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A Simple Stochastic Approach to Debt Sustainability Applied to Lebanon

Julian di Giovanni and Edward Gardner

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Prepared by Julian di Giovanni and Edward Gardner

Authorized for distribution by Edward Gardner

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Abstract

This paper applies a simple probabilistic approach to debt sustainability analysis to the case of Lebanon. The paper derives "fan charts" to depict the probability distribution of the government debt to GDP ratio under a medium-term adjustment scenario, as a result of shocks to GDP growth and interest rates. The distribution of shocks is derived from the past shocks to these variables and the related variance covariance. Because we are interested in assessing the sustainability of a particular policy scenario, we do not consider independent fiscal policy shocks or the endogenous policy response to shocks.

JEL Classification Numbers: E62, H63, C15

Keywords: Public debt sustainability, risk analysis, Monte Carlo, fan charts

Authors' E-Mail Addresses: JdiGiovanni@imf.org; EGardner@imf.org

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I. INTRODUCTION

Lebanon's large public debt overhang (177 percent of GDP in 2006) is the country's core macroeconomic vulnerability. For years now, Lebanon has been able to sustain government debt-to-GDP ratios which are well beyond levels generally deemed sustainable. The key enabling factor has been the ability of the domestic commercial banks to finance the government by tapping into a vast pool of expatriate and regional investors, as seen in the parallel increase of government debt and bank deposits—the latter reaching 283 percent of GDP in 2007. In a context of ample global, and especially regional, liquidity, the market has been willing to hold and absorb new debt, even as Lebanon was buffeted by financial shocks triggered by the assassination of former prime minister Hariri in 2005 and the conflict with Israel in 2006.[1] Notwithstanding Lebanon's unusually high debt "tolerance," the market's willingness to hold the debt cannot be divorced indefinitely from debt sustainability considerations.

There is broad agreement that debt sustainability should be the guiding principle of Lebanon's medium-term fiscal strategy, but defining sustainability operationally remains very difficult.[2] In the theoretical literature, debt sustainability is often defined by the government's ability to pursue its fiscal policy stance into the future without threatening solvency.[3] In more formal terms, debt may be deemed sustainable as long the government operates within its intertemporal budget constraint. The difficulty in turning this principle into an operational guideline stems from the fact that the intertemporal budget constraint depends to a large extent on endogenous policy choices, such as future capacity to tax or cut spending. Reinhart, Rogoff, and Savastano (2003) and Manasse, Roubini, and Schimmelpfennig (2005) have attempted to identify empirically thresholds beyond which countries are prone to suffer debt crises. By such measures, however, Lebanon should have faced a crisis long ago. Where policies have been anchored to explicit debt targets, such as under the EU's Stability and Growth Pact, the choice of targets tend to be based on political and practical considerations more that analytical ones.

In the absence of an identifiable debt sustainability threshold, debt sustainability analysis, as typically carried out in the Fund, has focused instead on debt dynamics. Under this approach, the government can be considered to be operating within its budget constraint as long as the expected fiscal policy stance keeps the debt-to-GDP ratio on a stable (or declining) path. As pointed out by Celasun, Debrun, and Ostry (2006), there are serious shortcomings to this

[1] A market perspective on the reasons for Lebanon's resilience to market shocks is discussed in Schimmelpfennig and Gardner (2007).

[2] See for example, Chalk and Hemming (2000).

[3] Solvency is typically defined as the absence of outright default or coercive restructuring, but does not necessarily exclude the option of inflating away government debt (cf. Celasun and others, 2006). This option is limited in Lebanon by extensive dollarization of debt.

approach. First, the conditions under which the debt-to-GDP ratio behaves over time are not deterministic but stochastic. The government may have control over its policy setting, but the debt path also depends on macroeconomic conditions that are outside of its control, i.e., GDP growth, interest rates and the exchange rate. Second, even if debt is declining, a high *level* of debt and its rollover create a risk that liquidity (or other) shocks will unravel into a debt crisis. Without a handle on the probability distribution of liquidity shocks, it is impossible to assign a probability to this risk. Until now, the liquidity shocks faced by Lebanon have been "small enough" to be absorbed by the international reserve buffer. The degree to which a country is exposed to liquidity shocks depends not only on the level of the debt but also on the nature of its investor base. While Lebanon's investor base has been relatively stable in the face of shocks, the fact that debt is essentially backed by short term deposits creates a large potential rollover risk.

In the exercise described below we introduce a stochastic dimension to the traditional debt sustainability analysis based on medium-term debt dynamics. The approach is akin to that of Celasun and others (2006), but differs in terms of the range of variables subjected to stochastic shocks and in terms of the "central scenario" around which confidence intervals are built. Section II describes in greater detail the macroeconomic and fiscal setting, and offers some international comparisons. Section III describes the methodology. Section IV presents the simulation results. Section V concludes.

II. Lebanon's Debt Dynamics

Emerging out of the civil war in 1991 with a government debt burden of around 50 percent of GDP, Lebanon saw a steady increase in that ratio in the following decade as the cost of reconstruction and pacification outpaced revenue efforts. Against a primary deficit averaging 7.8 percent of GDP in the period 1992–97, debt dynamics were adversely affected by the growing gap between real GDP growth (5.6 percent annual average in 1994–97) and the real interest on Treasury bills (7 percent annual average over the same period). A fiscal consolidation effort began in 1997, but a weakening growth performance kept the debt ratio on a rapidly ascending path until 2001, despite very sizeable improvements in the primary balance. A pickup in GDP growth, a gradual increase in the primary fiscal surplus, and a further lowering of borrowing costs (related to the soft financing received from donors and commercial banks at the Paris II donor conference of November 2002) all contributed to a near stabilization of the debt ratio until 2004. Since then, a number of adverse factors contributed to a 10 percentage point increase in the debt ratio. These include: the economic slowdown related to domestic political instability and the 2006 conflict with Israel, increases in the interest bill (due to rising spreads and the maturing of Paris II zero interest credits); and the fiscal impact of higher oil prices.

The dollarization of government debt grew steadily as Lebanon returned to international bond markets in 1994, and now stands at around 50 percent. With the move to a de facto exchange rate peg to the U.S. dollar in 1998, dollarization arguably helped reduce borrowing costs and contributed to the gradual lowering of the effective interest rate on government debt, albeit at the cost of higher balance sheet vulnerabilities. The spread between two-year domestic T-bills and Eurobonds (averaging five-year maturity) averaged 3.6 percent over the period 1998–2007.[4] Although issued on the international market, Eurobonds are held mostly by domestic banks as counterpart to their foreign currency deposits, with international investors playing a marginal role.

Lebanon's dedicated investor base has contributed to insulate Lebanon from the effects of the financial crises that have hit emerging market economies since the 1990s. The stock of deposits that essentially backs the government debt has been remarkably stable in the face of weakening fundamentals, and largely immune to international financial shocks. This unique strength of Lebanon among large debtor countries is reflected in an estimated pass through of international interest rate changes that is lower than that of other emerging markets.[5] It is also reflected in a lower volatility of market interest rates, compared with most emerging market economies, as seen in the table below. Table 1 presents the (annualized) standard deviation of monthly changes of the real interest rate for a broad set of emerging markets over the period of analysis.[6] The table is broken down into four major geographical areas: Latin American/Caribbean, Eastern Europe, the Middle East/Africa, and Asia. The standard deviation of Lebanon's shot-term interest rate is only 1.2 percent per year, which is the second lowest of the group. This low volatility is surprising given the major shocks Lebanon witnessed over the sample period, but also reflects the central bank's ability to manage the interest rate in the face of volatile capital flows.

