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The Dynamics of the Term Structure of Interest Rates in the United States in Light of the Financial Crisis of 2007–10

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The Dynamics of the Term Structure of Interest Rates in the United States in Light of the Financial Crisis of 2007–10

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Abstract

This paper assesses the dynamics of the term structure of interest rates in the United States in light of the financial crisis in 2007–10. In particular, this paper assesses the dynamics of the term structure of U.S. Treasury security yields in light of economic and financial events and the monetary policy response since the inception of the crisis in mid-2007. To this end, this paper relies on estimates of the term structure using Nelson-Siegel models that make use of unobservable or latent factors and macroeconomic variables. The paper concludes that both the latent factors and macroeconomic variables explain the dynamics of the term structure of interest rates, and the expectations of the impact on macroeconomic variables of changes in financial factors, and vice versa, have changed little with the financial crisis.

JEL Classification Numbers: G12; E43; E44; E58

Keywords: Term structure of interest rates; yield curve; U.S. Treasury security yields, interest rates; bond yields; United States; financial crisis

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[1]This paper has benefited from comments by Oya Celasun, Ying He, Herman Kamil, Christopher Towe, and many conversations with colleagues in the Monetary and Capital Markets Department and Western Hemisphere Department of the IMF.

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I. INTRODUCTION

The term structure of interest rates in the United States has gone through significant changes in the context of the financial crisis of 2007–10. This reflects, among others, the lowering of the key policy rate of the United States, or federal funds rate, to a range of 0 to ¼ percent, Federal Reserve purchases of U.S. Treasury securities of different maturities, and market reactions to these policy actions. Not surprisingly, in this context, the shape of the term structure has changed dramatically. The level of the term structure has shifted downward noticeably, and the slope and curvature of the term structure have changed perceptibly.

This paper assesses the dynamics of the term structure of interest rates in the United States in light of the financial crisis. In particular, this paper assesses how the dynamics of the term structure of interest rates, proxied by U.S. Treasury security yields as in the financial economics literature, have changed in light of the economic and financial events and the monetary policy response since the inception of the crisis in mid-2007. To this end, this paper relies on an estimation of the term structure that makes use of unobservable or latent factors and key macroeconomic variables. This paper also investigates the impact on macroeconomic dynamics of changes in financial factors by exploring the responses of macroeconomic variables to shocks to the factors of the term structure, and vice versa. To undertake this assessment, the paper relies on estimations of the term structure using the Nelson-Siegel Models (NSMs). As is well known, the NSMs are robust in explaining the performance of the term structure.

The paper is organized as follows. Section II summarizes the response of the Federal Reserve to the financial crisis. In light of the importance of the NSM to assess the performance of the term structure, section III describes briefly the general NSM. After briefly describing the data used in this paper, section IV shows a number of estimates of the term structure of U.S. Treasury securities, while providing an assessment of the term structure of yields of U.S. Treasury securities and the relationship of the term structure and macroeconomic variables. Section V offers a conclusion.

II. THE FEDERAL RESERVE'S RESPONSE TO THE FINANCIAL CRISIS

In light of its mandate to foster maximum employment and price stability, the Federal Reserve responded aggressively to the financial crisis that started in mid-2007. As noted by Federal Reserve Chairman Bernanke (2011), the Federal Reserve (2010) and Ceccheti (2009), the Federal Reserve lowered the target federal funds rate from 5¼ percent to a range of zero to ¼ percent. The Federal Reserve also cut the primary lending rate or discount rate from 100 basis points to 50 basis points above the federal funds rate, while increasing the term discount lending from overnight to a maximum of 30 days. In addition, the Federal Reserve adopted three set of actions to provide liquidity to financial institutions, while fostering improved conditions in financial markets.

- The first set of actions involved the provision of short-term liquidity to banks and nonbank financial institutions through the traditional discount window and newly created facilities, namely the Term Auction Facility (TAF), the Primary Dealer Credit Facility (PDCF), the Term Structure Lending Facility (TSLF), and the temporary liquidity swap arrangement between the Federal Reserve and other central banks. The TAF auctioned funds for a certain term, initially $20 billion to $30 billion, then $50 billion, and subsequently $75 billion per auction for terms of 28 to 35 days. The TSLF allowed dealers to exchange less liquid collateral for more liquid Treasury collateral, which was easier to finance. The PDF made it possible for borrowers, essentially investment banks and brokers, to borrow from the Federal Reserve by pledging a broad set of collateral. The Federal Reserve wound down both the PDCF and TSLF.

- The second set of actions involved the provision of liquidity directly to borrowers and investors in key credit markets. These actions included the newly created Asset-Backed Commercial Paper Money Market Facility (AMLF), the Commercial Paper Fund Facility (CPFF), the Money Market Investor Funding Facility (MMIFF), and the Term-Asset Securities Loan Facility (TALF). The Federal Reserve wound down virtually all these actions.

- The third set of actions involved the expansion of traditional tools of open market operations through the purchase of longer-term securities. In November 2008, the Federal Reserve announced the purchase of up to $100 billion of government-sponsored enterprises (GSE) debt and up to $500 billion in mortgage-backed securities. In March 2009, the Federal Reserve announced the purchase of up to $300 billion of longer-term Treasury securities in addition to increasing its purchases of GSE debt and mortgage-backed securities of up to $300 billion and $1.25 trillion, respectively. As a result of these actions, from December 2008 to March 2010, the Federal Reserve purchased $1.7 trillion in medium- and long-term Treasury, agency, and agency mortgage-backed securities.

- To complement these actions, in August 2010 the Federal Reserve began to reinvest the proceeds from all securities that matured or were redeemed in longer-term securities, with a view to keeping the size of security holdings broadly constant. In November 2010, the Federal Reserve announced a plan to purchase $600 billion in longer-term securities by mid-2011.

In response to these actions by the Federal Reserve, not surprisingly, the term structure of interest rates has taken on many shapes since mid 2007. For many years prior to the start of the crisis, the Federal Reserve had relied on the federal funds rate as the key policy rate, adjusting this rate, as necessary, to achieve the goals of monetary policy. The medium- and long-term interest rates, as averages of expected future short-term interest rates, moved in response to changes in both the key policy rate and market expectations about future short-term interest rates. To achieve their intended objectives, say, to influence the spending and decisions of households and businesses, respectively, the changes in the federal funds rate depended on their impact on medium- and long-term interest rates. In light of the severity of the financial crisis, the Federal Reserve has employed not only the target federal funds rate, but also alternative tools to ease monetary conditions as noted above.[2] In so doing, the Federal Reserve has sought to influence to an even greater extent than before the crisis all interest rates along the term structure. In this light, the question then becomes: How have the dynamics of the term structure of interest rates changed in light of the financial crisis of 2007-10? Before answering this question, it is first necessary to explain the characteristics of the term structure models that employ latent factors and macroeconomic variables.

III. TERM STRUCTURE MODELS

A. Background

As is well known, the term structure depicts a set of yields on U.S. Treasury securities of different maturities. The set of yields suggest the presence of a relationship among short-, medium- and long-term yields. This relationship does not appear stable over time, particularly because the term structure exhibits different shapes at different moments. Nevertheless, as Diebold and Li (2006) note, changes in the term structure follow certain patterns. The NSM captures these patterns, while reproducing the historical average shape of the term structure. The NSM model also accounts for the existence of unobservable, or latent, factors and their associated factor loadings and key macroeconomic variables that underlie U.S. Treasury security yields (Diebold and Li, 2006).

B. Yield-Only Nelson Siegel Model

As Gasha et al. (2010) note, the NSM successfully fits the term structure of U.S. Treasury security yields, while capturing the dynamics of the term structure. The NSM provides a tractable framework to fit the term structure by approximating the forward rate curve by a constant plus a polynomial times an exponential decay term given by[3]

(1) $$f_t(\tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}\lambda_t e^{-\lambda_t\tau}$$

where $f_t(\tau)$ is the instantaneous forward rate. This yields a corresponding term structure

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[2]Put differently, the aggressive response of the Federal Reserve reflected, in line with the Taylor rule, subdued inflation pressures and rising unemployment. As Rudebusch and Wu (2009) show, it is straightforward to put this policy response in a macro-finance model that gives a macroeconomic interpretation to the level and the slope of the term structure models.

