

IMF Working Paper

Macrofinancial Linkages of the Strategic Asset Allocation of Commodity-Based Sovereign Wealth Funds

*Aaron Brown, Michael Papaioannou, and
Iva Petrova*

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Monetary and Capital Markets Department

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Prepared by Aaron Brown, Michael Papaioannou and Iva Petrova¹

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Abstract

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This paper analyses the links between the investment strategies of a commodity-based SWF and the macroeconomic framework of the owner country. We examine some basic macrofinancial linkages of an SWF's strategic asset allocation (SAA) strategies with regard to the government budget, monetary policy, and exchange rate movements. Based on a simple Markowitz-model framework, which integrates the specific objectives and constraints facing an SWF and the country's specific characteristics and macroeconomic vulnerabilities (especially in relation to commodity prices and prospective defined liabilities), we derive an SAA. The asset-liability methodology that is applied in the selection of an SWF SAA also allows assessing whether (i) the SAA adequately takes into account the country-specific risks and vulnerabilities, and (ii) its objectives and macrofinancial constraints are consistent. Some analytical and practical issues in determining an SAA model are also discussed, along with key effects of a financial crisis.

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Author's E-Mail Address: Aaron.Brown@gov.ab.ca; mpapaioannou@imf.org;
ipetrova@imf.org

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I. INTRODUCTION

The assessment of the domestic macrofinancial linkages of the strategic asset allocation (SAA) of commodity-based sovereign wealth funds (SWFs) often constitutes an integral part of their design and establishment. Many commodity-rich countries that accumulated sizable sovereign assets in the past few years have set up different SWF-type arrangements with objectives related to fiscal stabilization, intergenerational saving transfers, support for future pension liabilities and social priorities (IWG, 2008).² Within the institutional framework that an SWF operates, the macrofinancial linkages of its investment strategies that need to be analyzed pertain to its relation to the government budget, monetary policy and exchange rate movements. We use a simple Markowitz-model framework that integrates the specific objectives and constraints of an SWF and the country's specific characteristics and macroeconomic vulnerabilities, especially with respect to commodity prices and prospective defined liabilities, to derive the optimal SAA. Thus, this methodology is key in providing an SAA that adequately incorporates these macrofinancial links.

In most cases, an SWF's SAA and investment guidelines are specified by the government (the owner of the SWF). In this context, our goal is to develop a model-based SAA framework to assess the macrofinancial linkages of an SWF's SAA, given the SWF's specific objectives and constraints. In particular, we describe the analytical assumptions based on the funding and spending rules for an SWF, select a set of asset classes for the investment decision process based primarily on the objectives of the SWF, and specify the Markowitz-type SAA model for the choice of the specific assets. Based on this analysis, we develop an illustrative asset allocation. It should be noted that the standard mean-variance framework to be used in selecting an SWF's SAA allows us also to assess whether its objectives and macrofinancial constraints are consistent.

In identifying eligible asset classes for the SAA we assume that the SWF invests only in foreign assets so as to reduce Dutch disease effects. Investing the assets in the often shallow domestic capital market, could potentially create local asset price bubbles and put upward pressure on inflation. However, the decision to invest externally may expose the SWF to exchange rate risk. The level of risk tolerance, therefore, needs to be explicitly addressed along with the diversification of assets within the permissible investment universe of the SWF.

In determining the SAA framework of an SWF, its asset-liability structure should also be taken into account. For example, while an asset-liability framework would be used for any SWF with defined liabilities, an asset-only approach would be sufficient for a commodity-based stabilization SWF, assuming that commodity reserves (in the ground) dominate

² For the purposes of this paper, SWFs are defined as in IWG, 2008. SWFs are special purpose investment funds or arrangements, owned by the general government. Created by the general government for macroeconomic purposes, SWFs hold, manage, or administer assets to achieve financial objectives, and employ a set of investment strategies which include investing in foreign financial assets. The SWFs are commonly established out of balance of payments surpluses, official foreign currency operations, the proceeds of privatizations, fiscal surpluses, and or receipts resulting from commodity exports.

financial assets in its asset portfolio. The local currency is used as a numeraire currency in calculating the investment performance of such an SWF, as government fiscal accounts are usually denominated in local currency. Further, the SAA of a fiscal stabilization SWF would be most consistent with its objectives if it has an allocation of fixed-income instruments of a counter-cyclical economy.³ Equities and high-grade corporate bonds of counter-cyclical economies could also be considered as eligible instruments in the investment policies of such SWFs. However, but we show that these instruments may expose a pure stabilization SWF to too much risks and may be more appropriate for SWFs that also seek savings objectives.⁴

The remaining of the paper is organized as follows: Section II discusses the main institutional arrangements pertaining to an SWF. The objectives of an SWF may include fiscal stabilization, saving for future generations, supplementing pension expenditures, and financing for social priorities. An SWF is intended to invest in foreign assets so as to mitigate Dutch disease effects and avoid inflationary pressures. Section III presents some macrofinancial linkages in the SAA of SWFs, focusing on the linkage between SWF investment decisions and fiscal policy, while monetary and exchange rate policy linkages are also discussed briefly. Section IV describes the SAA methodology for deciding on the optimal asset allocations in the context of the macroeconomic framework. We employ a standard Markowitz model, which uses an asset-liability management framework to determine the optimal asset allocation. The model assumes specific funding and spending rules for the SWF, as well as a set of investment asset classes to be considered. Section V outlines several practical aspects of developing an SWF's SAA framework, including the selection of asset classes, benchmarking and numeraire currency, while Section VI presents some illustrative calculations. Section VII discusses SAA considerations in periods of financial distress. Section VIII provides a summary and concluding remarks on SAA implementation issues.

II. INSTITUTIONAL ARRANGEMENTS OF AN SWF

The following section provides a broad review of (i) the roles, responsibilities and institutional arrangements of common SWFs; (ii) their risk management framework and investment strategy development; and (iii) issues in transparency and accountability.

Roles, responsibilities, and institutional arrangements

The roles and responsibilities pertaining to an SWF should be clearly defined. The owner of the funds is often the state, which usually delegates responsibility to the Ministry of Finance for the management of the SWF's investments, the asset returns, and the planning stage of the investment process, including the determination of the SAA. If this is the case, the MoF

³ This implies seeking higher financial returns when the domestic economy is in a recession. Commodity prices, especially in recent years, have co-moved with oil price increases due to global demand and speculative trading. High oil prices have empirically adversely impacted GDP in developed countries (Barrell, 2007).

⁴ We limit our illustration to these asset classes, but if we add riskier assets in the portfolio, the list should also include assets like real estate, infrastructure, and private equity.

may also be responsible for monitoring the investments, supervising the administration of the funds, selecting external managers, and preparing and publication of reports on the state of the funds. In this regard the MoF may establish specific guidelines and instructions for the custody of the funds, as well as for the selection and monitoring of external managers.

A fiscal agency arrangement for managing an SWF is commonly put in place to entrust the investment execution of its funds to an entity that has adequate institutional capacity to manage assets assigned to it. The entity responsible for the implementation of the actual investments follows the MoF investment mandate for the SWF.

Risk management framework and investment strategy development

A dynamic investment process generally follows three distinct phases: (i) a planning phase, when the investment strategy is linked to the fund's (macroeconomic) objectives and the owner's expression of risk tolerance; (ii) an execution phase, when the related investment mandates and the SAA are actually effected in the chosen asset classes; and (iii) the feedback phase, when the owner of the assets considers the performance of the assets and attributes and appraises the components of performance, including an assessment of assigned internal and external managers.

Using qualified and internationally reputed portfolio consultants in the development of the SAA is a sound decision, as they provide a wide range of services, including asset allocation support, performance evaluation, and manager screening and selection support. Consultants have cost-efficient access to investment technology, as well as extensive asset class and manager return databases. Consultants can aid in the interpretation and calculation of performance, as well as portfolio construction analysis to help determine the appropriate mix of managers.

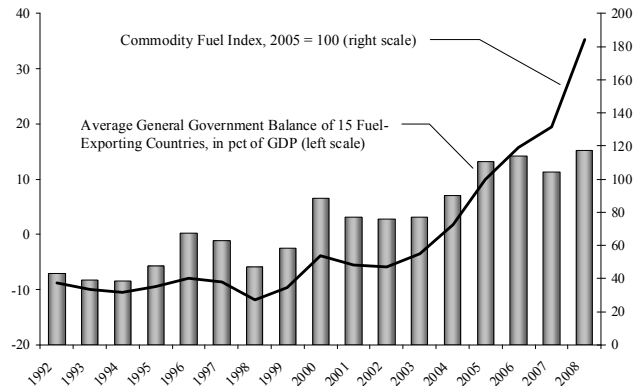
Regarding the implementation of the investment strategy, cross-country experience suggests that the central bank often has access to relevant system and human resources and relevant capacity in the management of low-risk asset classes, such as sovereign bonds and money market assets. However, only in a few cases (e.g., Norway) has the central bank developed expertise in managing riskier asset classes, such as equities and corporate bonds. Thus, international good practice calls for a country without this expertise to externalize the management of equities, corporate bonds, and alternative asset classes—such as real estate or infrastructure—to external fund managers. Regardless of such arrangements, the entity that manages the SWF should implement a solid risk management framework.

