

# IMF Working Paper

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## Overborrowing, Financial Crises and 'Macro-prudential' Policy\*

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**IMF Working Paper**

Research Department

**Overborrowing, Financial Crises and ‘Macro-prudential’ Policy\***

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**Abstract**

This paper studies overborrowing, financial crises and macro-prudential policy in an equilibrium model of business cycles and asset prices with collateral constraints. Agents in a decentralized competitive equilibrium do not internalize the negative effects of asset fire-sales on the value of other agents' assets and hence they borrow too much" ex ante, compared with a constrained social planner who internalizes these effects. Average debt and leverage ratios are slightly larger in the competitive equilibrium, but the incidence and magnitude of financial crises are much larger. Excess asset returns, Sharpe ratios and the market price of risk are also much larger. State-contingent taxes on debt and dividends of about 1 and -0.5 percent on average respectively support the planner's allocations as a competitive equilibrium and increase social welfare.

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Contents	Page
I. Introduction .....	3
II. Competitive Equilibrium.....	5
2.1 Private Optimality Conditions.....	7
2.2 Recursive Competitive Equilibrium.....	10
III. Constrained-Efficient Equilibrium.....	11
3.1 Equilibrium without Collateral Constraint.....	11
3.2 Recursive Constrained-Efficient Equilibrium.....	12
3.3 Comparison of Equilibria & ‘Macro-Prudential’ Policy.....	14
IV. Quantitative Analysis.....	16
4.1 Calibration.....	16
4.2 Borrowing decisions.....	18
4.3 Asset Returns.....	23
4.4 Incidence and Magnitude of Financial Crises.....	28
4.5 Long-Run Business Cycles.....	32
4.6 Properties of Macro-Prudential Policies.....	35
4.7 Welfare Effects.....	38
4.8 Sensitivity Analysis.....	41
V. Conclusions.....	45
 <b>Tables</b>	
1. Calibration.....	17
2. Asset Pricing Moments.....	27
3. Long Run Moments.....	34
4. Long Run Moments of Macro-Prudential Policies.....	14
 <b>Figures</b>	
1. Bond Decision Rules (left panel) and Land Pricing Function (Right Panel) for a Negative Two-Standard-Deviations TFP Shock.....	20
2. Comparison of Debt Dynamics.....	22
3. Ergodic Distribution of Leverage.....	28
4. Ergodic Cumulative Distribution of Land Returns.....	25
5. Event Analysis: Percentage Differences Relative to Unconditional Averages.....	30
6. Forecast of Expected Dividends and Land Returns.....	32
7. Event Analysis: Macr-Prudential Policies’.....	37
8. Welfare Costs of the Credit Externality for a Two-Standard-Deviations TFP Shock.....	40
 <b>References</b> .....	48
 <b>Appendix</b>	
1. Numerical Solution Method.....	51

# 1 Introduction

A common argument in narratives of the causes of the 2008 global financial crisis is that economic agents “borrowed too much.” The notion of “overborrowing,” however, is often vaguely defined or presented as a value judgment on borrowing decisions, in light of the obvious fact that a prolonged credit boom ended in collapse. This lack of clarity makes it difficult to answer two key questions: First, is overborrowing a significant macroeconomic problem, in terms of causing financial crises and playing a central role in driving macro dynamics during both ordinary business cycles and crises episodes? Second, are the so-called “macro-prudential” policy instruments effective to contain overborrowing and reduce financial fragility, and if so what are their main quantitative features?

In this paper, we answer these questions using a dynamic stochastic general equilibrium model of asset prices and business cycles with credit frictions. We provide a formal definition of overborrowing and use quantitative methods to determine how much overborrowing the model predicts and how it affects business cycles, financial crises, and social welfare. We also compute a state-contingent schedule of taxes on debt and dividends that can solve the overborrowing problem.

Our definition of overborrowing is in line with the one used in the academic literature (e.g. Lorenzoni, 2008, Korinek, 2009, Bianchi, 2009): The difference between the amount of credit that an agent obtains acting atomistically in an environment with a given set of credit frictions, and the amount obtained by a social planner who faces the same frictions but internalizes the general-equilibrium effects of its borrowing decisions. In the model, the credit friction is in the form of a collateral constraint on debt that has two important features. First, it drives a wedge between the marginal costs and benefits of borrowing considered by individual agents and those faced by a social planner. Second, when the constraint binds, it triggers Irving Fisher’s classic debt-deflation financial amplification mechanism, which causes a financial crisis.

This paper contributes to the literature by providing a quantitative assessment of overborrowing in an equilibrium model of business cycles and asset prices. The model is similar to those examined by Mendoza and Smith (2006) and Mendoza (2010). These studies showed

that cyclical dynamics in a competitive equilibrium lead to periods of expansion in which leverage ratios raise enough so that the collateral constraint becomes binding, triggering a Fisherian deflation that causes sharp declines in credit, asset prices, and macroeconomic aggregates.<sup>1</sup> In this paper, we study instead the efficiency properties of the competitive equilibrium, by comparing its allocations with those attained by a benevolent social planner subject to the same credit frictions as agents in the competitive equilibrium. Thus, while those previous studies focused on the amplification and asymmetry of the responses of macro variables to aggregate shocks, we focus here on the differences between competitive equilibria and constrained social optima.

In the model, the collateral constraint limits private agents not to borrow more than a fraction of the market value of their collateral assets, which take the form of an asset in fixed aggregate supply (e.g. land). Private agents take the price of this asset as given, and hence a “systemic credit externality” arises, because they do not internalize that, when the collateral constraint binds, fire-sales of assets cause a Fisherian debt-deflation spiral that causes asset prices to decline and the economy’s borrowing ability to shrink in an endogenous feedback loop. Moreover, when the constraint binds, production plans are also affected, because working capital financing is needed in order to pay for a fraction of labor costs, and working capital loans are also subject to the collateral constraint. As a result, when the credit constraint binds output falls because of a sudden increase in the effective cost of labor. This affects dividend streams and therefore equilibrium asset prices, and introduces an additional vehicle for the credit externality to operate, because private agents do not internalize the supply-side effects of their borrowing decisions.

We conduct a quantitative analysis in a version of the model calibrated to U.S. data. The results show that financial crises in the competitive equilibrium are significantly more frequent and more severe than in the constrained-efficient equilibrium. The incidence of financial crises is about three times larger. Asset prices drop about 25 percent in a typical crisis in the decentralized equilibrium, versus 5 percent in the constrained-efficient equilibrium. Output drops about 50 percent more, because the fall in asset prices reduces access

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<sup>1</sup>This is also related to the classic work on financial accelerators by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) and the more recent quantitative literature on this topic as in the work of Jermann and Quadrini (2009).

to working capital financing. The more severe asset price collapses also generate a “fat tail” in the distribution of asset returns in the decentralized equilibrium, which causes the price of risk to rise 1.5 times and excess returns to rise by 5 times, in both tranquil times and crisis times. The social planner can replicate exactly the constrained-efficient allocations in a decentralized equilibrium by imposing taxes on debt and dividends of about 1 and -0.5 percent on average respectively

The existing macro literature on credit externalities provides important background for our work. The externality we study is a pecuniary externality similar to those examined in the theoretical studies of Caballero and Krishnamurthy (2001), Lorenzoni (2008), and Korinek (2009). The pecuniary externality in these papers arises because financial constraints that depend on market prices generate amplification effects, which are not internalized by private agents. The literature on participation constraints in credit markets initiated by the work of Kehoe and Levine (1993) has also examined the role of inefficiencies that arise because of endogenous borrowing constraints. In particular, Jeske (2006) showed that if there is discrimination against foreign creditors, private agents have a stronger incentive to default than a planner who internalizes the effects of borrowing decisions on the domestic interest rate, which affects the tightness of the participation constraint. Wright (2006) then showed that as a consequence of this externality, subsidies on capital flows restore constrained efficiency.

Our work is also related to the quantitative studies of macro-prudential policy by Bianchi (2009) and Benigno, Chen, Otrok, Rebucci, and Young (2009). These authors studied a credit externality at work in the model of emerging markets crises of Mendoza (2002), in which agents do not internalize the effect of their individual debt plans on the market price of nontradable goods relative to tradables, which influences their ability to borrow from abroad. Bianchi examined how this externality leads to excessive debt accumulation and showed that a tax on debt can restore constrained efficiency and reduce the vulnerability to financial crises. Benigno et al. studied how the effects of the overborrowing externality are reduced when the planner has access to instruments that can affect directly labor allocations

during crises.<sup>2</sup>

Our analysis differs from the above quantitative studies in that we focus on asset prices as a key factor driving debt dynamics and the credit externality, instead of the relative price of nontradables. This is important because private debt contracts, particularly mortgage loans like those that drove the high household leverage ratios of many industrial countries in the years leading to the 2008 crisis, use assets as collateral. Moreover, from a theoretical standpoint, a collateral constraint linked to asset prices introduces forward-looking effects that are absent when using a credit constraint linked to goods prices. In particular, expectations of a future financial crisis affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. In addition, our model also differs in that we study a production economy in which working capital financing is subject to the collateral constraint. As a result, the credit externality distorts production plans and dividend rates, and thus again asset prices.

More recently, the quantitative studies by Nikolov (2009) and Jeanne and Korinek (2010) examine other models of macro-prudential policy in which assets serve as collateral.<sup>3</sup> Nikolov found that simple rules that impose tighter collateral requirements may not be welfare-improving in a setup in which consumption is a linear function that is not influenced by precautionary savings. In contrast, precautionary savings are critical determinants of optimal borrowing decisions in our model, because of the strong non-linear amplification effects caused by the Fisherian debt-deflation dynamics, and for the same reason we find that debt taxes are welfare improving. Jeanne and Korinek construct estimates of a Pigouvian debt tax in a model in which output follows an exogenous Markov-switching process and individual credit is limited to the sum of a fraction of aggregate, rather than individual, asset holdings plus a constant term. In their calibration analysis, this second term dominates and the probability of crises matches the exogenous probability of a low-output regime, and as result the tax cannot alter the frequency of crises and has small effects on their magnitude.<sup>4</sup> In contrast, in our model the probability of crises and their output dynamics are endogenous,

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<sup>2</sup>In a related paper Benigno et al. (2009) found that intervening during financial crisis by subsidizing nontradable goods leads to large welfare gains.

<sup>3</sup>Galati and Moessner (2010) conduct an exhaustive survey of the growing literature in research and policy circles on macro-prudential policy.

<sup>4</sup>They also examined the existence of deterministic cycles in a non-stochastic version of the model.

and macro-prudential policy reduces sharply the incidence and magnitude of crises.

Our results also contrast with the findings of Uribe (2006). He found that an environment in which agents do not internalize an aggregate borrowing limit yields identical borrowing decisions to an environment in which the borrowing limit is internalized.<sup>5</sup> An essential difference in our analysis is that the social planner internalizes not only the borrowing limit but also the price effects that arise from borrowing decisions. Still, our results showing small differences in average debt ratios across competitive and constrained-efficient equilibria are in line with his findings.

The rest of the paper is organized as follows: Section 2 presents the analytical framework. Section 3 analyzes constrained efficiency. Section 4 presents the quantitative analysis. Section 5 provides conclusions.

## 2 Competitive Equilibrium

We follow Mendoza (2010) in specifying the economic environment in terms of firm-household units who make production and consumption decisions. Preferences are given by:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t)) \right] \quad (1)$$

In this expression,  $E(\cdot)$  is the expectations operator,  $\beta$  is the subjective discount factor,  $n_t$  is labor supply and  $c_t$  is consumption. The period utility function  $u(\cdot)$  is assumed to have the constant-relative-risk-aversion (CRRA) form. The argument of  $u(\cdot)$  is the composite commodity  $c_t - G(n_t)$  defined by Greenwood, Hercowitz, and Huffman (1988).  $G(n)$  is a convex, strictly increasing and continuously differentiable function that measures the disutility of labor supply. This formulation of preferences removes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only.

Each household can combine land and labor services purchased from other households to produce final goods using a production technology such that  $y = \varepsilon_t F(k_t, h_t)$ , where  $F$

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<sup>5</sup>He provided analytical results for a canonical endowment economy model with a credit constraint where there is an exact equivalence between the two sets of allocations. In addition, he examined a model in which the exact equivalence of his first example does not hold, but still overborrowing is negligible.



is a decreasing-returns-to-scale production function,  $k_t$  represents individual land holdings,  $h_t$  represents labor demand and  $\varepsilon_t$  is a productivity shock, which has compact support and follows a finite-state, stationary Markov process. Individual profits from this production activity are therefore given by  $\varepsilon_t F(k_t, h_t) - w_t h_t$ .

The budget constraint faced by the representative firm-household is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + w_t n_t + [\varepsilon_t F(k_t, h_t) - w_t h_t] \quad (2)$$

where  $b_t$  denotes holdings of one-period, non-state-contingent discount bonds at the beginning of date  $t$ ,  $q_t$  is the market price of land,  $R_t$  is the real interest rate, and  $w_t$  is the wage rate.

The interest rate is assumed to be exogenous. This is equivalent to assuming that the economy is a price-taker in world credit markets, as in other studies of the U.S. financial crisis like those of Boz and Mendoza (2010), Corbae and Quintin (2009) and Howitt (2010), or alternatively it implies that the model can be interpreted as a partial-equilibrium model of the household sector. This assumption is adopted for simplicity, but is also in line with the evidence indicating that in the era of financial globalization even the U.S. risk-free rate has been significantly influenced by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998. Warnock and Warnock (2009) provide econometric evidence of the significant effect of foreign capital inflows on U.S. T-bill rates since the mid 1980s. Mendoza and Quadrini (2009) document that about 1/2 of the surge in net credit in the U.S. economy since then was financed by foreign capital inflows, and more than half of the stock of U.S. treasury bills is now owned by foreign agents.

Household-firms are subject to a working capital constraint. In particular, they are required to borrow a fraction  $\theta$  of the wages bill  $w_t h_t$  at the beginning of the period and have to repay at the end of the period. In the conventional working capital setup, a cash-in-advance-like motive for holding funds to pay for inputs implies that the wages bill carries a financing cost determined by the inter-period interest rate. In contrast, here we simply assume that working capital funds are within-period loans. Hence, the interest rate on

working capital is zero, as in some recent studies on the business cycle implications of working capital and credit frictions (e.g. Chen and Song (2009)). We follow this approach so as to show that the effects of working capital in our model hinge only on the need to provide collateral for working capital loans, as explained below, and not on the effect of interest rate fluctuations on effective labor costs.<sup>6</sup>

As in Mendoza (2010), agents face a collateral constraint that limits total debt, including both intertemporal debt and atemporal working capital loans, not to exceed a fraction  $\kappa$  of the market value of asset holdings (i.e.  $\kappa$  imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t} + \theta w_t h_t \leq \kappa q_t k_{t+1} \quad (3)$$

Following Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999), we interpret this constraint as resulting from an environment where limited enforcement prevents lenders to collect more than a fraction  $\kappa$  of the value of a defaulting debtor's assets, but we abstract from modeling the contractual relationship explicitly.

## 2.1 Private Optimality Conditions

In the competitive equilibrium, agents maximize (1) subject to (2) and (3) taking land prices and wages as given. This maximization problem yields the following optimality conditions for each date  $t$ :

$$w_t = G'(n_t) \quad (4)$$

$$\varepsilon_t F_h(k_t, h_t) = w_t [1 + \theta \mu_t / u'(t)] \quad (5)$$

$$u'(t) = \beta R E_t [u'(t+1)] + \mu_t \quad (6)$$

$$q_t (u'(t) - \mu_t \kappa) = \beta E_t [u'(t+1) (\varepsilon_{t+1} F_k(k_{t+1}, h_{t+1}) + q_{t+1})] \quad (7)$$

where  $\mu_t \geq 0$  is the Lagrange multiplier on the collateral constraint.