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[4] We use data until end-of-month October 2007 to compute this average.

[5] See Poddar and others (2006).

[6] See Section III for details on why we look at monthly changes of the interest rate and how the standard deviation is annualized.

Table 1. Standard Deviation of Changes in
Monthly Real Short-Term Interest Rates, 1998–2007
(Annualized, in percent)

Argentina	46.3	Bulgaria	10.5	Egypt	3.0	China,P.R.:Hong Kor	5.8
Brazil	8.7	Hungary	2.5	Jordan	2.1	India	74.2
Chile	4.4	Poland	3.7	Lebanon	1.2	Indonesia	19.7
Colombia	9.6	Turkey	163.7	Morocco	2.2	Korea	2.8
Dominican Republic	10.4			South Africa	2.6	Malaysia	1.7
El Salvador	7.8			Tunisia	0.9	Pakistan	9.0
Mexico	7.0					Philippines	2.8
Venezuela, Rep. Bol.	22.1					Singapore	2.1
						Sri Lanka	10.5
						Thailand	5.4

Notes: All rates are deposit rates, except for Pakistan and Tunisia, where money market rates are only available. Bulgaria's standard deviation is calculated over 1999–2007 given a large spike in 1998. Data source is the *International Financial Statistics*.

Still, in the period since 1993, Lebanon did experience periods of financial stress, linked to diminishing depositor confidence. For the most part, financial stress (characterized by a slowdown, or outright reversal, of deposit inflows) was triggered by domestic political tensions and relations with Israel, rather than economic developments. During these events, the drying up of bank financing forced the government to resort to central bank financing, which put pressure on international reserves. However, despite the very short-term maturity of time deposits, none of these episodes led to bank runs which could have triggered a full blown banking, balance of payments and debt crisis. The authorities' financial response and the expectation of foreign financial assistance in 2002–03 and again in 2006–07 helped restore confidence.

III. Methodology

To frame the discussion on methodology, the core equation to consider is the debt accumulation process:

$$d_{t+1} = (1 + r_t - g_t)d_t - p_t, \tag{1}$$

where d is the debt-to-GDP ratio, r is the effective real interest rate on the debt, g is real GDP growth, and p is the primary surplus-to-GDP ratio. The debt ratio is also affected by one-off privatization receipts, assumed to come in 2008 (30.8 percent of GDP).[7]

[7] The primary fiscal balance is adjusted to reflect the associated loss of revenue from the privatized enterprises.

The construction of the confidence intervals for the scenario values of Lebanon's debt-to-GDP ratio over 2008–12 follows simulation methods that have recently been exploited in the literature on debt sustainability (Garcia and Rigobon, 2004; Mendoza and Oviedo, 2004; and Hostland and Karam, 2005). We greatly simplify the analysis by (i) limiting the structure imposed on the data and (ii) abstracting from the endogeneity of fiscal policy (in response to shocks). Though such simplifications limit the richness of the model, they make it possible to overcome limitations in the data and the uncertainty associated with fiscal policy responses.

Our methodology follows two steps: extraction of the covariance structure of shocks, followed by a Monte Carlo simulation that feeds these shocks into the debt accumulation equation (1). The covariance of shocks to r, g, and p is extracted from historical monthly data over the period 1998–2007. The Monte Carlo simulation is conducted 10,000 times over the forecast period 2008–12, thus enabling us to construct confidence intervals around the "central" scenario. We further make different assumptions concerning how the shocks enter the debt equation. Section A elaborates on the construction of the variance-covariance matrix of shocks, and section B describes the simulation in more detail.

Unlike similar exercises, we apply stochastic techniques to a normative (adjustment) scenario rather than to the baseline (unchanged policies) scenario. The reason is that the baseline scenario is itself already unsustainable and adding a stochastic element to it is of limited interest. The more interesting question in the case of Lebanon lies in the probability distribution of an adjustment scenario intended to reverse explosive debt dynamics—the scenario used here is the one contained in the Lebanon—Staff Report for the 2007 Article IV Consultation (IMF, 2007).[8] This allows us, for instance to estimate the probability that the debt ratio will remain on a downward path in the face of market shocks. Accordingly, the central scenario is not extrapolated from the time series properties of the stochastic variables and from an endogenous policy reaction function, but is based on an independently projected path for interest rates, GDP growth, and fiscal primary surplus, and privatization. This difference in approach, in turn, requires a different means to identifying innovations and constructing the associated variance-covariance matrix, as described in the next section.

Also, reflecting both the currency mix of the debt and data deficiencies, we impose more structure to the derivation of the innovations to the effective interest rate on debt (r) that other similar exercises. Specifically, we derive innovations to r from the shocks to market rates at which the debt is refinanced over time in both local currency and foreign currency (the U.S. dollar). Innovations to the cost of dollar financing are extracted from secondary market Eurobond yields. For the purpose of this exercise, all foreign exchange debt is

[8] The scenario is based on the adjustment strategy which the authorities presented at the Paris International Donor Conference of January 25, 2007 (Paris III).

assumed to be denominated in U.S. dollars (as nearly all of it is) and financed through the Eurobond market. Innovations to the cost of domestic currency borrowing are more difficult to identify: The treasury bill (T-bill) rate displays very limited volatility, reflecting the fact that T-bill auctions are generally quantity auctions, and that there is no functioning secondary market. In this context, market volatility does not manifest itself directly in prices, but in variations in the quantities of T-bills placed in the market, until pressures build up to the point where a change in rates becomes necessary.[9] In the absence of a market-determined T-bill rate, we extract innovations to the marginal cost of government financing from local currency deposit rates. This approach can be justified on two levels: (i) since banks are the main buyers of government T-bills the return they require from the government is ultimately determined by their cost of funds; and (ii) looking forward, the authorities have accepted that interest rates need to respond more flexibly to market conditions, so that pressures from the market should henceforth be reflected more immediately in the T-bill market.

A. Construction of the Variance-Covariance Matrix of Shocks

Shocks to r and g are derived from the variance-covariance matrix (Σ) of shocks of five (monthly) data series. These are: (i) the 12-month moving average of the growth rate of the Coincident Indicator (a proxy for GDP growth), (ii) the domestic Lebanese pound (LL) deposit rate, (iii) the domestic U.S. dollar deposit rate, (iv) the five-year Eurobond rate, and (v) the three-month U.S. dollar LIBOR rate. As mentioned above, the "central" scenario (around which we build confidence intervals) is not derived from the data itself (such as a system of equations), but is set to equal an independent set of projections. Consistent with this approach, we do not extract innovations from an estimated system of equations (e.g., the non-structural VAR framework used by Garcia and Rigobon, 2004), but we define a shock as the first difference in the monthly variable; i.e., for any variable x, the shock e at month m is defined as:

$$e_m^x \equiv x_m - x_{m-1} \tag{2}$$

Modeling the shocks for the interest rate series in this way (i.e., as though interest rates were random walks) is consistent with the fact that a unit root for these variables cannot be rejected in our data.[10] The growth rate of the coincident indicator also has strong autocorrelation built into it given the moving-average process used in its calculation, and the use of equation (2) is thus suitable.

