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## Original Sin and Procylical Fiscal Policy: Two Sides of the Same Coin?

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**IMF Working Paper**

Policy Development and Review Department

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**Abstract**

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The paper develops a simple model of sovereign debt where default both through direct repudiation and through inflation are possible and give rise to (endogenous) constraints on the currency composition and the level of public debt. This set up allows to show that procyclicality of fiscal policy in EMEs can arise as a by-product of the “original sin” and both can be explained by the presence of weak monetary institutions which cannot commit to price stability. The paper suggests that, as monetary institutions in EMEs strengthen, the “original sin” would fade away and the cyclical properties of fiscal policy would improve.

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# 1 Introduction

Recent empirical studies have shown that the cyclical patterns of fiscal policy in developing economies not only differ from those of mature economies, but also from the neoclassical paradigm of Barro (1979) and Lucas and Stokey (1983). In particular, while government spending in developed countries is typically acyclical, it displays a high degree of procyclicality in emerging markets (Table 1). This stylized fact, documented by Braun (2001), Gavin, Hausmann, Perotti and Talvi (1996), Gavin and Perotti (1997), Talvi and Vegh (2000) and Kaminsky, Reinhart and Vegh (2004) have led to recent theoretical work seeking to explain these apparently puzzling differences (Table 1).

Most of this work has focused on two areas: political economy models of public expenditure, and capital market imperfections. The first strand, led by the work of Lane and Tornell (1999) and Talvi and Vegh (2000), has stressed the role of political pressures on public spending during good times as a source of fiscal procyclicality. Talvi and Vegh (2000) show that when fiscal surpluses lead to political pressures to increase spending, the optimal fiscal stance becomes procyclical. They suggest that this effect can explain the distinctive pattern of fiscal policy in developing countries, as they typically show higher volatility of the tax base allowing for stronger political pressure. Lane and Tornell (1999) argue that fiscal procyclicality can result from common pool problems, which are typically more pronounced in emerging markets as these economies tend to have a higher degree of political fragmentation. This strand of work suggests that efforts aimed at improving fiscal policy in emerging markets should concentrate in designing adequate fiscal institutional arrangements. The second line of research has stressed the role of capital market imperfections suggesting that fiscal procyclicality is the result of insufficient insurance instruments. It is argued that differences in the degree of market completeness and access to external borrowing can explain the distinctive pattern of fiscal policy in emerging markets. In particular, Gavin, Hausman and Perotti (1996) suggest that, in the presence of borrowing constraints, volatility of the tax base and the differences in the efficiency of the tax system can lead to fiscal procyclicality. Riascos and Vegh (2003) argue that fiscal procyclicality results from financial underdevelopment (market incompleteness) suggesting that the solution lies in developing financial markets. However, market incompleteness is taken as given and the reasons behind this aspect are left unexplored. As in Gavin et al (1996), a critical assumption to account for the different fiscal pattern in emerging markets is a higher volatility of the tax base (higher volatility of fundamentals). An implication of this line of research is that providing a broader menu of financial instruments would improve the ability of sovereigns to undertake an appropriate fiscal stance at each stage of the cycle.

Notably, none of the attempts to explain fiscal procyclicality has looked into the issue of endogeneity of market incompleteness, despite the fact that the well-known problem of “original sin”<sup>2</sup> is a form of endogenous market incompleteness. As a result, the role of monetary policy in affecting the structure of public debt and constraining fiscal policy has been overlooked.

This paper argues that different cyclical patterns of fiscal policy in emerging markets may stem from the inability of the monetary authority to commit to price stability (in many cases due to fiscal dominance), as the latter affects the currency structure of public debt, and reduces the “insurance” instruments available to the government. The paper develops the idea that the degree of market completeness is endogenous and influenced by monetary actions. Further, it exploits the notion that local currency debt instruments enhance the ability to conduct counter-cyclical policies by providing insurance against real shocks that are associated to currency depreciations.

From a technical perspective, the paper develops a simple perfect foresight model of a small open economy with a flexible exchange rate and a government that faces (endogenous) borrowing constraints that follow from the possibility of default on sovereign debt. The model is used to compare two polar cases: the cases of commitment and no-commitment of the monetary authority with respect to the rate of money growth. It is shown that the inability of the monetary authority to commit to a pre-announced policy results, in equilibrium, in a problem of “original sin;” that is, the inability of the government to borrow in local currency. More importantly, the currency structure of public debt is shown to play a key role in the design of fiscal policy as negative shocks are typically associated to depreciations of the domestic currency. In this environment, nominal liabilities provide some insurance to the government since the real value of its liabilities falls in the event of negative shocks. Given the existence of borrowing constraints, which follow from the possibility of strategic default by the government, the negative correlation between shocks and the real value of debt plays a key role by providing space for borrowing in bad states of nature and enhancing the ability to smooth public spending over time.

Unlike previous work, these results do not rely on differences in the magnitude of the shocks or the exogenous degree of financial development, but on the behavior of the monetary authority. In fact, it is shown that there exists a whole range of values for productivity shocks for which the currency structure of public debt determines whether

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<sup>2</sup>The “original sin” problem refers to the incapacity of most developing countries to issue liabilities in their own currencies. See Eichengreen, Hausman and Panizza (2003) and Hausman and Panizza (2003).

the optimal fiscal policy is acyclical, as in Barro (1979), or procyclical as the evidence in emerging markets as shown. The model indicates that fiscal procyclicality is likely to be a by-product of the well-known problem of original sin, and ultimately rooted in weak monetary institutions. It suggests that, despite financial underdevelopment, EMEs could enhance the cyclical properties of fiscal policy by working toward strengthening monetary institutions (improve commitment to price stability) and thereby also reducing original sin.

The paper is organized as follows: Section 2 describes the basic environment and set up the government's Ramsey problem; Section 3 characterizes optimal fiscal and monetary policy in a stationary economy distinguishing between the commitment and no-commitment cases to study the effect of credibility on the structure of public debt; and Section 4 introduces a temporary shock and characterizes the conditions under which fiscal policy is procyclical. Finally, Section 5 concludes with some remarks.

## 2 Model

Consider a small open economy populated by a continuum of identical households, who are endowed with perfect foresight and derive utility from consuming a homogeneous, perishable and tradable good, as well as a public good supplied by the government. It is assumed that households have perfect access to capital markets and face no borrowing constraints as they are able to use their whole stream of income as collateral. Interest parity is assumed to hold:

$$i_t = (1 + r)(1 + \varepsilon_t^e) - 1 \quad (1)$$

where  $i_t$  is the nominal interest rate and  $\varepsilon_t^e \equiv \frac{\Delta E_t^e}{E_{t-1}}$  the expected exchange rate depreciation rate. The government, on the other hand, is assumed to be credit constrained since, as it is usual in sovereign lending, debt contracts are not enforceable like in the private sector. In the event of a default, the government is assumed to be excluded from capital markets, and so unable to borrow again. In addition, defaulting on public debt is assumed to be fiscally costly, inducing a fiscal cost of  $\theta$  units of consumption good in each period to the fiscal authority.<sup>3</sup> The government can default at any point in time, and it will do so whenever welfare is maximized under this option ( $V^R > V^D$ ). This condition will imply that there exist an endogenous upper bound on sovereign debt.