Condition (4) is the individual's labor supply condition, which equates the marginal disutility of labor with the wage rate. Condition (5) is the labor demand condition, which equates

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<sup>6</sup>We could also change to the standard setup, but in our calibration,  $\theta = 0.14$  and  $R = 1.028$ , and hence working capital loans would add 0.4 percent to the cost of labor implying that our findings would remain largely unchanged.

the marginal productivity of labor with the effective marginal cost of hiring labor. The latter includes the extra financing cost  $\theta\mu_t/u'(t)$  in the states of nature in which the collateral constraint on working capital binds. The last two conditions are the Euler equations for bonds and land respectively. When the collateral constraint binds, condition (6) implies that the marginal utility of reallocating consumption to the present exceeds the expected marginal utility cost of borrowing in the bond market by an amount equal to the shadow price of relaxing the credit constraint. Condition (7) equates the marginal cost of an extra unit of land investment with its marginal gain. The marginal cost nets out from the marginal utility of foregone current consumption a fraction  $\kappa$  of the shadow value of the credit constraint, because the additional unit of land holdings contributes to relax the borrowing limit.

Condition (7) yields the following forward solution for land prices:

$$q_t = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{i=0}^j m_{t+1+i} \right) d_{t+j+1} \right], \quad m_{t+1+i} \equiv \frac{\beta u'(t+1+i)}{u'(t+i) - \mu_{t+i}\kappa}, \quad d_t \equiv \varepsilon_t F_k(k_t, h_t) \quad (8)$$

Thus, we obtain what seems a standard asset pricing condition stating that, at equilibrium, the date- $t$  price of land is equal to the expected present value of the future stream of dividends discounted using the stochastic discount factors  $m_{t+1+i}$ , for  $i = 0, \dots, \infty$ . The key difference with the standard asset pricing condition, however, is that the discount factors are adjusted to account for the shadow value of relaxing the credit constraint by purchasing an extra unit of land whenever the collateral constraint binds (at any date  $t+i$  for  $i = 0, \dots, \infty$ ).

Combining (6), (7) and the definition of asset returns ( $R_{t+1}^q \equiv \frac{d_{t+1}+q_{t+1}}{q_t}$ ), it follows that the expected excess return on land relative to bonds (i.e. the equity premium),  $R_t^{ep} \equiv E_t(R_{t+1}^q - R)$ , satisfies the following condition:

$$R_t^{ep} = \frac{\mu_t(1 - \kappa)}{(u'(t) - \mu_t\kappa)E_t[m_{t+1}]} - \frac{cov_t(m_{t+1}, R_{t+1}^q)}{E_t[m_{t+1}]}, \quad (9)$$

where  $cov_t(m_{t+1}, R_{t+1}^q)$  is the date- $t$  conditional covariance between  $m_{t+1}$  and  $R_{t+1}^q$ .

Following Mendoza and Smith (2006), we characterize the first term in the right-hand-side of (9) as the direct (first-order) effect of the collateral constraint on the equity premium, which reflects the fact that a binding collateral constraint exerts pressure to fire-sell land,

depressing its current price.<sup>7</sup> There is also an indirect (second-order) effect given by the fact that  $cov_t(m_{t+1}, R_{t+1}^q)$  is likely to become more negative when there is a possibility of a binding credit constraint, because the collateral constraint makes it harder for agents to smooth consumption.

Given the definitions of the Sharpe ratio ( $S_t \equiv \frac{R_t^{ep}}{\sigma_t(R_{t+1}^q)}$ ) and the price of risk ( $s_t \equiv \sigma_t(m_{t+1})/E_t m_{t+1}$ ), we can rewrite the expected excess return and the Sharpe ratio as:

$$R_t^{ep} = S_t \sigma_t(R_{t+1}^q), \quad S_t = \frac{\mu_t(1 - \kappa)}{(u'(t) - \mu_t \kappa) E_t [m_{t+1}] \sigma_t(R_{t+1}^q)} - \rho_t(R_{t+1}^q, m_{t+1}) s_t \quad (10)$$

where  $\sigma_t(R_{t+1}^q)$  is the date- $t$  conditional standard deviation of land returns and  $\rho_t(R_{t+1}^q, m_{t+1})$  is the conditional correlation between  $R_{t+1}^q$  and  $m_{t+1}$ . Thus, the collateral constraint has direct and indirect effects on the Sharpe ratio analogous to those it has on the equity premium. The indirect effect reduces to the usual expression in terms of the product of the price of risk and the correlation between asset returns and the stochastic discount factor. The direct effect is normalized by the variance of land returns. These relationships will be useful later to study the quantitative effects of the credit externality on asset pricing.

Since  $q_t E_t [R_{t+1}^q] \equiv E_t [d_{t+1} + q_{t+1}]$ , we can rewrite the asset pricing condition in this way:

$$q_t = E_t \sum_{j=0}^{\infty} \left( \prod_{i=0}^j E_{t+i} R_{t+1+i}^q \right)^{-1} d_{t+j+1}, \quad (11)$$

Notice that (9) and (11) imply that a binding collateral constraint at date  $t$  implies an increase in expected excess land returns and a drop in asset prices at  $t$ . Moreover, since expected returns exceed the risk free rate whenever the collateral constraint is expected to bind at any future date, asset prices at  $t$  are affected by collateral constraint not just when the constraints binds at  $t$ , but whenever it is expected to bind at any future date.

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<sup>7</sup>Notice that this effect vanishes when  $\kappa = 1$ , because when 100 percent of the value of land can be collateralized, the shadow value of relaxing the constraint by acquiring an extra unit of land equals the shadow value of relaxing it by reducing the debt by one unit.

## 2.2 Recursive Competitive Equilibrium

The competitive equilibrium is defined by stochastic sequences of allocations  $\{c_t, k_{t+1}, b_{t+1}, h_t, n_t\}_{t=0}^{\infty}$  and prices  $\{q_t, w_t\}_{t=0}^{\infty}$  such that: (A) agents maximize utility (1) subject to the sequence of budget and credit constraints given by (2) and (3) for  $t = 0, \dots, \infty$ , taking as given  $\{q_t, w_t\}_{t=0}^{\infty}$ ; (B) the markets of goods, labor and land clear at each date  $t$ . Since land is in fixed supply  $\bar{K}$ , the market-clearing condition for land is  $k_t = \bar{K}$ . The market clearing condition in the goods and labor markets are  $c_t + \frac{b_{t+1}}{R} = \varepsilon_t F(\bar{K}, n_t) + b_t$  and  $h_t = n_t$  respectively.

We now characterize the competitive equilibrium in recursive form. The state variables for a particular individual's optimization problem at time  $t$  are the individual bond holdings ( $b$ ), aggregate bond holdings ( $B$ ), individual land holdings ( $k$ ), and the TFP realization ( $\varepsilon$ ). Aggregate land holdings are not carried as a state variable because land is in fixed supply. Denoting by  $\Gamma(B, \varepsilon)$  the agents' perceived law of motion of aggregate bonds and  $q(B, \varepsilon)$  and  $w(B, \varepsilon)$  the pricing functions for land and labor respectively, the agents' recursive optimization problem is:

$$\begin{aligned} V(b, k, B, \varepsilon) &= \max_{b', k', c, n, h} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} [V(b', k', B', \varepsilon')] & (12) \\ \text{s.t.} \quad q(B, \varepsilon)k' + c + \frac{b'}{R} &= q(B, \varepsilon)k + b + w(B, \varepsilon)n + [\varepsilon F(k, h) - w(B, \varepsilon)h] \\ B' &= \Gamma(B, \varepsilon) \\ -\frac{b'}{R} + \theta w(B, \varepsilon)h &\leq \kappa q(B, \varepsilon)k' \end{aligned}$$

The solution to this problem is characterized by the decision rules  $\hat{b}'(b, k, B, \varepsilon)$ ,  $\hat{k}'(b, k, B, \varepsilon)$ ,  $\hat{c}(b, k, B, \varepsilon)$ ,  $\hat{n}(b, k, B, \varepsilon)$  and  $\hat{h}(b, k, B, \varepsilon)$ . The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by  $\hat{b}'(B, \bar{K}, B, \varepsilon)$ . In a recursive rational expectations equilibrium, as defined below, the actual and perceived laws of motion must coincide.

### Definition 1 (*Recursive Competitive Equilibrium*)

A recursive competitive equilibrium is defined by an asset pricing function  $q(B, \varepsilon)$ , a pricing function for labor  $w(B, \varepsilon)$ , a perceived law of motion for aggregate bond holdings  $\Gamma(B, \varepsilon)$ , and

a set of decision rules  $\left\{ \hat{b}'(b, k, B, \varepsilon), \hat{k}'(b, k, B, \varepsilon), \hat{c}(b, k, B, \varepsilon), \hat{n}(b, k, B, \varepsilon), \hat{h}(b, k, B, \varepsilon) \right\}$  with associated value function  $V(b, k, B, \varepsilon)$  such that:

1.  $\left\{ \hat{b}'(b, k, B, \varepsilon), \hat{k}'(b, k, B, \varepsilon), \hat{c}(b, k, B, \varepsilon), \hat{n}(b, k, B, \varepsilon), \hat{h}(b, k, B, \varepsilon) \right\}$  and  $V(b, k, B, \varepsilon)$  solve the agents' recursive optimization problem, taking as given  $q(B, \varepsilon), w(B, \varepsilon)$  and  $\Gamma(B, \varepsilon)$ .
2. The perceived law of motion for aggregate bonds is consistent with the actual law of motion:  $\Gamma(B, \varepsilon) = \hat{b}'(B, \bar{K}, B, \varepsilon)$ .
3. Wages satisfy  $w(B, \varepsilon) = G'(\hat{n}(B, \bar{K}, B, \varepsilon))$  and land prices satisfy  $q(B, \varepsilon) = E_{\varepsilon'|\varepsilon} \left\{ \frac{\beta u'(\hat{c}(\Gamma(B, \varepsilon), \bar{K}, \Gamma(B, \varepsilon), \varepsilon')) [\varepsilon' F_k(\bar{K}, \hat{n}(\Gamma(B, \varepsilon), \bar{K}, \Gamma(B, \varepsilon), \varepsilon')) + q(\Gamma(B, \varepsilon), \varepsilon')])}{u'(\hat{c}(B, \bar{K}, B, \varepsilon)) - \kappa \max[0, u'(\hat{c}(B, \bar{K}, B, \varepsilon)) - \beta R E_{\varepsilon'|\varepsilon} u'(\hat{c}(\Gamma(B, \varepsilon), \bar{K}, \Gamma(B, \varepsilon), \varepsilon'))]} \right\}$
4. Goods, labor and asset markets clear:  $\frac{\hat{b}'(B, \bar{K}, B, \varepsilon)}{R} + \hat{c}(B, \bar{K}, B, \varepsilon) = \varepsilon F(\bar{K}, \hat{n}(B, \bar{K}, B, \varepsilon)) + B$ ,  $\hat{n}(B, \bar{K}, B, \varepsilon) = \hat{h}(B, \bar{K}, B, \varepsilon)$  and  $\hat{k}(B, \bar{K}, B, \varepsilon) = \bar{K}$

### 3 Constrained-Efficient Equilibrium

#### 3.1 Equilibrium without collateral constraint

We start studying the efficiency properties of the competitive equilibrium by briefly characterizing an efficient equilibrium in the absence of the collateral constraint (3). The allocations of this equilibrium can be represented as the solution to the following standard planning problem:

$$H(B, \varepsilon) = \max_{b', c, n} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} [H(B', \varepsilon')] \quad (13)$$

$$s.t. \quad c + \frac{B'}{R} = \varepsilon F(\bar{K}, n) + B$$

and subject also to either this problem's *natural debt limit*, which is defined by  $B' > \frac{\varepsilon^{\min} F(\bar{K}, n^*(\varepsilon^{\min}))}{R-1}$ , where  $\varepsilon^{\min}$  is the lowest possible realization of TFP and  $n^*(\varepsilon^{\min})$  is the optimal labor allocation that solves  $\varepsilon^{\min} F_n(\bar{K}, n) = G'(n)$ , or to a tighter ad-hoc time- and state-invariant borrowing limit.

The common strategy followed in quantitative studies of the macro effects of collateral constraints (see, for example, Mendoza and Smith, 2006 and Mendoza, 2010) is to compare

the allocations of the competitive equilibrium with the Fisherian collateral constraint with those arising from the above benchmark case. The competitive equilibria with and without the collateral constraint differ in that in the former private agents borrow *less* (since the collateral constraint limits the amount they can borrow and also because they build precautionary savings to self-insure against the risk of the occasionally binding credit constraint), and there is financial amplification of the effects of the underlying exogenous shocks (since binding collateral constraints produce large recessions and drops in asset prices). Compared with the constrained-efficient equilibrium we define next, however, we will show that the competitive equilibrium with collateral constraints displays overborrowing (i.e. agents borrow *more* than in the constrained-efficient equilibrium ).

### 3.2 Recursive Constrained-Efficient Equilibrium

We study a benevolent social planner who maximizes the agents' utility subject to the resource constraint, the collateral constraint and the same menu of assets of the competitive equilibrium.<sup>8</sup> In particular, we consider a social planner that is constrained to have the same “borrowing ability” (the same market-determined value of collateral assets  $\kappa q(B, \varepsilon)\bar{K}$ ) at every given state as agents in the decentralized equilibrium, but with the key difference that the planner internalizes the effects of its borrowing decisions on the market prices of assets and labor.<sup>9</sup>

The recursive problem of the social planner is defined as follows:

$$\begin{aligned}
 W(B, \varepsilon) &= \max_{B', c, n} u(c - G(n)) + \beta E_{\varepsilon'|\varepsilon} [W(B', \varepsilon')] & (14) \\
 \text{s.t.} \quad c + \frac{B'}{R} &= \varepsilon F(\bar{K}, n) + B \\
 -\frac{B'}{R} + \theta w(B, \varepsilon)n &\leq \kappa q(B, \varepsilon)\bar{K}
 \end{aligned}$$

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<sup>8</sup>We refer to the social planner's equilibrium and constrained-efficient equilibrium interchangeably. Our focus is on second-best allocations, so when we refer to the social planner's choices it should be understood that we mean the constrained social planner.

<sup>9</sup>We could also allow the social planner to manipulate the borrowing ability state by state (i.e., by allowing the planner to alter  $\kappa q(B, \varepsilon)\bar{K}$ ). Allowing for this possibility can potentially increase the welfare losses resulting from the externality but the macroeconomic effects are similar. In addition, since asset prices are forward-looking, this would create a time-inconsistency problem in the planner's problem. Allowing the planner to commit to future actions would lead the planner to internalize not only how today's choice of debt affects tomorrow's asset prices but also how it affects asset prices and the tightness of collateral constraints in previous periods.

where  $q(B, \varepsilon)$  is the equilibrium pricing function obtained in the competitive equilibrium. Wages can be treated in a similar fashion, but it is easier to decentralize the planner's allocations as a competitive equilibrium if we assume that the planner takes wages as given and wages need to satisfy  $w(B, \varepsilon) = G'(n)$ .<sup>10</sup> Under this assumption, we impose the optimality condition of labor supply as a condition that the constrained-efficient equilibrium must satisfy, in addition to solving problem (14) for given wages.