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[9] Gaps between the government's financing need and the placement of T-bills are absorbed either through direct central bank financing of the government or through movements in the government's cash accounts.

[10] Note that we use real interest rates, which are defined as the nominal rate minus a 12-month moving average of year-to-year inflation.

Innovations in the five data series reflect underlying shocks to the structure of the Lebanese and world economies. In our simplified approach, we remain silent on this underlying structure and on how structural shocks pass-through to the five economic variables we analyze. However, the covariance of innovations captures some of these relationships empirically in a simplified manner. In particular, the impact of innovations to the U.S. dollar interest rate is taken into account through its covariance with Lebanese interest rates and GDP growth.

B. Monte Carlo Simulation

Armed with the covariance matrix of shocks, Σ, we next turn to constructing the confidence intervals for the debt-to-GDP ratio, based on the following steps:

1. We randomly draw i.i.d. monthly shocks to the five variables over 2008–12. The shocks are assumed to be jointly-normally distributed with mean zero and variance-covariance matrix Σ.

2. We sum the monthly shocks to yield annual shocks for the innovations to growth, and the effective interest rates on LL debt and on U.S. dollar debt (which take the form of Eurobonds primarily), which are then combined into an effective interest rate on total debt (r). The procedure is as follows:

   a. **Growth shock:** total annual shock at year t = $\boldsymbol{\varepsilon}_t^g = \sum_{m=1}^{12} \varepsilon_m^g$. The annual shock is the sum of monthly (m) shocks to growth during the year t, where m = 1 is January of year t and m = 12 is December of year t.

   b. **Shock to the effective interest rate on LL debt:** total annual shock at year t = $\boldsymbol{\varepsilon}_t^{LL}$, where the definition is derived from the assumption that T-bills are issued evenly through the year and all have a two-year maturity (as most do). Accordingly:

$$\boldsymbol{\varepsilon}_t^{LL} = \begin{cases} \frac{1}{2}\sum_{m=1}^{12} \varepsilon_m^{LL} & \text{if } t = 2008 \\ \sum_{m=-11}^{12} \varepsilon_m^{LL} & \text{if } t \geq 2009 \end{cases}$$

As discussed above, the shock to the LL deposit rate is used as a proxy for the T-bill rate. Here, m = –11 is January of year t–1. Because of the assumption of a two-year maturity, innovations in 2008 impact the effective LL interest rates with a weight of ½ in 2008. By end-2009, all of the local currency debt has been rolled over and is therefore exposed to interest rates shocks. This assumption on the maturity structure of debt imposes some persistence in the effective LL interest rate over time, since shocks to the interest rates in the year in which the debt is issued carry over to the following period.

c. **Shock to the effective interest rate on dollar debt:** total annual shock at year $t = \boldsymbol{\varepsilon}_t^{EURO}$, where the definition is derived from the assumption that Eurobonds are issued evenly through the year and that all have a maturity of 5 years. Accordingly:

$$\boldsymbol{\varepsilon}_t^{EURO} = \begin{cases} \frac{1}{5}\sum_{m=1}^{12} \varepsilon_m^{EURO} & \text{if } t = 2008 \\ \frac{2}{5}\sum_{m=-11}^{12} \varepsilon_m^{EURO} & \text{if } t = 2009 \\ \frac{3}{5}\sum_{m=-23}^{12} \varepsilon_m^{EURO} & \text{if } t = 2010 \\ \frac{4}{5}\sum_{m=-35}^{12} \varepsilon_m^{EURO} & \text{if } t = 2011 \\ \sum_{m=-47}^{12} \varepsilon_m^{EURO} & \text{if } t = 2012 \end{cases}$$

Thus the marginal Eurobond rate feeds into the effective interest rate on dollar debt as a five-year moving average. Here, m = –11, –23, –35, and –47 present January of year t–1, t–2, t–3, and t–4, respectively. Again, this implies that the effective rate reflects market shocks incrementally over five years, and only by 2012 it is fully exposed to market shocks. As in the case of the LL debt above, the maturity structure also implies built-in persistence for the Eurobond rate, since shocks to the interest rate in the year in which the debt is issued carry over to the following four years.

d. **Effective real interest rate shock:** The effective real interest rate in Lebanon, r, is calculated as a weighted average of the LL and U.S. dollar effective rates based on the assumption that the degree of dollarization remains unchanged at its present level of around 50 percent. Thus:

$$\boldsymbol{\varepsilon}_t^{r} = 0.5 \times \boldsymbol{\varepsilon}_t^{LL} + 0.5 \times \boldsymbol{\varepsilon}_t^{EURO}$$

e. **Primary surplus shock:** As discussed earlier, the simulations are based on a predetermined fiscal policy path that is not subject to exogenous fiscal shocks and that does not respond to macro developments through an endogenous policy reaction function—e.g., an increase in debt due to adverse interest rate or GDP shock does not trigger an offsetting fiscal policy adjustment. Nonetheless, we do allow for shocks to GDP to affect the primary balance through their impact on tax revenue. Based on the structure of revenue and expenditure, the innovation to the primary balance (as a ratio to GDP) ε_t^p can be expressed as follows:[11]

$$\boldsymbol{\varepsilon}_t^p = 0.2 \times \boldsymbol{\varepsilon}_t^g$$

3. Using the annual shocks derived above, we calculate the distribution of *r*, *g*, and *p* under alternative assumptions about the duration of the annual shocks:[12]

Assumption 1: shocks to growth (and by implication the primary fiscal surplus) as well as interest rates are taken to be *temporary*. Therefore, *r*, *g* and *p* differ from their scenario value at year *t* only by the annual shock realized at *t*. Thus, for *t* = 2008–12:

$$\begin{aligned} g_t &= \bar{g}_t + \boldsymbol{\varepsilon}_t^g \\ p_t &= \bar{p}_t + \boldsymbol{\varepsilon}_t^p \\ r_t &= \bar{r}_t + \boldsymbol{\varepsilon}_t^r, \end{aligned} \tag{3}$$

where $\bar{r}_t$, $\bar{g}_t$, and $\bar{p}_t$ are the scenario values.

Assumption 2: shocks to the growth rate (and by implication to the primary fiscal surplus) are still taken to be *temporary*, but shocks to the interest rate are assumed to be *permanent*. As such, this is clearly an extreme case assumption. Therefore, the realized value of the interest rates at time *t* now depends on their past value (and thus past shocks) after 2008. Thus, for *t* = 2008–12:

$$\begin{aligned} g_t &= \bar{g}_t + \boldsymbol{\varepsilon}_t^g \\ p_t &= \bar{p}_t + \boldsymbol{\varepsilon}_t^p \\ r_t &= \begin{cases} \bar{r}_t + \boldsymbol{\varepsilon}_t^r & \text{if } t = 2008 \\ r_{t-1} + \boldsymbol{\varepsilon}_t^r & \text{if } t \geq 2009 \end{cases} \end{aligned} \tag{4}$$

4. Applying equation (1), the value of *d* in 2007, and the values of r_t, g_t, and p_t—from either (3) or (4), we calculate the path of d_t over 2008–12. We also include two one-off deductions to d_t in 2008 due to privatization receipts.[13]

[11] See the Appendix for the derivation of this formula.

