WP/10/74

IMF Working Paper

Asset Securitization and Optimal Retention

John Kiff and Michael Kisser

INTERNATIONAL MONETARY FUND

 WP/10/74

IMF Working Paper

Monetary and Capital Markets

Asset Securitization and Optimal Retention*

Prepared by John Kiff and Michael Kisser

Authorized for distribution by Laura Kodres

March 2010

Abstract

This paper builds on recent research by Fender and Mitchell (2009) who show that if financial institutions securitize loans, retaining an interest in the equity tranche does not always induce the securitizer to diligently screen borrowers ex ante. We first determine the conditions under which this scenario becomes binding and further illustrate the implications for capital requirements. We then propose an extension to the existing model and also solve for optimal retention size. This also allows us to capture feedback effects from capital requirements into the maximization problem. Preliminary results show that equity tranche retention continues to best incentivize loan screening.

JEL Classification Numbers: G18, G21, G24, G28

Keywords: Securitization

Author's E-Mail Address: jkiff@imf.org

---

* This paper was written while Michael Kisser participated in the Internship Program at the International Monetary Fund in Washington, DC. We thank Laura Kodres, Corrado Macchiarelli, Jodi Scarlata, and Liliana Schumacher for helpful discussions. Finally, we are grateful to Marco Espinosa and Juan Solé.

Contents

1 Introduction

In the aftermath of the global financial crisis, both the U.S. Government and the European Union Parliament have been pushing to force securitizers to retain economic exposure in securitized assets in order to better align their interests with those of investors. While it seems that authorities are focusing on vertical slice retention, i.e. retaining equal amounts of each tranche in the securitization structure, Fender and Mitchell (2009) have shown that under certain circumstances mezzanine tranche retention might incentivize more diligent screening than both equity and vertical slice retention.[1]

Fender and Mitchell (2009) model a principal-agent problem in which an originating institution can extend loans to borrowers and then has the option of securitizing the loan portfolio by selling different tranches to individual investors. While the initial proportion of "good" and "bad" borrowers is given exogenously, the originator can exert costly screening effort before the loan is made to increase his expected return. Fender and Mitchell (2009) then compare optimal screening effort of the originator across different retention mechanisms. Their model shows that under some economic conditions, mezzanine tranche retention might induce the originator to exert more screening effort than if he retained interest in the equity tranche.

While these conditions indeed become binding for reasonable scenarios, it turns out that the implications also depend on the "thickness" of the retained tranche. Thus, as Fender and Mitchell (2009) argue, if the retained equity tranche is thick enough, then retaining the equity tranche is likely to best align incentives between the originator and outside investors. However, by not knowing ex-ante how much exposure the originator aims to retain, the complexity of a regulatory response increases dramatically.

This paper aims to contribute to recent public discussions and academic research on several dimensions. The first part reflects analysis which has been undertaken for the IMF (2009) and illustrates whether and when the technical conditions of Fender and Mitchell (2009) become binding. Second, we analyze the model and thereby include more realistic features such as the need to hold different amounts of capital for various retention mechanisms and consequently also create a simple structured product. Third, we then propose an extension to tbe existing model and also solve for optimal retention

[1]Securitization is a process in which different assets or portfolios of cash flow generating securities are pooled together and then sold to third parties. This paper focuses on structured finance which further implies that cash flows of the entire portfolio are tranched into several slices which differ with respect to their risk-return characteristics. Senior tranches are paid first and offer the lowest risk-return combination and are thus similar to a debt claim. On the other hand, junior tranches are paid last only in case there is any residual left, thereby making the payment structure similar to an equity claim. More details can be found in Jobst (2008).

size. This substantially facilitates policy recommendations and regulation. In addition, the extension allows us to capture feedback effects stemming from capital requirements which in turn reduce expected profits. Preliminary results show that under this more realistic maximization problem, equity retention best incentivizes diligent loan screening. Nevertheless, more work is needed to account for the feasibility of creating a structured product before finalizing any policy implications.

There is a broad literature discussing the benefits and costs of credit risk transfer (CRT) mechanisms. Focusing on the interaction between CRT and financial stability, Allen and Carletti (2006) show that CRT has an ambiguous effect on the fragility of the financial system. On one hand it is beneficial as it allows banks to better diversify their risks. However, CRT can also lead to contagion across different sectors in the financial system and thereby to a reduction in total welfare. Wagner and Marsh (2006) find that CRT can improve financial stability, specifically if the financial sector is fragile and the credit risk transfer is made from banks to non-banks. On the other hand, Wagner (2007a) shows that while CRT improves asset liquidity for banks, it also incentivizes them to take on higher risks which then offsets the positive impact of higher liquidity. Similarly, Wagner (2007b) finds that higher portfolio diversification due to the use of CRT can reduce financial stability and thereby confirms the ambiguous implications for total welfare.

This paper relates to literature dealing with the interaction between CRT and informational asymmetries. The "lemons problem", as coined by Akerlof (1970), shows that markets may break down in the context of informational asymmetries. Leland and Pyle (1977) use a signaling model to show how agency costs can be mitigated in the context of a partial firm sale. They model an entrepreneur with superior information regarding future prospects of assets in place who wants to sell part of his holdings to diversify risk. The entrepreneur can signal quality by retaining a larger fraction of the asset and thereby mitigate the agency problem. Diamond (1984) presents a model of financial intermediation based on delegated monitoring and finds that debt is the optimal contract. Also Gale and Hellwig (1985) show that a standard debt contract is optimal in the context of informational asymmetries. Innes (1990) models a principal-agent problem between a risk-neutral entrepreneur with access to an investment project who makes an unobservable effort choice influencing the probability of success and an outside investor who provides the necessary funding for the project. Given limited liability of the entrepreneur, Innes (1990) shows that debt financing is the optimal contract in the context of unobservable and noncontractable effort choice. Gorton and Pennacchi (1995) focus on the subsequent adverse selection problem a bank faces if it engages in loan sales. However, they show that implicit contract features such as the retention of part of the loan and or implicit guarantees against default can make loan sales possible and thereby reduce agency problems.

Focusing on CRT mechanisms, Morrison (2005) shows that credit derivatives may destroy the signaling value of debt and thereby cause disintermediation and lower welfare. On the other hand, Niccolo and Pelizzon (2006) demonstrate that even in the context of credit risk transfer, banks may signal their own types by using first-to-default contracts. Also, DeMarzo (2005) shows that if assets are not only pooled but also tranched into different risk categories, banks can signal the quality of the sold loan portfolio by retaining interest in the equity tranche, thereby confirming optimality of a (standard) debt contract. Chiesa (2008) models a loan originating bank which needs outside financing and extends the setup to allow for a systemic risk component. Because a high return does not necessarily mean that the bank has engaged in monitoring but instead can be the result of a favorable realization of the systemic risk factor, Chiesa (2008) is able to show that a pure debt contract is not optimal whereas CRT with limited credit enhancements enhances loan monitoring and expands financial intermediation. Fender and Mitchell (2009) build on Chiesa (2008), Duffie (2008) and Innes (1990) and develop a simple screening model in which a loan originating institution can screen borrowers, for a cost, and has the option to securitize its loan portfolio. Their focus is not on deriving an optimal profit maximizing contract for the originator but on comparing implied screening effort across different retention mechanisms.

The paper proceeds as follows. Section [2] presents the model of Fender and Mitchell (2009) and presents the underlying analysis used for the Global Financial Stability Report, IMF (2009). Section [3] extends the existing model and also solves for optimal retention size. Section [4] concludes.

2 Discussion of Fender and Mitchell (2009)

We first give a brief summary of a model proposed by Fender and Mitchell (2009) which analyzes incentives of a lender to diligently screen borrowers if the lender has the option to securitize the loan portfolio and sell part of it to outside investors. In a nutshell, they find that under some conditions equity retention by the lender does not always align incentives properly and can be dominated by mezzanine or vertical slice retention.

To test the relevance of their findings, we then check whether these technical dominance conditions actually becoming binding for reasonable parameter values. We then relate the results to capital requirements by creating a simple structured product and calculating corresponding capital requirements.

2.1 Summary of the Model

Fender and Mitchell (2009) model an originating institution which extends loans to borrowers for the total amount of Z and then has the option of securitizing the loan portfolio and selling different tranches to individual investors.[2] Outside investors and the originator are assumed to be risk-neutral and the risk-free interest rate is set to zero. A performing loan returns $R > 1$ while there is zero recovery for a defaulting loan. It should be noted that for what follows, the terms "originator" and "securitizer" are used interchangeably.

The simplified economy consists of good and bad borrowers who differ in their ability to repay the loan. The proportion of "good" borrowers is captured by the parameter θ which is given exogenously. To increase chances of lending to a high quality borrower, the originator can exert costly screening effort e to increase the probability of lending to a good quality borrower. Denoting α_G and α_B as the revised probabilities of lending to good and bad borrowers, we can see that

$$\alpha_G(e) = \min\left[\theta + e, 1\right] \tag{1}$$

$$\alpha_B(e) = \max\left[1 - \theta - e, 0\right] \tag{2}$$

where the min and max function follow from the fact that probabilities are bounded between zero and one. Screening borrowers is costly, which is captured by the cost function $c(e)$ which is assumed to be increasing in e and convex, i.e. $c(0) = 0$, $c'(e) > 0$ and $c''(e) > 0$. As Fender and Mitchell (2009) do not specify a functional form of the cost function, we assume quadratic costs, i.e. $c(e) = \frac{\phi}{2}e^2$ to

[2] As they show, Z can be normalized to one without loss of generality.

account for the convexity in effort costs. This assumption can be also found in Carletti (2004) and Duffie (2008).[3].

It is further assumed that at the time when the loans are extended, the originator has already decided if and in what form the loan portfolio will be securitized. Effort level is chosen accordingly and then different tranches of the portfolio are sold to outside investors. Outside investors are only willing to make payments equal to the expected value of future cash flows conditional on the optimal effort level of the originator.