Using the envelope theorem on the first-order conditions of problem (14) and imposing the labor supply optimality condition, we obtain the following optimality conditions for the constrained-efficient equilibrium:

$$u'(t) = \beta RE_t [u'(t+1) + \mu_{t+1}\psi_{t+1}] + \mu_t, \quad \psi_{t+1} \equiv \kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}} \quad (15)$$

$$\varepsilon_t F_n(\bar{K}, n_t) = G'(n_t) [1 + \theta \mu_t / u'(t)] \quad (16)$$

The key difference between the competitive equilibrium and the constrained-efficient allocations follows from examining the Euler equations for bond holdings in both problems. In particular, the term  $\mu_{t+1}\psi_{t+1}$  in condition (15) represents the additional marginal benefit of savings considered by the social planner at date  $t$ , because the planner takes into account how an extra unit of bond holdings alters the tightness of the credit constraint through its effects on the prices of land and labor at  $t+1$ . Note that, since  $\frac{\partial q_{t+1}}{\partial b_{t+1}} > 0$  and  $\frac{\partial w_{t+1}}{\partial b_{t+1}} \geq 0$ ,  $\psi_{t+1}$  is the difference of two opposing effects and hence its sign is in principle ambiguous. The term  $\frac{\partial q_{t+1}}{\partial b_{t+1}}$  is positive, because an increase in net worth increases demand for land and land is in fixed supply. The term  $\frac{\partial w_{t+1}}{\partial b_{t+1}}$  is positive, because the effective cost of hiring labor increases when the collateral constraint binds, reducing labor demand and pushing wages down. We found, however, that the value of  $\psi_{t+1}$  is positive in all our quantitative experiments with baseline parameter values and variations around them, and this is because  $\frac{\partial q_{t+1}}{\partial b_{t+1}}$  is large and positive when the credit constraint binds due the effects of the Fisherian debt-deflation mechanism.

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<sup>10</sup>This implies that the social planner does not internalize the direct effects of choosing the contemporaneous labor allocation on contemporaneous wages. We have also investigated the possibility of having the planner internalize these effects but results are very similar. This occurs again because our calibrated interest rate and working capital requirement are very small.



**Definition 2** (*Recursive Constrained-Efficient Equilibrium*)

The recursive constrained-efficient equilibrium is given by a set of decision rules  $\{\hat{B}'(B, \varepsilon), \hat{c}(B, \varepsilon), \hat{n}(B, \varepsilon)\}$  with associated value function  $W(B, \varepsilon)$ , and wages  $w(B, \varepsilon)$  such that:

1.  $\{\hat{B}'(B, \varepsilon), \hat{c}(B, \varepsilon), \hat{n}(B, \varepsilon)\}$  and  $W(B, \varepsilon)$  solve the planner's recursive optimization problem, taking as given  $w(B, \varepsilon)$  and the competitive equilibrium's asset pricing function  $q(B, \varepsilon)$ .
2. Wages satisfy  $w(B, \varepsilon) = G'(\hat{n}(B, \varepsilon))$ .

**3.3 Comparison of Equilibria & ‘Macro-prudential’ Policy**

Using a simple variational argument, we can show that the allocations of the competitive equilibrium are inefficient, in the sense that they violate the conditions that support the constrained-efficient equilibrium. In particular, private agents undervalue net worth in periods during which the collateral constraint binds. To see this, consider first the marginal utility of an increase in individual bond holdings. By the envelope theorem, in the competitive equilibrium this can be written as  $\frac{\partial V}{\partial b} = u'(t)$ . For the constrained-efficient economy, however, the marginal benefit of an increase in bond holdings takes into account the fact that prices are affected by the increase in bond holdings, and is therefore given by  $\frac{\partial W}{\partial b} = u'(t) + \psi_t \mu_t$ . If the collateral constraint does not bind,  $\mu_t = 0$  and the two expressions coincide. If the collateral constraint binds, the social benefits of a higher level of bonds include the extra term given by  $\psi_t \mu_t$ , because one more unit of aggregate bonds increases the inter-period ability to borrow by  $\psi_t$  which has a marginal value of  $\mu_t$ .

The above argument explains why bond holdings are valued differently by the planner and the private agents “ex post,” when the collateral constraint binds. Since both the planner and the agents are forward looking, however, it follows that those differences in valuation lead to differences in the private and social benefits of debt accumulation “ex ante,” when the constraint is not binding. Consider the marginal cost of increasing the level of debt at date  $t$  evaluated at the competitive equilibrium in a state in which the constraint is not binding. This cost is given by the discounted expected marginal utility from the implied reduction in consumption next period  $\beta RE [u'(t + 1)]$ . In contrast, the social planner internalizes the

effect by which the larger debt reduces tomorrow's borrowing ability by  $\psi_{t+1}$ , and hence the marginal cost of borrowing at period  $t$  that is not internalized by private agents is given by  $\beta RE_t \left[ \mu_{t+1} \left( \kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}} \right) \right]$ .

We now show that the planner can implement the constrained-efficient allocations as a competitive equilibrium in the decentralized economy by introducing a macro-prudential policy that taxes debt and dividends (the latter can turn into a subsidy too, as we show in the next Section, but we refer to it generically as a tax).<sup>11</sup> In particular, the planner can do this by constructing state-contingent schedules of taxes on bond purchases ( $\tau_t$ ) and on land dividends ( $\delta_t$ ), with the total cost (revenues) financed (rebated) as lump-sum taxes (transfers). The tax on bonds ensures that the planner's optimal plans for consumption and bond holdings are consistent with the Euler equation for bonds in the competitive equilibrium. This requires setting the tax to  $\tau_t = E_t \mu_{t+1} \psi_{t+1} / E_t u'(t+1)$ . The tax on land dividends ensures that these optimal plans and the pricing function  $q(B, \varepsilon)$  are consistent with the private agents' Euler equation for land holdings.

The Euler equations of the competitive equilibrium with the macro-prudential policy in place become:

$$u'(t) = \beta R(1 + \tau_t) E_t [u'(t+1)] + \mu_t \quad (17)$$

$$q_t(u'(t) - \mu_t \kappa) = \beta E_t [u'(t+1) (\varepsilon_{t+1} F_k(k_{t+1}, n_{t+1})(1 + \delta_{t+1}) + q_{t+1})] \quad (18)$$

By combining these two Euler equations we can derive the expected excess return on land paid in the land market under the macro-prudential policy. In this case, after-tax returns on land and bonds are defined as  $\tilde{R}_{t+1}^q \equiv \frac{d_{t+1}(1+\delta_{t+1})+q_{t+1}}{q_t}$  and  $\tilde{R}_{t+1} \equiv R(1 + \tau_t)$  respectively, and the after-tax expected equity premium reduces to an expression analogous to that of the decentralized equilibrium:

$$\tilde{R}_t^{ep} = \frac{\mu_t(1 - \kappa)}{E_t [(u'(t) - \mu_t \kappa)m_{t+1}]} - \frac{Cov_t(m_{t+1}, \tilde{R}_{t+1}^q)}{E_t [m_{t+1}]} \quad (19)$$

This excess return also has a corresponding interpretation in terms of the Sharpe ratio, the

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<sup>11</sup>See Bianchi (2009) for other decentralizations using capital and liquidity requirements and loan-to-value ratios.

price of risk, and the correlation between land returns and the pricing kernel as in the case of the competitive equilibrium without macro-prudential policy.

It follows from comparing the expressions for  $R_t^{ep}$  and  $\tilde{R}_t^{ep}$  that differences in the after-tax expected equity premia of the competitive equilibria with and without macro-prudential policy are determined by differences in the direct and indirect effects of the credit constraint in the two environments. As shown in the next Section, these effects are stronger in the decentralized equilibrium without policy intervention, in which the inefficiencies of the credit externality are not addressed. Intuitively, higher leverage and debt in this environment imply that the constraint binds more often, which strengthens the direct effect. In addition, lower net worth implies that the stochastic discount factor covaries more strongly with the excess return on land, which strengthens the indirect effect. Notice also that dividends in the constrained-efficient allocations are discounted at a rate which depends positively on the tax on debt. This premium is required by the social planner so that the excess returns reflect the social costs of borrowing.

## 4 Quantitative Analysis

### 4.1 Calibration

We calibrate the model to annual frequency using data from the U.S. economy. The functional forms for preferences and technology are the following:

$$u(c - G(n)) = \frac{\left[ c - \varkappa \frac{n^{1+\omega}}{1+\omega} \right]^{1-\sigma} - 1}{1-\sigma} \quad \omega > 0, \sigma > 1 \quad (20)$$

$$F(k, h) = \varepsilon k^{\alpha_K} h^{\alpha_h}, \quad \alpha_K, \alpha_h \geq 0 \quad \alpha_K + \alpha_h < 1 \quad (21)$$

The real interest rate is set to  $R - 1 = 0.028$  per year, which is the ex-post average real interest rate on U.S. three-month T-bills during the period 1980-2005. We set  $\sigma = 2$ , which is a standard value in quantitative DSGE models. The parameter  $\varkappa$  is inessential and is set so that mean hours are equal to 1, which requires  $\varkappa = 0.64$ . Aggregate land is normalized to  $\bar{K} = 1$  without loss of generality and the share of labor in output  $\alpha_h$  is equal to 0.64,

the standard value. The Frisch elasticity of labor supply ( $1/\omega$ ) is set equal to 1, in line with evidence by Kimball and Shapiro (2008).

We follow Schmitt-Grohe and Uribe (2007) in taking M1 money balances in possession of firms as a proxy for working capital. Based on the observations that about two-thirds of M1 are held by firms (Mulligan, 1997) and that M1 was on average about 14 percent of annual GDP over the period 1980 to 2009, we calibrate the working capital-GDP ratio to be  $(2/3)0.14 = 0.093$ . Given the 64 percent labor share in production, and assuming the collateral constraint does not bind, we obtain  $\theta = 0.093/0.64 = 0.146$ .

The value of  $\beta$  is set to 0.96, which is also a standard value but in addition it supports an average household debt-income ratio in a range that is in line with U.S. data from the Federal Reserve's *Flow of Funds* database. Before the mid-1990s this ratio was stable at about 30 percent. Since then and until just before the 2008 crisis, it rose steadily to a peak of almost 70 percent. By comparison, the average debt-income ratio in the stochastic steady-state of our baseline calibration is 38 percent. A mean debt ratio of 38 percent is sensible because 70 percent was an extreme at the peak of a credit boom and 30 percent is an average from a period before the substantial financial innovation of recent years.

**Table 1:** Calibration

		<i>Source / target</i>
Interest rate	$R - 1 = 0.028$	U.S. data
Risk aversion	$\sigma = 2$	Standard DSGE value
Share of labor	$\alpha_n = 0.64$	U.S. data
Labor disutility coefficient	$\chi = 0.64$	Normalization
Frisch elasticity parameter	$\omega = 1$	Kimball and Shapiro (2008)
Supply of land	$\bar{K} = 1$	Normalization
Working capital coefficient	$\theta = 0.14$	Working Capital-GDP=9%
Discount factor	$\beta = 0.96$	Debt-GDP ratio= 38%
Collateral coefficient	$\kappa = 0.36$	Frequency of Crisis = 3%
Share of land	$\alpha_K = 0.05$	Housing-GDP ratio = 1.35
TFP process	$\sigma_\varepsilon = 0.014, \rho_\varepsilon = 0.53$	Std. dev. and autoc. of U.S. GDP

The values of  $\kappa$ ,  $\alpha_K$  and the TFP process are calibrated to match targets from U.S. data by simulating the model. We set  $\alpha_K$  so as to match the average ratio of housing to GDP at current prices, which is equal to 1.35. The value of housing is taken from the *Flow of Funds*,

and is measured as real state tangible assets owned by households (reported in Table B.100, row 4). The model matches the 1.35 ratio when we set  $\alpha_K = 0.05$ .<sup>12</sup>

TFP shocks follow a log-normal AR(1) process  $\log(\varepsilon_t) = \rho \log(\varepsilon_{t-1}) + \eta_t$ . We construct a discrete approximation to this process using the quadrature procedure of Tauchen and Hussey (1991) using 15 nodes. The values of  $\sigma_\varepsilon$  and  $\rho$  are set so that the standard deviation and first-order autocorrelation of the output series produced by the model match the corresponding moments for the cyclical component of U.S. GDP in the sample period 1947-2007 (which are 2.1 percent and 0.5 respectively). This procedure yields  $\sigma_\varepsilon = 0.014$  and  $\rho = 0.53$ .

Finally, we set the value of  $\kappa$  so as to match the frequency of financial crises in U.S. data. We define a financial crisis as an event in which both the credit constraint binds and there is a decrease in credit of more than one standard deviation. Then, we set  $\kappa$  so that financial crises in the baseline model simulation occur about 3 percent of the time, which is consistent with the fact that the U.S. has experienced three major financial crises in the last hundred years.<sup>13</sup> This yields the value of  $\kappa = 0.36$ .

We recognize that several of the parameter values are subject of debate (e.g. there is a fair amount of disagreement about the Frisch elasticity of labor supply), or relate to variables that do not have a clear analog in the data (as is the case with  $\kappa$  or  $\theta$ ). Hence, we will perform sensitivity analysis to examine the robustness of our results to changes in the model's key parameters.

## 4.2 Borrowing decisions

We start the quantitative analysis by exploring the effects of the credit externality on optimal borrowing plans. The solution method is described in the Appendix. Since mean output is normalized to 1, all quantities can be interpreted as fractions of mean output.

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<sup>12</sup> $\alpha_K$  represents the share of fixed assets in GDP, and not the standard share of capital income in GDP. There is little empirical evidence about the value of this parameter, with estimates that vary depending, for example, on whether we consider land used for residential or commercial purposes, or owned by government at different levels. We could also calibrate  $\alpha_K$  using the fact that, in a deterministic steady state where the collateral constraint does not bind, the value-of-land-GDP ratio is equal to  $\alpha_K/(R-1)$ , which would imply  $\alpha_K = 1.35(0.028) = 0.038$ . This yields very similar results as  $\alpha_K = 0.05$ .

<sup>13</sup> The three crises correspond to the Great Depression, the Savings and Loans Crisis and the Great Recession (see Reinhart and Rogoff (2008)). While a century may be a short sample for estimating accurately the probability of a rare event in one country, Mendoza (2010) estimates a probability of about 3.6 percent for financial crises using a similar definition but applied to all emerging economies using data since 1980.