Finally, if a country has more than one SWF with different objectives, each SWF should be treated as a separate investment problem. However, this does not necessarily imply that the funds should be managed separately. The funds could be combined operationally as one pool of money, but they will require separate accounting and reporting, which requires that they be separated at least on that level.

Transparency and accountability

Transparency and accountability are very important to retain public confidence, especially in the face of difficult global market conditions. The link between the legislation for an SWF and the SAA for these funds may not be easily communicated to the public (a problem faced by most countries). However, cross-country experience suggests that it is important for the general public to understand that in order to achieve higher long-term returns; there will be times when the return on assets may be negative.

Figure 1. Commodity Prices and Budget Balance



Source: IMF, WEO.

Professional web pages are a standard communication tool. SWF websites often contain detailed information, including the legal and institutional framework, and in some cases, the current SAA and performance information.⁵ Furthermore, it is a good practice to publish portfolio updates regarding performance and asset mix, and possibly reports of detailed security level holdings, at least quarterly—some SWFs do so monthly. Publication of key information in English lowers the language barrier for international observers and allows quick and easy comparison with similar SWFs.

To enhance the public understanding of the operation and performance of an SWF and reduce the political economy/reputational risk for the authorities of potential SWF losses, it is important that the government have a dialogue with the public regarding the risky nature of financial investments. Portfolio returns expressed in local currency are easier to understand and can help mitigate myopic focus and hindsight analysis. Ex-post disclosures and analysis could emphasize that portfolio performance be viewed in the context of the objectives of the SWFs and their long-term horizon.

III. MACROFINANCIAL LINKAGES OF THE SWF MANAGEMENT

It has become a common practice to portion commodity revenues in funds with specific macroeconomic objectives to further reduce fiscal cyclicality (stabilization fund), provide for future government contingent liabilities (e.g., pension reserve fund), or meet the government's social priorities (e.g., educational fund). Notwithstanding their main macroeconomic objectives, SWFs have significant other effects on macroeconomic policy. The focus in this analysis is on commodity-based stabilization SWFs, which are designed to

⁵ See Hammer, Kunzel, and Petrova, 2008; Das, Lu, Mulder, and Sy, 2009.

fit closely the respective country's policy framework by minimizing the distortions that large and volatile commodity flows might cause to the fiscal accounts, inflation, and the exchange rate.

A. Government Budget

We describe the case of a stabilization fund with funding and withdrawal rules that are derived from a fiscal rule consistent with the Permanent Income Hypothesis. Under such a rule, government expenditures are equivalent to long-term government revenues minus a structural balance target, as a certain percent of GDP, every year.⁶ If the budget outcome is a surplus, it is invested in the SWF, while a deficit is financed by the SWF.⁷

The stabilization features of the SWF are determined by the parameters of the fiscal rule and the amplitude of the revenue gap (i.e., the actual minus long-term revenues). First, it should be acknowledged that the assumptions used to calculate the structural revenue—and hence the structural expenditures—will have a strong impact on the calculation of the revenue gap. The commodity price drop that would lead to a negative revenue gap and would trigger use of resources from the SWF would depend on these assumptions.

Second, the structural balance provides a built-in fiscal cushion and reduces the likelihood of using SWF resources if the target is a surplus. A large positive structural balance target (surplus), would mean that the SWF would have to be tapped only in very extreme cases in order to offset very large revenue shortfalls. In such cases, the SWF would serve as a savings fund since the stabilization function is sufficiently well performed by the structural balance. A small structural balance target or an allowance for a structural deficit would place a great demand on the SWF as a stabilization fund since it would have to be used more often. Therefore, while the stabilization function of the SWF is to counteract the revenue gap, the structural balance target determines the threshold at which use of SWF resources will be triggered.

The revenue gap in a commodity-based country has two main components: a cyclical effect resulting from commodity revenues and a cyclical effect resulting from other, including other tax, revenues. Since the commodity price effect often dominates (Figure 1), and the two

⁶ The Permanent Income Hypothesis developed by Friedman (1957) states that income, and consumption, can be decomposed into a permanent (long-term) component, which could be interpreted as the annuity value of total wealth, and a transitory (cyclical) component, which averages to zero over time. Based on this concept, the IMF calculates—for 20 industrialized countries—structural budget balances that reflect government expenditures and revenues when the economy is operating at a level of capacity utilization with low and stable inflation (Hagemann, 1999). For oil exporting countries, an optimal fiscal strategy based on the permanent income hypothesis is described in Engel and Valdes (2000).

⁷ Note that the actual budget balance depends upon the cyclicity of government revenues, reflecting the revenue shortfall relative to the long-term revenues:

$$\begin{aligned} \text{Budget Balance} &= \text{Revenues} - \text{Expenditures} = \text{Revenues} - (\text{Long-term Revenues} - \text{Structural Balance Target}) \\ &= \text{Revenues} - (\text{Long-term Revenues} - k_s \times \text{GDP}) \\ &= \text{Revenue Gap} + k_s \times \text{GDP} \end{aligned}$$

where k_s is an annual structural fiscal balance (savings) target rate (See section IV).

effects are likely to be highly correlated, the SWF will generally be used to neutralize the commodity price movements. The SAA of the SWF will, therefore, contain assets that have negative correlation with the respective commodity price. The selection of the SAA becomes more nuanced if the SWF, by intention or design, has multiple objectives, such as funding future social programs or other government liabilities (see below).

B. Monetary Policy

The country's monetary policy could be affected by the SWF management in various ways. The volatility of fiscal revenues, and commodity revenues specifically, presents a challenge for monetary authorities as it may affect the volatility of aggregate demand and lead to excessive inflation and real exchange rate fluctuation. This occurs either through the channel of government spending, by generating positive wealth effects as a result of domestic government investments, or through additional liquidity and private-sector lending enabled by government deposits.

The funding and withdrawal rules of a stabilization SWF, in combination with a fiscal rule, often seek to minimize such effects on domestic demand. Using a stabilization SWF to cover fiscal liabilities during cyclical downturns ensures that SWF resources will enter the economy during periods that private demand is relatively weak, and the absorption capacity for additional public resources is correspondingly strong. As the stabilization SWF may reduce the amplitude of budget balances, the transmission mechanism of monetary policy is likely to be enhanced.

C. Exchange Rate Issues

Investing the SWF's resources abroad tend to reduce their exchange rate impact. If the SWF's resources are to be saved abroad and mining companies and exporters are allowed to pay tax liabilities in dollars, this policy could prevent commodity export revenues from affecting the value of the exchange rate. In particular, it could mitigate the real appreciation associated with increases in commodity prices and commodity-driven GDP growth, as well as reduce the impact of any exchange-rate reversals.

Under a flexible exchange rate regime, the (nominal) exchange rate of the local currency vis-à-vis the U.S. dollar mitigates negative terms-of-trade shocks related to the (nominal) price of the endowed commodity, which could have direct implication on the objective, use, and SAA of a stabilization SWF. As in most commodity exporting countries, the local currency tends to depreciate when the commodity price falls. Then, the SWF interest returns measured in domestic currency are boosted when the domestic currency depreciates, providing an additional cushion to the budget.

However, if the authorities intervene to prevent a substantial exchange rate depreciation stemming from a negative terms-of-trade shock, the country may experience substantial deterioration in commodity exports and fiscal revenues. If automatic stabilizers are at work, the attendant fiscal deficit may need to be financed with SWF resources. Hence, a fixed exchange rate regime furthers the argument for investing the resources of an SWF in assets

that have negative correlation with domestic GDP growth. In general, the authorities' tolerance for exchange rate adjustment will have an impact on the SWF's SAA.

D. Public Sector Balance Sheet Effects

High commodity revenues are typically associated with excess liquidity in the domestic banking system and challenging liquidity management conditions. Particularly in countries where the exchange rate regime is not fully flexible, the central bank may incur substantial sterilization costs during commodity price booms. In such countries, the need to support a sufficiently well-capitalized central bank may assume urgency during periods of high commodity prices and appreciation pressures, so as to preserve the effectiveness of monetary policy implementation.⁸ While yielding large government revenues, commodity booms are likely to be associated with substantial carry costs of international reserves and low central bank profitability.⁹ Therefore, from an overall sovereign balance perspective, it is more feasible to replenish the central bank capital during such periods. If commodity prices are high and monies are appropriated for central bank recapitalization, less will be available for placing with an SWF. Therefore, the decision to recapitalize the central bank can be viewed as a use of SWF's resources and, in turn, will impact the SWF SAA.

IV. AN ANALYTICAL MODEL FOR THE SAA DETERMINATION OF A STABILIZATION SWF

The SAA methodology determines the investments and accordingly allows for an assessment of the management of an SWF and its macro-financial linkages. The main purpose of this methodology is to allocate investments so that the objective of the fund is achieved as efficiently as possible, while trading off relevant gains and relevant risks (Scherer and Gintschel, 2008). Hence, any analysis of optimal SAA starts with establishing explicit objective(s) of an SWF. The objective of the fund will identify which components of the sovereign's wealth, or which cash flows, should be taken into account when analyzing whether some asset risks are more relevant than others for the owner country.

