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Stress Tests for Defined Benefit Pension Plans – A Primer

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Stress Tests for Defined Benefit Pension Plans – A Primer

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Abstract

Stress testing is a useful and increasingly popular, yet sometimes misunderstood, method of analyzing the resilience of financial systems to adverse events. This paper aims to help demystify stress tests and illustrate their strengths and weaknesses. Using an Excel-based template with institution-specific data, readers are walked through the basics of liability valuation and stress testing of assets and liabilities of a typical defined benefit plan.

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Glossary

ABO	Accrued Benefit Obligation
AL	Actuarial Liability
DB	Defined Benefit
DC	Defined Contribution
EAOcd	Entry Age Obligation – Constant Dollar
EAOcp	Entry Age Obligation – Constant Percent
EET	Exempt-Exempt-Taxed
EU	European Union
IFRS	International Financial Reporting Standards
IOPRP	Institutions for Occupational Retirement Provision
NC	Normal Cost
OECD	Organization for Economic Cooperation and Development
PBOcd	Projected Benefit Obligation – Constant Dollar
PBOcp	Projected Benefit Obligation – Constant Percent
RBO	Retirement Benefit Obligation
TEE	Taxed-Exempt-Exempt
TER	Terminal Funding Method
VaR	Value at Risk

I. INTRODUCTION

1. This paper develops a simple methodology for stress testing the funding ratio of defined benefit pension plans. It is aimed at a non actuarial audience with limited understanding of pension terminology, and it is therefore verbose. At the same time it provides a detailed discussion of benefit and cost allocation methods for individuals who want to set the template in the context of alternative methodologies.

2. The remainder of this paper is structured as follows. Section II discusses stress testing for pension plans in an international context as one of the tools of risk based supervision. Section III introduces basic actuarial factors and functions involved in the estimation of actuarial liabilities of defined benefit (DB) plans. Section IV presents the pros and cons of alternative methods used to calculate pension liabilities. Sections III and IV contain a detailed discussion of the building blocks of the functions used subsequently in the accompanying template “Model.xls”. Individuals not familiar with these concepts may find Appendix I useful to review basic pension terminology. Individuals familiar with these concepts can safely skip these sections and refer to Appendix III where the mechanics of the accompanying template are discussed. Section V takes a real case of a DB pension plan and builds a model plan to conduct simple single factor stress tests of assets and liabilities. Finally, conclusions and a brief discussion of possible extension are contained in Section VI.

II. WHAT JURISDICTIONS DO IN THE AREA OF STRESS TESTING

3. For supervisory authorities, stress testing is one of the elements of the general framework of risk based supervision. Such elements include regulatory requirements in support of, and specific techniques for, quantitative risk assessment.

A. Regulatory Framework

4. Some regulatory requirements limit the investment risks that the plan can assume. For instance, *investment rules* restrict the investable universe for the asset manager as a way to improve asset quality and mix, reduce volatility, and increase liquidity. In some cases, the regulations establish quantitative limits, compliance with which can be assessed as part of the supervisory process. In other cases, this is achieved without quantitative limits and through the more flexible concept of the “prudent person” rule.

5. Other regulatory requirements limit the types and amount of insurance provided to beneficiaries. *Benefits regulations* ensure that plan sponsor do not use the plan as a tax avoidance mechanism and that adequate benefits are provided to plan members. Typically, private occupational DB plans provide/guarantee four types of benefits: retirement benefits, vested benefits, disability benefits, and death benefits that are either directly or indirectly regulated through general regulations or plan by laws.

Retirement benefit rules

6. Typically, individual plan members are eligible for retirement benefits upon retirement, if they meet age and, in some cases, minimum service requirements. Plan rules and/or country regulations might define the age at which an individual can be eligible to become a member of a pension plan, and such age does not necessarily coincide with the date of hire. Similarly, plan rules and/or country regulations define the *normal retirement age* at which an individual is eligible for retirement benefits. Typically, this is defined as the earliest age at which retirement can take place without any reduction in benefits. In addition, many plans allow individuals to receive reduced benefits at an *early retirement age*. The reduction in benefits can be *actuarially fair*, *more than actuarially fair* (*subsidized*) or *less than actuarially fair,* depending on whether plan rules are neutral between normal and early retirement or whether they were designed to encourage or discourage early retirement.[1] Unreduced early retirement benefits are sometimes available to individuals with lengthy periods of service.

7. Retirement benefit amounts are defined through the aid of benefit formulas. The most common type of formula provides, for a given accrual factor, units of benefits proportional to a *pensionable salary* and for each *year of credited service*. The accrual factor is typically expressed as a fixed percentage of the pensionable salary. The pensionable salary is typically defined as any fraction (greater, or more often, smaller than 100 percent)[2] of either the last total remuneration or some form of a career average (last, or best, *x* years up to a full career average). Finally, the years of credited service are typically the years during which the employee contributes to the plan and for non-contributory plans, the years of service. For instance, a typical benefit formula might define the retirement benefit as 2 percent of the average of the final five years' base salary times the number of years of service.

Disability benefits rules

8. Disability benefits often take the form life annuity with variable eligibility rules. Life annuities are usually paid to disabled employees beginning at either the early retirement age, the plan normal retirement age, or even just a few months after the accident depending on the severity of the disability and the plan's eligibility requirements. Alternatively, some plans allow for accrual of additional retirement benefits but waive contribution requirements.[3] The

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[1] Notice that in many countries, regulations prohibit less than actuarially fair benefit deductions for early retirement. The US is an example.

[2] Typically, international organization paying salaries net of income tax have pensionable salaries that are higher than net remuneration. The pensionable wage in many plans for civil servants is usually a fraction of total remuneration.

[3] Premium waiver coverage is very common in The Netherlands, for instance.

availability of either form of benefit often depends on other disability plans provided by the state or the plan sponsor. Eligibility rules vary across plans, but almost inevitably they include a minimum age and/or service requirement.

9. The benefit amount is calculated by applying the retirement benefit formula at the time of disability. However, it is often the case that benefits take the form of flat amounts or that the projected end of service salary is used in the benefit formula when the annuity is deferred to the normal retirement age.[4]

Death benefits rules

10. Death benefits take various forms and their values can vary considerably. They can be lump sums calculated as multiples of the salary of the employee or some flat dollar amount for non active employees. Plans sometimes provide for a life annuity payable to the surviving spouses, either immediately or at the normal retirement age of the deceased employee. Eligibility rules vary again but, in general, eligibility for death benefits coincides with the vesting of retirement benefits. When an annuity is paid, this is often a fraction of the vested benefit of the deceased worker.[5]

Vested benefits rules

11. The aforementioned benefits can be subject to vesting requirements. An employee has *vested benefits* if their payment at retirement is no longer contingent on her continued membership of the plan. The *vesting schedule* can provide for *full immediate vesting* (in which the employee acquires full rights to accruing benefits as soon as she joins the plan), *full vesting after x years* of credited service (in which the employee has no rights to accruing benefits in the first *x* years of membership and has full rights afterwards), or *graded vesting* (in which retirement benefits are vested gradually, with any type of schedule, in the first *x* years of membership, when full vesting is reached). Regulations often require that benefits be fully vested after a fairly short period, such as two or five years, to ensure that individuals whose employment terminates prior to retirement receive some value from their membership of the plan.

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[4] With notable exceptions, as in the US, where service salary cannot be projected for disability benefits.

[5] For instance, in the Netherlands, death benefits calculated as multiples of the employee's salary are only common in accident and dismemberment insurance, not pensions. In pension plans, death benefits take the form of temporary or life survivor benefits calculated as a percentage of the old age benefit that the deceased would have obtained if s/he had reached the normal retirement age.

Valuation rules

12. Valuation rules contribute to the financial viability of the plan. *Valuation rules* are designed to ensure that the plan has valued its assets and liabilities appropriately, and that its assets will be sufficient to meet its liabilities both currently and prospectively.

13. Assets can be marked to market or marked to costs. *Market based valuation* is in principle simple but in practice certain types of assets might not be traded regularly in the market. Therefore, methodologies need to be applied so that valuation can take place. In some jurisdictions, asset valuation requirements are set out in regulations while in others, the valuation requirements are established by reference to professional standards which can be local or international, like the International Financial Reporting Standards (IFRS). When assets are *valued at amortized costs*, market volatility is smoothed over potentially many years, and regulations often define the amortization period as well as the smoothing technique.

14. Liability valuation is a far more complex matter. This requires the use of many *actuarial assumptions*, together with information on a plan's membership and rules, and it can be based on any of many *actuarial valuation methods* Actuarial assumptions relate to factors like mortality, termination, retirement, inflation, discount rate, and wage increases. These are discussed in detail in section III. Actuarial valuation methods are used to produce a value of the liabilities for the balance sheet as well as the *funding policy*: i.e., the way the costs of a pension plan are allocated among the years in which they are generated. We will discuss in detail valuation and costing methods in Section IV and we will see that these can potentially be infinite.

15. Assets and liabilities can be valued in many ways, but what matters is that they are valued in a consistent way. There is general trend for the adoption of market based valuation methods. However, a large debate is still alive on the pros and cons of market based versus amortized cost based valuations. On the one hand, market based valuations enhance transparency, especially when it is necessary to have a valuation for liquidation purposes. However, marking to market increases the volatility of balance sheet items and therefore, costing. On the other hand, smoothing techniques may be appropriate when liabilities are far in the future and produce more stable funding patterns that sponsors prefer for budgeting purposes. The usefulness of valuation results for both financial and risk assessment is generally enhanced if the basis used to value the assets is consistent with the basis used to value the liabilities. For example, assets and liabilities might both be valued using market values and market interest rates, or they might both be valued using some smoothing mechanisms. Typically, regulations limit the choice of asset and liability valuation methods.[6]

[6] For instance, in the US. liability valuation for financial reporting purposes can only be conducted using the projected unit credit method (what we will call *PBOcd*, in this text) while for funding purposes, regulation

(continued…)

16. Valuation can be based on a closed group or an open group (see also Box 1). *Open group valuation* methods are typically used for social security plans. These methods assume that the plan has an infinite life and variables of interest (solvency, financial performance, demographics, et cetera) are projected over many years. These valuations assume that current plan members will eventually die and that new individuals, still unborn at the valuation date, will eventually join. In other words, these valuations require a demographic module to project the age distribution of the plan membership over time. *Closed group valuation* methods consider only the plan membership at the valuation date and are typically used for private pension plans.

17. Finally, valuation can be on a going concern or on a termination basis (see also Box 1). A *going concern valuation* assumes that the plan continues to exist in the future with plan members accruing future benefits and the plan accruing future income. Plan actuaries might make optimistic (pessimistic) assumptions on future experience factors, thus overstating (understating) the current financial viability of the plan. *Plan termination valuations* assume that the plan is closed at the day of the valuation and neither benefits nor income are accrued.[7] Accrued obligations can then be settled at the valuation date or following the plan termination rules. These valuations provide a clearer picture of the financial position of a pension plan should it need to cease operations, for example, because of the insolvency of the plan sponsor. In some jurisdictions, both types of valuations are required and typically, every three years.

Solvency rules

18. Solvency rules are quite diverse across countries (Table 1). Solvency rules have the objective of ensuring that a pension plan has sufficient assets to meet its liabilities. The regulatory requirements for solvency purposes are quite diverse across countries due to the lack of international regulatory standards on the assumptions and methodologies used to determine assets and liabilities. For instance, expected liability cash flows are based on current salaries in some countries, while in other countries they are based on salaries projected to the normal retirement age. Alternatively, some countries require regulatory funding levels to be calculated using a market discount rate (such as the yield on government bonds) such as Belgium, Canada, and Japan. Others require a fixed discount rate (Finland, Ireland, Germany, Norway, Portugal, and Spain) or a rate equal to the future expected return

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allows for a series of methods (including all the ones we discuss in this text). Smoothing techniques can be used for asset valuation so that investment gains and losses can be recognized over time, reducing volatility on the plan balance sheet.

[7] Plan termination valuation is not permitted for funding purposes in the US and allowed only for financial reporting. Canada, instead, requires both types of valuation.

on plan assets (the United Kingdom). These different provisions will produce (*ceteris paribus*) different levels of liabilities with different sensitivities to same stress test scenarios.

Table 1. Regulatory Provisions for Liability Valuation Vary Considerably Across Countries

Country	Accrued Liabilities/Technical Provisions	Discount Rate and Other Economic Assumptions
Belgium	The calculation of technical provisions must be prudent and take into account the risk profile of the pension fund (IORP). Furthermore, the technical provisions must at least equal the vested reserves, which are determined by the pension plan rules and the Social and Labor law. When Belgian social legislation is applicable the technical provisions must at least be the maximum of vested rights as defined in the plan rules and own contributions accumulated with an interest rate of 3.75 percent. Minimum vested rights are calculated on the basis of current salaries with an interest rate of 6 percent and specific mortality tables (MR 88-90 table for males and the FR 88-90 table for females).	Belgian prudential legislation: the discount rate for the calculation of the technical provisions has to be chosen in a prudent manner and taking into account: (i) the return on covering assets as well as future returns and/or (ii) the return on bonds of a Member State or on other high-quality bonds.
Canada[8]	Plan termination liability (current unit credit).	Interest rate of x percent per annum for 10 years and y percent per annum thereafter. The rate "x" is equal to the annualized market yield on seven-year Government of Canada benchmark bonds plus 90 basis points. The rate "y" is a more complicated blend of market yields on such seven-year bonds and on long term Government of Canada benchmark bonds, again plus 90 basis points. Lower interest rates apply when the plan provides indexation of pensions; the formulas are specified in the CIA Standards of Practice.
Finland	Accrued benefits calculated under current unit credit method.	3.5 percent-3.8 percent depending on the plan.

[8]The information for Canada in this table applies to defined benefit pension plans regulated at the federal level. Provincially regulated plans may have different requirements, particularly for the maximum allowable amortization period.

Country	Accrued Liabilities/Technical Provisions	Discount Rate and Other Economic Assumptions
Germany	The technical provisions are the present value of the future liabilities minus the present value of the future premiums. The valuation of liabilities includes salary increases or inflation revaluation between the valuation date and retirement age if these are included in the pension promise.	The maximum discount rate for *Pensionskassen* and *Pensionsfonds* (if the latter offer insurance-like guarantees) is currently 2.25 percent for new schemes. *Pensionsfonds* can use market interest rates on a best estimate basis if they offer no insurance-like guarantees.
Ireland	Plan termination liability (current unit credit), including mandatory revaluation of benefits with 4 percent cap, until retirement.	(a) a pre-retirement discount rate of 7.50 percent; (b) a long term post-retirement discount rate of 4.50 percent; (c) a pre-retirement price inflation rate of 2.00 percent; and (d) a post-retirement long term rate of price inflation of 2.00 percent.
Japan	Plan termination liability (current unit credit).	80-120 percent of 10-year government bonds issued during the previous 5 years.
Netherlands	Accrued benefits calculated under current unit credit method.	Discount rate for the valuation of liabilities is based on swap rates. Smoothing is allowed for determining contributions.
Norway	Accrued benefits calculated under current unit credit method.	4 percent discount rate until 1993. For contributions due after 1 January 2004 and pension funds established after 1993 the maximum rate is 3 percent, 2.75 percent for new contracts after 2006.
Portugal	Accrued benefits calculated under current unit credit method. If indexing of pensions is contractually guaranteed, then an allowance for the effect of future indexing must be included in the calculation of the accrued liabilities.	4.50 percent.
Spain	Projected Benefit Obligation (including salaries at retirement - projected unit credit method).	4 percent discount rate. Inflation assumption of 1.5-2.0 percent.
Switzerland	Accrued benefits calculated under current unit credit method.	
United Kingdom	Accrued benefits must be calculated on a prudent basis	The discount rate in the UK can broadly be described by the following equation: discount rate = risk free rate + risk premium A proxy such as a government bond yield is typically used for the spread over the risk free rate is assumed, typically based on: the time horizon of liabilities; the potential for additional investment return; and a prudence adjustment, based on the employer's covenant.

Country	Accrued Liabilities/Technical Provisions	Discount Rate and Other Economic Assumptions
United States (Single Employer Plans)	Accrued benefits calculated using projected unit credit method for financial reporting. Other methods are allowed for funding purposes.	For financial reporting: modified yield curve (three segments) based on a two-year average of top three levels of high-grade corporate bonds of appropriate duration. For funding methods: IRS determined rate plus allowed spread for inflation.

Source: Adapted from Yermo and Severinson (2010).

19. In addition, solvency rules typically require assets to exceed liabilities by a solvency margin. Sometimes, the level of assurance is increased by requiring assets to exceed liabilities by a *solvency margin*. The solvency margin might be calculated as a *simple percentage of liabilities*, or be *risk weighted*, or *stress test related*.

20. Risk weighted solvency margins require the definition of asset and liability weights. This approach is consistent with the risk-based capital requirements applicable to banks and insurers in many jurisdictions. With this approach, weights are attached to various proxies for the asset and liability risks and often just added to one another to determine the total required solvency margin, or sometimes combined using a non linear formula. For instance, The Netherlands requires a solvency margin of 5 percent of technical provisions (following the EU IOPRP Directive) and the standard model defines the various solvency buffers associated with each type of risk (interest risk, equity and real estate risk, currency risk, commodity risk, credit risk, and insurance risk), as well as how such factors are combined to derive the overall solvency margin (Table 2).

21. Stress tests are often used to complement risk weighted solvency margins. Risk weighted margins are affected by model risk and might not provide a reliable solvency estimate needed to withstand adverse conditions. Stress testing overcomes this weakness by requiring plans to calculate the additional amount of assets it would need to be able to meet its obligations under a prescribed stress scenario or scenarios. It is complementary to risk weighting as it generally uses the standard or accepted internal model to conduct the necessary calculations under the different scenarios of increasing but still plausible severity. Similarly to the banking and insurance world, the stress testing results are used by the supervisor to graduate the policy and supervisory responses with more intense scrutiny devoted to plans with poorer results. An example of a jurisdiction with such requirements is Denmark (Table 2).

Table 2. Risk Based Solvency Requirements for DB and DC Plans – Select Countries

Country	Treatment of Longevity Risk	Discount Factors	Minimum Solvency Requirements	Solvency Buffers
Netherlands	Group specific mortality table adjusted for predicted longevity improvements, plus buffer to address uncertainty in predicted values.	Market yield curve measured by Euro swap curve.	Minimum solvency margin of 5 percent of Technical Provisions (from EU IORP Directive) Measured once per year using current market values Maximum period for correction of deviations: 3 years (temporarily increased to 5 years after the 2008 financial crisis).	Maximum probability of under-funding within one year measured with stress test: 2.5 percent Solvency buffers determined by risk factors specific to each asset class. Example of risk factors include yearly decline in: Equity: 25-35 percent (depends on type) Currency: 20 percent Real Estate: 15 percent Maximum period for correction of deviations: 15 years.
Denmark	Fund-specific mortality table approved by actuary and supervisor Traffic light stress test includes assessment of the impact of a 5 percent improvement in longevity.	Market yield curve measured by Euro swap curve.	Solvency margin defined by EU Life Directive: 5 percent of Technical Provision plus 0.3 percent of risk bearing investments Measured every six months using current market values Period of correction from minimum required standards: One year.	Traffic light system is a stress test rather than part of the formal solvency rule, but results are taken into consideration in the supervisory assessment. Test defines three zones: green, yellow, and red Final outcome depends on whether entity remains solvent after test. Example (yearly variations) : Listed equity: Red 12 percent, Yellow 30 percent Interest rate (medium duration) Red +- 0.85 percent; Yellow +-1.2 percent.
Australia	No formal liabilities in DC plans.	No formal liabilities in DC plans.	No solvency requirements for DC plans.	No solvency requirements for DC plans.