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<sup>3</sup>This additional cost only serves the purpose of ensuring that the government does not default in steady state (the IC constraint holds). Similar results could be derived setting up an economy with balanced growth, but this would significantly complicate the solution of the model.

Finally, it is assumed that the government can borrow from external markets both on foreign currency (at an interest rate  $r$ ) and in local currency. Unlike demand for foreign currency instruments, which is perfectly elastic (as long as the IC constraint does not bind) the demand for local currency denominated debt ( $d$ ) is assumed to be increasing in domestic bond interest rates and decreasing in the depreciation rate. In particular:  $d = d(I_t^g)$ , with  $d'(I_t^g) > 0$  and  $I_t^g \equiv \frac{1+i_t^g}{1+\varepsilon_t} - (1+r)$ . This can be interpreted as a reduced form function derived from a portfolio optimization problem.

## 2.1 Households

Households derive utility from consuming private and public goods according to the following lifetime utility function:

$$V_0 \equiv \sum_{t=0}^{\infty} \beta^t U(c_t, g_t) \quad (2)$$

where  $\beta$  is the discount factor,  $c_t$  denotes consumption of the private (tradable) good and  $g_t$  denotes the public good. The utility function satisfies the usual concavity properties and it is separable in consumption of private and public goods:

$$U(c_t, g_t) = u(c_t) + v(g_t) \quad (3)$$

$$U_c(c_t, g_t) = u'(c_t) > 0 \quad ; \quad U_{cc}(c_t, g_t) = u''(c_t) < 0 \quad (4)$$

$$U_g(c_t, g_t) = v'(g_t) > 0 \quad ; \quad U_{gg}(c_t, g_t) = v''(g_t) < 0 \quad (5)$$

where  $u_i$  denotes  $\frac{du(i,j)}{di}$  and  $u_{ij}$  denotes  $\frac{d^2u(i,j)}{didi}$ . Households are endowed with one unit of labor, which they supply inelastically in domestic labor markets. They have access to external capital markets where they face a perfectly elastic supply of funds, and they can also save by holding cash balances. The household budget constraint is given by:

$$k_t^h + c_t + \frac{M_t^h}{E_t} = k_{t-1}^h(1+r) + \frac{M_{t-1}^h}{E_t} + w_t(1-\tau) + \pi_t^f + \tau_t^{MA} \quad (6)$$

where  $k_t^h$  denotes time  $t$  holdings of foreign bonds, which yield an interest  $r$  in each period;  $c_t$  denotes consumption of the private good,  $M_t^h$  denotes time  $t$  nominal cash balances,  $w_t$  denotes wage income on which taxes ( $\tau w$ ) are levied,  $\pi$  denotes dividends from firms and finally,  $\tau_t^{MA}$  denotes transfers from the government (monetary authority).



Define  $m_t^h \equiv \frac{M_t^h}{E_t}$  as real cash balances held by the households. Then households' financial wealth is given by:

$$a_t \equiv m_t^h + k_t^h \quad (7)$$

and the budget constraint can be re-written as :

$$a_t = a_{t-1}(1+r) + w_t(1-\tau) - c_t - m_{t-1}^h \left( \frac{i_t}{1+\varepsilon_t} \right) + \tau_t^{MA} + \pi_t^f \quad (8)$$

Finally, money is introduced in the model by assuming that households faces a cash in advance (CIA) constraint for the purchase of consumption goods:<sup>4</sup>

$$E_t c_t \leq \alpha M_{t-1}^h \quad (9)$$

Since the CIA constraint binds for any positive nominal interest rate, solving Equation (8) recursively yields the following household intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \frac{c_t(1+\alpha i_t)}{(1+r)^t} = \left[ a_0 + \lim_{t \rightarrow \infty} \frac{a_t}{(1+r)^t} \right] + \sum_{t=0}^{\infty} \frac{[w_t(1-\tau) + \Omega_t^f + \tau_t^{MA}]}{(1+r)^t} \quad (10)$$

where  $(1+\alpha i_t)$  denotes the time  $t$  effective price of consumption.

The household optimization problem consists therefore in maximizing Equation (2) subject to (10) and the usual No Ponzi game condition  $\lim_{t \rightarrow \infty} \frac{a_t}{(1+r)^t} \geq 0$ , taking  $\{i_t, \tau_t^{MA}, \pi_t^f\}$  and  $a_0$  as given. Assuming the discount factor satisfies  $\beta(1+r) = 1$  (this ensures the existence of a steady state) the first order conditions that characterize the solution to the household's problem are simply given by:

$$u'(c) = \lambda [1 + \alpha i_t] \quad (11)$$

$$\lim_{t \rightarrow \infty} \frac{a_t}{(1+r)^t} = 0 \quad (12)$$

where  $\lambda$  is the Lagrange multiplier associated to the intertemporal budget constraint (10). Equation (11) determines the optimal path of consumption. From the assumption of separability ( $u_{cg} = 0$ ) follows that independent of the path of government expenditure, private consumption is constant over time as long as the nominal interest rate is constant over time.<sup>5</sup> Equation (12), on the other hand, simply represents the usual transversality

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<sup>4</sup>Note that the CIA constraint can also be written as  $c_t \leq \alpha \frac{m_{t-1}^h}{1+\varepsilon_t}$ .

<sup>5</sup>The separability assumption allows us to separate the private sector problem from the government

condition which follows from imposing the No Ponzi game condition in the optimization problem.

Finally, using Equation (12) the household intertemporal budget constraint can be written as:

$$\sum_{t=0}^{\infty} \frac{c_t(1 + \alpha i_t)}{(1 + r)^t} = a_0 + \sum_{t=0}^{\infty} \frac{[w_t(1 - \tau) + \pi_t^f + \tau_t^{MA}]}{(1 + r)^t} \quad (13)$$

## 2.2 Firms

Firms are modeled in a simple fashion. They produce the homogeneous tradable good using labor as the only input according to the following production function:

$$y_t(l_t) = A_t (l_t)^\sigma \quad (14)$$

where  $0 < \sigma < 1$ ,  $l_t$  denotes labor and  $A_t > 0$  denotes a productivity parameter. Firms are assumed to pay taxes on profits, defined as  $\pi_t = y_t - w_t l_t$ . Thus, the firms' flow constraint is given by:

$$k_t^f = k_{t-1}^f(1 + r) + (y_t - w_t l_t)(1 - \tau) - \pi_t^f \quad (15)$$

where  $w_t$  denotes wage per unit of labor, as mentioned before,  $\pi_t^f$  are dividends paid to the shareholders (households);  $k_t^f$  are firm's holdings of foreign bonds; and  $\tau$  is the tax rate on profits. Solving Equation (15) recursively and using Equation (14) yields:

$$\sum_{t=0}^{\infty} \frac{\pi_t^f}{(1 + r)^t} = \left[ k_0^f + \lim_{t \rightarrow \infty} \frac{k_t^f}{(1 + r)^t} \right] + \sum_{t=0}^{\infty} \frac{[A_t (l_t)^\sigma - w_t l_t](1 - \tau)}{(1 + r)^t} \quad (16)$$

where the LHS represents the firm's market value, given by the discounted present value of dividends paid to the shareholders.

