a numerical interpretation of the idea that the time series data generating process can be generated using a mixture of stationary processes[11] each represented with different probability density functions (Hamilton, 1989, 1990, 1993 and 1994).[12] In this approach, the actual data is represented by a continuum jumping from a finite set of probability density functions, each representing a specific scenario or state of the economy. This section develops the switching regime process and presents the iterative expected maximization (EM) algorithm used to generate those density functions.[13]

Let's consider a variable y_t that comes from N different states ($s_t=1,...,N$), each one represented by its own probability density function $y \to N\left(\theta_{s_t}, \sigma_{s_t}^2\right)$. It is straightforward to define the associated density function as:

$$f\left(y_t \mid s_t = j, \Psi_{t-1}; \Gamma\right) = \frac{1}{\sqrt{2\pi\sigma_{s_t}^2}} . e^{-\frac{1}{2}\left[\frac{y_t - \theta_{s_t}}{\sigma_{s_t}}\right]^2}, \forall\ j = 1,2,....,N \tag{7}$$

where the unknown parameters are represented by vector $\Gamma = \left[\theta_1, \theta_2, ..., \theta_N, \sigma_1^2, \sigma_2^2, ...\sigma_N^2\right]'$. The random variable s_t is generated from some distribution function where the unconditional probability that s_t could be equal to j is denoted by π_j. Analytically:

$$P\left[s_t = j \mid \Psi_{t-1}; \Gamma\right] = \pi_j\ , \forall \text{j=1,2,...,N} \tag{8}$$

where now the set of conditional information Γ is expanded to include the probability vector π, whose non negative elements sum up one.

Using the Bayes' theorem[14] we can say that the joint probability for a random variable y_t and the state s_t is given by:

[11] Stationarity means a stable variance-covariance matrix.

[12] See Kim and Nelson (1999) for further details on these topics.

[13] Mizrach and Watkins (1999) mentioned that the EM algorithm is very robust contrasting the traditional hill-climbing gradient techniques. However it is highly computer intensive, which decreased its attractiveness. For a complete evaluation of alternative univariate non linear optimization routines, see Potter (1999).

[14] Bayes' theorem or Bayes' rule establish that:
$P\left[y_t, s_t = j \mid \Psi_{t-1}; \Gamma\right] = f\left(y_t \mid s_t = j; \Psi_{t-1}; \Gamma\right) \cdot f\left(s_t = j \mid \Psi_{t-1}; \Gamma\right)$.

$$p\left(y_t, s_t = j \mid \Psi_{t-1};\Gamma\right) = \frac{\pi_j}{\sqrt{2\pi\sigma_{s_t}^2}}.e^{-\frac{1}{2}\left[\frac{y_t-\theta_{s_t}}{\sigma_{s_t}}\right]^2}, \forall\ j = 1,2,....,N \tag{9}$$

Consequently, the unconditional distribution function for y_t will be represented by:

$$f\left(y_t \mid \Psi_{t-1};\Gamma\right) = \sum_{j=1}^{N} p\left[y_t, s_t = j \mid \Psi_{t-1};\Gamma\right] \tag{10}$$

As is usual, assuming independent and identically distributed (*iid*) observations for all t=1,2,3....,T, the optimizing equation can be represented by the natural logarithm of the joint density function or the likelihood function (LMLE) for the vector Γ:

$$\begin{aligned} \underset{\{\Gamma\}}{Max}\, \ell\left(\Gamma\right) &= \sum_{t=1}^{T} \ln f\left(y_t \mid \Psi_{t-1};\Gamma\right) \\ &= \sum_{t=1}^{T} \ln\left[\sum_{j=1}^{N} f\left(y_t \mid s_t = j, \Psi_{t-1};\Gamma\right)\cdot P\left[s_t = j \mid \Psi_{t-1};\Gamma\right]\right] \end{aligned}$$

$$\text{s.t. } \sum_{j=1}^{N}\pi_j = 1, \pi \geq 0, \forall j = 1,2,...,N$$

What's interesting about this methodology is that using the estimated coefficients we can compute the probability of being in each scenario/state. To compute these probabilities it is necessary to provide the observed y_t to the unconditional probability $\hat{\pi}_j$ as in:

$$P\left[s_t = j \mid \Psi_{t-1};\hat{\Gamma}\right] = \frac{P\left[y_t, s_t = j \mid \Psi_{t-1};\hat{\Gamma}\right]}{f\left(y_t \mid \Psi_{t-1};\hat{\Gamma}\right)} = \frac{\hat{\pi}_j \cdot f\left(y_t \mid s_t = j, \Psi_{t-1};\hat{\Gamma}\right)}{f\left(y_t \mid \Psi_{t-1};\hat{\Gamma}\right)} \tag{11}$$

The traditional optimization procedure to solve this problem consists in estimating the vector of coefficients of the log-linear transformation maximizing its logarithmic function through traditional gradient methods[15]. More precisely, Mizrach and Watkins (1999) mention that this kind of problem can be solved by two alternative methods. First methodology is related with the traditional Hill Climbing techniques using gradient numerical search algorithms. Standard procedures include Newton-Rampson (NR), Broyden, Fletcher, Goldfarb and Shanno (BFGS), and the Davidon-Fletcher-Powell (DFP)[16] methods.

A second methodology consists in the application of the Expected Maximization (EM) algorithm developed by Hamilton (1990, 1991).[17] The algorithm consists in a two-stage

[15] Johnson (2000) evaluates alternative optimization methods, considering the traditional hill-climbing techniques and the most advanced genetic algorithms optimization methods. It is shown that genetic algorithm methods are very efficient in finding the optimal parameter vector, although computer-intensive.

[16] All these methods are available in GAUSS and MATLAB libraries. A good description can be found in Hamilton (1994), Press et al. (1988), Thisted (1988) and Mittelhammer et al. (2000).

[17] Could be the case that the maximum likelihood function is infinite if some scenario's distribution mean is equal to any observation, where the variance of this state equals to zero. Hamilton (1991) uses a "pseudo-Bayesian" procedure to solve this problem, modifying the numerator and the denominator by some constant, to

(continued…)

procedure where the stopping rule is defined by some distance criteria applied to the estimated parameter vector $\hat{\Gamma}$ along the k[th]=min{k,K} iteration. First step builds the expectation (E) assuming a vector of parameters $\hat{\Gamma}^{(k-1)}$ for the k[th] iteration, while second stage maximizes (M) the log-likelihood function generating new $\hat{\Gamma}^{(k)}$ estimate.

The iterative procedure considers the following system of three equations, each one computing the mean, the volatility and the probability:

$$\hat{\theta}_j = \frac{\sum_{t=1}^{T} y_t \cdot P\left[s_t = j \mid \Psi_{t-1}; \hat{\Gamma}^{(k-1)}\right]}{\sum_{t=1}^{T} P\left[s_t = j \mid \Psi_{t-1}; \hat{\Gamma}^{(k-1)}\right]}, \forall j = 1,2,...,N$$

$$\hat{\sigma}_j^2 = \frac{\sum_{t=1}^{T} \left(y_t - \hat{\theta}_j\right)^2 \cdot P\left[s_t = j \mid \Psi_{t-1}; \hat{\Gamma}^{(k-1)}\right]}{\sum_{t=1}^{T} P\left[s_t = j \mid \Psi_{t-1}; \hat{\Gamma}^{(k-1)}\right]}, \forall j = 1,2,...,N$$

$$\hat{\pi}_j = \frac{1}{T}\sum_{t=1}^{T} P\left[s_t = j \mid \Psi_{t-1}; \hat{\Gamma}^{(k-1)}\right], \forall j = 1,2,...,N \tag{12}$$

where $P\left[s_t = j \mid \Psi_{t-1}; \hat{\Gamma}\right] = \dfrac{\hat{\pi}_j \cdot f\left(y_t \mid s_t = j, \Psi_{t-1}; \hat{\Gamma}\right)}{f\left(y_t \mid \Psi_{t-1}; \hat{\Gamma}\right)}$, with f representing the normal density function.

The following section applies this methodology to CAPDR countries.

A. Application to CAPDR: Identifying Potential Growth and Output Gap Volatility

According to this methodology, first we need to identify the number of states under analysis. We set the number of states or scenarios to three[18]: *i)* recession or low grow, *ii)* sustainable growth, and, *iii)* boom or overheating. The second scenario called "sustainable growth" will characterize the state in which the economy is growing at its potential or long term sustainable trend. The other two distributions will represent long-term unsustainable scenarios: economies cannot run continuously in recession or overheating (these are not "absorbing states" in transition matrices' taxonomy). Accordingly, fine tuning or structural policy measures are expected to be implemented by the authorities to help containing growth along those sustainable paths.

avoid this indeterminacy problem in the iterative system of equations. It was not necessary to implement this modification in our algorithm as indeterminacy was not an issue.