Country	Treatment of Longevity Risk	Discount Factors	Minimum Solvency Requirements	Solvency Buffers
Mexico	No formal liabilities in DC plans.	No formal liabilities in DC plans.	No formal solvency requirements, but Value at Risk (VaR) limit designed to limit downside risk for DC members Historic VaR calculated with rolling 550 day sample at 5 percent significance with different limits imposed on the two portfolios. Price vector provided by 2 independent vendors Higher risk portfolio: 1 percent maximum daily loss Standard risk portfolio: 0.6 percent maximum daily loss.	

Source: Adapted from Brunner *et al*. (2008).

B. Risk Monitoring Techniques[9]

22. Supervisors use various techniques for monitoring the various sources of risk affecting the solvency position of defined benefit plans. These include sensitivity testing using different actuarial factors, the analysis of sources of earnings, roll-forward calculations, value at risk calculations, duration and maturity gap analysis, and deterministic and stochastic stress testing.

23. Valuations with different actuarial factors. As it will become obvious in sections III and IV, different actuarial assumptions will produce different liability valuations with certain assumptions (like mortality and interest rate assumptions) having a large impact on valuations. Comparisons can be made across different plans or for the same plan over time. The objective of such comparisons is to identify inappropriate assumptions, which might have contributed to an unrealistically optimistic valuation and thus an increased risk of future financial difficulty. When used with alternative assumptions (linked to alternative macro or financial scenarios), the objective of this technique is to provide sensitivity testing of the extent to which the financial situation of a pension plan would be exposed to changes in key environmental factors.

24. Analysis of sources of earnings. The financial performance of a defined benefit plan is a function of the investment and actuarial performance. In the former case, a favorable (unfavorable) investment experience relative to underlying assumptions generates investment gains (losses). In the latter case, a favorable (unfavorable) experience of actuarial factors (mortality, termination, disability, retirement et cetera) relative to underlying assumptions generates actuarial gains (losses). By comparing actual experience to previous assumptions and recalculating the valuation using both new assumptions and those used in the previous

[9] I am indebted to Michael Hafeman for suggesting the taxonomy used in this section.

valuation, the sources of earnings (gains and losses) can be identified and quantified. The objective of such comparisons is to identify inappropriate assumptions and anticipate the likelihood that future financial and actuarial results will be adverse.

25. Roll-forward calculation. Roll-forward calculations can be used to roughly estimate the financial position of the pension plan under some types of scenarios that are of interest to the supervisor. With this technique, the financial position of a pension plan is projected from a valuation date to future dates by using information produced by the valuation method to roll-forward the values of the various balance sheet items. While in principle, the financial position can be projected many accounting periods into the future, the increasing projection error is likely to limit the usefulness of long-term roll-forward calculations.

26. Maturity gap analysis. The future cash flows of a pension plan can be projected, either as part of the valuation process or independent of it. Cash flows related to assets can be compared with those related to liabilities. The objective of this technique is to identify period by period maturity mismatches between assets and liabilities and their sensitivity to alternative underlying assumptions. The technique requires detailed information on the maturity of fixed income investments.

27. Duration and convexity analysis. A very parsimonious technique, duration analysis enables the comparison of changes in the value of liabilities with changes in the value of interest rate sensitive assets. However, the technique is limited to simple parallel shifts in the yield curve. More often, the curve twists (it becomes flatter or steeper) or changes in curvature (butterfly shifts). A first metric used to address this limitation is convexity. Convexity is a second-order term that measures the change in the duration estimate for a small change in rates. For instance, for a positive duration instrument with no embedded options, positive convexity means that the duration extends (increases) when interest rates fall, and the duration shortens (decreases) when interest rates rise.[10] Duration and convexity would take care of the asymmetric percentage change in prices of bonds when the yield curve shifts upwards or downwards. In other words, the two metrics together can measure interest rate risk for larger interest rate changes. Typically, in an asset-liability management strategy it is required that the convexity of assets be larger than the convexity of liabilities. If this is the case, when interest rates increase, the value of assets decreases by less than the value of liabilities while when interest rates decrease, the value of assets increases by more than the value of liabilities.

[10]All fixed cash-flow bonds have positive duration and positive convexity. Securities with embedded options may have regions with negative or reduced positive convexity. For example, home mortgages can have negative convexity as rates lower and increase the likelihood of prepayments, resulting in lower duration as rates fall, and convexity may turn positive from lower likelihood of prepayment or extension resulting in greater duration as rates rise.

28. Key rate duration and convexity analysis. Key rate, or partial duration, measure the local sensitivity to a shift in just a portion of the yield curve.[11] By matching partial durations of asset and liability portfolios it is possible to obtain a substantial degree of protection against nonparallel shifts in the yield curve. In addition, key rate shifts are constructed so that their sum equals a parallel shift and thus the sum of key rate durations is equal to effective duration for fixed cash flow instruments.[12] Similarly to the simpler case, key rate/partial convexities measure the change in the key rate/partial duration estimate for a small change in rates.

29. Value at Risk (VaR) analysis. VaR analysis is used to measure market risk on the asset side of the balance sheet. The technique measures the expected dollar loss from adverse market movements with a specified probability (confidence interval of, say, 97.5 percent) over a particular period of time (say, one day or more). The technique is clearly of little use to measure long term market risk but irrespectively, it is becoming popular among supervisors and plan managers alike.

30. Stress testing analysis. Stress testing involves calculating the financial position of a pension plan at either the current or a future valuation date on the basis of one or more defined adverse scenarios. Each scenario might vary the assumptions with respect to only one factor, such as interest rates, inflation, mortality, et cetera, or vary several factors at once. The objective of stress testing is often to help assess the financial ability of an entity to withstand the effects of various risk scenarios, should they occur. Two general flavors of stress testing are generally available: deterministic and stochastic stress testing. With *deterministic stress testing*, scenarios are defined a priori without any reference to their likelihood. With *stochastic stress testing* scenarios are randomly generated to produce a distribution of results on the basis of distributions of the underlying assumptions. While stochastic techniques appear to provide a richer set of results, their weakness lies exactly on how close to reality the assumed distributions are.

III. ACTUARIAL COST FACTORS

31. The calculation of DB pension liabilities relies on assumptions for key various actuarial cost factors. *Actuarial cost factors* are the elements/assumptions used by the *actuarial cost methods* to calculate the actuarial liabilities. We discuss the former in this section, while we dedicate the next section to the latter. Several actuarial cost factors are

[11]Key rate and partial durations are essentially the same concept. The only difference is that key rate durations are calculated by shocking the spot rate curve while partial durations are calculated by shocking the yield curve.

[12]See Ho (1992), Reitano (1992), Fabozzi and Fong (1994), and Dettatreya and Fabozzi (1995) for various alternative metrics that follow the approach just described.

considered in the calculation of liabilities. The key ones are decrement assumptions, related survival probabilities, salary assumptions, inflation, and discount rate assumptions.

A. Decrement Assumptions

32. Decrement assumptions describe how plan members' liabilities are affected by various risks/contingencies. For instance, retired, members are exposed only to mortality risk. However, active members are exposed to mortality, termination, disability, and retirement risk. In order to describe how members transit from one status to another, rates of decrement (given by available tables) are assumed for each contingency or risk. In other words, the rate of decrement refers to the proportion of the plan population in one status transiting to another status. In this paper we indicate with ${q'}_x^{(k)}$ the rate at which members of age x transit from one status to another within the period between x and $x + 1$. The prime indicates we are working with rates (as opposed to probabilities) and (k) indicates the type of contingency (mortality (m), termination (t), and retirement (r)).

Mortality assumptions

33. Mortality rates are one of the most important actuarial assumptions used in calculating the liabilities of DB plans. This because they affect all active and retired members since entry age and over a long period of time. These are typically indicated with ${q'}_x^{(m)}$ and for the solvency of the plan it is important that deaths be at least equal to their expected number according to the mortality assumption.

34. The impact of mortality on a DB plan depends on the participant status. Mortality eliminates the pension benefit obligation (it is "good" for the plan) but it may trigger other type of benefits. If the deceased had not vested any pension benefits then accumulated contributions are typically returned to survivors in the form of a cash lump sum. If pension benefits were vested then death benefits often take the form of immediate or deferred annuities with or without a cash lump sum.

35. Mortality assumptions depend on various factors like age, gender and occupational status. Assumed mortality rates increase with age until the maximum assumed age when the rate equals 1. Other factors that affect mortality rates are gender and occupation. For instance, females typically die at much lower rates than males of the same age while individuals in hazardous work categories (mining, army, police, *et cetera*) die at much higher rates than the average. It is important to keep these differences in mind as in many jurisdictions tables are mandated[13] (unisex, or population, say). The basis risk related to the

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[13]For instance, EU legislation does not allow plans to discriminate across genders and unisex tables need to be used. Additionally, in many jurisdictions, the mortality rates that can be used to deduct pension expenses from income tax are set in regulations while individual plans can deviate prescribed rates but cannot deduct from income tax the reserving in excess of regulatory requirements.

difference between the mandated and the covered population mortality rates can result in large unrecorded liabilities which the actuary needs to compensate for with extra provisioning.

36. Two key types of mortality tables are static (or period) and dynamic (or cohort) tables. Aggregate longevity increases over time and it is very difficult to estimate it as the rate of improvement will vary considerably depending on the country's starting point, and economic and social conditions. Static or period mortality rates give us today mortality rates of individuals of different ages x. Cohort mortality rates give us the mortality rates of individuals of different ages x, n years from now. Table A1 reports the USA 1996 male static annuitant mortality rates.

Termination assumptions

37. Termination rates are another key actuarial assumption. Unlike mortality rates, they affect only active members, but still for a potentially long period of time between the entry age and the retirement age. They are typically indicated with ${q'}^{(t)}_{x,y}$ and they simply tell us the rate at which active members of age x that entered the plan at age y leave the plan (without retiring) within the period x to $x + 1$.

38. The impact of termination rates on a DB plan depends on the participant's vesting status. For non-vested members, termination eliminates the pension benefit obligation (it is "good" for the plan). If the member has no vested benefits, the termination benefit will typically be only a cash lump sum return of the member's accumulated contributions.[14] If the member has vested benefits, these take the form of a deferred annuity with or without the option to receive all or a portion of the present value of the deferred annuity as a lump sum. Termination rates do not apply to retired members.

39. Termination assumptions depend on various factors like age, length of service, gender and occupational status. Unlike mortality rates they decrease over time until they are equal to zero when active members retire (or can retire in case early retirement is allowed). Two key factors affecting termination are age and length of service. For a given length of service, termination rates typically decrease with age: i.e., active members are less likely to be fired or to leave the employer voluntarily, the older they are. Also, for any given age, termination rates typically decrease with the length of service: i.e., active members are less likely to be fired or leave the employer voluntarily, the more experienced or the longer they have been employed. Hence, termination rates are typically divided in *ultimate termination rates* and *select termination rates*. The former rates are age dependent only, while the second depend on both the entry age and the length of service. As an example, Table A2 provides an

[14] In some jurisdictions the term "termination benefit" is used in lieu of "severance pay": i.e., a benefit triggered by dismissal.

example of termination rates for members of ages 20 to 64 and with a select period of five years.[15]

Retirement assumptions

40. Retirement decrement rates become relevant when the plan provides for early or delayed retirement. They affect only active members between the earliest and the latest permissible retirement ages and their effect is to start pension benefit payments. These are typically indicated with ${q'}_x^{(r)}$ and they simply tell us the rate at which active members of age x retire within the period x to $x + 1$. Generally, they increase until the maximum age at which retirement is allowed, when they are equal to 1 (i.e., everybody is assumed to retire at that age which in the case of delayed retirement is higher than the normal retirement age).

41. Retirement assumptions depend on a number of factors. One important factor is whether the reduction in accrued benefits due to early retirement is *actuarially fair* or not. Accrued benefits are payable in full at the normal retirement age and if early retirement is elected, they are typically reduced.[16] If this reduction is less than actuarially fair, then members are implicitly encouraged to retire early and early retirement rates are likely to be higher. If this reduction is more than actuarially fair, then members are implicitly encouraged to retire at the normal retirement age and early retirement rates are typically lower. Another important factor is when/if social security benefits are paid: in such case, it is common to see a large increase in retirement rates in that year. Other important influences on the retirement rates are gender, social expectations within a country, and the prevailing economic conditions. In practice, the actual schedule varies from plan to plan but it is common to observe a spike at the earliest effective retirement age and a hump at about the age associated with eligibility for social security benefits.[17]

Other decrement assumptions

42. A DB plan would typically have other decrement assumptions. For instance, when disability benefits are provided for, the rate at which members become disabled and then, depending on the nature of the disability benefits, tables on rates of death and continuance of disability could be relevant and would also need to be factored into the funding and costing

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[15]The table assumes a vesting period of 10 years, normal retirement age of 64 and early retirement age of 55. This is why termination rates for people aged at least 55 and with at least 10 years of services are assumed to be equal to zero.

[16]Some plans provide for unreduced early retirement benefits if other criteria are met, such as, age 50 or more with 30 or more years of service, or age plus years of service equal to 85 or more. Such criteria can have a very significant effect on the retirement decrement rates.

[17]Table A3 reports a plausible schedule, with these characteristics.

calculations. In this paper, we will not consider other decrement assumptions since doing so can greatly complicate the calculations, but is unlikely to materially affect the outcome of stress tests performed in the context of an FSAP.

B. Contingent, Composite Survival Probabilities

43. From the aforementioned decrement assumptions it is possible to derive the composite survival probabilities. The idea is simple: all the various contingencies to which individuals are exposed will affect their probability of survival in the plan and therefore, future liability cash flows. Composite survival probabilities are very important assumptions as they can affect cash flows into the very distant future, potentially more than 70 years away from the valuation date. Here, it is important to notice that we are moving from a decrement rate space to a decrement probability space. Therefore, we need to discuss how probabilities relate to rates in a multi decrement environment and how to construct composite survival probabilities.[18]

44. In a single decrement environment, the rate of decrement is equal to the probability of decrement. This is the case for retired individuals who are exposed only to the contingency of death (as far as the effects on benefits under the plan are concerned). For them, the rate at which people die in one year (as modeled by the mortality table) is equal to the probability of dying in the same year. More formally, if we indicate with ${q'}_x^{(k)}$ the rate of decrement for cause k at age x and with $q_x^{(k)}$ the probability of the same decrement, then ${q'}_x^{(k)} = q_x^{(k)}$

45. In a multiple decrement environment, the rate of decrement is higher than the probability of decrement. This is the case for active members who are exposed to multiple risks during the year. For them, the probability of dying (say) will be linked to the other decrement rates and will be smaller than the rate at which the population dies: i.e. ${q'}_x^{(k)} > q_x^{(k)}$. In practice, by assuming that rates are uniformly distributed over the period, decrement probabilities can be approximated with:

$$q_x^{(k)} \approx q_x^{\prime(k)} \prod_{j \neq k} \left(1 - \tfrac{1}{2} q_x^{\prime(j)}\right) \tag{1}$$

46. Composite survival probabilities give us the probability that an individual will survive from one period to the next. In a single decrement environment, they are simply the complement of the decrement probability. In a multiple decrement environment they are "composite" in the sense that they are the complement of the sum of the relevant decrement

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[18] The accompanying spreadsheet uses two decrement factors (mortality and termination) to derive composite survival probabilities for active members.

probabilities. More concretely, for an individual aged *x* exposed to mortality, termination and early retirement contingencies, the composite survival probability of surviving until age *x* +1 is given by:

$$p_x^{(T)} = 1 - \left(q_x^{(m)} + q_x^{(t)} + q_x^{(r)}\right) \quad (2)$$

where $q_x^{(k)}$ is defined in equation (1) and (*T*) indicates that we are in a multiple decrement world.

47. Contingent, composite survival probabilities can be used to derive the survival probability over more than one period. In particular, given the time independence of probabilities, for an individual aged *x*, the probability of surviving for the next *n* periods is simply given by:

$${}_np_x^{(T)} = \prod_{s=0}^{n-1} p_{x+s}^{(T)} \quad (3)$$

C. Salary Assumptions

48. The future level of salaries is also a key assumption affecting the liabilities of DB plans. Since benefits accrual can depend on future salaries, there needs to be a way to estimate them. In addition, since such estimations can span many years (like for mortality assumptions), their impact on funding and costing can be important.

49. Future salary assumptions depend on three key factors. These are merit increase assumptions, productivity improvement assumptions, and inflation assumptions. Merit increase assumptions are intended to reflect the increased contributions of members to the employer as they progress in their careers. An example of a cumulative merit increase salary scale is provided in Table A4, which assumes merit increases of about 4.5 percent for the first year for a worker entering at age 20, peaking at 5.4 percent at age 35 and declining to zero by age 64. Productivity assumptions vary across sectors but it is not uncommon to observe an assumption of 1 percent annual increases in productivity. Both merit and productivity assumptions are difficult to estimate as they can vary significantly from one employer to another and can change in response to changes in economic and labor market conditions. Finally, the most important assumption for future salaries is typically inflation.

50. Future wage assumptions are summarized by the wage function. Once the aforementioned assumptions are made, it is possible to estimate the future wage levels with the aid of the wage function. For any plan member of age *x* who joined the plan at age *y* with a beginning of the period wage $w_{y,y}$, future salaries can be calculated as:

$$w_{x,y} = w_{y,y} \frac{ms_{x,y}}{ms_{y,y}} [(1+\pi)(1+pr)]^{(x-y)} \tag{4}$$

where $w_{y,y}$ is the beginning of the period entry wage, $ms_{x,y}$ is the cumulative merit increase at age *x*, $ms_{y,y} = 1$ is the cumulative merit increase for service before age *y* (hence, = 1 to indicate no increase), π is the inflation assumption,[19] and *pr* is the productivity improvement assumption.

The wage function $wf_{x,y}$ gives the wage $w_{x,y}$ as a multiple of the entry wage $w_{y,y}$. More formally:

$$wf_{x,y} = \frac{w_{x,y}}{w_{y,y}} = \frac{ms_{x,y}}{ms_{y,y}} [(1+\pi)(1+pr)]^{(x-y)} \tag{5}$$

Table A5 reports the select and ultimate end of period wage function assumptions for the aforementioned merit scale and productivity assumptions, together with a 1 percent annual inflation assumption.