The firm's optimization problem consists therefore in maximizing (16) subject the No Ponzi game condition  $\lim_{t \rightarrow \infty} \frac{k_t^f}{(1 + r)^t} \geq 0$ , taking  $\{w_t\}$  and  $k_0^f$  as given. The first order conditions of this problem are simply given by:

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problem as it will become clear later.

$$\sigma A_t (l_t)^{\sigma-1} = w_t \quad (17)$$

$$\lim_{t \rightarrow \infty} \frac{k_t^f}{(1+r)^t} = 0 \quad (18)$$

Equation (17) implicitly defines the demand for labor as a function of the wage rate, while Equation (18) is the transversality condition for the firm, which again follows from imposing the No Ponzi game condition in the optimization problem.

## 2.3 Government

The government comprises both the fiscal and the monetary authority. The monetary authority sets monetary policy under a flexible exchange rate regime,<sup>6</sup> by setting the rate of money growth in each period. The fiscal authority, on the other hand, levies nondistortionary taxes on firms' profits and labor income.<sup>7</sup> Government expenditure takes the form of a public good that provides direct utility to the households as shown in Equation (2). In order to finance (temporarily high) spending, the government can borrow from external markets both in foreign currency (at an interest rate  $r$ ) or in domestic currency. As mentioned before, while the supply of funds in foreign currency is perfectly elastic (for Incentive Compatible levels of debt) the demand for local currency debt is not. As mentioned before, the demand for local currency instruments ( $d$ ) is assumed to be increasing in the nominal interest rate of these bonds ( $i_t^g$ ) and decreasing in the expected depreciation rate ( $\varepsilon_t^e$ ):

$$\frac{\partial d}{\partial i_t^g} = -\frac{d'(eI_t^g)}{(1 + \varepsilon_t^e)} \geq 0 \quad , \quad \frac{\partial d}{\partial \varepsilon_t^e} = \frac{d'(eI_t^g)}{(1 + \varepsilon_t^e)^2} \leq 0 \quad (19)$$

This specification departs from open economy models like in Lahiri and Vegh (2003), where domestic debt is held only by domestic agents who derive liquidity services from these securities. In this case, nominal debt is held by foreigners. A set up a la Lahiri and Vegh (2003) would deliver the same qualitative results but it would complicate the solution of the model since it would introduce a link between the household problem and the government taxation problem, which is shut down with this specification. This will become clear in the next section.

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<sup>6</sup>Similar results would hold in a model with a fixed exchange rate regime where the central bank sets the level of the exchange rate in each period (allowing for devaluations) instead of the money supply. However, this set up would additionally require the introduction of a nontradable good for the real exchange rate to adjust to shocks, as the nominal rate does in this exchange rate regime.

<sup>7</sup>Taxes are nondistortionary taxes in this economy as the supply of labor is perfectly inelastic.

As mentioned before, since sovereign debt contracts are not enforceable, the government can borrow from external sources (either in foreign or local currency) only as long as it has no incentives to repudiate on these obligations. In other words, the government's Ramsey problem, which is described in detail in the next section, must satisfy an Incentive Compatibility (IC) constraint of the form:

$$V_t^R \geq V_t^D \quad (20)$$

where  $V_t^R$  denotes welfare under sovereign debt repayment and  $V_t^D$  denotes welfare under default. As it is shown below, this IC constraint is equivalent to an endogenous upper bound on total public debt.

In the event of a default, the government loses its access to capital markets, forcing the fiscal budget to be balanced every period.<sup>8</sup> In addition, it is assumed that a default also results in a fiscal cost of  $\theta$  units of consumption good in each period.<sup>9</sup> It follows that in the event of a default government expenditure is given by:

$$g_t^D = \Gamma_t - \theta \quad (21)$$

which is clearly driven by the endogenous behavior of tax revenues ( $\Gamma_t$ ).

Provided that total public debt is below its upper bound level (the IC constraint holds) the government's budget constraint is given by:

$$b_t = b_{t-1}(1+r) + g_t - \Gamma_t + \tau_t^{MA} - \frac{M_t - M_{t-1}}{E_t} + \frac{D_{t-1}}{E_t}(1+i_{t-1}^g) - d_t \quad (22)$$

where  $b_t$  denotes foreign currency denominated debt,  $g_t$  is government expenditure in the form of the public good,  $\Gamma_t$  denotes tax revenues;  $D_{t-1}$  denotes the nominal value of previous period domestic currency debt,  $d_t$  denotes its real value, and  $\tau_t^{MA}$  denotes central bank's transfers to the household.

Money is introduced in the economy as a transfer to the household ("helicopter drop"), which implies that  $\tau_t^{MA} = \frac{M_t - M_{t-1}}{E_t}$ .

Notice also that tax revenues are given by

$$\Gamma_t = \tau w_t l_t + \tau [y_t - w_t l_t] = \tau y_t \quad (23)$$

so fiscal revenues are proportional to output in this set up. The tax rate  $\tau_t$  is assumed to be constant over time, so we focus on government expenditure as the fiscal

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<sup>8</sup>In a perfect foresight setting, a default is always an out-of-equilibrium event.

<sup>9</sup>This simply ensures that in stationary equilibrium the government does not default.

policy instrument.

Finally, the following timing of policy actions is assumed: at the beginning of each period, before resorting to capital markets to finance any fiscal imbalance, the MA announces a target for nominal money growth:

$$\mu_{t+1} \equiv \frac{M_{t+1} - M_t}{M_t} \quad (24)$$

This introduces, in a simple way, a potential problem of commitment since the government would have incentives to deviate from the announced policy in order to inflate nominal debt away. This commitment problem has been largely studied in the literature and it is the central argument to explain why governments in emerging markets are unable to borrow in their own currencies. The contribution of this paper is precisely to show that this same problem can explain the differences in cyclical patterns of fiscal policy between developed and developing countries.