[3]A forward rate $f_t(\tau,\tau^*)$ is the interest rate of a forward contract, set at time t, on an investment that is initiated τ periods into the future and that matures τ^* periods beyond the start date of the contract. The instantaneous forward rate $f_t(\tau)$ is obtained by letting the maturity of the contract go to zero.

$$(2) \qquad y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)$$

where $\beta_{1t}, \beta_{2t}, \ \beta_{3t}$ and λ_t are parameters and 1, $\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right)$ and $\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)$ are their loadings. The parameter λ_t controls both the exponential decay rate and the maturity at which the loading on β_{3t} reaches its maximum. Even though the NSM appears to be static, Diebold and Li (2006) interpret the parameters β_{1t}, β_{2t} and β_{3t} as dynamic latent factors. They show that these parameters can be construed as the level, slope, and curvature factors, respectively, particularly because their loadings are a constant, a decreasing function of τ, and a concave function of τ.[4]

As Gasha et al. (2010) discuss, this framework:

- provides a parsimonious approximation of the term structure, since the three loadings $\left[1, \left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) \text{ and } \left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)\right]$ give the model sufficient flexibility to reproduce a range of shapes of observed yield curves;

- generates a forward curve and term structure that start at the instantaneous rate $\beta_{1t} + \ \beta_{2t}$ and then level off at the finite infinite-maturity value of β_{1t}, which is constant;[5]

- makes it possible to interpret the three factors β_{1t}, β_{2t} and β_{3t} as long-, short- and medium-factors, respectively, in light of its three loadings $\left[1, \left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) \text{ and } \left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)\right]$;[6] and

- establishes that the time-series statistical properties of the three factors β_{1t}, β_{2t} and β_{3t} underlie the dynamic patterns of the term structure.

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[4] A heuristic interpretation of the factors along these lines is the following: (i) since yields at all maturities load identically on β_{1t}, an increase in β_{1t} increases all yields equally, changing the level of the yield curve; (ii) since short rates load more heavily on β_{2t}, an increase in β_{2t} raises short yields more than long yields, thereby changing the slope of the yield curve; and (iii) since short rates and long rates load minimally on β_{3t}, an increase in β_{3t} will increase medium-term yields, which load more heavily on it, increasing the yield curve curvature. An additional implication of the NS model is that $y_t(0) = \beta_{1t} + \beta_{2t}$, i.e., the instantaneous yield depends on both the level and the slope factors.

[5] These values are obtained by taking the limits of $y_t(\tau)$ as τ goes to zero and to infinity, respectively.

[6] To appreciate this interpretation, notice that the loading on β_{1t} is 1, which does not decay to zero in the limit; the loading on β_{2t} is $\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right)$, which starts at 1 but decays quickly and monotonically to 0; the loading on β_{3t} is $\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)$, which starts at 0, increases, and then decays to 0. This coincides with Diebold and Li (2006) interpretation of the three factors as level, slope and curvature.

Diebold, Rudebusch, and Aruoba (2006) argue that the state-space representation provides a powerful framework for analysis and estimation of dynamic models. As explained by Gasha et al. 2010, this representation provides a way of specifying a dynamic system, while making it possible to handle a wide range of time series models. It facilitates estimation, the extraction of latent term structure factors, and the testing of hypotheses about the dynamic interactions between the term structure and macroeconomic factors. The state-space representation is

(3) $$(F_t - \mu) = A(F_{t-1} - \mu) + \eta_t$$

(4) or $$F_t = \mu + AF_{t-1} + \eta_t$$

(5) $$y_t = \Lambda F_t + \varepsilon_t.$$

Equations (4) and (5) can be expressed as

(6) $$\begin{bmatrix}\beta_{1t}\\ \beta_{2t}\\ \beta_{3t}\end{bmatrix} = \begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix} + \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{23} & a_{33}\end{bmatrix}\begin{bmatrix}\beta_{1,t-1}\\ \beta_{2,t-1}\\ \beta_{3,t-1}\end{bmatrix} + \begin{bmatrix}\eta_{1t}\\ \eta_{2t}\\ \eta_{3t}\end{bmatrix}$$

(7) $$\begin{bmatrix}y_t(\tau_1)\\ \cdots\\ y_t(\tau_N)\end{bmatrix} = \begin{bmatrix}1 & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} - e^{-\lambda_t\tau_1}\\ \cdots & \cdots & \cdots\\ 1 & \frac{1-e^{-\lambda_t\tau_N}}{\lambda_t\tau_N} & \frac{1-e^{-\lambda_t\tau_N}}{\lambda_t\tau_N} - e^{-\lambda_t\tau_N}\end{bmatrix}\begin{bmatrix}\beta_{1t}\\ \beta_{2t}\\ \beta_{3t}\end{bmatrix} + \begin{bmatrix}\epsilon_{1t}\\ \cdots\\ \epsilon_{Nt}\end{bmatrix}$$

Equation (6), or the *transition equation*, governs the dynamics of the state vector, which, for the three-factor NSM, is given by the unobservable vector $F_t = (\beta_{1t}\ \ \beta_{2t}\ \ \beta_{3t})'$. As in Diebold and Li (2006), it is assumed that these time-varying factors follow a vector autoregressive process of first order, VAR (1), where the mean state vector μ is a 3x1 vector of coefficients, the transition matrix A is a 3x3 matrix of coefficients, and η_t is a white noise transition disturbance with a 3x3 non-diagonal covariance matrix Q.[7] Equation (7), or the *measurement equation*, is the specification of the term structure itself, and relates N observable yields to the three unobservable factors. The vector of yields Y_t, contains N different maturities $Y_t = [y_t(\tau_1)\ \ \cdots\ \ y_t(\tau_N)]'$. The measurement matrix Λ is an Nx3 matrix whose columns are the loadings associated with the respective factors, and ε_t is a white noise *measurement* disturbance with an NxN diagonal covariance matrix H. It is assumed, mainly to facilitate computations, that both disturbances are orthogonal to each other and to the initial state, F_0 . Formally,

(8) $$\begin{pmatrix}\eta_t\\ \varepsilon_t\end{pmatrix} \backsim WN\left[\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}Q & 0\\ 0 & H\end{pmatrix}\right]$$

where, $$\mathbb{E}(F_0\,\eta_t') = 0$$

[7]The VAR is expressed in terms of deviations from the mean since F_t is a covariance-stationary vector process.

$$\mathbb{E}(F_0\, \varepsilon_t') = 0.$$

In addition to computational tractability, these assumptions are essential to estimate both equations.

C. Yield-Macro Nelson Siegel Model

Recent latent factor models of the term structure have begun to incorporate explicitly macroeconomic factors. In this context, Diebold, Rudebusch, and Aruoba (2006) use a state-space representation to incorporate macroeconomic factors in a latent factor model of the term structure to analyze the potential bidirectional feedback between the term structure and the economy. They enhance the state vector to include some key macroeconomic variables associated with economic activity, monetary stance, and inflation, specifically manufacturing capacity utilization (CU_t), the federal funds rate (FFR_t), and annual price inflation($INFL_t$). In so doing, they offer an insight into the underlying economic forces that drive the evolution of interest rates.

In this light, the state-space representation takes on the form

(9) $$F_t = \mu + AF_{t-1} + \eta_t$$

(10) $$Y_t = \Lambda F_t + \varepsilon_t$$

where $F_t = (\beta_{1t}\ \beta_{2t}\ \beta_{3t}\ CU_t\ FFR_t\ INFL_t)'$, and the dimensions of μ, A, and η_t are increased accordingly, to 6x1, 6x6 and 6x1, respectively. The matrix Λ now contains six columns, of which the three leftmost include the loadings on the three yield factors, and the three rightmost contain only zeroes, indicating that the yields still load only on the yield curve factors. The transition disturbance covariance matrix *Q*, with increased dimension to 6x6, and the measurement disturbance covariance matrix *H* are non-diagonal and diagonal matrices, respectively.[8]

(11) $$\begin{bmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \\ CU_t \\ FFR_t \\ INFL_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{bmatrix} + \left[\begin{bmatrix} a_{11} & \cdots & a_{16} \\ \vdots & \ddots & \vdots \\ a_{61} & \cdots & a_{66} \end{bmatrix}\right] \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \\ CU_{t-1} \\ FFR_{t-1} \\ INFL_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \\ \eta_{5t} \\ \eta_{6t} \end{bmatrix}$$

[8] Diebold, Rudebusch, and Aruoba (2006) note that these macroeconomic variables represent the minimum set of fundamentals required to capture basic macroeconomic dynamics.

$$
(12)\qquad \begin{bmatrix} y_t(\tau_1) \\ \cdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} & \frac{1-e^{-\lambda_t\tau_1}}{\lambda_t\tau_1} - e^{-\lambda_t\tau_1} & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & & & \\ 1 & \frac{1-e^{-\lambda_t\tau_N}}{\lambda_t\tau_N} & \frac{1-e^{-\lambda_t\tau_N}}{\lambda_t\tau_N} - e^{-\lambda_t\tau_N} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \\ CU_t \\ FFR_t \\ INFL_t \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \cdots \\ \epsilon_{Nt} \end{bmatrix}
$$

As Diebold, Rudebusch, and Aruoba (2006) note, this framework opens the way to understand the nature of the interaction between the term structure and macroeconomic variables. Gasha et al. (2010) indicate that the Kalman filter makes it possible to estimate this state-space representation.