In general, salary at age *x* can be expressed as a function of the salary at age *z* since:

$$wf_{x,z} = \frac{wf_{x,y}}{wf_{z,y}} = \frac{ms_{x,y}}{ms_{z,y}} [(1+\pi)(1+pr)]^{(x-z)} \tag{6}$$

Finally, the *cumulative salary* of an individual of age *x* who joined the plan at age *y* (which will be used in the next section) is simply given by:

$$W_{x,y} = \sum_{s=y}^{x-1} w_{s,y} \tag{7}$$

D. Discount Rate Assumptions

51. Discount rates are, together with survival probabilities, one of the most critical assumptions in valuing DB liabilities. The reason being that they are used to discount expected future cash flows, which are often very distant in time, to the present time. For instance, the last pension payment for a new entrant could be more than 80 years in the future.

[19] Which does not need to be constant. In particular, when current levels of inflation are significantly different than long-term expectations, it is common to grade the assumptions over time.

52. There is a large debate on what the appropriate discount rate should be for pension liabilities. Traditionally, actuaries have argued that pension liabilities should be discounted at rates that reflect the expected long-term returns on a pension fund's assets, while financial economists (and increasingly some actuaries) have countered that pension liabilities should be discounted at market rates, either the risk-free government bond yields or the high-grade corporate bonds yields. This debate is being superseded by a requirement for consistent (market) valuation of assets and liabilities and the introduction of countercyclical/dynamic solvency buffers.[20] For our purpose, we allow for a very general formulation[21] of the rate v^n that discounts the cash flow expected to occur at the beginning of period *n* to time zero and given by:

$$v^n = \prod_{s=0}^{n-1} (1 + f_s)^{-s} \tag{8}$$

where f_s is the forward interest rate assumed for the s^{th} year. This, in turn, can be extracted from the government debt yield to maturity curve (ytm_s) by noticing that:[22]

$$(1 + f_s) = \frac{(1 + ytm_s)^s}{(1 + ytm_{s-1})^{s-1}} \tag{9}$$

An example of a discount rate function derived from the yield curve is provided in Table A6 where yield to maturity rates above period 30 are assumed to be constant.

IV. Actuarial Liabilities and Methods

53. Actuarial liabilities are estimated for each category of plan member. The two most important categories are typically retirees and active members. Other categories can include active members with past service benefits, active but disabled members, and terminated

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[20]In addition, there is a debate regarding the appropriate discount rate to be used for financial reporting versus funding standards. Regulations would typically require some form of market rate for financial reporting standards, while they would allow for a more stable rate for funding standards. See Vittas (2010) for a discussion.

[21]In other words we do not enter in the debate of the appropriate discount rate. In our template we simply allow for any type of rate whether market determined or fixed in regulation.

[22]The use of the forward rates, rather than the spot rates, in calculating discount rates implies an underestimation of the discount rate curve. This stems from the fact that yield rates typically contain an illiquidity premium to compensate for the higher risk involved in investing at longer maturities. Estimating such premium and correcting for it would be beyond the scope of this introductory text. For our purpose, we ignore the difference that is likely to be trivial in very liquid markets.

members with deferred benefits. For the sake of simplicity, we will only focus in this section on retired and active members.

A. Individual Actuarial Liabilities for Retired Members

54. The actuarial liability for a retired member can be calculated through the use of annuity functions. Basically this involves projecting the year by year contingent cash flows and discounting them through an annuity function that corresponds to the form of pension provided by the DB plan. DB plans provide a variety of annuities. These aim at insuring retirees against risks like longevity and inflation. They can cover also spouses and other beneficiaries, insurance can be extended for a certain period or for life, and they sometimes provide for participation in investment and mortality experience. In this section we make use of the definitions of contingent survival probabilities and discount rate functions developed in section III and limit our discussion on how to value standard types of annuities provided in DB plans.

Straight life annuities

55. A straight life nominal annuity insures individuals against longevity and investment risks only. It provides periodical benefit payments fixed in nominal terms B starting with the normal retirement[23] age r = 55 until death. The present value of these expected cash flows is given by:

$$B\ddot{a}_r = \sum_{s=0}^{\infty} B\ {}_{s}p_r^{(m)} v^s \qquad (10)$$

where $\ddot{a}_r$ is the present value of a straight life annuity due (i.e., payable at the beginning of each period), ${}_{s}p_r^{(m)}$ is the contingent survival probability for an individual retired at the normal retirement age r defined in equation (3) and v^s is the discount rate for period s defined in equation (8).

56. A straight life real annuity insures also against inflation risk. In this case, the value of the periodic benefit payment is not constant anymore. After the initial payment of B, it increases with inflation at a rate π.[24] Its present value is given by:

[23]Throughout section IV and V we calculate individual liabilities assuming that plan members all retire at the normal retirement age. In practice, for valuing liabilities, one needs to begin with the actual retirement age of each retiree. We discuss later the impact of this simplification which is also made in the accompanying template "Model.xls".

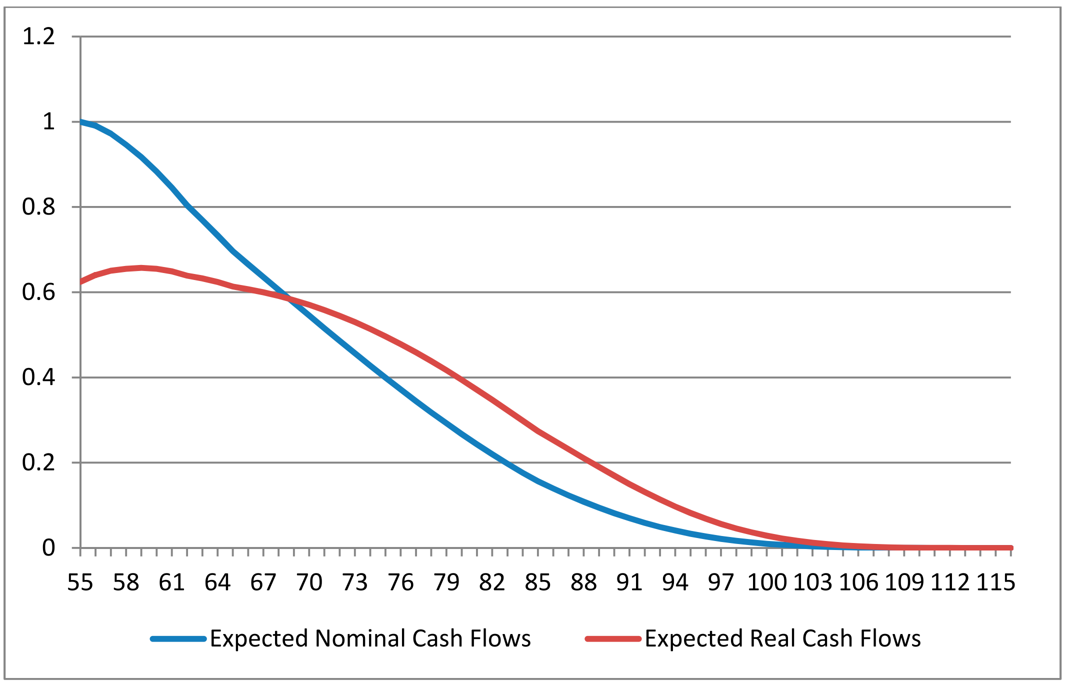
[24] For example, the rate of increase might be one-half the inflation rate or it may be discretionary or ad hoc. For instance, many pension plans that provide indexed pensions in the US do not provide full indexation or

(continued…)

$$B\ddot{a}_r^{\pi} = \sum_{s=0}^{\infty} B(1+\pi)^s \ {}_sp_r^{(m)} v^s \tag{11}$$

57. Initial periodic payments of real annuities are much lower than nominal annuities that have the same present value. Figure 1 reports the expected cash flows for a nominal straight life annuity due paying $B = 1$ and a real annuity starting at the normal retirement age $r = 55$. Calculations use the survival probabilities and the discount function defined in Table A1 and Table A6 and an inflation assumption of 3.5 percent. The areas under the curves are nothing more than the sum of these expected cash flows and given by (10) and by (11), respectively. Since the present values of the two annuities must be the same,[25] it follows that the initial payments from the real annuity must be lower than nominal payments (Figure 2).

Figure 1. Straight Life Annuities – Expected Nominal and Real Cash Flows



Note: Calculations assume: $r = 55$, $\pi = 0.035$, $B = 1$ for nominal annuity, $B = 0.6246$ for real annuity with same present value, mortality rates defined in Table A1, and discount rate defined in Table A6.

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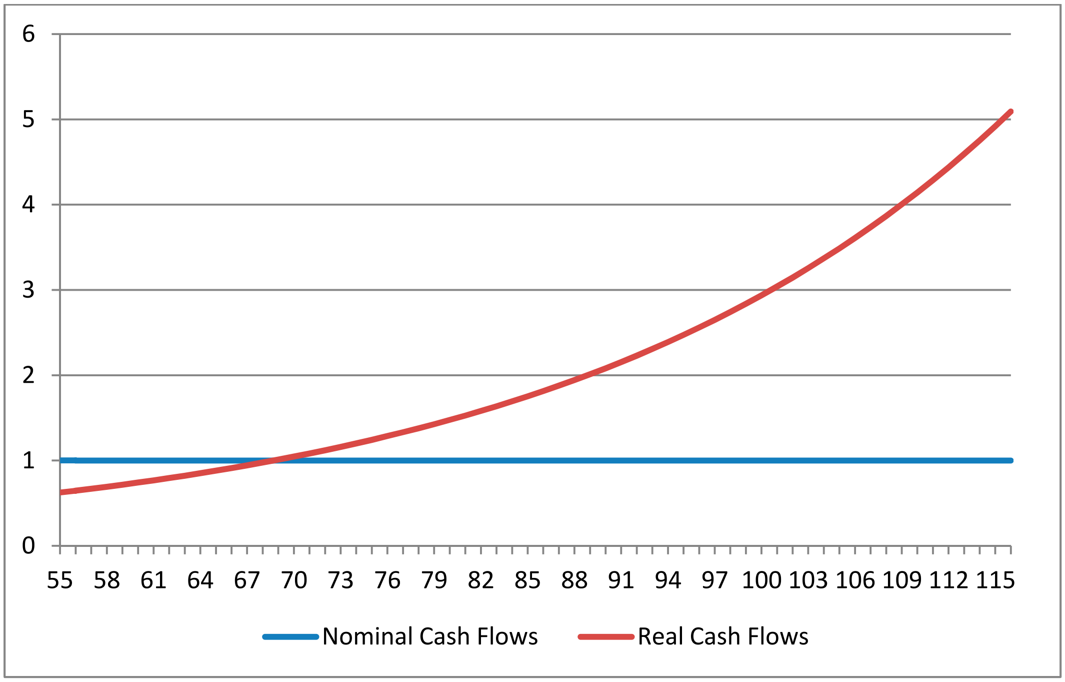
automatic cost of living (COLA) adjustment. As discussed in section V, we will be able to account for this possibility by assuming alternative inflation indexation rates for annuities in the accompanying template "Model.xls".

[25]Intuitively, if $\ddot{a}_r$ is the premium that would buy a straight nominal annuity that pays periodically $B = 1$, then the same premium would buy a real annuity that pays less (initially, although increasing with inflation).

Figure 2. Straight Life Annuities – Nominal and Real Cash Flows per Surviving Annuitant



Note: Calculations assume: $r = 55$, $\pi = 0.035$, $B = 1$ for nominal annuity, $B = 0.6246$ for real annuity with same present value, mortality rates defined in Table A1, and discount rate defined in Table A6.

Joint life annuities

58. Joint life annuities provide the same type of longevity insurance as straight annuities but to more than one beneficiary. The present value of a nominal joint life annuity due that pays B to two individuals of age x and z, respectively, when both are alive and that pays αB when either of the two beneficiaries is dead is given by:

$$B\,{}^{\alpha}\ddot{a}_{x,z} = \sum_{s=0}^{\infty} B \begin{bmatrix} {}_sp_x^{(m)}\ {}_sp_z^{(m)} + \\ \alpha\ {}_sp_x^{(m)}\left(1 - \ {}_sp_z^{(m)}\right) + \\ \alpha\ {}_sp_z^{(m)}\left(1 - \ {}_sp_x^{(m)}\right) \end{bmatrix} v^s \qquad (12)$$

59. Variations of joint life annuities also exist. The most common variation is an annuity due that pays the full pension when either both beneficiaries, or when only the primary beneficiary (the retiree), are alive but that pays only a fraction α of the pension to the surviving spouse. In this case, the present value is given by:

$$B\,{}^{\alpha}\ddot{a}_{\underbrace{x}_{1},z} = \sum_{s=0}^{\infty} B \begin{bmatrix} {}_{s}p_{x}^{(m)}\ {}_{s}p_{z}^{(m)} + \\ {}_{s}p_{x}^{(m)}\left(1 - {}_{s}p_{z}^{(m)}\right) + \\ \alpha\ {}_{s}p_{z}^{(m)}\left(1 - {}_{s}p_{x}^{(m)}\right) \end{bmatrix} v^{s} \tag{13}$$

where 1 under the subscript x indicates the primary beneficiary. Of course, many more modifications can be imagined. However, these are not discussed here as this would be beyond the scope of this introductory presentation.[26]

B. Individual Actuarial Liabilities for Active Members

60. The calculation of the actuarial liability in respect of active members depends on the *plan benefit formula or function* and on the *actuarial cost method* used. The calculation of actuarial liabilities for active members is a considerably more complex affair than the calculation of actuarial liabilities for retired members. This depends on the plan benefit formula and on the actuarial cost method used. Unfortunately, many benefit formulae exist and many cost methods can be used. We discuss these two concepts in turn.

Benefit functions

61. The *accrued benefit function* is used to determine the level of benefits paid at retirement. It sums the amount of benefits that individuals have accrued over time before retirement. Hence, if we define by $b_{x,y}$ the amount of benefits accrued between age x and age $x + 1$ by a plan member that entered at age y (sometimes called "accrual factor"), the accrued benefit function is simply given by the sum of past accrued benefits:

$$B_{x,y} = \sum_{s=y}^{x-1} b_{s,y} \tag{14}$$

As previously mentioned, accrued benefit functions can take many forms but three are more common among DB plans: the flat dollar, the career average and the final average salary.

Flat dollar function

62. Flat dollar benefit accruals are fixed in dollar levels and the amount of pension depends only on the length of the working career. These types of benefits are based upon the philosophy that after a certain number of years of service, each member should receive the

[26]In some plans, joint life annuities are provided on an actuarially-equivalent basis, so for valuation of active members' liabilities, the proportion married and the spousal age difference would not be important assumptions. However, for retirees, the actual form of annuity should be valued.

same benefit at the normal retirement age. Benefits accrued are not related to the salaries or entry age of plan members. In other words, they expressed in flat dollar amounts and are only proportional to the number of years of service. For an individual with attained age *x* who joined the plan at age *y*, the flat dollar benefit function is given by:

$$B_x = (x - y)b \tag{15}$$

Career average salary function

63. Career average benefits are based on the average compensation over the whole working career. With career average benefits, plan members are credited benefits every year on the basis of the wage during that year. Typically, the accrual factor is fixed for the whole plan member population[27] as a percentage of the pensionable salary ($b_{x,y} = bw_{x,y}$) for each year of credited service.[28] Hence, the accrued benefit function for an individual aged *x* who joined the plan at age *y* is given by:

$$\begin{aligned} B_{x,y} &= \sum_{s=y}^{x-1} b_{x,y} = \sum_{s=y}^{x-1} w_{s,y} b = \\ &= b(x-y)\frac{W_{x,y}}{(x-y)} = b(x-y)\overline{w_{x,y}} \end{aligned} \tag{16}$$

where $\overline{w_{x,y}}$ is the average salary over full career $(x - y)$ for an individual of age *x* who joined the plan at age *y*.

Final average salary function

64. Final average salary benefits are based on the average compensation over a period defined by the plan rules. In its simplest case, plan members are credited units of benefits during their career through a given accrual factor and these will be applied to their average pensionable salary during a specified period (say, 1, or 3, or 5, or 10 years immediately prior to retirement or termination).[29] In a more complex case, the average pensionable salary is calculated over the years of highest compensation (rather than the last years prior to retirement), which might not need to be consecutive. Another variant is to average the

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[27]But this may not need the case when the plan has been reformed or when different career streams are given different accrual factors.

[28]For simplicity, we ignore here the possibility that pensionable salary can be different from total remuneration and that years of credited service can be deferent from years of service.

[29]For instance, with a 2 percent accrual factor, an average salary during the final 5 years of US$100,000, and 40 years of service, the retired member would receive a pension worth US$80,000.

highest compensation over *n* years in the final *m* > *n* years of employment (rather than over the full career). If the average is calculated over the last five years immediately prior retirement or termination, the accrued benefit function is given by:

$$B_{x,y} = \sum_{s=y}^{x-1} b_{x,y} = b(x-y)\frac{\sum_{s=x-5}^{x-1} w_{s,y}}{5} = b(x-y)\overline{w_{x,y:5}} \tag{17}$$

where $\overline{w_{x,y:5}}$ is the final five years average salary.

C. Actuarial Cost Methods

65. The calculation of liabilities for active members is more complex than the calculations for retired members. The reason is that active members are expected to accrue additional benefits before retiring, which will depend on future service and, except in the case of a flat benefit plan, future salaries. Depending on which actuarial cost method is used, these projected benefits may or may not be taken into consideration.

66. *Actuarial cost methods* are used by the plan actuary to estimate liabilities for active members. *Actuarial cost methods* define how expected costs of a pension plan are allocated over the period of active membership. These methods (also known as "*funding methods*") generate an annual *normal cost* ($NC_{x,y}$) representing the present value of the benefits earned under the plan that have been allocated to that year.[30] $NC_{x,y}$ is for the year in respect of a member is given by:

$$NC^*_{x,y} = \left(b^*_{x,y}\right)\left({}_{r-x}p_x^{(T)}\right)\left(v^{r-x}\right)\ddot{a}_r \tag{18}$$

where $b^*_{x,y}$ is the benefit accrual allocated to the period between attained age *x* and attained age *x* + 1 for a member who joined the plan at age *y* and stemming from the use of a cost method yet to be defined (and therefore, marked by the asterisk "*"), ${}_{r-x}p_x^{(T)}$ is the composite probability that the individual aged *x* will survive until the normal retirement age *r*, v^{r-x} is the discount factor used to calculate the present value of the accrued benefits, and

[30]Unfortunately, things are not as simple as many other types of costs should be also considered when calculating the actuarial liability. Three are more common: supplemental costs, past service costs, and ancillary costs. The first type of cost is generated when the experience of the plan deviates from the assumed cost factors creating actuarial gains or losses. The second type of cost is generated when the plan recognizes liabilities for past service prior to the introduction of the plan (or an amendment of the plan that applies improvements to past service). The third is associated with ancillary benefits like death, disability, etc. (in addition to retirement/pension benefits). For sake of simplicity we will not discuss these types of costs and we consider only normal costs associated with retirement benefits based on the normal retirement age *r*.