Finally, following Chiesa (2008) Fender and Mitchell introduce a systemic risk factor and assume that the economy can be either in a high or a low state and that the corresponding probabilities are given by p_L and p_H. The state of the economy has a distinct impact on each borrower. In the high state, the good borrower is always able to repay the loan while the bad borrower may default with a probability $PD_B(H)$. In the low state, the bad borrower always defaults with probability one while the good borrower only does so with a probability of $PD_G(L)$.

Fender and Mitchell (2009) then derive the optimal screening level of the securitizer across different retention mechanisms. In each case, the originator maximizes its expected profits by trading-off screening benefits and costs. The maximization problem can be summarized as follows

$$\max_e \Pi(e) = \Omega S + F(e) - c(e) - 1 \tag{3}$$

where Π denotes expected profits, S is the fairly priced upfront payment received from outside investors, $F(e)$ are future revenues from the retained credit exposure, and 1 is the normalized total amount of the loan portfolio. In fact, it may be more correct to write $S = S(e^*)$, i.e. investors make an up-front payment which takes into account the optimal effort of the originator.

The variable Ω captures monetary benefits from securitization and acts as a scaling factor. Specifically it is assumed that $\Omega > 1$. The intuition for this monetary benefit is that securitization may free up capital or may lead to an increase in current profit. As Fender and Mitchell (2009) argue, some source of these benefits is linked to the design of compensation schemes which reward management for short-term profits and/or is due to accounting treatment which allows for recognition of profits upon the sale of the securitized portfolio.

[3]Specifically, we set the value of ϕ equal to one to make first order conditions comparable to Fender and Mitchell (2009).

Fender and Mitchell (2009) then compare different retention mechanisms and investigate how optimal effort levels differ depending on whether the originator retains the mezzanine or equity tranche or a vertical slice of the loan portfolio.

2.1.1 Vertical slice retention

In the case of vertical slice retention, the originator retains an equal slice in each of the different tranches which implies the retention of a proportional interest in the entire loan portfolio. A special case of vertical tranche retention is $\nu = 100$ percent, in which the whole loan pool is retained. The maximization problem under vertical slice retention is given by:

$$\begin{aligned} \max_{e} \ \pi_{\nu}(e) &= \Omega S_{\nu} + \nu p_L \left[(1 - PD_G(L)) \, R\alpha_G(e) \right] \\ &+ \nu p_H \left[R\alpha_G(e) + (1 - PD_B(H)) \, R\alpha_B(e) \right] - c(e) - 1 \end{aligned} \tag{4}$$

where ν denotes the size of the vertical slice. The securitizer's optimal effort level is determined by the following first-order condition

$$e^*_{\nu} = \min \left\{ \nu R \left[p_L \left(1 - PD_G(L) \right) + p_H PD_B(H) \right], 1 - \theta \right\}$$

2.1.2 Equity tranche retention

Similarly, one can write down the maximization problem for the case the originator retains the equity tranche, i.e.

$$\begin{aligned} \max_{e} \ \pi_E(e) &= \Omega S_E + \max \left\{ (1 - PD_G(L)) \, R\alpha_G(e) - B_1, 0 \right\} p_L \\ &+ \max \left\{ R\alpha_G(e) + (1 - PD_B(H)) \, R\alpha_B(e) - B_1, 0 \right\} p_H \\ &- c(e) - 1 \end{aligned} \tag{5}$$

where t denotes the size or 'thickness' of the equity tranche and $B_1 = (1 - t)R$ are the promised payments to senior and mezzanine tranche holders. The use of the max function follows from the limited liability feature of equity holders. In this case, the determination of optimal effort levels is more complex because of the kinks introduced by the tranching (see below and Fender and Mitchell (2009)).

2.1.3 Mezzanine tranche retention

The last retention mechanism analyzed by Fender and Mitchell (2009) is given by mezzanine tranche retention, where it is assumed that the equity and mezzanine tranches are equally thick. In this case, the maximization problem is as follows.

$$\begin{aligned}\max_e \ \Pi_M(e) &= \Omega S_M + \min\left[\max\left\{(1-PD_G(L))\,R\alpha_G(e) - B_2, 0\right\}, B_M\right] p_L \\ &+ \min\left[\max\left\{R\alpha_G(e) + (1-PD_B(H))\,R\alpha_B(e) - B_2, 0\right\}, B_M\right] p_H \\ &- c(e) - 1 \qquad (6)\end{aligned}$$

where $B_2 = (1-2t)R$ are the promised payments to senior tranche holders and $B_M = tR$ is the maximum payment which mezzanine tranche holders can expect to receive (see below and Fender and Mitchell (2009) for the optimality conditions).

2.1.4 Optimal retention conditions

Fender and Mitchell (2009) solve the maximization problem for each retention mechanism and find that mezzanine retention incentivizes more screening than equity retention if the equity tranche is expected to be exhausted in the low state of nature and if

$$p_L(1-PD_G(L)) > p_H PD_B(H) \qquad (7)$$

This condition implies that mezzanine retention incentivizes more screening effort than equity retention if the relative benefit of screening in the low state of nature, as measured by $p_L(1-PD_G(L))$, is higher than the relative benefits of screening in the high state of nature. Finally, retaining a vertical slice only incentivizes more screening effort than both equity and mezzanine retention if again the equity tranche is expected to be exhausted in the low state of nature and if the retained slice ν is greater than

$$\nu > \frac{\max\left[p_L(1-PD_G(L)), p_H PD_B(H)\right]}{\left[p_L\,(1-PD_G(L)) + p_H PD_B(H)\right]} \qquad (8)$$

which, by definition of equation [8], is rather unlikely unless the vertical slice is very thick. Summing up, two conditions have to be satisfied such that retention of the mezzanine tranche results in higher screening effort. First, equity holders must expect to lose their investment in the low state of nature. Second, the technical conditions given in equation [7] must be satisfied. The next section thus investigates how likely it is that both conditions jointly hold.

2.2 Numerical Analysis

The objective of this section is to characterize under which circumstances the dominance of the mezzanine tranche or vertical slice becomes practically relevant.[4]

[4] This section displays results of the analysis which has been undertaken for the October 2009 Global Financial Stability Report, see IMF (2009).

We try to answer this question by presenting different stylized examples. In the first one, we look at a market which mostly consists of good borrowers, a *high* quality market. On the other hand, the second example then focuses on a *low* quality market in a bad economic environment. For each case, we illustrate when the technical conditions under which equity retention incentivizes less effort become binding. Finally, we try to analyze the impact of capital charges by explicitly relating screening effort to implied capital requirements.

The basic setup for the calculations is taken from Fender and Mitchell (2009). For simplicity it is assumed that the probability of the high quality borrower defaulting in the low state is equal to the probability that the low quality borrower defaults in the high state, i.e. $PD_B(H) = PD_G(L)$. Further, promised payments to both mezzanine and senior tranche holders are given by $B_1 = (1-t)R$ which consist of expected payments to senior tranche holders of $B_2 = (1-2t)R$ and payments of mezzanine tranche holders of $B_M = tR$. We also set the ϕ in the cost function to one, and since the Fender and Mitchell (2009) paper does not specify the gross return R, we set $R = 1.05$.

2.2.1 High Quality Market

To illustrate results for a relatively safe market, we start by assuming that 2 out of 3 borrowers are of high quality. Further, no directional assumption is made regarding the future state of the economy, i.e. $p_H = p_L = 0.5$. Figure [1] shows implied effort levels under equity and mezzanine tranche retention for different levels of default probabilities and different sizes of the retained tranche. Specifically, the figure displays the difference between implied effort levels under equity and mezzanine tranche retention. A positive value means that equity induces higher screening effort whereas a negative value corresponds to the case when mezzanine tranche retention dominates. It can be seen that for most parameter values, equity retention induces higher screening effort than retaining interest in the mezzanine tranche. In fact, there are some combinations between tranche thickness and default probability where mezzanine retention dominates equity retention.

This can be further visualized by focusing on a two dimensional graph. Figure [2] shows relative effort levels, defined as comparing implied effort levels to the effort level the originator would exert if the entire loan portfolio were retained, for both equity (solid blue line) and mezzanine (dashed green line) retention when the tranche thickness is held constant at 25 percent.[5] One can see that for default

[5]The reason for the slight discrepancy of Figure [2] with Figure 5 in Box 2.6 of the October 2009 Global Financial Stability report is due to the fact that this calculation corrects for an error in Fender and Mitchell (2009). To check whether the equity tranche is expected to be exhausted in the low state of nature, Fender and Mitchell (2009) suggest to check whether $\nu R\left[p_L\left(1-PD_G(L)\right)+p_H PD_B(H)\right] \geq \hat{e}$. Instead, the correct test would be to check whether $\min\left\{\nu R\left[p_L\left(1-PD_G(L)\right)+p_H PD_B(H)\right], 1-\theta\right\} \geq \hat{e}$.

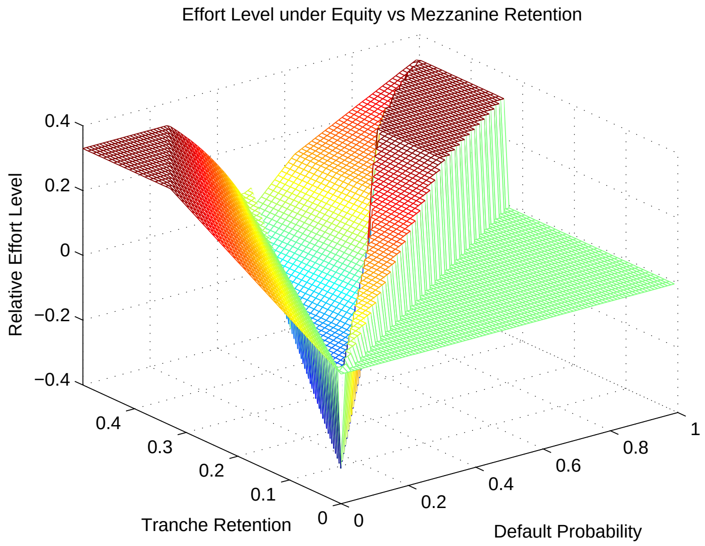

Figure 1: Difference in Implied Effort Levels

probabilities of around 20 to 40 percent, mezzanine retention generates higher screening effort than if the originator retained the equity tranche.

