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Life Expectancy and Income Convergence in the World: A Dynamic General Equilibrium Analysis

Kenichi Ueda

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Life Expectancy and Income Convergence in the World: A Dynamic General Equilibrium Analysis

Prepared by Kenichi Ueda[1]

Authorized for distribution by Stijn Claessens

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Abstract

There is world-wide convergence in life expectancy, despite little convergence in GDP per capita. If one values longer life much more than material happiness, the world living standards may this have already converged substantially. This paper introduces the concept of the dynastic general equilibrium value of life to measure welfare gains from the increase in life expectancy. A calibration study finds sizable welfare gains, but these gains hardly mitigate the large inequality among countries. A conventional GDP-based measure remains a good approximation for (non) convergence in world living standards, even when adjusted for changes in life expectancy.

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Author's E-Mail Address: kueda@imf.org

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I. INTRODUCTION

Although per capita GDPs of developing countries have not converged much toward those of advanced economies, life expectancy in developing countries has improved to a comparable level, as shown in Figure 1.[2] This challenges one of the biggest puzzles in the economic growth literature, that is, the lack of convergence in world living standards. Indeed, Becker, Phillipson, and Soares (2006, BPS hereafter) point out that the substantial convergence has already occurred in the longevity-corrected income. However, they compute an increase in the value of life based on one-generation, partial-equilibrium concept, using estimates in the existing labor literature (e.g., Viscusi and Aldy, 2003). This concept is theoretically inconsistent with the typical growth theory, upon which the non-convergence puzzle relies.

I introduce a coherent concept of welfare, the dynastic general equilibrium value of life, to evaluate the welfare gains from increase in life expectancy in the context of economic growth process. This value of life concept is different from the one a subliterature of labor economics has been working on. In the existing literature, the value of life is based on a partial equilibrium concept: a willingness to accept a higher wage for a marginal increase in the fatality rate.[3] This calculation does not take into account the general equilibrium effects of possible economy-wide acceleration in accumulation of physical and human capital that results from population-wide extension of longevity. Thus, the general equilibrium value may well be larger than the partial equilibrium value.[4] However, a dynastic consideration may lower the value of life, since the existing literature does not take into account the substitutability of descendants for the current generation from the dynastic point of view.[5]

Although it is impossible to discuss longevity with a standard assumption of infinitely lived households in a neoclassical growth theory, Barro (1974) and Becker and Barro (1988) provide a theoretical foundation for this commonly used assumption and I follow their treatments: parents have altruism towards their children. To focus on the longevity issue, the fertility choice is given and population is assumed to be stationary. As such, a dynastic model with generational changes becomes equivalent to a model with infinitely lived households.

In the next section, after I set up the model, I prove first that the dynastic general equilibrium value of life is zero under a canonical neoclassical growth theory, which is based on the

[2]Life expectancy of the middle-income countries has increased about 24 years (about 46 to 70) over the 1960-2004 period, but only about 10 years (69 to 79) for the advanced economies. Partly due to the AIDS epidemic, low-income countries experience less convergence, but still they achieved about 16 years' (43 to 59) increase in life expectancy. See Deaton (2006) for detailed analysis and a literature review.

[3]Specifically, it is the coefficient on the fatality rate in a regression of wages of various occupations after controlling for key characteristics of workers and jobs such as age, gender, and so forth (e.g., wage difference between miners versus waiters at fast food restaurants).

[4]There are several other general equilibrium effects (for example, effects on population dynamics) that determine availability of per capita physical capital. I will revisit these issues later.

[5]Rosen (1988) points out this as a potential problem for his calculation of the value of life essentially based on an one-generation model.

Barro-Becker assumption on altruism and the standard laws of motion of physical and human capital. This result should not be surprising, since the Barro-Becker assumption makes the future generation's utility a perfect substitute for the current generation's.

Second, I prove that, with a slightly more realistic assumption, the dynastic general equilibrium value of life is positive and sizable, much larger than, for example, typical welfare gains from eliminating business cycle. Here, although I am not unaware of debates about altruism, I keep the Barro-Becker assumption (perfect altruism) and focus rather on human capital accumulation. Specifically, I assume that the depreciation of human capital over generations is much larger than within the same person. In this formulation, unlike in the existing literature, the value of life stems not from extra utilities obtained in additional years, but from the economization of depreciation by less frequent changes of generations over time within a dynasty.[6]

In Section III, I report quantitative assessment. I compute the dynastic general equilibrium value of life with key parameter values calibrated to actual economic growth and life insurance coverage. Then, I construct the full income, corrected for increase in the dynastic general equilibrium value of life, for 96 countries for 1990 and 2000. The dynastic general equilibrium value of life turns out sizable and makes the full income show convergence better than the GDP per capita. However, the gains in convergence are small, less than half of those BPS report, implying that the GDP-based measure is a good approximation of the true nature of the world income inequality.

In Section IV, I prove that a more general model with imperfect altruism does not alter any of the results based on perfect altruism. This is because a slight change in discount rates over generations does not affect the main economic mechanism, that is, the current generation optimizes the consumption sequence over the descendants given life expectancy. Section V concludes.

II. Model and Qualitative Results

A. Technology and Preference

There is a continuum of dynasty with measure one. In any period, a dynasty consists of only one individual. In other words, there is no overlap of generations within a dynasty.[7] Death is

[6]Of course, there are many other reasons to live longer and I will discuss potential alternative models throughout the paper. However, I try to keep the model as simple as possible to focus on presenting the concept of the dynastic general equilibrium value of life, as well as computing the value to compare with the existing, partial equilibrium value.

[7]An alternative interpretation is that children do overlap with adults but when children become adults, their parents die, and only adults' utilities matter for a dynasty. In this interpretation, I still need to assume the same length of overlapping periods. Although this assumption might not be realistic, it captures the real world trend: As the life expectancy becomes longer, the mother's age at having the first child becomes older. The optimal length of overlapping period could be analyzed in terms of trade-off between the utility gains through spending

modeled here as deterministic for the sake of simplicity, but the case with stochastic death can be analyzed in the same way and discussed extensively.

The production function is Cobb-Douglas with physical capital $k_t \in \mathbb{R}_+$ and human capital $h_t \in \mathbb{R}_+$:

$$y_t = f(k_t, h_t) = Ak_t^{\alpha} h_t^{1-\alpha}, \tag{1}$$

where $\alpha \in (0, 1)$ denotes the capital share.

Physical capital k_t evolves as

$$k_{t+1} = (1 - \delta_k)k_t + i_{kt}, \tag{2}$$

where $\delta_k \in (0, 1)$ is the depreciation rate and $i_{kt} \in \mathbb{R}$ is the investment.[8] When a parent dies, her child inherits the physical capital with the same depreciation δ_k.

Human capital h_t follows a similar law of motion, but depreciation over generations, denoted by $\delta_o \in (0, 1)$, may be larger than depreciation within a single person, denoted by $\delta_w \in (0, 1)$. In sum, the law of motion of human capital is

$$\begin{aligned} h_{t+1} &= (1 - \delta_w)h_t + i_{ht}, \quad \text{for } t \neq nT \text{ and;} \\ &= (1 - \delta_o)h_t + i_{ht}, \quad \text{for } t = nT, \end{aligned} \tag{3}$$

where n is a positive integer representing the n-th generation of a dynasty and $i_{ht} \in \mathbb{R}_+$ is the investment in human capital.