$\ddot{a}_r$ is the present value at the normal retirement age *r* of the annuity provided by the plan (this can be any of the ones discussed in the previous section).

67. In general, the sum of the normal costs up to the attained age provides the *actuarial liability* under the given cost method. Hence, the actuarial liability for an individual aged *x* who joined the plan at age *y* can be defined as:

$$\begin{aligned} AL^*_{x,y} &= \sum_{s=y}^{x-1} NC^*_{s,y} = {}_{r-x}p_x^{(T)} v^{r-x} \ddot{a}_r \sum_{s=y}^{x-1} b^*_{s,y} = \\ &= \left(B^*_{x,y}\right)\left({}_{r-x}p_x^{(T)}\right)\left(v^{r-x}\right)\ddot{a}_r \end{aligned} \tag{19}$$

where $B^*_{x,y}$ is the value of accrued benefit up to the attained age *x* for a member who joined the plan at age *y* and stemming from the use of a cost method yet to be defined (and therefore, marked by the asterisk "*").

68. Two general categories of cost methods are more common: (i) *benefit allocation methods*; and (ii) *cost allocation methods*. The two methods are radically different and will produce very different values of actuarial liabilities for the same set of underlying data and assumptions as shown later. In general, benefit allocation methods define how to allocate the total benefits that will be earned by a person to the various years of service, and then value the annual allocations. Cost allocation methods value the cost of the total benefits that will be earned by a person, and then allocate the total cost among the various years of service. As a result, they produce different values of actuarial liabilities $AL^*_{x,y}$.

Benefit allocation methods

69. Benefit allocation methods allocate units of benefits to the various plan years. With these methods, the actuary would calculate the present value of accrued benefits that all plan members may become entitled to (if they survive) either due to past service (i.e., rendered up to the valuation date) or also adding future service (i.e., projected until retirement). Two types of benefit allocation methods described here: the unit credit method and the projected unit credit method.

70. The *unit credit method* considers only past service. The accrual function $b_{x,y}$ is determined by the benefit function/formula defined by the plan (typically, flat dollar). The accrued benefit function $B_{x,y}$ is simply the sum of the accruals between entry age *y* and attained age *x*. Finally, the actuarial liability is called *accrued benefit obligation* (*ABO*) which is equal to the present value of the benefits accrued up to the attained age *x*. Hence:

$$
\begin{aligned}
b_{x,y} &= b_{x,y} \\
B_{x,y} &= \sum_{s=y}^{x-1} b_{s,y} \\
ABO_{x,y} &= \left(B_{x,y}\right)_{r-x}p_x^{(T)} v^{r-x} \ddot{a}_r
\end{aligned} \tag{20}
$$

71. The *projected unit credit method* considers also future service until the normal retirement age. The accrual function $b_{x,y}$ is determined by applying the benefit function/formula defined by the plan (flat dollar, career average salary or final average salary). The accrued benefit function $B_{r,y}$ is simply the sum of the accruals between entry age *y* and the normal retirement age *r*. The method therefore produces an actuarial liability which is equal to the present value of the benefits that will be accrued at the normal retirement age evaluated with the accrued benefit function/formula defined by the plan. Given the definition we will call this liability *retirement benefit obligation* (*RBO*).[31] Hence:

$$
\begin{aligned}
b_{x,y} &= b_{x,y} \\
B_{x,y} &= \sum_{s=y}^{r-1} b_{s,y} = B_{r,y} \\
RBO_{x,y} &= \left(B_{r,y}\right)_{r-x}p_x^{(T)} v^{r-x} \ddot{a}_r
\end{aligned} \tag{21}
$$

72. The *RBO* method is not used in practice as it breaks any relationship between costs and service rendered. The *RBO* method (also known as *initial funding method*) forces the sponsor to reserve for the present value of all possible benefits accruable until retirement right at entry age *y*, independently of whether the plan member will remain on payroll or not until then. This goes against rational actuarial and accounting practice, which attempt to allocate pension costs in reasonable relationship to the services rendered. Hence, it is not used in practice. Nonetheless, it is a useful concept as we will see that all other accepted methods are essentially a weighted function of the *RBO* method.

73. For this reason, regulations allow to prorate the RBO obligation for funding purposes. Since the *RBO* liability is often much larger than the *ABO* liability and since it is not certain that workers will remain on payroll until retirement, regulations allow plan managers to prorate the *RBO* liability over time. What proration achieves is to allocate projected benefits equally to all years of service. Notice that accrued benefit functions are very flat during the early years of an individual career and very steep towards retirement. Prorating reduces the

[31] We owe this very intuitive *RBO* label to Milevsky (2006). This is also known as the present value of future benefits (PVFB). Notice, that the way we use "projected unit credit" method in this text to indicate the general class of methods that project benefits until the normal retirement age can be misleading in the US. In this country this term is generally used to indicate the constant dollar variation that we will discuss later.

amount of reserves that the sponsor has to allocate towards retirement by anticipating factors that are expected to increase the ultimate level of benefits and recognizing their cost throughout the working years. Proration of the *RBO* liability produces an actuarial liability called *projected benefit obligation* (*PBO*).

74. There are two benefit allocation proration methods generally used: the *constant dollar* and the *constant percent* methods. With the constant dollar pro rata method, the accrual factor $b_{x,y}$ is constant and defined as a fixed share of retirement benefits: i.e., $b_{x,y} = B_{r,y}/(r-y)$. Hence the accrued benefits $B_{x,y}$ at age *x* is a simply the product of the constant accrual benefit and the number of years between attained and entry age: i.e., $B_{x,y} = (x-y)B_{r,y}/(r-y)$. Hence:

$$\begin{aligned} b_{x,y} &= \frac{B_{r,y}}{(r-y)} \\ B_{x,y} &= \frac{(\mathrm{x}-\mathrm{y})}{(r-y)} B_{r,y} \\ PBOcd_{x,y} &= \frac{(\mathrm{x}-\mathrm{y})}{(r-y)} B_{r,y\,r-x}p_x^{(T)} v^{r-x} \ddot{a}_r = \frac{(\mathrm{x}-\mathrm{y})}{(r-y)} RBO_{x,y} \end{aligned} \tag{22}$$

with the constant percent pro rata method, the accrual factor $b_{x,y}$ is a fraction of accrued benefits at retirement and the fraction is defined as a constant percent the individual salary. Hence:

$$\begin{aligned} b_{x,y} &= \frac{w_{x,y}}{W_{r,y}} B_{r,y} \\ B_{x,y} &= \frac{W_{x,y}}{W_{r,y}} B_{r,y} \\ PBOcp_{x,y} &= \frac{W_{x,y}}{W_{r,y}} B_{r,y\,r-x}p_x^{(T)} v^{r-x} \ddot{a}_r = \frac{W_{x,y}}{W_{r,y}} RBO_{x,y} \end{aligned} \tag{23}$$

Cost allocation methods

75. Cost allocation methods are fundamentally different from, and more complex than, benefit allocation methods. Rather than assigning units of benefits to specific years of service, cost allocation methods take the total costs of the benefits as the amount to be allocated to the various years of service. Their derivation is also a little more complex than the benefit allocation methods.

76. Two main variants exist: the *constant dollar method* or the *constant percent method*. The constant dollar method allocates a constant share of the cost of the *RBO* liability while the constant percent method allocates a fixed percent of the employee's salary to each year of past service starting from the entry age *y* until the attained age *x*.

77. Both methods use the present value at entry age *y* of a *temporary annuity.* A temporary annuity is simply an annuity provided for a fixed period of time or until death, whichever comes first.[32] For our purposes we need two types of temporary annuities: a *constant benefit annuity* for the constant dollar method and a *salary based annuity* for the constant percent method.

The present value of a *constant benefit temporary annuity* is nothing more than the present value at entry age *y* of a temporary annuity that pays a constant (one) unit of benefit ($B = 1$) until the normal retirement age *r* and it is given by:

$$\ddot{a}^{T}_{y:\overline{r-y}|} = \sum_{s=y}^{r-1} {}_{s-y}p_y^{(T)} v^{s-y} \tag{24}$$

where $\overline{r-y}|$ indicates the total number of periods that the annuity can be paid and the superscript *T* indicates that we are in a multiple decrement world.

The present value of a *constant salary based temporary annuity* is nothing more than the present value at entry age *y* of a temporary annuity that pays multiples w_s/w_y of the entry age salary until the normal retirement age *r* and it is given by:

$$^{w}\ddot{a}^{T}_{y:\overline{r-y}|} = \sum_{s=y}^{r-1} \frac{w_s}{w_y} {}_{s-y}p_y^{(T)} v^{s-y} \tag{25}$$

The fact that the annuity is evaluated at entry age explains why these cost methods are also called *entry age cost allocation methods*.

78. The *constant dollar cost allocation method* (*EAOcd*) allocates costs on the basis of the constant dollar period certain annuity. In this case, the accrual function $b_{x,y}$ is a fraction of the final *RBO* liability where such fraction is given by the ratio of the present value at entry age *y* of one unit of benefits ($B = 1$) paid at the attained age *x* through a temporary annuity over the present value at entry age *y* of a temporary annuity paying one unit of benefits ($B = 1$) between the entry age *y* and the normal retirement age *r*. Hence:

[32]Hence, it is different from a *period certain annuity* which pays the annuity for a fixed period no matter what.

$$b_{x,y} = \frac{{}_{x-y}p_y^{(T)} v^{x-y}}{\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y}$$

$$B_{x,y} = \sum_{s=y}^{x-1} b_{x,y} = \sum_{s=y}^{x-1} \frac{{}_{s-y}p_y^{(T)} v^{x-y}}{\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y} = \frac{\ddot{a}^T_{y:\overline{x-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y} \tag{26}$$

$$EAOcd_{x,y} = \frac{\ddot{a}^T_{y:\overline{x-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y\,r-x}p_x^{(T)} v^{r-x} \ddot{a}_r = \frac{\ddot{a}^T_{y:\overline{x-y}|}}{\ddot{a}^T_{y:\overline{r-y}|}} RBO_{x,y}$$

79. The *constant percent cost allocation method* (*EAOcp*) allocates costs on the basis of the salary based period certain annuity. In this case, the accrual function $b_{x,y}$ is a fraction of the final *RBO* liability where such fraction is given by the ratio of the present value at entry age *y* of salary multiples w_x/w_y paid at the attained age *x* through a temporary annuity over the present value at entry age *y* of a temporary annuity paying salary multiples w_x/w_y between the entry age *y* and the normal retirement age *r*. Hence:

$$b_{x,y} = \frac{\frac{w_{x,y}}{w_{y,y}}\,{}_{x-y}p_y^{(T)} v^{x-y}}{\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y}$$

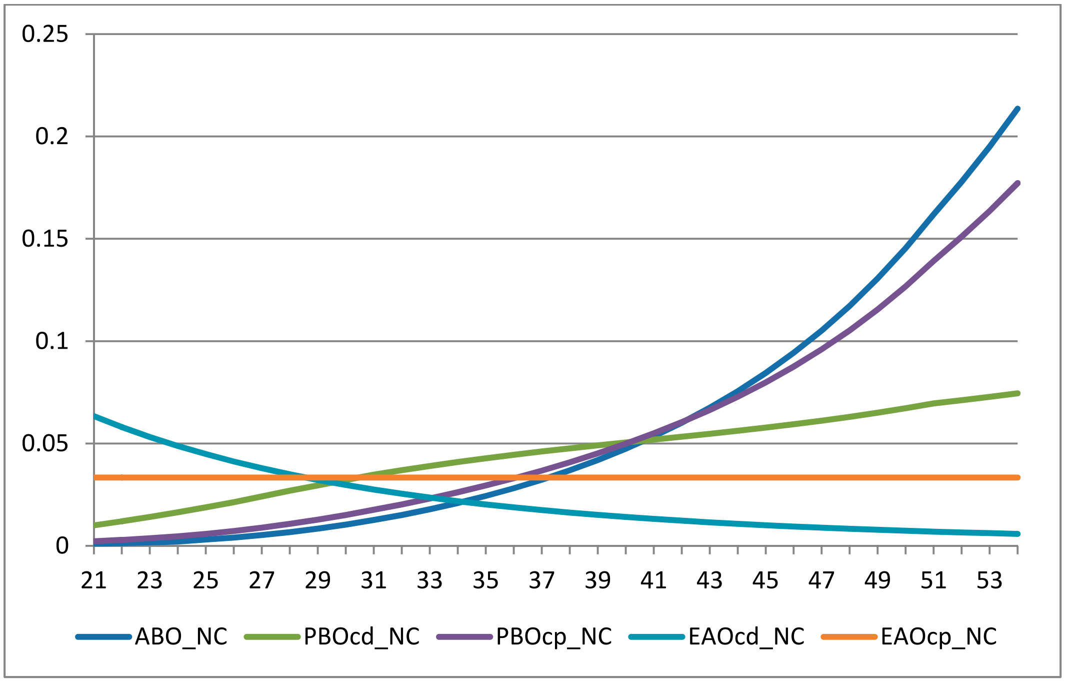
$$B_{x,y} = \sum_{s=y}^{x-1} b_{x,y} = \sum_{s=y}^{x-1} \frac{\frac{w_{x,y}}{w_{y,y}}\,{}_{x-y}p_y^{(T)} v^{x-y}}{{}^{w}\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y} = \frac{{}^{w}\ddot{a}^T_{y:\overline{x-y}|}}{{}^{w}\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y} \tag{27}$$

$$EAOcp_{x,y} = \frac{{}^{w}\ddot{a}^T_{y:\overline{x-y}|}}{{}^{w}\ddot{a}^T_{y:\overline{r-y}|}} B_{r,y\,r-x}p_x^{(T)} v^{r-x} \ddot{a}_r = \frac{{}^{w}\ddot{a}^T_{y:\overline{x-y}|}}{{}^{w}\ddot{a}^T_{y:\overline{r-y}|}} RBO_{x,y}$$

D. Comparing Benefit and Cost Allocation Methods

80. Alternative costs methods can produce vastly different individual normal costs. As it should now be obvious, individual costs methods amortize the *RBO* obligation of a member at entry age ($RBO_{r,y}$) over the members working life. The choice of methods will produce different patterns of normal costs. One the one hand, benefit allocation methods will follow more closely the accrual function with normal costs increasing over time, potentially becoming as steep as the accrual function. On the other hand, cost allocation methods, can produce normal costs that are more stable over time and sometimes, decreasing over time.

Figure 3. Individual Normal Costs – Percentage of Salary 1/



1/ Calculations assume: y = 20, r = 55, pr = 0.01, π = 0.035, b = 0.01, five year average final salary formula, nominal straight life annuity, mortality rates defined in Table A1, termination rates defined in Table A2, merit scale defined in Table A4, and discount rate defined in Table A6.

81. This can be seen in Figure 3 where normal costs for the different cost methods discussed in this section are reported as a share of wage at the attained age. The ABO normal costs increases exponentially over time implying that the sponsor needs to transfer to the pension fund an exponentially increasing share of an individual's salary. Increasing cost patterns are also characteristic of all benefit allocation methods while cost allocation methods produce more stable cost patterns. Indeed an entry age cost allocation method prorated as a percentage of salary ($EAOcp_{x,y}$) produces by definition a stable cost pattern over time. Finally, Table 3 reports the accrued benefit functions for these methods to show the source of the difference in normal costs.

Table 3. Accrued Benefits of Main Actuarial Cost Methods 1/

Benefit Allocation Methods	
$B^*_{x,y} = B_{x,y}$	Accrued Benefit Obligation (*ABO*)
$B^*_{x,y} = \frac{(x-y)}{(r-y)} B_{r,y}$	Projected Benefit Obligation Prorated – Constant Dollar (*PBOcd*)
$B^*_{x,y} = \frac{W_{x,y}}{W_{r,y}} B_{r,y}$	Projected Benefit Obligation Prorated – Constant Percent (*PBOcp*)
Cost Allocation Methods	
$B^*_{x,y} = \frac{\ddot{a}^T_{y:\overline{x-y}\rvert}}{\ddot{a}^T_{y:\overline{r-y}\rvert}} B_{r,y}$	Entry Age Obligation Prorated – Constant Dollar (*EAOcd*)
$B^*_{x,y} = \frac{{}^{w}\ddot{a}^T_{y:\overline{x-y}\rvert}}{{}^{w}\ddot{a}^T_{y:\overline{r-y}\rvert}} B_{r,y}$	Entry Age Obligation Prorated – Constant Percent (*EAOcp*)
Other Methods /1	
$B^*_{x,y} = 0 \; \forall \, x < r$ $B^*_{x,y} = B_{r,y} \; x = r$	Terminal Funding (*TER*)
$B^*_{x,y} = B_{r,y}$	Retirement Benefit Obligation (*RBO*) (InitiEfunding)

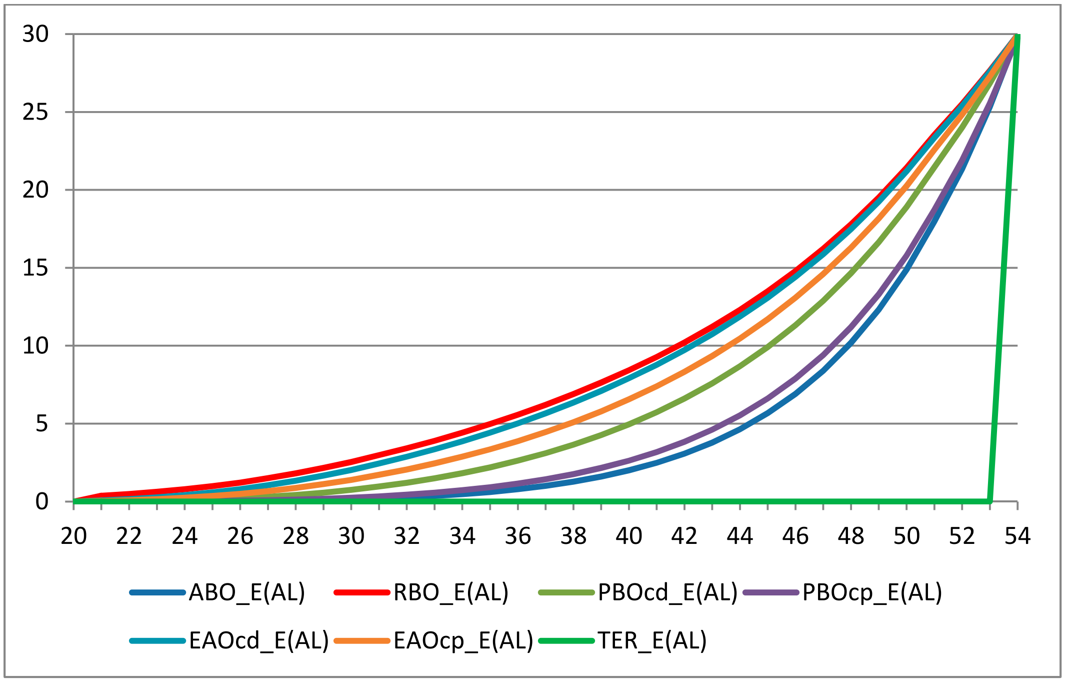
1/ Not used in practice but reported here as theoretical polar cases of all other methods.