The fiscal rule, which defines how monies are channelled into the SWF and how its funds are intended to be spent, and the resulting cash flows are operational images of the SWF objectives, as defined by its owner. We will limit our analysis to the cash flows which are part of the stated objectives for an SWF. The arrangement for the SWF to receive cash inflows can work on a net or gross basis. Analysing the components of a net arrangement helps in tuning the asset allocation to the purpose of the SWF (Sharpe and Tint, 1990).

⁸ Recent studies have found that the financial strength of the central bank is important to achieve the desired inflation outcome (Stella and Lonnberg, 2008).

⁹ A central bank with large carry costs during upturns may not be able to replenish its capital during downturns and currency depreciations due to other costs. If a downturn is severe and is combined with a banking crisis, the central bank may face substantial losses related to its lender-of-last-resort function, capital injections in commercial banks, and liquidity facilities.

We present an SAA model for a stabilization SWF with an objective to finance fiscal deficits and specific future social programs.¹⁰ The funding rule of the SWF, i.e., the local currency amount going into it, can be formulated as follows:

$$F_t = R_t - E_t - \max\left(k_1, \min\left(\frac{R_t - E_t}{GDP_t}, (k_2 + k_3)\right)\right) \times Y_t \quad (1)$$

where

F_t - a budget stabilization SWF's funding in period t ;

R_t - actual total budget revenue in period t ;

E_t - actual expenditure in period t ;

Y_t - GDP in period t ;

k_1 - lower mandatory limit of funding a designated social program or fund from the effective fiscal balance, in percent of GDP.

k_2 - upper limit of funding a designated social program or fund from the effective fiscal balance, in percent of GDP.

k_3 - discretionary contribution to capitalize the central bank, in percent of GDP.

A surplus savings fiscal rule could be expressed as:

$$E_t = R_t^{LT} - k_s Y_t \quad (2)$$

where

k_s - the annual structural balance (savings) target rate;

R_t^{LT} - long-term revenue in period t .

Then, we can express the stabilization SWF's funding, F_t , in relation to the revenue gap, $R_t - R_t^{LT}$. Contributions in the stabilization SWF will be positive if the revenue gap exceeds a certain threshold, as shown below:

¹⁰ The use of the term "social program" is general and refers also to tranching out other SWFs, such as pension-reserve SWF.

$$F_t > 0$$

$$R_t - E_t > \max\left(k_1, \min\left(\frac{R_t - E_t}{Y_t} + k_s, (k_2 + k_3)\right)\right) \times Y_t$$

$$\text{Revenue gap} = R_t - R_t^{LT} > \left[\max\left(k_1, \min\left(\frac{R_t - E_t}{Y_t} + k_s, (k_2 + k_3)\right)\right) - k_s \right] \times Y_t \quad (3)$$

$$R_t - R_t^{LT} > \max\left((k_1 - k_s), \min\left(\frac{R_t - R_t^{LT}}{Y_{t-1}}, (k_2 + k_3 - k_s)\right)\right) \times Y_t$$

However, if the revenue gap, $R_t - R_t^{LT}$, is below $-(k_s - k_1)Y_t$, the stabilization SWF's monies will have to be used to finance other possible contributions in social programs or a fund.¹¹

$$F_t = R_t - R_t^{LT} + k_s Y_t - \max\left(k_1, \min\left(\frac{R_t - R_t^{LT}}{Y_t} + k_s, (k_2 + k_3)\right)\right) \times Y_t$$

$$= R_t - R_t^{LT} - \max\left((k_1 - k_s), \min\left(\frac{R_t - R_t^{LT}}{Y_{t-1}}, (k_2 + k_3 - k_s)\right)\right) \times Y_t \quad (4)$$

$$= \tilde{R}_t - \max\left((k_1 - k_s), \min\left(\frac{\tilde{R}_t}{Y_t}, (k_2 + k_3 - k_s)\right)\right) \times Y_t$$

where

$$\tilde{R}_t - \text{revenue gap, } R_t - R_t^{LT}, \text{ in period } t.$$

The “net return” on the stabilization SWF, Z_t , with regard to the optimal tradeoff between expected risk and returns can be found as follows. The total stabilization SWF value, S_t , is given by

$$S_t = A_t + C_t - L_t \quad (5)$$

where

S_t - the stabilization SWF value at time t ;

A_t - initially endowed stock of SWF assets, valued at time t ;

¹¹ $R_t - R_t^{LT} = R_t - (E_t + k_s Y_t) = R_t - E_t - k_s Y_t$ or $R_t - E_t = (R_t - R_t^{LT}) + k_s Y_t$, i.e., Budget Balance = Revenue Gap + Structural Balance Target.

C_t -revenue-related asset component at time t , which is the present value of government revenue surpluses;

L_t - liability component at time t , which is the present value of the conditional outlays from the stabilization SWF.

Since the SAA is set with a view to the expected liability stream, we define the stabilization SWF's total return, $\frac{S_t}{S_{t-1}}$, as:

$$\begin{aligned} \frac{S_t}{S_{t-1}} &= \frac{A_t}{S_{t-1}} + \frac{C_t}{S_{t-1}} - \frac{L_t}{S_{t-1}} = \\ &= \frac{A_{t-1}}{S_{t-1}} \frac{A_t}{A_{t-1}} + \frac{\tilde{R}_t + k_s Y_t}{S_{t-1}} - \frac{\left(p_1 k_1 + p_2 \left(\frac{\tilde{R}_t}{Y_t} + k_s \right) + p_3 (k_2 + k_3) \right) \times Y_t}{S_{t-1}} \end{aligned} \quad (6)$$

where

p_1 - probability that the revenue gap is smaller than $(k_1 - k_s) \times Y$.

p_3 - probability that the revenue gap is greater than $(k_2 + k_3 - k_s) \times Y$.

$p_2 = 1 - p_1 - p_3$ probability that the revenue gap will be somewhere in between.¹²

Note that the liability component, L_t , is conditional on the size of the revenue gap, \tilde{R}_t .

Hence, in a low-revenue state (revenue gap is below $(k_1 - k_s)$) transfers are made only to a designated social program. If the revenue gap is between $(k_1 - k_s)$ and $(k_2 + k_3 - k_s)$, mandatory payments are to the designated social program, and, possibly, for the discretionary capitalization of the central bank. Finally, in the high-revenue state, the maximum payments are made to both the designated social program/fund and the central bank.

The long-term return on a stabilization SWF's assets is transferred to the budget, while the excess return is capitalized. Therefore, the net return on the stabilization SWF, Z_t , can be written as:

¹² A scenario with negative net outflows from the designated fund for a social program or for supporting the national pension scheme is possible under extreme circumstances, which we do not consider in this model.

$$Z_t = \frac{A_{t-1}}{S_{t-1}} \tilde{R}_{s,t} + \frac{(1-p_2)\tilde{R}_t}{S_{t-1}} - \frac{(p_1(k_1 - k_s) + p_3(k_2 + k_3 - k_s))Y_t^*}{S_{t-1}} \times \frac{Y_t}{Y_t^*} \quad (7)$$

where

$$Z_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

Y_t^* - long-term (potential) GDP.

$\tilde{R}_{s,t}$ - the deviation of the SWF's net interest and capital gain return (financial asset return) from its long-term value.

We define the objective function for a stabilization SWF as the variance of Z_t , which is to be minimized subject to constraints in a Markowitz framework. Hence, the minimization of the objective function of the SWF is equivalent to the minimization of the variance of Z_t . In this analysis, we exclude those components that are unaffected by the choice of assets, as well as scaling factors that will not affect the optimal asset allocation, and assume that the expected value of the excess return of the stabilization SWF is zero, i.e., $E(\tilde{R}_{s,t}) = 0$.

The variance of Z is equal to:

$$\begin{aligned} \text{var}(Z_t) = & \left(\frac{A_{t-1}}{S_{t-1}} \right) \text{var} \left(\frac{\tilde{R}_{s,t}}{S_{t-1}} \right) + 2(1-p_2) \frac{Y_t^*}{S_{t-1}} \text{cov} \left(\frac{\tilde{R}_{s,t}}{S_{t-1}}, \frac{\tilde{R}_t}{Y_t^*} \right) \\ & - 2(p_1(k_1 - k_s) + p_3(k_2 + k_3 - k_s)) \left(\frac{Y_t^*}{S_{t-1}} \right) \text{cov} \left(\frac{\tilde{R}_{s,t}}{S_{t-1}}, g_t \right) \end{aligned} \quad (8)$$

where

g_t - GDP gap, $\frac{GDP_t - Y_t^*}{Y_t^*}$, between the actual and the potential GDP in period t.

$\frac{\tilde{R}_{s,t}}{S_{t-1}}$ - the deviation of the stabilization SWF's net interest and capital gain return (financial asset return) from its long-term value in percent of the stabilization SWF's assets.