[12] Although we are simulating shocks to the real interest rate, we place a floor of –2 percent for the real rate consistent with: (i) the assumption that nominal interest rates cannot be negative; and (ii) the scenario's assumed constant inflation rate of 2 percent.

5. We repeat steps 1-4 10,000 times and extract the 5^{th} 25^{th}, 75^{th}, and 95^{th} percentiles of the values of d_t to construct empirical confidence intervals for 2008–12 under each of the two assumptions about the interest rate process.

IV. Simulation Results

Before presenting the confidence intervals for the debt ratio *d,* we provide below some summary statistics and distributions of the simulated series of *r*, *g*, and *d*.

A. Summary Statistics and Simulation Distributions

Summary statistics

Table 2 presents the volatility of the shocks found in the data in the period 1998–2007. We present both the standard deviations of the monthly series as well as the annualized standard deviations.

Table 2. Standard Deviation of Shocks, 1998–2007
(In percent)

	Variable	Monthly	Annualized
	Growth	0.62	2.13
Interest Rates	LL deposit	0.35	1.21
	U.S. $ deposit	0.33	1.13
	Eurobond	0.58	2.00
	LIBOR	0.39	1.37

Note: The annual value is calculated by assuming that the monthly innovations in the data are uncorrelated over time, and thus equals $\sqrt{12}$ times the standard deviation of the monthly series.

The volatility of growth shocks is quite high, which is in part due to the fact that the Coincident Indicator, which is used as a proxy for Real GDP, is a fairly volatile series. However, the smoothing procedure applied to these data yield sensible correlations with the annual GDP data. The shocks to the two deposit rates behave similarly, though the LL deposit rate is slightly more volatile. This is not surprising since the currency-risk premium was most probably not constant over the sample period. The Eurobond innovations are the most volatile. Finally, shocks to the LIBOR rate are moderate.

[13] The value of these transactions are based on price earnings assumptions discussed with the authorities.

The variance of the individual variables is only part of the information that we need to conduct the simulation; the co-movement of the shocks is also a key ingredient. Table 3 presents the correlation matrix of the five variables used for the simulations.

Table 3. Correlation Matrix of Shocks
(In percent)

			Interest Rates			
		Growth	LL Deposit	U.S. $ Deposit	Eurobond	LIBOR
	Growth	1				
Interest Rates	LL deposit	-0.08	1			
Interest Rates	U.S. $ deposit	-0.13	0.74	1		
Interest Rates	Eurobond	-0.14	0.54	0.51	1	
Interest Rates	LIBOR	-0.23	0.64	0.88	0.45	1

Note: this table presents the correlation of shocks, which are based on the correlation of first-differenced data over the period 1998–2007.

The reported correlations are sensible. Shocks to the interest rates are all negatively correlated with shocks to growth. However, note that the growth correlation with the LIBOR rate is the largest (in absolute terms). This could reflect the fact that our calculated innovations may still be picking up endogenous policy or market responses. Therefore, the use of LIBOR adds useful exogenous variation in the data. The interest rate shocks are all positively correlated, and, as expected, the short rates (deposit and LIBOR rates) have the highest correlations.

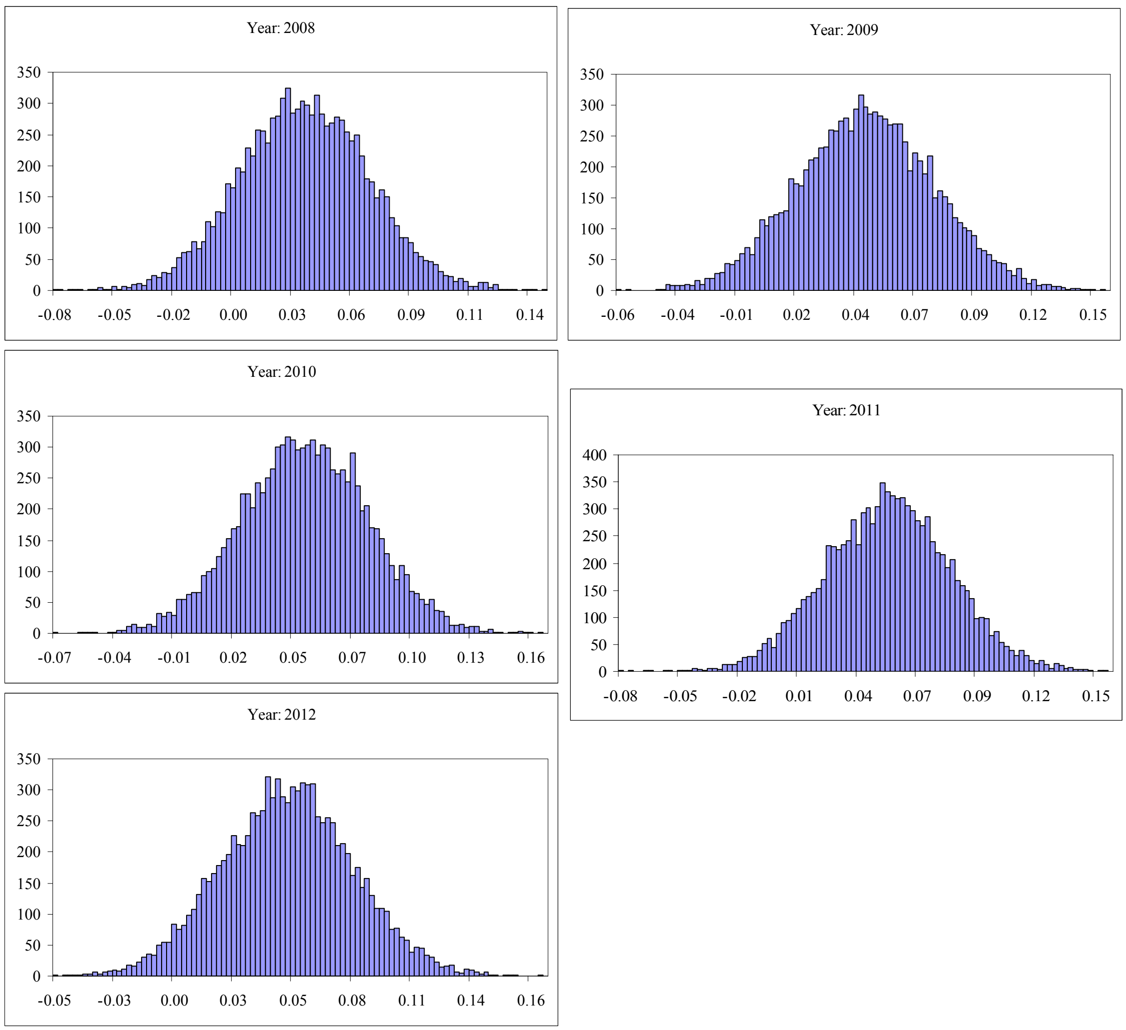
Simulation distributions

Figures 1a–1e present histograms of the simulated data for *g* (it is the same for either assumptions on *r*), for *r* (for both assumptions), and for *d* (for both assumptions) for the six years simulated.[14] In examining these histograms, it is important to note that they are built around the scenario values of *r*, *g*, and *d*. Therefore, the means of the distributions will vary over time. However, it is the variance of the distribution which is of most interest.