82. Aggregate cost patterns are independent of the individual cost patterns. Notice that *aggregate cost patterns* will not follow *individual cost patterns* as they depend on the composition of the plan membership. If the average age and average salary of the active members does not increase over time, the aggregate cost will be stable over time. This means that for a plan with stable population, the choice of the cost method is in the end dictated by the value of liability it produces, as well as the philosophy of the sponsor regarding prefunding, the type of benefit formula, tax restrictions, regulatory requirements, and professional standards.

83. The difference in normal costs across methods naturally produces different values of individual actuarial liabilities. Since different cost methods amortize the *RBO* liability in different ways, they will of course produce very different values of actuarial liabilities (Figure 4). The lowest value is produced by the *ABO* method, which does not recognize expected future benefit accrual. The highest is produced by the *RBO* method, which fully recognizes accrued (past) and accruable (future) benefits. All prorated methods (whether benefit or cost allocation methods) produce intermediate values of actuarial liabilities. In Figure 4 we also added a new type of actuarial liability that is often called *terminal funding* method (*TER*). With this method, the sponsor reserves for the accrued liability only at the normal retirement age, when the liability is due with probability one. This method is not

allowed by regulation as it amounts to a plan with zero reserves for active members all the time (this would be a pay-as-you-go plan).[33]

Figure 4. Individual Actuarial Liabilities – Various Cost Methods 1/



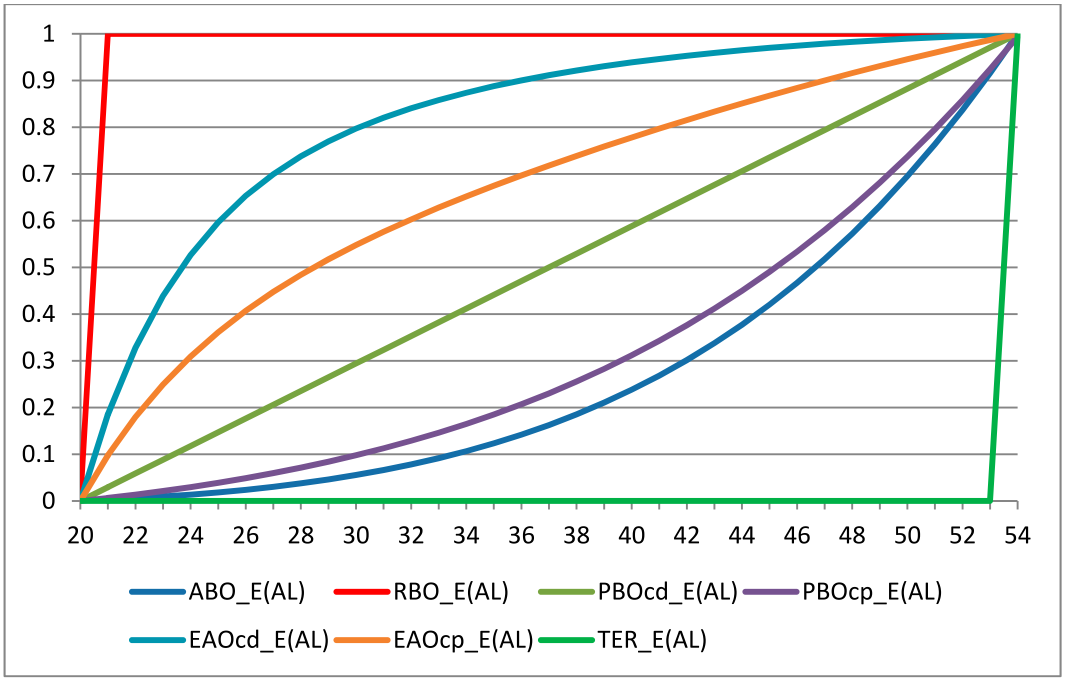
1/ Calculations assume: y = 20, r = 55, pr = 0.01, π = 0.035, b = 0.01, five year average final salary formula, nominal straight life annuity, mortality rates defined in Table A1, termination rates defined in Table A2, merit scale defined in Table A4, and discount rate defined in Table A6.

84. All methods produce an individual actuarial liability that is a fraction of the *RBO* obligation. It should now be clear that an infinite number of cost methods can be designed having at the two extremes: the *TER* and the *RBO* methods. The *TER* method hides the costs until retirement; it might be efficient for the sponsor (if liquid assets are available when obligation is due) but it is highly risky for the plan member (as no collateral is set aside for the pension promise). The opposite is true for the RBO method. All other methods produce an actuarial liability that is a share of the *RBO* liability. Figure 5 reports the level of the individual actuarial liabilities produced by the various methods discussed as a share of the *RBO* liability. Notice how the *ABO* and *PBOcp* methods tend to backload reserving for the sponsor while *EAOcd* and *EAOcp* methods tend to front load reserving for the sponsor. Finally, the *PBOcd* method can be seen as an intermediate method between the *RBO* and the

[33] Accrual accounting, such as IFRS, does not allow for this regardless of when the liability might be funded.

TER methods as it produces an individual actuarial liability that is linearly increasing over time as a share of the *RBO* liability. Table 4 reports the same shares in a more formal way.

Figure 5. Actuarial Liabilities as a Share of RBO 1/



1/ Note: Calculations assume: y = 20, r = 55, pr = 0.01, π = 0.035, b = 0.01, five year average final salary formula, nominal straight life annuity, mortality rates defined in Table A1, termination rates defined in Table A2, merit scale defined in Table A4, and discount rate defined in Table A6.

E. From Individual to Aggregate Values

85. Aggregate liabilities for the whole plan can be calculated in two different ways. Under the *individual method*, the actuary calculates the individual liabilities and other functions (as done in this section) and then sums them across all different individual members. Under the *aggregate method*, no reference is made to individuals and values are calculated on an aggregate basis by averaging values across all individuals or specified cohorts. In the next section we will use an aggregate method by annual cohorts as this allows us to be much more parsimonious in terms of data requirement. The template can also use only average information on cohorts of more than one year.[34]

[34] An aggregate approach is acceptable if the data necessary for individual calculations is unavailable or if the purpose of the valuation does not require much precision (such as a stress test done as part of an FSAP). However, individual calculations are greatly preferable.

Table 4. Actuarial Liabilities as a Share of *RBO*

Benefit Allocation Methods	
$ABO_{x,y} = \frac{B_{x,y}}{B_{r,y}} RBO_{x,y}$	Accrued Benefit Obligation (*ABO*)
$PBOcd_{x,y} = \frac{x-y}{r-y} RBO_{x,y}$	Projected Benefit Obligation Prorated – Constant Dollar (*PBOcd*)
$PBOcp_{x,y} = \frac{W_{x,y}}{W_{r,y}} RBO_{x,y}$	Projected Benefit Obligation Prorated – Constant Percent (*PBOcp*)
Cost Allocation Methods	
$EAOcd_{x,y} = \frac{\ddot{a}^T_{y:\overline{x-y}\vert}}{\ddot{a}^T_{y:\overline{r-y}\vert}} RBO_{x,y}$	Entry Age Obligation Prorated – Constant Dollar (*EAOcd*)
$EAOcp_{x,y} = \frac{{}^{w}\ddot{a}^T_{y:\overline{x-y}\vert}}{{}^{w}\ddot{a}^T_{y:\overline{r-y}\vert}} RBO_{x,y}$	Entry Age Obligation Prorated – Constant Percent (*EAOcp*)
Other Methods /1	
$TER_{x,y} = 0 \; \forall \; x < r$ $TER_{x,y} = RBO_{x,y} \; x = r$	Terminal Funding (*TER*)
$RBO_{x,y} = RBO_{x,y}$	Retirement Benefit Obligation (*RBO*) (Initial Funding)

1/ Not used in practice but reported here as theoretical polar cases of all other methods.

V. Stress Test Methodology

86. This section refers to a real life plan to discuss a possible stress test methodology implemented with the accompanying template “Model.xls”. The model plan uses the definitions developed in the previous sections and the underlying data and assumptions are taken from a real DB plan. Even for a simple plan like the one chosen, several simplifications were made relative to what was discussed in earlier sections in order to make the template easy to use in an operational context.

A. The Real Pension Plan

87. The real plan has the following characteristics:

- **Benefits.** The plan provides for retirement, death, and disability benefits with various withdrawal options in the form of single gender specific and/or joint straight life real inflation indexed annuities or cash lump sums.

- **Retirement benefits.** Retirement benefits represent the largest liability of the plan and are based on a final 5 year average salary formula with an accrual rate of 1 percent. Entry age varies from a minimum of y = 20 to the normal retirement age

$r = 55$. Early retirement is allowed starting with age $x = 45$ and in a few cases delayed retirement was granted. We know that early retirement reductions are less than actuarially fair but no information is available on benefit accrual in the case of delayed retirement.

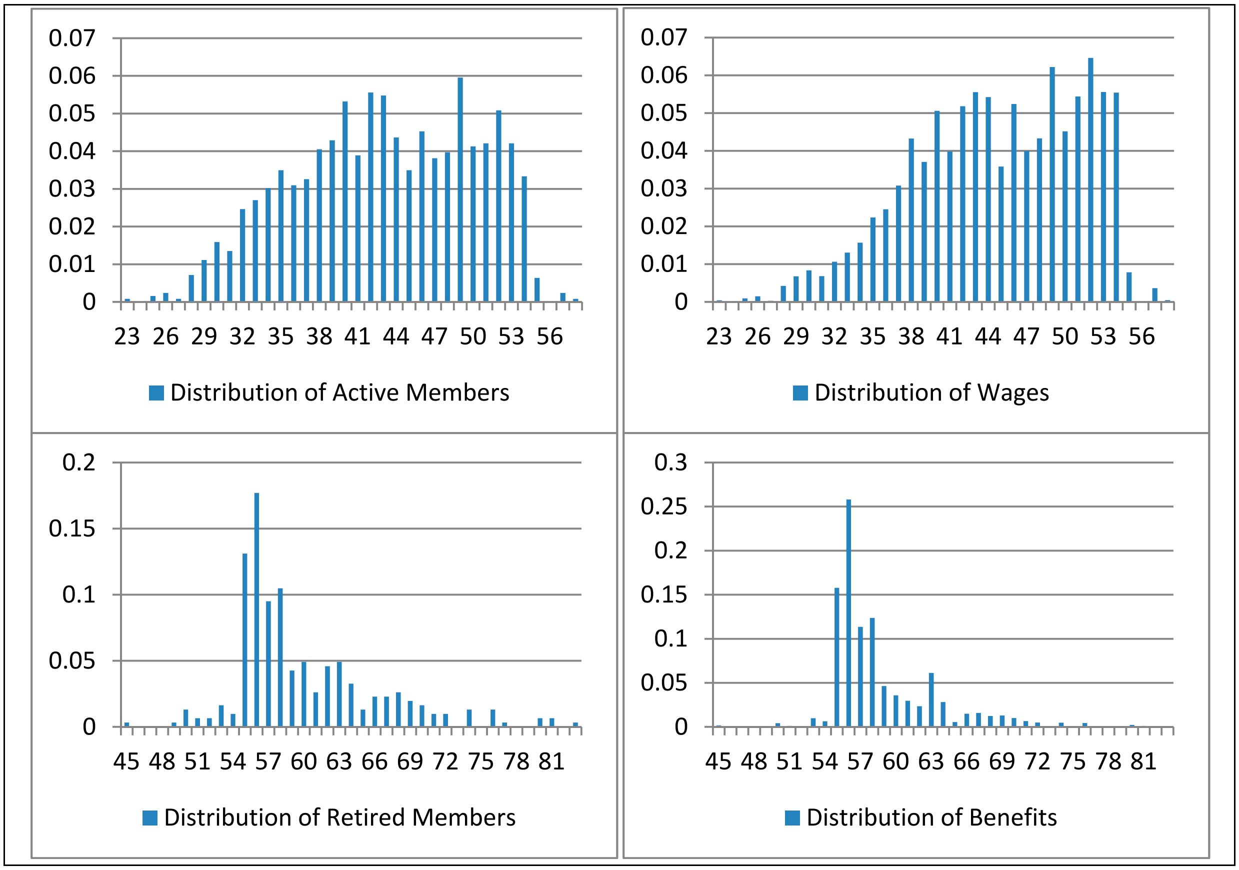
- **Assets and liabilities.** The investment portfolio is reported by asset classes and economic sectors, as well as domestic and foreign. Total assets amount to LCU3,773 million. The plan actuary uses a *PBOcd* actuarial method to calculate active members' individual actuarial liabilities. These are added up to yield the plan aggregate liability. The latest actuarial report shows liabilities amounting to LCU4,000 million.

- **Solvency regulation.** No minimum solvency margin is required in the jurisdiction. The scant rules issued simply require that a fund has assets in excess of liabilities to be considered "funded". The reported solvency ratio is 94 percent.

- **Actuarial cost factors.** The plan actuary uses the male mortality and termination rates defined in Table A8. In addition, we could obtain inflation, salary increase and discount rate assumptions reported in Table A7. We could not obtain information on other decrement factors as the actuarial report was not available.

- **Plan membership.** The relevant distributions are reported in Figure 6. The top left quadrant reports the density distribution of the number of active workers with two peaks around ages 42 and 52. The top right quadrant reports the density distribution of the wage remuneration of active workers with two peaks around ages 40 and 52. The bottom left and right quadrants report the density distributions of the number of retired workers and their pensions, with a large concentration soon after the normal retirement age of 55. Notice that the active distributions include individuals active beyond the normal retirement age and the retired distributions include individuals who retired before the normal retirement age. Actual data is reported in Table A8.

Figure 6. Active and Retired Member Distributions



B. The Model Pension Plan

88. On the basis of the information collected we constructed a model plan with the following characteristics:[35]

- **Benefits**. We consider only retirement benefits in the form of a single life inflation indexed annuity.

- **Retirement benefits.** Retirement benefits are calculated on the basis of a final salary formula with a constant accrual rate. Effective entry and retirement ages are assumed the same for all members and set at $y = 20$ and $r = 55$.

- **Assets and liabilities.** Regarding assets, we use investment portfolio data reported by the real plan. Regarding liabilities we use an aggregate (by annual cohorts) *PBOcd* method to calculate active member liabilities. Notice that we are not interested in the

[35]Appendix III contains a detailed description on how the accompanying template works. Table A7 and Table A8 summarize the actuarial cost factors used.

level of the actuarial liabilities, *per se*, but in their change for given shocks in the actuarial factors. Once the model plan's actuarial liabilities are estimated, they are rescaled to coincide with the liabilities calculated by the plan actuary. Once the shock is applied to the actuarial factors and the change in the model plan's actuarial liability is calculated, this same change is applied to the real plan's actuarial liability to calculate the change in the funding ratio. The error made by moving from the change in the model liability to the change in the plan liability is immaterial for our purpose.

- **Actuarial cost factors.** Since we could not obtain additional information on actuarial cost factors, we only use male mortality for all members and termination rates defined in Table A8. Wage projections are calculated using actual merit scale, inflation, and productivity assumptions reported in Table A7.

- **Plan membership.** We use real plan distributions reported in Figure 6. Due to the cutoff date imposed by our normal retirement assumption at age $r = 55$, early retirees are ignored in our calculations while late retirees are assumed to have retired at the normal retirement age: all members are assumed to have full career service.

C. Impact of Simplifications Made

89. Already in this very simple plan we had to make several simplifications, which do not reduce the usefulness of the accompanying template. The simplifications made might result in either an overestimation of real liabilities or an underestimation. Typically, however, they should have an ambiguous impact on liability estimation:

- Simplifications that imply an underestimation of true liabilities. We only consider pension benefits and disregard ancillary benefits like death and disability. We also ignore lump sum commutation which can result in large underestimation of retirees' liabilities.[36] We also consider only single life annuities and disregard joint life annuities. This underestimates the annuity factor for active members and therefore, produces a smaller valuation of their liabilities. Naturally, retired members' liabilities are also underestimated and such underestimation could be significant if many retired members have joint life annuities and we value them as single life annuities.

- Simplifications that imply an overestimation of true liabilities. We assume that all plan members join the plan at entry age y with immediate vesting. However, in reality some individuals would enter later and not necessarily with immediate vesting,

[36] In the US, for instance, the discount rate used to calculate lump sum commutation is neither the one used for financial reporting nor the one used for funding purposes. It is linked to the Fed rate and established year by year by the IRS. Such disconnection can result in very large unfunded liabilities, depending on the monetary policy stance.

therefore accruing smaller vested benefits by the time they terminate employment or retire.[37] In addition, we only consider final salary pensions that imply an overestimation of true liabilities relative to the more common case of career average salary pensions. Finally, we consider full indexation of pension rights to inflation and full longevity insurance but in many jurisdictions these indexations are more and more often conditional on the performance of the plan so that inflation, longevity and investment risk are shared between providers and retirees. Where these forms of risk sharing are present, our model provides an overestimation of liabilities.

- Simplifications that have an ambiguous impact. We assume that active members all retire at the normal retirement age of *r*. Depending on the degree of actuarial fairness of early retirement provisions, the model estimation may be higher or lower than the plan liabilities. We also assume that the pensionable salary (the base of the accrual rate) is equal to total remuneration. This overestimates (underestimates) liabilities when pensionable salary is in fact lower (higher) than total remuneration.

- Plan termination or continuation valuation. The choice of method (*PBOcd*) implies that we are conducting a plan continuation (ongoing concern) valuation as opposed to a plan termination valuation. This may differ from the regulatory requirements in the local jurisdictions (Box 1) where the template is used.

90. The simplifications used in our template do not affect the validity of the model for stress testing purposes. The simplifications made are immaterial for our purpose. Again, we are not interested in the absolute level of liabilities the estimation of which is best left to the plan actuary. Here, we are merely interested in their rate of change for given shocks. As it will be seen in this section, we will rescale our model liabilities to coincide with the true liabilities and apply the model rate of change stemming from the shock to the true liabilities to obtain the change in the funding ratio.

[37] Also, the calculations assume that all benefit payments are made annually, at the beginning of the year. On average, they will be made one-half year later.

Box 1. Continuation and Termination Valuation Methods

Within closed group methods there are two major subgroups (intermediate alternatives are also possible): *plan continuation methods* (also known as ongoing concern methods) and *plan termination methods*. Plan termination methods assume that the plan is terminated and that plan members do not accrue any more benefits. Plan continuation methods assume that the plan is terminated to new entrants only and that plan members continue accruing benefits under the (now closed) plan.