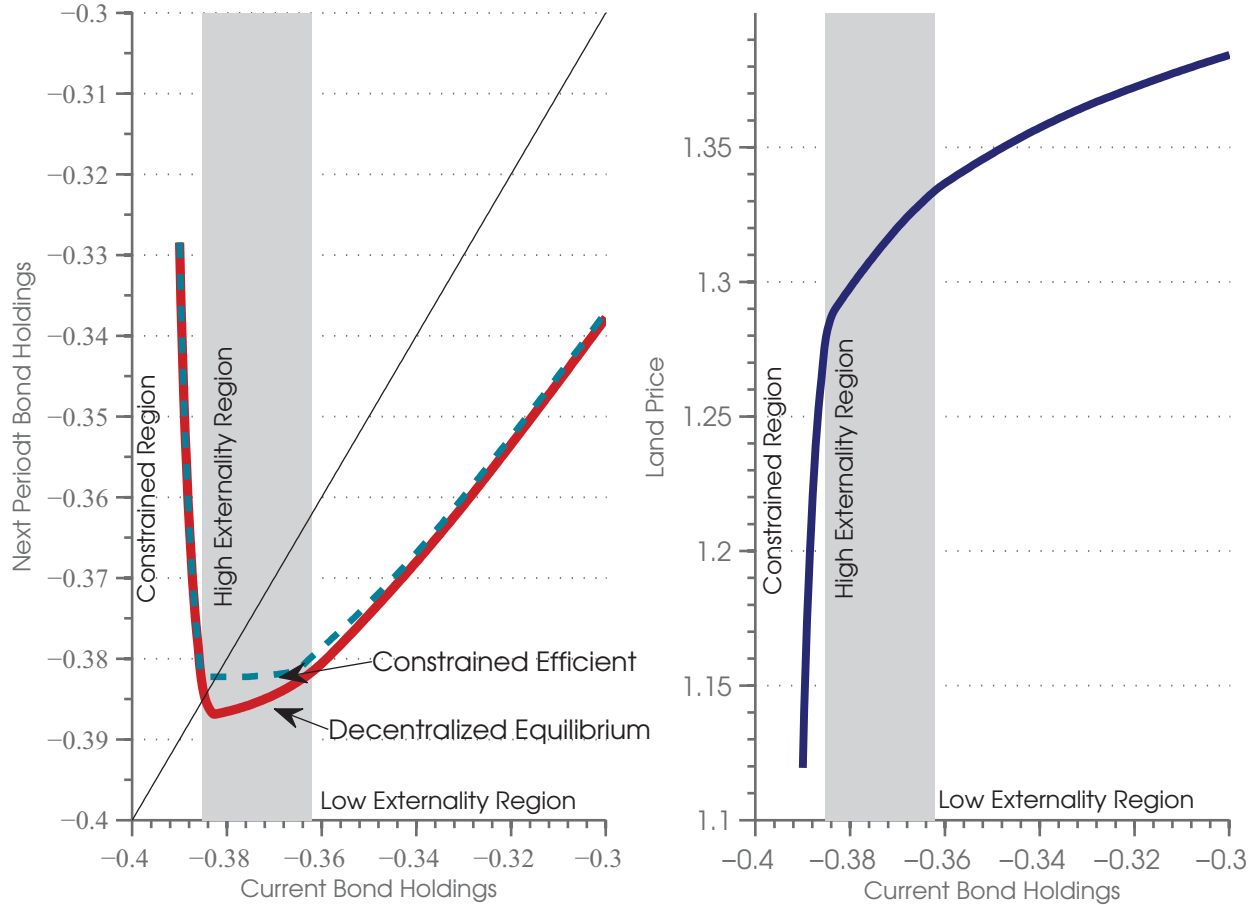
The two panels of Figure 1 show the bond decision rules ( $b'$ ) of private agents and the social planner as a function of  $b$  (left panel) as well as the pricing function for land (right panel), both for a negative two-standard-deviations TFP shock. The key point is to note that the Fisherian deflation mechanism generates non-monotonic bond decision rules, instead of the typical monotonically increasing decision rules. The point at which bond decision rules switch slope corresponds to the value of  $b$  at which the collateral constraint holds with equality but does not bind. To the right of this point, the collateral constraint does not bind and the bond decision rules are upward-sloping. To the left of this point, the bond decision rules are *decreasing* in  $b$ , because a reduction in current bond holdings results in a sharp reduction in the price of land, as can be seen in the right panel, and tightens the borrowing constraint, thus increasing  $b'$ .

As in Bianchi (2009), we can separate the bond decision rules in the left panel of Figure 1 into three regions: a “constrained region,” a “high-externality region” and a “low-externality region.” The “constrained region” is given by the range of  $b$  in the horizontal axis with sufficiently high initial debt (i.e. low  $b$ ) such that the collateral constraint binds in the constrained-efficient equilibrium. This is the range with  $b \leq -0.385$ . In this region, the collateral constraint binds in both constrained-efficient and competitive equilibria, because the credit externality implies that the constraint starts binding at higher values of  $b$  in the latter than in the former, as we show below.

By construction, *the total amount of debt* (i.e. the sum of bond holdings and working capital) in the constrained region is the same under the constrained-efficient allocations and the competitive equilibrium. If working capital were not subject to the collateral constraint, the two *bond decision rules* would also be identical. But with working capital in the constraint the two can differ. This is because the effective cost of labor differs between the two equilibria, since the increase in the marginal financing cost of labor when the constraint binds,  $\theta\mu_t/u'(t)$ , is different. These differences, however, are very small in the numerical experiments, and thus the bond decision rules are approximately the same in the constrained region.<sup>14</sup>

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<sup>14</sup>The choice of  $b'$  becomes slightly higher for the social planner as  $b$  gets closer to the upper bound of the constrained region, because the deleveraging that occurs around this point is small enough for the probability of a binding credit constraint next period to be strictly positive. As a result, for given allocations, conditions (15) and (6) imply that  $\mu$  is lower in the constrained-efficient allocations.



**Figure 1:** Bond Decision Rules (left panel) and Land Pricing Function (right panel) for a Negative Two-standard-deviations TFP Shock

The high-externality region is located to the right of the constrained region, and it includes the interval  $-0.385 < b < -0.363$ . Here, the social planner chooses uniformly higher bond positions (lower debt) than private agents, because of the different incentives driving the decisions of the two when the constrained region is near. In fact, private agents hit the credit constraint at  $b = -0.383$ , while at that initial  $b$  the social planner still retains some borrowing capacity. Moreover, this region is characterized by “financial instability,” in the sense that the levels of debt chosen for  $t + 1$  are high enough so that a negative TFP shock of standard magnitude in that period can lead to a binding credit constraint that leads to large falls in consumption, output, land prices and credit. We will show later that this is also the region of the state space in which the planner uses actively its macro-prudential policy to manage the inefficiencies of the competitive equilibrium.

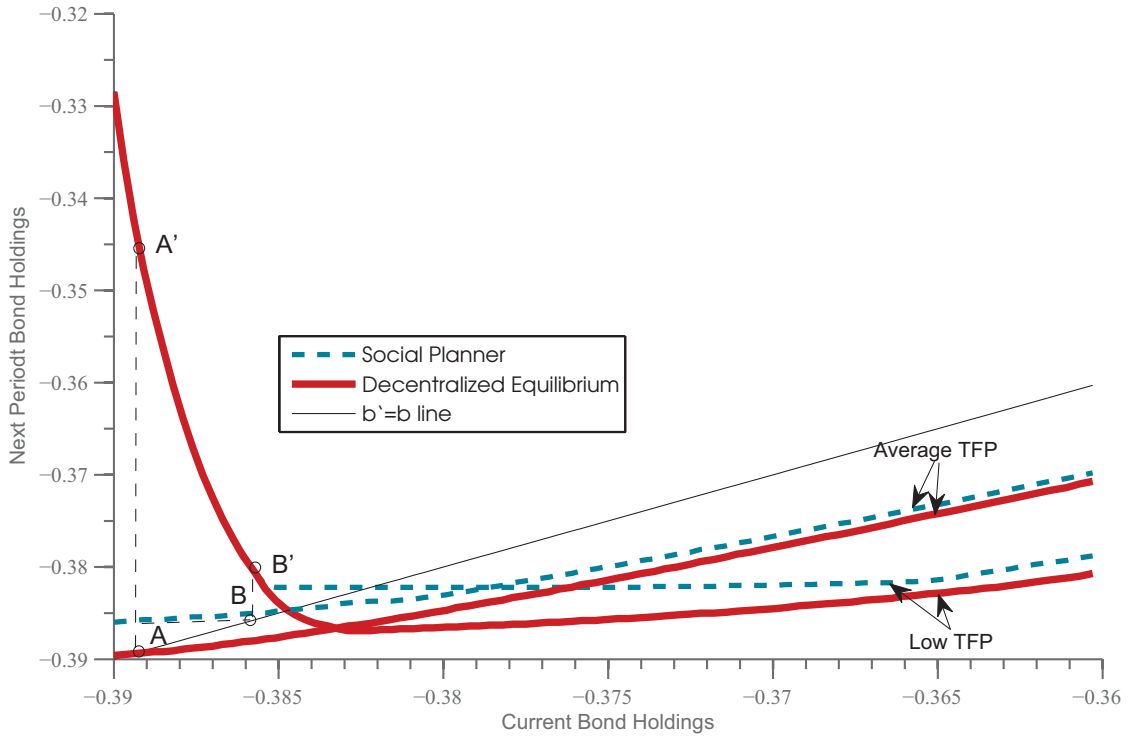
The low-externality region is the interval for which  $b \geq -0.363$ . In this region, the probability of a binding constraint next period is zero for both the social planner and the competitive equilibrium. The bond decision rules still differ, however, because expected marginal utilities differ for the two equilibria. But the social planner does not set a tax on debt, because negative shocks cannot lead to a binding credit constraint in the following period.

The long-run probabilities with which the constrained-efficient (competitive) economy visits the three regions of the bond decision rules are 2 (4) percent for the constrained region, 69 (70) percent for the high-externality region, and 29 (27) percent for the low-externality region. Both economies spend more than 2/3rds of the time in the high-externality region, but the prudential actions of the social planner reduce the probability of entering in the constrained region by a half. Later we will show that this is reflected also in financial crises that are much less frequent and less severe than in the competitive equilibrium.

The larger debt (i.e. lower bond) choices of private agents relative to the social planner, particularly in the high-externality region, constitute our first measure of the overborrowing effect at work in the competitive equilibrium. The social planner accumulates extra precautionary savings above and beyond what private individuals consider optimal in order to self-insure against the risk of financial crises. This effect is quantitatively small in terms of the difference between the two decision rules, but this does not mean that its macroeconomic effects are negligible. Later in this Section we illustrate this point by comparing financial crises events in the two economies. In addition, the fact that small differences in borrowing decisions lead to major differences when a crisis hits can be illustrated using Figure 2 to study further the dynamics implicit in the bond decision rules.

Figure 2 shows bond decision rules for the social planner and the competitive equilibrium over the range  $(-0.39, -0.36)$  for two TFP scenarios: average TFP and TFP two-standard-deviations below the mean. The ray from the origin is the  $b' = b$  line. We use a narrower range than in Figure 1 to “zoom in” and highlight the differences in decision rules. Assume both economies start at a value of  $b$  such that at average TFP the debt of agents in the competitive equilibrium would remain unchanged (this is point  $A$  with  $b = -0.389$ ). If the TFP realization is indeed the average, private agents in the decentralized equilibrium keep

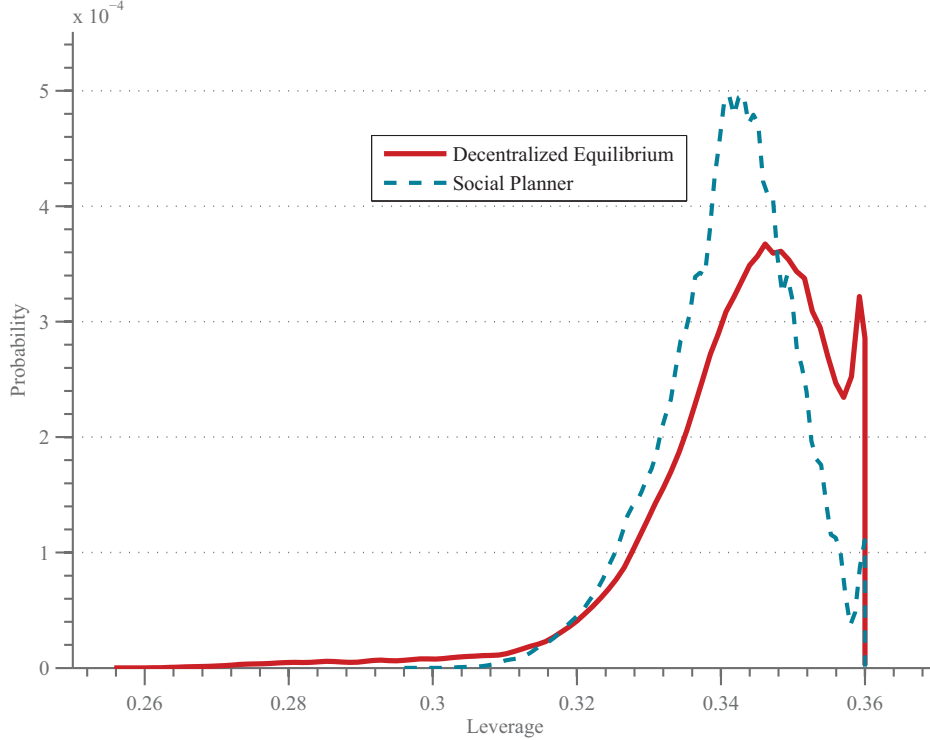




**Figure 2:** Comparison of Debt Dynamics

that level of debt. On the other hand, the social planner builds precautionary savings and reduces its debt to point  $B$  with  $b = -0.386$ . Hence, the next period the two economies start at the debt levels in  $A$  and  $B$  respectively. Assume now that at this time TFP falls by two standard deviations. Now we can see the large dynamic implications of the small differences in the bond decision rules of the two economies: The competitive equilibrium suffers a major correction caused by the Fisherian deflation mechanism. The collateral constraint becomes binding and the economy is forced to a large deleveraging that results in a sharp reduction in debt (an increase in  $b$  to  $-0.347$  at point  $A'$ ). Consumption falls leading to a drop in the stochastic discount factor and a drop in asset prices. In contrast, the social planner, while also facing a binding credit constraint, adjusts its debt marginally to just about  $b = -0.379$  at point  $B'$ . This was possible for the social planner because, taking into account the risk of a Fisherian deflation *and* internalizing its price dynamics, the planner chose to borrow less than agents in the decentralized equilibrium a period earlier.

Overborrowing can also be assessed by comparing the long-run distributions of debt and



**Figure 3:** Ergodic Distribution of Leverage ( $\frac{-b_{t+1} + \theta w_t h_t}{q_t K}$ )

leverage across the competitive and constrained-efficient equilibria. The fact that the planner accumulates more precautionary savings implies that its ergodic distribution concentrates less probability at higher leverage ratios than in the competitive equilibrium. Figure 3 shows the ergodic distributions of leverage ratios (measured as  $\frac{-b_{t+1} + \theta w_t h_t}{q_t K}$ ) in the two economies. The maximum leverage ratio in both economies is given by  $\kappa$  but notice that the decentralized equilibrium concentrates higher probabilities in higher levels of leverage. Comparing averages across these ergodic distributions, however, mean leverage ratios differ by less than 1 percent. Hence, overborrowing is relatively small again if measured by comparing differences in unconditional long-run averages of leverage ratios.<sup>15</sup>

### 4.3 Asset Returns

Overborrowing has important quantitative implications for asset returns and their determinants. Figure 4 shows the long-run distributions of land returns for the competitive

<sup>15</sup>Measuring “ex ante” leverage as  $\frac{-b_t + \theta w_t h_t}{q_t K}$ , we find that leverage ratios in the competitive equilibrium can exceed the maximum of those for the planner 3 percent of the time and by up to 12 percentage points.

equilibrium and the social planner. The key difference in these distributions is that the one for the competitive equilibrium features fatter tails. In particular, there is a sharply fatter left tail in the competitive equilibrium, for which the 99th percentile of returns is about -17.5 percent, v. -1.6 percent in the constrained-efficient equilibrium. The fatter left tail in the competitive equilibrium corresponds to states in which a negative TFP shock hits when agents have a relatively high level of debt. Intuitively, as a negative TFP shock hits, expected dividends decrease and this puts downward pressure on asset returns.<sup>16</sup> In addition, if the collateral constraint becomes binding, asset fire-sales lead to a further drop in asset prices.