[14] Note that these are empirical histograms, and thus report the frequency of values.

Figure 1a plots the simulated distributions for *g* for the six years. As expected, the distributions look very similar over time, although it is centered on different means given the scenario's assumption of increasing GDP growth over 2008–12.[15]

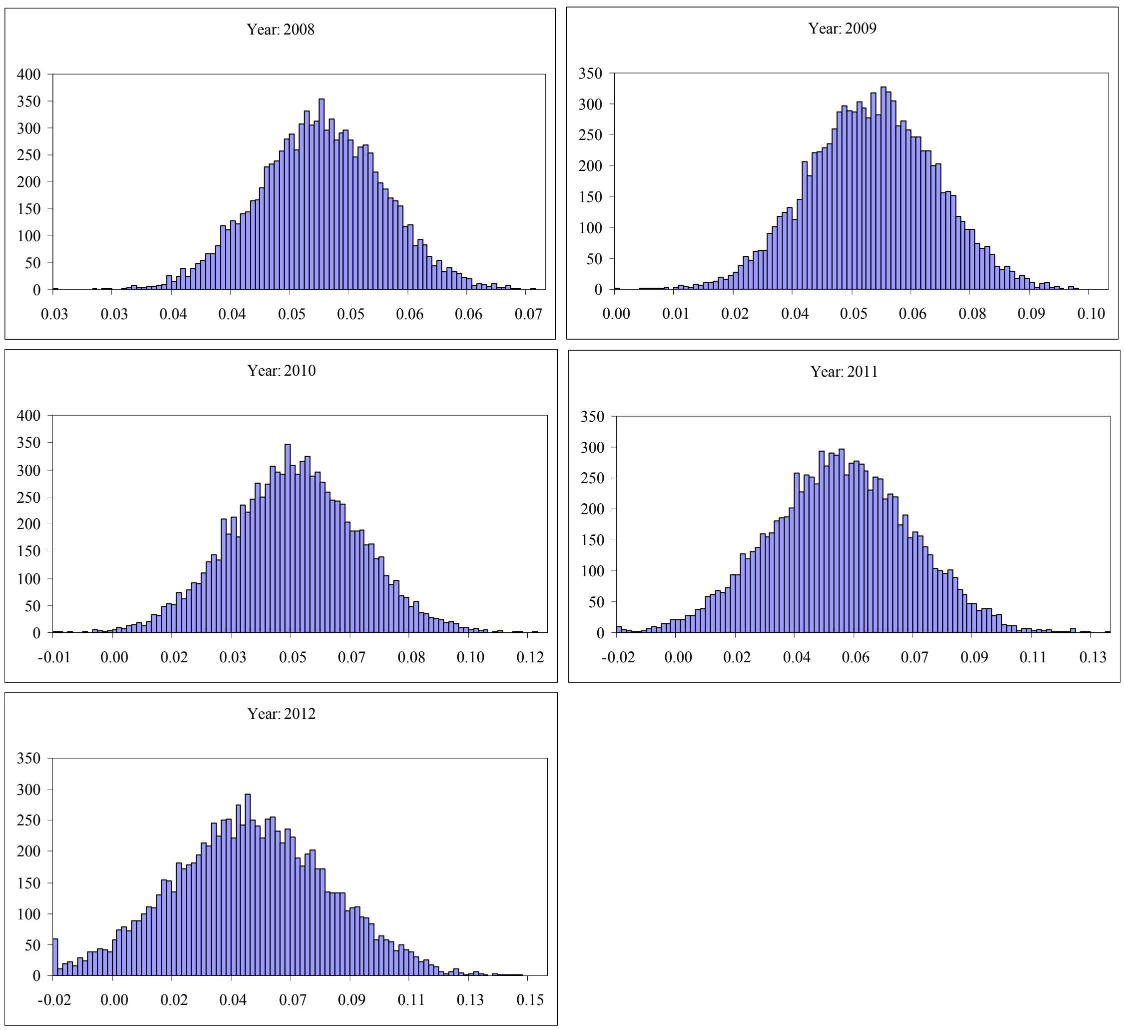
Figure 1a. Distribution of Simulated Values of Real GDP Growth Rate (*g*): 2008–12



[15] The standard deviations of the distribution for the five years are 2.98, 3.01, 2.97, 2.97, and 3.00 percent, respectively.

Figure 1b plots the simulated distributions for *r* for the six years for the *temporary* shocks assumption. Even though the shocks are temporary, the variance of the distribution still increases with time. This is because of the moving-average components of the LL deposit and Eurobond rate. Therefore, *r* picks up more uncertainty with time.[16] Also note the increase of realizations of *r* at the lower bound of –2 percent in the later years.