Assuming that the correlation of the financial asset return with the revenue gap is equal to the correlation of the financial asset return and the GDP gap:

$$\text{var}(Z_t) = \left(\frac{A_{t-1}}{S_{t-1}} \right) \text{var} \left(\frac{\tilde{R}_{s,t}}{S_{t-1}} \right) + \delta \left[\left(\frac{Y_t^*}{S_{t-1}} \right) \text{cov} \left(\frac{\tilde{R}_{s,t}}{S_{t-1}}, \frac{\tilde{R}_t}{Y_t^*} \right) \right] \quad (9)$$

where

$$\delta = 2 \left((1 - p_2) - (p_1(k_1 - k_s) + p_3(k_2 + k_3 - k_s)) \left(\text{var}(g_t) / \text{var} \left(\frac{\tilde{R}_{T,t}}{Y_t^*} \right) \right)^{0.5} \right)$$

Hence, we have a setting where assets of the stabilization SWF should be used to hedge the government revenue gap. We can interpret equation (9) as a risk penalty to those assets which are highly correlated with the government non-interest revenue gap. However, in high-revenue states, the greater likelihood to capitalize the central bank and make larger payments for social programs implies that such payments could be aided by greater revenue from financial assets, thereby mitigating the risk penalty. The greater the excess revenue of the government, the greater its capacity to recapitalize the central bank and contribute to social programs. This is because a revenue shortfall—relative to the structural revenue—caused by a decline in the owner country's commodity prices and other factors affecting economic activity must be offset by a revenue surplus from financial assets.

Finally, we decompose the SWF's financial asset return—and its deviation from the long-term value respectively—into a return stemming from the performance of the assets in portfolio in the currencies that they are denominated, and a return stemming from exchange rate movements.

$$\text{var}(Z_t) = \left(\frac{A_{t-1}}{S_{t-1}} \right) \text{var}(\tilde{r}_{s,t}^f + \tilde{e}) + \delta \left(\frac{Y_t^*}{S_{t-1}} \right) \left[\text{cov} \left(\tilde{r}_{s,t}^f, \frac{\tilde{R}_{T,t}}{Y_t^*} \right) + \text{cov} \left(\tilde{e}, \frac{\tilde{R}_{T,t}}{Y_t^*} \right) \right] \quad (10)$$

where

$\tilde{r}_{s,t}^f$ - is the deviation of the stabilization SWF's financial asset return from its long-term value in the currency of denomination of the assets in the SWF portfolio;

\tilde{e} - deviation of the exchange rate vis-à-vis the currencies of denomination of the assets in the SWF portfolio from its long-term value.

Therefore, the stabilization SWF's portfolio should contain assets that are negatively correlated with the revenue gap and, under the covariance assumptions mentioned above, the owner country's GDP growth. This implies that the stabilization SWF's assets must be negatively correlated with commodity prices and other factors that boost economic activity in the SWF owner country.¹³ In particular, the variance of the SWF portfolio returns—

¹³ We assume that the path of fiscal expenditures based on the permanent income hypothesis ensures minimum variance of expenditures. Based on this assumption, the SAA seeks to ensure that the SWF's excess return (i.e., over and above its long-term path) stabilizes the financing of fiscal expenditures. Therefore, determining the SAA is a two-step process. First, as the right hand side of equation 9 on page 44 shows, the SWF's return must be countercyclical to fiscal revenues (i.e., economic growth). The mapping of equation 9 onto the SAA involves selecting assets with countercyclical returns. Second, once such assets have been selected, the actual allocation should minimize the variance of the portfolio (as per the left hand side of equation 9).

expressed in SWF owner country's currency—stemming from the variation of the exchange rate between the SWF owner country and the country of the SWF asset investment plus the variation of the SWF returns in the currency of the country of asset investment, should be negatively correlated with the SWF owner country's revenue gap.

Equation (10) brings an additional dimension to the SAA choice, as both the exchange rate risk and the covariance of the exchange rate with the revenue gap have an impact on the objective function. Therefore, while the SWF portfolio should contain assets denominated in currencies that are negatively correlated with the SWF owner country revenue gap, it should also contain assets for which the volatility of the exchange rate is as low as possible.

V. PRACTICAL ASPECTS OF AN SAA METHODOLOGY

Based on the methodology proposed above, this section presents some considerations regarding the macrofinancial linkages in relation to an SWF's SAA framework. It should be recognized that SAA is a dynamic process and that countries do change their asset allocation if macroeconomic conditions or financial market expectations change significantly. Best practices suggest that the SAA be subject to an annual review process, while the framework and methodologies should be reviewed regularly but on a longer cycle (3–5 year cycle). Nonetheless, the framework presented in this paper uses one-period mean-variance optimization under a standard assumption of normal distribution. This is a good starting point as an SAA methodology, but as it precludes dynamic optimization, it may not be the best framework in the case of SWFs with long investment horizon. In such cases, dynamic optimization models are more appropriate.¹⁴

SWF objectives

The main purpose of the SAA is to allocate investments so that the objective of the fund is achieved as efficiently as possible, while trading off relevant gains and risks. Hence, any analysis of an optimal SAA starts with making the purpose of the fund, or its objectives, as explicit as possible. The objective of the fund, and its relevant macro-financial linkages, will identify which components of the sovereign's wealth, or which cash flows, should be taken into account when analyzing whether some asset risks are more relevant than others.

If a particular commodity-based country has multiple pools of assets with different objectives, then each can be treated as a separate SAA problem as different objectives likely translate to different return requirements and risk tolerances. Nonetheless, from the perspective of a holistic sovereign asset-liability management, it would be best to exploit complementarities in setting the objectives of the funds. This implies that the funds may not need to be managed separately. They could still be combined operationally as one pool of money, with the SAA assessment done separately.

¹⁴ See Claessens and Kreuser (2009).

Portfolio benchmarks

Setting portfolio benchmarks also tie the asset allocation to the overall goal of the fund. If there is an implicit return objective that needs to be met—e.g., because the SWF has a savings purpose—it would be appropriate to set the benchmark in terms of expected returns (in percent). However, if the SWF’s objective is to minimize expenditure volatility, the primary objective should not be a return requirement, but rather a volatility target, and a quantifiable measure of the desired volatility should be calculated and monitored on a regular basis. For more than one SWF, an investment policy document could specify explicitly an overall portfolio benchmark.¹⁵

Numeraire currency

The relevance of returns and risks in financial assets depends on the total state-contingent wealth of the investor. A first step in identifying the relevant risk is to choose a numeraire currency for measuring the performance of the SWF. A second step is to identify the eligible asset classes and analyse the correlation structure between returns of eligible assets and any risk factors driving the total wealth of the investor, when measured in the appropriate numeraire.

The choice of numeraire currency should reflect the unit of calculation in which the performance of the SWF is most appropriately measured—depending on the objective of the SWF. For a stabilization SWF, whose objective is to smooth fiscal expenditures that are measured in local currency (the unit of measure for the government budget), the numeraire should be the local currency. However, if the public is more accustomed to reporting financial asset performance in real terms or another currency, the performance of an SWF could also be reported in that currency.¹⁶

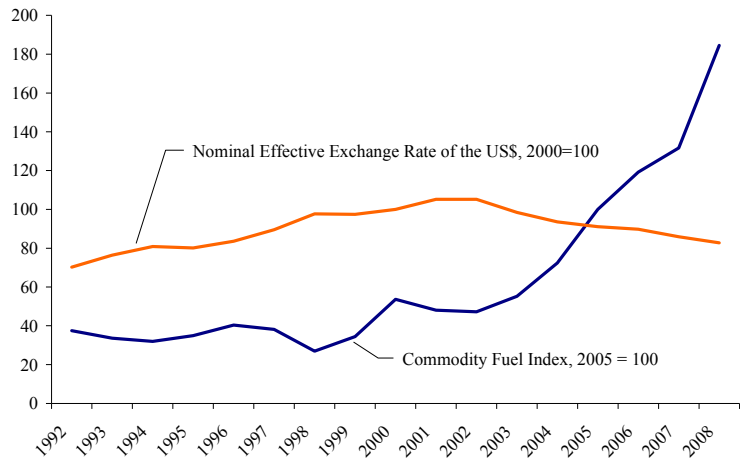
An asset-only framework is sufficient for the SAA of a stabilization SWF. In the case of a large size of unextracted commodity relative to financial assets (Sharpe and Tint, 1990), such uneven balance sheets are best viewed in the asset only space. From a macro-financial perspective, the stabilization SWF’s resources need to be invested in counter-cyclical foreign

¹⁵ Risk management guidelines often establish deviation limits around that benchmark, which could be expressed in maximum percentage deviation for any given asset class and a tracking error or VaR limit for overall deviation.