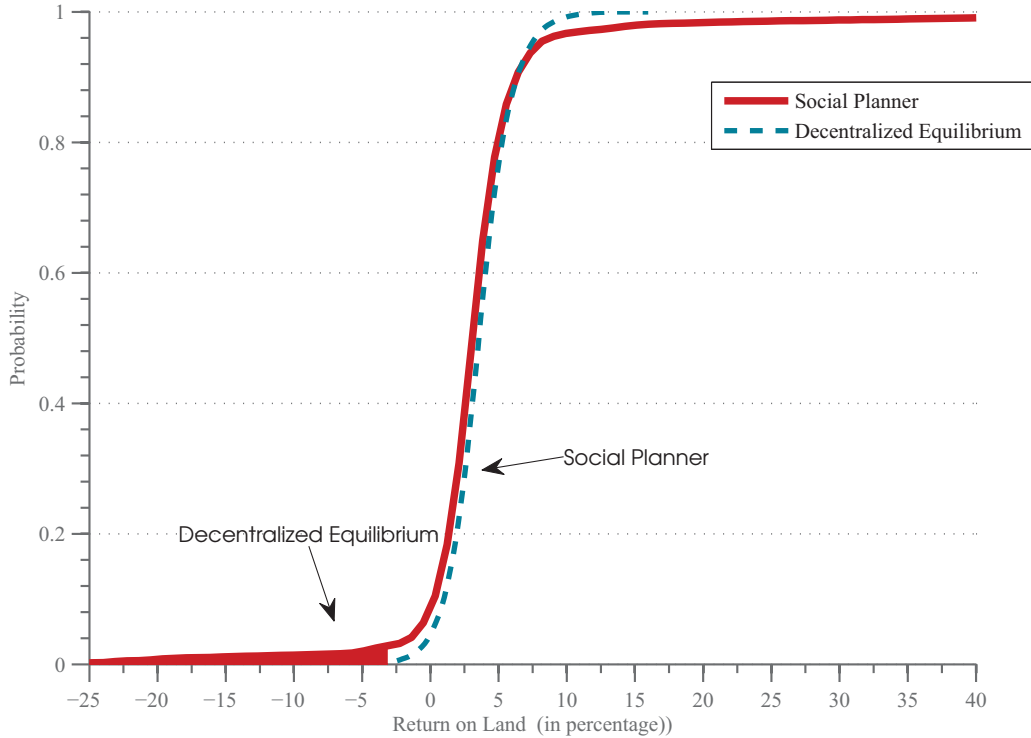
We show below that the fatter tails of the distribution of asset returns, and the associated time-varying risk of financial crises, have substantial effects on the risk premium. These features of our model are similar to those examined in the literature on asset pricing and “disasters” (see Barro, 2009). Note, however, that this literature generally treats financial disasters as resulting from exogenous stochastic processes with fat tails and time-varying volatility, whereas in our setup financial crises and their time-varying risk are both endogenous.<sup>17</sup> The underlying shocks driving the model are standard TFP shocks, even in periods of financial crises. In our model, as in Mendoza (2010), financial crises are endogenous outcomes that occur when shocks of standard magnitudes trigger a Fisherian deflation. Table 2 reports statistics that characterize the main properties of asset returns in the constrained-efficient and competitive equilibria. We also report statistics for a competitive equilibrium in which land in the collateral constraint is valued at a fixed price equal to the average price across the ergodic distribution  $\bar{q}$  (i.e. the credit constraint becomes  $-\frac{b_{t+1}}{R_t} + \theta w_t n_t \leq \kappa \bar{q} k_{t+1}$ ).<sup>18</sup> This fixed-price scenario allows us to compare the properties of asset returns in the competitive and social planner equilibria with a setup in which a collateral constraint exists but the Fisherian deflation channel and the credit externality are removed.

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<sup>16</sup>Similarly, the fatter right-tail in the distribution of returns of the competitive equilibrium corresponds to periods with positive TFP shocks, which were preceded by low asset prices due to fire sales.

<sup>17</sup>The literature on disasters typically uses Epstein-Zin preferences so as to be able to match the large observed equity premia. Here we use standard CRRA preferences with a risk aversion coefficient of 2, and as we show later, we can obtain larger risk premia than in the typical CRRA setup without credit frictions. Moreover, we obtain realistically large risk premia when the credit constraint binds.

<sup>18</sup> Because the asset is in fixed supply, these allocations would be the same if we use instead an ad-hoc borrowing limit such that  $-\frac{b_{t+1}}{R_t} + \theta w_t n_t \leq \kappa \bar{q} \bar{K}$ . The price of land, however, would be lower since with the ad-hoc borrowing constraint land does not have collateral value.



**Figure 4:** Ergodic Cumulative Distribution of Land Returns

Table 2 lists expected excess returns (i.e. the equity risk premia), the direct and indirect (covariance) effects of the credit constraint on excess returns, the (log) standard deviation of returns, the price of risk, and the Sharpe ratio. These moments are reported for the unconditional long-run distributions of each model economy, as well as for distributions conditional on the collateral constraint being binding and not binding.

The mean unconditional excess return is 1.09 percent in the competitive equilibrium v. only 0.17 percent in the constrained-efficient equilibrium and 0.86 percent in the fixed-price economy. The risk premium in the competitive equilibrium is large, about half as large as the risk-free rate. The fact that the other two economies produce lower premia indicates that the high premium of the competitive equilibrium is the combined result of the Fisherian deflation mechanism and the inefficiencies induced by the credit externality. Note also that the high premium produced by our model contrasts sharply with the findings of Heaton and Lucas (1996), who found that credit frictions without the Fisherian deflation mechanism do

not produce large premia, unless transactions costs are very large.<sup>19</sup>

The excess returns conditional on the collateral constraint not binding in the constrained-efficient and fixed-price economies are in line with those obtained in classic asset pricing models that display the “equity premium puzzle.” The equity premia we obtained in these two scenarios are driven only by the covariance effect, as in the classic models, and they are negligible: 0.03 percent in the fixed-price economy and 0.06 percent in the constrained-efficient economy. This is natural because, without the constraint binding and with the effects of the credit externality and the Fisherian deflation removed or weakened, the model is in the same class as those that display the equity premium puzzle. In contrast, our baseline competitive economy yields a 0.23 percent premium conditional on the constraint not binding, which is small relative to data estimates that range from 6 to 18 percent, but 4 to 8 times larger than in the other two economies.

Conditional on the collateral constraint being binding, mean excess returns in the competitive equilibrium are nearly 14 percent, 4.86 percent for the social planner, and 1.29 percent in the fixed-price economy. Interestingly, the lowest unconditional premium is the one for the constrained-efficient economy (0.17 percent), but conditional on the constraint binding, the lowest premium is the one for the fixed-price economy (1.29 percent). This is because on one hand the Fisherian deflation effect is still at work when the collateral constraint binds in the constrained-efficient economy, but not in the fixed-price economy, while on the other hand the constrained-efficient economy has a lower probability of hitting the collateral constraint (so that the higher premium when the constraint binds does not weigh heavily when computing the unconditional average). In turn, the probability of hitting the collateral constraint is higher for the fixed-price economy, because the incentive to build precautionary savings is weaker when there is no Fisherian amplification.

The unconditional direct and covariance effects of the collateral constraint on excess returns are significantly stronger in the competitive equilibrium than in the constrained-efficient and fixed-price economies, and even more so if we compare them conditional on the

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<sup>19</sup>The unconditional premium in the fixed price economy, at 0.86 percent, is not trivial, but note that it results from the fact that the constraint binds with very high probability, given the smaller incentives to accumulate precautionary savings. The risk premium in the unconstrained region of the fixed-price model is only 0.03 percent, v. 0.23 in our baseline model.

**Table 2:** Asset Pricing Moments

	Excess Return	Direct Effect	Covariance Effect	$s_t$	$\sigma_t(R_{t+1}^q)$	$S_t$
<b>Decentralized Equilibrium</b>						
Unconditional	1.09	0.87	0.22	5.22	3.05	0.79
Constrained	13.94	13.78	0.16	4.05	2.71	11.75
Unconstrained	0.23	0.00	0.23	5.3	3.08	0.05
<b>Constrained-Efficient Equilibrium</b>						
Unconditional	0.17	0.11	0.06	2.88	1.85	0.08
Constrained	4.86	4.80	0.06	3.02	2.07	2.38
Unconstrained	0.06	0.00	0.06	2.86	1.84	0.03
<b>Fixed Price Equilibrium</b>						
Unconditional	0.86	0.82	0.04	2.59	1.69	0.46
Constrained	1.29	1.23	0.05	2.81	1.84	0.69
Unconstrained	0.03	0.00	0.03	2.16	1.39	0.02

Note: The table reports averages of the conditional excess return after taxes, the direct effect, the covariance effect, the price of risk  $s_t$ , the (log) volatility of the return of land denoted  $\sigma_t(R_{t+1}^q)$ , and the Sharpe ratio. All numbers except the Sharpe ratios are in percentage

constraint being binding. Again, the direct and covariance effects are larger in the competitive equilibrium because of the effects of the overborrowing externality and the Fisherian deflation mechanism.

In terms of the decomposition of excess returns based on condition (10), Table 2 shows that the unconditional average of the price of risk is about twice as large in the decentralized equilibrium than in the constrained-efficient and fixed-price economies. This reflects the fact that consumption, and therefore the pricing kernel, fluctuate significantly more in the decentralized equilibrium. The Sharpe ratio and the variability of land returns are also much larger in the competitive equilibrium. The increase in the former indicates, however, that the mean excess return rises significantly more than the variability of returns, which indicates that risk-taking is “overcompensated” in the competitive equilibrium (relative to the compensation it receives when the social planner internalizes the credit externality). Note also that the correlations between land returns and the stochastic discount factor, not shown in the Table, are very similar under the three equilibria and very close to 1. This is important because it implies that the differences in excess returns and Sharpe ratios cannot

be attributed to differences in this correlation.

#### 4.4 Incidence and Magnitude of Financial Crises

We show now that overborrowing in the competitive equilibrium increases the incidence and severity of financial crises. To demonstrate this result we construct an event analysis of financial crises with simulated data obtained by performing long (100,000-period) stochastic time-series simulations of the competitive, constrained-efficient and fixed-price economies, removing the first 1,000 periods. A financial crisis episode is defined as a period in which the credit constraint binds and this causes a decrease in credit that exceeds one standard deviation of the first-difference of credit in the corresponding ergodic distribution.

The event analysis exercise is important also because it sheds light on whether the model can produce financial crises with realistic features, which is a key first step in order to make the case for treating the normative implications of the model as relevant. We show here that, while we did not aim to build a rich equilibrium business cycle model so we could keep the analysis of the externality tractable, and hence our match to the data is not perfect, the model does produce financial crises with realistic features in terms of abrupt, large declines in allocations, credit, and land prices, and it supports non-crisis output fluctuations in line with observed U.S. business cycles. Moreover, studies more focused on matching data from financial crisis events have shown that the Fisherian deflation mechanism can do well at explaining crisis dynamics nested within realistic long-run business cycle co-movements (see Mendoza (2010)).

The first important result of the event analysis is that the incidence of financial crises is significantly higher in the competitive equilibrium. We calibrated  $\kappa$  so that the competitive economy experiences financial crises with a long-run probability of 3.0 percent. But with the same  $\kappa$ , financial crises occur in the constrained-efficient economy only with 0.9 percent probability in the long run. Thus, the credit externality increases the frequency of financial crises by a factor of 3.33.<sup>20</sup>

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<sup>20</sup>We could also define crises in the constrained-efficient equilibrium by using the value of the credit threshold obtained in the competitive equilibrium. However, with this criterion we would obtain an even lower probability of crises, because credit declines equal to at least one standard deviation of the first-difference of credit in the decentralized equilibrium are zero-probability events in the constrained efficient equilibrium.

The second important result is that financial crises are more severe in the competitive equilibrium. This is illustrated in the event analysis plots shown in Figure 5. The event windows are for total credit, consumption, labor, output, TFP and land prices, all expressed as deviations from long-run averages. These event dynamics are shown for the decentralized, constrained-efficient, and fixed-price economies.

We construct comparable event windows for the three scenarios following this procedure: First we identify financial crisis events in the competitive equilibrium, and isolate five-year event windows centered in the period in which the crisis takes place. That is, each event window includes five years, the two years before the crisis, the year of the crisis, and the two years after. Second, we calculate the median TFP shock across all of these event windows in each year  $t - 2$  to  $t + 2$ , and the median initial debt at  $t - 2$ . This determines an initial value for bonds and a five-year sequence of TFP realizations. Third, we feed this sequence of shocks and initial value of bonds to the decisions rules of each model economy and compute the corresponding endogenous variables plotted in Figure 5. By proceeding in this way, we ensure that the event dynamics for the three equilibria are simulated using the same initial state and the same sequence of shocks.<sup>21</sup>

The features of financial crises at date  $t$  in the competitive economy are in line with the results in Mendoza (2010): The debt-deflation mechanism produces financial crises characterized by sharp declines in credit, consumption, asset prices and output.

The five macro variables illustrated in the event windows show similar dynamics across the three economies in the two years before the financial crisis. When the crisis hits, however, the collapses observed in the competitive equilibrium are much larger. Credit falls about 20 percentage points more, and two years after the crisis the credit stock of the competitive equilibrium remains 10 percentage points below that of the social planner.<sup>22</sup> Consumption, asset prices, and output also fall much more sharply in the competitive equilibrium than in the planner's equilibrium. The declines in consumption and asset prices are particularly

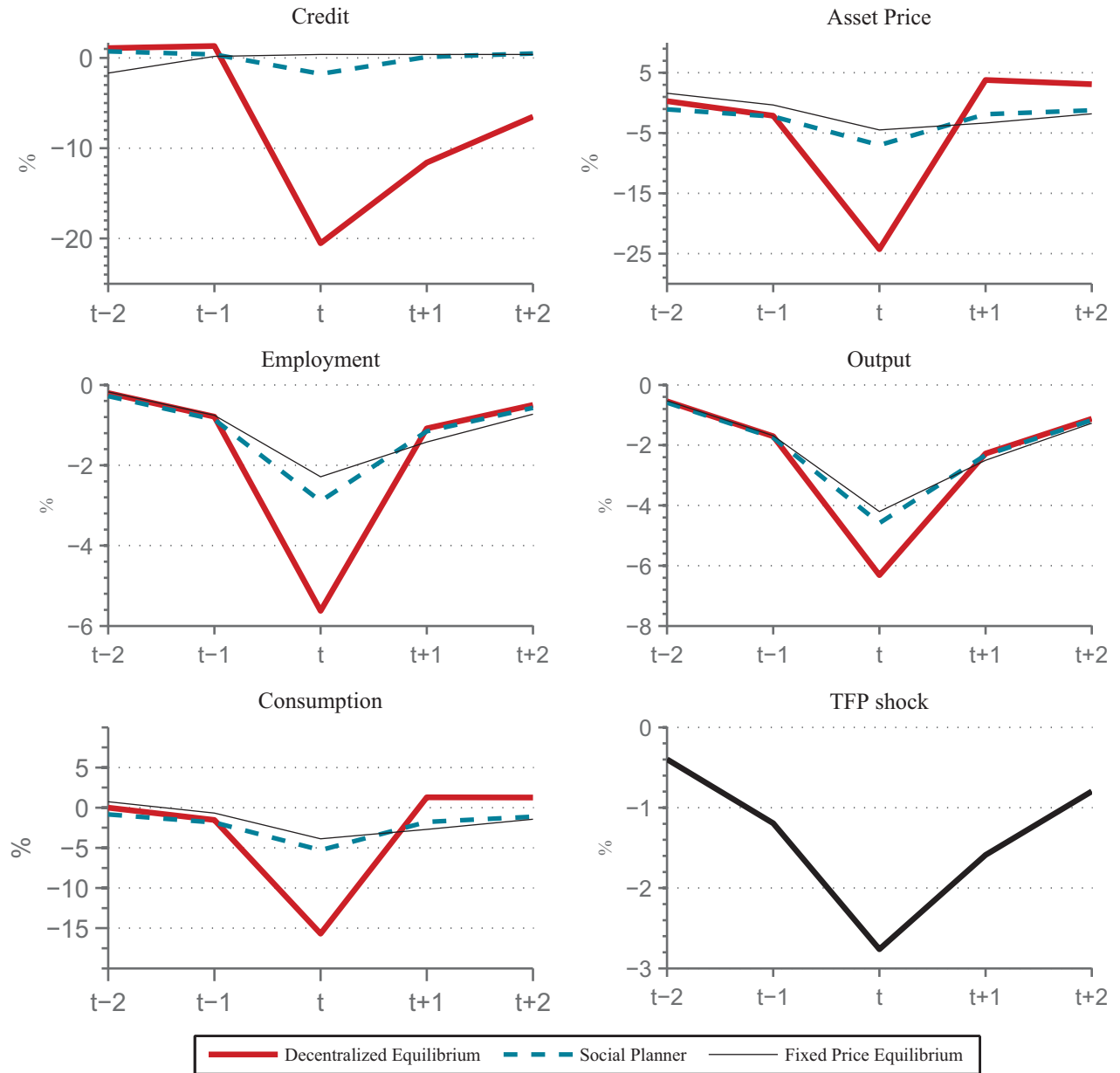
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<sup>21</sup>The sequence of TFP shocks is 0.9960, 0.9881, 0.9724, 0.9841, 0.9920 and the initial level of debt is 1.6 percent above the average.