IV. ESTIMATIONS OF THE TERM STRUCTURE

After briefly summarizing the data used in this paper, this section presents estimations of the term structure of U.S. Treasury security yields using the NSMs, and provides an assessment of the performance of the term structure.

A. Data

As in Gasha et al. (2010), this paper uses U.S. Treasury security yields and macroeconomic variables. The yields are annualized zero-coupon bond nominal yields continuously compounded. The yields, obtained from Bloomberg, are monthly observations of U.S. Treasury securities of 9 maturities—3, 6, 12, 24, 36, 48, 60, 84 and 120 months—for the period of 1972:1 to 2010:11. The macroeconomic variables include (i) the inflation variable, or the annual percentage change in the monthly price deflator for personal consumption expenditures; (ii) the real economic activity relative to potential, manufacturing capacity utilization; and (iii) the monetary policy instrument, or the monthly average federal funds rate.

B. Yield-Only Nelson Siegel Model

As Gasha et al. (2010) note, the term structure, including the crisis period, exhibits the following characteristics:

- The average term structure is upward sloping and concave.
- The term structure takes on a variety of shapes through time, including upward sloping, downward sloping, humped, and inverted humped.
- The term structure has shifted downward noticeably in the context of the Fed policy, among others, to lower the fed funds rate to nearly zero.
- The level of the term structure is highly persistent as it exhibits a small variation relative to its mean. The slope of the term structure is less persistent than the level of

> the term structure, with the slope being highly variable relative to its mean. The curvature is the least persistent of all three factors as it displays the largest variability relative to its mean.

A three-factor, yield-only NSM model fits well the term structure of U.S. Treasury security yields for the periods of 1972:1-2007:6 and 1972:1-2010:11 (Figure 1). As Tables 1 and 2 show, the estimated means and standard deviations of the residuals of the measurement equation are small for all maturities of U.S. Treasury securities for both periods. In line with Diebold and Li (2006), the residual sample autocorrelations, particularly with ρ(1) and ρ(12), indicate that pricing errors are persistent, possibly because of tax and liquidity effects. A goodness-of-fit test, measured by the Chi-square test statistic,[9] confirms that the observed term structure differs little from the estimated term structure (Table 3). Reflecting the goodness of the fit of U.S. Treasury securities at any maturity, Figure 2 shows that the observed term structure and estimated term structure for both three-month and five-year U.S. Treasury securities are virtually the same, or nearly overlap. In this context, not surprisingly, the average term structure across all maturities of U.S. Treasury securities fits well the observed yields over the entire estimation period (Figure 3). Figure 4 displays the three estimated factors for the periods under analysis.[10] The results also suggest that the yields-only NSM provides an effective framework to estimate the term structure across different states of the business cycle.

In conclusion, the estimations of the term structure of U.S. Treasury security yields capture well the dynamics of the observed term structure of interest rates in light of the financial crisis. These estimations encapsulate the changes in the term structure as a result the Federal Reserve's actions and changes in expectations of short-term future interest rates. The estimations confirm that the yield-factors of the term structure of interest rates―level, slope and curvature―provide a good representation of the term structure, even in light of the financial crisis. By way of example, as Figure 3 shows, the estimations capture well the impact of the monetary policy thrust in the United States in response to the economic and financial events since mid 2007, namely a downward shift in the term structure and a flattening of the slope. The estimation also picks up the subsequent increase in the slope of the term structure increases and the decline in curvature.

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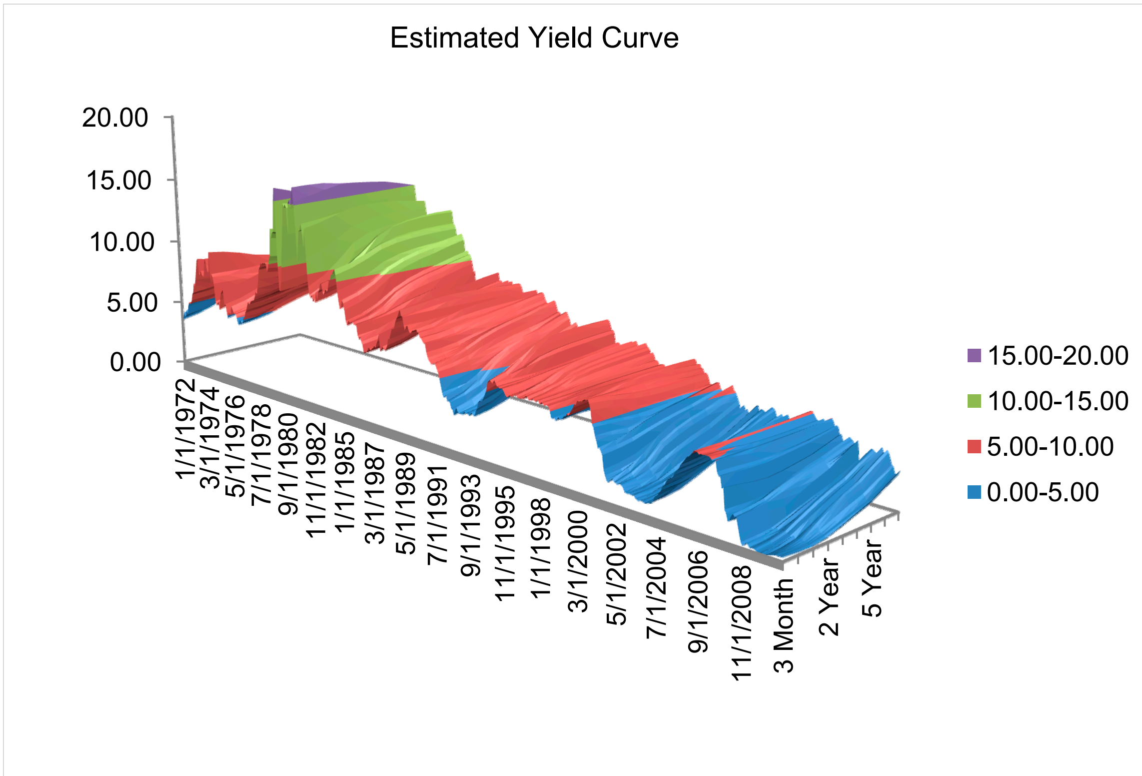
[9]The Chi-square statistic is defined as the square of the difference between the observed term structure and the estimated term structure divided by the variance of the observed yields. The null hypothesis states that the observed term structure is the same as the estimated term structure.

[10]Note that the slope of the term structure is actually -β_{2t}, i.e., the program estimates it as the negative of the slope.

Figure 1. Observed Yield Curves



Source: Fund staff estimates.

Table 1. United States: Yield-Only Model, Yield Curve Residuals, 1972:1-2007:6

Maturity (Months)	Mean	Std. Dev.	Minimum	Maximum	ρ(1)	ρ(12)	ρ(30)
3	-0.079	0.134	-0.846	0.306	0.706	0.297	0.123
6	0.033	0.066	-0.179	0.372	0.613	0.301	0.114
12	0.051	0.125	-0.299	0.621	0.704	0.283	0.000
24	0.022	0.059	-0.157	0.298	0.683	0.090	-0.048
36	-0.011	0.033	-0.214	0.112	0.477	0.168	0.105
48	-0.015	0.046	-0.162	0.141	0.770	0.095	0.062
60	-0.023	0.044	-0.186	0.137	0.681	0.121	-0.145
84	0.014	0.036	-0.078	0.188	0.678	0.212	0.079
120	0.012	0.072	-0.188	0.287	0.759	0.372	0.022

Source: Fund staff estimates.