¹⁶ In the case of a stabilization SWF, measuring performance in real terms would give a sense of the real value performance of the SWF. However, financial returns are typically measured in nominal terms, because the proceeds of financial assets are used for expenditures in nominal terms. Therefore, performance expressed in real terms may be useful only for analytical purposes and not for comparison with nominal budget expenditures. Furthermore, if the country has low and stable inflation, performance measurement in nominal local currency terms is not significantly biased by high levels of domestic inflation. In the case of a savings SWF, an objective of maintaining the purchasing power of the SWF in domestic currency (i.e., a real return target) would suggest that performance be measured in real terms. An objective of maintaining the external purchasing power of the SWF would suggest choosing a basket of currencies as a numeraire and measuring the performance in terms of this currency basket (McCauley, 2008).

assets that would tend to offset the adverse impact of lower commodity prices on the fiscal accounts.¹⁷ The ideal assets would be those that are likely to display very low or negative correlation to the owner country's business cycle. Hence, the SAA will be biased toward assets of countries that move counter-cyclically to the owner country, such as U.S. dollar assets in the case of SWFs of oil exporting countries (Figure 2). Nonetheless, these assets should also offer attractive valuation, income, and long-term growth fundamentals in their own right. Finally, a stabilization fund should be invested in shorter duration assets, as the resources of this type of SWF may need to be used in the short to medium term.

Figure 2. Correlation between Fuel Prices and the Value of the U.S. Dollar



Source: IMF: WEO and IFS.

VI. ILLUSTRATIVE APPLICATION¹⁸

We present a methodology for the assessment of the management of SWFs and their macro-financial linkages, the salient features of the SAA optimization and an illustrative portfolio optimization model output.

Markowitz model

The base for many SAA models is the Markowitz model. In its basic form, the model examines the interactions between asset classes and chooses combinations that minimize the volatility (standard deviation) proxying the risk for a given level of expected return.

The standard Markowitz model uses three inputs to determine an optimal portfolio: expected return, standard deviation, and correlations between various asset classes. The expected return is often treated as a single point estimate, but in reality, is the expected mean of a distribution, which in the Markowitz model is assumed to be normal. Hence, in the model used for this illustration we have produced uncertainty bands to recognize the fact that the

¹⁷ In other words, when commodity prices are high (and the stabilization SWF's returns lower by construction) and if k_3 percent of GDP is allocated for bank capitalization, the lower returns from the stabilization SAA will be mitigated.

¹⁸ This section uses an optimizer developed by Ryan Buble and Rod Babineau.

model parameters are estimates and have varying degrees of uncertainty. The model thus provides both an optimal return and a worst-case return for comparison.

Model inputs

The model is extremely sensitive to the inputs, suggesting that a large amount of time, research and discussion should be committed to choosing them.

Expected Return. The expected return, which is effectively the mean of the distribution of potential returns for the asset class, is a long-term assumption. There are a number of ways to choose this input, each having merits and flaws. Some possibilities are:

- *Long-term historical data* works relatively well for well-established and mature asset classes, such as U.S. equities. However, regardless of how long a data set is used, it is ultimately backward-looking and potentially of limited use for SAA purposes if current data deviate significantly from historical averages. Moreover, for asset classes that are less established or for countries where there has been a major economic event like a credit default or an exchange rate regime change, this data could be meaningless as a forward-looking tool.
- *Risk premium model.* Based on a basic long-run expectation for inflation in a given country, one can use historical risk premiums to build expected returns for most asset classes. This method is easier to implement for publicly traded securities like stocks and bonds, but can also be applied to private asset classes.
- *GDP growth model.* One of the key fundamental drivers of stock prices is corporate earnings growth. Over a long period of time, there is a strong correlation between corporate earnings growth and real GDP growth, as the same factors that drive business growth also drive GDP growth.¹⁹ To obtain an expected return using this method, three items are needed: a GDP growth forecast, an inflation forecast and a historical dividend payout ratio. Historically in the U.S., the dividend payout ratio has been close to 2 percent. Therefore if U.S. GDP growth is forecasted at 2 percent and inflation is forecasted at 2.5 percent, the sum of these three factors (6.5 percent) can be used as an equity long horizon expected return assumption in the model. This can be used as a base figure to derive other assumptions.

Expected Volatility. Setting the expected volatility (the expected standard deviation of the assumed return distribution) presents the same challenges as the expected return. If historical returns are used, then historical volatility should be chosen for consistency reasons. If a forecasting approach for returns, such as the GDP growth model, is used, then a forward looking volatility forecast will be more appropriate.

¹⁹ See Maginn et al. (2007).

Expected Correlation. As with the two previous inputs, the model is extremely sensitive to correlations between asset classes. In this case however, a historical correlations over an acceptable time period may be more appropriate as opposed to a forward looking model, since predicting correlations may not be a viable task. An examination of recent correlation trends would be useful to see if historical correlations hold, or an adjustment needs to be made.

Asset/liability problem

In the context of a commodity-based SWF, incorporating the underlying wealth-driving asset is problematic. Using oil reserves as an example, the value of oil in the ground in most cases will be vastly larger than the collection of financial assets being modelled. To include the value of oil reserves as an asset in the model would cause the model to be dominated by oil and will either force the financial assets into a short oil position or render the analysis meaningless all together.

A solution to this problem is to pre-select asset classes based on their relationship to the wealth-driving asset. As discussed in Sharpe and Tint (1990), even when financial assets are small relative to total assets (financial assets vs. assets in the ground), the covariance structure across asset and liabilities provides useful insights in the asset selection through an SAA. As shown analytically above, the SAA for a stabilization SWF should include assets exhibiting counter-cyclicality to the owner economy's business cycle. Continuing with the oil example, if the goal is to minimize volatility in the portfolio due to oil prices, then analysis can be done to select asset classes or currencies which are uncorrelated with or countercyclical to the price of oil.

If the chosen asset classes satisfy the intended budget stabilization objectives and inherently meet the SWF's goal of diversifying away from the wealth-driving asset, then the resultant portfolio will also meet that latter goal.

Illustrative model output

We discuss two variations of the model above—for the case of a country that has chosen the oil price as a numeraire, and the case of a country that has chosen the domestic currency as a numeraire to measure the SWF performance.

Price-of-oil numeraire

We assume that the SWF country's economic activity and fiscal revenues are highly correlated with the oil price. In this case, instead of selecting domestic currency, the country may select the oil price as a numeraire. This would be particularly relevant for countries that do not have a diversified economic base and the oil price would guide the selection of an appropriate universe of investment instruments.

We follow a top down approach by first searching for countries the SWF would invest in, whose basic economic indicators (exchange rate and GDP growth) are negatively correlated with the price of oil. The search is restricted to those countries that are likely to have deep and liquid financial markets, i.e., OECD member and applicant countries.

We conduct several correlation tests at this stage, using quarterly data from the International Financial Statistics for the first quarter of 1989 to the fourth quarter of 2008. First, the correlation of the effective exchange rates—both nominal and real—for these countries with the price of oil reveals that the U.S. dollar has the greatest negative covariance with the price of oil (Table 1). Other countries' currencies that showed negative covariance were Japan and Switzerland.

Table 1. Covariance Tests of Exchange Rates and Oil Price

NEER	Statistic	3 Spot Price	Dubai Spot Price	Texas Spot Price
Germany	Cov	0.000	0.000	0.000
	t-stat	1.091	1.049	1.202
Japan	Cov	-0.001	-0.001	-0.001
	t-stat	-1.043	-1.118	-1.073
Switzerland	Cov	-0.000	-0.000	-0.000
	t-stat	-0.468	-0.289	-0.554
UK	Cov	0.001 **	0.001 ***	0.001 **
	t-stat	2.645	2.717	2.517
US	Cov	-0.002 ***	-0.002 ***	-0.002 ***
	t-stat	-5.986	-5.714	-6.072
<hr/>				
REER				
Austria	Cov	0.000	0.000	0.000
	t-stat	-0.877	-1.042	-0.630
Denmark	Cov	0.000	0.000	0.000
	t-stat	-0.367	-0.359	-0.255
Germany	Cov	0.001 **	0.001 ***	0.001 **
	t-stat	2.598	2.815	2.428
Japan	Cov	-0.001	-0.001	-0.001
	t-stat	-1.304	-1.387	-1.325
Switzerland	Cov	0.000	0.000	0.000
	t-stat	-1.121	-1.023	-1.133
UK	Cov	0.001 **	0.001 **	0.001 **
	t-stat	2.150	2.213	2.042
US	Cov	-0.002 ***	-0.002 ***	-0.002 ***
	t-stat	-5.201	-5.031	-5.233

Source: IFS, and authors' calculations.

Note: *** indicates significance at 1 percent level, ** at 5 percent, and * at 10 percent.

Second, the covariance of GDP growth with the oil price shows that Germany could be included in the investment universe (Table 2). Finally, the covariance between the output gap and the price of oil selects Germany and Japan, although the covariance is not significantly different from zero.

Table 2. Covariance Tests of GDP and Oil Price

GDP Growth	Statistic	3 Spot Price		Dubai Spot Price		Texas Spot Price	
Germany	Cov	-0.001	**	-0.002	***	-0.001	**
	t-stat	-2.377		-2.691		-2.094	
Japan	Cov	0.000	***	0.000	***	0.000	***
	t-stat	2.742		2.791		2.719	
UK	Cov	0.000		0.000		0.000	
	t-stat	1.418		1.253		1.650	
US	Cov	0.000	**	0.000	**	0.000	**
	t-stat	2.137		2.330		2.047	
<hr/>							
GDP Gap							
Germany	Cov	-0.007		-0.009		-0.005	
	t-stat	-0.286		-0.332		-0.210	
Japan	Cov	-0.008		-0.008		-0.004	
	t-stat	-0.207		-0.189		-0.116	
UK	Cov	0.061	**	0.060	**	0.060	**
	t-stat	2.548		2.349		2.633	
US	Cov	0.076	***	0.080	***	0.072	***
	t-stat	3.727		3.689		3.647	

Source: IFS, and authors' calculations.