Focusing again on Figure [1], one can also see that there is a triangle on the lower right where the difference in screening levels is zero. This is due to Assumption 1 in Fender and Mitchell (2009) which requires that even for zero screening effort the equity tranche never gets exhausted in the high state of nature, thereby excluding some combinations of default probability and tranche thickness for which this assumption is not feasible. It should be noted that the assumption does not come without loss of generality. As soon as it is optimal under equity and mezzanine retention not to exert any screening effort, then vertical slice retention can dominate even if the retained slice is thin.

2.2.2 Low Quality Market

The stylized example of a low quality market is characterized by assuming that the entire loan market consists of 80 percent bad borrowers and 20 percent good borrowers. Further, there is an 80 percent probability of a downturn occurring during the life of the loan. Figure [3] displays the difference in implied effort levels under both equity and mezzanine tranche retention.

The specific combinations of default probability and tranche thickness show decisively whether equity or mezzanine tranche retention incentives more effort. The lower the size of the retained tranche, the more likely it is that mezzanine retention dominates. Also, the higher the default probability, the

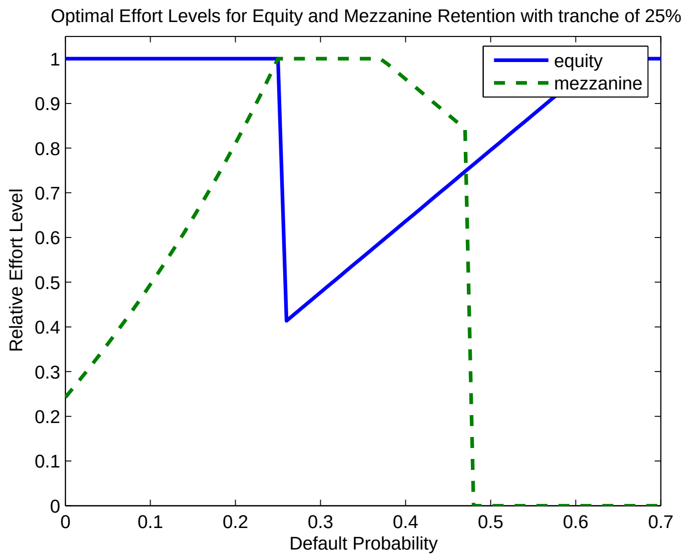

Figure 2: Relative Effort Levels under Equity and Mezzanine Retention

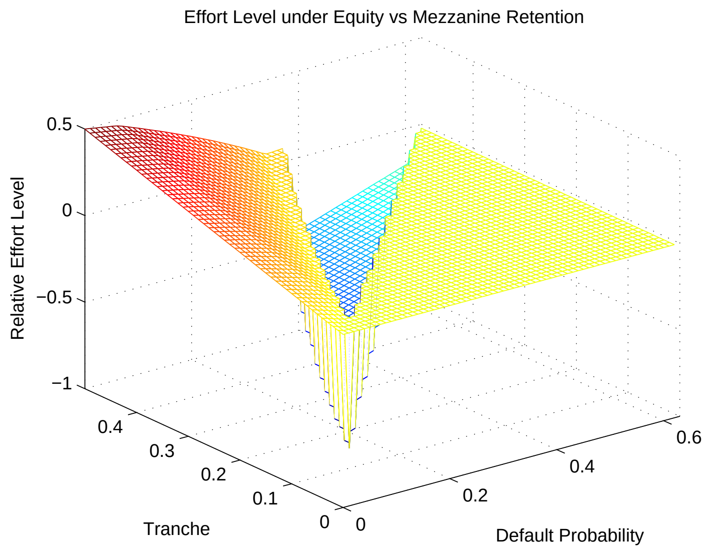

Figure 3: Difference in Implied Effort Levels

greater is the chance the mezzanine retention incentives more effort than equity retention.

2.2.3 Impact on Capital Requirements

Financial institutions pay capital charges on the credit risk they retain on their balances which in turn depend on the probability of default of the entire loan portfolio and on the riskiness of the individual tranches. Given the relevance of the issue, we thus try to assess the interaction between screening effort and capital requirements by running some calculations on a simple structured product. We therefore derive implied capital charges by relating the outcomes of the model to capital requirements following the standardized Basel II approach.

Having calculated optimal effort levels for the different maximization problems, the first step specifically involves calculating the probability of default of the entire portfolio (TPD_i), assuming the three possible retention mechanisms. This is done by evaluating

$$TPD_i = \alpha_B(e_i^*)\left[p_L + PDp_H\right] + \alpha_G(e_i^*)PDp_L \tag{9}$$

where i denotes the equity, mezzanine and vertical slice retention schemes.

Knowing the default probability of the entire portfolio, the next step consists in calculating the corresponding expected loss of the portfolio and the individual tranches which in turn is used to rate the structured product. To do so, we apply Moody's binomial extension technique (BET) to the calculated default probability to generate a loss distribution for a portfolio of 1,000 equal-sized loans. Further, we assume a 100 percent loss severity on defaulted loans and a uniform 10 percent default correlation.[6] The loss distribution is then applied to a simple three-tranche example based on this portfolio and assuming a ten-year term to maturity, to calculate expected losses for each tranche. These expected losses are then used to back out credit ratings based on the Moody's idealized expected loss tables.

The capital charge calculation example assumes that the exogenous probability of making a good loan is 60 percent and the probability of the low state is 50 percent. In order to make results easily comparable to the previous section, we assume that the probability of a good loan defaulting in a low state and a bad loan defaulting in a high state are always identical.

[6]See Fender and Kiff (2005) for implementation details. Although more accurate loss distributions can be calculated with Monte Carlo methods, the binomial expansion technique is a useful approximation for illustrating basic structured credit economics.

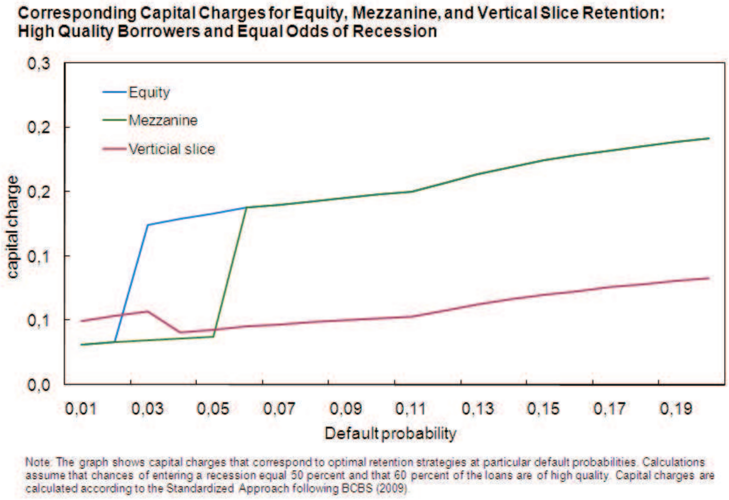

Figure 4: Capital Charges

Under the Basel II standardized approach, capital charges (CC) are calculated for a simple three-tranche structure constructed as above and comprised of a senior tranche rated A- or higher and equal-sized mezzanine and equity tranches. The minimum regulatory capital requirement on the retained tranche(s) is equal to 8 percent of the risk-weighted par value(s). For example, the par value of any retained AAA-rated tranche is 20 percent weight, and any tranche rated below BB- is 1,250 percent. Further details about the risk weights can be found in IMF (2009). No credit enhancements are considered. The formula below summarizes the calculation of the capital charges

$$CC_i = 0.08 \cdot RW_i \cdot t \tag{10}$$

Figure [4] shows the capital charges over a range of default probabilities associated with retention of the equity or mezzanine tranches, or a vertical slice of the same size. It shows that the capital requirements for equity and mezzanine retention are lower than those for vertical slice retention when the conditional default probabilities are very low (up to 2 percent). However, capital charges are lower for vertical slice retention for higher default probabilities (greater than 5 percent). It is clear that the charges on equity tranches will usually be higher because they almost always draw the maximum RW (1,250 percent) whereas the mezzanine tranche draws the 350 percent RW associated with BB rated tranches, at least up to the 5 percent default probability in this case. (At default probabilities greater than 5 percent, the mezzanine tranche draws a 1,250 percent RW.) The illustration suggests that some important feedback effects are missing from this simple model. In other words, a more realistic model would make capital charges part of the effort level optimization calculations.

2.2.4 Intermediate Summary

The preceding examples have shown that the choice of the dominant retention mechanism that incentivizes the most screening mainly depends on the quality of the loan pool and the assumption regarding the future state of the economy. However, we have also observed that that the devil is in the details in the sense that optimal screening effort within a given scenario also depends on the combination of default probability and thickness of the retained tranche.

Specifically, the smaller the size of the retained tranche, the more likely it is that equity retention does not induce the highest possible screening effort. Thus policymakers not only have to assess the quality of the loan pool and make assumptions regarding the future state of the economy, but they also would have to judge how much exposure the originator decides to retain in order to compare different retention mechanisms. Finally, we have also seen that as the different retention mechanisms imply different capital charges, this effect should be incorporated by making capital charges part of the maximization problem and would have to be considered alongside retention in any regulation.

In what follows, we therefore propose an extension to the model of Fender and Mitchell (2009) which allows us to derive both optimal screening effort and optimal thickness of the retained interest.