[18] It seems natural in our application set the number of states equals to three. In our model, each state can be easily identifiable with a specific macroeconomic policy stance: for overheating (recession), macroeconomic policy should be contractionary (expansive). Finally, under sustainable growth patterns, macroeconomic policy should aim to be neutral and under this scenario the economy is growing at its potential.

Once we applied the switching model, we obtain the mixture distribution for growth. The following table describes the convergence values for mean growth, volatility and the unconditional probabilities for each scenario, after the algorithm stopped (k^* iterations).[19]

Table 2. Switching Model: Three States for the Economy

	Convergence Results for each Scenario		
	Moderate Growth or Recession	**Sustainable Growth**	**Overheating**
		Growth (%)	
CAPDR	**0.32**	**4.77**	**7.76**
Costa Rica	2.43	5.00	8.32
Dominican Republic	1.45	6.65	10.59
El Salvador	-3.10	2.78	6.85
Guatemala	0.55	3.42	5.64
Honduras	-1.77	3.65	6.13
Nicaragua	-0.17	4.04	6.48
Panama	2.86	7.86	10.32
		Std. Deviation (%)	
CAPDR	**0.90**	**0.93**	**0.57**
Costa Rica	1.95	1.13	0.57
Dominican Republic	1.09	1.52	0.08
El Salvador	0.00	1.07	0.63
Guatemala	0.00	0.69	0.64
Honduras	0.35	0.71	0.33
Nicaragua	0.77	0.89	0.50
Panama	2.15	0.49	1.21
		Unconditional Probability (%)	
CAPDR	**23.42**	**57.67**	**18.90**
Costa Rica	45.93	26.80	27.27
Dominican Republic	26.20	64.47	9.33
El Salvador	4.76	76.46	18.78
Guatemala	4.76	84.55	10.69
Honduras	14.29	52.63	33.08
Nicaragua	23.81	64.14	12.05
Panama	44.23	34.66	21.11

Estimates of potential growth in Table 2 are at least diverse, with regional potential average of about 4.8 percent (in overheating, regional growth is obviously higher however presents less volatility). The growth estimates for the recessionary state are not always negative given that in countries such as Panama, Costa Rica and Dominican Republic, average growth were not negatives. On the opposite, El Salvador, Honduras and Nicaragua experienced episodes of negative growth.

In addition, the table suggest some support for the possibility that for almost ten years (57.7 percent of the sample), CAPDR economies experienced sustainable growth patterns, and

[19] In the GAUSS code (see appendix) the iterations are indexed with the letter m. It was considered a maximum of k=10000 iterations, however convergence was achieved earlier.

only four years (23.4 percent of the sample) of episodes associated with recessionary states. The remaining years are associated to overheating.

It is possible to compute the probability density functions for the three scenarios for each of the CAPDR countries (Table 2).[20] This is reported in Figure 5. The distributions on the right of each chart (red-dashed line) represent the probability density function of an overheating economy. For the Dominican Republic the leptokurtosis is evident in this state given the low probability of this event (9.3 percent of the sample, about 1½ years). For this scenario, Costa Rica and Honduras also exhibit similar features.

Left-hand side distributions in Figure 5 represent probability density functions for the low-growth scenario. El Salvador and Honduras show a clear negative figure for this scenario with average growths of -3.1 and -1.8 percent, respectively. Nicaragua presents a slightly negative average growth (-0.2 percent) with a standard deviation of about 0.8 percent, a bit below CAPDR (0.9 percent). Under this scenario, leptokurtosis is also present in El Salvador and Guatemala, with very low volatilities.

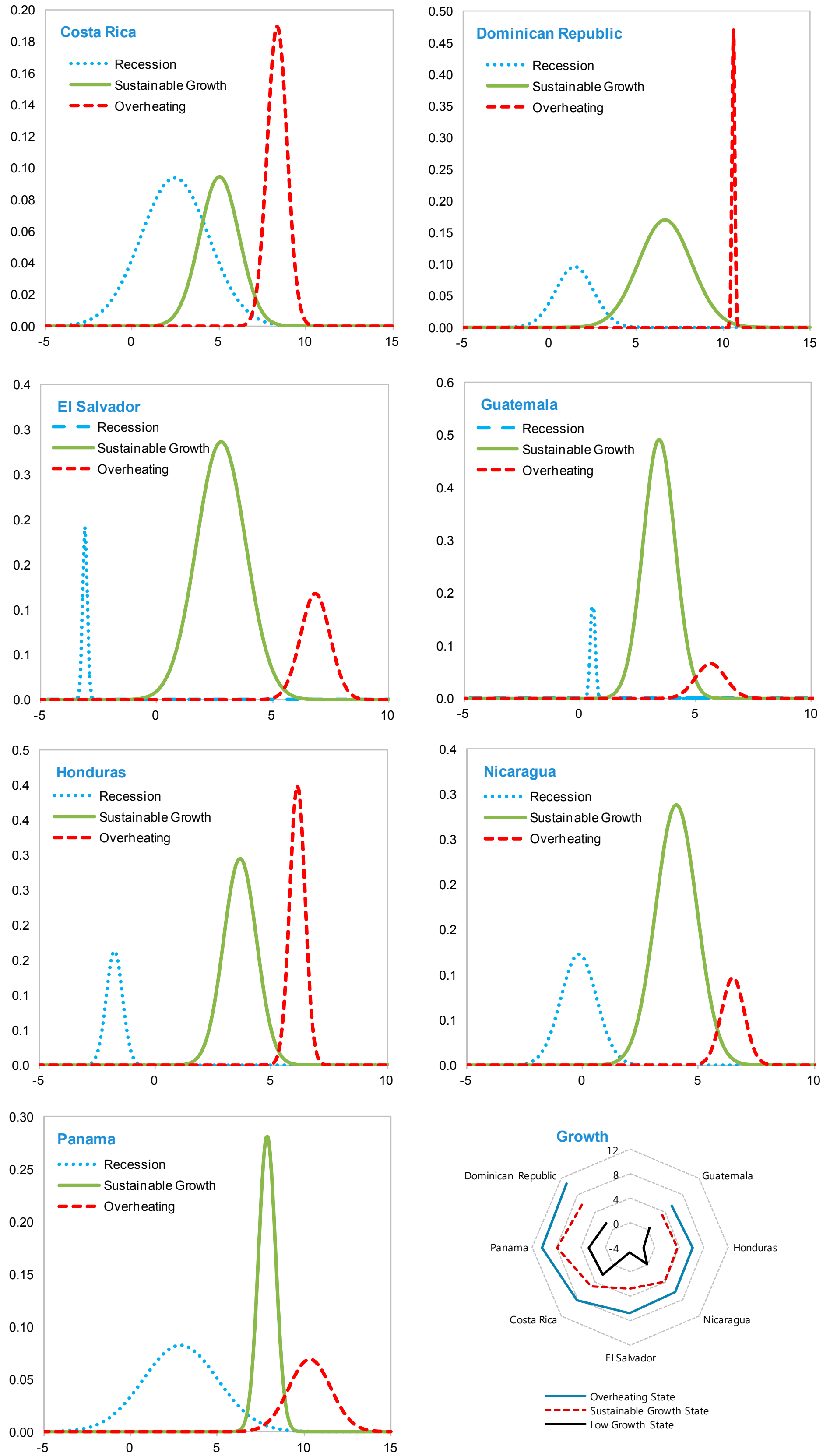
In our discussion, the relevant analysis should be centered on the sustainable growth distribution (probability density functions located in the middle of the charts in Figure 5). By construction, these distributions represent growth in economies without inflationary pressures (in theory, because this is not included in the model). Here authorities should take a neutral stance in term of monetary-fiscal measures. In a normal economy (a textbook case) monetary policy should be neutral, with interest rate aligned with the long term inflation (which is the targeted inflation) and long term real interest rate (which could be approximated to the real GDP per capita growth).

Dominican Republic, Panama and Costa Rica reported average growth rates of/above 5 percent, above the other economies. Nicaragua has an average potential growth of about 4 percent with a standard deviation of 0.9 percent; Honduras presents a potential growth of 3.7 percent with the lowest standard deviation in the region (0.7 percent); Guatemala reports a potential growth of about 3.4 percent with a volatility of about 0.7 percent; finally, El Salvador reports the lowest potential growth under this scenario with a 2.8 percent and a standard deviation of about 1.1 percent, slightly above the CAPDR volatility (0.9 percent).

In terms of unconditional probabilities, the region reports a 58 percent of chances to be in this scenario, clearly above the other two events (23.4 percent for recession and 19 percent for overheating). However, the evidence of leptokurtosis in the distributions for Costa Rica and Panama was also reflected in their probabilities: these are the only countries with probabilities below 50 percent (35 and 27 percent respectively). Remarkable are the cases of Guatemala and El Salvador reporting very high probabilities (85 and 77 percent, respectively).