I assume a perfect life insurance market in which an individual pays a premium $\pi_t b_t$, with $\pi_t \in \mathbb{R}_+$ for $b_t \in \mathbb{R}$ benefits for her child in case she dies.[9] The budget constraint is thus,

$$\begin{aligned} c_t + i_{kt} + i_{ht} &= r_t k_t + w_t h_t - \pi_t b_t, \quad \text{for } t \neq nT \text{ and;} \\ c_t + i_{kt} + i_{ht} &= r_t k_t + w_t h_t + b_t, \quad \text{for } t = nT. \end{aligned} \tag{4}$$

where $r_t \in \mathbb{R}_+$ is a rental rate of physical capital and $w_t \in \mathbb{R}_+$ is the wage rate.

The life expectancy is denoted by T. A person discounts her own future utility by β and has per annum altruism γ towards her descendants. Given the budget constraint (4), a household

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time with children and the cost of sharing the same income and time with children. However, this extended model would not be likely to affect the main findings of this paper.

[8]For the sake of technical simplicity, I assume that physical capital can be transformed back to consumption goods freely and used to invest in the human capital. Later, I will prove that there is an optimal human-to-physical-capital ratio, and this assumption makes it possible for the optimal ratio to be always achieved.

[9]Typically, benefits are positive, but in the case with little altruism, they may be negative. This can happen when the parent generation consumes more than their lifetime income, for example, by issuing government bonds. In this sense, this paper is a natural extension of the Barro (1974) paper on the analysis of government bonds and creation of wealth.

maximizes the following dynastic utility:

$$\max_{\{c_t\}_{t=1}^{t=\infty}} u(c_1) + \beta u(c_2) + \cdots + \beta^T u(c_T) + \gamma^{T+1} u(c_{T+1}) + \gamma^{T+1} \beta u(c_{T+2}) + \cdots , \tag{5}$$

where $u : \mathbb{R}_+ \to \mathbb{R}_+$ is the period utility function with standard properties, $u' > 0$, $u'' < 0$, $\lim_{c\to 0} u'(c) = \infty$, and $\lim_{c\to\infty} u'(c) = 0$.

The dynastic general equilibrium value of life can be measured only when there is a change in life expectancy. The increase in the dynastical general equilibrium value of life is defined as the difference between the dynasty's utility (5) under a specific life expectancy T and its utility under an improved life expectancy. Moreover, I will define it in a parametric form later with more detailed discussions.

B. Representative Agent

To be consistent with a standard growth model, I assume that an individual's subjective discount rate is the same as her altruism parameter. This assumption follows Barro (1974) and an exogenous fertility case of Becker and Barro (1988).

Assumption 1. *[Barro-Becker, Perfect Altruism]*

$$\gamma = \beta.$$

Assumption 1 states that a person cares for her descendants' utility as much as her own future utility. Obviously, under this assumption, a household maximizes the following dynastic utility:

$$\max_{\{c_t\}_{t=1}^{t=\infty}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t). \tag{6}$$

The only possible drop of income within a dynasty occurs when a parent dies and human capital depreciates at rate δ_o, larger than the usual rate δ_w. Using a life insurance scheme, it is easy to smooth out the human capital evolution without affecting the budget constraints. Specifically, the benefit, net of the premium, should be set to compensate the extra depreciation resulting from the death of a parent,

$$b_t = (\delta_o - \delta_w) h_t - \pi_t b_t. \tag{7}$$

The competitive market of this life insurance makes the premium actuarially fair. Let $\rho_t \in [0, 1]$ represent the replacement ratio of the population, the portion of dynasties that change generations. Then the actuarially fair condition can be derived to equate the expected benefits and the expected premiums,

$$\rho_t b_t = (1 - \rho_t) \pi_t b_t, \tag{8}$$

or equivalently,

$$\pi_t = \frac{\rho_t}{1-\rho_t}. \tag{9}$$

By combining these two equations (7) and (9), the benefits can be expressed solely in terms of exogenous parameter values and the state variables,

$$b_t = (1-\rho_t)(\delta_o - \delta_w)h_t, \tag{10}$$

and

$$\pi_t b_t = \rho_t(\delta_o - \delta_w)h_t. \tag{11}$$

With this life insurance scheme, the inter-generational human capital transmission becomes the same as the intra-generational human capital formation, enabling a dynasty to keep the same levels of consumption and human and physical capital investment in any period. The economy can then be regarded as a representative agent economy. In aggregate, the insurance premium and benefits are canceled out each other in the representative agent's budget constraint. The aggregate human capital evolves simply as

$$H_{t+1} = (1-(1-\rho_t)\delta_w - \rho_t\delta_o)H_t + I_{ht}, \tag{12}$$

where aggregate variables are denoted by capital letters.

In general, ρ_t is a function of size distribution of age groups as well as potentially different life expectancy for each age group. However, for the sake of simplicity, I assume a stationary environment in which longevity is the same for any age cohort, the initial physical and human capital are equal across all households, and the initial population within each age cohort is identical. Thus, the replacement ratio is always the same, $\rho_t = 1/T$.

Note that the stochastic death can be modeled in the same manner and I use stochastic interpretation interchangeably. Here, to be equivalent with deterministic death case, I assume a stochastically stationary environment. Specifically, I assume that ρ_t is the same death probability for everyone who is alive in period t; that the younger generation automatically replaces the older upon its death; and that the size of each age cohort is the same in the initial period. These assumptions imply that the same distribution over age groups is preserved for all periods.

C. Neutrality of Longevity under Neoclassical Assumptions

I would like to point out first that the dynastic general equilibrium value of life in a standard neoclassical growth model is zero. In this regard, from the viewpoint of a standard growth theory, it appears unwise to correct the GDP numbers by adjusting for any value from the increase in life expectancy. This, of course, does not imply that, in a different growth model, which I will explore later, it makes sense to take into account the increase in life expectancy in evaluating living standards. The point here is that the value of life in the context of economic growth should be analyzed in a manner consistent with an underlying growth model.

By a standard growth theory, I mean two assumptions: Assumption 1 (perfect altruism) and

Assumption 2. *[Smooth Human Capital Transmission]*

$$\delta_o = \delta_w.$$

Assumption 2 states that a human capital transfer between a parent and a child is the same as the law of motion within the same person.

Theorem 1. *[Neutrality of Longevity] Under the standard neoclassical growth assumptions 1 and 2, the dynastic general equilibrium value of life is zero.*

Proof. Under Assumption 1, a parent's utility is perfectly substitutable by a child's utility. Under Assumption 2, there is no loss in having frequent replacement of generations within a dynasty. For example, consider one economy in which all dynasties replace generations every 30 years and another in which all dynasties replace generations every 60 years. In both economies, given the same initial physical and human capital, the dynastic value is the same. *Q.E.D.*

Note that, because the proof runs exactly the same, this proposition is valid with any other typical production functions, for example, a Cobb-Douglas production function with decreasing returns to scale and a more general production function with a constant elasticity of substitution between physical and human capital.