Our template uses a *PBOcd* valuation method and it is easy to define the continuation and termination liabilities for an individual. For retirees they are the same; again, the choice of method affects only liabilities for active members. Hence, for an individual of attained age $x \geq r$ that joined the plan at the entry age y the actuarial liability is given by the product between the accrued benefits at the normal retirement age r $\left(B_{r,y}\right)$ and the annuitization factor $(\ddot{a}_x^{\pi})$:

$$AL(R) = B_{r,y}\ddot{a}_x^{\pi}$$

For active members, continuation and termination method liabilities are different. The continuation liability for an individual of attained age $x \in [y, r)$ is given by the product of the service prorated share of the accrued benefits at the normal retirement age r $\left(\frac{x-y}{r-y}B_{r,y}\right)$, the contingent composite survival probability until retirement $\left({}_{r-x}p_x^{(T)}\right)$, the annuitization factor at retirement $(\ddot{a}_r^{\pi})$, and the discount rate (v^{r-x}) needed to calculate the present value at the valuation date:

$${}^{cont}AL(A) = \frac{x-y}{r-y}B_{r,y}\ {}_{r-x}p_x^{(T)}v^{r-x}\ \ddot{a}_r^{\pi}$$

The termination liability for an individual of attained age $x \in [y, r)$ is given by the product of the accrued benefits at the time of valuation $\left(B_{x,y}\right)$, the contingent simple survival probability until retirement $\left({}_{r-x}p_x^{(m)}\right)$, the annuitization factor at retirement $(\ddot{a}_r^{\pi})$, and the discount rate (v^{r-x}) needed to calculate the present value at the valuation date:

$${}^{term}AL(A) = B_{x,y}\ {}_{r-x}p_x^{(m)}v^{r-x}\ \ddot{a}_r^{\pi}$$

Notice that in the termination liability we assume no future benefit accrual. Therefore, we do not need wage projections, termination rates, or ancillary benefits (if provided). Hence, (in this specific case) the *PBOcd* method collapses to the *ABO* method. In general however, comparing the two valuation methods is not easy as actuarial assumptions used in either method are typically different. For instance, regulations (and logic) typically restrict the use of discount rates to an expected rate of return on assets in the case of plan continuation methods, and to the interest rate at the time of valuation to approximating the rate at which the plan sponsor could “sell” the liability to an insurance carrier, in the case of plan termination methods. For this reason, many jurisdictions require the use of both valuation methods for reporting prudential and reporting standards.

D. Stress Testing the Funding Position – Asset Shocks

91. This section uses the template “Model.xls” to conduct single factor stress test of the plan funding ratio. The plan is reasonably healthy, with a support ratio of 413 percent and a funding ratio of 94 percent. However, we observe that the plan is exposed to concentration risk in key asset classes and suspect that the assumptions used by the actuary to calculate

liabilities are overly optimistic. For instance, the level interest rate of 9 percent used for discounting cash flows appears excessively high in relation to market interest rates and the mortality table used is static and not dynamic. In addition, discussions with the actuary have revealed that the termination rates used have not been updated for many years: the economy is booming and the sponsor is retaining far more staff than what it used to when the termination rates were estimated. Finally, we have initial signs that the economy is overheating and inflation expectations are increasing. We conduct both asset and actuarial factor shocks to see how sensitive the funding ratio is to market risk (on the asset side) and to the actuarial assumptions used for liability valuation purposes (on the liability side).

92. Asset stress tests focus on concentration risk. We collected information on the investment portfolio of the model plan in sheet BAL. The data is disaggregated by type of issuer (government, financial sector, and real sector; both domestic and foreign) and by type of investment vehicle. The analysis can design any type of scenario (macro, credit, or other) that impacts on asset values which will translate into a change in the funding ratio. For our purposes we assume up to a 25 percent symmetrical shock in the value of assets, issued by the government, financial sector, real sector, in the value of bonds, stocks and foreign assets (amounting to a change in the exchange rate).[38] Results are reported in Table 5. Plan assets are fairly diversified, with marginally higher concentration in real sector investments, but almost equally divided among bonds and stocks. FX risk is unhedged and a 25 percent local currency appreciation (or a decrease in FX denominated assets by the same amount) would reduce the funding ratio from 94 percent to 88 percent.

Table 5. Model Plan – Stress Tests (Asset Concentration Risk)

Mkt. shock	Govt.	Financial	Real	Bonds	Stocks	FX	TOT
-25%	89.74	89.99	87.47	82.12	83.06	88.10	70.75
-20%	90.66	90.86	88.84	84.56	85.31	89.35	75.46
-15%	91.57	91.73	90.21	87.00	87.56	90.59	80.18
-10%	92.49	92.59	91.58	89.44	89.82	91.84	84.90
-5%	93.41	93.46	92.96	91.89	92.07	93.08	89.61
0%	**94.33**	**94.33**	**94.33**	**94.33**	**94.33**	**94.33**	**94.33**
+5%	95.25	95.20	95.70	96.77	96.58	95.57	99.05
+10%	96.17	96.06	97.07	99.21	98.84	96.82	103.76
+15%	97.08	96.93	98.44	101.65	101.09	98.07	108.48
+20%	98.00	97.80	99.82	104.10	103.35	99.31	113.19
+25%	98.92	98.67	101.19	106.54	105.60	100.56	117.91

[38] This is a pretty rough test. Flat percentage changes are more commonly used to test the effects of changes in equities and real estate, but for bonds and mortgages it would be more common to calculate or estimate the average duration of the portfolio and then test the effects of a given change in interest rates. For this exercise, information on duration of fixed income instruments was not available.

E. Stress Testing the Funding Position – Liability Shocks

93. Liability stress tests are more varied and the results can differ significantly depending on the specific circumstances of the pension plan. The critical issue about liabilities is that their valuation relies so critically on assumptions that may not be validated ex post by experience. The mismatch between assumptions and reality creates unfunded liabilities that might develop very slowly over time and become evident only when it is too late for the sponsor to easily – if at all – remedy the situation. Hence, adequate liability measurement becomes critical even in the short term.[39] We consider here interest rate, inflation, longevity, and termination rate shocks.

Interest rate shock

94. We assume two types of shocks: (i) we substitute the level 9 percent interest rate used for discounting future cash flows with more reasonable level rates; and (ii) we derive market discount rates from a AAA government debt yield curve, which we then shock. In general, changes in the discount rate will impact the present value of retirees' liabilities by changing v^s and $\ddot{a}_x^{\pi}$, in equation (29)[40] and the present value of active members' liabilities by changing v^{r-x} and $\ddot{a}_r^{\pi}$ in equation (33).[41] We expect liabilities for active members to be more sensitive to interest rate changes due to their higher duration: total liability duration is 15 years, versus 17 years for active members and 10 years for retired members.

95. Interest rate shocks have a large impact on liabilities. If the interest rate assumption decreases from 9 percent to 4 percent, the funding ratio decreases from 94 to 38 percent, or an average 11 percent for every percentage point change in the interest rate (Table 6).

Table 6. Model Plan – Stress Tests (Level Interest Rates)

f_s	9%	8%	7%	6%	5%	4%
FR	**94.33**	80.76	68.33	57.11	47.08	38.25

96. With AAA market rates, the funding ratio is much smaller than reported. When we derive a market discount rate from a AAA government debt yield curve (in our case, the end

[39]According to a recent survey, MetLife (2010) finds that the five risk factors ranked highest in importance in the UK were Measurement of Technical Provisions/Liabilities, Longevity Risk, Employer Covenant, Investment Management Style and Funding Deficits, whereas in the US the "most important" risk factors were Liability Measurement, Underfunding of Liabilities, Plan Governance, Asset Allocation and Advisor Risk.

[40]Equation reported in Appendix III.

[41]Equation reported in Appendix III.

of 2009 US domestic debt curve[42]), the funding ratio is around 33 percent (Table 7). This dramatic change in the solvency position underlines the importance of having a reasonable discount factor to value liabilities. The plan actuary has assumed that a 9 percent average rate of return on assets yields a 94 percent funding ratio. However, the risk free rate at 30 years is around 4 percent and even if the equity risk premium is realized over the full duration of liabilities, the plan will never be able to meet its obligation. Finally, if we assume parallel shifts of the yield curve of up to 150 basis points[43] and derive corresponding discount rate assumptions, the plan termination funding ratio varies between 26 and 39 percent (Table 7).

Table 7. Model Plan – Stress Tests (Shifts in the Yield Curve)

Shock	-150bps	-100bps	-50bps	0	+50bps	+100bps	+150bps
FR	26.52	28.51	30.59	**32.70**	34.86	37.04	39.24

Inflation shock

97. Inflation shocks impact the values of annuities and wage projections. Inflation impacts $(1+\pi^a)$ in equation (29)[44] and therefore the annuitization factor $\ddot{a}_r^\pi$ in equation (33).[45] In addition, it impacts $(1+\pi^w)$ in equation (31)[46] and therefore, the value of accrued benefits $B_{r,x}$ in equation (33).[47] Table 8 reports the sensitivity of the funding ratios to changes in inflation assumptions used to project wages and to calculate the annuity factor. A 100bps increase in the inflation assumption for annuity valuation π^a reduces the funding ratio from 94 percent to 85 percent. A 100bps increase in the inflation assumption for wage projections π^w reduces the funding ratio from 94 to 89 percent. A 100bps increase in both inflation assumptions reduces the funding ratio from 94 to 80 percent.[48]

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[42] This is done for illustrative purposes only: the relevant yield curve for the jurisdiction where the template is applied should be used.

[43] Negative shifts are not strictly parallel at shorter maturities as yield rates cannot become negative.

[44] Equation reported in Appendix III.

[45] Equation reported in Appendix III.

[46] Equation reported in Appendix III.

[47] Equation reported in Appendix III.

[48] In principle there is no reason why we should have two different inflation assumptions for wage projections and annuity. The only reason why this is done here is that, typically, plans do not provide full inflation indexation. By disconnecting inflation assumptions for wages and annuities, we are able to test the impact of alternative inflation annuity guarantees, while maintaining full inflation indexation in wage projections.

Table 8. Model Plan – Stress Tests (Inflation Shocks)

	-150bps	-100bps	-50bps	π^a	+50bps	+100bps	+150bps	ε_{FR,π^a}
-150bps	118.44	112.85	107.36	101.97	96.70	91.53	86.49	-10.24
-100bps	115.45	110.01	104.66	99.40	94.26	89.22	84.31	-10.24
-50bps	112.49	107.19	101.97	96.86	91.84	86.93	82.14	-10.24
π^w	109.56	104.39	99.31	**94.33**	89.44	84.67	80.00	-10.25
+50bps	106.66	101.63	96.68	91.83	87.07	82.42	77.87	-10.25
+100bps	103.78	98.88	94.07	89.35	84.72	80.19	75.77	-10.25
+150bps	100.94	96.17	91.49	86.90	82.39	77.99	73.69	-10.25
ε_{FR,π^w}	5.28	5.28	5.28	5.28	5.28	5.28	5.28	

98. The funding ratio is more sensitive to inflation assumptions used for the projection of increases in pension payments than for wage projections. Notice that the elasticity of the funding ratio, with respect to the inflation assumption for annuity valuation ε_{FR,π^a}, is much larger than the elasticity with respect to the inflation assumption used for wage projections ε_{FR,π^w}. This is due to the fact that π^a increases both $\ddot{a}_r^{\pi}$ and $B_{r,x}$ in equation (33).[49]

Longevity shock

99. We model longevity shocks by projecting mortality improvements over a number of years. In order to capture longevity improvements, not captured by the static table used by the plan actuary, we project over a number of improvement years *t* the period mortality rates. The projected mortality rate in calendar year *t*, ${}^{PRJ}_{t}q'^{(m)}_{x}$, is the rate from the static table multiplied by $(1-r_x)^t$. Thus:

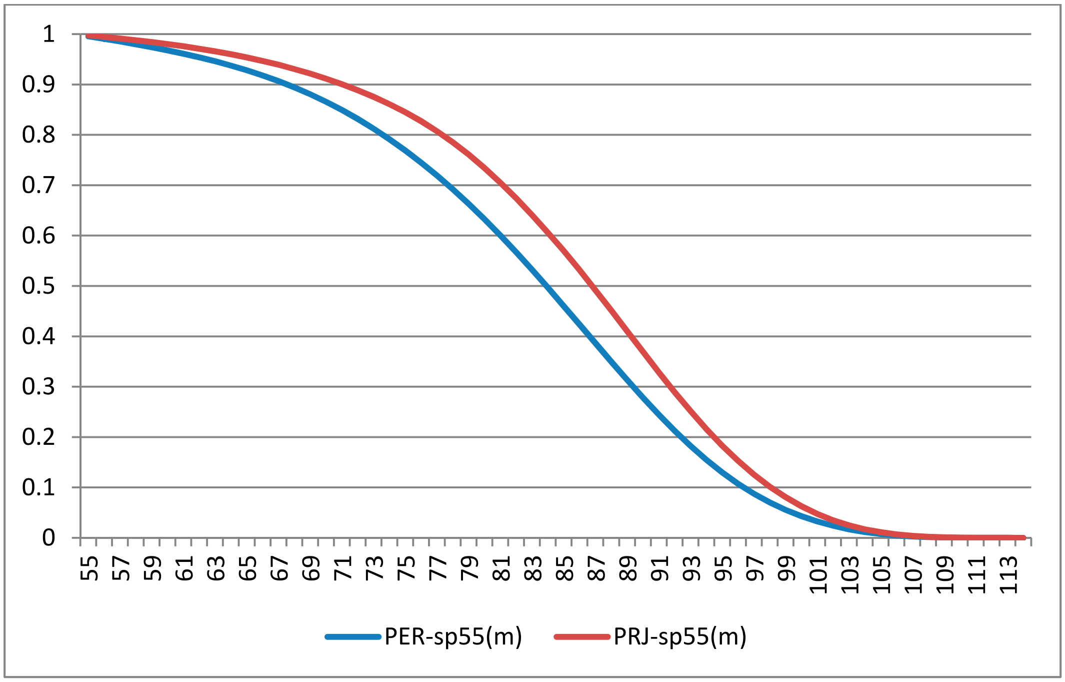
$$ {}^{PRJ}_{t}q'^{(m)}_{x} = {}^{PER}q'^{(m)}_{x}(1-r_x)^t \tag{28}$$

where r_x represents annual rates of mortality the multiplicative improvement in longevity for each cohort x. As an example, Figure 7 reports the change in the conditional survival probabilities for an individual aged 55 applying the US mortality improvement rates used for the 1994 Group Annuity Reserving Table, and projecting the static rates for $t = 30$ years.[50] The area between the two curves is the change in life expectancy at age 55 ($\Delta e_{55} = 2.70$).

[49]Equation reported in Appendix III.

[50]These rates are as good as any, and in any case, they should be discussed with the local actuary and amended as needed.

Figure 7. Impact of Longevity Improvements on Survival Probabilities



100. Longevity improvements translate into higher in life expectancy at different ages. For given r_x, an increasing number of improvement years translate into higher values of life expectancy. Table 9 reports the increase in life expectancy at the plan normal retirement age of 55 (e_{55}) from 28.39 to 33.96 years, and the related decrease in the funding ration between 94.33 to 85.26 for increasing number of improvement years between 0 and 70.

Table 9. Model Plan – Stress Tests (Longevity Shocks)

t	0	30	40	50	60	70
e_{55}	**28.39**	31.09	31.88	32.62	33.31	33.96
FR	**94.33**	89.46	88.21	87.10	86.13	85.26

Termination rate shock

101. Changes in termination rate assumptions will only affect liabilities of active members. A recent report of the personnel department concludes that retention rates have increased by at least 10 percent across the board. On the basis of this information, we investigate the impact on the funding ratio of decreases in termination rates of between 10 and 30 percent. Results are reported in Table 10.

Table 10. Model Plan – Stress Tests (Termination Shocks)

$q_x^{(t)}$	-0%	-10%	-15%	-20%	-25%	-30%
FR	**94.33**	92.18	91.11	90.04	88.97	87.91

VI. Conclusions and Other Considerations

102. This paper described the basic mechanics of DB plan liability valuation and how to conduct simple stress tests of the solvency ratio, using the accompanying Excel template. We reviewed the essential building blocks of liability valuation of DB plans. We then constructed an Excel template to analyze changes in funding ratio of DB plans for alternative values of actuarial factors and asset values. The accompanying template "Model.xls" uses a last salary DB formula and a projected benefit obligation constant dollar (*PBOcd*) actuarial method to value liabilities. The template is by no means a substitute for a proper actuarial evaluation, but the simplifications introduced do not affect its usefulness to evaluate sensitivity of the funding ratio.

103. Section V provides an example of the stress tests that can be conducted with the template and extensions are of course possible. The stress test methodology used in this note is parsimonious and aimed at quantifying key risk exposures by non actuarial analysts. As already mentioned, it is not a substitute for a full actuarial evaluation. However, it can be extended in several ways:

- **Refinements.** On the asset side, it would be worthwhile to extend the analysis to identify sources of risk stemming from interest rate shocks at various maturities, and credit risk shocks of large exposure, or of the sponsor (ability to pay contributions), so as to improve the connection of the stress tests with macro scenarios. On the liability side, the methodology could be refined by: (i) improving the granularity of the age, wage and pension distributions; (ii) considering additional "decrement factors" beyond the mortality tables such as the distribution for entry into and exit from the labor force (retirement, disability, voluntary unemployment, et cetera); (iii) reflecting gender and types of pensions in the calculations; and (iv) also considering the possibility of decreases in longevity due to health, famine or natural catastrophe events. Finally, other tests could be conducted to assess the impact of plan changes. Indeed the template lends itself to study various parametric reforms such as changes in accrual rate, retirement ages, any actuarial assumption, *et cetera*.

- **Liquidity shocks.** Data on asset liquidity was not used in the template. Asset shocks should also include tests on the portion of assets that might need to be used to cover short term liabilities. These types of shocks are potentially more severe than the ones considered, as they affect the ability of the plan to meet short term liabilities (rather than long term) and could force plan managers to sell assets at (potentially) distressed prices, further undermining the ability to meet long term liabilities. These shocks are very important for closed plans, with no active members accruing benefits or plans with very low support ratios.

- **Multi asset shocks.** Multi asset (factor) shocks have not been considered. This would require the estimation of asset classes return correlations.

- **Asset-liability correlations.** Asset shocks have been considered in isolation of liability shocks. When liabilities are discounted using a market yield curve, this becomes an unreasonable simplification. In such a case, the analyst should attempt to offset changes in liabilities, with changes in the value of the portion of assets which are interest rate sensitive. This of course would require knowing the durations of these assets.

- **Expected cash flow analysis.** The analysis conducted in section V is merely focused on changes in the funding ratio. An alternatively, potentially appealing way to present the same results is to analyze the shocks in terms of impact on future cash flows to see until when assets (always on a termination basis) are enough to meet liabilities. The template already produces expected liability cash flows, and it would be possible to project asset cash flows, by assuming future rates of returns on assets, dispositions of assets, and the allocation of future cash flows to different types of assets.

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Appendix I. Basic Pension Concepts

This appendix covers the basic pension definitions and the tax treatment of pension savings. The terms “plan”, “fund”, and “firm” are often used interchangeably, creating much confusion even among experts. OECD (2005) proposes a taxonomy that is gradually being adopted as standard and which provides some badly needed housekeeping in this area. In what follows, we summarize such taxonomy and discuss pension plans, pension funds, and pension firms in this order. At the end of this appendix we also provide a summary discussion of the tax treatment of pension savings.