[20] Given the mixture approach, the envolvent function for these three distributions is called mixture distribution (not reported) and should integrate 1 (100 percent). Each one of the distributions reported across the country charts do not integrate 1 because there were weighted using the unconditional probability reported in the third segment of Table 2. However the envolvent function it does.

Figure 5. Mixture and Density Functions for each State of the Economy: CAPDR



B. Identifying Conditional Probabilities

Mixture approach allows generating probabilities of being in each state or scenario conditional on actual GDP, for all CAPDR countries. The implemented procedure does not impose any corner solutions for the probabilities (0 percent or 100 percent), in such a way that it is possible that given all the available information for any particular date, the probabilities differs from those extreme values. Even when each distribution function overlaps with others scenario distributions, the procedure is efficient in identifying the state in which the economy is.

Conditional probabilities are reported in Figure 6, along with a heat-map at the bottom, to be explained and discussed later. Certainly, they are by construction in line with the unconditional probabilities presented in Table 2 and also with the output gaps reported in Figure 7. The decomposition from the unconditional (all sample) to the conditional probabilities were built using economies' output.

In general, there are jumps from one distribution to another as we should expect. The persistence of some states or scenarios, measured and observed through the value of the corresponding probability, is the result of the actual output path. To better understanding of the outcomes, let's discuss some of the results in detail. In Costa Rica for instance, there are three episodes of overheating: 1992–93, 1998–99, and 2006–07. After the last episode of overheating, the international financial crisis hit the country increasing the probability of being in a recessionary state. As a consequence of this, the probability of being in a recessionary state increases to almost 100 percent in 2008, reaching 100 percent by 2009. For 2010—11 the negative effects of the international crisis decrease and the economy moves into a scenario of sustainable growth.

Now we will focus the analysis in the last segment of the sample. The Dominican Republic was facing sustainable growth patterns during 2007–08. Even when the international financial crisis was spread worldwide, its immediate impact on this country was minor. This result is supported from the probabilities of being in a sustainable or a recessionary state: 50 percent each scenario in 2009. Next couple of years, probabilities switched in favor of the sustainable state with a probability of about 95 percent.

The assessment for El Salvador, Guatemala, Honduras and Nicaragua is similar for the last segment of the sample. Countries were experiencing sustainable growth patterns by 2008. However after the crisis, all economies switched immediately to recessionary states. The probability of being in sustainable growth paths decreased to zero while the chance of being in a recession was one hundred percent. The negative impact was transitory, as all the economies recovered its sustainable status by 2010–11.

The story in Panama is somehow different. During 2007–08 the economy was clearly overheated, and during 2009 the economy moved transitorily to a low-growth growth scenario, recovering the next year. By 2011, Panama was the only economy in the CAPDR region with overheating signs as the probability of this event was 100 percent.

The results of our analysis show that, although all economies were affected by the international financial crisis, all of them recovered very quickly reporting scenarios of sustainable growth for 2010–11. The only country experiencing an overheating status by 2011 is Panama.

The following section develops an indicator that summarizes the three probabilities into one.[21] This innovative concept will help to understand the performance of the regional potential growth.

C. Conditional Probabilities and Cycle Indicator Function

It is useful to build a comprehensive indicator to follow the performance of the economy using as inputs the probabilities calculated in the previous section. Specifically, the following index is computed to analyze the overall picture of growth using as inputs the probabilities presented in Figure 6 and calculated from the switching model.

Let's define *F* the cycle indicator function and consider a sigmoid transformation of an artificial factor ξ, as follows:

$$F = \frac{e^{\xi}}{1+e^{\xi}} \tag{13}$$

where the artificial factor ξ is defined based on the three probabilities calculated in the switching exercise as follows:

$$\xi = \frac{P[\text{Overheating}] - P[\text{Recession}]}{P[\text{Sustainable Growth}]} \tag{14}$$

By construction we know that the cycle indicator function *F* belongs to the interval [0,1]. If the function *F* takes a value of 0.5 means that the economy is showing signs of sustainable growth, but if *F* moves towards 0(1) signs of recession (overheating) emerges. With this definition at hand, we can assess the overall situation of the economy.

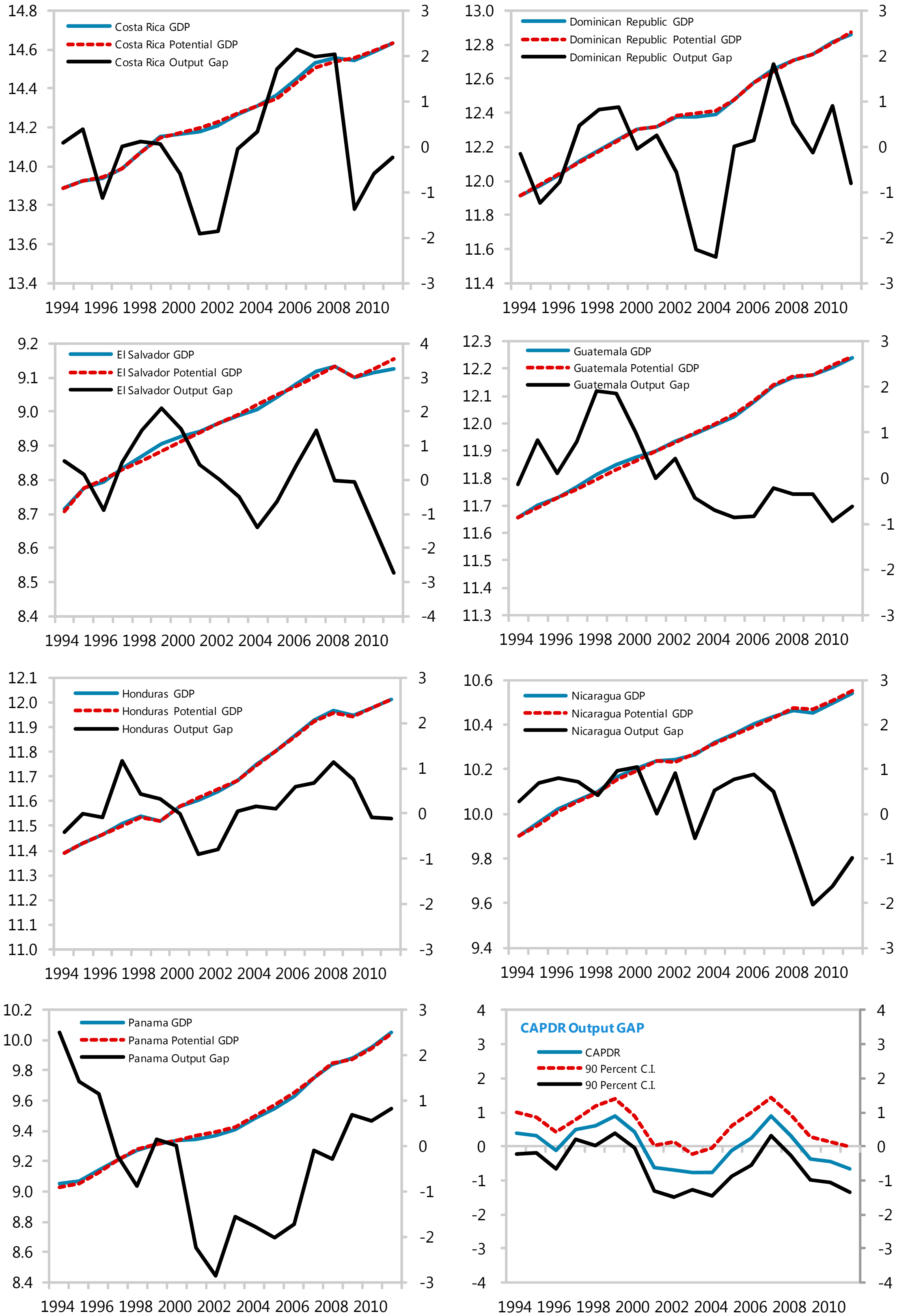
Instead of presenting the charts for the CAPDR economies, a summarized heat-map was built (bottom-right chart on Figure 6). This illustration points out eventual "inflationary pressures" (or overheating economies) just by looking at the color of the cells, generated using the output growth dynamic according to the computed switching probabilities. The color spectrum goes from blue, which indicates a recessionary scenario, to red depicting an overheating state. As we discussed in the previous section, most of the economies were experiencing sustainable growth patterns by 2008, with the exception of the overheating situation in Panama (red shade) and the cooling scenario in the Costa Rica (blue cell). Once the international financial crisis hit the region in 2009, all the economies went into a recessionary state (blue cells in the chart) to quickly recover and reach sustainable growth patterns by 2010–11. The only exception of this recovery is Panama which reaches an overheating scenario during 2011.