Finally it is assumed, without loss of generality, that the nominal interest rate on domestic debt is constant over time ( $i_t^g = i^g$  for all  $t$ ), so that the real domestic interest rate is simply determined by the depreciation rate ( $\varepsilon_t$ ).

## 2.4 Equilibrium Path

A competitive equilibrium is a sequence for quantities  $\{c_t, k_t^h, b_t, d_t, m_t^h, g_t, k_t^f, l_t, \pi_t^f\}$  and prices  $\{E_t, i_t, i_t^g, w_t\}$  such that: i)  $\{c_t, m_t^h, k_t^h\}$  solve the households' maximization problem given  $\{E_t, i_t, w_t\}$  and  $\{g_t, \pi_t^f\}$ ; ii)  $\{l_t\}$  solves firms' maximization problem given  $\{w_t\}$ ; iii) the government intertemporal budget constraint holds; and iv) markets clear:  $l_t = 1$ ,  $m_t^h = m_t$ .

As agents are rational and there is no uncertainty in the economy, it follows that they perfectly anticipate the depreciation rate  $\varepsilon_t^e = \varepsilon_t$ . Then the nominal interest rate is given by:

$$i_t = (1 + r)(1 + \varepsilon_t) - 1 \quad (25)$$

Furthermore, it can be shown that in equilibrium the depreciation rate  $\varepsilon_t$  must equal the rate of money growth ( $\varepsilon_t = \mu_t$ ), as it is standard in models with flexible exchange rates.<sup>10</sup>

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<sup>10</sup>From the definition of real cash balances ( $m_t \equiv M_t/E_t$ ) follows that  $m_t = m_{t-1} \left[ \frac{1+\mu_t}{1+\varepsilon_t} \right]$ . Then combining this equation with the interest parity condition and equation (11) and (9) yields:

From the equilibrium condition for labor markets follows that  $l_t = 1$ , thus output is given by  $y_t = A_t$ . Combining the household's budget constraint (8) and the firm's budget constraint (15), and using the fact that  $\tau_t^{MA} = \frac{M_t - M_{t-1}}{E_t}$ , the household intertemporal budget constraint (10) can be written as:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = k_0^p + \sum_{t=0}^{\infty} \frac{[A_t(1-\tau)]}{(1+r)^t} \quad (27)$$

where  $k_0^p \equiv (k_0^h + k_0^f)$  denotes private sector holdings of foreign assets. It is clear from this equation, Equation (11) and the assumption of separability of the utility function, that the optimal path of private consumption can only be affected by an intertemporal distortion due to an irregular path of nominal interest rates.<sup>11</sup>

On the other hand, using  $\varepsilon_t = \mu_t$  and  $\tau_t^{MA} = \frac{M_t - M_{t-1}}{E_t}$  to substitute in the government budget constraint yields the fiscal constraint:

$$k_t^g = k_{t-1}^g(1+r) - g_t + \tau A_t - d_{t-1} \left[ \frac{1+i^g}{1+\mu_t} - (1+r) \right] \quad (28)$$

where  $k_t^g \equiv -(b_t + d_t)$  denotes government net external position. Notice that at time  $t$ , the stock of nominal debt  $d_{t-1}$  and the interest rate  $i^g$  are given. This implies that, by increasing the rate of money growth  $\mu_t$  (and thus depreciating the currency) the government can reduce the real value of public liabilities which implies a transfer of resources from foreigners to domestic households. This creates the potential commitment problem that, in equilibrium, will affect the currency composition of public debt.

Finally, define  $k_t \equiv k_t^p + k_t^g$  as the country net external position. Then combining Equations (27), and (28) yields the resource constraint of the economy:

$$k_t = k_{t-1}(1+r) + y_t - c_t - g_t + d_{t-1} I_{t-1}^g \quad (29)$$

Notice that this resource constraint differs from the classical case only by its last term, which represents the social benefit (cost) of borrowing from abroad in domestic currency.

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$$m_t = m_{t-1} \left[ \frac{1+\mu_t}{1-r-1/\alpha+u_c(m_{t-1}/\alpha)/\lambda\alpha} \right] \quad (26)$$

which is an unstable equation around the steady state. Thus, it follows that in a nondivergent path for real cash balances it must be the case that the depreciation rate equals the rate of money growth ( $\varepsilon_t = \mu_t$ ).

<sup>11</sup>It will be shown, however, that the path of interest rates is constant in equilibrium since the optimal rate of money growth is constant as well.

Finally, note that the equilibrium path is not yet fully specified since we have not determined the actual paths of the rate of money growth  $\{\mu_t\}$  and government expenditure  $\{g_t\}$ , which are in fact policy instruments that affect the path of consumption as it is clear from Equations (11) and (25), as well as the path of government debt  $b_t$  and  $d_t$ . In the next section, we study the how the path of  $\mu_t$  and  $g_t$  are set.

### 3 The Ramsey Problem

We study two polar cases to show that the ability of the monetary authority to commit to a certain rate of money growth  $\mu_t$  affects the currency composition of public debt and ultimately restricts fiscal policy.

#### 3.1 The Commitment Case

Consider the case of a monetary authority that announces a rate of money growth  $\mu_t = \mu_t^A$  (before borrowing) and commit to this policy. Since in equilibrium  $\varepsilon_t = \mu_t$ , the announcement about money growth rate is credible then  $\varepsilon_t^e = \mu_t^A$ ,  $i_t = (1+r)(1+\mu_t^A) - 1$  and  ${}^e I_t^g = (1+r) - \frac{1+i^g}{1+\mu_t^A}$ . Then the Ramsey Problem of the government can be written as a simple dynamic programming problem in the following way:

$$V^R(k^p, k^g, \mu^A) = \max_{(c, g, k^{p'}, k^{g'}, \mu^{A'})} \{u(c) + v(g) + \beta V^R(k^{p'}, k^{g'}, \mu^{A'})\} \quad (30)$$

$$k^{p'} + c \leq k^p(1+r) + A(1-\tau) \quad (31)$$

$$u'(c) = \lambda [1 + \alpha i] \quad (32)$$

$$k^{g'} + g \leq k^g(1+r) + A\tau + d(I^g) [I^g] \quad (33)$$

$$I^g = (1+r) - \frac{1+i^g}{1+\mu^A} \quad (34)$$

$$V^D(k^{p'}) \leq V^R(k^{p'}, k^{g'}, \mu^{A'}) \quad (35)$$

Equation (31) is the household budget constraint once we take into account the solution to the optimization problem of the firm and the central bank transfer policy  $\tau_t^{MA} = \frac{M_t - M_t}{E_t}$ . From this equation and Equation (33) follows that the government does not collect inflation tax precisely because of the central bank's transfer policy. This allows us to focus solely on the effect that a currency depreciation has on the real value of domestic debt, as a source of income for the government. Equation (32) follows from the household's first order condition and ensures that the optimal path of consumption in the