Table 2. United States: Yield-Only Model, Yield Curve Residuals, 1972:1-2010:11

Maturity (Months)	Mean	Std. Dev.	Minimum	Maximum	ρ(1)	ρ(12)	ρ(30)
3	-0.088	0.153	-0.953	0.342	0.715	0.282	0.134
6	0.028	0.057	-0.154	0.321	0.580	0.286	0.092
12	0.051	0.128	-0.340	0.617	0.704	0.275	0.012
24	0.023	0.067	-0.189	0.344	0.697	0.093	0.001
36	-0.009	0.031	-0.198	0.100	0.458	0.101	0.091
48	-0.015	0.051	-0.297	0.151	0.770	0.087	0.039
60	-0.021	0.048	-0.192	0.134	0.697	0.116	-0.125
84	0.013	0.040	-0.157	0.192	0.651	0.164	0.069
120	0.007	0.070	-0.182	0.250	0.770	0.275	0.040

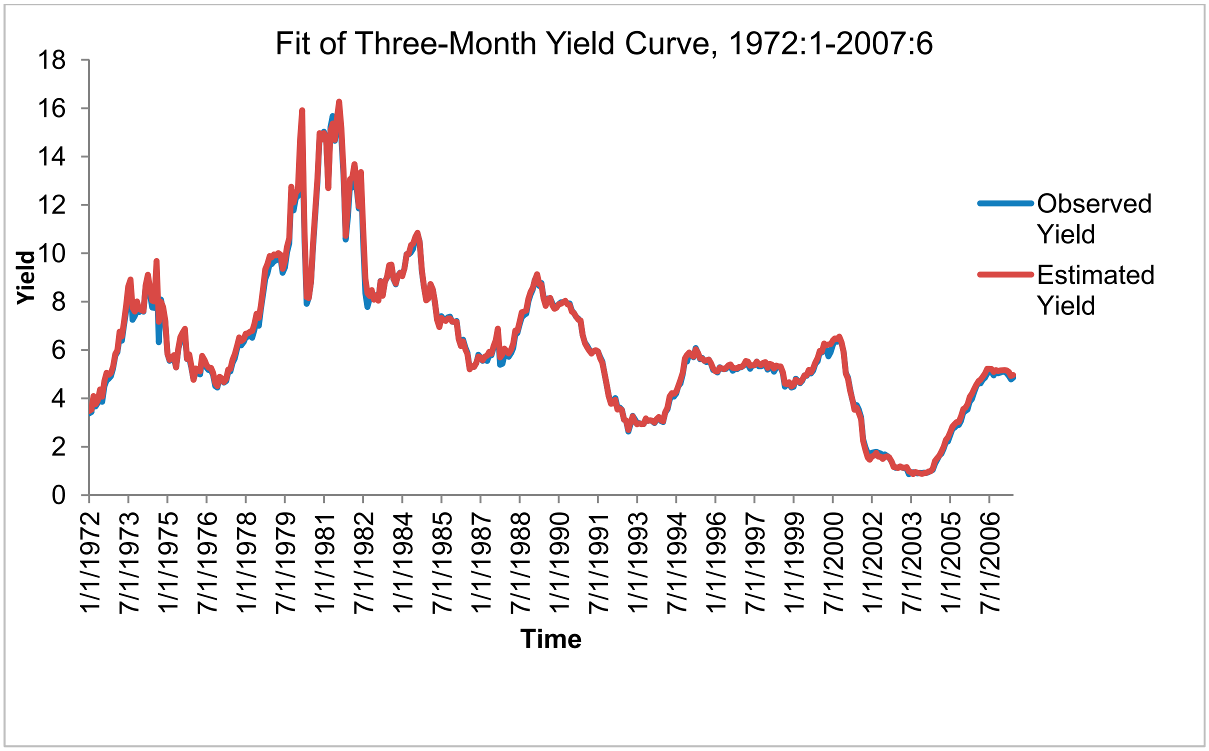
Source: Fund staff estimates.

Table 3. Goodness of Fit of the Yield-Only NSM

Chi-square Test of Fit, 1972:1-2007:6	
	Value
SSE	27.5367
Chi-square	6.5550
DF	3831
Chi-square Test of Fit, 1972:1-2010:11	
	Value
SSE	33.6330
Chi-square	7.1332
DF	4200

Source: Fund staff estimates.

Figure 2. Performance Evaluation of Model



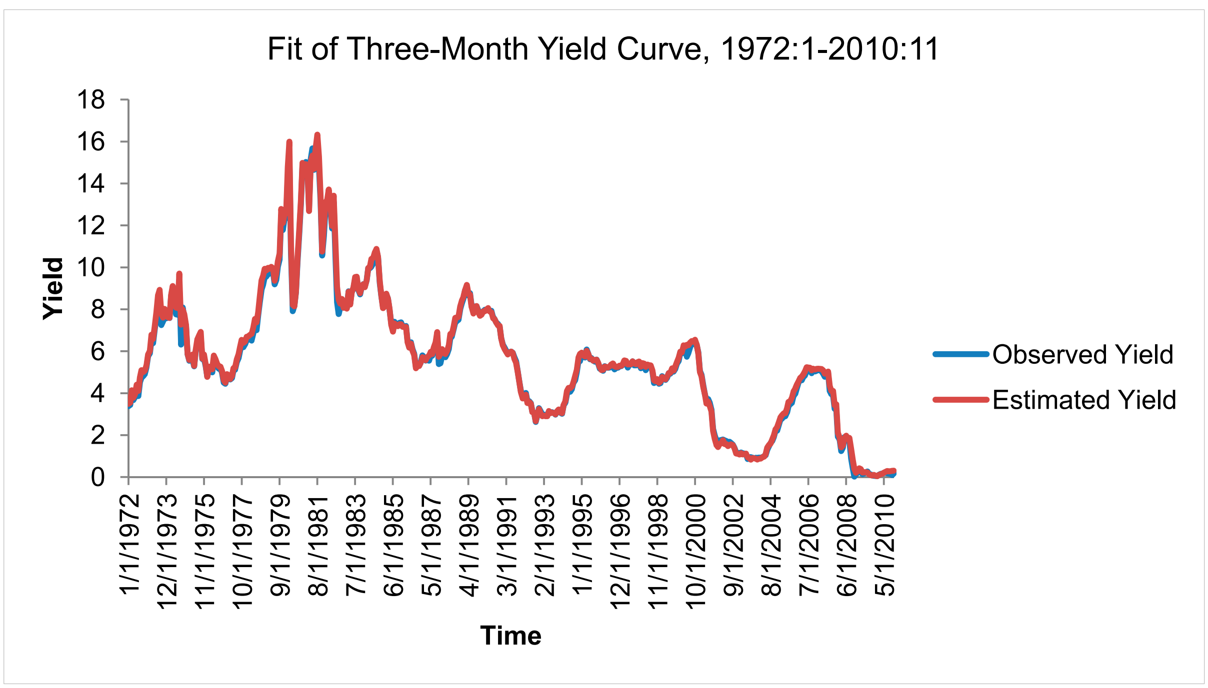
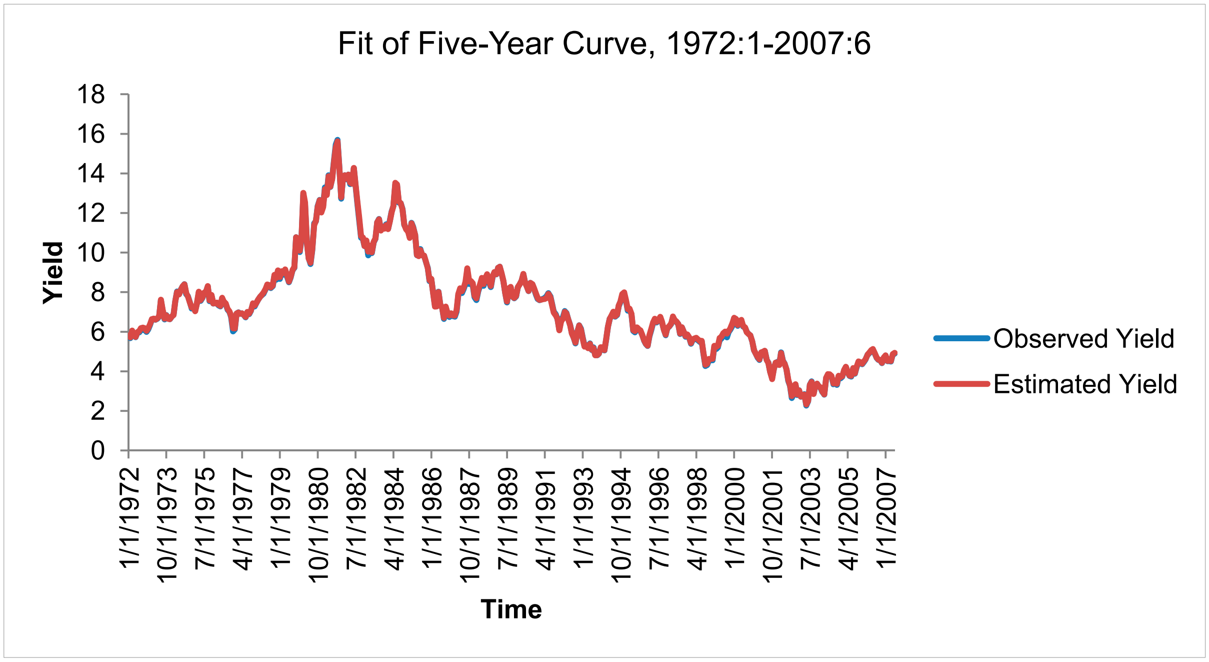
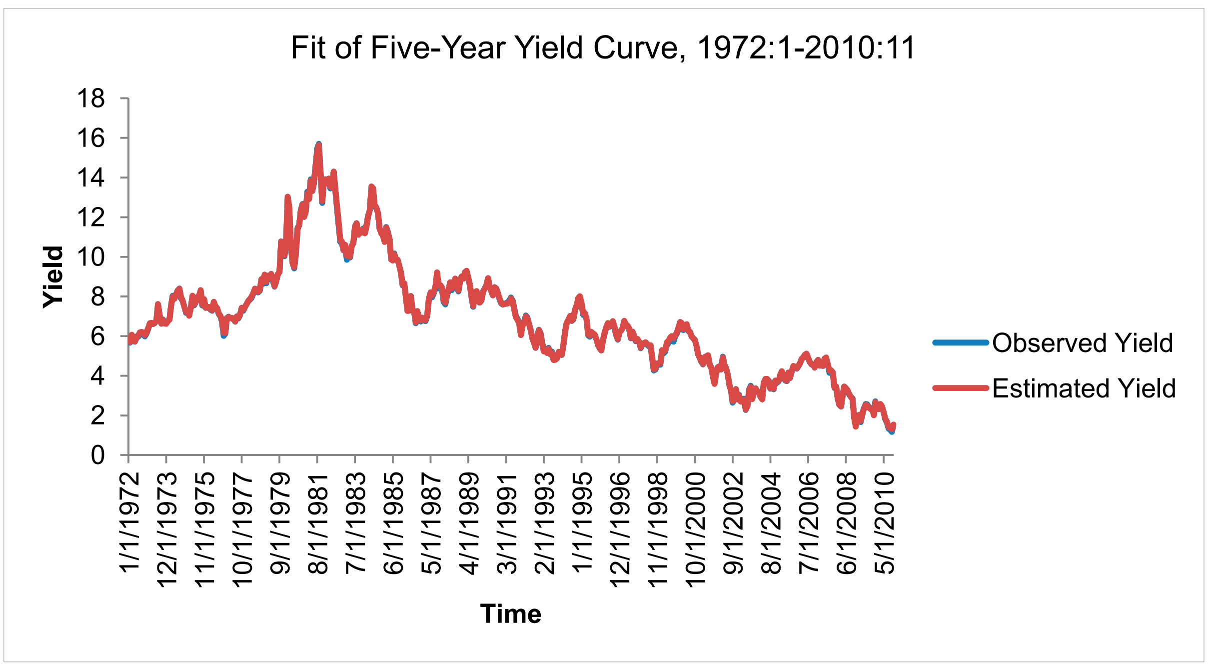