[21] This indicator would be useful to assess monetary policies during the cycles.

Figure 6. CAPDR: Probabilities for each Scenario and Heat Map–Switching Model

/1 Computed using the Global Indicator function F explained in the text. Red cell indicates overheating, blue: recessionary state, and yelow: sustainable growth.

Figure 7. CAPDR: Potential Output and Output Gap–Switching Model



The main conclusion of this analysis is that potential output growth is about 4.8 percent for the region, with a standard deviation of about 1 percent, implying that most of the countries report a potential growth between 3.8 and 5.8 percent. However, some heterogeneous results can be observed beyond the one-standard deviation confidence interval: economies such as El Salvador reports potential growth of about 2.8 percent while economies such as Panama and the Dominican Republic present potential growth over 6 percent. As a general conclusion it is possible to assert that after the crisis hit the region in 2009, all CAPDR economies experienced a fast recovery by 2010–11, going through patterns of sustainable growth. Only Panama seems to be suffering from overheating in 2011.

The following section presents the last approach to assess and measure potential growth and output gaps: the state-space model.

IV. State-Space Models

This section develops the state-space approach to identify and decompose the observed GDP growth in two components: the potential output and the output gap. The general structure of the model is represented by two blocks of equations which characterize the state space system: the measurement and the state equations.

$$y_t = H\mathrm{B}_t + Ax_t + \varepsilon_t^y \tag{15}$$

$$\mathrm{B}_t = \Gamma_0 + \Gamma_1 \mathrm{B}_{t-1} + \varepsilon_t \tag{16}$$

Equation (15) represents the dynamic of the measurement variables defined by y_t (log of GDP) explained by a vector of observed exogenous variables x_t, a vector of unobserved state variables B_t and an *iid* error term $\varepsilon_t^y \xrightarrow{iid} N(0, \Theta)$. For one measured variable the variance covariance matrix is defined by the scalar $\Theta = \sigma_y^2 < \infty$ and should be estimated by maximum likelihood (ML) procedures.

The dynamic of the state variables is represented by the state equation (16). As is standard, the error term is assumed to be uncorrelated with the error term of the measurement equation (15), and in general is represented by a data generating process (DGP) centered in zero, normally distributed, and with a diagonal variance covariance matrix Q:

$$\varepsilon_t \xrightarrow{iid} N(0, Q) \tag{17}$$

The ML estimation of the state space representation (15)-(17) is performed using the Kalman filter method (KF). This is a recursive process based on two stages: prediction and correction. For prediction we use some prior information on estimates of the parameters Γ_0, Γ_1, H and A, and the variance covariance matrices Θ and Q, while for the correction, we use the posteriors on the estimates and the variance covariance matrix. The Kalman factor makes use of prior information to generate the posteriors, and this learning procedure is repeated iteratively until all the sample data is analyzed. In the following sections we present the two specifications applied to the CAPDR economies.

A. Model I: Deterministic Drift

The State Space structure of the system can be represented by one measurement equation that links the current values of output $\{y_t\}$ with two state variables: potential output and output gap, represented by $\{y_t^p, ygap_t\}$ respectively.

$$y_t = y_t^p + ygap_t \tag{18}$$

$$y_t^p = \overline{\mu} + y_{t-1}^p \tag{19}$$

$$ygap_t = \rho_1 ygap_{t-1} + \rho_2 ygap_{t-2} + \varepsilon_t^{ygap} \tag{20}$$

$$\varepsilon_t^{ygap} \xrightarrow{iid} N\left(0, \sigma_{ygap}^2\right)$$

To mapping this model to the State-Space representation (15)-(16) we need to re write the system as:

$$\begin{aligned}
&H = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \\
&\mathrm{B}_t = \begin{bmatrix} y_t^p \\ ygap_t \\ ygap_{t-1} \end{bmatrix} \\
&A = x = \Theta = 0 \\
&\Gamma_0 = \begin{bmatrix} \overline{\mu} \\ 0 \\ 0 \end{bmatrix}; \ \Gamma_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_1 & \rho_2 \\ 0 & 1 & 0 \end{bmatrix}
\end{aligned} \tag{21}$$

Potential output follows a random walk with deterministic drift or trend while the output gap is represented by a stable AR(2)[22]. All the variables are in logs and the residuals follows a Gaussian white noise. Rewriting the system, the dynamic of the state variables can be summarized by the following format which represents the transition equations:

$$\begin{bmatrix} y_t^p \\ ygap_t \\ ygap_{t-1} \end{bmatrix} = \begin{bmatrix} \overline{\mu} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_1 & \rho_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} y_{t-1}^p \\ ygap_{t-1} \\ ygap_{t-2} \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{ygap} \\ 0 \end{bmatrix} \tag{22}$$

The variance covariance matrix of the independent residuals of the transition system is as follows:

[22] Stationarity condition requires that all the roots of the AR (2) differential equation must be outside the unit circle, which implies that: $|\rho_2| < 1, \rho_1 + \rho_2 < 1, \rho_1 - \rho_2 < 1$, simultaneously. This stationarity condition must be imposed in the Kalman procedure.

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma^2_{ygap} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{23}$$

In summary, the four parameters to estimate are: $\{\rho_1, \rho_2, \bar{\mu}, \sigma^2_{ygap}\}$.

B. Model II: Drift with Mean Reversion

Very similar to the previous model, the State Space structure of the system will be represented by one measurement equation that links the current values of output $\{y_t\}$ with two state variables $\{y^p_t, ygap_t\}$. However, in this representation, potential output follows a random walk with drift or trend where the process governing the drift follows a mean reversal dynamics with long term steady state $\bar{\mu}$ and with an adjustment coefficient $\beta \in (0, 1)$:

$$y_t = y^p_t + ygap_t \tag{24}$$

$$y^p_t = \mu_{t-1} + y^p_{t-1} \tag{25}$$

$$ygap_t = \rho_1 ygap_{t-1} + \rho_2 ygap_{t-2} + \varepsilon^{ygap}_t \tag{26}$$

$$\varepsilon^{ygap}_t \xrightarrow{iid} N\left(0, \sigma^2_{ygap}\right)$$

$$\mu_t = (1-\beta)\bar{\mu} + \beta\mu_{t-1} \tag{27}$$

The dynamics of the state variables can be summarized by the following matrix system which represents the transition equations:

$$\begin{bmatrix} y^p_t \\ ygap_t \\ ygap_{t-1} \\ \mu_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (1-\beta)\bar{\mu} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \rho_1 & \rho_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix} \cdot \begin{bmatrix} y^p_{t-1} \\ ygap_{t-1} \\ ygap_{t-2} \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon^{ygap}_t \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

The variance covariance matrix of the independent residuals of the transition system is as follows:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{ygap} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{29}$$

In conclusion, the parameters to estimate under this representation are: $\{\rho_1, \rho_2, \bar{\mu}, \beta, \sigma^2_{ygap}\}$.

The following section reports the estimation of the coefficients associated to the two state-space models developed previously.

C. Estimation and Empirical Results[23]

This section presents the results of the methodology that decomposes output into potential output and output gap, considering the models discussed above. Table 3 shows a summary for both the basic and extended specification (Models I and II).

To begin, we estimate the simple version of the State-Space model, called Deterministic Drift (Model I). Panama is the greatest-potential growth country in the region with 5.6 percent, and Nicaragua is the lowest-growth country (3.3 percent). Furthermore, Panama and the Dominican Republic share the highest volatilities in the region with an output gap standard deviation of about 2.4 percent. The lowest-volatility country is Guatemala with 0.8 percent. In this specification all the coefficients are statistically significant at 5 percent of confidence.

The alternative specification (Model II) adds dynamic to the drift allowing for mean reversion in the process (see last section for details). As in the previous model, almost all the coefficients of this representation are statistically significant (with the exception of some β).[24] For all countries, the steady-state potential growth was higher (parameter μ) and also statistically significant. For the first model CAPDR average potential growth was 4.37 percent while for the second specification, potential growth is 4.68 percent.[25] As average growth increases also does volatility. Output gap volatility was about 1.96 percent in Model I while 2.1 percent for the second model.

Finally, the velocity in which output returns to its trend after a shock could be measured through β. For most countries is either low and/or statistically insignificant. The exception is Nicaragua with a β equals to 0.3824, meaning that after a shock output will return to its steady growth in just about 2½ years (inverse of 0.3824). The first and second order coefficients for the output gap are barely constant for the region (simple average) across models (1.04 and -0.51 for model I, and 1.14 and -0.45 for model II).