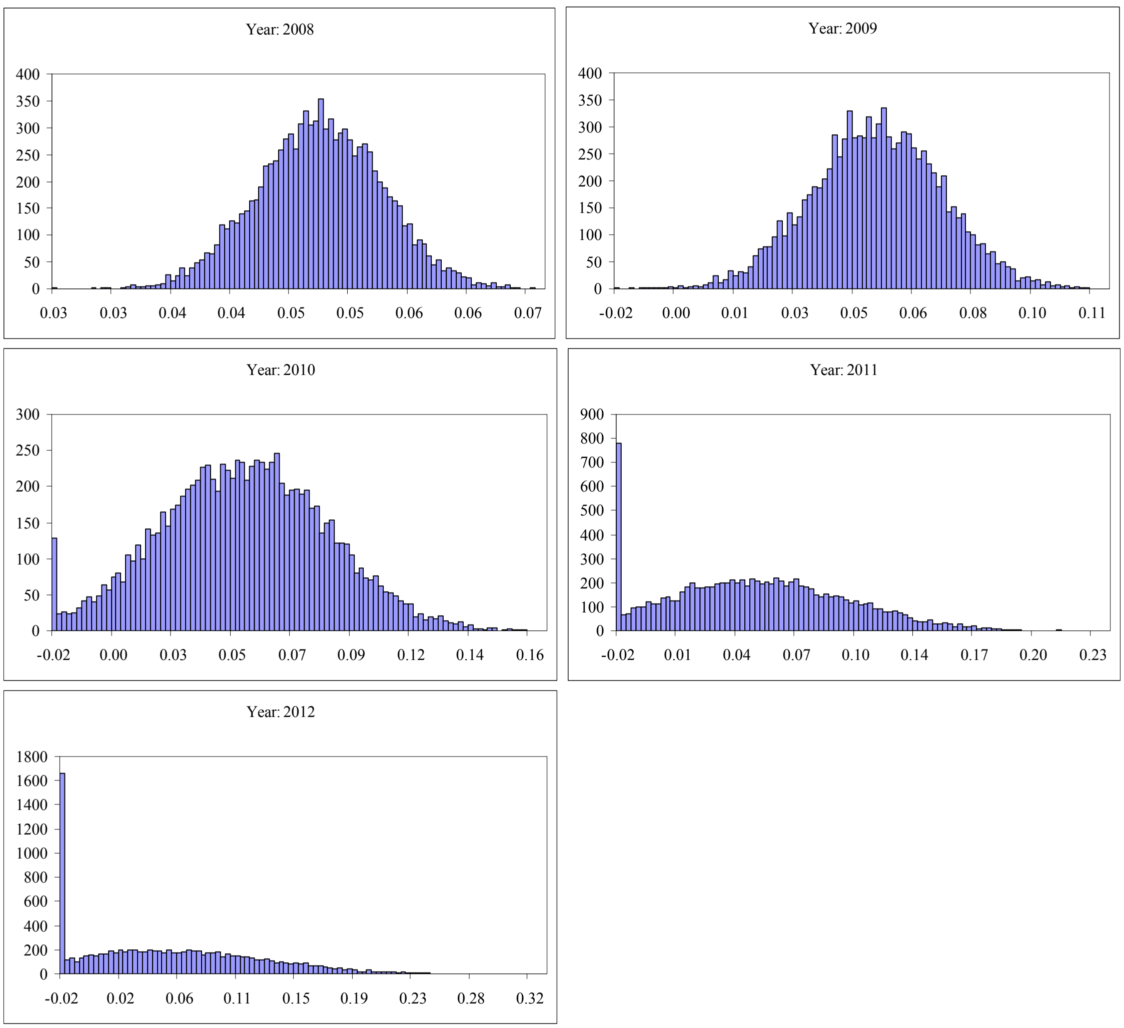
Figure 1b. Distribution of Simulated Values of Effective Interest Rate (*r*): Temporary Shocks, 2008–12



[16] The standard deviations of the distribution for the five years are 0.50, 1.41, 1.77, 2.24, and 2.80 percent, respectively.

Figure 1c plots the simulated distributions for r for the six years for the *permanent* shocks assumption. In this case the variance of the distribution is increasing over time for two reasons: first, because of the moving-average components built into r as for the temporary shocks; and second, because the shocks in period t are carried into the following periods given the autoregressive process of r.[17] In this case, the lower bound of –2 percent starts to contribute to a more sizeable part of the distribution in later years.

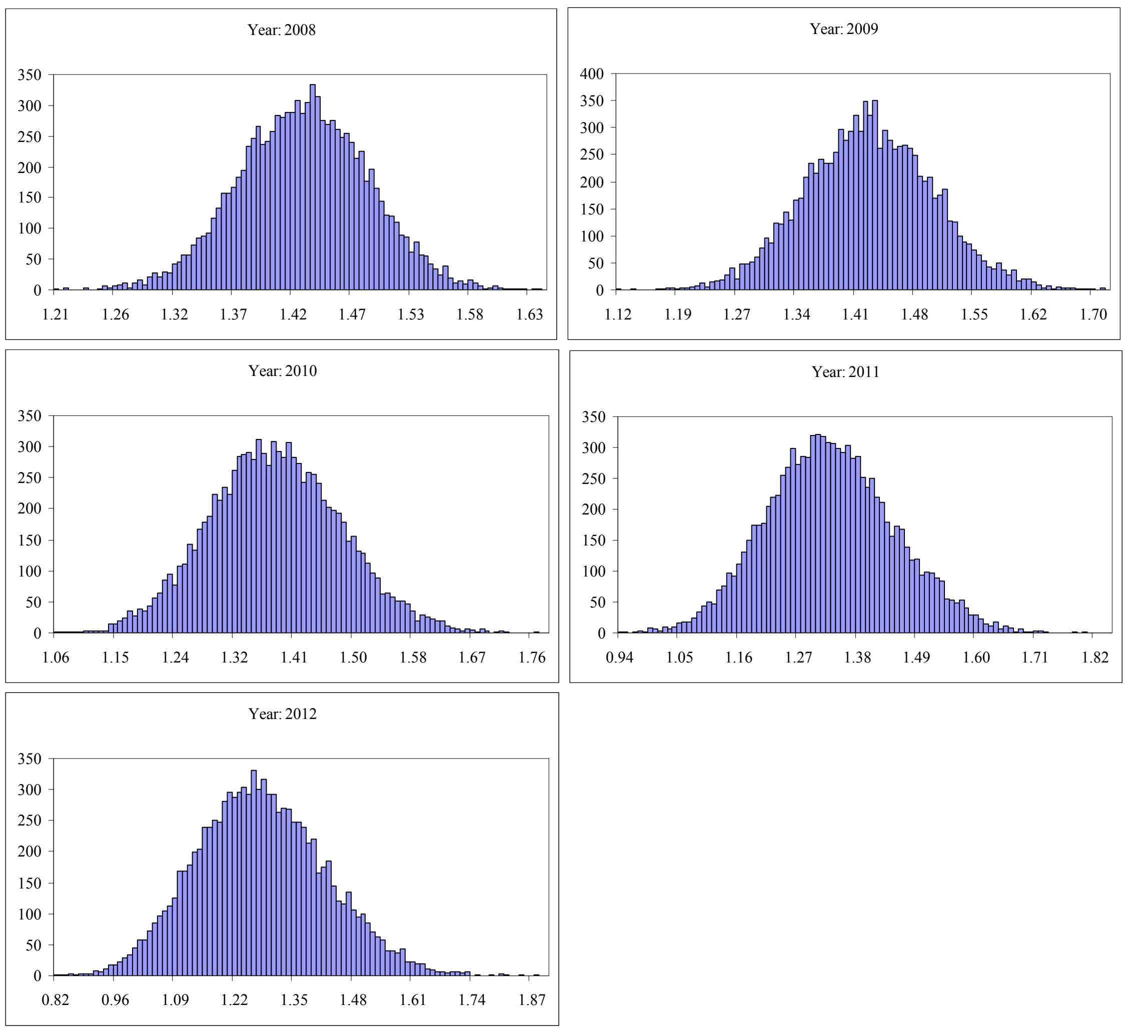
Figure 1c. Distribution of Simulated Values of Effective Interest Rate (r): Permanent Shocks, 2008–12



[17] The standard deviations of the distribution for the five years are 0.50, 1.79, 3.15, 4.63, and 6.25 percent, respectively.

Figure 1d plots the simulated distributions for *d* for the six years for the *temporary* interest rate shocks assumption. Even though the shocks are temporary, the variance of the distribution still increases with time.[18] Besides the moving-average component of r, d_t depends on its past values (see equation (1)). Therefore, the shocks are accumulated (non-linearly) over time as well. Thus, even though the distributions are bell-shaped, they would not be normally distributed, as suggested by the observed skewness in the histogram of 2011.