D. Positive Value of Life with Costly Human Capital Transfer

Now, I would like to consider a model that is a little more realistic than the standard growth theory, but I will keep the model as simple as possible to introduce a new concept of value of life. Specifically, I question the validity of Assumption 2, the same depreciation rates of human capital within the same person and across generations. In reality, human capital transmission from a parent to a child is more costly than a memory loss within the same person.[10] I am also not unaware of the debate on Assumption 1, the degree of parental altruism (see, for example, Altonji, Hayashi, and Kotlikoff, 1997), but I will keep the

[10]For the sake of simplicity, I assume here that the new adult's human capital is a linear function of the parents'. The best way to model this deeper would be that everyone is assumed to be endowed with the same basic human capital. Then, education investment in childhood determines the general human capital for a new adult, who then continues to acquire more specialized human capital in college and on the job. However, the children's human capital level would have a positive association with the parents', as long as the education costs must be covered, at least partially, by parents' income, which depends on their human capital levels. As such, implication of my model would not differ much from a more sophisticated model in terms of the growth process and the welfare implication.

Barro-Becker assumption (perfect altruism) for now, as it serves as a foundation for many growth models with infinite lived households.[11] In sum, I alter only the assumption on human capital depreciation as follows.

Assumption 3. *[Costly Human Capital Transfer]* $\delta_o > \delta_w$.

Proposition 1 (Non-Neutrality of Longevity)**.** *Under Assumptions 1 and 3, the dynastic general equilibrium value of life is strictly positive.*

Proof. In any period when generation changes, human capital depreciates larger than usual. This makes insurance premiums higher and human capital accumulation becomes slower, as easily seen in equation (12). *Q.E.D.*

This proposition also holds with any typical production functions, as is the case with Theorem 1. More importantly, I would like to point out that the whole source of the value of life in this paper stems from economizing depreciation cost when transferring human capital over generations, in contrast with a conventional direct effect of longevity on the value of life by extra consumption in extended years of living (Rosen, 1988). This is because utility of the current generation is substitutable with utility of the future generation in a model with dynastic altruism. Still, the dynastic general equilibrium value of life can be large, as the extended longevity could bring faster accumulation of physical and human capital.

III. Quantitative Assessment

A. Computable Form

While Proposition 1 assures a positive value of life qualitatively, I will show here a quantitative assessment. Before doing so, I will rewrite the model in a computable form.

Let δ_h denote the composite depreciation rate, that is,

$$\delta_h \equiv (1-\rho_t)\delta_w + \rho_t\delta_o. \tag{13}$$

Obviously, lower longevity (i.e., higher replacement of generations) implies higher depreciation. Using this, the law of motion of human capital for a representative agent is expressed as

$$H_{t+1} = (1-\delta_h)H_t + I_{ht}. \tag{14}$$

Note that the law of motion of physical capital is the same as for the individual level (2), that is, for the representative agent,

$$K_{t+1} = (1-\delta_k)K_t + I_{kt}. \tag{15}$$

[11] The case with imperfect altruism will be discussed later, in Section IV.

Under the Barro-Becker assumption, the representative consumer's maximization problem can be rewritten as a dynamic programming problem using the value function, $W : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$, omitting the time subscripts but with superscript $+$ to denote the value of the next period:

$$W(K,H) = \max_{I_k, I_h, c, K^+, H^+} u(c) + \beta W(K^+, H^+), \tag{16}$$

subject to the resource constraint,

$$c + I_k + I_h = AK^\alpha H^{1-\alpha}, \tag{17}$$

and the law of motion of physical capital, equation (15), and that of human capital, equation (14).

The optimal human-to-physical-capital ratio H/K is uniquely determined by equating the marginal returns of human and physical capital net of depreciation (see Appendix I. A for the derivation):

$$(1-\alpha)A\left(\frac{H}{K}\right)^{-\alpha} - \delta_h = \alpha A \left(\frac{H}{K}\right)^{1-\alpha} - \delta_k \tag{18}$$

or equivalently,

$$(1-\alpha)A - \alpha A \frac{H}{K} = (\delta_h - \delta_k)\left(\frac{H}{K}\right)^\alpha. \tag{19}$$

The right-hand side of this equation is linearly decreasing with the human-to-physical-capital ratio, H/K, from the positive intercept $(1 - \alpha A)$, while the left-hand side is monotonically increasing with the ratio H/K from 0. It is easy to see that the equilibrium human-to-physical-capital ratio, which I denote by x, is decreasing with δ_h (i.e., $\partial x/\partial \delta_h < 0$), that is, increasing with longevity (i.e., $\partial x/\partial T > 0$). At the limit, if $\delta_h \rightarrow \delta_k$, then $x \rightarrow (1-\alpha)/\alpha$, which is a widely known ratio.

For the sake of simplicity, I assume that the economy always achieves this optimal human-to-physical-capital ratio x. Indeed, as long as the adjustment does not require the reduction of capital more than depreciation, households make this adjustment instantly and achieve the optimal ratio (see, for example, McGrattan, 1998). Given $H = xK$, I can rewrite the system of equations as a one-capital model, which is easier to analyze.

In each period, a specific amount of human capital should be invested for one unit of physical capital to keep the same optimal ratio x. Specifically, I_h must satisfy

$$xK^+ = (1-\delta_h)xK + I_h. \tag{20}$$

Using the law of motion of physical capital (15), the optimal human capital investment is identified as

$$I_h = xI_k + x(\delta_h - \delta_k)K. \tag{21}$$

Thus, the feasibility constraint that keeps the optimal human-to-physical-capital ratio is expressed as

$$c + (1+x)I_k + x(\delta_h - \delta_k)K = Ax^{1-\alpha}K. \tag{22}$$

A representative household now solves the one-dimensional value function, $V : \mathbb{R}_+ \to \mathbb{R}_+$,

$$V(K) = \max_{I_k, c, K^+} u(c) + \beta V(K^+), \tag{23}$$

subject to the budget constraint (22) and the law of motion of physical capital (15).

Using a constant relative risk aversion (CRRA) utility function, $c^{1-\sigma}/(1-\sigma)$, with risk aversion parameter $\sigma \in \mathbb{R}_{++}$, the optimal consumption growth can be represented as the following Euler equation (see Appendix I. B for the derivation):

$$\frac{(u^-)'}{u'} = \left(\frac{c}{c^-}\right)^\sigma = \beta\left(A\frac{x^{1-\alpha}}{1+x} - \frac{x}{1+x}(\delta_h - \delta_k) + (1-\delta_k)\right) = \beta G(x, \delta_h). \tag{24}$$

For the sake of simplicity, I assume that the ranges of parameter values ensure the perpetual consumption growth, $\beta G(x, \delta_h) > 1$. Also, to keep the value from exploding to ∞, I restrict my attention to the parameter values that satisfy $\beta(\beta G(x, \delta_h))^{1/\sigma} < 1$.[12]

B. Benchmark Parameter Values

Quantitative assessment is shown under specific benchmark parameter values, which are summarized in Table 1. Many are standard parameters in the business cycle literature and I use typical values for them. Namely, the discount rate $\beta = 0.96$, the relative risk aversion $\sigma = 1.2$, the capital share $\alpha = 1/3$, and the physical capital depreciation rate $\delta = 0.05$.

Other parameters are specific to this paper. Namely, the total factor productivity $A = 0.25$, the human capital depreciation rate of a single person $\delta_w = 0.02$, and that between parents and children $\delta_o = 0.7$. I pick those values by calibrating the model to match the reasonable range of consumption growth with the historical value for the long-run U.S. growth experience, which is about 2 percent. As shown in the third row of Table 2, the growth rates are about this target level.