<sup>22</sup>The model overestimates the drop in credit relative to what we have observed so far in the U.S. crisis (which as of the third quarter of 2010 reached about 7 percent of GDP) . One reason for this is that in the model, credit is in the form of one-period bonds, whereas in the data, loans have on average a much larger maturity. In addition, our model does not take into account the strong policy intervention that took place with the aim to prevent what would have been a larger credit crunch.



larger (-16 percent v. -5 percent for consumption and -24 percent v. -7 percent for land prices). The asset price collapse also plays an important role in explaining the more pronounced decline in credit in the competitive equilibrium, because it reflects the outcome of the Fisherian deflation mechanism. Output falls by 2 percentage points more, and labor falls almost 3 percentage points more, because of the higher shadow cost of hiring labor due to the effect of the tighter binding credit constraint on access to working capital.



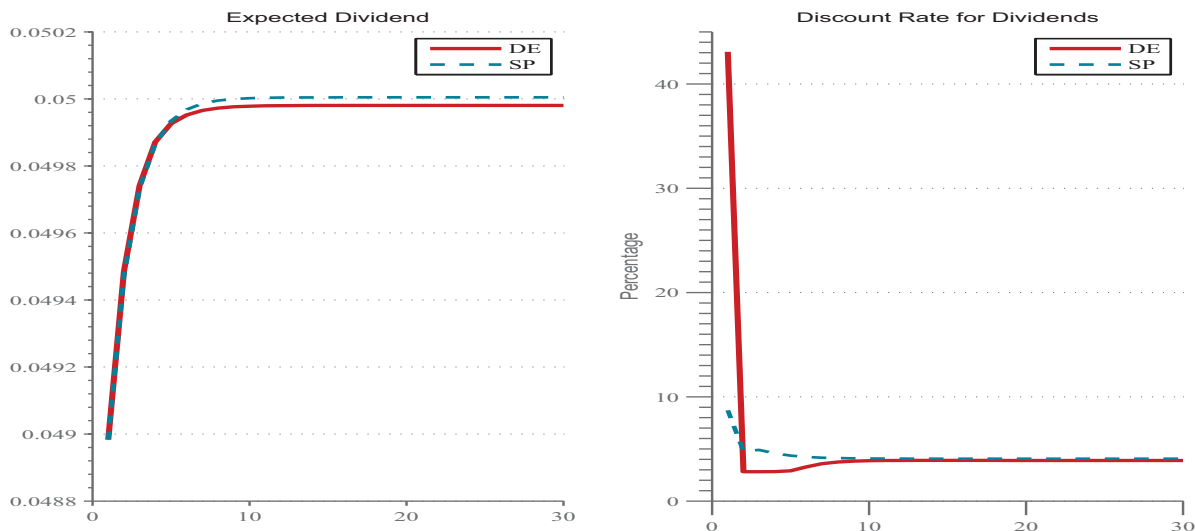
**Figure 5:** Event Analysis: percentage differences relative to unconditional averages

The event analysis results can also be used to illustrate the relative significance of the wage and land price components in the externality term  $\psi_t \equiv \kappa \bar{K} \frac{\partial q_t}{\partial b_t} - \theta n_t \frac{\partial w_t}{\partial b_t}$  identified in condition (15). Given the unitary Frisch elasticity of labor supply, wages decrease one-to-one with labor (and hence the event plot for wages would be identical to the one shown for employment in Figure 5). As a result, the extent to which the drop in wages can help relax the collateral constraint is very limited. Wages and employment fall about 6 percent at date  $t$ , and with a working capital coefficient of  $\theta = 0.14$ , this means that the effect of the drop in wages in the borrowing capacity is  $0.14(1 - 0.06)0.06 = 0.79$  percent. On the other hand, given that  $\bar{K} = 1$  and that asset prices fall about 25 percent below trend at date  $t$ , and since  $\kappa = 0.36$ , the effect of land fire sales on the collateral constraint is  $0.36(0.25) = 9$  percent. Thus, the land price effect of the externality is about 10 times bigger than the wage effect. This finding will play an important role in our quantitative analysis of the features of the macro-prudential policy later in this section.

The fixed-price economy displays very little amplification given that the economy is free from the Fisherian deflation mechanism. Credit increases slightly at date  $t$  in order to smooth consumption and remains steady in the following periods. The fact that land is valued at the average price, and not the market price, contributes to mitigate the drop in the price of land, since it remains relatively more attractive as a source of collateral.

To gain more intuition on why land prices drop more because of the credit externality, we plot in Figure 6 the projected conditional sequences of future dividends and land returns on land up to 30 periods ahead of a financial crisis that occurs at date  $t = 0$  (conditional on information available on that date). These are the sequences of dividends and returns used to compute the present values of dividends that determine the equilibrium land price at  $t$  in the event analysis of Figure 5. The expected land returns start very high when the crisis hits in both competitive and constrained-efficient equilibria, but significantly more for the former (at about 40 percent) than the latter (at 10 percent). On the other hand, expected dividends do not differ significantly, and therefore we conclude that the sharp change in the pricing kernel reflected in the surge in projected land returns when the crisis hits is what drives the large differences in the drop of asset prices.

The large deleveraging that takes places when a financial crisis occurs in the competitive



**Figure 6:** Forecast of expected dividends and land returns.

equilibrium implies that projected land returns for the immediate future (i.e. the first 6 periods after the crisis) drop significantly. Returns are also projected to fall for the social planner, but at a lower pace, so that in fact the planner projects higher land returns than agents in the competitive equilibrium for a few periods. Projected dividends for the same immediate future after the crisis are slightly smaller than the long-run average of 0.05 in both economies because of the persistence of the TFP shock. In the long-run, expected dividends are slightly higher for the social planner, because the marginal productivity of land drops less during the financial crisis as a result of the lower amount of debt. Notice also that the planner projects to discount dividends with a slightly higher land return in the long run, because the tax on debt more than offsets the fact that the risk premium of the planner is lower (recall that we are comparing after-tax returns as defined in Section 2). This arises because the tax on debt makes bonds relatively more attractive and this leads in equilibrium to a higher required return on land.

#### 4.5 Long-Run Business Cycles

Table 3 reports the long-run business cycle moments of the competitive, constrained-efficient and fixed-price equilibria, which are computed using each economy's ergodic distribution. The credit externality at work in the competitive equilibrium produces higher business cycle variability in output and labor, and especially in consumption, compared

with the constrained-efficient and fixed-price economies. The high variability of consumption and credit are consistent with the results in Bianchi (2009), but we find in addition that the credit externality produces a moderate increase in the variability of labor and a substantial increase in the variability of land prices and leverage. Notice that the variability in consumption is higher than the variability of output in the decentralized equilibrium which is not the case in U.S. data. However, if we exclude the crisis periods, the ratio of the variability of consumption to the variability of GDP would be 0.87 (compared with 0.88 in annual U.S. data from 1960 to 2007).

It may seem puzzling that we can obtain non-trivial differences in long-run business cycle moments even though financial crises are a low probability event in the competitive equilibrium. To explain this result, it is useful to go back to Figure 1. This plot shows that even during normal business cycles the optimal plans of the competitive and constrained-efficient equilibria differ, and this is particularly the case in the high-externality region. Because the economy spends about 70 percent of the time in this region, where private agents borrow more and are more exposed to the risk of financial crises, long-run business cycle moments differ. In addition, the larger effects that occur during crises have a non-trivial effect on long-run moments. This is particularly noticeable in the case of consumption where the variability drops from 2.7 to 1.7 in the decentralized equilibrium when we exclude the crises episodes.

The business cycle moments for consumption, output and labor in the constrained-efficient economy are about the same as those of the fixed-price economy. This occurs even though the constrained-efficient economy is subject to the Fisherian deflation mechanism and the fixed-price economy is not. The reason for this is because the social planner accumulates extra precautionary savings, which compensate for the sudden change in the borrowing ability when the credit constraint binds. The constraint binds less often and when it does it has weak effects on macro variables. On the other hand, the constrained-efficient economy does display lower variability in leverage and land prices than the fixed-price economy, and this occurs because the social planner internalizes how a drop in the price tightens the collateral constraint. The output correlations of leverage, credit, and land prices also differ significantly across the model economies. The GDP correlations of leverage and credit are

**Table 3:** Long Run Moments

	Standard Deviation			Correlation with GDP			Autocorrelation		
	DE	SP	FP	DE	SP	FP	DE	SP	FP
Output	2.10	1.98	1.97	1.00	1.00	1.00	0.50	0.51	0.51
Consumption	2.71	1.87	1.85	0.86	0.99	0.99	0.23	0.56	0.57
Employment	1.25	1.02	0.98	0.97	1.00	1.00	0.42	0.50	0.51
Leverage	3.92	2.72	3.80	-0.57	-0.93	-0.95	0.59	0.69	0.71
Total Credit	3.55	0.95	0.76	0.27	-0.35	-0.42	0.58	0.77	0.81
Land Price	3.95	2.24	3.48	0.79	0.97	0.97	0.16	0.56	0.60
Working capital	2.48	2.04	1.97	0.97	1.00	1.00	0.42	0.50	0.51

Note: ‘DE’ represents the decentralized equilibrium, ‘SP’ represents the social planner, ‘FP’ represents an economy with land valued at a fixed price equal to the average of the price of land in the competitive equilibrium.

significantly higher in the competitive equilibrium, while the correlation between the price of land and GDP is lower. The model without credit frictions would have a natural tendency to produce countercyclical credit because consumption-smoothing agents want to save in good times and borrow in bad times. This effect still dominates in the constrained-efficient and fixed-price economies, but in the competitive equilibrium the collateral constraint and the Fisherian deflation hamper consumption smoothing enough to produce procyclical credit and a higher GDP-leverage correlation. Similarly, the GDP-land price correlation is nearly perfect when the Fisherian deflation mechanism is weakened (constrained-efficient case) or removed (fixed-price case), but falls to about 0.8 in the competitive equilibrium. Because of the strong procyclicality of land prices, leverage is countercyclical with a GDP correlation of -0.57. This is in line with the countercyclicality of household leverage in U.S. data, although the correlation is lower than in the data (the correlation between the ratio of net household debt to the value of residential land and GDP is -0.25 at the business cycle frequency).<sup>23</sup>

In terms of the first-order autocorrelations, the competitive equilibrium displays lower autocorrelations in all its variables compared to both constrained-efficient and fixed-price equilibria. This occurs because crises in the competitive equilibrium are characterized by

<sup>23</sup>Two caveats on this point. First, at lower frequencies the correlation is positive. As Boz and Mendoza (2010) report, the household leverage ratio rose together with GDP, land prices and debt between 1997 and 2007. Second, the countercyclicality of leverage for the household sector differs sharply from the strong procyclicality of leverage in the financial sector (see Adrian and Shin (2010)).

deep but not very prolonged recessions.

#### 4.6 Properties of Macro-prudential Policies

Table 4 shows the statistical moments that characterize the state-contingent schedules of taxes on debt and dividends by which the social planner decentralizes the constrained-efficient allocations as a competitive equilibrium. To make the two comparable, we express the dividend tax as a percent of the price of land.

The unconditional average of the debt tax is 1.07 percent, v. 0.09 when the constraint binds and 1.09 when it does not. The tax remains positive, albeit small, on average when the collateral constraint binds, because in some these states the social planner wants to allocate borrowing ability across bonds and working capital in a way that differs from the competitive equilibrium. If there is a positive probability that the credit constraint will bind again next period, the social planner allocates less debt capacity to bonds and more to working capital. As a result, a tax on debt remains necessary in a subset of the constrained region. Note, however, that these states are not associated with financial crisis events in our simulations. They correspond to events in which the collateral constraint binds but the deleveraging that occurs is not strong enough for a crisis to occur.

The debt tax fluctuates about 2/3rds as much as GDP and is positively correlated with leverage, i.e.  $\frac{-b_{t+1} + \theta w_t n_t}{q_t K}$ . This is consistent with the macro-prudential rationale behind the tax: The tax is high when leverage is building up and low when the economy is deleveraging. Note, however, that since leverage itself is negatively correlated with GDP, the tax also has a negative GDP correlation. When the constraint binds, the correlation between the tax and leverage is zero by construction, because leverage remains constant at the value of  $\kappa$ .