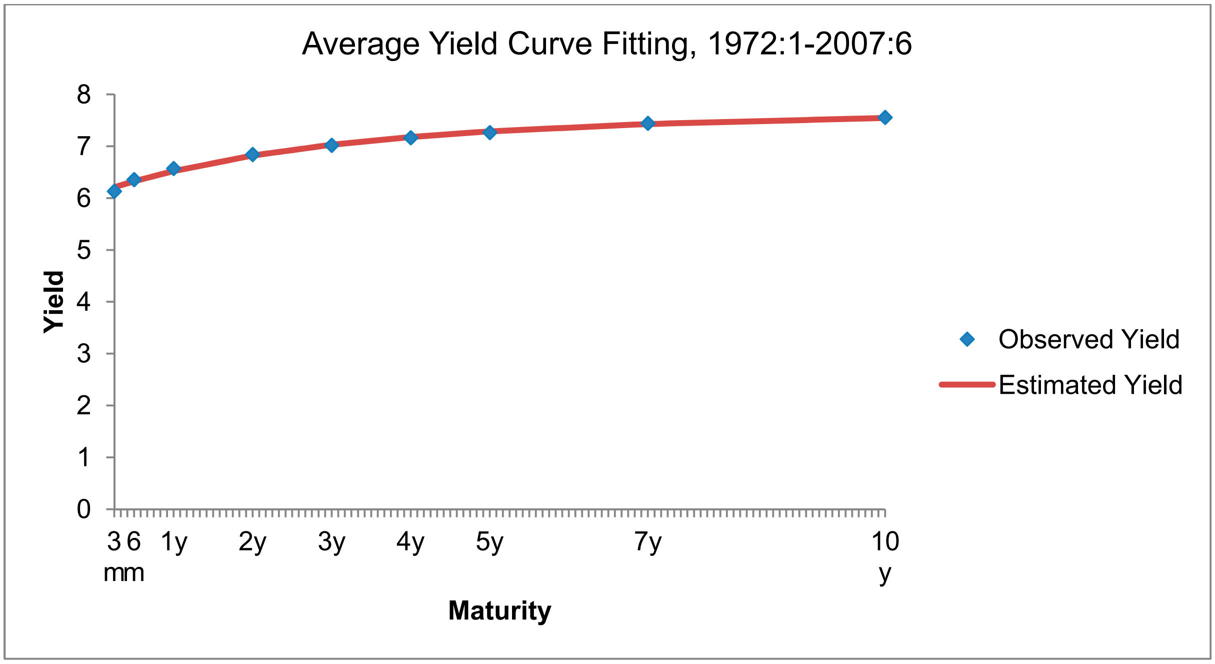
Figure 2. Performance Evaluation of Model (continued)

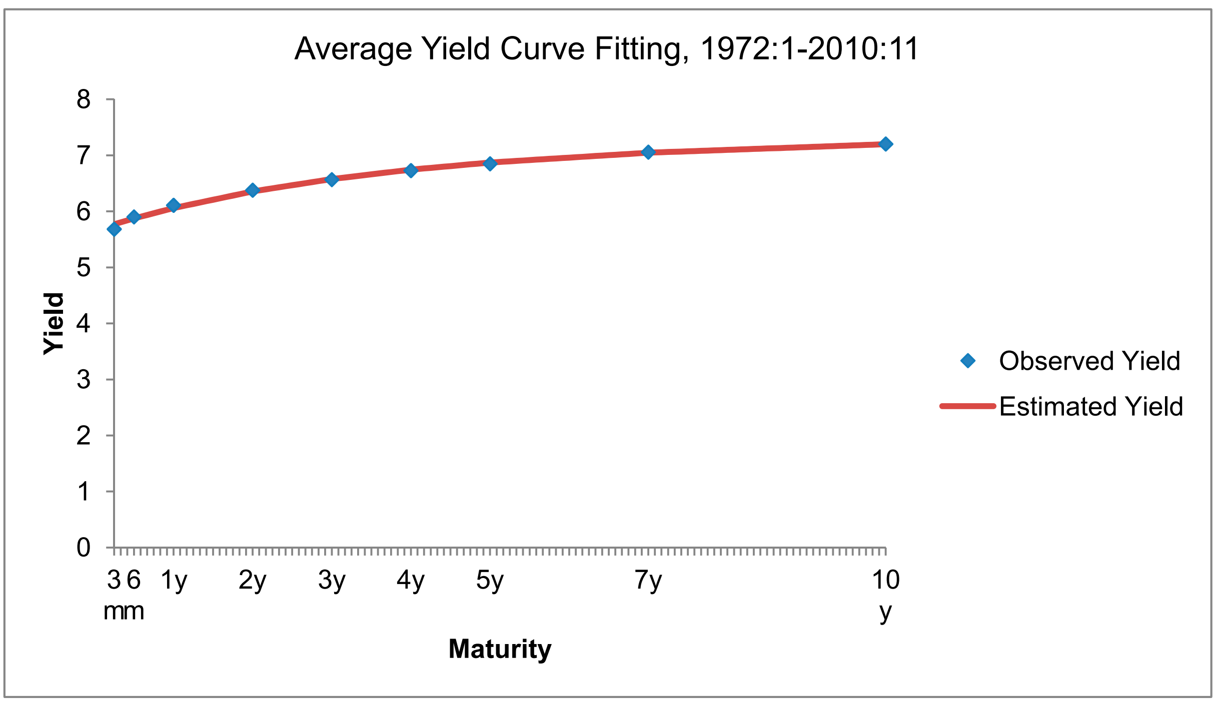




Source: Fund staff estimates.