Pension plans

A *pension plan*[51] is a legal contract having an explicit retirement objective. The contract may be part of a broader employment contract, it may be defined in the plan rules or documents by the *plan sponsor*, or it may be required by law.[52] The parameters of the pension plan (such as contribution rates, eventual guarantees, retirement age, types of benefits, et cetera) may be mandated by law, or statute, or defined in the plan rules or documents by the employer, or defined in special laws or regulations. These parameters are often pre-requisites for the plan to be able to obtain special tax treatment. Finally, pension plans may also offer additional benefits such as disability, sickness, and survivor benefits.[53]

Plans can be *public* or *private*. In public plans, the general government (that is central, state, and local governments, including social security institutions) administers the plan and its assets, and pays pension benefits. Social security and similar schemes are the typical examples of public plans. Their purpose is to provide minimum benefits at retirement (with or without longevity insurance) for the population at large (or at least the formal sector). In private plans, an entity other than general government administers the assets during the accumulation phase, and/or administers the payment of pension benefits. Typically, private pension plans are managed by the employer acting as the plan sponsor, a pension entity or a private sector provider; they may complement or substitute for social security schemes and, in most countries, they include plans for public sector or other special categories of workers.

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[51]The term “plan” is more common in North America while the term “scheme” is more common in the United Kingdom and many of its former colonies.

[52]Pension plans are typically sponsored by the employer, but not always. For example, negotiated multi-employer plans may be established by unions to cover their members. Hence, the more general term “plan sponsor”.

[53]Most pension plans also offer withdrawal benefits, which might be payable in one or a combination of a cash lump sum, an amount transferrable to another pension plan or individual retirement plan, or a deferred pension. The form of withdrawal benefit available to an individual can depend on the plan rules, legislation, and the circumstances of the individual (such as age and years of service at the date of termination).

Plans can be *occupational* or *individual*. Participation in occupational plans is linked to an employment relationship between the plan member and the entity that establishes the plan (the plan sponsor). Occupational plans may be established by a single employer or a group of employers (i.e., industry associations), sometimes in conjunction with labor associations (i.e., a trade union). Participation in individual or personal plans is not linked to an employment relationship, and under this category all sorts of products are marketed by the financial industry to capture voluntary savings while offering tax exemption.

Plans can be *mandatory* or *voluntary*. Mandatory plans are plans that have to be established by law, or to which participation of workers is mandatory. The typical examples of these types of plans are the traditional social security schemes or private sector mandatory plans in Latin America or Eastern Europe. However, private pension plans can be voluntary for the employer but mandatory for the employee, once the employer has decided to sponsor one.

Plans can be *defined benefit* (DB), *defined contribution* (DC), or *hybrid*. This distinction is based on the nature of the pension promise. DB plans insure longevity risk: i.e., the risk of living longer than expected and outliving one's assets. In other words, DB plans give retirees a life annuity (not necessarily, but typically), the amount of which is typically a function of the pensionable salary[54] (which, itself, could be a function of the pre-retirement or some career average salary), an accrual factor (the rate at which pension benefits accumulate with the passage of time), and the number of years of contributions or work under the plan.[55] DC plans do not insure longevity risk, but typically, provide a cash balance at retirement, which is a function of the contribution paid into the plan and the investment returns generated by these assets over time. The key difference between a DB plan and a DC plan is that investment and longevity risks (and, sometimes, inflation risks) are typically borne by the sponsor in a DB plan, while they are typically borne by the worker in a DC plan.[56] Hybrid plans have basically a mixture of the two aforementioned types of financing mechanisms; in some jurisdictions (i.e., the US) these are considered DB plans.

Plans can be *funded* or *unfunded*. This distinction relates to the amount of assets available to meet future liabilities. By definition, DC plans are fully funded, since their liabilities amount to the current value of assets allocated to members. However, for DB plans and DC plans with guarantees, an actuary needs to calculate the value of assets and liabilities, which are the present value of future benefits. If the value of assets is higher (lower) than the value of

[54]But not always, as in the case of flat benefit plans.

[55]Annuities do not need to be fixed, but they can be linked to inflation or the average wage growth of the covered population, some other investment return index, or even the funding status (see next paragraph) of the plan (in Brazil, these last types of plans are called Variable Contribution (VC) plans).

[56]Sometimes plans provide for risk sharing mechanisms between the provider and the beneficiary. For simplicity, DC plans with financial or biometric guarantees are considered DB plans in what follows.

liabilities, DB plans are considered fully funded (partially funded).[57] Traditionally, many public plans have few or no assets and current contributions are used to pay current benefits, i.e., they are traditionally pay-as-you-go (PAYG) financed. However, some OECD countries have long had partial pre-funding of public pension liabilities, or have more recently replaced these plans by private pension plans. In Asia and many African countries, it is more common to find public plans (often called provident funds) with assets providing retirement benefits with little or no longevity insurance.

Pension funds

Pension funds are pools of savings accumulated during the working life of individuals. At any given point in time, they are the cumulative sum of the flow of the employer and employee contributions and investment income, net of the cumulative sum of benefits and administration expenses paid.

Autonomous pension funds are legally separated from the plan sponsor, taking the form of either a special purpose legal entity (a pension entity), or a separate account managed by a financial institution on behalf of the plan/fund members. Pension funds that support personal pension plans are, by definition, autonomous. Both in occupational and personal pension plans, the plan/fund members have a legal or beneficial right, or some other contractual claim, against the assets held in the autonomous pension fund representing the financial collateral of their benefit promise. These arrangements provide the highest degree of protection to the beneficiary from bankruptcy of the sponsor (especially when an independent custodian is involved).

Non-autonomous pension funds are not legally separated from the plan sponsor but stay on its balance sheet. For instance, they may take the form of a reserve in the plan sponsor's balance sheet (as for the German Direktzsusage system), often disappearing to finance working capital. Alternatively, they may be held in legally separated vehicles, but are the property of the plan sponsor (in the form of financial reserves) and not of the beneficiary. In the case of non-autonomous pension funds, pension plan members have no legal claim on the pension fund assets. These arrangements provide the lowest degree of protection to the beneficiary from bankruptcy of the sponsor, since the sponsor can use pension assets to fund its business.[58]

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[57] The valuation of pension assets and liabilities is an extremely complex exercise function of often obscure accounting and actuarial standards that vary from country to country.

[58] Accounting rules, such as IFRS, might require the plan sponsor to report the liabilities and costs of such a pension plan in its financial statements. This can provide information useful for stress testing, even for a plan that has no separate financial statements of its own.

In many countries, pension products are directly sold by insurance companies. Insured pension funds are bought by the sponsors on behalf of their workers through group policies or directly by individuals. This situation excludes cases where an insurance firm acts as plan administrator or asset manager of a plan with an autonomous pension fund established by the sponsor. These insured pension fund arrangements provide protection to the beneficiary from bankruptcy of the sponsor, but expose the beneficiary to bankruptcy of the insurer, since assets are segregated from the balance sheet of the sponsor but merged with the assets of the insurer.[59]

There could also be single or multiple claimants on pension funds. Collective pension funds pool assets of pension plans of different plan sponsors. Group pension funds pool assets of unconnected individuals and/or companies in the same pension plan. Collective and group funds are commonly found in occupational corporate pension plans. Individual pension funds do not pool assets of multiple sponsors or beneficiaries, and are typically based on an individual account (with assets invested in units of pooled investment funds). Examples of these funds can be found in Latin America and Eastern Europe (in the form of occupational, mandatory funds) or in the United States (in the form of occupational, voluntary personal accounts, also widely known as 401(k) plans). Finally, pension funds can be closed or open, if they do or do not restrict membership to a specific group of individuals (like a given company workforce, professional association, or industry group). Open pension funds are found in Latin America and Eastern Europe, where funds compete for market share with other pension funds.

Pension entities

Four types of entities are typically involved in the governance structure of pension arrangements: the pension plan manager, the pension fund manager, the custodian, and external auditors.[60] The pension plan manager is often a special-purpose legal entity such as a trust, foundation, or a corporate entity that owns and may also control the pension fund on behalf of the pension plan/fund members. Plan members may have either a legal or a beneficial ownership right over the pension fund, or a contractual claim against the special purpose entity with respect to their rights to the pension fund assets. The typical functions of the plan manager are to collect contributions, maintain records, manage assets, and pay benefits. However, the plan manager can hire service providers for many of these services

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[59]The provision of retirement products by insurers is highly regulated in many jurisdictions for the purpose of consumer protection. Often, insurers need to keep assets in segregated accounts and pension beneficiaries are considered priority claimants in case of liquidation of the insurer.

[60]These do not need to be corporate entities: the plan manager might be a board made up of sponsor and member representatives. The fund might be managed by the same board. The auditor might be an individual practitioner.

and, often, some of these services are publicly provided. For instance, collection of contributions in mandatory DC plans in many Latin American and Eastern European countries is often centralized and executed by the tax authority, alongside the collection of labor income taxes.

Often, plan managers hire external pension fund managers to manage all or part of the assets in the fund. This is typically the case when the governing body develops an investment policy that includes complex mandates that need specialized asset management skills; like alternative investments, currency hedging, et cetera. Hiring an external asset manager can also be beneficial for shared mandates, as a way to benchmark in-house management skills, and provide sponsors or beneficiaries with higher risk-adjusted expected rates of return on assets. Many corporate DB plans are simply mangers of managers and limit their in house investments to treasury operations.

The custodian is a critical firm in the governance structure of pension fund arrangements, also for whistle blowing. Seldom is the case that allows the plan manager to provide custodial services. This is not good international practice as it undermines consumer protection. The role of the custodian is to process all trade settlement payments necessitated by the asset manager activity, to reconcile transactions, and hold in custody securities records for each manager. In addition, the custodian typically reports to the plan manager, its operations are subject of the internal audits, and it contributes to the valuation of assets and liabilities. In sophisticated jurisdictions, the valuation of the assets and liabilities of a pension fund might be performed every business day by the pension plan manager, on the basis of (i) information from the custodian bank for all completed transactions with the fund's assets of the previous business day; (ii) accounting record of the fund's liabilities and transactions, with its assets for the previous business day; (iii) information on the assets' market prices on the previous business day; and (iv) determination of a fair value for assets and liabilities which do not have a market price, by using applicable methods reviewed by the supervisory authorities.[61] Finally, the custodian is sometimes used by the supervisor as a whistle blower to signal serious breaches of contribution[62] or investment regulations.

External auditors can also be used as whistle blowers in various circumstances. Auditors review all financial statements, IT systems, internal controls, the reconciliation of values with the custodian and the accounting, and actuarial assumptions used by the plan manager. The pension supervisor often uses auditors as an essential tool to discharge its responsibilities, and to leverage on its scarce human capital resources. To this end, regulations typically (i) allow the supervisor to call the auditors for clarifications ,without need for approval of the

[61]Daily valuation is common for defined contribution plans, but certainly not for defined benefit plans.

[62]Such as the need to make contributions according to the valuation and plan provisions, within a specified deadline, such as seven days after month end.

pension plan's board or management; (ii) they grant access to the auditors' working papers; and (iii) require auditors to report serious breaches of regulation and prudential guidelines directly to the supervisor, especially in the case of suspected money-laundering activities.

Finally, pension firms can be *public* or *private*. Similarly to other legal entities, this depends on whether they are subject to public or private law.

Tax incentives

Savings through pension plans often come with tax advantages. It is important to give savings, through pension plan, an equitable and consistent tax treatment in order to promote long term savings accumulation for retirement. International experience and basic economic logic (see below) show that appropriate tax treatment is when savings are taxed only once. There are two alternative ways to achieve this. The first is to make contributions from income that has been subjected to income tax, and to exempt from income tax the investment income and the distribution of plan benefits – this is known as the Taxed-Exempt-Exempt (TEE) alternative. The second way is to exempt from income tax both contributions and investment income, while making plan benefits liable to income tax – this is known as the Exempt-Exempt-Taxed (EET) alternative.

Expenditure tax regimes are better at promoting long term savings than comprehensive income tax regimes. The two alternatives described above (TEE and EET) are *expenditure tax regimes,* where the post-tax rate of return is expected to equal in present value terms the pre-tax rate of return, and therefore, consumption is taxed at the same rate now and in the future. In contrast, TTE and ETT are *comprehensive income-tax regimes,* which tax income equally regardless of the source. These regimes (TTE and ETT) treat equally the different uses to which income may be put (saving is seen as just another commodity, like consumption), and hence, maintain neutrality between consumption and savings. An *expenditure tax regime* (TEE or EET), which maintains tax neutrality of consumption over time, is usually preferred since it avoids the double taxation of savings and encourages the accumulation of contractual savings for retirement purposes. Thus, EET or TEE are preferable to either TTE or ETT (or even worse TTT), and are prevalent as a best practice in several advanced economies.

The choice between the two alternative expenditure tax regimes (TEE, EET) is usually dictated by fiscal considerations. TEE and EET regimes are in general not equivalent as, other things being equal, taxation will be lower in EET than in TEE owing to tax deferral. The introduction of an expenditure tax regime for contractual savings has both a static and a dynamic impact on Government tax revenues. The static analysis of the impact on the tax revenue flow to the Government would require: (i) information on the amount of contributions made to the different retirement agencies and life insurance companies by employers and employees; (ii) the tax treatment of such contributions; (iii) the average investment return of the different retirement funds and life insurance companies; (iv) the tax treatment of such returns; and (v) estimates of the present value of the future streams of benefits disbursed to retirees and their tax treatments. Nevertheless, the dynamic impact could be substantially different from the static impact. Deferment itself (i.e., EET) means that more pre-tax income is available for current investment and accumulation, which is likely to translate into higher economic growth. If the economy is expected to grow at a substantial rate, and contributions are indexed to inflation, an EET scheme may actually provide a higher present value tax revenue income than a TEE scheme. Furthermore, the EET scheme is more credible than the alternative TEE. This is because the TEE scheme will always entail uncertainty as to whether Government would, in the future, tax benefits as well.

Appendix II. General Actuarial Assumptions

Table A1. USA Male Annuitant Mortality Rates (t887 – 1996)

x	$q_x^{(m)}$	x	$q_x^{(m)}$	x	$q_x^{(m)}$	x	$q_x^{(m)}$
0	0.002080	30	0.000694	60	0.006428	90	0.112208
1	0.000815	31	0.000699	61	0.006933	91	0.121402
2	0.000454	32	0.000700	62	0.007520	92	0.131017
3	0.000367	33	0.000701	63	0.008207	93	0.141030
4	0.000321	34	0.000702	64	0.009008	94	0.151422
5	0.000291	35	0.000704	65	0.009940	95	0.162179
6	0.000270	36	0.000719	66	0.011016	96	0.173279
7	0.000257	37	0.000749	67	0.012251	97	0.184706
8	0.000294	38	0.000796	68	0.013657	98	0.196946
9	0.000325	39	0.000864	69	0.015233	99	0.210484
10	0.000350	40	0.000953	70	0.016979	100	0.225806
11	0.000371	41	0.001065	71	0.018891	101	0.243398
12	0.000388	42	0.001201	72	0.020967	102	0.263745
13	0.000402	43	0.001362	73	0.023209	103	0.287334
14	0.000414	44	0.001547	74	0.025644	104	0.314649
15	0.000425	45	0.001752	75	0.028304	105	0.346177
16	0.000437	46	0.001974	76	0.031220	106	0.382403
17	0.000449	47	0.002211	77	0.034425	107	0.423813
18	0.000463	48	0.002460	78	0.037948	108	0.470893
19	0.000480	49	0.002721	79	0.041812	109	0.524128
20	0.000499	50	0.002994	80	0.046037	110	0.584004
21	0.000519	51	0.003279	81	0.050643	111	0.651007
22	0.000542	52	0.003576	82	0.055651	112	0.725622
23	0.000566	53	0.003884	83	0.061080	113	0.808336
24	0.000592	54	0.004203	84	0.066948	114	0.899633
25	0.000616	55	0.004534	85	0.073275	115	1.000000
26	0.000639	56	0.004876	86	0.080076		
27	0.000659	57	0.005228	87	0.087370		
28	0.000675	58	0.005593	88	0.095169		
29	0.000687	59	0.005988	89	0.103455		

Table A2. Select and Ultimate Termination Rates (Various Entry Ages)

x	$q'^{(t)}_{x,20}$	$q'^{(t)}_{x,25}$	$q'^{(t)}_{x,30}$	$q'^{(t)}_{x,35}$	$q'^{(t)}_{x,40}$	$q'^{(t)}_{x,45}$	$q'^{(t)}_{x,50}$	$q'^{(t)}_{x,55}$	$q'^{(t)}_{x,60}$
20	0.246913								
21	0.227261								
22	0.209647								
23	0.193147								
24	0.177861								
25	0.163588	0.214564							
26	0.154278	0.177513							
27	0.138179	0.152452							
28	0.126942	0.135648							
29	0.116617	0.122185							
30	0.107226	0.107226	0.172689						
31	0.098598	0.098598	0.141418						
32	0.097280	0.091158	0.117427						
33	0.083717	0.083717	0.097788						
34	0.077340	0.077340	0.082412						
35	0.071678	0.071678	0.071678	0.129676					
36	0.066693	0.066693	0.066693	0.125460					
37	0.062155	0.062155	0.062155	0.083860					
38	0.058273	0.058273	0.058273	0.069241					
39	0.054765	0.054765	0.054765	0.059328					
40	0.051830	0.051830	0.051830	0.051830	0.095359				
41	0.049299	0.049299	0.049299	0.049299	0.076237				
42	0.047173	0.047173	0.047173	0.047173	0.062358				
43	0.045351	0.045351	0.045351	0.045351	0.053247				
44	0.043833	0.043833	0.043833	0.043833	0.047173				
45	0.042618	0.042618	0.042618	0.042618	0.042618	0.069444			
46	0.041543	0.041543	0.041543	0.041543	0.041543	0.055373			
47	0.040694	0.040694	0.040694	0.046945	0.040694	0.046869			
48	0.039885	0.039885	0.039885	0.039885	0.039885	0.042517			
49	0.039277	0.039277	0.039277	0.039277	0.039277	0.040398			
50	0.038670	0.038670	0.038670	0.038670	0.038670	0.038670	0.054462		
51	0.038625	0.038625	0.038625	0.038625	0.038625	0.038625	0.046768		
52	0.037455	0.037455	0.037455	0.037455	0.037455	0.037455	0.042213		
53	0.036645	0.036645	0.036645	0.036645	0.036645	0.036645	0.039589		
54	0.035835	0.035835	0.035835	0.035835	0.035835	0.035835	0.037556		
55	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.034924	0.052843	
56	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.033796	0.042415	
57	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.032292	0.036342	
58	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.030571	0.032799	
59	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.028446	0.030653	
60	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.026117	0.056150
61	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.023283	0.034722
62	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.019942	0.026117
63	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.016197	0.021448
64	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.011945	0.012856

Table A3. Early Retirement Rates

x	$q'^{(r)}_x$
55	0.051
56	0.051
57	0.051
58	0.051
59	0.051
60	0.202
61	0.303
62	0.405
63	0.304
64	0.304
65	1.000