In conclusion, state-space models offer a good tool to decompose output in potential output and output gap. The models considered in this section gave almost similar measures for the potential output levels. Some downside bias could be emerging in the basic specification which makes us prefer the second model; here, the potential growth is always higher across-countries. The same result applies for the output gap volatility given that for the second specification the CAPDR volatility is also higher.

---

[23] The GAUSS codes used in this section are available upon request. Convergence issues compelled us to exclude El Salvador from the sample.

[24] The CAPDR average β is about -0.04.

[25] These are simple averages, not reported in the table.

Figure 8 reports potential output and output gaps considering the second specification. In the aggregate figures, this approach offers no major differences in comparison with the other two methodologies. However its flexibility to include in the model more “economics” is huge.

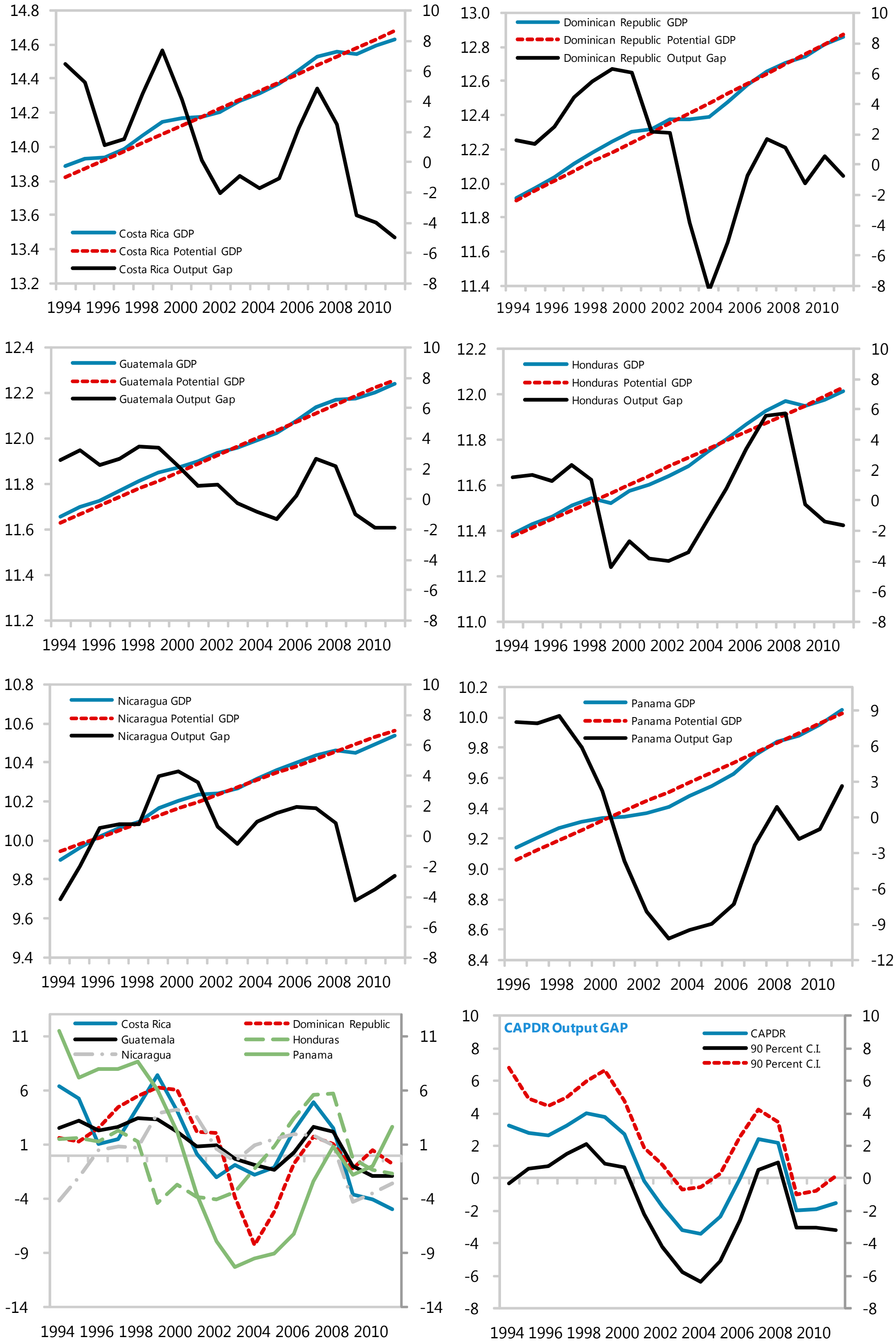
Some final conclusions are worth emphasizing here. First, average output growth for the CAPDR is about 4.7 percent with an output gap volatility of about 2.1 percent. Second, output gap seems to be well represented by an AR(2), whose autocorrelation coefficients were consistently statistically significant in both specifications. Third, consistently with the previous approaches, Panama is the highest-potential growth country, and it is the only overheated economy in 2011 (last two charts, Figure 8). All the remaining economies present insignificant output gaps for 2010–11. Finally, the output cycle is about 8–10 years even though the confidence interval is wider than in the previous approaches.

Table 3. Relation between Growth and Volatility: State-Space Models

	Model I: Deterministic Drift				Model II: Mean Reversion				
Country Coefficients	μ	σ_{GAP}	ρ_1	ρ_2	μ	β	σ_{GAP}	ρ_1	ρ_2
CAPDR	4.3674	1.9591	1.0362	-0.5060	4.6804	-0.0369	2.0956	1.1357	-0.4467
Costa Rica	4.7936	1.9591	0.9434	-0.6612	5.0285	-0.2941	2.1663	1.2373	-0.5732
	(95.40)	(6.50)	(5.43)	(-3.61)	(37.33)	(-1.10)	(6.46)	(6.22)	(-2.86)
Dominican Republic	5.4203	2.3996	0.8368	-0.3922	5.7108	-0.3127	2.2442	1.0904	-0.4350
	(72.51)	(6.49)	(4.27)	(-2.02)	(46.40)	(-1.60)	(6.51)	(5.19)	(-2.14)
Guatemala	3.5171	0.8160	0.9011	-0.6202	3.6914	-0.1645	1.1714	1.1419	-0.3578
	(170.2)	(6.47)	(5.11)	(-3.34)	(25.80)	(-0.39)	(6.39)	(2.62)	(-0.81)
Honduras	3.6204	2.2575	0.8631	-0.2202	3.8503	-0.0968	2.3260	0.8933	-0.2617
	(35.69)	(6.46)	(4.33)	-(1.18)	(33.90)	(-0.87)	(6.52)	(4.39)	(-1.40)
Nicaragua	3.2543	1.8817	1.2215	-0.4581	3.6580	0.3824	1.6553	1.0596	-0.4589
	(25.72)	(6.48)	(6.44)	(-2.40)	(23.90)	(2.03)	(6.49)	(5.48)	(-2.22)
Panama	5.5989	2.4407	1.4511	-0.6839	6.1435	0.2644	3.0106	1.3916	-0.5936
	(28.88)	(6.48)	(8.32)	(-3.80)	(8.72)	(0.34)	(6.47)	(6.18)	(-3.07)

Note: Numbers in parentheses are t statistics. For CAPDR simple average is reported.

Figure 8. CAPDR: Potential Output and Output Gap–State-Space Model



V. Summary of the Models

This section presents a summary of the estimations but now focusing in the CAPDR region. Results are broadly consistent with Sosa et at. (2013), and IMF (2013). Average potential growth for the region is about 4.4 percent (this is computing the average for the three approaches), with an output gap volatility of about 1.9 percent. El Salvador is the lowest-growth country with an average of 2.6 percent, followed by Guatemala with 3.5 percent. On the top of the list are Dominican Republic and Panama, growing at 5.9 and 6.5 percent.