government problem is implementable in a decentralized equilibrium. Equation (33) is the fiscal constraint, following from (22). Equation (34) simply denotes the opportunity cost of holding domestic debt, as explained before. Finally, Equation (35) is the Incentive Compatibility constraint necessary to ensure that the government does not default on external debt. Notice that  $V^D$  does not depend on the level of debt of the government or the announcement of the depreciation rate precisely because in the event of a default, all public debt is defaulted. However, as we have assumed that debt contracts are enforceable in the private sector, even in the case of a sovereign default welfare is a function of the private sector foreign position  $k^{p'}$ . Furthermore, as we have assumed that the tax rate  $\tau$  is constant over time and since the utility function is separable in private and public good consumption ( $u_{cg} = 0$ ), it follows from Equations (31) and (32) that the optimal path of private consumption can only be affected by monetary policy to the extent that it distorts the path of nominal interest rates. If the optimal path of inflation is constant, then the private sector problem is completely independent of the government problem and, as a result, it can be shown that  $V_{k^{p'}}^D(k^{p'}) = V_{k^{p'}}^R(k^{p'}, k^{g'}, \mu^{A'})$ .

The first order conditions that characterize the solution to this optimization problem, in a stationary economy, are given by:

$$u'(c_t) = u'(c_{t+1}) \quad (36)$$

$$u'(g_t) = u'(g_{t+1})[1 + (1+r)\delta_t] \quad (37)$$

$$\eta_{d_t, I_t^g} = 1 \quad (38)$$

$$\delta_t[V_{t+1}^R - V_{t+1}^D] = 0 \quad (39)$$

where  $\eta_{d_t, I_t^g} \equiv \frac{\partial d_t/d_t}{\partial I_t^g/I_t^g}$  denotes the elasticity of the demand for domestic debt with respect to the nominal interest rate. Equations (36) and (37) define the optimal path of consumption of the private and the public good, respectively. It is clear from these Equations that  $c_t$  is always constant over time, while the path of  $g_t$  depends on whether the IC constraint binds or not. The former implies, as follows from Equations (32), (25) and (26) that the rate of money growth ( $\mu_t$ ), the depreciation rate ( $\varepsilon_t$ ), and the nominal interest rate ( $i_t$ ) are constant over time. Equation (38) on the other hand, defines the actual level for the rate of money growth. Given  $i^g$ , the optimal level of money growth is the one that maximizes the benefits from borrowing in domestic currency ( $d_t I_t^g$ ), which is exactly the level at which the elasticity of demand is equal to one (the top of the Laffer curve).

Thus, in equilibrium, there exist an optimal positive level of local currency denominated debt and therefore an optimal currency composition of public debt. Define



$\rho_t \equiv -d_t/k_t^g$  as the proportion of public debt denominated in domestic currency, it follows that the steady state value of  $\rho_{ss}$  is given by:

$$\rho_{ss}^* = -\frac{d(I^{g*})}{k_0^g} \quad (40)$$

From the assumption of separability of the utility function, we know that  $V_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}) + \sum_{j=0}^{\infty} \beta^j v(g_{t+j})$ ; and, since the path of private consumption is the same under repayment or default, it follows that the IC constraint can simply be written as:

$$\sum_{j=0}^{\infty} \beta^j v(g_{t+j}^R) \geq \sum_{j=0}^{\infty} \beta^j v(g_{t+j}^D) \quad (41)$$

where  $\sum_{j=0}^{\infty} \beta^j v(g_{t+j}^D) = \sum_{j=0}^{\infty} \beta^j v(A_t \tau - \theta)$  as follows from Equation (21). That is, the IC constraint only depends on fiscal policy (the path of government expenditure) precisely because we have separated the private sector from the public sector problem.

### 3.2 No Commitment

Consider now the case of a monetary authority that cannot credibly commit to a certain policy. In this case, the announcement  $\mu^{A'}$  is irrelevant and it does not affect the demand for domestic currency debt instruments.

$$V^R(k^p, k^g, \mu^A) = \max_{(c, g, k^{p'}, k^{g'}, \mu, \mu^{A'})} \{u(c) + v(g) + \beta V^R(k^{p'}, k^{g'}, \mu^{A'})\} \quad (42)$$

$$k^{p'} + c \leq k^p(1+r) + A(1-\tau) \quad (43)$$

$$u'(c) = \lambda[1 + \alpha i] \quad (44)$$

$$k^{g'} + g \leq k^g(1+r) + A\tau + d({}^e I^g) [I^g] \quad (45)$$

$$I^g = (1+r) - \frac{1+i^g}{1+\mu} \quad (46)$$

$${}^e I^g = (1+r) - \frac{1+i^g}{1+\mu^e} \quad (47)$$

$$V^D(k^{p'}) \leq V^R(k^{p'}, k^{g'}, \mu^{A'}) \quad (48)$$

Notice that the problem is exactly as before except for the fact that now  $d({}^e I^g)$  does not depend on the previous announcement  $\mu^A$  but on the expectation of the depreciation

rate  $\mu^e$ . Disregarding reputational equilibria, the latter implies that the government takes  $d({}^e I^g)$  as given when choosing the depreciation rate at time  $t$ .

The first order conditions that characterize the solution to this optimization problem are given by:

$$u'(c_t) = u'(c_{t+1}) \quad (49)$$

$$u'(g_t) = u'(g_{t+1})[1 + (1+r)\delta_t] \quad (50)$$

$$d_t({}^e I^g) \left[ \frac{1+i^g}{(1+\mu_t)^2} \right] = 0 \quad (51)$$

$$\delta_t[V_{t+1}^R - V_{t+1}^D] = 0 \quad (52)$$

As before, Equations (49) and (50) define the optimal path of consumption of the private and the public good, from which it is clear that  $c_t$  is constant over time while  $g_t$  is constant when the IC constraint does not bind and increasing through time when it does. It follows that the rate of money growth ( $\mu_t$ ), the depreciation rate ( $\varepsilon_t$ ), and the nominal interest rate ( $i_t$ ) are also constant over time, and independent of the path of  $g_t$ . Equation (51) on the other hand, defines the actual level for the rate of money growth. Notice that for any given positive level of domestic debt ( $d_t > 0$ ) the benefit of increasing the depreciation rate is positive implying that there is no interior solution (it is optimal to inflate debt away completely). This implies in fact that the equilibrium depreciation rate is the one that turns the demand for domestic bonds equal to zero. That is:

$$d_t({}^e I^g) = 0 \quad \Leftrightarrow \quad \varepsilon_t^e = \frac{(1+r) - \bar{I}^g}{(1+i^g)} \quad \text{for all } t \quad (53)$$

Therefore, the proportion of local currency denominated debt is in this case:  $\rho = 0$ .

Finally it can be shown that the actual path of government expenditure depends on the path of output in the economy. The following proposition states an important result which will be useful for the next section.