At the same time, the benchmark parameter values are also taken to be consistent with the life insurance coverage per annual income in the U.S. data, which is about 6 on average in the U.S., according to Hong and Ríos-Rull (2006).[13] In the model of this paper, the life insurance benefits b_t per income y_t depend on the probability of death ρ_t. As $H_t = xK_t = x^\alpha Y_t/A$, the premium can be rewritten from equation (10) to the following:

$$\frac{b_t}{Y_t} = (1-\rho_t)(\delta_o - \delta_w)\frac{x_t^\alpha}{A}. \tag{25}$$

[12] These are standard assumptions in the growth literature. See, for example, Townsend and Ueda (2007).

[13] Based on 1990 data from Stanford Research Institute and 1992 data from the Survey of Consumer Finances, they estimate the face value of life insurance at about 4 to 8 for ages between 30 to 60. It is hump-shaped with the peak in early 40s.

In this formula, only the optimal human-to-physical-capital ratio, x, varies over time and across countries, but it is always about 12 under the benchmark parameter values (see the fourth row of Table 2). Thus, the benchmark parameter values provide for life insurance coverage about 6 times higher than the annual income for any country and year (see the second row of Table 2).[14]

C. Dynastic General Equilibrium Value of Life

Quantitatively, I define the dynastic general equilibrium value of life as the wealth transfer that compensates for a possible welfare increase resulting from a change in longevity at the steady state. Specifically, let $V_\xi(K)$ denote the value of the value function (i.e., the discounted sum of period utilities over time for a dynasty) with the state variable K under life expectancy $T = \xi$. Suppose some policies can increase the longevity from $T = \xi$ to $T = \tilde{\xi}$. Then, the percentage increase in the dynastic general equilibrium value of life τ is defined as

$$V_\xi(K(1+\tau)) = V_{\tilde{\xi}}(K). \tag{26}$$

Since the Euler equation (24) implies the linear savings function and the constant growth in consumption, as well as in physical and human capital, the value function can be expressed almost analytically, given a constant value of the optimal human-to-physical-capital ratio, x, which can be obtained numerically. Let s denote the equilibrium constant savings rate and g denote the equilibrium constant growth rate. Then the value function can be expressed as

$$\begin{aligned} V(K) &= u(c) + \beta u(gc) + \beta^2 u(g^2 c) + \cdots \\ &= u(c) + \beta g^{1-\sigma} u(c) + \beta^2 g^{2(1-\sigma)} u(c) + \cdots \\ &= \frac{1}{1-\beta g^{1-\sigma}} \frac{c^{1-\sigma}}{1-\sigma} \\ &= \frac{((1-s)Ax^{1-\alpha})^{1-\sigma}}{(1-\beta g^{1-\sigma})(1-\sigma)} K^{1-\sigma}. \end{aligned} \tag{27}$$

Note that the multiplier on $K^{1-\sigma}$ is constant, regardless of the level of capital. I let Ψ denote it:

$$V(K) = \Psi K^{1-\sigma}. \tag{28}$$

After numerically obtaining the coefficient Ψ_ξ for the specific longevity $T = \xi$ and $\Psi_{\tilde{\xi}}$ for $T = \tilde{\xi}$, I can calculate the ratio of the latter to the former, denoted by $1 + \triangle$. Then, the increase in the dynastic general equilibrium value of life τ can be obtained as follows:

$$\Psi_{\tilde{\xi}} K^{1-\sigma} = (\Psi_\xi(1+\triangle))K^{1-\sigma} = \Psi_\xi (K(1+\triangle)^{\frac{1}{1-\sigma}})^{1-\sigma}. \tag{29}$$

[14] Table 2 shows in the second and the fourth rows that the optimal human-to-physical-capital ratio x and life insurance benefits b depend on life expectancy but also that variations in x and b are almost negligible. For life expectancy 60, $x = 11.51$ and $\rho = 1/60$, and thus the equilibrium life insurance benefit, as a ratio to income, is $6.04 = (1 - 1/60)(0.7 - 0.02)(11.51^{1/3})/0.25$.

That is,

$$1 + \tau = (1 + \triangle)^{\frac{1}{1-\sigma}} \tag{30}$$

This wealth transfer τ is equivalent to the increase in annual output and thus permanent consumption, typically used as the measure of welfare gains in the business cycle literature, as initiated by Lucas (1987). This is because the model exhibits the endogenous linear growth by the assumption $g = \beta G(x, \delta_h) > 1$. With the constant growth rate, one-time transfer of wealth of τ percent always creates τ percent higher levels of wealth, income, and consumption thereafter.

The welfare gains from increase in life expectancy turn out to be sizable. The first row in Table 2 reports the wealth transfer τ, as the implied increase in dynastic general equilibrium value of life after 40. For example, if the representative agent's life expectancy increased from 40 to 60, it is equivalent to a 12.2 percent increase in permanent consumption. Apparently, the longer the life expectancy, the higher the welfare gain. However, the marginal gain becomes smaller as the life expectancy rises, which I discuss further later. Note that a mere 0.5 percent increase in permanent consumption is considered large in the business cycle literature as well as in a policy evaluation for economic development (e.g., Townsend and Ueda, 2007).

D. Sensitivity Analysis

To check sensitivity of the results, I compute the dynastic general equilibrium value of life with various parameter values (Table 3). A higher discount rate, $\beta = 0.99$, gives a higher value of life, because the future gains from infrequent changes of generations are valued more. But the consumption growth is too high, about 4.5 to 5.0 percent. Using similar reasoning, a lower intertemporal elasticity of substitution, $\sigma = 2$, gives a lower value of life. However, the consumption growth is now too low, about 1 to 1.5 percent. A higher depreciation rate within a generation, $\delta_w = 0.04$, brings a higher value of life by producing a consumption growth more sensitive to the increase in life expectancy. Again, the consumption growth is too low, less than 1 percent. A higher depreciation rate over generations, $\delta_o = 0.9$, gives a similar result, albeit with less impact. With this change, the consumption growth is within the reasonable range, but now the equilibrium life insurance benefits rise to about 7.5–8.0 times more than the annual income, not in line with the empirical estimates.

In summary, the choice of parameter values should be limited in the vicinity of the benchmark values, on which I focus below. The sensitivity analysis reveals that the dynastic general equilibrium value of life varies with key parameter values. But at the same time, slight changes in parameter values lead to substantial alterations in consumption growth and equilibrium life insurance, to which the model is calibrated.

E. Income Convergence

The first row of Table 2 reports the increase in the dynastic general equilibrium value of life under the benchmark parameter values but with marginal improvements decreasing. For a low-income country, in which the life expectancy increased from about 40 in 1960 to 60 in 2005 (triangles in Figure 1), the implied increase in value of life is as much as 12.2 percent of consumption level each year. For a high-income country, in which the life expectancy increased from about 70 in 1960 to 80 in 2005 (circles in Figure 1), the implied increase in the value of life is 2.5 percent (i.e., $1.1875/1.1591$).

The difference in the marginal increase in the value of life contributes to reduction of full income variation among countries, once the increase in the value of life is taken into account. This convergence effect varies substantially with parameter values. However, as noted above, the choice of parameter values is limited to the vicinity of the benchmark parameter values, to generate the growth rates at about 2 percent and the equilibrium life insurance benefits to income ratio at about 6.