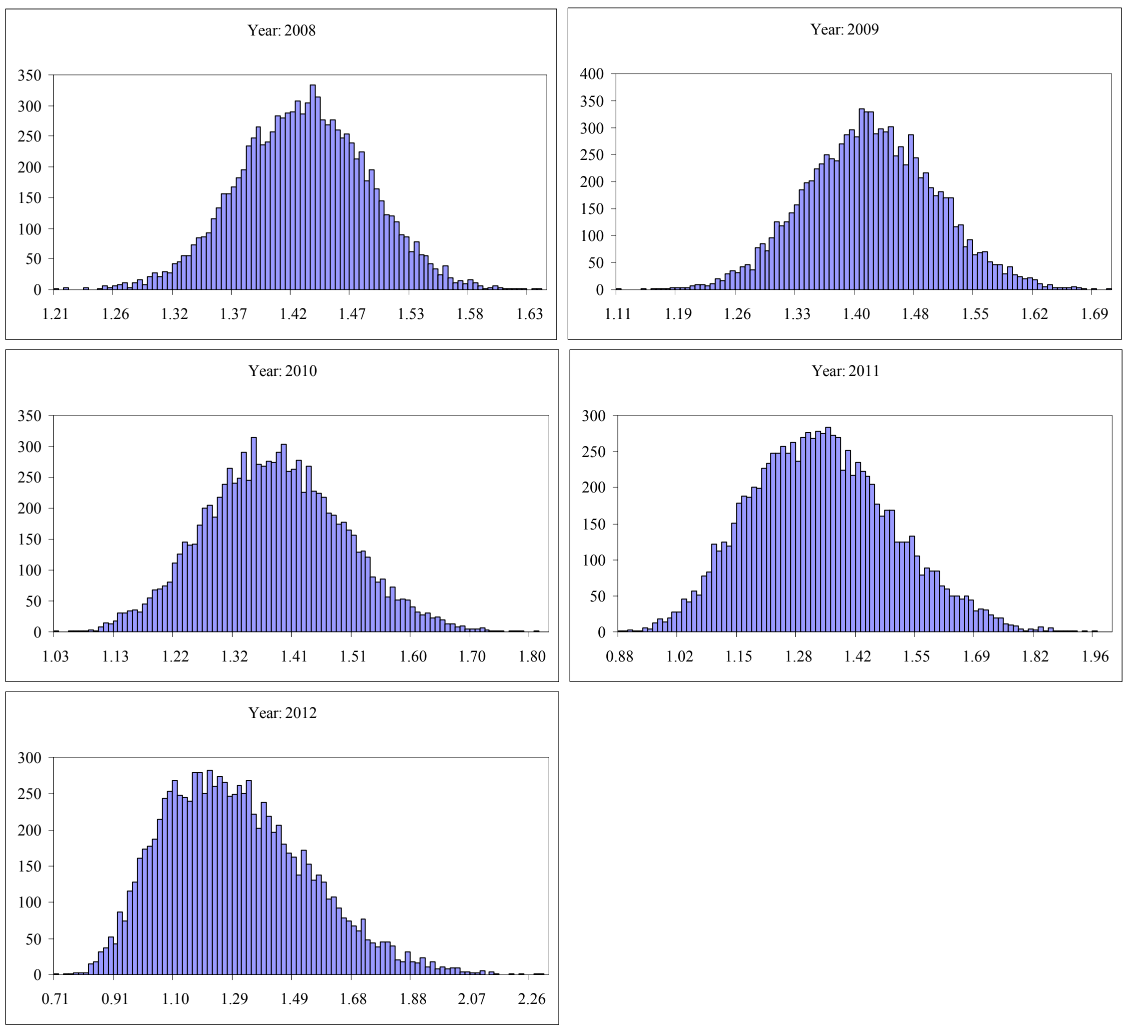
Figure 1d. Distribution of Simulated Values of Debt-to-GDP Ratio (*d*): Temporary Shocks, 2008–12



[18] The standard deviations of the distribution for the five years are 5.70, 7.71, 9.63, 11.79, and 14.26 percent, respectively.

Figure 1e plots the simulated distributions for *d* for the six years for the *permanent* interest rate shocks assumption. Not only does the variance of the distribution increase with time,[19] but the distribution starts to skew prominently to the left in the latter periods. This skewness reflects *r* hitting the lower bound more frequently. This non-normal distribution also points to the danger of reflecting uncertainty in a country's debt-to-GDP ratio by simply looking at historical standard deviations of the ratio.

Figure 1e. Distribution of Simulated Values of Debt-to-GDP Ratio (*d*): Permanent Shocks, 2008–12



[19] The standard deviations of the distribution for the five years are 5.70, 7.88, 11.01, 16.08, and 23.43 percent, respectively.

B. Fan Charts

The scenario and confidence interval values for the debt ratio are shown below, under the assumption that interest rates shocks are temporary (Figure 2a) or permanent (Figure 2b). We present both sets of confidence intervals because there are advantages and disadvantages with both. On the one hand, the unit root properties of the monthly interest rate variables suggest that the "permanent shock" assumption may more closely describe the "true" underlying stochastic process. On the other hand, the lack of mean reversion under the "permanent shock" assumption produces a distribution of effective interest rates that lacks realism in the outer years, as seen in the histogram of the real effective interest rate above. In the event, the "true" stochastic process lies probably in between these two extreme cases, i.e., there is likely to be more permanence in interest rate shocks that allowed under the "temporary" shock assumption, but also some mean reversion over time.

With these caveats in mind, the fan charts suggest that, under the adjustment scenario, there is a relatively high probability that the debt ratio will remain on a downward path, despite possible adverse shocks to GDP growth and interest rate. Under the assumption that shocks are only temporary, there is less than a 5 percent probability that the debt ratio will fail to decline by 2012, i.e., the 95th percentile corresponds to a nearly stable debt ratio, and in less than 5 percent of the cases debt would be rising again at some point by 2012. Even under the most adverse assumption of permanent interest rate shocks, the probability that the debt ratio will begin rising again by 2012 is around 25 percent, i.e., the 75th percentile in Figure 2b begins rising around 2012.