**Table 4:** Long Run Moments of Macro-prudential Policies

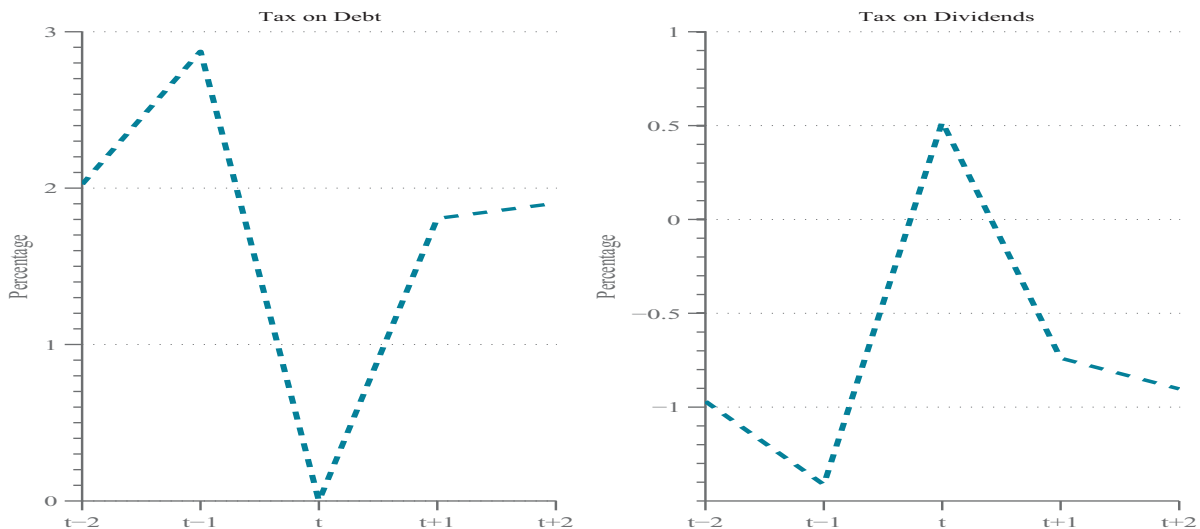
	Average		Standard Deviation		Correlation with Leverage	
	Debt Tax	Dividend Tax	Debt Tax	Dividend Tax	Debt Tax	Dividend Tax
Unconditional	1.07	-0.46	1.41	0.62	0.73	-0.64
Constrained	0.09	0.52	0.41	0.04	0.0	0.0
Unconstrained	1.09	-0.49	1.40	0.61	0.81	-0.79

The unconditional average of the dividend tax is negative (i.e. it is a subsidy), and it is very small at about -0.46 percent: when the constraint binds it is on average about 0.52 percent (v. a -0.49 percent average when the constraint does not bind). The fact that on average the planner requires a subsidy on dividends may seem puzzling, given that land is less risky in the regulated decentralized equilibrium, as we have shown above. There is another effect at work, however, because the debt tax puts downward pressure on land prices by making bonds relatively more attractive than land, and this effect turns out to be quantitatively larger. Thus, since by definition the constrained-efficient allocations are required to support the same land pricing function of the competitive economy without policy intervention, the planner calls for a dividend subsidy on average in order to offset the effect of the debt tax on land prices. The variability of the tax on dividends is 0.62 percentage points, less than 1/3rds the variability of GDP. The correlation between this tax on dividends and leverage is negative in the unconstrained region reflecting the negative correlation between the tax on debt and the tax on land explained above.

The dynamics of the debt and dividend taxes around crisis events are shown in Figure 7. The debt tax is high relative to its average, at about 2.7 percent, at  $t - 2$  and  $t - 1$ , and this again reflects the macro-prudential nature of these taxes: Their goal is to reduce borrowing so as to mitigate the magnitude of the financial crisis if bad shocks occur. At date  $t$  the debt tax falls to zero, and it rises again at  $t + 1$  and  $t + 2$  to about 2 percent. The latter occurs because this close to the crisis the economy still remains financially fragile (i.e. there is still a non-zero probability of agents becoming credit constrained next period). The tax on dividends follows a similar pattern. Dividends are subsidized at a similar rate before and after financial crises events, but they are actually taxed when crises occur. The reason is again that the social planner needs to support the same pricing function of the competitive equilibrium that would arise without policy intervention. Hence, with the tax on debt falling to almost zero, there is pressure for land prices to be higher than what that pricing function calls for, and hence dividends need to be taxed to offset this effect.

The macro-prudential behavior of the debt tax is very intuitive and follows easily from the precautionary behavior of the planner we have described. On the other hand, the tax on dividends and its dynamic behavior seem less intuitive and harder to sell as a policy rule (i.e.

the notion of proposing to tax dividends at the through of a financial crisis is bound to be unpopular). The two policy instruments are required, however, in order to implement exactly the allocations of the constrained social planner as a decentralized competitive equilibrium. Moreover, the planner’s allocations are guaranteed to attain a level of welfare at least as high as that of the competitive equilibrium without macro-prudential policy, since this equilibrium remains feasible to the social planner. If one takes the debt tax and not the tax on dividends, one cannot guarantee this Pareto improvement. Indeed, we solved a variant of the model in which we introduced the optimal schedule of debt taxes but left the tax on dividends out, and found that average welfare is actually lower than without policy intervention by -0.02 percent. This occurs because welfare in the states of nature in which the constraint is already binding is lower than without policy intervention.<sup>24</sup> Hence, while our results may provide a justification for the use of macro-prudential policies, they also provide a warning because selective use of macro-prudential policies (i.e. partial implementation of the policy instruments indicated by the model) can reduce welfare in some states of nature. In this experiment this happens because the selective use of the debt tax without the tax on dividends lowers asset prices in some states of nature, and reduces welfare in those states by reducing the value of collateral.



**Figure 7:** Event Analysis: Macroprudential Policies

<sup>24</sup>If we reduce the debt tax we can obtain again average welfare gains, which again illustrates the interdependence of macroprudential policies.



Jeanne and Korinek (2010) also compute a schedule of macroprudential taxes on debt to correct a similar externality that arises because of a collateral constraint that depends on asset prices. In their constraint, however, the agents' borrowing capacity is determined by the aggregate level of assets and by a linear state- and time-invariant term (i.e. their borrowing constraint is defined as  $\frac{b_{t+1}}{R} \geq -\kappa q_t \bar{K} - \psi$ ). The fact that their constraint depends on aggregate rather than individual asset holdings, as in our model, matters because it implies that agents do not value additional asset holdings as a mechanism to manage their borrowing ability.<sup>25</sup> But more importantly, leaving aside this difference, they calibrate parameter values to  $\kappa = 0.046$ ,  $\psi = 3.07$  and  $q_t \bar{K} = 4.8$ , which imply that the effects of the credit constraint are driven mainly by  $\psi$ , and only less than 7 percent ( $0.07 = 0.046 * 4.8 / (0.046 * 4.8 + 3.07)$ ) of the borrowing ability depends on the value of asset holdings. As a result, the Fisherian deflation effect and the credit externality are weak, and hence they find that macroprudential policy lessens the macro effects of financial crises much less than in our setup. The asset price drop is reduced from 12.3 to 10.3 percent, and the consumption drop is reduced from 6.2 to 5.2 percent (compared with declines from 24 to 7 and 16 to 5 percent respectively in our model). Moreover, they model the stochastic process of dividends as an exogenous, regime-switching Markov chain such that the probability of a crisis (i.e. binding credit constraint) coincides with the probability of a bad realization of dividends, implying that the probability of busts is unaffected by macroprudential regulation. Thus, in their setup macro-prudential policy is much less effective at reducing the magnitude of financial crises and has no impact on their incidence.

#### 4.7 Welfare Effects

We move next to explore the welfare implications of the credit externality. To this end, we calculate welfare costs as compensating consumption variations for each state of nature that make agents indifferent between the allocations of the competitive equilibrium and the constrained-efficient allocations. Formally, for a given initial state  $(B, \varepsilon)$  at date 0, the

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<sup>25</sup>To illustrate this point, we recomputed our model assuming that the borrowing constraint depends on the aggregate value of assets, as in their setup. Because assets do not have individual value as collateral, asset prices drop even more during crises, and this leads private agents to accumulate more precautionary savings, which results in crises having zero-probability in the long-run under both competitive and constrained-efficient equilibria for our baseline calibration.

welfare cost is computed as the value of  $\gamma$  such that the following condition holds:

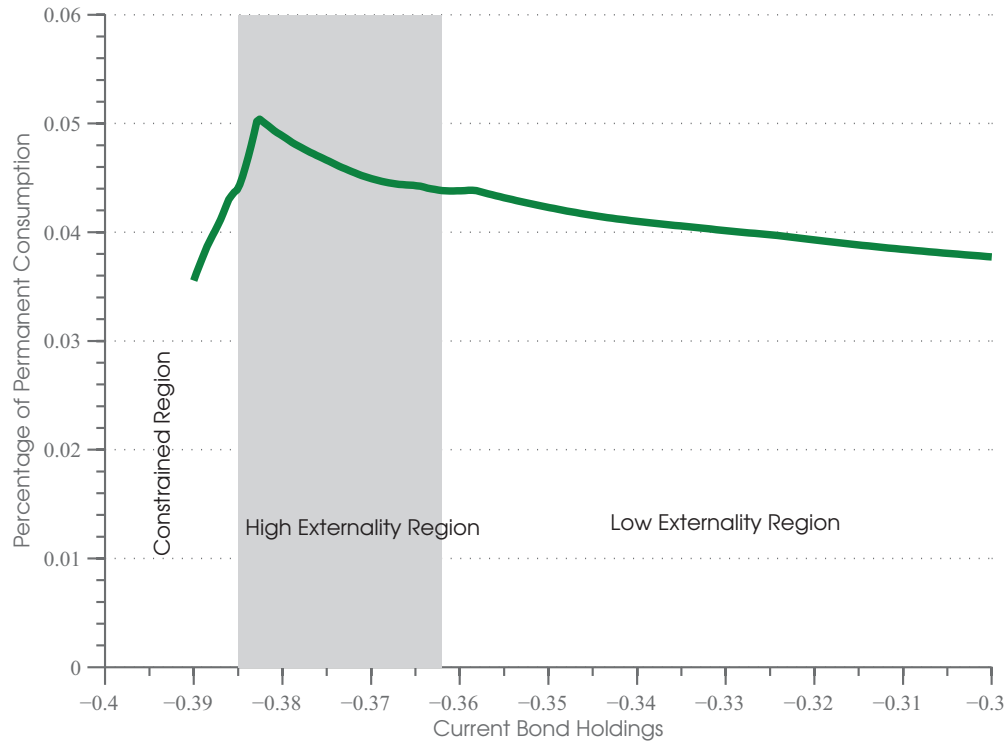
$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1 + \gamma) - G(n_t^{DE})) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(n_t^{SP})) \quad (22)$$

where the superscript DE denotes allocations in the decentralized competitive equilibrium and the superscript SP denotes the social planner's allocations. Note that these welfare costs reflect also the welfare gains that would be obtained by introducing the social planner's optimal debt and dividend tax policies, which by construction implement the constrained-efficient allocations as a competitive equilibrium.

The welfare losses of the DE arise from two sources. The first source is the higher variability of consumption, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. The second is the efficiency loss in production that occurs due to the effect of the credit friction on working capital. Without the working capital constraint, the marginal disutility of labor equals the marginal product of labor. With the working capital constraint, however, the shadow cost of employing labor rises when the constraint binds, and this drives a wedge between the marginal product of labor and its marginal disutility. Again, since the collateral constraint binds more often in the DE than in the SP, this implies a larger efficiency loss.

Figure 6 plots the welfare costs of the credit externality as a function of  $b$  for a negative, two-standard-deviations TFP shock. These welfare costs approximate a bell shape skewed to the left. This is due to the differences in the optimal plans of the social planner vis-a-vis private agents in the decentralized equilibrium. Recall that in the constrained region, the current allocations of the decentralized equilibrium essentially coincide with those of the constrained-efficient economy, as described in Figure 1. Therefore, in this region the welfare gains from implementing the constrained-efficient allocations only arise from how future allocations will differ. On the other hand, in the high-externality region, the constrained-efficient allocations differ sharply from those of the decentralized equilibrium, and this generally enlarges the welfare losses caused by the credit externality. Notice that, since the constrained-efficient allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the social planner, but these are far out-

weighed by less vulnerability to sharp decreases in future consumption during financial crises. Finally, as the level of debt is decreased further and the economy enters the low-externality region, financial crises are unlikely and the welfare costs of the inefficiency decrease.



**Figure 8:** Welfare Costs of the Credit Externality for a two-standard-deviations TFP Shock

The unconditional average welfare cost over the decentralized equilibrium’s ergodic distribution of bonds and TFP is 0.046 percentage points of permanent consumption. This contrasts with Bianchi (2009) who found welfare costs about 3 times larger. Note, however, that our results are in line with his if we express the welfare costs as a fraction of the variability of consumption. Consumption was more volatile in his setup because he examined a calibration to data for emerging economies, which are more volatile than the United States.

The fact that welfare losses from the externality are small although the differences in consumption variability are large is related to the well-known Lucas result that models with CRRA utility, trend-stationary income, and no idiosyncratic uncertainty produce low welfare costs from consumption fluctuations. Moreover, the efficiency loss in the supply-side when the constraint binds produces low welfare costs on average because those losses have a low

probability in the ergodic distribution.

#### 4.8 Sensitivity Analysis

We examine now how the quantitative effects of the credit externality change as we vary the values of the model's key parameters. Table 5 shows the main model statistics for different values of  $\sigma$ ,  $\kappa$ ,  $\omega$  and  $\theta$ . The Table shows the unconditional averages of the tax on debt and the welfare loss, the covariance effect on excess returns, the probability of financial crises, and the impact effects of a financial crisis on key macroeconomic variables. In all of these experiments, only the parameter listed in the first column changes and the rest of the parameters remain at their baseline calibration values.

The results of the sensitivity analysis reported in Table 5 can be understood more easily by referring to the externality term derived in Section 2: The wedge between the social and private marginal costs of debt that separate competitive and constrained-efficient equilibria,  $\beta RE_t \left[ \mu_{t+1} \left( \kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}} \right) \right]$ . For given  $\beta$  and  $R$ , the magnitude of the externality is given by the expected product of two terms: the shadow value of relaxing the credit constraint,  $\mu_{t+1}$ , and the associated price effects  $\kappa \bar{K} \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}}$  that determine the effects of the externality on the ability to borrow when the constraint binds. As explained earlier, the price effects are driven mostly by  $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ , because of the documented large asset price declines when the collateral constraint binds. It follows therefore, that the quantitative implications of the credit externality must depend mainly on the parameters that affect  $\mu_{t+1}$  and  $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ , as well as those that affect the probability of hitting the constraint.

The coefficient of relative risk aversion  $\sigma$  plays a key role because it affects both  $\mu_{t+1}$  and  $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ . A high  $\sigma$  implies a low intertemporal elasticity of substitution in consumption, and therefore a high value from relaxing the constraint since a binding constraint hinders the ability to smooth consumption across time. A high  $\sigma$  also makes the stochastic discount factors more sensitive to changes in consumption, and therefore makes the price of land react more to changes in bond holdings. Accordingly, rising  $\sigma$  from 2 to 2.5 rises the welfare costs of the credit externality by a factor of 5, and widens the differences in the covariance effects across the competitive and constrained-efficient equilibria. In fact, the covariance effect in the decentralized equilibrium increases from 0.22 to 0.37 whereas for the constrained

efficient allocations the increase is from 0.06 to 0.08. Stronger precautionary savings reduce the probability of crises in the competitive equilibrium, and financial crises become a zero-probability event in the constrained-efficient equilibrium. Conversely, reducing  $\sigma$  to 1 makes the externality extremely small, measured either by differences in the incidence or severity of financial crises.<sup>26</sup>

The collateral coefficient  $\kappa$  also plays an important role because it alters the effect of land price changes on the borrowing ability. A higher  $\kappa$  implies that, for a given price response, the change in the collateral value becomes larger. Thus, this effect makes the externality term larger. On the other hand, a higher  $\kappa$  has two additional effects that go in the opposite direction. First, a higher  $\kappa$  implies that the direct effect of the collateral constraint on the land price is weaker, leading to a lower fall in the price of land during fire sales. Second, a higher  $\kappa$  makes the constraint less likely to bind, reducing the externality. The effects of changes in  $\kappa$  are clearly non-monotonic. If  $\kappa$  is equal to zero, there is no effect of prices on the borrowing-ability. At the same time, for high enough values of  $\kappa$ , the constraint never binds. In both cases, the externality does not play any role. Quantitatively, Table 5 shows that small changes in  $\kappa$  are positively associated with the size of the inefficiency. In particular, an increase in  $\kappa$  from the baseline value of 0.36 to 0.40 increases the welfare cost of the inefficiency by a factor of 6 and financial crises again become a zero-probability event in the constrained-efficient equilibrium.