3 Optimal Retention and Screening Policy

The previous section has illustrated that the implications regarding the impact on screening if retaining one tranche or another, not only depend on assumptions regarding the quality of the loan pool and the future state of the economy but also on the thickness of the retained credit exposure. As Fender and Mitchell (2009) argue, if the retained equity tranche is thick enough, there might be no conflict of interest as incentives are perfectly aligned.

One open question of the model introduced by Fender and Mitchell (2009) is that it does not endogenize the choice of the retention size and thus complicates potential policy recommendations. Regulators not only have to make assumptions regarding the future state of the economy and the quality of borrowers but they also have to consider the effect of different tranche thicknesses on optimal screening decisions.

We therefore propose a simple extension which allows us to also solve for the optimal size or thickness of the retained credit exposure. To do so, we assume that whenever the originator sells more senior tranches and those tranches are expected to be fully paid back, then the originator is able to value the corresponding upfront payments at a premium to its notional amount. On the other hand, if the originator sells tranches with higher risk than what is actually retained (i.e. sells equity and keeps the mezzanine tranche), then no premium can be realized with respect to the more risky tranche. This can be rationalized by the existence of informational asymmetries between the originator and outside investors which are assumed to neutralize any accounting or compensation related benefit of securitization in this context.

Before formally introducing the assumption, let's define the expected cash flows of the entire loan portfolio in the low and the high state of nature of the economy.

$$P_L(e) \equiv \left[(1 - PD_G(L))\,(\theta + e)\right] R \tag{11}$$

$$P_H(e) \equiv \left[(\theta + e) + (1 - PD_B(H))\,(1 - \theta - e)\right] R \tag{12}$$

It follows that the expected cash flows of the entire portfolio are given by $P(e) = p_L P_L(e) + p_H P_H(e)$. In order to facilitate exposition of the results, we further set $PD_G(L) = PD_B(H) = PD$.

Assumption 1 *If the originator sells more senior tranches to outside investors and if in expectation those tranches are to be fully repaid, then the originator is able to profit from market imperfections such as early recognition of profits or short-term linked compensation by valuing the corresponding upfront payments at a premium to its notional amount.*

Specifically, the originator values payments from senior and mezzanine tranche holders in the total amount of $R(1-t)$ at

$$S_E(t) = (1 - t^2)R \tag{13}$$

where the subscript E denotes the fact that in this case the originator retains the equity tranche. In a similar fashion, payments from senior tranche holders in the amount of $R(1-2t)$ are valued at

$$S_M(t) = (1 - 2t^2)R \tag{14}$$

where the subscript M denotes the fact that in this case the originator retains the mezzanine tranche.

One possible criticism of Assumption [1] might be that the choice of the functional form regarding the subjective valuation of the originator is somehow ad-hoc. However, this specific functional form allows us to capture a natural trade-off between benefits from securitization and agency costs due to informational asymmetries. Besides, the intuition is similar to Jensen and Meckling (1976) who illustrate a trade-off between agency costs of equity and agency costs of debt in the capital structure choice of a single firm.

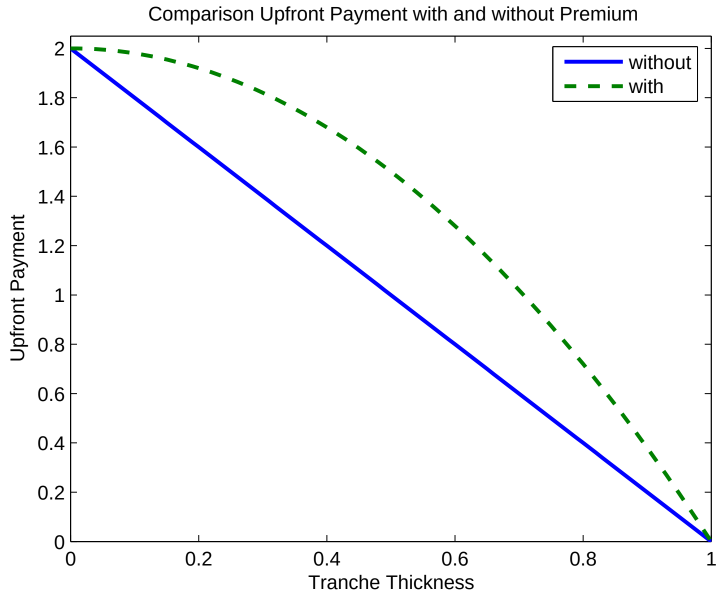

Figure 5: Visualizing the Intuition of Assumption [1]

Figure [5] visualizes the intuition underlying Assumption [1]. The solid blue line plots the fixed upfront payment $R(1-t)$ for different values of t where R is set to 2. The dashed green line displays values corresponding to the concave function $(1-t^2)R$. It can be seen that starting with a fully retained loan portfolio, i.e. $t = 1$, the initial marginal benefit of securitizing is very high and it subsequently decreases the more of the loan portfolio is sold to outside investors. We can also see that if the entire portfolio is sold to outside investors, i.e. $t = 0$, the net benefits from securitization are zero such that the originator is only able to value the upfront payment at its notional value. The assumption thus captures the intuition that initially securitizing is beneficial to the originator but the less credit exposure is retained with the originator, the more agency costs relating to asymmetric information dominate and eliminate the marginal gain from securitizing an additional unit.

Additionally, we have seen in the previous section that different retention mechanisms have different effects when associated with capital requirements which is why we also specifically introduce capital charges into the model. Specifically, we assume a linear cost function for capital requirements.

Assumption 2 *To capture the various capital charges associated with each of the retention mechanisms, we assume the following linear cost function*

$$k(t) = k_i \cdot t \tag{15}$$

where k_i captures capital charges, conditional on the specific retention mechanism. Specifically we *assume that* $k_i = 0.08 \times RW_i \times (R-1)$ *where* RW_i *denotes the risk weight under equity or mezzanine retention and* $(R-1)$ *captures the opportunity costs of the capital charges.*[7]

Combining assumptions [1] and [2] with the model of Fender and Mitchell (2009), one is now able to write down the following general maximization problem.

$$\max_{e,t} \Pi(e,t) = S(t) + F(e,t) - c(e) - k(t) - 1 \tag{16}$$

It should be noted that no additional Ω is needed as the benefits and agency costs from securitization are already incorporated into the functional form for $S(t)$. For what follows, we will now present the profit maximization problem under the case of equity retention and compare it to the case when the securitizer retains the mezzanine tranche.

3.1 Equity Retention

In the case of equity retention, expected profits of the retained credit exposure are given by

$$F(e,t) = p_L \max\{P_L(e) - B_1(t), 0\} + p_H \max\{P_H(e) - B_1(t), 0\} \tag{17}$$

Claims of outside investors are given by $p_L \min\{P_L(e^*), B_1(t)\}$ in the low economic state and $p_H \min\{P_H(e^*), B_1(t)\}$ in the high state. In order to derive optimal screening effort, we compare two cases. Under the first one, the equity tranche never gets exhausted whereas in the second case, equity tranche holders lose their investment in the low economic state.

3.1.1 Case E1: No exhaustion of equity tranche

Proposition [1] summarizes the maximization problem of the originator in the case of equity tranche retention.

Proposition 1 *Under case E1, the maximization problem of the securitizer is as follows*

$$\begin{aligned} \max_{e,t} \pi_{E1}(e,t) &= (1-t^2)R + p_L[P_L(e) - B_1(t)] + p_H[P_H(e) - B_1(t)] \\ &- \frac{\phi}{2}e^2 - k(t) - 1 \qquad (18) \\ s.t.\ B_1(t) &\leq P_L(e) \qquad (19) \\ e &\leq 1-\theta \qquad (20) \end{aligned}$$

where [19] is the positive payoff constraint of equity tranche holders in the low state of nature and [20] considers the fact that the probability of making a good loan is bounded by one.

[7] The intuition for this assumption is as follows. As the originator is required to retain capital in its balance sheet it does not lose the entire amount but only the option to originate even more loans. Thus, the true costs of capital requirements are foregone profits or opportunity costs.

If the positive payoff constraint is not binding, i.e. the solution is unconstrained then expected profits are maximized by choosing e^* and t^* as follows.

$$\begin{aligned} e^* &= \min\left[(p_L(1-PD)+p_H PD)\,\frac{R}{\phi}, 1-\theta\right] \\ t^* &= \frac{1}{2}-\frac{k}{2R} \end{aligned} \tag{21}$$

Otherwise, optimal effort level and retention size are given by

$$\begin{aligned} e^* &= \min\left[\frac{k(1-PD)+R\left[2(1-PD)-p_H(1-2PD)-2(PD-1)^2\theta\right]}{\phi+2(PD-1)^2R}, 1-\theta\right] \\ t^* &= \max\left[\frac{\phi-k(PD-1)^2+(PD-1)(2PD-1)p_H R+(PD-1)\phi\theta}{\phi+2(PD-1)^2R}, PD\right] \end{aligned} \tag{22}$$

Proof: Results follow directly from calculating the corresponding first order conditions for each of the distinct cases.

In case the positive payoff constraint is not binding, the optimal effort level is the same as the one derived in Fender and Mitchell (2009). Thus, if one expects the equity tranche not to be exhausted in the future, then equity tranche retention induces optimal screening effort as the originator would exert the same amount of effort as if the entire loan portfolio was kept on balance sheet. One can also see that without additional constraints regarding capital charges (i.e. $k = 0$), it would be actually optimal to retain 50 percent of the loan portfolio. This also holds true if the maximum effort constraint is binding.

Focusing on the second set of solutions, one can see that if the positive payoff constraint becomes binding, the originator chooses a specific combination between e^* and t^* to guarantee that the payoff in the low state is non-negative. In fact, if the originator exerts an effort level of $(1-\theta)$, then it is optimal to set the tranche thickness equal to the default probability to avoid a negative payoff in the low state of nature.