Table A4. Cumulative Wage Merit Scale – Multiples of Entry Age 20

x	*Scale*	x	*Scale*	x	*Scale*
20	1.0000	35	1.7582	50	2.4780
21	1.0456	36	1.8119	51	2.5154
22	1.0921	37	1.8645	52	2.5508
23	1.1397	38	1.9171	53	2.5842
24	1.1883	39	1.9698	54	2.6156
25	1.2369	40	2.0204	55	2.6440
26	1.2875	41	2.0720	56	2.6713
27	1.3381	42	2.1216	57	2.6946
28	1.3887	43	2.1712	58	2.7169
29	1.4414	44	2.2188	59	2.7361
30	1.4930	45	2.2654	60	2.7523
31	1.5456	46	2.3109	61	2.7665
32	1.5993	47	2.3555	62	2.7776
33	1.6519	48	2.3980	63	2.7857
34	1.7056	49	2.4395	64	2.7908

Table A5. Select and Ultimate Wage Function Assumptions

x	$wf_{x,20}$	$wf_{x,30}$	$wf_{x,40}$	$wf_{x,50}$	$wf_{x,60}$
20	1.0000				
21	1.0930				
22	1.1934				
23	1.3019				
24	1.4190				
25	1.5440				
26	1.6800				
27	1.8253				
28	1.9802				
29	2.1485				
30	2.3263	1.0000			
31	2.5176	1.0822			
32	2.7231	1.1706			
33	2.9403	1.2639			
34	3.1735	1.3641			
35	3.4198	1.4700			
36	3.6839	1.5836			
37	3.9629	1.7035			
38	4.2596	1.8310			
39	4.5750	1.9666			
40	4.9054	2.1086	1.0000		
41	5.2589	2.2606	1.0721		
42	5.6289	2.4197	1.1475		
43	6.0218	2.5885	1.2276		
44	6.4328	2.7652	1.3114		
45	6.8657	2.9513	1.3996		
46	7.3214	3.1471	1.4925		
47	7.8009	3.3533	1.5903		
48	8.3019	3.5686	1.6924		
49	8.8285	3.7950	1.7998		
50	9.3744	4.0297	1.9111	1.0000	
51	9.9477	4.2761	2.0279	1.0612	
52	10.5453	4.5330	2.1497	1.1249	
53	11.1679	4.8006	2.2767	1.1913	
54	11.8161	5.0793	2.4088	1.2605	
55	12.4858	5.3671	2.5453	1.3319	
56	13.1870	5.6685	2.6883	1.4067	
57	13.9052	5.9773	2.8347	1.4833	
58	14.6559	6.3000	2.9877	1.5634	
59	15.4290	6.6323	3.1453	1.6459	
60	16.2242	6.9741	3.3074	1.7307	1.0000
61	17.0473	7.3279	3.4752	1.8185	1.0507
62	17.8921	7.6911	3.6475	1.9086	1.1028
63	18.7581	8.0633	3.8240	2.0010	1.1562
64	19.6444	8.4443	4.0047	2.0955	1.2108

Table A6. Discount Rate Assumptions

period	y_s	$(1+f_s)$	v^n
0	0.00000	1.00000	1.00000
1	0.00466	1.00466	0.99536
2	0.00938	1.01413	0.98150
3	0.01370	1.02238	0.96002
4	0.01673	1.02590	0.93578
5	0.01977	1.03202	0.90674
6	0.02279	1.03801	0.87354
7	0.02581	1.04410	0.83664
8	0.02727	1.03759	0.80633
9	0.02874	1.04054	0.77491
10	0.03021	1.04349	0.74261
11	0.03066	1.03521	0.71735
12	0.03111	1.03612	0.69234
13	0.03157	1.03704	0.66762
14	0.03202	1.03795	0.64321
15	0.03248	1.03886	0.61915
16	0.03293	1.03977	0.59547
17	0.03339	1.04068	0.57219
18	0.03384	1.04159	0.54934
19	0.03429	1.04251	0.52695
20	0.03475	1.04342	0.50502
21	0.03520	1.04433	0.48358
22	0.03566	1.04524	0.46265
23	0.03611	1.04616	0.44224
24	0.03657	1.04707	0.42236
25	0.03702	1.04798	0.40302
26	0.03747	1.04890	0.38423
27	0.03793	1.04981	0.36600
28	0.03838	1.05072	0.34833
29	0.03884	1.05164	0.33123
30	0.03929	1.05255	0.31469
31	0.03929	1.03929	0.30279
32	0.03929	1.03929	0.29135
33	0.03929	1.03929	0.28033
34	0.03929	1.03929	0.26973
35	0.03929	1.03929	0.25954

Appendix III. The Accompanying Template (Model.xls) and Assumptions

The accompanying Model.xls template contains the following assumptions:

- **Basic Assumptions** (Table A7 below). Entry age is set at age $y = 20$ (Input!D12) and normal retirement age at $r = 55$ (Input!D13).

- **Decrement assumptions** (Table A8 below). Mortality rates ${q'}_x^{(m)}$ (Input!I11:I136) are derived from the 1996 US male annuitant table (t887). Termination rates ${q'}_x^{(t)}$ (Input!J11:J136) are assumed between age 20 and 54. Mortality and termination rates are used to calculate the mortality probabilities $q_x^{(m)}$, the termination probabilities $q_x^{(t)}$, and the composite survival probabilities $p_x^{(T)}$ following the methodology discussed in section III.[63]

- **Salary assumptions** (Table A7 and Table A8). Wage growth is set at about 7 percent and composed of 3 percent average merit increase (Input!T11:T136), 3.5 percent inflation ($\pi^w = 0.035$ in Input!D18), and 1 percent productivity improvements ($pr = 0.01$ in Input!D17).

- **Discount rate assumptions** (Table A8). A level 9 percent interest rate is assumed for the purpose of discounting liabilities. Nonetheless, the template allows for almost any type of discount curve derived from 7 easily obtainable yield-to-maturity rates (Input!O11:O136) following the methodology discussed in section III, equations (8) and (9).

- **Retirement benefit assumptions** (Table A7). Retirement benefits are calculated on the basis of a final salary formula with constant accrual rate ($b = 0.01$ in Input!D11). Effective entry and retirement ages are assumed the same for all members and set at $y = 20$ and $r = 55$. We consider only retirement benefits in the form of a single life inflation indexed annuity, with the inflation assumption defined as $\pi^a = 0.035$ in Input!D20.

- **Distributions of active and retired members** (Table A8). For each age cohort, we collected the number of workers (Input!V11:V136) and retirees (Input!W11:W136), as well as the cohort wage remuneration (Input!X11:X136) and cohort retirement benefit paid (Input!Y11:Y136). This data is used to calculate the total number of workers (Input!V6), retirees (Input!W6), remuneration (Input!X6), retirement benefits (Input!Y6) and their density distributions (Input!Z11:Z136 to Input!AC11:AC136).

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[63] $q_x^{(m)}$ and $q_x^{(t)}$ are calculated using equation (1) and rendered in Input!K11:K136 and Input!L11:L136, respectively. $p_x^{(T)}$ is calculated using equation (2) and rendered in Input!M11:M136.

Actuarial liabilities for retired cohorts

Actuarial liabilities for retired cohorts are calculated in sheet AL-R. This is done in two steps:

In the first step, we calculate for each cohort $x \in [r, \infty)$ (in row 'AL-R'!F8:CH8) the present value of \$1 real life annuity using equation (11). This is given by:

$$\ddot{a}_x^{\pi} = \sum_{s=0}^{\infty} (1+\pi^a)^s \; {}_s p_x^{(m)} v^s \tag{29}$$

where the inflation term structure $(1+\pi^a)^s$ is rendered in 'AL-R'!B11:B136, the discount factor curve v^s is derived using equation (8) and rendered in 'AL-R'!C11:C136, the conditional probabilities of survival ${}_s p_x^{(m)}$ are derived using equation (3) and rendered in matrix 'AL-R'!F11:CH136, and the present value $\ddot{a}_x^{\pi}$ is rendered in row 'AL-R'!F7:CH7.

In the second step, we aggregate the actuarial liabilities for all retired cohorts. This is given by:

$$AL(R) = \sum_{x=r}^{\infty} [(RN)(RB)(RNd_x)(RBd_x)]\ddot{a}_x^{\pi} \tag{30}$$

and rendered in 'AL-R'!D1.

Actuarial liabilities for active cohorts

Actuarial liabilities for active cohorts are calculated in sheets Brx and AL-A. Among the actuarial cost methods described in section IV, we use the projected unit credit, constant dollar, method (*PBOcd*). Again, there are two steps involved:

In the first step, we calculate in the Brx sheet the accrued benefit at retirement for each active cohort. This is done by first projecting wages until retirement for all active cohorts. These are needed since we are performing a projected benefit obligation constant dollar evaluation as discussed in section III, equation (6). Hence, in matrix Brx!F11:CH136, we project for each cohort $x \in [y, r)$ (in row Brx!F8:CH8) the future wages for each period $s \in [x, r)$ (in column Brx!A11:A136) as a multiple of today's wage using the formula:

$$wf_{s,x} = \frac{wf_{s,y}}{wf_{x,y}} = \frac{ms_{s,y}}{ms_{x,y}} [(1+\pi^w)(1+pr)]^{(s-x)} \tag{31}$$

We then calculate for each cohort in row Brx!F1:CH1 the total accrued benefits at retirement, using the plan final salary benefit formula and the distributions of active members and salaries:

$$B_{r,x} = b(r-y)wf_{r,x}[(AN)(AW)(ANd_x)(AWd_x)] \tag{32}$$

where $wf_{r,x} = max\left(wf_{s,x}\right)$.[64]

In the second step, we aggregate in the sheet AL-A the actuarial liabilities for all active cohorts. As discussed in section IV, these are given by the product of the present value of life annuity at retirement $\left({}_{r-x}p_x^{(T)} v^{r-x} \ddot{a}_r^{\pi}\right)$, the accrued benefit at retirement $\left(B_{r,x}\right)$, and the constant dollar method prorated $(x-y)/(r-y)$. The plan actuarial liabilities for active cohorts are nothing less than the sum of cohort liabilities across all active cohorts rendered in 'AL-A'!D1. Thus:

$$AL(A) = \sum_{x=y}^{r-1} \frac{(x-y)}{(r-y)} B_{r,x}\left({}_{r-x}p_x^{(T)} v^{r-x} \ddot{a}_r^{\pi}\right) \tag{33}$$

where ${}_{r-x}p_x^{(T)}$ is the conditional composite probability of survival between age *x* and age *r* rendered in matrix 'AL-A'!E11:BM136 and row 'AL-A'!E3:BM3, v^{r-x} is the discount rate for $(r-x)$ periods rendered in column 'AL-A'!C11:C136 and row 'AL-A'!F4:CH4, and $\ddot{a}_r^{\pi}$ is the present value of a real life annuity at the normal retirement age rendered in cell 'AL-R'!F7.

Finally, total model plan liabilities are calculated in sheet AL-TOT. These are nothing less than the sum of the actuarial liabilities for active and retired members: i.e., $AL(TOT) = AL(R) + AL(A)$.

[64]The template can be easily changed by substituting $max\left(wf_{s,x}\right)$ with $\sum_{\mathrm{j}=0}^{\mathrm{n}-1} \frac{wf_{r-j,x}}{\mathrm{n}}$ to implement a final *n* years' average salary formula.

Table A7. Actuarial Factors Used in the Model Plan (Cells)

Symbol	Value	Input Cells	Assumption
y	20	Input!D12	Entry Age
r	55	Input!D13	Normal Retirement Age
x_{max}	115	Input!D14	Max Age
pr	1.0%	Input!D17	Labor Productivity
π^{w}	3.5%	Input!D18	Inflation (wages)
π^{a}	3.5%	Input!D21	Inflation (annuities)
b	1.0%	Input!D22	Accrual Rate

Table A8. Actuarial Factors Used in the Model Plan (Series)

x	$q'^{(m)}_{x}$	$q'^{(t)}_{x}$	ytm_s	ms_x	r_x	AN_x	RN_x	AW_x	RB_x
0	0.002080		0.09		0.02				
1	0.000815		0.09		0.02				
2	0.000454		0.09		0.02				
3	0.000367		0.09		0.02				
4	0.000321		0.09		0.02				
5	0.000291		0.09		0.02				
6	0.000270		0.09		0.02				
7	0.000257		0.09		0.02				
8	0.000294		0.09		0.02				
9	0.000325		0.09		0.02				
10	0.000350		0.09		0.02				
11	0.000371		0.09		0.02				
12	0.000388		0.09		0.02				
13	0.000402		0.09		0.019				
14	0.000414		0.09		0.019				
15	0.000425		0.09		0.019				
16	0.000437		0.09		0.019				
17	0.000449		0.09		0.019				
18	0.000463		0.09		0.019				
19	0.000480		0.09		0.019				
20	0.000499	0.246913	0.09	1	0.018				
21	0.000519	0.227261	0.09	1.0455535	0.017				
22	0.000542	0.209647	0.09	1.0921193	0.015				
23	0.000566	0.193147	0.09	1.1396974	0.013	1		13,051.2	
24	0.000592	0.177861	0.09	1.1882878	0.01				
25	0.000616	0.163588	0.09	1.2368782	0.006	2		26,645.7	
26	0.000639	0.154278	0.09	1.2874932	0.005	3		42,413.2	
27	0.000659	0.138179	0.09	1.3381082	0.005	1		9,350.3	
28	0.000675	0.126942	0.09	1.3887232	0.005	9		121,732.7	
29	0.000687	0.116617	0.09	1.4413628	0.005	14		194,015.8	
30	0.000694	0.107226	0.09	1.4929901	0.005	20		238,983.9	
31	0.000699	0.098598	0.09	1.5456297	0.005	17		195,757.2	
32	0.000700	0.090728	0.09	1.5992816	0.005	31		305,389.9	
33	0.000701	0.083717	0.09	1.6519212	0.005	34		373,748.8	
34	0.000702	0.077340	0.09	1.7055731	0.005	38		448,630.4	
35	0.000704	0.071678	0.09	1.7582127	0.005	44		640,414.5	
36	0.000719	0.066693	0.09	1.8118646	0.005	39		702,237.6	
37	0.000749	0.062155	0.09	1.8645042	0.006	41		882,063.2	

x	$q'^{(m)}_x$	$q'^{(t)}_x$	ytm_s	ms_x	r_x	AN_x	RN_x	AW_x	RB_x
38	0.000796	0.058273	0.09	1.9171438	0.007	51		1,238,367.8	
39	0.000864	0.054765	0.09	1.9697834	0.008	54		1,061,260.2	
40	0.000953	0.051830	0.09	2.0203984	0.009	67		1,448,272.7	
41	0.001065	0.049299	0.09	2.0720257	0.01	49		1,140,185.0	
42	0.001201	0.047173	0.09	2.1216284	0.011	70		1,482,635.0	
43	0.001362	0.045351	0.09	2.1712311	0.012	69		1,589,866.5	
44	0.001547	0.043833	0.09	2.2188092	0.013	55		1,552,165.0	
45	0.001752	0.042618	0.09	2.265375	0.014	44	1	1,025,525.9	4,129.0
46	0.001974	0.041543	0.09	2.3109285	0.015	57		1,500,607.7	
47	0.002211	0.040694	0.09	2.3554697	0.016	48		1,144,279.7	
48	0.002460	0.039885	0.09	2.3979863	0.017	50		1,239,180.9	
49	0.002721	0.039277	0.09	2.4394906	0.018	75	1	1,779,703.7	1,391.7
50	0.002994	0.038670	0.09	2.477958	0.019	52	4	1,293,359.4	151.1
51	0.003279	0.038625	0.09	2.5154131	0.02	53	2	1,556,602.2	345.2
52	0.003576	0.037455	0.09	2.5508436	0.02	64	2	1,848,848.7	2,138.7
53	0.003884	0.036645	0.09	2.5842495	0.02	53	5	1,590,645.0	24,468.5
54	0.004203	0.035835	0.09	2.6156308	0.019	42	3	1,585,986.7	15,729.3
55	0.004534		0.09		0.018	8	40	225,327.3	38,941.3
56	0.004876		0.09		0.017		54		636,361.5
57	0.005228		0.09		0.016	3	29	105,313.1	279,947.7
58	0.005593		0.09		0.016	1	32	14,368.8	35,187.6
59	0.005988		0.09		0.016		13		114,217.0
60	0.006428		0.09		0.015		15		88,488.6
61	0.006933		0.09		0.015		8		72,969.6
62	0.007520		0.09		0.014		14		5,797.4
63	0.008207		0.09		0.014		15		1,592.2
64	0.009008		0.09		0.014		10		69,779.9
65	0.009940		0.09		0.013		4		13,472.2
66	0.011016		0.09		0.013		7		3,789.5
67	0.012251		0.09		0.014		7		38,979.3
68	0.013657		0.09		0.014		8		3,479.0
69	0.015233		0.09		0.015		6		3,249.4
70	0.016979		0.09		0.015		5		2,461.3
71	0.018891		0.09		0.015		3		1,627.4
72	0.020967		0.09		0.015		3		12,221.4
73	0.023209		0.09		0.015				
74	0.025644		0.09		0.014		4		1,182.3
75	0.028304		0.09		0.014				
76	0.031220		0.09		0.013		4		1,112.9
77	0.034425		0.09		0.012		1		1,534.3
78	0.037948		0.09		0.011				
79	0.041812		0.09		0.01				
80	0.046037		0.09		0.009		2		5,484.5
81	0.050643		0.09		0.008		2		299.5
82	0.055651		0.09		0.008				
83	0.061080		0.09		0.007		1		1,534.3
84	0.066948		0.09		0.007				
85	0.073275		0.09		0.007				
86	0.080076		0.09		0.006				
87	0.087370		0.09		0.005				
88	0.095169		0.09		0.005				
89	0.103455		0.09		0.004				
90	0.112208		0.09		0.004				
91	0.121402		0.09		0.003				

x	$q'^{(m)}_x$	$q'^{(t)}_x$	ytm_s	ms_x	r_x	AN_x	RN_x	AW_x	RB_x
92	0.131017		0.09		0.003				
93	0.141030		0.09		0.003				
94	0.151422		0.09		0.002				
95	0.162179		0.09		0.002				
96	0.173279		0.09		0.002				
97	0.184706		0.09		0.001				
98	0.196946		0.09		0.001				
99	0.210484		0.09		0.001				
100	0.225806		0.09		0.000				
101	0.243398		0.09		0.000				
102	0.263745		0.09		0.000				
103	0.287334		0.09		0.000				
104	0.314649		0.09		0.000				
105	0.346177		0.09		0.000				
106	0.382403		0.09		0.000				
107	0.423813		0.09		0.000				
108	0.470893		0.09		0.000				
109	0.524128		0.09		0.000				
110	0.584004		0.09		0.000				
111	0.651007		0.09		0.000				
112	0.725622		0.09		0.000				
113	0.808336		0.09		0.000				
114	0.899633		0.09		0.000				
115	1.000000		0.09		0.000				