I apply the benchmark case to the actual data for each country and compare the results with BPS. Sample countries (96 countries, comprising more than 82 percent of the world population) and methodology are the same as in BPS. Income per capita is from Penn World Table 6.1, namely, real GDP per capita in 1996 international prices, adjusted for terms of trade. Population is also from Penn World Table 6.1. Life expectancy at birth is from the World Development Indicators (WDI), World Bank. As for the inequality and convergence measures, I calculate standard measures following BPS. Namely, they are the relative mean deviation, the coefficient of variation, the standard deviation of log values, the Gini coefficient, and the regression to the mean. The regression to the mean is the coefficient of a regression of the change in the natural log of income over the period on the initial 1960 level. I calculate all the convergence measures in tables, except for those under *memorandum (BPS)*, which are replicated from BPS as a reference. Full income measures incorporate gains in life expectancy with 1960 as the base year.[15]

Note that in some of the tables not replicated here, BPS show nonlinear convergence effects depending on the initial levels and changes in life expectancy and income. This comes from the fact that the benefit of increase in longevity is essentially the direct gain in the form of the additional utility that results from extension of life. Based on a one-generation model, the calibration requires an appropriate choice of the absolute level of utility (Rosen, 1988). In contrast, the dynastic general equilibrium value of life proposed here is free from the affine transformation, as the benefit from the increase in longevity stems from a decline in the aggregate human capital depreciation rate.[16]

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[15]The convergence results using the real GDP per capita are slightly different between BPS and my calculation, although both BPS and I use the same data source and code *ainequal* in Stata. This is because the WDI, *ainequal* code, and Stata program I use are as of August 2007, updated since the publication of BPS—I owe Rodorigo Soares for this clarification.

[16]Indeed, $log(log(1+\tau))$ is almost linear. The wealth compensation transfer for increase in life expectancy from 30 is approximated by $-2.0719 + 0.5960 * LifeExpectancy$ using 5-year-interval ages between 30 and

Finally, I compute the overall effects on income convergence by accounting for changes in life expectancy based on the dynastic general equilibrium value of life introduced in this paper as well as those based on the partial equilibrium value reported in BPS. The overall effects are reported in columns under Improvements in Table 4. The specific formula is as follows:

$$1 - \frac{\text{measure using full income per capita}}{\text{measure using GDP per capita}}, \tag{31}$$

except for the regression-to-the-mean measure, for which the denominator and numerator are flipped to account for the fact that, unlike other measures, a higher absolute value means a better convergence.

Increase in life expectancy does not appear to substantially change the view of inequality in world living standards based on GDP per capita only. Improvements in convergence in full income is small when corrected for the increase in the dynastic general equilibrium value of life. Compared with the partial equilibrium correction by BPS, improvements in income convergence is less than half in most measures.

IV. Case with Imperfect Altruism

Now I consider a more general case in which altruism is imperfect. When people care about descendants less than about themselves, the dynastic general equilibrium value of life might be higher than the perfect altruism case. However, this is not the case, at least in the framework of this paper. Indeed, I will show below that all the main results hold for this case, although some technical difficulties emerge.

Suppose current generations care more about themselves than about their descendants. The altruistic parameter γ is now,

Assumption 4. $0 < \gamma < \beta$.

Proposition 2. *Under Assumption 4:*
(i) The consumption growth rate within a generation is g, the same as in the case with perfect altruism;
(ii) The consumption growth rate over a generation is ϕg, that is, $\phi \equiv (\gamma/\beta)^{1/\sigma}$ times lower than within a generation;
(iii) All households keep the same optimal ratio of human to physical capital x as in the case with perfect altruism;
(iv) Human and physical capitals grow at the same rate g, as consumption.

See the proof in Appendix III. More importantly, there is no need to calibrate this case, as shown in the next proposition.

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80 (i.e., $30, 35, 40, \cdots, 80$) as regressors. The fitted values are used to calculate the full income for all the sample countries. The errors predicting transfer are minimal: 1.5 percent at the lowest end, 30, and within 0.6 percent range for all other approximation nodes (i.e., $35, 40, 45, \cdots, 80$).

Because survivors' growth rate is different from that of the newly born, there will be nondegenerated distribution of capital, income, and consumption in any period. However, Proposition 2 shows that consumption growth for survivors is g for any level of capital holdings and it is ϕg for the newly born, again, at any level of capital holdings. As such, I can still write a representative agent problem using the aggregate consumption and capital to solve for the equilibrium sequence of aggregate consumption, income, and human and physical capital. Then, based on the beginning-of-period capital allocation and the age status, the individual allocation can be distributed according to a linear function of the aggregate values. The allocation based on this social planner's problem is easily proven to coincide with the competitive equilibrium allocation. (See, for example, Eichenbaum and Hansen, 1990, for discussion on a more general linear expenditure system and the representative agent model.)

Each individual maximizes her dynasty's utility, given the equilibrium evolution of price system, namely the returns on physical capital $R_k = \alpha A(H/K)^{1-\alpha}$ and on human capital $R_h = (1-\alpha)A(H/K)^{\alpha}$. Using the optimal human-to-physical-capital ratio, the value function can be again expressed in terms of physical capital only, similar to $V(K)$ in the perfect altruism case. For the imperfect altruism case, the individual k can be different from average K and thus a value function can be expressed as $v(k,K)$.

$$v(k,K) = \max_{b,c,\hat{c},i_k,\hat{i}_k,k^+,\hat{k}^+} (1-\rho)\beta(u(c) + v(k^+,K^+)) + \rho\gamma(u(\hat{c}) + v(\hat{k}^+,K^+)), \qquad (32)$$

subject to two *ex post* budget constraints: for survivors,

$$c + (1+x)i_k + x(\delta_w - \delta_k)k = (R_k(K) + xR_h(K))k - \pi b; \qquad (33)$$

and for the newly born,

$$\hat{c} + (1+x)\hat{i}_k + x(\delta_o - \delta_k)k = (R_k(K) + xR_h(K))k + b, \qquad (34)$$

where $\wedge$-bearing variables represent the variables for the newly born. Note that the timing is slightly different from the representative agent value function with perfect altruism. The value here (32) is measured just before an individual decides on the insurance purchase.

Given the price system, which is determined by the aggregate variables, the individual value function is homogeneous in the individual capital level k.[17] Thus, I can write $v(\hat{k}^+,K^+) = v(\phi k^+,K^+) = \zeta v(k^+,K^+)$. Note that, since the consumption of the newly born is ϕ times less than the others, her physical capital and thereby human capital are also ϕ times less in the equilibrium: $\hat{k}^+ = \phi k^+$ and $u(\hat{c}) = u(\phi c) = \phi^{1-\sigma}u(c)$. Using these, $v(k,K)$ can be expressed as

$$v(k,K) = \max_{b,c,,k^+} (1-\rho)\beta(u(c) + v(k^+,K^+)) + \rho\gamma\phi^{1-\sigma}u(c) + \rho\gamma\zeta v(k^+,K^+) \qquad (35)$$

[17] The optimal decisions on consumption and investments relative to the size of k are unchanged when multiplying k by any scaler.

or equivalently,

$$v(k,K) = \max_{b,c,,k^+} ((1-\rho)\beta + \rho\gamma\phi^{1-\sigma})u(c) + ((1-\rho)\beta + \rho\gamma\zeta)v(k^+, K^+). \tag{36}$$

Let

$$\tilde{\beta} \equiv (1-\rho)\beta + \rho\gamma\phi^{1-\sigma}$$

and

$$\tilde{\gamma} \equiv \frac{(1-\rho)\beta + \rho\gamma\zeta}{(1-\rho)\beta + \rho\gamma\phi^{1-\sigma}}.$$