We can visualize the results by calculating implied effort levels under both solution sets for the case of the low quality market described in the previous section.[8] The solid blue line in Figure [6] shows implied effort levels under the first solution set given by equation [21], whereas the dashed green line

[8]Specifically, the probability that the economy will be in a low state is set to 80 percent and the proportion of high quality loans is assumed to be 20 percent. Further, the gross return R is still set to 1.05 and the parameter ϕ equals one.

displays effort levels according to equation [22]. The thick red line including the asterisks combines the two and displays the final optimal effort level which considers whether the positive payoff constraint is binding or not. It can be seen that in both cases the maximum effort level is capped at 0.8, thereby satisfying the maximum effort constraint. However, due to the positive payoff constraint in the low state of nature the originator would exert higher effort level for a wider range of default probabilities, as indicated by the red line.

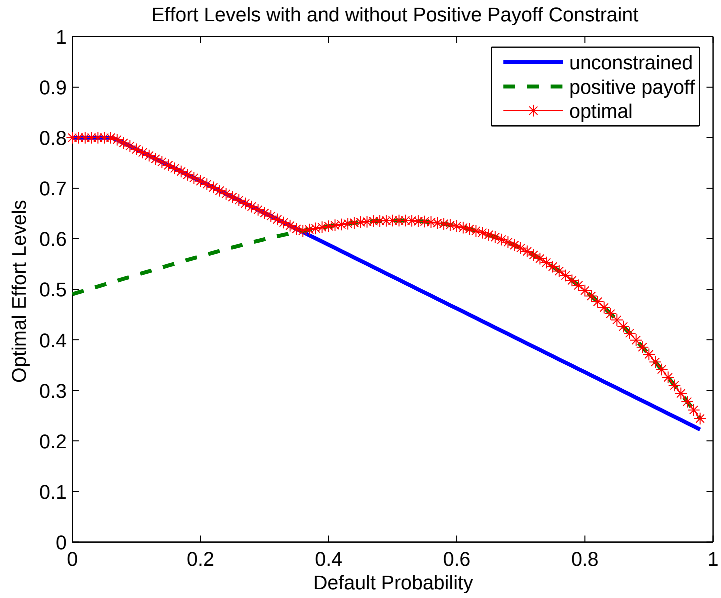

Figure 6: Effort levels under equity retention with and without positive payoff constraint

We can also analyze optimal implied tranche thickness. The solid blue line in Figure [7] displays optimal implied tranche thickness under the first solution set whereas the dashed green line corresponds to the second one. Again, the red line including the asterisks combines the two and correctly considers whether the positive payoff constraint is fulfilled. It can be seen that as long as the unconstrained solution is feasible, it is optimal to retain slightly more than 40 percent of the credit exposure. However, as soon as default probabilities exceed 40 percent, the originator has to increase its credit exposure to guarantee a non-negative payoff in the low state of nature.

The ultimate question is whether the originator would choose to exert maximum screening effort for high levels of default probability in order to break-even in the low state of nature or or whether he would instead use his limited liability option. While it is interesting to see the dynamics at work, enforcing a non-negative payoff in the low state of nature might not be a realistic scenario as it does

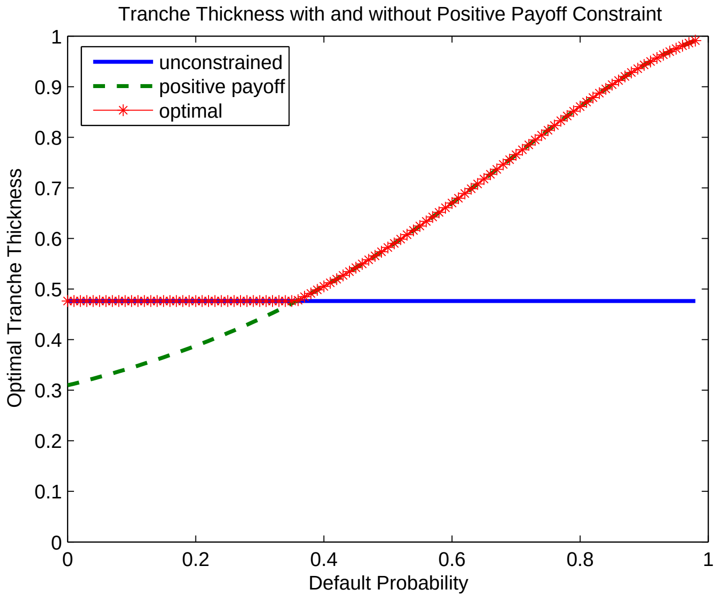

Figure 7: Tranche thickness under equity retention with and without positive payoff constraint

not capture the possibility of the equity tranche holder to simply walk away from his obligations. Therefore, the next section discusses profit maximization in case the originator only maximizes over the expected cash flows received in the high state of nature.

3.1.2 Case E2: Equity gets exhausted in low state of economy

In case equity tranche holders expect to lose their investment in the low state of the economy, the originator is only able to value those upfront payments at a premium that relate to the high state of the economy.

Proposition 2 *Under case E2, the maximization problem of the securitizer is as follows*

$$\begin{aligned} \max_{e,t}\ \pi_{E2}(e,t) &= p_L P_L(e^*) + p_H(1-t^2)R + p_H\left[P_H(e) - B_1(t)\right] - \frac{\phi}{2}e^2 \\ &- k(t) - 1 \qquad (23) \\ s.t.\ B_1(t) &\leq P_H(e) \qquad (24) \\ e &\leq 1-\theta \qquad (25) \end{aligned}$$

where [24] is the positive payoff constraint of equity tranche holders in the high state of nature and [25] considers the fact that the probability of making a good loan is bounded by one.

If the positive payoff constraint is not binding, i.e. the solution is unconstrained then expected profits are maximized by choosing e^* and t^* as follows.

$$
\begin{aligned}
e^* &= \min\left[p_H PD\frac{R}{\phi}, 1-\theta\right] \\
t^* &= \frac{1}{2} - \frac{k}{2Rph}
\end{aligned} \tag{26}
$$

Otherwise,

$$
\begin{aligned}
e^* &= \min\left[\frac{PD\left[k + 2PDp_H R(1-\theta)\right]}{\phi + 2PD^2 p_H R}, 1-\theta\right] \\
t^* &= \max\left[\frac{PD(\phi(1-\theta) - kPD)}{\phi + 2PD^2 p_H R}, 0\right]
\end{aligned} \tag{27}
$$

Proof: Follows directly from calculating the corresponding first order conditions for each of the distinct cases.

It can be seen that consistent with Fender and Mitchell (2009), equity holders will exert less screening effort than for the preceding case. Optimal tranche thickness is given by holding a 50 percent equity tranche less an adjustment for capital costs in case the positive payoff constraint is not binding. Otherwise, effort level and tranche size are chosen to ensure that the payoff relating to the credit exposure is non-negative.

We can plot again the implied effort levels according to solution set [26] which is given by the solid blue line and compare it to the case when we also consider the positive payoff constraint, as indicated by the dashed green line. The thick red line including the asterisks combines the two and displays optimal effort choice. The pattern is similar as before. If the originator wants to ensure that his payment in the high state is nonnegative, then he must increase his effort level. Nevertheless, it can be seen that optimal screening effort is substantially lower than for the preceding case.

Looking at the implied tranche thickness, Figure [9] displays thickness levels according to both solution sets. The red line including the asterisks combines both cases and correctly considers the positive payoff constraint. It is interesting to observe that the optimal tranche thickness is lower than in the preceding case. Nevertheless, after reaching a certain threshold, the originator has to increase the amount of credit he retains to ensure that the positive payoff constraint is not violated.

In order to derive the final effort level under equity retention we have to compare the value of the profit function under the different equity maximization problems.

3.1.3 Intermediate Summary

We have seen that different combinations of effort level and retention size maximize the originator's profit function, depending on whether positive payoff constraints in the high and low state of nature

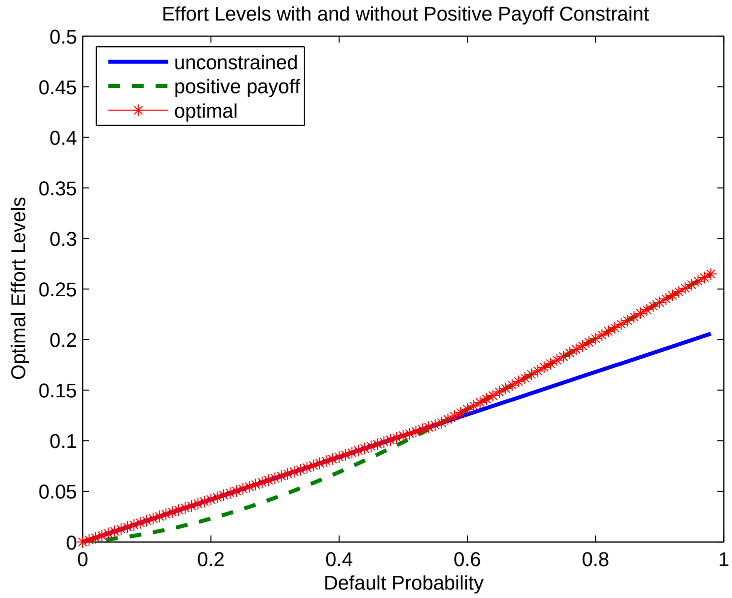

Figure 8: Effort levels under equity retention with and without positive payoff constraint

combined with the maximum effort constraint become binding. The final choice regarding optimal effort and retention combines cases E1 and E2 by giving equity holders the option to default in the low state of nature.

In other words, the two maximization problems do not account for the option of the originator to walk away from his obligations in the low state and make use of his limited liability option. Thus, the final choice is given by comparing expected profits under both regimes. If expected profit under Case E1, i.e. the equity tranche never gets exhausted, is higher than under case E2, then optimal effort and tranche thickness are implied by case E1. Else, effort and tranche thickness are chosen according to the solution of case E2. Figure [10] shows implied optimal effort levels (solid blue line) and optimal tranche thickness (dashed green line) accounting for the level of expected profit.