Note: *** indicates significance at 1 percent level, ** at 5 percent, and * at 10 percent.

We then perform a second level of correlation tests to verify that asset returns are also negatively correlated with the price of oil (Table 3). In these tests, we include U.S. government bond returns, corporate bond returns and equity returns, as well as government bond returns and equity returns of three countries with deep and liquid financial markets—U.K., Japan, and Germany. The time period covers first quarter of 1989 to first quarter of 2009.

We observe that only U.S. asset returns show negative correlation with the price of oil, and among the U.S. assets, only the return on government bonds is significantly negatively correlated.

Table 3. Covariance Tests of Asset Returns and Oil Price

Total Return Index (TRI)		Statistic	3 Spot Price	Dubai Spot Price	Texas Spot Price
S&P Germany	A	Cov	0.011	-0.001	0.017
		t-stat	0.276	-0.021	0.451
BOFA ML German Gov't Bond	B	Cov	0.028 *	0.028 *	0.029 **
		t-stat	1.864	1.806	2.017
S&P Japan	C	Cov	0.007	-0.005	0.013
		t-stat	0.195	-0.124	0.375
JPM Jap Gov't Bond	D	Cov	0.002	0.001	0.002
		t-stat	0.151	0.071	0.143
S&P UK	E	Cov	0.013	0.008	0.016
		t-stat	0.552	0.307	0.703
JPM UK Gov't Bond	F	Cov	0.021	0.022	0.022 *
		t-stat	1.564	1.574	1.693
BOFA ML US Corp	G	Cov	-0.001	-0.001	-0.001
		t-stat	-0.958	-1.065	-0.937
S&P 500	H	Cov	0.008	0.008	0.008
		t-stat	1.433	1.242	1.531
JPM US Gov't Bond	I	Cov	-0.003 **	-0.004 **	-0.003 **
		t-stat	-2.183	-2.297	-2.060

Source: IFS, Bloomberg, Datastream, and authors' calculations.

Note: *** indicates significance at 1 percent level, ** at 5 percent, and * at 10 percent.

Based on these tests, only U.S. assets show robustness to different criteria.²⁰ In what follows below, the illustrative output is generated using only U.S. assets: equities, government universe bonds, inflation-linked bonds, and high-grade corporate universe bonds.²¹ The assumptions for these asset classes are summarized in Table 4.²² In this process, we choose broadly defined asset classes and use the most common market proxies. Thus, for U.S. Equities, we use the S&P 500 index, which is a broad large-cap market index that is generally accepted as the standard proxy for the U.S. stock market. However, in real-world

²⁰ In practice, instead of selecting the currency with the lowest correlation with oil, it might be preferred to diversify over several currencies, even if the additional currencies have higher correlations with oil. See the section on domestic currency numeraire.

²¹ We do not model deposit-like cash instruments explicitly. For a stabilization SWF, short-term fixed government bonds are considered adequate substitutes for deposits (they are cash equivalents). If cash was included, the optimization tool would avoid any allocation to it because it has the lowest return profile. We also do not see a benefit of modeling a broader choice of asset classes for a stabilization SWF. If an SWF has specific exposures that need to be hedged, a broader asset class selection might be warranted.

²² The historical return and standard deviation figures in Table 1 refer to annualized quarterly U.S. dollar performance during the Q1 1989-Q1 2009.

applications, these asset classes can be narrowed further. For example, U.S. Equities could be broken into large-, mid- and small-cap, and the indexes could be separated into Value and Growth.

Table 4. Asset Class Assumptions

	Asset Class	Historical Return (%)	Standard Deviation
I	U.S. Equities (S&P 500 TRI)	8%	14.4%
II	U.S. Government Bonds (JPM TRI)	4%	4.5%
III	U.S. Inflation-linked Bonds (Barclays TIPS)	2.75%	5.7%
IV	U.S. High Grade Corporate Bonds (BOFA ML TRI)	5%	5.3%

Correlations	I	II	III	IV
I	1			
II	-0.346	1		
III	-0.228	0.861	1	
IV	0.035	0.092	0.042	1

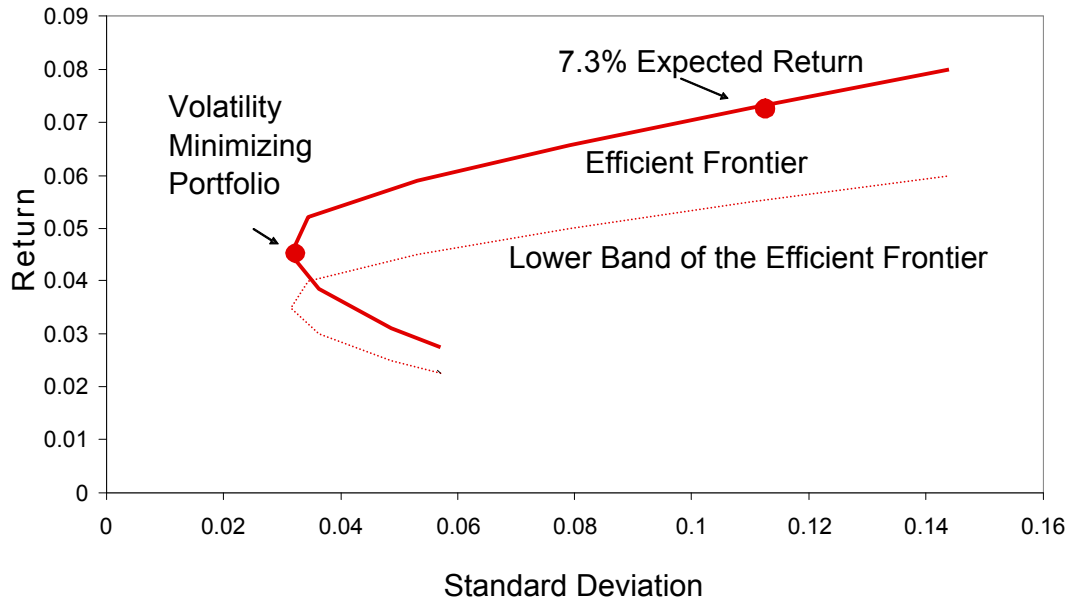
Sources: Bloomberg, and authors' calculations.

The results are illustrated in Figure 3. The thick line depicts the efficient frontier while the dashed line is a minimum return (lower-band) frontier using various confidence levels for the asset classes. In this case, the confidence level of equities is ± 2 percent, while for other assets is ± 1 percent.²³ Two portfolios are highlighted:

- **Minimum volatility portfolio.** The portfolio has a nominal expected return of 4.5 percent and an expected volatility of 3.2 percent. This portfolio consists of 100 percent U.S. government bonds. This outcome is expected as intuitively, government bonds should have the lowest expected volatility of the asset classes used. Furthermore, U.S. government bond yields are negatively correlated with the price of oil, thereby representing an appropriate choice for a stabilization fund.
- **A portfolio based on a return objective.** For illustration, we have selected a portfolio with an expected return of 7.3 percent and an expected volatility of 11.0 percent. It consists of 76 percent U.S. large-cap equities, 2 percent U.S. inflation-linked bonds, and 22 percent U.S. high-grade corporate bonds. Such a portfolio would be rather unusual for a pure stabilization SWF; nonetheless, it demonstrates the extent to which the portfolio could be reallocated if the fund has a mixture of stabilization and savings objectives.

²³ It should be noted that these levels are subjective, and an SWF should set its own bands.

Figure 3. Efficient Frontier with Price-of-Oil Numeraire



In this case, the model places a heavy weight on equities, so as to generate a high overall portfolio return. The choice of two types of bonds indicates that there is a diversification benefit to holding both nominal and inflation-linked bonds, even if the recommended allocation to inflation-linked bonds is rather small at 2 percent. Also, if a stabilization SWF has a return objective, some assets could be allowed to be cyclical—such as equities and inflation-linked bonds—in order to attain the return objective, provided that the overall exchange-rate adjusted SAA portfolio return is countercyclical to the domestic economy's growth.²⁴

In Figure 3, to generate the lower-band frontier, the model looks for the average minimum expected total portfolio return within the specified uncertainty levels. The gap between the two lines illustrates the possible long-range outcomes if the expected return falls between the mean and the lower bound. Possible returns could exceed the mean and thus increase portfolio returns; however, this upper bound is not depicted in Figure 3, as investors are generally not worried about the upside.

In the left hand section of the graph, where the portfolios are mostly comprised of U.S. government bonds—whose returns have a low volatility—the lines are quite close to each other. As riskier asset classes—such as equities and corporate bonds—are added to the portfolio, which introduces more uncertainty, the gap between the two lines widens. The illustration is important as it is often overlooked that the expected returns used in the model

²⁴ In this illustrative example, the portfolio is countercyclical only when its return is adjusted for exchange rate changes relative to the U.S. dollar.

represent the mean of a distribution of returns and that capital markets are highly unpredictable.