**Proposition 1** *For any interval of time  $[t, t+j]$  in which the path of  $A_t$  is constant, the optimal path of government expenditure is constant.*

**Proof.** See appendix. ■

Using this result in the next section, we pin down the actual path of government expenditure for a stationary economy, and following this we show how the fiscal policy in the event of a negative shock is affected by monetary policy.

## 4 A Stationary Economy

Consider a constant path for the productivity parameter ( $A_t = A$  for all  $t$ ) and output ( $y_t$ ) so that there is a stationary equilibrium, and assume that the initial government external position satisfies  $k_0^g > -\theta/r$ , which ensures that the government does not default in steady state. From Equations (36) and (49) follows that the optimal path of private consumption is constant over time. Thus, using Equation (34) we can fully specify the path of private consumption as:

$$c_t = rk_0^p + A(1 - \tau) \quad (54)$$

which hold for the two cases under study. On the other hand, since  $A_t = A$  for all  $t$ , from Proposition 1 follows that the path of government consumption is constant over time.

Notice that Equations (38) and (51) imply different equilibrium levels of money growth rate and level of domestic debt. The following propositions collect the main results:

**Proposition 2** *Assume  $k_0^g > -\theta/r$ . If  $A_t = A$  for all  $t$ , the path of government expenditure ( $g_t$ ) is constant over time. The stock of domestic debt for the commitment cases is  $d^* = d(I^{g^*})$  while for the no-commitment case is  $d^* = 0$ .*

**Proof.** See appendix. ■

Finally, using the government budget constraint we can solve for the actual level of government expenditure for both cases:

$$g_t = rk_0^g + A\tau + d(I^g) [I^g] \quad (55)$$

where, as mentioned before,  $d(I^g) = d(I^{g^*})$  in the case of a credible monetary policy and  $d(I^{g^*}) = 0$  otherwise.

It is straight forward to show that welfare in the case of inconsistent monetary policy is lower than in the commitment case, precisely as the benefits from borrowing in domestic currency vanish when the monetary authority cannot commit to a certain policy.

More important, these results show how the ability of the monetary authority to commit to a certain policy can affect the structure of public debt in terms of the currency denomination. This will in turn affect the optimal fiscal policy in the event of a shock, as it is shown in the next section.

## 5 A Temporary Shock

Consider an (unexpected) temporary shock that takes place at time  $t = 0$  and last for  $T$  periods, such that the productivity parameter (and so output) follows:

$$A_t = \begin{cases} A^L & \text{for } t \leq T \\ A^H & \text{for } t > T \end{cases} \quad (56)$$

where  $A^L < A^H$  and  $A^H$  is the steady state value of the productivity parameter.

From Equations (36) and (49) we still have that private consumption is constant over time ( $c_t = c_0$ ). Then, using the household budget constraint we can pin down the level of  $c_0$  following the shock as:

$$c_0 = rk_0^p + A_H(1 - \tau) - [A_H - A_L](1 - \tau)[1 - (1 + r)^{-T}] \quad (57)$$

where the last term denotes the welfare loss in terms of private consumption associated to the productivity shock.

As before, we know that the path of government expenditure is increasing over time if and only if the borrowing constraint binds. Proposition 1 has shown that for any period in which output is constant the government expenditure must also be constant since the IC constraint does not bind. This implies that  $g_t$  is constant both for all  $t > T$  and for all  $t \leq T$ , since output is constant in these two periods. ( $y_t = A_L$  for  $t \leq T$  and  $y_t = A_H$  for  $t > T$ ). Define  $g_1$  as the expenditure level for all  $t \leq T$  and  $g_2$  for all  $t > T$ . From the government budget constraint it follows that the path of government consumption for  $t > T$  is given by:

$$g_2 = rk_{T+1}^g + \tau A^H + d(I^g)[I^g] \quad (58)$$

where  $k_{T+1}^g$  is the government net external position at time  $T + 1$ ; and  $d(I^g) = d(I^{g*})$  for the credible policy case and  $d(I^g) = 0$  otherwise. Since government expenditure is also constant for  $t \leq T$ , it follows that for  $t \leq T$  the government budget constraint is given by:

$$k_t^g = (1 + r)k_{t-1}^g - g_1 + \tau A_L + dI^g \quad (59)$$

Solving this equation recursively yields:

$$k_{T+1}^g = k_0^g(1 + r)^{T+1} + [\tau A_L + dI^g - g_1] \left[ \frac{(1 + r)^{T+1} - 1}{r} \right] \quad (60)$$

And combining Equations (58) and (60), we can finally pin down the conditions under which the fiscal policy is procyclical ( $g_1 < g_2$ ).

**Proposition 3** *There exists a threshold value  $\underline{k}^g$  such that  $k_0^g < \underline{k}^g$  is a necessary and sufficient condition for government expenditure to be procyclical. The threshold value is given by:*

$$\underline{k}^g = -\frac{\theta + dI^g - \tau(A_H - A_L)[1 - (1+r)^{-T}]}{r} \quad (61)$$

**Proof.** See appendix. ■

This proposition simply states that, if the net external position of the government is too low (highly indebted), the government is unable to smooth public expenditure over time since this would require a large accumulation of debt, which is not incentive compatible.

As expected, the threshold value  $\underline{k}^g$  is decreasing in the cost of defaulting  $\theta$ , and increasing both in the magnitude of the output fall ( $A_H - A_L$ ) and the persistence of the shock ( $T$ )<sup>12</sup>. The former result follows from the fact that a larger cost of default works as a commitment device; the later result follows from the fact that the larger the output contraction and the more persistent the shock are, the more the government needs to borrow.

To pin down the actual path of government expenditure when the fiscal policy is procyclical, recall that this happens when the IC constraint binds at time  $t = T$ . This implies that:

$$g_2 = \tau A_H - \theta \quad (62)$$

Then, solving the government budget constraint yields:

$$g_1 = [\tau A_L + dI^g] + \frac{rk_0^g + (\theta + dI^g)(1+r)^{-T}}{[1 - (1+r)^T]} \quad (63)$$

from which it is clear that the path of  $g_t$  depends on the initial net foreign position of the government ( $k_0^g$ ). Notice however that  $k_0^g$  is endogenous since it depends on  $E_0$  when there exists nominal liabilities. Furthermore, notice that in steady state the government net foreign position must satisfy  $k_t^g = k_{t-1}^g + d_{t-1} \left[ \frac{E_t - E_{t-1}(1+\mu)}{E_t} \right]$  where the last term in brackets is equal to zero unless there is an unexpected shock that depreciates

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<sup>12</sup>  $\frac{dk^g}{d\theta} = -\frac{1}{r}$ ,  $\frac{dk^g}{d(A_H - A_L)} = \frac{\tau[1 - (1+r)^{-T}]}{r}$  and  $\frac{dk^g}{dT} = \frac{\tau(A_H - A_L)(1+r)^{-T} \ln(1+r)}{r}$

(or appreciates) the domestic currency. Then, using the definition of  $\rho$  this Equation simplifies to:

$$k_t^g = k_{t-1}^g \left[ 1 - \rho_{t-1} \left( 1 - \frac{E_{t-1}(1 + \mu)}{E_t} \right) \right] \quad (64)$$

If there is no unexpected shock the exchange rate follows  $E_t = E_{t-1}(1 + \mu)$  and therefore the net foreign position of the government remains constant. However, in the event of an unexpected shock that induces a consumption fall, the exchange rate would depreciate on impact inducing a change in the value of government nominal liabilities.