We can see that for default levels of up to 10 percent, retention of the equity tranche delivers the maximum possible effort level which can be exerted. While implied effort then starts to decrease for default probabilities of up to 40 percent, we can still see that the positive payoff constraint is not binding as the implied tranche thickness stays constant. Then the originator must increase the retained credit exposure to ensure that the payoff in the low state is nonnegative. However, only when default probabilities exceed 80 percent, it makes sense for the originator to use of his limited liability option in the low state of nature, as indicated by the drop in effort and tranche thickness.

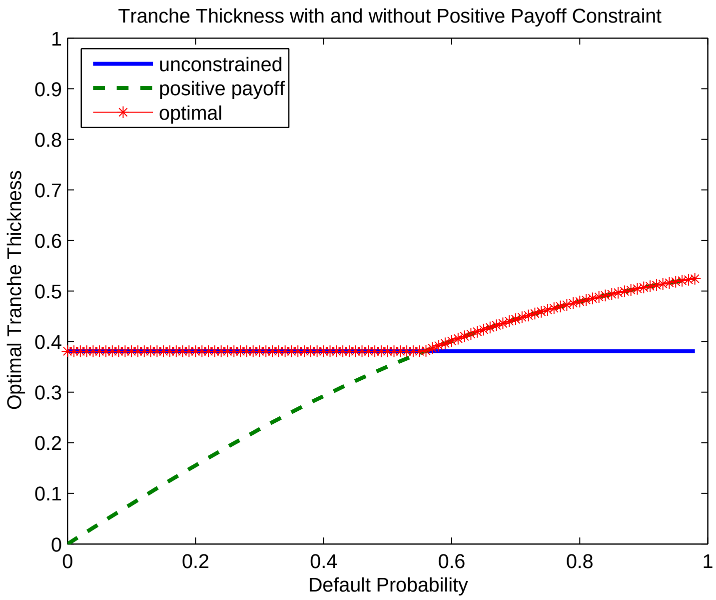

Figure 9: Tranche thickness under equity retention with and without positive payoff constraint

While this is an interesting result, our ultimate interest lies in answering whether mezzanine retention induces higher screening effort than equity retention and whether this outcome is still practically feasible.

3.2 Mezzanine Retention

Retaining interest in the mezzanine tranche is similar to holding subordinated or junior debt. Thus the payment structure is given by

$$\begin{aligned} F(e,t) &= \min\left[\max\left\{P_L(e) - B_2(t), 0\right\}, tR\right] p_L \\ &+ \min\left[\max\left\{P_H(e) - B_2(t), 0\right\}, tR\right] p_H \end{aligned} \tag{28}$$

To characterize the solution, we will make use of the argument that the payoff in the high state of nature is always larger than the payoff in the low state, i.e. $P_H(e) > P_L(e)$. Similar to Fender and Mitchell (2009), we can calculate the maximum effort level the originator is willing to exert in the low state of nature. Denoting $\hat{e}(t)$ as the maximum effort level in the low state of nature, it follows that

$$\hat{e}(t) = \frac{1-t}{1-PD} - \theta \tag{29}$$

Confining attention to effort levels $e \in [0, \hat{e}(t)]$, the expected value of the equity tranche, denoted by

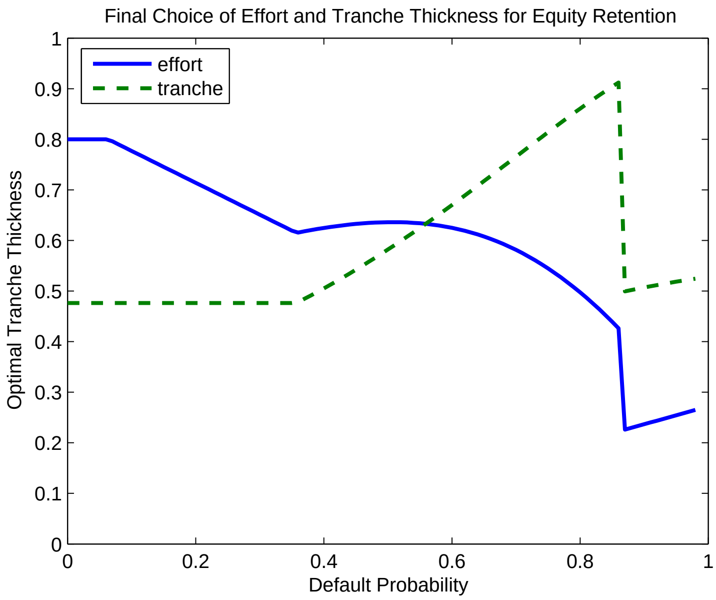

Figure 10: Optimal Effort Level and Tranche Thickness for Equity Retention

C is given by

$$C(t) = 0 + p_H\left(P_H(e^*) - B_1(t)\right) \tag{30}$$

as long as the equity tranche does not get exhausted in the high state of nature. Denoting D as the expected value of the senior tranche it follows that $D(t) = (1-2t)R$. Using this together with Assumptions [1] and [2], Proposition [3] is as follows.

Proposition 3 *Under mezzanine tranche retention, if the equity tranche is not expected to be exhausted in the high state of nature, the maximization problem of the originator is given by*

$$\begin{aligned} \max_{e,t}\ \pi_M(e,t) &= C(t) + (1-2t^2)R + p_L\left(P_L(e) - B_2(t)\right) + p_H tR \\ &- \frac{\phi}{2}e^2 - k(t) - 1 \qquad (31) \\ s.t.\ B_2(t) &\leq P_L(e) \qquad (32) \\ P_L(e) &\leq B_1(t) \qquad (33) \\ e &\leq 1-\theta \qquad (34) \end{aligned}$$

where [32] is the positive payoff constraint of mezzanine tranche holders in the low state of nature, [33] is the maximum payoff constraint of mezzanine tranche holders in the low state of nature and [34] considers the fact that the probability of making a good loan is bounded by one.

A total of 8 cases have to be analyzed out of which the following are relevant. In case the positive and maximum payoff constraint are non binding, i.e. the solution is unconstrained then expected profits are maximized by choosing e^* and t^* as follows

$$e^* = \min\left[(1-PD)p_L\frac{R}{\phi}, 1-\theta\right] \tag{35}$$

$$t^* = \frac{1}{2} - \frac{k}{4R} \tag{36}$$

On the other hand, if the positive payoff constraint is binding but the maximum payoff constraint is not, then

$$e^* = \min\left[\frac{k(1-PD)+(1-PD)R2(p_L-\theta(1-PD))}{2(\phi+(PD-1)^2R)}, 1-\theta\right] \tag{37}$$

$$t^* = \max\left[\frac{2\phi - k(PD-1)^2+(PD-1)^2p_HR - 2\theta(1-PD)}{4(\phi+(PD-1)^2R)}, \frac{PD}{2}\right] \tag{38}$$

Finally, if only the maximum payoff constraint is binding, then the solution is given by

$$e^* = \frac{k(1-PD)+(PD-1)R\left[-4+2p_H+p_L+4\theta(1-PD)\right]}{1+4(PD-1)^2R} \tag{39}$$

$$t^* = \frac{1-k(PD-1)^2+(PD-1)^2\left[2p_H+p_LR\right]-\theta(1-PD)}{1+4(PD-1)^2R} \tag{40}$$

Proof: See appendix.

It can be seen that for the unconstrained case, the solution regarding optimal effort choice is again similar to Fender and Mitchell (2009). Similar to before, we visualize results by focusing on the stylized example of the low quality loan market in a bad economic environment, as introduced in Section [2]. Figure [11] plots implied effort level for the case of mezzanine retention. The solid blue line which starts at 0.8 characterizes the unconstrained solution, as given in equation [35]. On the other hand, the dashed green line describes the solution given by [37] whereas the dash-dotted cyan line corresponds to effort level chosen under [39].

We can see that by accounting for both the positive and maximum payoff constraint, the solution is given by the red thick line including the asterisks and corresponds to equation [39] for low to intermediate levels of default probability. In other words, as long as the idiosyncratic risk relatively low, the maximum payoff constraint becomes binding. This is due to the fact that holding the mezzanine tranche is equivalent to investing in junior debt, thus promised payments are capped at some predefined level. This is why it is optimal to exert less effort than under the unconstrained case. Only when the default probabilities are sufficiently high, i.e. 40 percent or higher, then the maximum payoff constraint is not binding and the solution is given by [35]. The reason why the solution is not shown for default

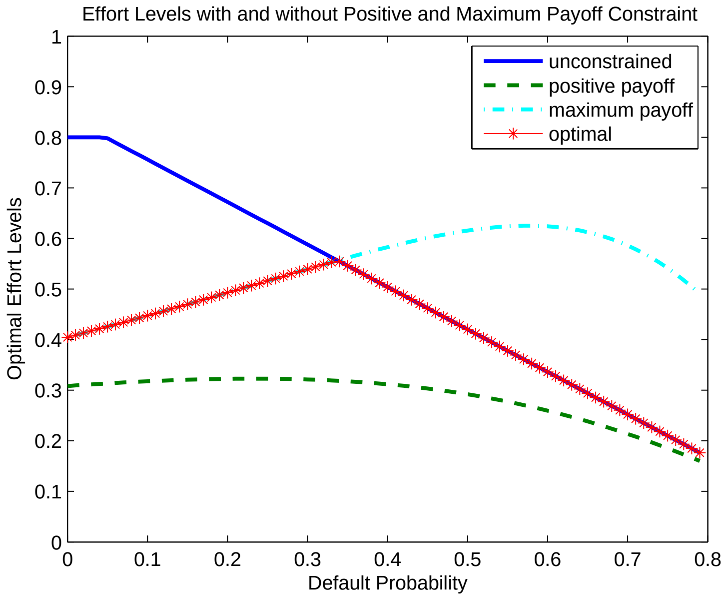

Figure 11: Implied Effort Level for Mezzanine Retention

probabilities exceeding 80 percent is because then the equity tranche would also be exhausted in the high state of nature, thereby implying that the maximization problem of the mezzanine tranche holder would need to be modified. For reasons of simplicity and due to the fact that idiosyncratic risks higher than 80 percent combined with the negative prospects of the low quality market are not too realistic, we do not pursue this issue further here.