Domestic currency numeraire

We now turn to the case of a domestic currency numeraire, using quarterly OECD data for the GDP gap of two oil producers—Norway and Mexico—and the total return on bonds and equity for Germany, Japan, U.K., and the U.S (as numbered in Table 1). Since the numeraire is the domestic currency and in order to be consistent with the GDP gap data, all returns are converted in domestic currency using the relevant real exchange rates.

Notice that this exercise uses very simplified assumptions, thereby claiming no direct application to the specific countries. In particular, relating this analysis to the model in Section V, we assume that the size of the SWF assets is about equal to potential GDP; the country has a policy of maintaining a structural fiscal balance; there are no social programs that drain potential SWF flows; and the size of the revenue gap is equivalent to the size of the GDP gap. We denote the GDP gap data N_{gap} (Norway) and M_{gap} (Mexico) to abstract the results from the countries.

Upon inspecting the covariance matrix of the first country's GDP gap (N_{gap}) with the selected assets, we observe that all fixed income assets, except for the U.S. government bonds are negatively correlated with the GDP gap (Table 5). Also, the German government bonds have the lowest variance, followed by the U.K. and the U.S. government bonds. It could therefore, be expected that these assets will be included in the portfolio.

In the case of the second country, we find all equities are negatively correlated with the GDP gap. Nonetheless, due to the low volatility of the return of U.K. and U.S. government bonds, the portfolio may also contain these assets.

Table 5. GDP Gap and Asset Return Covariance

Ngap	Expected Return	Standard Deviation	A	B	C	D	E	F	G	H	I	NGAP
A	7.72%	0.269	0.072									
B	6.56%	0.082	-0.001	0.007								
C	-3.92%	0.265	0.034	0.002	0.070							
D	4.57%	0.134	0.001	0.005	0.015	0.018						
E	10.03%	0.171	0.036	-0.001	0.025	0.005	0.029					
F	9.07%	0.094	0.001	0.003	0.003	0.006	0.007	0.009				
G	8.55%	0.109	0.002	0.004	0.008	0.010	0.005	0.007	0.012			
H	10.04%	0.183	0.039	0.000	0.030	0.006	0.027	0.005	0.008	0.033		
I	6.81%	0.107	0.000	0.004	0.006	0.010	0.003	0.007	0.012	0.006	0.011	
NGAP	5.02%	0.059	0.000	-0.001	0.000	-0.001	-0.001	-0.001	0.000	-0.001	0.000	0.003

Mgap	Expected Return	Standard Deviation	A	B	C	D	E	F	G	H	I	MGAP
A	6.69%	0.261	0.068									
B	5.20%	0.187	0.010	0.035								
C	-4.87%	0.255	0.030	0.013	0.065							
D	3.23%	0.183	0.006	0.028	0.020	0.034						
E	7.24%	0.189	0.037	0.016	0.025	0.015	0.036					
F	7.69%	0.164	0.008	0.026	0.010	0.023	0.019	0.027				
G	9.26%	0.194	0.007	0.031	0.014	0.028	0.017	0.026	0.038			
H	7.90%	0.183	0.036	0.014	0.028	0.013	0.030	0.014	0.017	0.034		
I	5.45%	0.145	0.002	0.023	0.009	0.021	0.011	0.019	0.028	0.011	0.021	
MGAP	-0.37%	0.054	-0.002	0.000	-0.002	0.001	-0.002	0.000	0.000	-0.002	0.000	0.003

Source: IFS, Bloomberg, Datastream, and authors' calculations.

The results confirm these observations (Table 6). In the case of the first country, the SAA is concentrated in German government bonds (52 percent), U.K. government bonds (21 percent) and U.S. government bonds (20 percent), and allocates a smaller share to German equity (8 percent). Would the cyclicity of the economy (revenue gap) not have factored in the optimization, the minimum variance portfolio would have given greater weight to German government bonds (60 percent) and a very small weight of U.S. government bonds. A return target (e.g., 8 percent) would shift the allocation toward U.S. corporate bonds (14 percent) and U.K. government bonds (42 percent).

In the case of the second country, a large share is allocated in U.S. government bonds (77 percent) and German equity (19 percent), with a small allocation to U.K. government bonds (4 percent). Would the GDP gap not have been considered, the allocation of U.K. equity would have doubled, and some allocation would have been given to U.S. equity. Given a return target (e.g., 7 percent), the allocation would shift significantly toward U.K. government bonds (37 percent) and U.S. equity (22 percent).

Finally, smoothing government revenues comes at a cost (Table 4 and Figure 4). In both examples, the variance of the portfolio is greater and the return is smaller when the objective

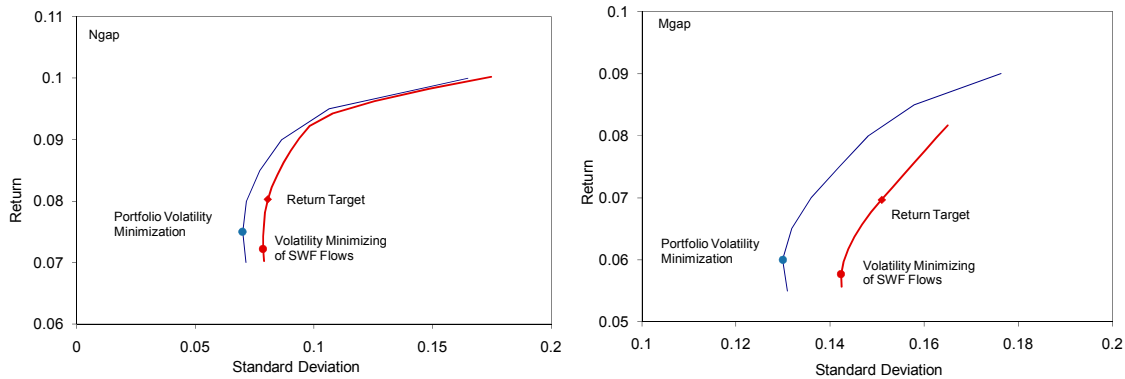
of the SWF is to smooth revenues, rather than simply minimize the variance of the SWF asset portfolio.

Table 6. Illustrative SAA with Domestic Currency Numeraire

Ngap	Expected Return	Standard Deviation	A	B	C	D	E	F	G	H	I
Volatility Minimizing of SWF Flows	7.2%	7.9%	8.0%	51.7%	0.0%	0.0%	0.0%	20.8%	0.0%	0.0%	19.5%
Return Target	8.0%	8.1%	6.4%	35.8%	0.0%	0.0%	0.0%	42.1%	14.4%	1.3%	0.0%
Portfolio Variance Minimization	7.5%	7.0%	4.6%	59.8%	0.0%	0.0%	4.3%	27.6%	0.0%	1.1%	2.6%

Mgap	Expected Return	Standard Deviation	A	B	C	D	E	F	G	H	I
Volatility Minimizing of SWF Flows	5.8%	14.2%	19.3%	0.0%	0.0%	0.0%	0.0%	3.5%	0.0%	0.0%	77.2%
Return Target	7.0%	15.1%	7.2%	0.0%	0.0%	0.0%	0.0%	36.6%	1.8%	22.0%	32.4%
Portfolio variance minimization	6.0%	13.0%	17.9%	0.0%	0.0%	0.0%	0.0%	6.8%	0.0%	7.2%	68.1%

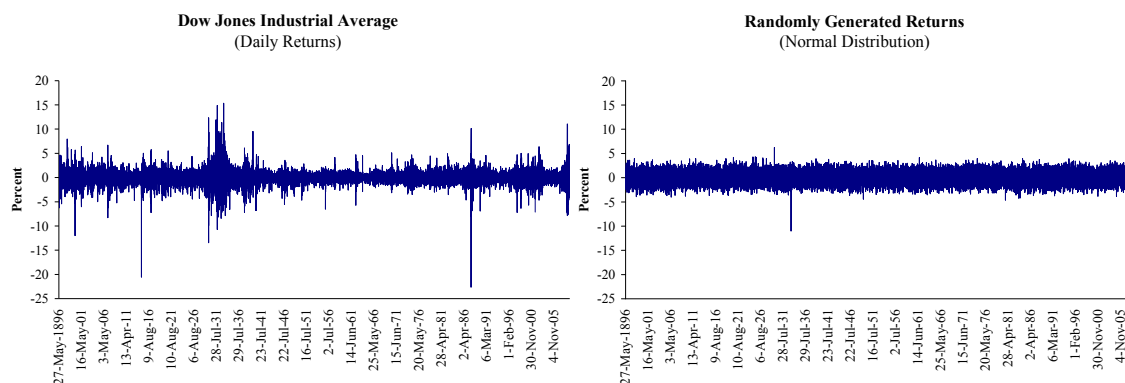
Figure 4. Efficient Frontiers with Domestic Currency Numeraire



VII. SAA CONSIDERATIONS UNDER CRISIS PERIODS

The standard Markowitz model that typically forms the basis for an SAA uses three inputs to determine an optimal portfolio: expected returns, standard deviation, and correlations between various asset classes. The expected return is often treated as a single point estimate, but in reality, it is the expected mean of a distribution. The Markowitz model assumes that the distribution is normal. However, history has shown otherwise. Since 1896 large positive and negative spikes, which would be extraordinarily rare under a normal distribution, have been relatively frequent (Figure 5).