Substitute these notations in the above and obtain

$$v(k,K) = \max_{b,c,,k^+} \tilde{\beta}u(c) + \tilde{\beta}\tilde{\gamma}v(k^+, K^+). \tag{37}$$

I can take a household with mean wealth $k = K$ as a representative agent. Then, I can rewrite the value function as if it is a value for a representative agent, using $\tilde{V}(K) = v(K,K)$ with $K = \int k\Omega(dk)$ where $\Omega(k)$ is the distribution of physical capital k. I maintain the stationarity assumption for the deterministic death case. Specifically, I assume the same as in the perfect altruism case: The relative sizes of human and physical capital are different but depend only on age, and the distribution of the age group is uniform and time-invariant. As for the stochastic death case, technically, the economy is never stationary. When a death shock hits one dynasty, it is optimal for the dynasty to choose to deplete some physical and human capital with incomplete life insurance. This dynasty still faces the same probability of death ρ in the next period as other dynasties do. Because the growth rate is linear for consumption and human and physical capital, its process is not mean reverting and thus its cross-section distributions diverge with time. However, we can still use a representative agent model and solve for aggregate (mean) allocation of consumption and investments over time, and then allocate those aggregate quantities to each dynasty as linear functions of their physical capital holding at the beginning of each period. Again, this is the competitive equilibrium path (Eichenbaum and Hansen, 1990).

Proposition 3. *Increase in the dynastic general equilibrium value of life is independent of the degree of altruism.*

This is formally proven in Appendix III. The value function $\tilde{V}(K)$ takes the form of $\kappa V(k)$ for a constant κ and thus the wealth transfer τ, defined in (30) to equate the values under the different life expectancies, does not vary with κ. Intuitively, even if households do not care as much about descendants as in the perfect altruism case, they optimize consumption plans over generations so that they behave almost as if they live forever, with a slightly different discount rate. Although the different discount rate changes the value of capital for a given life expectancy, it does not affect the current wealth transfer to compensate a possible lower life expectancy. Note also that Appendix III proves $\zeta = \phi^{1-\sigma}$ and $\tilde{\gamma} = 1$ as a corollary.

Even if the increase in dynastic general equilibrium value of life is the same, life insurance coverage may well be different if parents do not care about children much. This is qualitatively true, but the sensitivity analysis of calibration study has shown that the parameter values cannot be so different from the benchmark values. As for the altruism parameter, it cannot be so imperfect. Indeed $\gamma \geq 0.9\beta$ is a plausible parameter value. Under this *near perfect* altruism, the life insurance coverage is still about 6 under otherwise benchmark parameter values. Also, the growth rate is almost the same as in the perfect altruism case.

More specifically, the growth rate (the common growth rate for consumption, human capital, and physical capital over generations) is ϕ times lower than the rate within a generation (Proposition 2). Given the same h and k, the newly born consumes $\hat{c} = \phi c$ and have the next-period human capital $\hat{h} = \phi h$ and the next-period physical capital $\hat{k} = \phi k$. The budget constraint for the newly born can be transformed from (34) into

$$\phi c + \phi k^{+} - (1 - \delta_k)k + \phi h^{+} - (1 - \delta_o)h = R_k k + R_h h + b. \tag{38}$$

Subtracting both sides of the budget constraint (38) from the budget constraint for survivors (33), the life insurance benefits are determined by

$$(1 + \pi)b = (\delta_o - \delta_w)h - (1 - \phi)(c + h^{+} + k^{+}), \tag{39}$$

or equivalently, letting s denote the savings rate and g the growth rate under the perfect altruism case,

$$\frac{b_t}{Y_t} = (1 - \rho_t)\left((\delta_o - \delta_w) - (1 - \phi)\left(\frac{s + g}{x} + g\right)\right)\frac{x_t^{\alpha}}{A}. \tag{40}$$

It is easy to calibrate the population average growth rate to about 2 percent by slightly adjusting β. However, compared to the perfect altruism case (25), the equation (40) shows that the life insurance benefit has an additional term,

$$-(1 - \phi)\left(\frac{s + g}{x} + g\right).$$

Note that when the altruism parameter γ approaches β, the perfect altruism case, then ϕ approaches 1 and the additional term vanishes, making the formula equal to the one under the perfect altruism case (25). If the absolute value of the additional term is positive but small, say, within 0.1, then the life-insurance-benefits-to-income ratio also remains about 6. This is a plausible case in which the altruism parameter is more than 90 percent of the value under perfect altruism (i.e., $\gamma \geq 0.9\beta$), given other parameters set at the benchmark values.

Finally, I would like to discuss the wage premium observed for a risky job. This is a premium that a person demands for an early death, given the exogenously given life expectancy for everyone else, including her descendants. An early death of a person with measure zero does not affect the aggregate dynamics. For her descendants, the consumption sequence is the same as those of the other dynasties—perfect smoothing under the perfect altruism case and imperfect smoothing but optimized sequence under the imperfect altruism case—under the specific economy-wide life expectancy. As such, sudden early death does not bring any loss for her descendants. Even for herself, early death does not bring any loss in the case of perfect altruism, as the children are perfect substitutes. Therefore, there must be no wage premium under the perfect altruism assumption.

In the case with imperfect altruism, however, the wage premium exists, because a person cares for herself more than her descendants. To see this, take a derivative of $v(k, K)$ (35) with respect to ρ, ignoring all the effects of economy-wide life expectancy and substituting ζ by $\phi^{1-\sigma}$:

$$\begin{aligned}\frac{\partial v(k,K)}{\partial \rho} &= (\gamma\phi^{1-\sigma} - \beta)(u(c) + v(k^{+}, K^{+})) \\ &= \frac{\gamma\phi^{1-\sigma} - \beta}{\tilde{\beta}} v(k, K).\end{aligned} \tag{41}$$

This wage premium for the representative agent is expressed when evaluating $v(k, K)$ at $k = K$. It is positive if the level of $\tilde{V}(K)$ is positive. As $\tilde{V}(K)$ increases in K, the observed wage premium in U.S. dollars is easily obtained under any parameter values by adjusting the "exchange rate" between the model unit K and U.S. dollars. Note that this adjustment or fitting is similar to what is proposed by Rosen (1988) and widely used in the literature—in the partial equilibrium setting, it is the intercept term in the (period) utility function that is set freely to obtain the observed wage premium without changing any qualitative results and most of quantitative results. Note again, however, that this partial-equilibrium value of life is not an adequate measure of the true welfare gains from an economy-wide increase in life expectancy.

V. Concluding Remarks

I introduced a new concept of value of life, the dynastic general equilibrium concept, and showed how to compute it. This concept is consistent with the standard neoclassical growth theory. As such, it can serve as a reference measure for evaluating the increase in the life expectancy in the growth experience of the world. Theoretically, the value is shown to be positive in a realistic model. It is different from the partial-equilibrium, one-generation calculation of the value of life upon which the existing literature relies.

The calibration study shows that the welfare gains from increase in life expectancy are sizable, much larger than typical estimates of welfare gains from eliminating the business cycle in the U.S. However, improvements in world income inequality by accounting for the increase in the dynastic general equilibrium value of life are small. Under the calibrated set of parameter values, the effect on convergence in the world living standards corrected for the dynastic general equilibrium value of life is less than half the existing, partial-equilibrium-based estimates. Overall, a GDP-based measure of world income convergence appears to be a good approximation for convergence in living standards even when adjusted for differences in life expectancy.