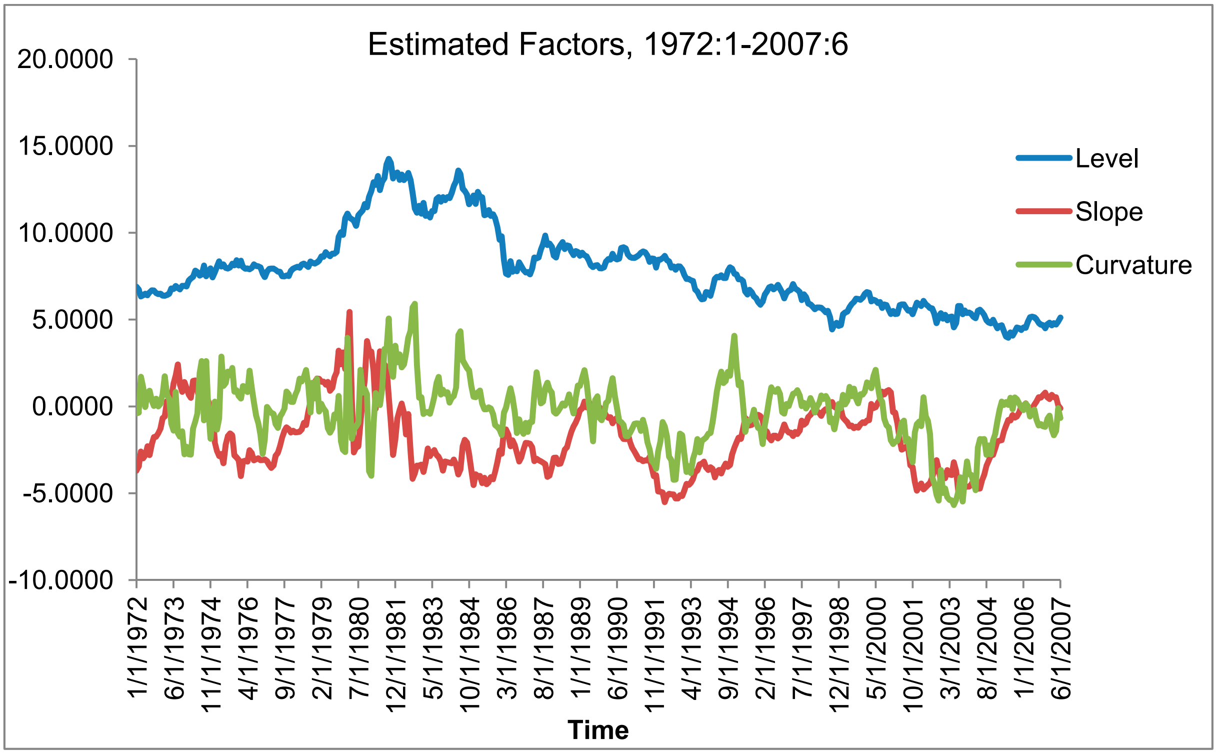
Figure 3. Observed and Estimated Average Yield Curve

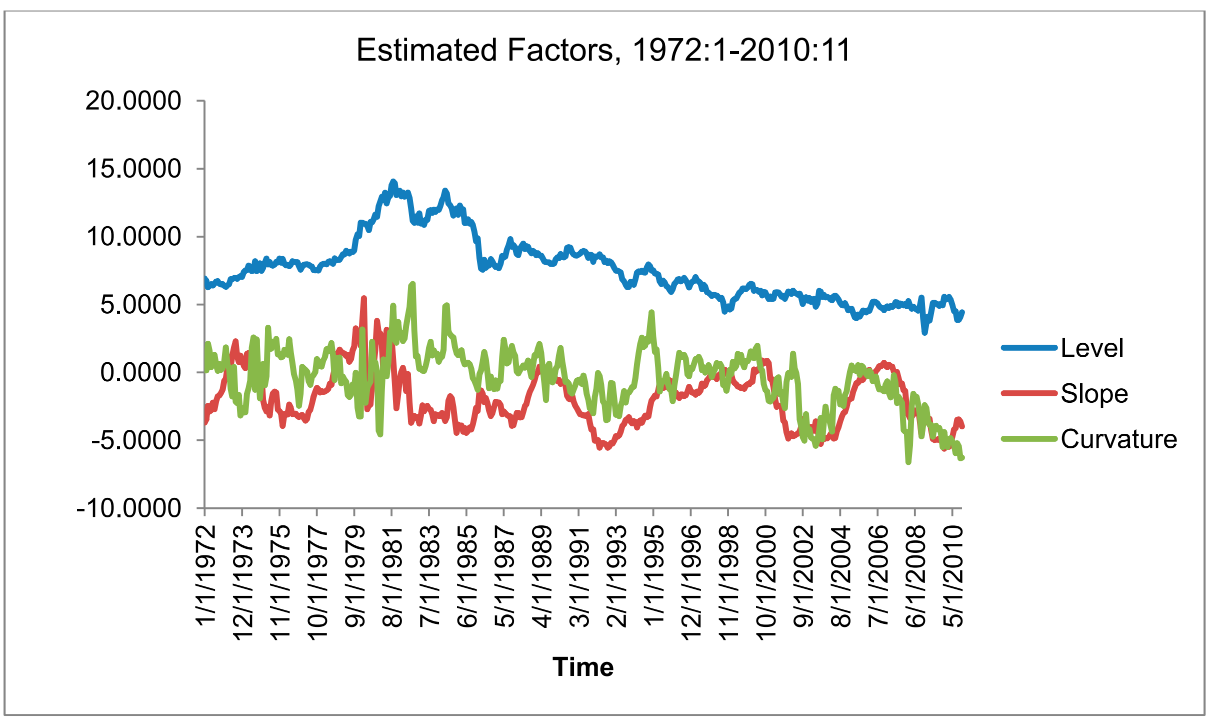




Source: Fund staff estimates.

Figure 4. Estimates of the Level, Slope and Curvature in the Yields-Only Model





Source: Fund staff estimates.

C. Yield-Macro Nelson Siegel Model

The extension of the NSM to include macroeconomic factors makes it possible to capture the dynamic interactions between the term structure and macroeconomy. In particular, the extension of the NSM to include three macroeconomic factors—manufacturing capacity utilization (CU_t), the federal funds rate, (FFR_t), and annual price inflation, (INF_t)—opens the way to explore the feedback between the term structure and macroeconomy and vice versa. As Diebold, Piazzesi and Rudebusch (2006) argue, these three variables represent the minimum set of variables to capture macroeconomic dynamics.

The yield-macro NSM fits the term structure of U.S. Treasury security yields well for the periods of 1972:1-2007:6 and 1972:1-2010:11. As Tables 4 and 5 indicate, the estimated means and standard deviations of the residuals of the measurement equation are small for all maturities of U.S. Treasury securities for both periods. The measure of goodness of fit, or the Chi square test, shows that the observed term structure is remarkably close to the estimated term structure (Table 6). As in the case of the yield-only NSM, the characteristics of the estimated term structure using the yield-macro NSM are essentially the same for the periods of 1972:1-2007:6 and 1972:1-2010:11. The term structure estimated by the yield-macro NSM is upward sloping and concave, while taking on a variety of shapes. The yield-macro NSM again proves that it is very robust to estimate the yield curve of U.S. Treasury securities.