Figure 5. Daily DJIA Returns



Note: The chart on the left hand side depicts the returns of the Dow Jones Industrial Average (DJIA) since 1896. The one on the right hand side shows a randomly generated series of returns using a normal distribution with the same mean and standard deviation as the DJIA.

Source: Dowjones.

The main implications of the current global financial crisis on SWF SAA can be summarized as follows:

- ***In times of severe financial distress the correlations between asset classes tend to move towards one.*** Since mid-September 2008, all asset classes except for government bonds have trended down sharply. The implied high asset class correlation suggests that a portfolio loses some of its diversification benefits and may experience large declines.
- ***Esoteric risks in the portfolio should be explicitly identified.*** The broad term “portfolio risk” is often given to portfolio volatility, as measured by the standard deviation of returns. However, this may oversimplify risk. A number of risks that a portfolio faces—such as credit risk or other single-factor risks—are easily quantifiable and straightforward to monitor and control by limiting exposure. Nevertheless, the current crisis has shown that portfolios also have hidden risks that are extremely difficult to define, monitor and control. For example, many institutional investors found that their portfolios had significant liquidity risk, which was very difficult to assess as it largely depended on macroeconomic factors external to the portfolio. A risk like liquidity risk is not captured by traditional portfolio risk measures or reflected in a Value-at-Risk or other risk budgeting systems, and is thus difficult to prepare for or protect against. Overall, the current experience has proven that any comprehensive risk management framework should try to address the more esoteric risks that a portfolio may face.
- ***Downside protection needs to be implemented at both the portfolio and manager level.*** If the investment philosophy has a priority of limiting the downside risk, the following measures can be taken:
 - At the portfolio level, factor exposure limits need to be implemented to prevent managers from overloading in one area or the portfolio becoming overloaded through a series of manager exposures.

- If using derivatives to synthetically replicate exposures, one should be cognizant of the underlying securities and the added fixed income exposure that they bring. Liquidity considerations should be taken into account, while derivatives must be cash settled with counterparties on a regular basis.
 - For example, at the portfolio level a put option strategy to “lock in” a floor to portfolio losses is another option. However, this type of strategy does not come without a cost and will drag performance in good years.
 - Targeting low beta active managers or those that have a proven track record of limiting downside risk may help mitigate portfolio losses.
 - Using a risk-budgeting approach and taking risk only in asset classes where there is a significant premium to be made does not necessarily limit risk, but it makes for a more prudent use of risk.
 - Rigorous stress testing of both asset mixes and managers does not prevent risk, but makes the investor more informed as to what might happen in times of duress.
- ***Investors tend to be more loss averse than risk averse.*** Risk tolerances are often set during normal, or largely positive, times in the market. While investors appear to be comfortable with potential losses from owning risky assets, they are reluctant to accept actual losses. Therefore, there is often a disconnect between investors’ appetite for risk and their loss tolerance, with the loss tolerance being much lower. This implies headline risk, which is an important consideration when setting the SAA, especially for public funds. It puts an even greater emphasis on the downside protection, as mentioned above, and on the need to educate the public about the possibility of negative SWF performance.

Finally, it should be emphasized that while a Markowitz-based strategic asset allocation model will break down in times of market distress, it is important to set the strategic allocation with a view that in the long-term markets will, for the most part, be normal. Protection from sharp market downturns needs to come through portfolio implementation (such as manager selection and exposure limits) and ongoing monitoring. The owner of an SWF, as any other investor, needs to carefully assess the acceptable level of risk and then take steps to ensure that the portfolio risk does not exceed this level.

VIII. CONCLUDING REMARKS

Overall, an SWF’s SAA should be guided by its objectives. With respect to the SAA, an SWF with distinct and separate objectives and characteristics should have a distinct and separate SAA. If an SWF seeks to meet an implicit return objective, it should have a primary return objective. However, if an SWF’s policy purpose is to smooth the fiscal expenditure, it should not have a primary return objective, but instead a volatility objective.

In analyzing the SAA for a stabilization SWF, the asset choice should be treated as an asset only problem. The pertinent macroeconomic linkages of the fund's SAA suggest selection of assets that have a negative correlation with the owner country's economic growth.

Furthermore, the selection of the numeraire has an important impact on the SAA. We show that when the numeraire is the price of oil, the SAA would be heavily biased toward U.S. assets. This is especially likely to be the case for SWFs of oil producers with a narrow economic base and exchange rate pegs to the U.S. dollar. When the numeraire is the domestic currency, the impact of the exchange rate could be significant. Therefore, while searching for assets whose return is negatively correlated with economic activity remains a key consideration, in some cases stabilizing fiscal expenditures through SWF investment flows may be achieved by simply choosing assets of a country for which the exchange rate volatility is as low as possible.

In normal times, the most critical step in SAA is deciding about the inputs and the expected return over the investment horizon. Once this has been done, the risk appetite must be established. However, if a financial crisis hits, even the best crafted parameter inputs may become irrelevant. Financial crises could render diversification meaningless, as asset class correlations tend to converge to one, thus impacting an SAA. SWFs like other investors, should always seek to go beyond the model and try to identify and assess portfolio risks that are not easily quantifiable. Nevertheless, it is advisable to build a portfolio that performs well in most states of the world, and address risks separately, so as not to build a portfolio that is mediocre in all states of the world. To this end, the mean-variance Markowitz model is a good starting point for deriving a stabilization SWF's optimal SAA.

Finally, for an SWF that needs to maximize country welfare over a long horizon, dynamic optimization is more appropriate, in particular with regard to accounting for state-contingent behavior of asset prices. Going forward, extending the model in this direction would provide a reference framework for other types of SWFs, especially savings and pension-reserve funds. Also, extending the analysis to include a decomposition of the variance of the SWF portfolio return into a component driven by exchange rate volatility and a component driven by the SWF asset performance in foreign currency would help identify the sources for domestic stabilization of the revenue gaps.

REFERENCES

- Barrell, R., 2007, "Report on the Project LINK Meeting," Chinese Academy of Social Sciences, Beijing, May.
- Claessens, Stijn and Jerome Kreuser, 2009, "Strategic Investment and Risk Management for Sovereign Wealth Funds," Paper presented at the Joint BIS/ECB/World Bank Public Investors Conference on Strategic Asset Allocation for Central Banks & Sovereign Wealth Funds at ECB, Frankfurt am Main, November 24–25, 2008.
- Das, Udaibir S., Yinqiu Lu, Christian Mulder, and Amadou Sy, 2009, "Setting up a Sovereign Wealth Fund: Some Policy and Operational Considerations" IMF Working Paper WP/09/179 (Washington: International Monetary Fund).
- Engel, E., and R. Valdes, 2000, "Optimal Fiscal Strategy for Oil Exporting Countries," IMF Working Paper WP/00/118 (Washington: International Monetary Fund).
- Goldfarb, D., and G. Iyengar, 2003, "Robust Portfolio Selection Problems," *Journal of Mathematics of Operations*, Vol. 28, pp. 1–38.
- Hagemann R., 1999, "The Structural Budget Balance - The IMF's Methodology," IMF Working Paper WP/99/95 (Washington: International Monetary Fund).
- Hammer, C., P. Kunzel, and I. Petrova, 2008, "Sovereign Wealth Funds: Current Institutional and Operational Practices," IMF Working Paper WP/08/254 (Washington: International Monetary Fund).
- IWG—International Working Group of Sovereign Wealth Funds, 2008, *Sovereign Wealth Funds—Generally Accepted Principles and Practices—“Santiago Principles.”*
- Stella, Peter, and Ake Lonnberg, 2008, "Issues in Central Bank Finance and Independence," IMF Working Paper 08/37. Available via the Internet: <http://www.imf.org/external/pubs/ft/wp/2008/wp0837.pdf>
- Maginn, J. L., D. L. Tuttle, D. W. McLeavey, and J. E. Pinto, eds., 2007, "Managing Investment Portfolios: A Dynamic Process," CFA Institute Investment Series (Wiley, 3rd edition).
- McCauley, Robert, 2008, "Choosing the Currency Numeraire in Managing Official Reserves," in *RBS Reserve Management Trends*, Central Banking Publications Ltd. (London).
- Scherer, B., and A. Gintschel, 2008, "Optimal Asset Allocation for Sovereign Wealth Funds," *Journal of Asset Management*, Vol. 9, No. 3, pp. 215–238.
- Rozanov, A., 2007, "Sovereign Wealth Funds: Defining Liabilities," State Street Global Advisors Note, Available via the Internet: <http://www.ssga.com/weblogic/OIGPublicationsServlet?AUTHORS=950,1892670>
- Sharpe, W. F., and L. G. Tint, 1990, "Liabilities – A New Approach," *Journal of Portfolio Management*, winter, pp. 5–10.