Figure 9. Potential Growth: average of the three methodologies

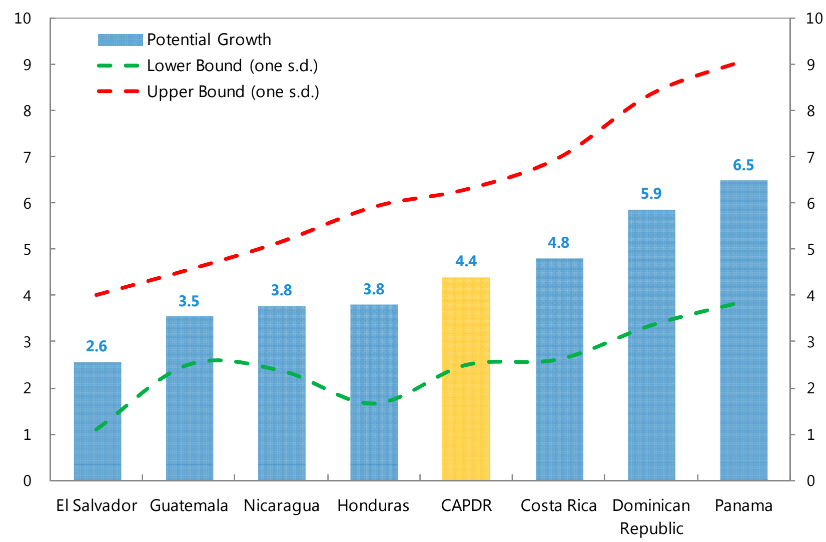


Table 4. Three Approaches: Growth and Output Gap Volatilities in CAPDR

	GDP		HP Filter		HP Filter[1/]		Production Function		Switching[2/]		State-Space[3/]		CAPDR	
	μ	σ_{Growth}	μ	σ_{GAP}	μ	σ_{GAP}	μ	σ_{GAP}	μ	σ_{GAP}	μ	σ_{GAP}	μ	σ_{GAP}
CAPDR	**4.2**	**2.5**	**4.3**	**2.5**	**4.1**	**2.1**	**4.3**	**2.4**	**4.8**	**0.9**	**4.7**	**2.1**	**4.4**	**1.9**
Costa Rica	4.5	2.9	4.6	2.8	4.2	2.4	4.5	2.6	5.0	1.1	5.0	2.2	4.8	2.2
Dominican Republic	5.3	3.5	5.6	3.1	5.4	1.0	5.5	3.1	6.6	1.5	5.7	2.2	5.9	2.5
El Salvador	2.5	2.0	2.5	1.7	1.8	2.7	2.4	1.6	2.8	1.1			2.6	1.5
Guatemala	3.5	1.3	3.5	1.2	3.3	1.5			3.4	0.7	3.7	1.2	3.5	1.0
Honduras	3.8	2.5	3.8	2.8	3.5	3.1	3.8	2.7	3.7	0.7	3.9	2.3	3.8	2.1
Nicaragua	3.8	2.0	3.7	1.6	3.3	2.0	3.6	1.4	4.0	0.9	3.7	1.7	3.8	1.4
Panama	6.1	3.3	6.0	3.9	7.3	1.9	5.9	3.0	7.9	0.5	6.1	3.0	6.5	2.6

[1/] As reference. Computed using 2008-2011 data.
[2/] Sustainable State.
[3/] Mean Reversion Model.

The main conclusion in measuring potential output growth is the robustness found in the three approaches. Without having a theoretical measure to define potential output and consequently output gap, we found in these three methods a source of consistency with some minor divergences and heterogeneities (Figure 10). For Costa Rica and El Salvador, the potential growth is very similar; however Panama is the country with the lowest robustness across the three approaches. The other countries report output growths with some variability.

Even though potential output is robustly measured in the three approaches, resulting is very similar frequencies; the output gap variability is different depending on the model used to get the potential output gap (see Figure 11). Between the production function approach and the state-space differences are not so remarkable; however the switching model reports consistently the lowest output gap volatilities. This is because the approach distinguishes and captures with high precision the event of sustainable growth, filtering the output gap cycle from the unsustainable events such as overheating and recessionary states. This clear demarcation is not present in the other two methods.

Figure 10. CAPDR: Potential Growth across Models

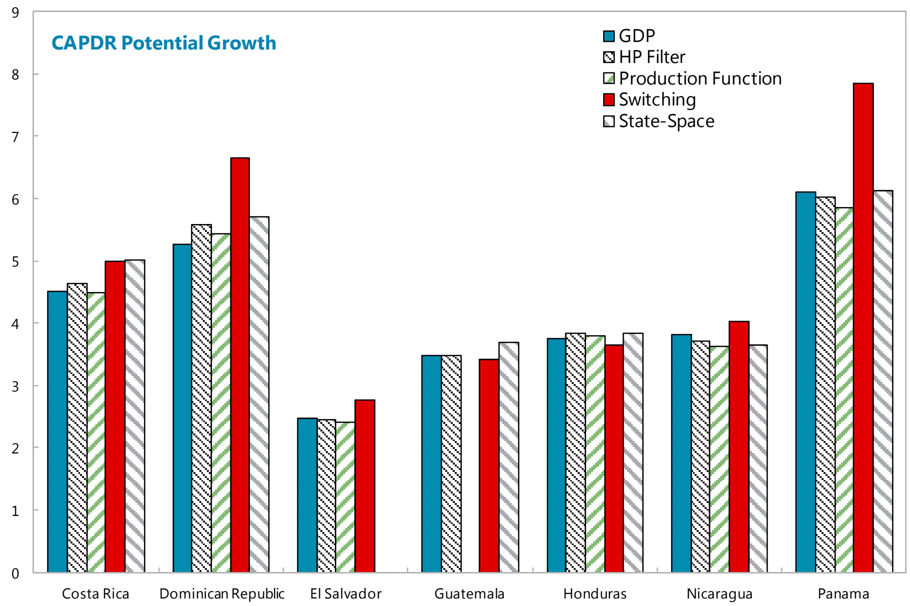
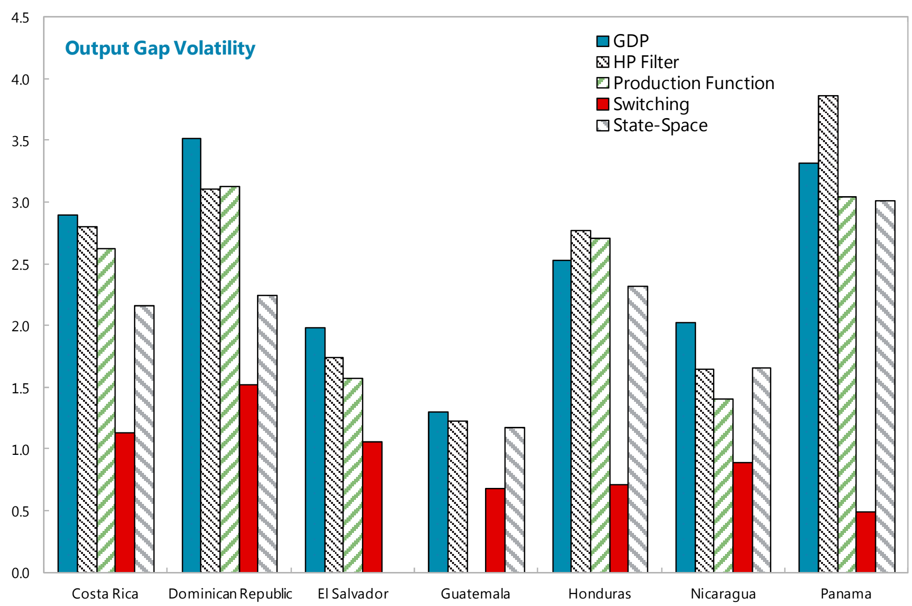


Figure 11. CAPDR: Output Gap Volatilities across Models



VI. CONCLUSIONS

This paper develops three methodologies to measure potential output, potential growth and output gap. The production function approach, the Switching methodology and the state-space models where applied to CAPDR countries.

Using annual data from 1994–11, we have shown that the output business cycle is about eight years, with an average potential growth of about 4.6 percent, and an output gap volatility of about 1.8 percent.

Moreover, the negative effect of the financial crisis vanished after one year. All the countries report closed output gaps for 2010–11, with the exception of Panama who presents overheating signs during 2011.

We sustained that the lowest-potential growth country is El Salvador, with an average of about 2.7 percent and with a standard deviation for the output gap of about 1.3 percent. On the other side of the spectrum, we found that the highest-potential growth country was Panama, with an average growth of about 6.7 percent and an output gap standard deviation of about 2.2 percent, the second volatility in the region after Dominican Republic (2.4 percent).

Finally, while there are significant differences in potential growth across countries, the results were robust to the alternative methods. Accountability, quality of institutions, policy implementation (Swiston and Barrot, 2011), slow technology diffusion (Howitt, 2000), resource misallocation and selection (Hsieh and Klenow, 2009; Bartelsman et al., 2013), and radical institutional reforms could be some of the key elements to explain differences in potential growth. This seems as a fertile area for future research.