Interpretation of the two concepts of value of life is quite different. On the one hand, the dynastic general equilibrium value of life should be used to evaluate the value of new medicine in advanced economies, or the value of new sanitation system in developing countries, which reduce the fatality rate almost exogenously to individual decisions. On the other hand, the traditional concept, the partial-equilibrium, one -generation value of life should be used to evaluate the value of life of a person who takes risky jobs, pays fitness gym fees, and cares about better nutrition, given the economy-wide life expectancy. Note that Acemoglu and Johnson (2006) and Deaton (2006) argue that most of the changes in life expectancy in the world stem from sources exogenous to each individual, that is, improvements of environments in which people live, technological advancement of medicine, and epidemics of diseases (e.g., AIDS).

There are several caveats, since I intentionally keep the model simple and look at only the steady state. In particular, when constructing a more realistic model in the future, it will be important to take into account life cycle effects, stochastic death, and endogenous choices on fertility and longevity. In addition, any transitional dynamics created by changes in longevity would potentially generate implications different from those obtained under the steady state.[18] Especially, the life cycle and population dynamics are important: in reality, age itself affects productivity and changes in age distribution affect physical to human capital in efficiency terms.[19]

The model generates somewhat unsuccessful predictions in terms of consumption growth and human-to-physical capital ratio. Contrary to an empirical finding by Acemoglu and Johnson (2006), the model predicts that a longer life expectancy brings a higher growth rate and higher human-to-physical capital ratio as Table 2 reports. However, these effects in the model are very small and can be easily offset by other forces. The findings by Acemoglu and Johnson (2006) are likely to be combined results of life cycle effects, endogenous fertility choice, and transitional population dynamics (see also Grossman, 1972, Ehrlich and Lui, 1991, and Young, 2005). With unexpected fall of mortality rate, population increased more than the optimal and per capita income dropped at least for the short term. This mechanism can more than offset the positive effect described here: The longer life expectancy of a generation brings a lower human capital depreciation rate in aggregate, so that a dynasty member has more incentive to save, in particular in the form of human capital investment. Obviously, a future work is warranted to develop a model to trace the actual data better.

Too much emphasis on value of life creates a cynicism that life is much more important than material happiness, and thus convergence in life expectancy is sufficient for convergence in the living standards in the world. On the other hand, ignoring value of life in the economic growth process is an obvious mistake, since life expectancy is one of commonly used,

[18]Moreover, I omit within-country heterogeneity in income inequality following BPS. However, considering it would not change the conclusion much. Although Deaton (2005) has a different view, Sala-i-Martin (2006) notes that, for 1970 to 2000, the reduction in across-country inequality is the major source of the reduction in overall world income distribution, more than offsetting the increase in the within-country inequality.

[19]In the empirical studies, the age effect is shown to be important. Murphy and Topel (2006) shows that the value of life-year converges to zero for the old, as the income becomes zero. Also see Kniesner and Viscusi (2005).

valuable measures of living standards. To resolve the philosophical dilemma, evaluation of an increase in life expectancy must be scientific, based on a rigorous theory with support of actual data. In this regard, there remains much to be done, but I hope this paper serves as an important step toward understanding the evolution of living standards in the world.

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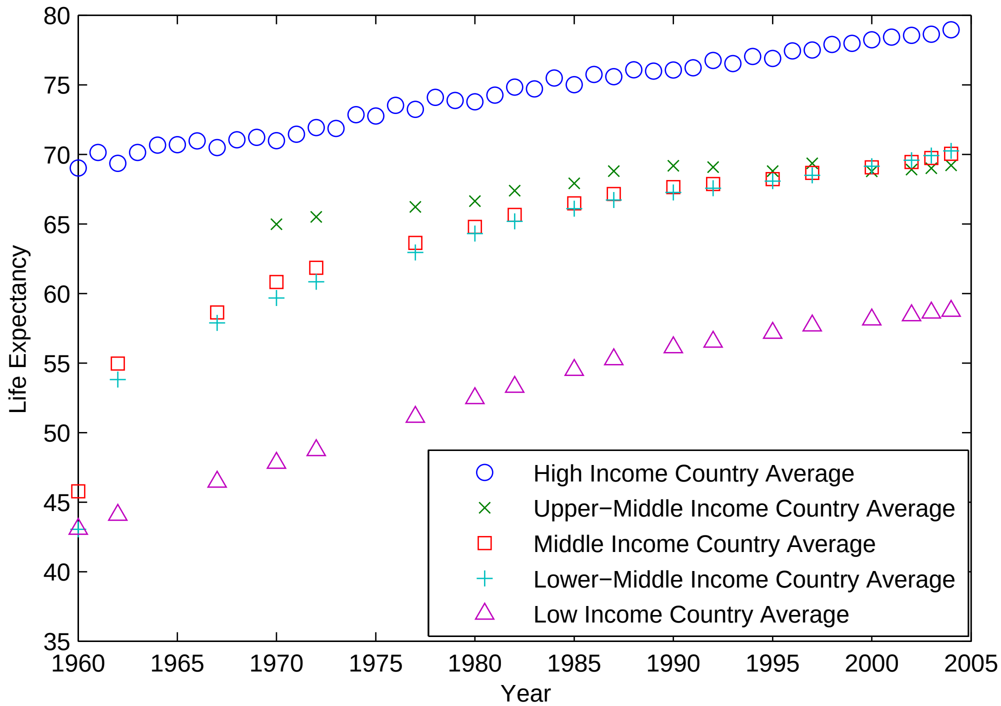
Figure 1. Evolution of Life Expectancy



Note: The data is from the WDI online edition as of March, 2007. Categories of countries are also defined in the WDI. Not all the data points are available, depending on country categories.

Table 1. Parameter Values

	Benchmark	Higher β	Higher σ	Higher δ_w	Higher δ_o
β	0.96	0.99			
σ	1.2		2.0		
α	1/3				
δ_k	0.05				
δ_w	0.02			0.04	
δ_o	0.7				0.9
A	0.25				

Table 2. Benchmark Quantitative Assessment

Life Expectancy	40	50	60	70	80	∞
Implied Increase in Dynastic G.E. Value of Life from 40 (% increase in annual income)	N/A	7.18	12.20	15.91	18.75	40.33
Life Insurance Benefits (ratio to annual income)	5.96	6.00	6.04	6.06	6.07	N/A
Consumption Growth (%)	1.80	2.03	2.19	2.30	2.38	2.97
Optimal H/K Ratio	11.35	11.44	11.51	11.55	11.58	11.82