From the cash in advance constraint (9) we know that  $\alpha c_t E_t = M_{t-1}$  which implies that:

$$\frac{E_{t-1}(1 + \mu_t)}{E_t} = \frac{c_t}{c_{t-1}} \quad (65)$$

Combining Equations (64) and (65) we can express the government net external position following a shock as:

$$k_0^g = k_{ss}^g \left[ 1 - \rho_{ss} \left( 1 - \frac{c_0}{c_{ss}} \right) \right] \quad (66)$$

where  $ss$  denotes the steady state value, and  $c_{ss}$  and  $c_0$  are given by Equations (54) and (57), respectively.

Finally, combining the result of Proposition 3 and the last equation, we can find conditions under which the fiscal policy is procyclical in terms of the (steady state) currency composition of public debt.

**Proposition 4** *A necessary and sufficient condition for the fiscal policy to be procyclical is:  $\rho_{ss} < \left( \frac{k_{ss}^g - k^g}{k_{ss}^g} \right) \left( \frac{c_{ss}}{c_{ss} - c_0} \right)$ .*

**Proof.** See appendix. ■

That is, if the share of nominal debt in total public debt is sufficiently small the fiscal policy is procyclical. Notice however that  $k^g$  and  $c_0$  are functions of  $A_L$ , that is they depend on the magnitude of the shock. The next corollary characterizes the fiscal policy for different shocks, distinguishing between the cases of commitment and no-commitment.

**Corollary 5** *There exist threshold values  $\underline{A}_L$  and  $\bar{A}_L$  such that: i) if  $A_L > \bar{A}_L$  the path of  $g_t$  is constant no matter how the monetary authority behaves; ii) if  $A_L < \underline{A}_L$  fiscal policy is procyclical no matter how the monetary authority behaves; iii) if  $\underline{A}_L < A_L < \bar{A}_L$  fiscal policy is acyclical if the monetary authority can commit to price stability procyclical otherwise.*

The threshold values solve:

$$\rho^* = \left( \frac{k_{ss}^g - k^g(\underline{A}_L)}{k_{ss}^g} \right) \left( \frac{c_{ss}}{c_{ss} - c_0(\underline{A}_L)} \right) \quad (67)$$

and

$$k_{ss}^g = k^g(\bar{A}_L) \quad (68)$$

This corollary states the central point of the paper: there exists a range of values for the productivity shock such that the currency composition of public debt determines whether the fiscal policy is procyclical or not. As such, a government with a monetary authority that cannot commit to a certain policy is forced to undertake a procyclical fiscal policy while a government with a committed monetary authority is able to perfectly smooth government expenditure over time (fiscal policy a la Barro (1979)).

## 6 Concluding Remarks

The paper has developed a simple monetary model of an small open economy with endogenous borrowing constraints, where the set of available debt instruments to the government is affected by the behavior of the monetary authority. As a result, monetary policy influences the real value of public debt, and its response to different shocks, therefore limiting the scope for counter-cyclical fiscal policy.

The model helps to bring together two well-known stylized facts which underscore the differences between mature and developing economies with respect to the currency-structure of public debt (“original sin”) and the cyclical properties of fiscal policy (procyclicality of fiscal policy in emerging countries). It suggests therefore that any effort to improve cyclical aspects of fiscal policy in emerging markets could largely benefit from strengthening monetary institutions toward committing to price stability. Similarly, it suggests that, as monetary institutions in EMEs improve and “original sin” fades away, the cyclical properties in these economies are likely to become more similar to those observed in mature economies.

**Table 1. Correlation of Government Expenditure and Real GDP**

| Countries          | Central Government |                             |                                   |                                   | General Government Expenditure |
|--------------------|--------------------|-----------------------------|-----------------------------------|-----------------------------------|--------------------------------|
|                    | Expenditure        | Expenditure minus Interests | Expenditure on Goods and Services | Expenditure on Wages and Salaries |                                |
|                    | HP Filter          |                             |                                   |                                   |                                |
| OECD               | -0.13              | -0.05                       | -0.06                             | -0.15*                            | -0.06                          |
| Middle-High Income | 0.38*              | 0.10                        | 0.08                              | 0.01                              | 0.43*                          |
| Middle-Low Income  | 0.22*              | 0.13                        | 0.07                              | 0.03                              | 0.20*                          |
| Low Income         | 0.38*              | 0.24*                       | 0.54*                             | 0.59*                             | 0.37*                          |
|                    | Band-Pass Filter   |                             |                                   |                                   |                                |
| OECD               | -0.05              | -0.15*                      | -0.11                             | -0.20*                            | -0.02                          |
| Middle-High Income | 0.53*              | 0.19*                       | 0.23*                             | 0.13                              | 0.44*                          |
| Middle-Low Income  | 0.29*              | 0.29*                       | 0.26*                             | 0.23*                             | 0.23*                          |
| Low Income         | 0.46*              | 0.42*                       | 0.53*                             | 0.59*                             | 0.34*                          |

Source: Kaminsky, Reinhart and Vegh (2004).

\* Indicates statistical significance at 10 percent level.



## Appendix A

**Proof.** [Proposition1 ] We will prove that the IC constraint cannot bind if output follows a constant path ( $A_t = A$ ) by contradiction. Assume the constraint binds for any two consecutive periods  $t$  and  $t + 1$ . Notice first that from Equations (36) (or (49)) we know that private consumption  $c_t$  follows a constant path. Furthermore, solving Equation (27) we can solve for the optimal consumption path under repayment:

$$c_t^R = rk_0^p + r \sum_{t=0}^{\infty} \frac{[A_t(1 - \tau)]}{(1 + r)^t} \quad (69)$$

which is exactly equal to the consumption level under default, since we have assumed that  $\tau$  is constant and a default does not affect private agents access to capital markets. It follows therefore that:

$$\sum_{j=0}^{\infty} \frac{u(c_{t+j}^R)}{(1 + r)^j} = \sum_{j=0}^{\infty} \frac{u(c_{t+j}^D)}{(1 + r)^j} \quad (70)$$

and thus the IC constraint can be written as:

$$\sum_{j=0}^{\infty} \frac{v(g_{t+j}^R)}{(1 + r)^j} \geq \sum_{j=0}^{\infty} \frac{v(g_{t+j}^D)}{(1 + r)^j} \quad (71)$$