APPENDIX

A. Switching Model: Gauss Code

GAUSS Code to estimate the switching model using the EM algorithm under three states of the nature. The variable “y” represents real GDP growth. This code was run in GAUSS8.0.6.

```
@ Gauss Code for Markov Switching with three Scenarios: Christian Johnson, 2013© @
max=10000;
s=3;
ps=zeros(s,rows(y));
tol=.000001;
mu=(meanc(y)+2*stdc(y))*ones(1,max)|(meanc(y)-0*stdc(y))*ones(1,max)|
                (meanc(y)-2*stdc(y))*ones(1,max);
sd2=stdc(y)^2*1*ones(1,max)|stdc(y)^2*1*ones(1,max)|stdc(y)^2*1*ones(1,max);
q=1/s+zeros(s,max);
psnum=ps;
theta=zeros(max,3*s);

m=2;
do while m lt max;
        t=1;
        do while t le rows(y);

        j=1;
        do while j le s;
                psnum[j,t]=q[j,m]*1/sqrt(2*pi*sd2[j,m])*exp(-.5*((y[t]-mu[j,m])^2/sd2[j,m]));
                j=j+1;
        endo;

        j=1;
        do while j le s;
                ps[j,t]=psnum[j,t]/sumc(psnum[.,t]);
                j=j+1;
        endo;

        t=t+1;
        endo;

        j=1;
        do while j le s;
                mu[j,m] =ps[j,.]*y/sumc(ps[j,.]');
                sd2[j,m]=ps[j,.]*(y-mu[j,m])^2/sumc(ps[j,.]');
                q[j,m]  =1/rows(y)*sumc(ps[j,.]');
        j=j+1;
        endo;

        theta[m,.]=mu[1,m]~mu[2,m]~mu[3,m]~sd2[1,m]~sd2[2,m]~sd2[3,m]~q[1,m]~q[2,m]~q[3,m];
        dif=sumc((theta[m,.]'-theta[m-1,.]')^2);
        if dif le tol;goto a10;endif;
        m=m+1;
        mu[1,m]=mu[1,m-1];mu[2,m]=mu[2,m-1];mu[3,m]=mu[3,m-1];
        sd2[1,m]=sd2[1,m-1];sd2[2,m]=sd2[2,m-1];sd2[3,m]=sd2[3,m-1];
        q[1,m]=q[1,m-1];q[2,m]=q[2,m-1];q[3,m]=q[3,m-1];
endo;
```

B. State-Space Model: Gauss Code

GAUSS Code to estimate the State-Space models for CAPDR countries. The variable "y" represents real GDP growth of the chosen country. This code was run in GAUSS8.0.6.

```
@ ML estimation Kalman Filter for Output GAP CAPDR: Christian Johnson, 2013©  @
new;
cls;
rndseed 1234567890;

library pgraph,cml;
#include cml.ext;
cmlset;


_pmcolor={0,0,0,0,0,0,0,0,15};
_pcolor={1,4,1,5};
_pdate="";
_plwidth={.5 .5 .5 2.5};
_pltype=0;

"
                                    Ingrese Variable a ser Estudiada:

                           1        :        Costa Rica

                           2        :        Dominican Republic

                           3        :        El Salvador

                           4        :        Guatemala

                           5        :        Honduras

                           6        :        Nicaragua (old data base 1994)

                           7        :        Panama

                           8        :        Nicaragua (new data base 2006)

";


country=con(1,1);



@Costa Rica,Dominican Republic,El Salvador,Guatemala,Honduras,Nicaragua,Panama,Nicaragua(Base 2006)@
@ sample 1990-2011@
serie=seqa(1991,1,(2011-1990)*1);
y={
857.5047          122.271  4.8009   100.2174 77.189   19.091   6.6448   69111.4
876.9106          123.426  4.9726   103.224  79.699   19.061   7.2706   69002.8
957.1656          136.402  5.3476   107.9723 84.182   19.138   7.867    69281.6
1028.1268         146.2538 5.7419   111.6433 89.426   19.061   8.2962   69002.8
1076.7652         149.6224 6.0892   115.4806 88.2609  20.0084  8.5326   72432.5
1118.9839         157.8421 6.4787   120.5787 91.8618  21.1913  8.6821   76714.6
1128.9047         169.0984 6.5892   123.9518 95.149   22.5357  9.3221   81581.6
1191.8771         182.6335 6.869    129.0921 99.9005  23.4296  9.9244   84817.7
1291.9691         195.4372 7.1265   135.0105 102.7992 24.2992  10.653   87965.8
1398.1973         208.5615 7.3723   139.9940 100.8568 26.0089  11.0703  94155.1
1423.3605         220.3590 7.531    143.5330 106.6542 27.0757  11.3709  98016.9
1438.6815         224.3458 7.6597   146.9778 109.5586 27.8774  11.4362  100919.1
1480.4347         237.3314 7.839    152.6609 113.6718 28.0875  11.6911  101679.9
1575.2493         236.7301 8.0193   156.5245 118.8405 28.7955  12.1828  104243.0
1642.3464         239.8359 8.1677   161.4582 126.2470 30.3252  13.0992  109780.6
1739.021          262.0513 8.4587   166.7220 133.8858 31.6239  14.0412  114481.8
1891.7008         290.0152 8.7896   175.6912 142.7890 32.9369  15.2386  119235.2
2041.8137         314.5929 9.1271   186.7669 151.6775 34.1369  17.0844  125231.5
2097.5884         331.1267 9.2434   192.8949 157.9199 35.0788  18.8129  128864.1
2076.2827         342.5642 8.9538   193.9506 154.5546 34.5634  19.5384  126998.6
```

```
2173.4502         369.1170 9.0815   199.3483 158.8414 36.1120  21.0219  130987.4
2263.9518         385.6642 9.2086   206.8958 164.5828 37.8093  23.2457  137602.3
};
y=y*1000;
y=reshape(y,22,8);
y=y[.,country];

y;

"
                                Input Model:

                        1       :       SS with Deterministic Drift

                        2       :       SS with Stochastic Drift


";

ser=con(1,1);

if ser eq 1;goto z10;endif;
if ser eq 2;goto z20;endif;


@============================== SS Deterministic Model ===============@
z10:

y=100*ln(y);
nobs=rows(y);
ini=2;
p_ini=1e-5;



@ESTIMATION: KALMAN FILTER ML CAPDR      @
@========================================================@
proc lnlk1(par,y);

    local f, q, h, nobs, b_all, b, b1,
        k, p, p1, yhat, e, lnl, t, var_y, invar_y, r, mu, p_ini;

    f=  (1 ~0 ~0)|
        (0~ par[1]~ par[2])|
        (0~ 1 ~0);
    h=1~1~0;
    q=  (0 ~0 ~0)|
        (0 ~par[3]^2 ~0)|
        (0 ~0 ~0);
    mu=(par[4]|0|0);

    nobs=rows(y);
    lnl=zeros(nobs,1);
    r=0;
    p_ini=1e-5;

        t=ini;
        do while t le nobs;

        @ Time Update : Prediction @

        if t eq ini;

                b1=y[ini-1]|zeros(rows(f)-1,1);
                p1=eye(rows(f))*p_ini;
                b_all=b1;
        endif;

        b=mu+f*b1;
```

```
        p=f*p1*f'+q;

        @ Measurement Update : Correction @
        k=p*h'*inv(h*p*h'+r);
        b1=b+k*(y[t]-h*b);
            @Accumulation@
            b_all=b_all~b1;
        p1=p-k*h*p;

        lnl[t]=-0.5*ln(2*pi)-0.5*ln(det(h*p*h'+r))-0.5*((y[t]-h*b)^2)/(h*p*h'+r);

      t=t+1;
    endo;

retp(sumc(lnl[ini:nobs]));

endp;

startv={1.2, -0.3, 1.8, 4};
__title="Kalman Filter in Country";
_cml_Bounds={-2 2, -0.9999 0.9999, 0.0001 9, -10 10};
_cml_C={-1 -1  0 0 , 1 -1  0 0 };
_cml_D=0.99999*(-1| -1);
_cml_GridSearchRadius=0.0001;
_cml_GradOrder = 3;
_cml_LineSearch=2; @ 1** - STEP* - HALF** - BRENT** - BHHHSTEP @
_cml_Algorithm=3;   @ BFGS - DFP - NEWTON* - BHHH @
_cml_ParNames="rho 1"|"rho 2"|"s_ygap"|"mu";


{par_out, f_out, g_out, cov_out, retcode} = CMLPrt(CML(y,0,&lnlk1,startv));
par_out~sqrt(diag(cov_out))~par_out./sqrt(diag(cov_out));
cov_out;

"   Likelihooh Value is         :";;(nobs-ini)*f_out;
"   Retcode                     :";;retcode;
"   Estimated Parameters:";par_out;

@=====================================@
@           SIMULATIONS               @
@=====================================@

    par=par_out;

    f=  (1 ~0 ~0)|
        (0~ par[1]~ par[2])|
        (0~ 1 ~0);
    h=1~1~0;
    q=  (0 ~0 ~0)|
        (0 ~par[3]^2 ~0)|
        (0 ~0 ~0);
    mu=(par[4]|0|0);

    nobs=rows(y);
    lnl=zeros(nobs,1);
    r=0;
    p_ini=1e-5;

        t=ini;
        do while t le nobs;

        @ Time Update : Prediction @

        if t eq ini;

                b1=y[ini-1]|zeros(rows(f)-1,1);
                p1=eye(rows(f))*p_ini;
                b_all=b1;
        endif;
```