Similarly, one can also compare implied tranche thickness. Focusing on the red thick line including the asterisks in Figure [12] we can see again that as long as default probabilities are low, it is optimal to retain less credit exposure than implied by the unconstrained solution which is given by the solid blue line. This can be explained by the fact that for low to intermediate levels of default probability, the risk that the mezzanine tranche gets exhausted is relatively low, thereby enabling the securitizer to reduce credit exposure and benefit from the value premium relating to the upfront payment. However, after default probabilities reach a certain threshold of 40 percent, the solution is again given by [35].

The ultimate question of interest is whether a profit maximizing originator would choose to retain the equity or mezzanine tranche and what the corresponding implications for screening effort would then be. It turns out that by comparing profits under both retention mechanisms, a rational securitizer would choose to retain the mezzanine tranche for this specific example. Figure [13] shows implied screening level under both equity retention (dashed green line) and mezzanine retention (solid blue

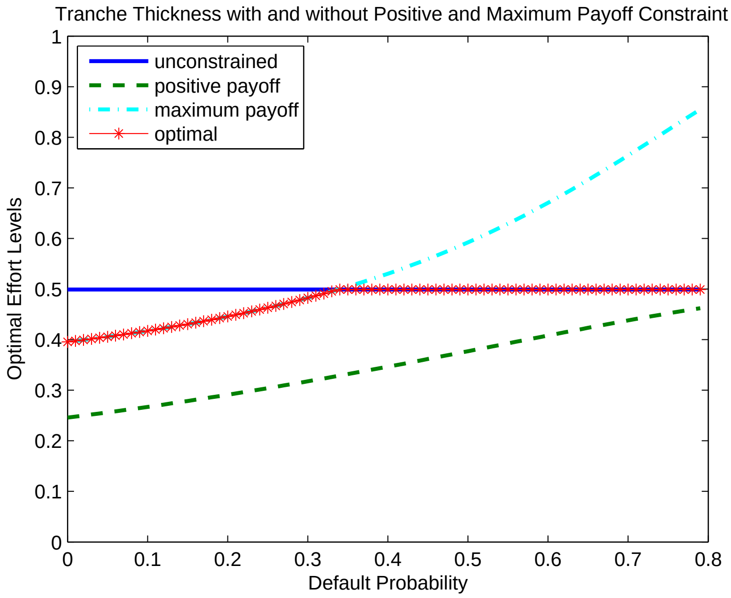

Figure 12: Implied Tranche Thickness for Mezzanine Retention

line). The thick line indicates the retention choice of a profit maximizing securitizer. Contrary to the previous section, it can be seen that the screening level under mezzanine retention is actually lower than if the securitizer had retained the equity tranche. This is because for low to intermediate levels of default probability, the maximum payoff constraint under mezzanine retention becomes binding such that the unconstrained solution is not feasible. However, as it has been illustrated in the previous section, this is exactly the area where mezzanine retention potentially dominates equity retention. The fact that the originator prefers to retain the mezzanine tranche is due to the fact that the retained credit exposure is higher for equity retention as can be seen in Figure [14]. This is because it helps equity tranche holders to reduce the probability that their tranche gets exhausted in the low state of nature. However, this also decreases the value of the upfront payment which is why total profit is lower under equity retention.

We have seen that if the securitizer can choose screening effort and the size of the retained credit exposure to maximize profits, implications regarding optimal effort level and the corresponding dominance of one tranche with respect to another one might change. However, we want to emphasize that this result is still specific to the example chosen and it also depends on the assumption regarding the functional form of the upfront payment. More importantly, while the model realistically includes capital charges in the maximization problem, we have not yet addressed the question whether a structured product can be created with the implied effort level and tranche thickness. In other words, a full

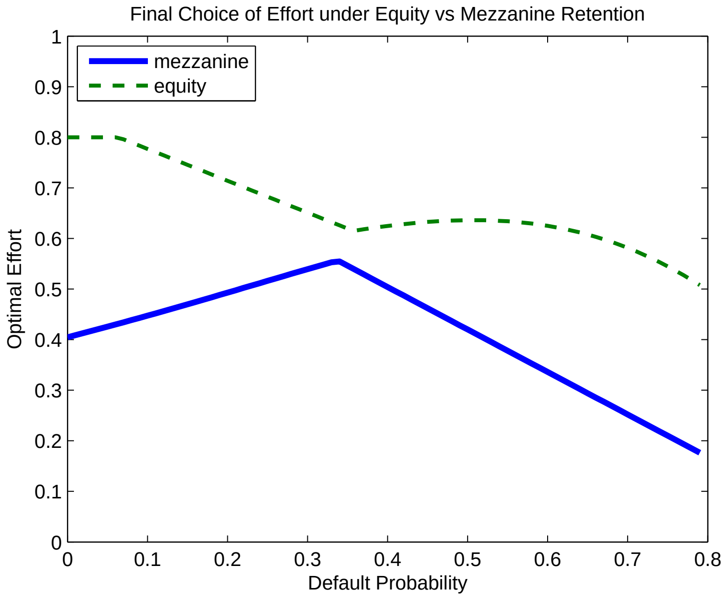

Figure 13: Implied Effort Levels under Equity vs Mezzanine Retention

maximization problem would need to consider an additional constraint regarding the tranche thickness which accounts for the feasibility of a structured product before giving final policy implications.

4 Summary and Outlook

This paper investigates optimal retention policy and screening effort in the context of asset securitization. We start by analyzing a recently proposed by model by Fender and Mitchell (2009) who find that under certain technical conditions, retaining interest in the mezzanine tranche induces an originator to exert more screening effort than if he held the equity tranche.

Calculating two different stylized examples, we find that this result depends on the specific combination of economic conditions, quality of the loan pool, default probabilities and the thickness of the retained tranches. We also calculate corresponding capital charges by relating the results of the Fender and Mitchell Model to a simple structured product. We find that while equity retention may generate the highest screening effort, it unsurprisingly comes at the cost of high capital charges.

We then propose a theoretical extension by also solving for the optimal retention size or tranche thickness. This allows us to reduce the dimensionality problem from a policymaker's perspective as no assumptions regarding the tranche thickness is necessary. Additionally, we are able to introduce capital costs into the objective function to capture negative feedback effect of different capital requirements.

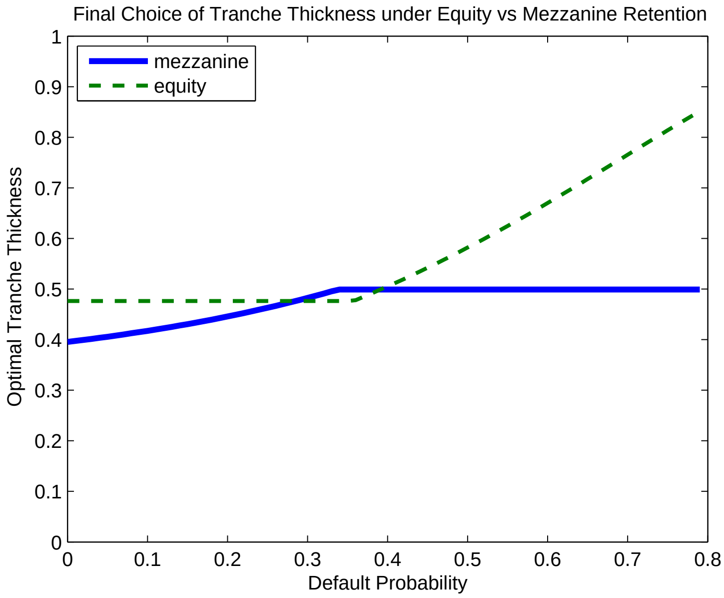

Figure 14: Implied Tranche Thickness under Equity vs Mezzanine Retention

We then focus on the example of a low quality market used in Section [2.2] to check whether the previous implications still hold. It turns out that if a securitizer can choose both effort and tranche thickness to maximize profits, then equity retention still generates higher effort levels for this specific example.

However, we want to stress that this result depends on the specific parameter values and that we have not yet considered the question of whether we can create a structured product with the implied effort levels and tranche thickness. Thus, future research will introduce an additional constraint regarding the tranche thickness into the maximization problem which then accounts for the feasibility of creating a structured product.

A Appendix

A.1 Proof of Proposition [3]

Proof. The maximization problem is given as follows.

$$\begin{aligned} \max_{e,t}\ \pi_M(e,t) &= C(t) + (1-2t^2)R + p_L\left(P_L(e) - B_2(t)\right) + p_H tR - \frac{\phi}{2}e^2 - ct \\ &- \ 1 - \lambda_1\left[B_2(t) - P_L(e)\right] - \lambda_2\left[P_L(e) - B_1(t)\right] - \lambda_3\left[e + \theta - 1\right] \end{aligned} \tag{41}$$

For the remaining cases, both $\lambda_1 = \lambda_2 > 0$. It immediately follows that as $\hat{t}(e) = 0$ as $B_2(t) = P_L(e)$ and $P_L(e) = B_1(t)$. This in turn implies that

$$e^* = \frac{1}{1-PD} - \theta \tag{42}$$

Because $\lambda_2 = 0$, we need that $e^* < 1 - \theta$ which only holds true if $PD = 0$. Because this case is economically not interesting, we do not present it in the proposition.