Table 4. United States: Yield-Macro Model, Yield Curve Residuals, 1972:1-2007:6

Maturity (Months)	Mean	Std. Dev.	Minimum	Maximum	ρ(1)	ρ(12)	ρ(30)
3	-0.079	0.135	-0.872	0.275	0.705	0.296	0.124
6	0.033	0.065	-0.202	0.357	0.599	0.298	0.118
12	0.051	0.125	-0.316	0.615	0.704	0.283	0.000
24	0.021	0.058	-0.156	0.299	0.679	0.085	-0.046
36	-0.012	0.033	-0.215	0.112	0.479	0.169	0.106
48	-0.015	0.046	-0.164	0.142	0.771	0.096	0.057
60	-0.023	0.044	-0.186	0.135	0.683	0.122	-0.147
84	0.014	0.036	-0.077	0.185	0.676	0.213	0.072
120	0.012	0.072	-0.192	0.286	0.760	0.371	0.021

Source: Fund staff estimates.

Table 5. United States: Yield-Macro Model, Yield Curve Residuals, 1972:1-2010:11

Maturity (Months)	Mean	Std. Dev.	Minimum	Maximum	ρ(1)	ρ(12)	ρ(30)
3	-0.088	0.153	-0.980	0.306	0.714	0.283	0.136
6	0.028	0.057	-0.181	0.306	0.561	0.279	0.096
12	0.051	0.127	-0.359	0.609	0.704	0.276	0.012
24	0.023	0.067	-0.190	0.345	0.694	0.091	0.003
36	-0.009	0.031	-0.197	0.102	0.459	0.101	0.089
48	-0.015	0.051	-0.299	0.151	0.769	0.085	0.034
60	-0.021	0.048	-0.193	0.131	0.699	0.117	-0.128
84	0.013	0.039	-0.157	0.189	0.646	0.162	0.064
120	0.007	0.071	-0.178	0.248	0.772	0.275	0.039

Source: Fund staff estimates.

Table 6: Goodness of Fit of the Yield-Macro NSM

Chi-square Test of Fit, 1972:1-2007:6	
	Value
SSE	27.5367
Chi-square	5.7951
DF	3828
Chi-square Test of Fit, 1972:1-2010:11	
	Value
SSE	33.6330
Chi-square	6.4420
DF	4197

Source: Fund staff estimates.

In sum, these estimations of the term structure of U.S. Treasury security yields successfully capture the dynamics of the term structure in the United States, particularly since the onset of the financial crisis in mid-2007. These estimations provide support for the central notion of conceptual macrofinance models that the dynamics of the term structure depend on the monetary policy stance and market expectations about short-term future interest rates (see Rudebusch and Wu, 2008, and Walsh, 2010). In the context of the framework of the NSMs, the estimations suggest that the latent factors of the term structure—level, slope, and curvature—and macroeconomic variables help explain the dynamics of the term structure over the estimation periods. Again, by way of example, in response to the Federal Reserve's actions to lower the target federal funds rate and use alternative tools to ease monetary policy and market reactions to these actions, the level of the term structure shifted downward initially during the financial crisis in mid 2007, while the slope flattened. However, the slope of the term structure has since increased, gradually at times, in light of the Federal Reserve's efforts to increase liquidity to foster maximum employment and market belief that the increase in liquidity may require an increase in the policy rate to contain eventual inflation pressures.

As Diebold, Rudesbusch and Aruoba (2006) suggest, impulse response functions from VARs facilitate the assessment of the dynamics of the yield-macro system. In this context, they note that it is possible to consider different groups of impulse responses. This paper focuses on the possible responses of the yield curve to shocks of the macroeconomic variables and the responses of the macroeconomic variables to term-structure factors.[11] As Figure 5 illustrates, the impulse functions show that:

- The level of the term structure responds directly to shocks to the macrovariables. However, the response of the level is statistically significant. This may indicate that the level responds to a surprise on the inflation front.

- The slope of the yield curve responds in a statistically significant way to positive shocks to capacity utilization and inflation. These responses appears to be consistent with a monetary policy that responds to positive output and inflation surprises that lead to a rise in the short end of the yield curve. The response of the slope to inflation is statistically insignificant.

- The curvature tends to show little response to shocks to the macroeconomic variables.

Also, as Figure 5 reveals, the impulse functions that summarize the response of the macroeconomic variables to shocks in the term-structure factors show the following:

- The three macroeconomic variables respond in a statistically significant way to a positive shock in the level. To the extent that the yield curve incorporates information about inflation expectations, these responses appear to be consistent with economic intuition. The increase in inflation in response to a shock in the level appears to reduce the expected real rate of interest, a process that stimulates economic activity. This, in turn, appears to prompt an increase in the fed funds rate.

[11] Diebold, Rudebusch and Aruoba (2006) suggest that the impulse responses from VARs take on a particular ordering of the variables, in particular $\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{1t}, CU_t, FFR_t$ and INF_t. The yield curve factors enter prior to the macroeconomic variables since they are dated at the beginning of the period.