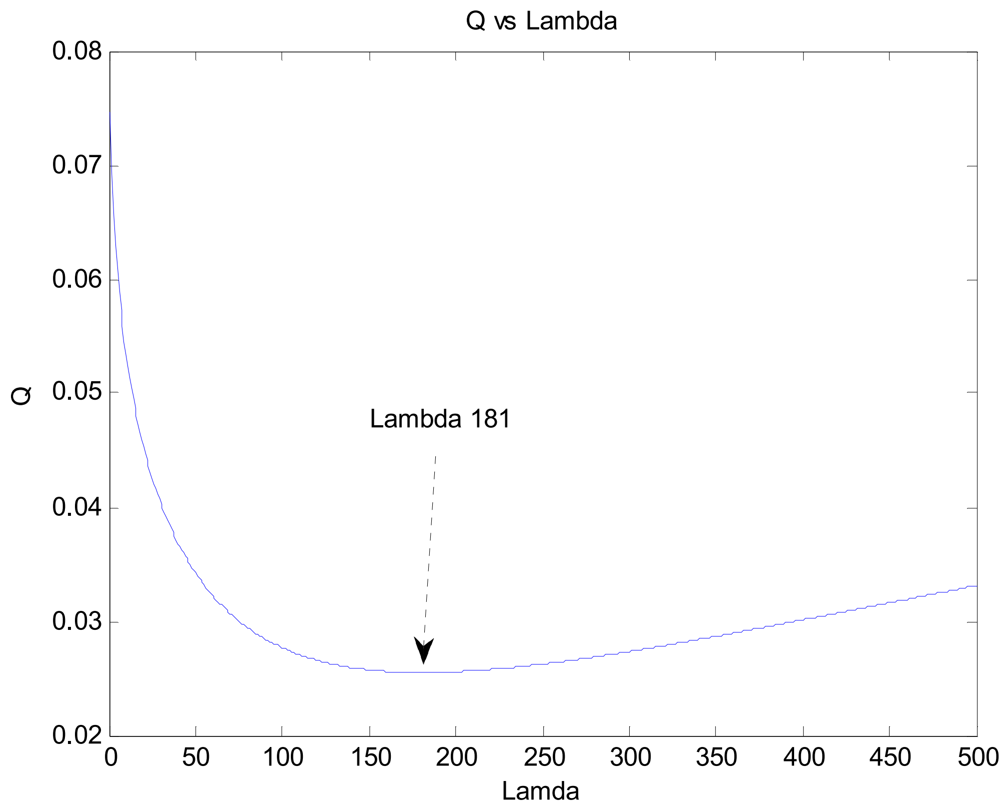
```
        b=mu+f*b1;
        p=f*p1*f'+q;

        @ Measurement Update : Correction @
        k=p*h'*inv(h*p*h'+r);
        b1=b+k*(y[t]-h*b);
            @Accumulation@
            b_all=b_all~b1;
        p1=p-k*h*p;


      t=t+1;
    endo;

b_all=b_all[.,ini:cols(b_all)]';b_all;

goto z100;

@=============================== SS Stochastic Model ===============@

z20:

y=100*ln(y);
nobs=rows(y);
ini=2;
p_ini=1e-5;



@ESTIMATION: KALMAN FILTER ML for CAPDR  @
@========================================================@
proc lnlk(par,y);

    local f, q, h, nobs, b_all, b, b1,
        k, p, p1, yhat, e, lnl, t, var_y, invar_y, r, mu, p_ini;

    f=  (1 ~0 ~0 ~1)|
        (0~ par[1]~ par[2] ~0)|
        (0~ 1 ~0 ~0)|
        (0 ~0 ~0 ~par[5]);
    h=1~1~0~0;
    q=  (0 ~0         ~0 ~0)|
        (0 ~par[3]^2 ~0 ~0)|
        (0 ~0         ~0 ~0)|
        (0 ~0         ~0 ~0);
    mu=(0|0|0|(1-par[5])*par[4]);

        nobs=rows(y);
    lnl=zeros(nobs,1);
    r=0;
    p_ini=1e-5;

        t=ini;
        do while t le nobs;

        @ Time Update : Prediction @

        if t eq ini;

                b1=y[ini-1]|zeros(rows(f)-1,1);
                p1=eye(rows(f))*p_ini;
                b_all=b1;
        endif;

        b=mu+f*b1;
        p=f*p1*f'+q;

        @ Measurement Update : Correction @
```

```
        k=p*h'*inv(h*p*h'+r);
        b1=b+k*(y[t]-h*b);
            @Accumulation@
          if t gt ini;  b_all=b_all~b1;
            p1=p-k*h*p;endif;

        lnl[t]=-0.5*ln(2*pi)-0.5*ln(det(h*p*h'+r))-0.5*((y[t]-h*b)^2)/(h*p*h'+r);

      t=t+1;
    endo;

retp(sumc(lnl[ini:nobs]));

endp;


startv={1.2, -0.3, 2, 4, 0.4};
__title="Kalman Filter in Country";
_cml_Bounds={-2 2, -0.9999 0.9999, 0.0001 9, -10 10, -1 2};
_cml_C={-1 -1  0 0 0  , 1 -1  0 0 0  };
_cml_D=0.99999*(-1| -1);
_cml_GridSearchRadius=0.00001;
_cml_GradOrder = 3;
_cml_LineSearch=2; @ 1** - STEP* - HALF** - BRENT** - BHHHSTEP @
_cml_Algorithm=3;   @ BFGS - DFP - NEWTON* - BHHH @
_cml_ParNames="rho 1"|"rho 2"|"s_ygap"|"mu"|"beta";

{par_out, f_out, g_out, cov_out, retcode} = CMLPrt(CML(y,0,&lnlk,startv));
par_out~sqrt(diag(cov_out))~par_out./sqrt(diag(cov_out));
cov_out;


"
    Likelihooh Value is         :";;(nobs-ini)*f_out;
"   Retcode                     :";;retcode;
"   Estimated Parameters:";par_out;


@=======================================@
@           SIMULATIONS                 @
@=======================================@

    par=par_out;

    f=  (1~0~0~1)|
        (0~par[1]~ par[2] ~0)|
        (0~1~0~0)|
        (0~0~0~par[5]);
    h=1~1~0~0;
    q=  (0~0~0~0)|
        (0 ~par[3]^2 ~0 ~0)|
        (0 ~0        ~0 ~0)|
        (0 ~0        ~0 ~0);
    mu=(0|0|0|(1-par[5])*par[4]);

        nobs=rows(y);
    lnl=zeros(nobs,1);
    r=0;
    p_ini=1e-5;

        t=ini;
        do while t le nobs;

        @ Time Update : Prediction @

        if t eq ini;

                b1=y[ini-1]|zeros(rows(f)-1,1);
                p1=eye(rows(f))*p_ini;
```

```
                b_all=b1;
        endif;

        b=mu+f*b1;
        p=f*p1*f'+q;

        @ Measurement Update : Correction @
        k=p*h'*inv(h*p*h'+r);
        b1=b+k*(y[t]-h*b);
            @Accumulation@
           if t gt ini; b_all=b_all~b1;
        p1=p-k*h*p;endif;


      t=t+1;
    endo;

b_all=b_all[.,ini-1:cols(b_all)]';b_all;


@============ Figures ===========================@
z100:

tt=rows(b_all);
ti=seqa(1990,1,22);
xtics(1991,2011,2,0);
gy=y[2:nobs]-y[1:nobs-1];
xlabel("Annual");

begwind;
window(2,2,0);
title("GDP Growth");
xy(ti[2:nobs],gy);
nextwind;
title("GDP and Its Trend Component");
_plegstr="GDP\Trend";
xy(ti[ini:nobs],exp(y[ini:nobs]/100)~1.1*exp(b_all[1:rows(b_all),1]/100));
nextwind;
title("Cyclical Component (Output Gap)");
ytics(-12,12,2,0);
bar(ti[ini:nobs],y[ini:nobs]-b_all[1:rows(b_all),1]);
nextwind;
title("Drift Component (Mu)");
if ser eq 1;xy(ti[ini+1*0:nobs],b_all[1:rows(b_all),2]);endif;
if ser eq 2;xy(ti[ini+1*0:nobs],b_all[1:rows(b_all),4]);endif;
endwind;

print "Potential Output";
exp(b_all[1:rows(b_all),1]/100);
```

C. Optimal Lambda using the Pedersen (2001, 2002) Method: Nicaragua

Figure 12. Q Loss Function and Optimal Lambda



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