■

Table B.1. Credit Ratings versus Idealized Expected Losses and Basel II Risk Weights *(in percent)*		
Credit Rating	Maximum Expected Loss for Each Rating Level	Risk Weight
AAA	0.0055	
AA+	0.0550	20
AA	0.1100	
AA-	0.2200	
A+	0.3850	
A	0.6600	50
A-	0.9900	
BBB+	1.4300	
BBB	1.9800	100
BBB-	3.3550	
BB+	5.1700	
BB	7.4250	350
BB-	9.7130	
B+ and lower	$\geq$ 12.2100	1,250
Sources: Yoshizawa (2003) for the idealized expected losses and BCBS (2009) for the risk weights		

A.2 Capital Charges

This annex shows how the Moody's binomial extension technique (BET) was used to produce Figure [4] which displays capital requirements for different retention schemes for a three-tranche structured credit product. Assuming a 100 percent loss-given-default, the BET is applied to the calculated default probability (TPD) to generate a loss distribution for a portfolio of 1,000 equal-sized loans. The loss distribution is then applied to the three-tranche example based on this portfolio and assuming a ten-year term to maturity, to calculate expected losses for each tranche. These expected losses are then used to back out credit ratings based on the Moodys idealized expected loss tables (Table B.1).

The capital charge calculation example in the main text assumes that the exogenous probability of making a good loan (θ) is 60 percent and the probability of the low state (p_L) is 50 percent and is applied to a hypothetical portfolio of 1,000 loans. In order to show the details of the capital charge calculation, assume that the probability of a good loan defaulting in a low state (PD_L) and a bad loan defaulting in a high state (PD_H) are both set to five percent. Hence, an effort level (e) of 20 percent will imply that the probability of making a good loan ($\alpha_G(e)$) increases from 60 to 80 percent, and a total default probability (TPD) of 12.5 percent.

If the loan defaults in this portfolio were uncorrelated, at this point the tranche-by-tranche expected loss calculations could be done with a huge spreadsheet comprised of (in this case) 1,000 binomial probabilities. However, the systemic risk factor (represented by p_L in this case) implies that the loan defaults will indeed be correlated, and the BET is used to produce reasonably accurate approximations of the true loss probability distributions under these conditions.

In this case, by assuming that the pairwise default correlations between the 1,000 loans are all equal to 10 percent, the actual portfolio can be replaced by a simpler portfolio of just 10 homogeneous uncorrelated loans. The mechanics of the calculation for this example ($p_L = 50$ percent, $\theta = 60$ percent, $e = 20$ percent, and $PD_L = PD_H = 5$ percent) are illustrated in Table B.2.

The first step in the calculation process was to determine the senior tranche size that would result in the expected loss for this tranche such that an A- rating is obtained according to Table B.1 (0.99 percent). This turns out to be $733.60 of the assumed $1,000 portfolio, which implies a size for both of the equal-sized equity and mezzanine tranches of $133.20. (The equity and mezzanine tranches are equal by assumption.)

This scenario puts the risk weights of the three tranches at 1,250, 1,250 and 50 percent, respectively, for the equity, mezzanine and senior tranches. However, this first iteration assumed a 20 percent screening effort level, whereas the optimal effort level will vary according to the retention scheme (equity, mezzanine or vertical slice) and size.

At the 5 percent PD level the optimal effort levels are 40.0 (equity retention), 31.2 (mezzanine) and 7.0 percent (vertical slice), and these effort levels were fed back into the rating / risk weight calculations to produce the total capital charges plotted in Figure 6. For example, the 31.2 percent effort level associated with mezzanine tranche retention decreases the TPD to 6.90 percent, which, as shown in Table B.3, reduces the senior tranche expected loss to 0.1755 percent (AA rating) and the mezzanine tranche to 8.7937 percent (BB). On the other hand, the 7.0 percent effort level associated with vertical slice retention would increase the TPD to 19.0 percent, which increases the senior tranche expected loss to 3.1939 percent (BBB) (not shown).

Table B.2. Calculation of Tranche Sizes (Steps 1 and 2) with Assumed 20 Percent Effort Level					
		Dollar Losses ($)			
Defaults	Probability (%) (1)	Total	Equity (2)	Mezzanine (3)	Senior (4)
0	26.31	0	0.00	0.00	0.00
1	37.58	100	100.00	0.00	0.00
2	24.16	200	133.20	66.80	0.00
3	9.20	300	133.20	133.20	33.60
4	2.30	400	133.20	133.20	133.60
5	0.39	500	133.20	133.20	233.60
6	0.05	600	133.20	133.20	333.60
7	0.00	700	133.20	133.20	433.60
8	0.00	800	133.20	133.20	533.60
9	0.00	900	133.20	133.20	633.60
10	0.00	1,000	133.20	133.20	733.60
Expected Loss ($)		125	85.68	32.06	7.26
Tranche Size ($)		1,000	133.20	133.20	733.60
Expected Loss/Tranche Size (%)		12.50	64.3254	24.0668	0.9900
Credit Rating			No Rating	B- to CCC	A-
Risk Weight (%)			1,250	1,250	50

Notes:

(1) Probability of n defaults = (10!/(n!(10-n)!))TPDˆn(1-TPD)ˆ(10-n)

(2) Loss(equity) = min{Total Loss, Size(equity)}

(3) Loss(mezzanine) = min{Total Loss - Loss(equity), Size(mezzanine)

(4) Loss(senior) = Total Loss - Loss(equity) - Loss(mezzanine)

The last step involves mapping the revised credit ratings and risk weights into the corresponding capital charges (CC) using the formula below.

$$CC = 0.08 \sum [t_{equity} RW_{equity} + t_{mezz} RW_{mezz} + t_{senior} RW senior]$$

where the ts are the relevant retained tranche sizes or "thicknesses." Table B.3 shows, for example, that the mezzanine tranche retention scenario results in a risk weight of 20 percent on the \$733.60 senior tranche, 350 percent on the \$133.20 mezzanine tranche and 1,250 percent on the \$133.20 equity tranche. However, only the mezzanine risk weight is relevant in this case, so the retained tranche capital charge will be \$37.30 (0.08 x \$133.20 x 350 percent).

Table B.3. Calculation of Tranche Sizes (Step 3) with Assumed 31.2 Percent Effort Level					
		Dollar Losses ($)			
Defaults	Probability (%) (1)	Total	Equity (2)	Mezzanine (3)	Senior (4)
0	48.92	0	0.00	0.00	0.00
1	36.26	100	100.00	0.00	0.00
2	12.09	200	133.20	66.80	0.00
3	2.39	300	133.20	133.20	33.60
4	0.31	400	133.20	133.20	133.60
5	0.03	500	133.20	133.20	233.60
6	0.00	600	133.20	133.20	333.60
7	0.00	700	133.20	133.20	433.60
8	0.00	800	133.20	133.20	533.60
9	0.00	900	133.20	133.20	633.60
10	0.00	1,000	133.20	133.20	733.60
Expected Loss ($)		69.00	56.00	11.71	1.29
Tranche Size ($)		1,000	133.20	133.20	733.60
Expected Loss/Tranche Size (%)		6.90	42.0420	8.7937	0.1755
Credit Rating			No Rating	BB	AA
Risk Weight (%)			1,250	350	20

Notes:

(1) Probability of n defaults = (10!/(n!(10-n)!))TPDˆn(1-TPD)ˆ(10-n)

(2) Loss(equity) = min{Total Loss, Size(equity)}

(3) Loss(mezzanine) = min{Total Loss - Loss(equity), Size(mezzanine)

(4) Loss(senior) = Total Loss - Loss(equity) - Loss(mezzanine)

References

G. Akerlof. The market for lemons: Quality uncertainty and market mechanism. *The Quarterly Journal of Economics*, 84:488–500, 1970.

Franklin Allen and Elena Carletti. Credit risk transfer and contagion. *Journal of Monetary Economics*, 53:89–111, 2006.

Elena Carletti. The structure of bank relationships, endogeneous monitoring, and loan rates. *Journal of Financial Intermediation*, 13:58–86, 2004.

Gabriella Chiesa. Optimal credit risk transfer, monitored finance and banks. *Journal of Financial Intermediation*, 17:464–477, 2008.

Peter M. DeMarzo. The pooling and tranching of securities: A model of informed intermediation. *Review of Financial Studies*, 18:1–34, 2005.

Douglas Diamond. Financial intermediation and delegated monitoring. *Review of Economic Studies*, 60:393–414, 1984.

Dareell Duffie. Innovations in credit risk transfer: Implications for financial stability. *BIS Working Paper*, 255, 2008.

Ingo Fender and John Kiff. Cdo rating methodology: some thoughts on model risk and its implications. *Journal of Credit Risk*, 1:37–57, 2005.

Ingo Fender and Janet Mitchell. Incentives and tranche retention in securitisation: A screening model. *Working Paper*, 2009.

Douglas Gale and Martin Hellwig. Incentive-compatible debt contracts: The one-period problem. *Review of Economic Studies*, 70:647–663, 1985.

Gary B. Gorton and George G. Pennacchi. Banks and loan sales marketing nonmarketable assets. *Journal of Monetary Economics*, 35:389–411, 1995.

IMF. Global financial stability report. 2009.

Robert D. Innes. Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, 52:45–67, 1990.

Michael Jensen and William Meckling. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3, 1976.

Andreas Jobst. What is securitization. *Finance and Development*, 3:48f., 2008.

Hayne E. Leland and David H. Pyle. Informational asymmetries, financial structure, and financial intermediation. *Journal of Finance*, 32:371–387, 1977.

Alan D. Morrison. Credit derivatives, disintermediation, and investment decisions. *Journal of Business*, 78:621–647, 2005.

Antonio Niccolo and Loriana Pelizzon. Credit derivatives: Capital requirements and strategic contracting. *Working Paper*, 2006.

Wolf Wagner. The liquidity of bank assets and banking stability. *Journal of Banking and Finance*, 31: 121–139, 2007a.

Wolf Wagner. Interbank diversification, liquidity shortages and banking crises. *Working Paper*, 2007b.

Wolf Wagner and Ian W. Marsh. Credit risk transfer and financial sector stability. *Journal of Financial Stability*, 2:173–193, 2006.