If the constraint binds at  $t$  and  $t + 1$ , then Equation (71) is satisfied with equality in both periods. Then, by simple manipulation we can show that:

$$v(g_t^R) = \sum_{j=0}^{\infty} \frac{v(g_{t+j}^D)}{(1 + r)^j} - \frac{1}{(1 + r)} \sum_{j=0}^{\infty} \frac{v(g_{t+1+j}^D)}{(1 + r)^j} \quad (72)$$

On the other hand, we know that under default government expenditure must satisfy  $g_t = \tau A^H + dI^g - \theta$  every period. Then:

$$\sum_{j=0}^{\infty} \frac{v(g_{t+j}^D)}{(1 + r)^j} = \sum_{j=0}^{\infty} \frac{v(\tau A_t + dI^g - \theta)}{(1 + r)^j} \quad (73)$$

Then, we can show that the RHS of Equation (72) is equal to  $\frac{v(\tau A_t + dI^g - \theta)}{(1 + r)}$ . It follows therefore that:

$$v(g_t^R) = \frac{[\tau A_t + dI^g - \theta]}{(1 + r)} \quad (74)$$

which is constant over time as long as output ( $A_t$ ) is constant over time. This implies

that  $\mu_t = 0$  as follows from Equation (37), which violates our initial assumption that the IC constraint binds in these two periods. ■

**Proof.** [Proposition 2] The proof of this proposition is straight forward. We have shown before that for any constant path of output the IC constraint cannot bind. Therefore, from Equations (36) and (49) follows that government expenditure is constant over time. Furthermore, using the fact that  $g_t$  is constant overtime and private consumption is equal both under repayment and under default (as shown in the previous proof), we can write the IC constraint as:

$$\frac{v(g^R)}{r} \geq \frac{v(g^D)}{r} \quad (75)$$

We know that  $g^D = \tau A - \theta$  and  $g^R = rk_0^g + \tau A + dI^g$ . It follows therefore that the IC constraint can be written as:

$$k_0^g \geq -\frac{(\theta + dI^g)}{r} \quad (76)$$

from which it is clear that the IC constraint is equivalent to an upper bound on public debt (lower bound on net position). It follows that, if  $k_0^g > -\frac{\theta}{r}$  the constraint does not bind for any of the two cases. This in fact ensures that there is no default in stationary equilibrium.

If this condition holds, then the IC constraint holds and the government is able to borrow, which implies that  $d$  can be positive if the monetary authority can commit to a monetary policy. Finally, from Equations (38) and (51) it is clear that  $d(I_t^g) = d(I_t^{g*})$  for the case of the credible monetary authority and  $d(I_t^g) = 0$  otherwise. ■

**Proof.** [Proposition 3] It was shown before that government expenditure is constant for  $t > T$  and for  $t < T$ . Call  $g_1$  the value of government expenditure for  $t < T$  and  $g_2$  for  $t > T$ . Then, government expenditure is procyclical if  $g_1 < g_2$ . From the Ramsey problem first order conditions follows that,  $g_1 < g_2$  if and only if  $\mu_T > 0$ . So  $g_1 < g_2$  if and only if:

$$\sum_{j=0}^{\infty} \frac{v(g_{T+1+j}^R)}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{v(g_{T+1+j}^D)}{(1+r)^j} \quad (77)$$

Using Equations (58) and (21) we can rewrite this equation as:

$$\frac{v(rk_{T+1}^g + \tau A_H + dI^g)}{r} \geq \frac{v(\tau A_H - \theta)}{r} \quad (78)$$

which holds if and only if:

$$k_{T+1}^g \geq -\frac{(\theta + dI^g)}{r} \quad (79)$$

Notice that if  $g_1 = g_2 = g$ , then from the government budget constraint we know that the level of government expenditure is given by:

$$g = rk_0^g + dI^g + \sum_{t=0}^{\infty} \frac{\tau A_t}{(1+r)^t} \quad (80)$$

$$= rk_0^g + dI^g + \tau A_L [1 - (1+r)^{-T}] + \tau A_H [(1+r)^{-T}] \quad (81)$$

Substituting into Equation (59) yields:

$$k_{T+1}^g = k_0^g + [\tau A_L - \tau A_H] [1 - (1+r)^{-T}] / r \quad (82)$$

Then, combining Equations (79) and (82) yields:

$$k_0^g \geq - \frac{(\theta + dI^g) + [\tau A_L - \tau A_H] [1 - (1+r)^{-T}]}{r} = \underline{k}^g \quad (83)$$

Then  $k_0^g < \underline{k}^g$  is a sufficient condition for the IC constraint to bind at time  $T + 1$  and induce a procyclical fiscal policy ( $g_1 < g_2$ ). This is also a necessary condition since otherwise  $g_1 = g_2 = g$  satisfy the first order conditions and the IC constraint. ■

**Proof.** [Proposition 4] This proof is straight forward. We have shown that  $k_0^g < \underline{k}^g$  is a necessary and sufficient condition for the fiscal policy to be procyclical. Using Equation (66) we can express this condition as:

$$\underline{k}^g > k_{ss}^g \left[ 1 - \rho_{ss} \left( 1 - \frac{c_0}{c_{ss}} \right) \right] \quad (84)$$

which can be re written as:

$$\rho_{ss} < \left( \frac{k_{ss}^g - \underline{k}^g}{k_{ss}^g} \right) \left( \frac{c_{ss}}{c_{ss} - c_0} \right) \quad (85)$$

which completes the proof. ■

**Proof.** [Corollary 5.] To prove this we simply need to show that the RHS of the inequality of Proposition 4 is decreasing in  $A_L$ . Notice that differentiating this expression with respect to  $A_L$  yields:

$$\frac{dRHS}{dA_L} = - \left( \frac{c_{ss}}{c_{ss} - c_0} \right) \frac{1}{k_{ss}^g} \frac{dk_{ss}^g}{dA_L} + \left( \frac{k_{ss}^g - \underline{k}^g}{k_{ss}^g} \right) \frac{c_{ss}}{(c_{ss} - c_0)^2} \frac{dc_0}{dA_L} \quad (86)$$

Then using Equations (54) and (57) we can simplify this equation to:

$$\frac{dRHS}{dA_L} = \frac{(1 - \tau)}{rk_{ss}^g} \frac{c_{ss}}{(c_{ss} - c_0)^2} [1 - (1 + r)^{-T}] [rk_{ss}^g + \theta + dI^g] \quad (87)$$

which is negative since we have assumed that  $0 > rk_{ss}^g > -\theta$  and we know that  $dI^g \geq 0$ . ■

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