Figure 2a. Scenario's Debt-to-GDP Ratio Fan Chart: Temporary Shocks, 2007–12

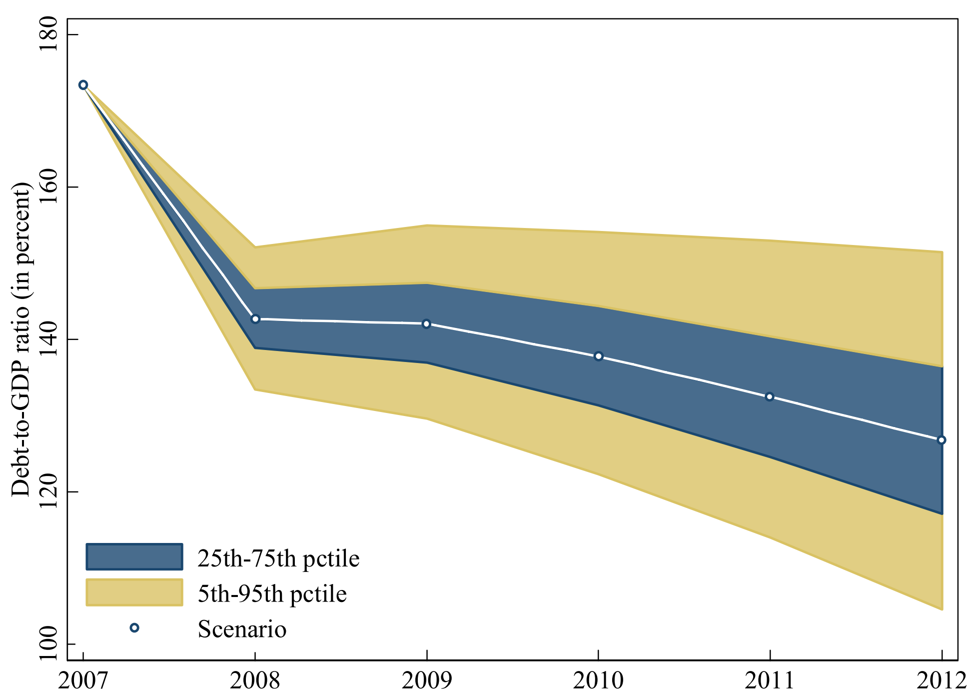
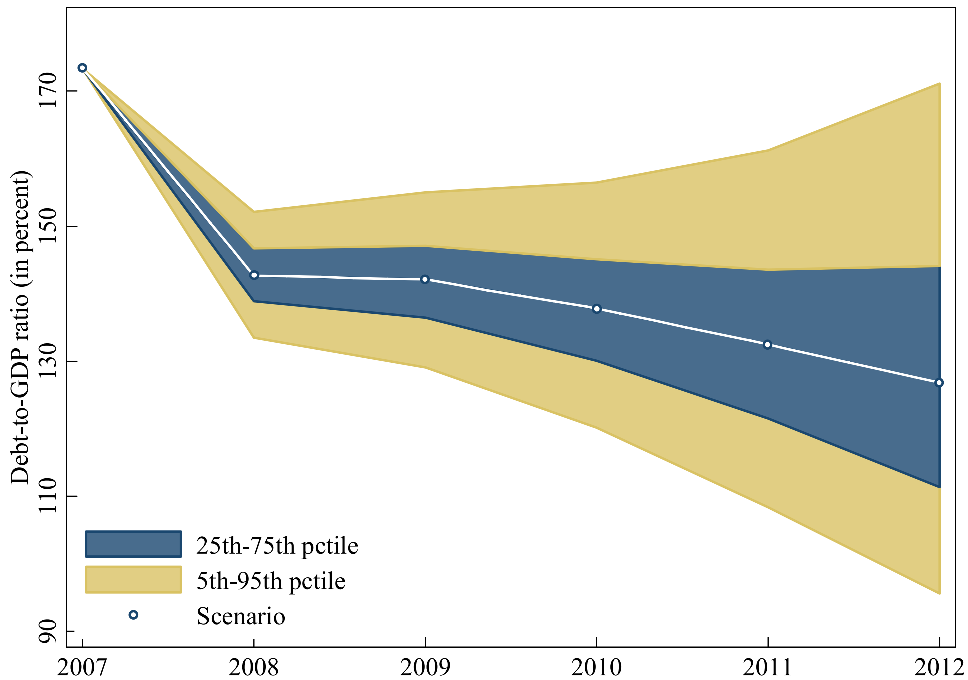


Figure 2b. Scenario's Debt-to-GDP Ratio Fan Chart: Permanent Shocks, 2007–12



V. Conclusion

In this exercise we add a stochastic dimension to debt sustainability analysis, by considering debt dynamics under the condition of stochastic GDP growth and interest rates. The distribution of shocks or innovations to GDP growth and interest rates, and their covariance, are estimated based on 1998–2007 data. Unlike other studies, such as Celasun and others (2006), we abstract here from endogenous interactions between the variables that determine debt dynamics, i.e., the primary balance, growth and interest rates. In our approach, the path of primary fiscal balance is predetermined (except for the endogenous response of tax revenues to changes in GDP), and there is no feedback from the debt ratio back to growth and interest rates. The choice of this simpler analytical framework was dictated by two considerations. First, unlike other studies that were intended to assess the debt sustainability of current fiscal policy, we focused on debt sustainability of a specific adjustment policy scenario that breaks from past policy. Accordingly, we could not extrapolate a policy reaction function from past behavior. Second, consistent with the assumed break in the policy framework relative to the past, we chose not to impose a structural model on the relationship between variables. In our approach, therefore, innovations are measured as the first differences in the interest rates and growth series. The distribution of the debt ratio around the "central" scenario is then derived based on the distribution of these innovations under alternative assumptions about the permanence of shocks.

In the particular case of Lebanon, the analysis suggests that the authorities' ambitious adjustment effort, as outlined in the Lebanon—Staff Report for the 2007 Article IV Consultation (IMF, 2007), has a reasonable probability of succeeding in placing the debt ratio on a steady downward path over the medium term, despite possible adverse shocks to growth and interest rates. The major limitation of this analysis is that it does allow for the possibility of adverse fiscal shocks, due for instance to political instability, the realization of contingent liabilities, or limitations in budgetary control. Nor does it consider, on the other hand, the possibility that fiscal policy would respond to counteract adverse shocks. However, these limitations cannot be easily overcome through statistical means: given the fragile debt dynamics under which Lebanon has operated for much of the recent past, the data are unlikely to validate the presence of an endogenous debt-stabilizing policy reaction function.

APPENDIX

The relationship between the shocks to the primary fiscal surplus and the shocks to GDP growth ($\boldsymbol{\varepsilon}_t^p = 0.2 \times \boldsymbol{\varepsilon}_t^g$) is derived as follows. First, we assume that government revenues (*T*) are a function of nominal GDP (*Y*):

$$T = \overline{T}e^{\alpha Y},$$

where $\overline{T}$ is a constant, and α is a revenue-income elasticity. Then, if *E* are nominal expenditures, we can write the primary surplus-to-output ratio, *p*, as:

$$p = \frac{\overline{T}e^{\alpha Y} - E}{Y} \tag{A5}$$

Next, take the derivative of *p* with respect to *Y* in (A5) to obtain:

$$\frac{dp}{dY} = \frac{\alpha t - p}{Y}, \tag{A6}$$

where *t* = *T/Y*. We are interested in the shock to *p*, so we re-write (A6) as:

$$dp = (\alpha t - p)\frac{dY}{Y} \equiv \phi \frac{dY}{Y},$$

which can be re-written (approximately) in discrete terms as:

$$\Delta p \approx \phi \frac{\Delta Y}{Y} = \phi g^y, \tag{A7}$$

where the last equality follows from the fact that we assume constant inflation in the scenario and simulation; therefore, the innovation to real and nominal GDP growth is identical. In turn, $\phi = 0.2$ is derived from the average values of *p* (0.05) and *t* (.25) over the projection period and the assumption of unit revenue elasticity (α =1)

In deriving the central scenario, the relationship between GDP growth and the primary surplus as specified in equation (A7) is already taken into account, i.e., the path of *p* in the central scenario reflects policy choices that incorporate information about projected growth. Accordingly, only innovations to growth will cause *p* to deviate from its scenario values, and will feed into innovations to *p* by a factor of ϕ.

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