Figure 5. Impulse-Response Functions

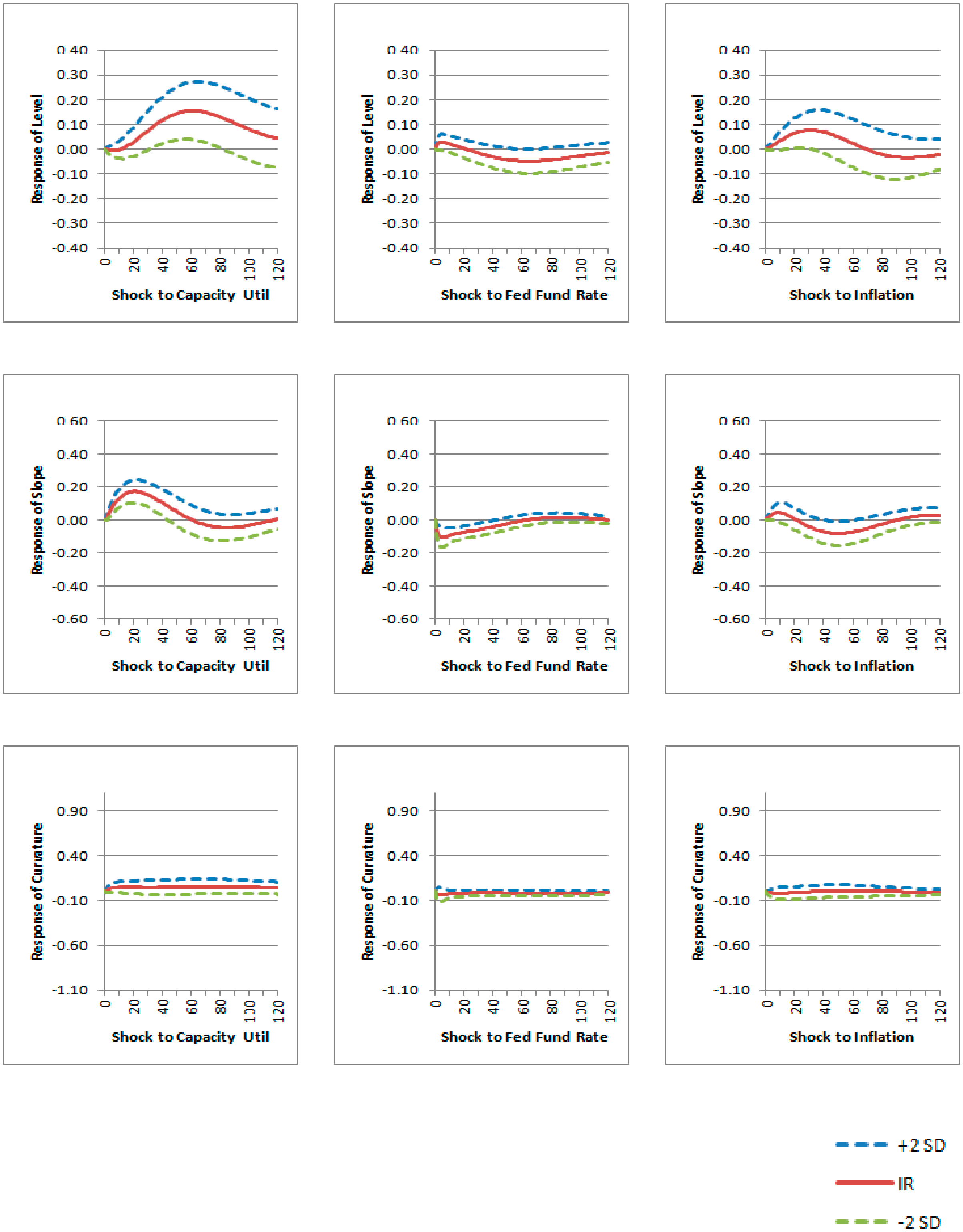
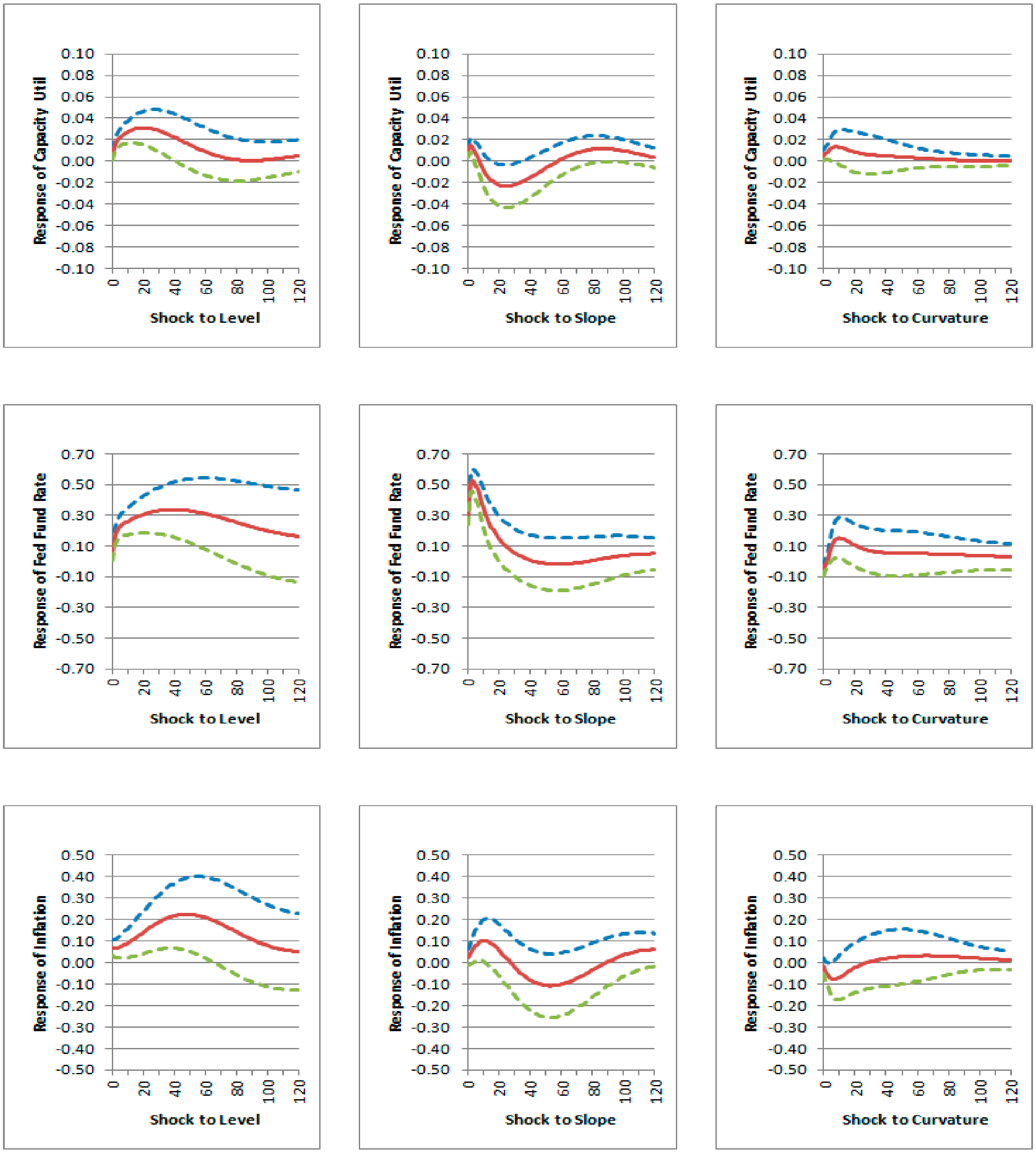


Figure 5. Impulse-Response Functions (continued)



Source: Fund staff estimates.

- As in Gasha et al. (2010) and Diebold, Rudebusch and Aruoba (2006), the fed funds rate and the slope of the yield curve have a close connection. After first increasing sharply, the fed funds rate declines in response to a shock in the slope of the term structure. The response of inflation to shocks to the slope factor is statistically insignificant.

- Macroeconomic variables show little response to a shock of the curvature.

In conclusion, the impulse responses suggest that there is a degree of bilateral feedback between the term-structure factors and the macroeconomic variables, and vice versa. The bilateral feedback from the term-structure factors to the macroeconomic variables appears to be stronger, a result that is consistent with previous studies (Gasha, et al., 2010; and Diebold, Rudebusch and Aruoba, 2006).

Variance decompositions provide an additional metric for analyzing the interactions of the term structure and the macroeconomy. Table 7 provides the variance decompositions for all yields of U.S Treasury securities for a 60-month period. The yield factors, namely the level, slope and curvature, account for an increasing share of the variance of the yields as the maturity of the U.S. Treasury securities rise. By way of example, the yield factors explain about 75 percent of the variance of the yields of three-month U.S. Treasury securities and 81 percent of this variance of the yields of 10-year U.S. Treasury securities, with the level explaining most of the variance. Of the macroeconomic factors, the capacity utilization explains about 20 percent of the variance of the yields of U.S. Treasury securities. However, the contribution of this factor to explain the variance of the yields declines as the maturity of the U.S. Treasury securities increases.

Table 7. Variance Decomposition

	Level	Slope	Curvature	Capacity Utilization	Fed Fund Rate	Inflation
3 Month	0.4792	0.2239	0.0458	0.2091	0.0278	0.0144
6 Month	0.4996	0.2055	0.0520	0.2032	0.0260	0.0137
1 Year	0.5315	0.1767	0.6320	0.1925	0.0230	0.0131
2 Year	0.5737	0.1418	0.0759	0.1762	0.0187	0.0138
3 Year	0.6012	0.1241	0.0771	0.1657	0.0161	0.0157
4 Year	0.6211	0.1151	0.0721	0.1590	0.0144	0.0182
5 Year	0.6360	0.1106	0.6470	0.1545	0.0134	0.0208
7 Year	0.6555	0.1075	0.0502	0.1491	0.0123	0.0255
10 Year	0.6702	0.1074	0.0352	0.1448	0.0116	0.0308

Source: Fund staff estimates.

V. CONCLUSIONS

This paper assesses the dynamics of the term structure of interest rates in the United States in light of the financial crisis in 2007-10. In particular, this paper assesses how the dynamics of the term structure of U.S. Treasury security yields have changed in light of the Federal Reserve's aggressive response to the financial crisis, and market expectations about future short-term interest rates. To this end, this paper relies on estimates of the term structure that make use of latent factors and key macroeconomic variables. This paper also investigates the impact on macroeconomic dynamics of changes in financial factors, and vice versa, by exploring the responses of macroeconomic variables to shocks to the factors of the term structure, and the impact on the factors to shocks of the macroeconomic variables, respectively. The paper relies on the Nelson-Siegel models to estimate the term structure, and to draw conclusions about the dynamics of the term structure and its relationship to key macroeconomic variables.

The estimation of the term structure of U.S. Treasury security yields successfully captures the dynamics of the term structure in the United States. The paper shows that the yield-only and yield-macro NSM models fit well the many shapes of the observed term structure during 1972:1-2007:6 and 1972:1-2010:11. In line with previous findings in the literature, this paper confirms that that it is possible to explain the variations across U.S. Treasury securities with different maturities over the estimation period in terms of three yield-factors, namely the level, slope and curvature. The level and slope of the yield curve exert an important influence in the dynamics of capacity utilization, inflation, and fed funds rate. In this context, the paper provides evidence that the expectations of the impact on macroeconomic variables of changes in financial variables, and vice versa, have changed little with the financial crisis. It also shows that the term structure appears to depend on the monetary policy stance and market expectations about future short-term interest rates. In addition, it confirms that the Nelson-Siegel models are sufficiently robust to explain the many shapes that the term structure has taken on over time, including since the inception of the financial crisis in mid-2007.

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