The above results have interesting policy implications. In particular, they suggest that while increasing credit access by rising  $\kappa$  may increase welfare relative to a more financially constrained environment, rising  $\kappa$  can also strengthen the effects of credit externalities and hence make macro-prudential policies more desirable (since the welfare cost of the externality also rises).

A high Frisch elasticity of labor supply ( $(1/\omega) = 1.2$ ) implies that output drops more when a negative shock hits. If the credit constraint binds, this implies that consumption falls more, which increases the marginal utility of consumption and raises the return rate at which future

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<sup>26</sup>Notice that the probability of a crisis in the competitive equilibrium becomes 10 percent, more than three times larger than the target employed in the baseline calibration due to the reduction in the level of precautionary savings.

dividends are discounted.<sup>27</sup> Moreover, everything else constant, a higher elasticity makes the externality term higher by weakening the effects of wages on the borrowing capacity. Hence, a higher elasticity of labor supply is associated with higher effects from the credit externality, captured especially by larger differences in the severity of financial crises, a higher probability of crises, and a larger welfare cost of the credit externality.

The fraction of wages that have to be paid in advance  $\theta$  plays a subtle role. On one hand, a larger  $\theta$  increases the shadow value of relaxing the credit constraint, since this implies a larger rise in the effective cost of hiring labor when the constraint binds. On the other hand, a larger  $\theta$  implies, *ceteris paribus*, a weaker effect on borrowing ability, since the reduction of wages that occurs when the collateral constraint binds has a positive effect on the ability to borrow. Quantitatively, increasing (decreasing)  $\theta$  by 5 percent increases (decreases) slightly the effects that reflect the size of the externality.

Changes in the volatility and autocorrelation of TFP do not have significant effects. Increasing the variability of TFP implies that financial crises are more likely to be triggered by a large shock. This results in larger amplification and a higher benefit from internalizing price effects. In general equilibrium, however, precautionary savings increase too, resulting in a lower probability of financial crises for both equilibria. Therefore, the overall effects on the externality of a change in the variability of TFP depend on the relative change in the

probability of financial crisis in both equilibria and the change in the severity of these episodes. An increase in the autocorrelation of TFP leads to more frequent financial crises for given bond decision rules. Again, in general equilibrium, precautionary savings increase making ambiguous the effect on the externality.

In terms of the optimal debt on tax, the results of the sensitivity analysis produce an important finding: The average debt tax of about 1.1 percent is largely robust to the parameter variations we considered. Except for the scenario that approximates logarithmic utility ( $\sigma = 1$ ), in all other scenarios included in Table 5 the mean tax ranges between 1.01 and 1.2 percent.

We consider now shocks that affect directly the collateral constraint by affecting the

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<sup>27</sup>The increase in leisure mitigates the decrease in the stochastic discount factor but does not compensate for the fall in consumption

**Table 5:** Sensitivity Analysis

	Average Effects on Impact during Financial Crises													
	Average Welfare		Covariance Effect		Crisis Probability		Consumption		Credit		Land Price		GDP	
	Tax	Loss	DE	SP	DE	SP	DE	SP	DE	SP	DE	SP	DE	SP
benchmark	1.1	0.05	0.22	0.06	3.0	0.9	-15.7	-5.3	-20.5	-1.7	-24.2	-7.0	-6.3	-4.6
$\sigma = 1$	0.6	0.001	0.03	0.03	9.3	7.8	-3.8	-3.3	-2.2	-1.4	-2.8	-2.4	-2.6	-2.6
$\sigma = 1.5$	1.0	0.01	0.08	0.04	5.7	2.5	-6.5	-3.1	-4.0	-0.2	-6.1	-2.5	-3.7	-2.9
$\sigma = 2.5$	1.2	0.24	0.37	0.08	2.0	0.0	-15.9	-4.2	-19.7	0.64	-30.3	-7.0	-5.8	-4.0
$\kappa = 0.32$	1.0	0.02	0.14	0.06	4.6	2.1	-8.8	-3.7	-9.4	0.4	-13.2	-5.1	-4.7	-3.6
$\kappa = 0.4$	1.2	0.29	0.34	0.06	2.2	0.0	-17.2	-4.2	-21.6	1.7	-27.9	-5.7	-5.7	-4.0
$1/\omega = 0.83$	1.0	0.03	0.16	0.05	3.7	1.1	-8.6	-3.4	-8.5	0.8	-12.3	-4.3	-4.4	-3.3
$1/\omega = 1.2$	1.1	0.12	0.27	0.06	3.9	2.3	-18.5	-4.9	-25.9	-1.0	-29.8	-6.3	-6.7	-4.6
$\theta = 0.13$	1.1	0.03	0.18	0.06	1.7	1.6	-15.3	-5.2	-20.0	-1.6	-23.7	-6.9	-6.2	-4.5
$\theta = 0.15$	1.1	0.05	0.21	0.06	2.8	1.2	-17.5	-5.4	-24.0	-2.0	-27.5	-7.1	-6.6	-4.6
$\sigma_\varepsilon = 0.010$	1.1	0.06	0.19	0.04	3.23	0.00	-13.9	-4.3	-19.3	-2.5	-21.9	-5.9	-4.9	-3.4
$\sigma_\varepsilon = 0.018$	1.0	0.05	0.26	0.08	2.51	0.16	-17.3	-6.9	-21.4	-1.9	-26.7	-9.0	-7.5	-5.9
$\rho_\varepsilon = 0.43$	1.1	0.05	0.22	0.05	2.78	0.00	-15.6	-4.9	-20.1	-0.9	-24.1	-6.4	-6.3	-4.5
$\rho_\varepsilon = 0.63$	1.1	0.05	0.23	0.06	2.74	1.17	-16.1	-5.2	-21.7	-2.0	-25.1	-6.9	-6.3	-4.5
Stochastic $\kappa$	1.1	0.04	0.25	0.06	3.02	1.22	-12.0	-3.8	-15.1	0.01	-18.3	-5.2	-5.3	-3.6

Note: 'DE' represents the decentralized equilibrium, 'SP' represents the social planner. The average tax on debt corresponds to the average value of the state contingent tax on debt to decentralize the constrained-efficient allocations. The covariance effect represents the unconditional average of the covariance effect. Consumption, credit, land prices and output are responses of these variables on impact during a financial crisis (see section 4.4 for a definition of the event analysis). The baseline parameter values are:  $R - 1 = 0.028$ ,  $\beta = 0.96$ ,  $\sigma = 2$ ,  $\alpha_h = 0.64$ ,  $\chi = 0.64$ ,  $\omega = 1$ ,  $\bar{K} = 1$ ,  $\theta = 0.14$ ,  $\kappa = 0.36$ ,  $\alpha_K = 0.05$ ,  $\sigma_\varepsilon = 0.014$ ,  $\rho_\varepsilon = 0.53$

extent to which agents can pledge assets as collateral. We consider a stochastic process for  $\kappa$  that follows a symmetric two-state Markov chain independent from shocks to TFP. In line with evidence from Mendoza and Terrones (2008) on the mean duration of credit booms in industrial countries, we calibrate the probabilities of the Markov chain so that the average duration of each state is 6 years. We keep the average value of  $\kappa$  as in our benchmark model and consider fluctuations of  $\kappa$  of 10 percent which is meant to be suggestive. As shown in Table 5, the effects of the externality remain largely unchanged to this modification.

Overall, the results of the sensitivity analysis show that parameter changes that weaken the model's financial amplification mechanism also weaken the magnitude of the externality. This results in smaller average taxes, smaller welfare costs and smaller differences in the incidence and severity of financial crises. The coefficient of risk aversion is particularly important also because it influences directly the price elasticity of asset demand, and hence it determines how much asset prices can be affected by the credit externality. This parameter plays a role akin to that of to the elasticity of substitution in consumption of tradables and non-tradables in Bianchi (2009), because in his model this elasticity drives the response of the price at which the collateral is valued. Accordingly, he found that the credit externality has significant effects only if the elasticity is sufficiently low.

## 5 Conclusion

This paper examined the positive and normative effects of a credit externality in a dynamic stochastic general equilibrium model in which a collateral constraint limits access to debt and working capital loans to a fraction of the market value of an asset in fixed supply (e.g. land). We compared the allocations and welfare attained by private agents in a competitive equilibrium in which agents face this constraint taking prices as given, with those attained by a constrained social planner that faces the same borrowing limits but takes into account how current borrowing choices affect future asset prices and wages. This planner internalizes the debt-deflation process that drives macroeconomic dynamics during financial crises, and hence borrows less in periods in which the collateral constraint does not bind, so as to weaken the debt-deflation process in the states in which the constraint becomes binding. Conversely, private agents overborrow in periods in which the constraint does not



bind, and hence are exposed to the stronger adverse effects of the debt-deflation mechanism when a financial crisis occurs.

The novelty of our analysis is in that it quantifies the effects of the credit externality in a setup in which the credit friction has effects on both aggregate demand and supply. The effects on demand are well-known from models with credit constraints: consumption drops as access to debt becomes constrained, and this induces an endogenous increase in excess returns that leads to a decline in asset prices. Because collateral is valued at market prices, the drop in asset prices tightens the collateral constraint further and leads to fire-sales of assets and a spiraling decline in asset prices, consumption and debt. On the supply side, production and labor demand are affected by the collateral constraint because firms buy labor using working capital loans that are limited by the collateral constraint, and hence when the constraint binds the effective cost of labor rises, so the demand for labor and output drops. This affects dividend rates and hence feeds back into asset prices. Previous studies in the macro/finance literature have shown how these mechanisms can produce financial crises with features similar to actual financial crises, but the literature had not conducted a quantitative analysis comparing constrained-efficient v. competitive equilibria in an equilibrium model of business cycles and asset prices.

We conducted a quantitative analysis in a version of the model calibrated to U.S. data. This analysis showed that, even though the credit externality results in only slightly larger average ratios of debt and leverage to output compared with the constrained-efficient allocations (i.e. overborrowing is not large), the credit externality does produce financial crises that are significantly more severe and more frequent than in the constrained-efficient equilibrium, and produces higher long-run business cycle variability. There are also important asset pricing implications. In particular, the credit externality and its associated higher macroeconomic volatility in the competitive equilibrium produce equity premia, Sharpe ratios, and market price of risk that are much larger than in the constrained-efficient equilibrium. We also found that the degree of risk aversion plays a key role in our results, because it is a key determinant of the response of asset prices to volatility in dividends and stochastic discount factors. For the credit externality to be important, these price responses need to be nontrivial, and we found that they are nontrivial already at commonly used risk aversion

parameters, and larger at larger risk aversion coefficients that are still in the range of existing estimates.

This analysis has important policy implications. In particular, the social planner can decentralize the constrained-efficient allocations as a competitive equilibrium by introducing an optimal schedule of state-contingent taxes on debt and dividends. By doing so, it can neutralize the adverse effects of the credit externality and produce an increase in social welfare. In our calibrated model, the tax on debt necessary to attain this outcome is about 1 percent on average. The tax is higher when the economy is building up leverage and becoming vulnerable to a financial crisis, but before a crisis actually occurs, so as to induce private agents to value more the accumulation of precautionary savings than they do in the competitive equilibrium without taxes.

These findings are relevant for the ongoing debate on the design of new financial regulation to prevent financial crises, which emphasizes the need for “macro-prudential” regulation. Our results lend support to this approach by showing that credit externalities associated with fire-sales of assets have large adverse macroeconomic effects. At the same time, however, we acknowledge that actual implementation of macro-prudential policies in financial markets remains a challenging task. In particular, the optimal design of these policies requires detailed information on a variety of credit constraints that private agents and the financial sector face, real-time data on their leverage positions, and access to a rich set of state-contingent policy instruments. Moreover, as we showed in this paper, implementing only a subset of the optimal policies because of these limitations (or limitations of the political process) can reduce welfare in some states.

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## Appendix: Numerical Solution Method

The computation of the competitive equilibrium requires solving for functions  $\mathcal{B}(b, \varepsilon), q(b, \varepsilon), \mathcal{C}(b, \varepsilon), \mathcal{N}(b, \varepsilon), \mu(b, \varepsilon)$  such that:

$$\mathcal{C}(b, \varepsilon) + \frac{\mathcal{B}(b, \varepsilon)}{R} = \varepsilon F(\bar{K}, \mathcal{N}(b, \varepsilon)) + b \quad (23)$$

$$-\frac{\mathcal{B}(b, \varepsilon)}{R} + \theta G'(\mathcal{N}(b, \varepsilon))\mathcal{N}(b, \varepsilon) \leq \kappa q(b, \varepsilon)\bar{K} \quad (24)$$

$$u'(t) = \beta R E_{\varepsilon'/\varepsilon} \left[ u'(\mathcal{C}(\mathcal{B}(b, \varepsilon), \varepsilon')) \right] + \mu(b, \varepsilon) \quad (25)$$

$$\varepsilon F_n(\bar{K}, \mathcal{N}(b, \varepsilon)) = G'(\mathcal{N}(b, \varepsilon))\mathcal{N}(b, \varepsilon)(1 + \theta\mu(b, \varepsilon)/u'(\mathcal{C}(b, \varepsilon))) \quad (26)$$

$$q(b, \varepsilon) = \frac{\beta E_{\varepsilon'/\varepsilon} \left[ u'(c(\mathcal{B}(b, \varepsilon), \varepsilon')) \varepsilon' F_k(\bar{K}, \mathcal{N}(\mathcal{B}(b, \varepsilon), \varepsilon')) + q(\mathcal{B}(b, \varepsilon), \varepsilon') \right]}{(u'(\mathcal{C}(b, \varepsilon)) - \mu(b, \varepsilon)\kappa)} \quad (27)$$

We solve the model using a time iteration algorithm developed by Coleman (1990) modified to address the occasionally binding endogenous constraint. The algorithm follows these steps:<sup>28</sup>

1. Generate a discrete grid for the economy's bond position  $G_b = \{b_1, b_2, \dots, b_M\}$  and the shock state space  $G_\varepsilon = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N\}$  and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.
2. Conjecture  $\mathcal{B}_K(b, \varepsilon), q_K(b, \varepsilon), \mathcal{C}_K(b, \varepsilon), \mathcal{N}_K(b, \varepsilon), \mu_K(b, \varepsilon)$  at time  $K \forall b \in G_b$  and  $\forall \varepsilon \in G_\varepsilon$ .
3. Set  $j = 1$
4. Solve for the values of  $\mathcal{B}_{K-j}(b, \varepsilon), q_{K-j}(b, \varepsilon), \mathcal{C}_{K-j}(b, \varepsilon), \mathcal{N}_{K-j}(b, \varepsilon), \mu_{K-j}(b, \varepsilon)$  at time  $K - j$  using (23),(24),(25),(26), (27) and  $\mathcal{B}_{K-j+1}(b, \varepsilon), q_{K-j+1}(b, \varepsilon), \mathcal{C}_{K-j+1}(b, \varepsilon), \mathcal{N}_{K-j+1}(b, \varepsilon), \mu_{K-j+1}(b, \varepsilon) \forall b \in G_b$  and  $\forall Y \in G_Y$ :

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<sup>28</sup>For the social planner's allocations, we use the same algorithm operating on the planner's optimality conditions.