Table 3. Sensitivity Analysis

Life Expectancy	40	50	60	70	80	∞
Higher β						
Implied Increase in Dynastic G.E. Value of Life from 40 (% increase in annual income)	N/A	15.15	26.26	34.71	41.35	95.70
Life Insurance Benefits (ratio to annual income)	5.96	6.00	6.04	6.06	6.07	N/A
Consumption Growth (%)	4.44	4.68	4.84	4.95	5.04	5.64
Optimal H/K Ratio	11.35	11.44	11.51	11.55	11.58	11.82
Higher σ						
Implied Increase in Dynastic G.E. Value of Life from 40 (% increase in annual income)	N/A	5.34	8.95	11.55	13.51	27.52
Life Insurance Benefits (ratio to annual income)	5.96	6.00	6.04	6.06	6.07	N/A
Consumption Growth (%)	1.07	1.21	1.31	1.37	1.42	1.77
Optimal H/K Ratio	11.35	11.44	11.51	11.55	11.58	11.82
Higher δ_w						
Implied Increase in Dynastic G.E. Value of Life from 40 (% increase in annual income)	N/A	33.78	40.31	45.13	48.84	77.06
Life Insurance Benefits (ratio to annual income)	5.69	5.73	5.76	5.79	5.80	N/A
Consumption Growth (%)	0.47	0.69	0.84	0.95	1.03	1.59
Optimal H/K Ratio	10.83	10.91	10.97	11.02	11.05	11.27
Higher δ_o						
Implied Increase in Dynastic G.E. Value of Life from 40 (% increase in annual income)	N/A	9.55	16.32	21.37	25.28	55.64
Life Insurance Benefits (ratio to annual income)	7.68	7.75	7.79	7.83	7.85	N/A
Consumption Growth (%)	1.46	1.76	1.96	2.10	2.21	2.97
Optimal H/K Ratio	11.21	11.33	11.41	11.47	11.51	11.82

Table 4. Convergence of Income and Full Income

	GDP per capita			Full Income		Improvements (%)	
	1960	1990	2000	1990	2000	1990	2000
Relative mean dev.	0.48	0.48	0.44	0.47	0.42	3.2	4.6
Coeff. of variation	1.24	1.31	1.23	1.26	1.18	3.4	4.5
Std. dev. of logs	1.03	1.01	0.97	0.98	0.95	3.2	1.8
Gini coeff.	0.57	0.57	0.54	0.56	0.52	2.7	3.4
Regression to the		-0.01	-0.13	-0.05	-0.18	82.1	24.5
mean over 1960		(0.82)	(0.03)	(0.22)	(0.01)		
(memorandum: BPS)							
Relative mean dev.	0.48	0.47	0.42	0.44	0.38	7.1	10.8
Coeff. of variation	1.23	1.25	1.17	1.17	1.05	6.9	10.3
Std. dev. of logs	1.02	1.03	0.96	0.98	0.95	5.3	1.5
Gini coeff.	0.51	0.52	0.49	0.49	0.46	4.9	6.4
Regression to the		-0.01	-0.13	-0.10	-0.26	93.1	49.3
mean over 1960		(0.86)	(0.01)	(0.02)	(0.00)		

Notes: Parenthesis in the rows of regression to the mean shows *p-value*. Income per capita is real GDP per capita in 1996 international prices, adjusted for terms of trade (Penn World Table 6.1). Full income is calculated by the author with 1960 as base year, incorporating gains in the dynastic general equilibrium value of life based on increase in life expectancy at birth (World Development Indicators, World Bank, On-line version as of August 2007). Inequality measures are weighted by country population (Penn World Table 6.1), generated by *ainequal* code with *aweight* option in Stata 9.2. Sample includes 96 countries, comprising more than 82 percent of the world population. Regression to the mean is the coefficient of a regression of the change in the natural log of income over the period on the 1960 level based on weighted regressions (*regression* code with *aweight* option in Stata 9.2). The numbers in parenthesis show the p-values. The figures under *memorandum (BPS)* show the numbers reported by BPS, except for the columns under Improvements, which I calculated.

APPENDIX I. SOLUTIONS

A. Optimal H/K Ratio

I assign Lagrange multipliers, μ_b, μ_h, and μ_k to three constraints (17), (14), and (15), respectively.

The first order condition with respect to c is

$$u'(c) = \mu_b, \tag{A1}$$

the condition with respect to I_k is

$$\mu_b = \mu_k, \tag{A2}$$

the condition with respect to I_h is

$$\mu_b = \mu_h, \tag{A3}$$

the condition with respect to K^+ is

$$\beta W_1^+ = \mu_k, \tag{A4}$$

and the condition with respect to H^+ is

$$\beta W_2^+ = \mu_h. \tag{A5}$$

From those first order conditions, it is clear that the marginal value of physical and human capitals are the same, that is,

$$\frac{W_1^+}{W_2^+} = \frac{\mu_k}{\mu_h} = 1. \tag{A6}$$

Moreover, the envelop theorem with respect to K provides

$$\mu_b \alpha A K^{\alpha-1} H^{1-\alpha} + \mu_k(1-\delta_k) = W_1, \tag{A7}$$

and with respect to H,

$$\mu_b(1-\alpha) A K^{\alpha} H^{-\alpha} + \mu_h(1-\delta_h) = W_2. \tag{A8}$$

Given the marginal-rate-of-transformation equation ($A6$), the left-hand sides of two equations (A7) and (A8) must be equal. Moreover, (A2) and (A3) imply that all Lagrange multipliers have equivalent values and I can simplify the combined condition of (A7) and (A8) to obtain equation (18).

B. Euler Equation

Using μ_{Vb} and μ_{Vk} as the Lagrange multipliers associated with two constraints, (22) and (15), respectively, we have first order conditions as below:

$$u'(c) = \mu_{Vb}, \tag{A9}$$

$$\mu_{Vb}(1+x) = \mu_{Vk}, \tag{A10}$$

and

$$\beta(V^{+})' = \mu_{Vk}. \tag{A11}$$

The envelop theorem provides an additional condition,

$$\mu_{Vb}(Ax^{1-\alpha} - x(\delta_h - \delta_k)) + \mu_{Vk}(1-\delta_k) = V'. \tag{A12}$$

Combining those conditions, we get

$$\mu_{Vb}(Ax^{1-\alpha} - x(\delta_h - \delta_k)) + \mu_{Vb}(1+x)(1-\delta_k) = \frac{\mu_{Vb}^{-}(1+x)}{\beta}, \tag{A13}$$

where superscript $-$ denotes the value of the previous period. Substituting the shadow price of the budget constraint μ_{Vb} by the marginal utility u' using (A9), I obtain the Euler equation (24).

Appendix II. Imperfect Altruism Case

A. Proof of Proposition 2

Since the current generation is replaced with probability ρ, individual decisions are made given the aggregate state variables, that is, physical capital K and human capital H, and associated gross returns on physical capital $R_k(K,H)$ and on human capital $R_h(K,H)$. Both depend only on aggregate capital levels—hereafter, I write R_k and R_h without (K,H) as long as it causes no confusion. Individuals also take insurance premium π as given.

With potentially different consumption levels among the newly born and survivors, it is necessary to consider the value function right before an individual knows whether she will live or die, but after all decisions for (contingent) consumption and investments are made. Let w denote this value function:

$$\begin{aligned} w(k,h,K,H) = \max_{b,c,\hat{c},k^{+},\hat{k}^{+},h,\hat{h}^{+}} &(1-\rho)\beta\left(u(c) + w(k^{+},h^{+},K^{+},H^{+})\right) \\ &+ \rho\gamma\left(u(\hat{c}) + w(\hat{k}^{+},\hat{h}^{+},K^{+},H^{+})\right), \end{aligned} \tag{A1}$$

subject to two budget constraints: for survivors,

$$c + k^{+} - (1-\delta_k)k + h^{+} - (1-\delta_w)h = R_k k + R_h h - \pi b; \tag{A2}$$

and for the newly born,

$$\hat{c} + \hat{k}^{+} - (1-\delta_k)k + \hat{h}^{+} - (1-\delta_o)h = R_k k + R_h h + b. \tag{A3}$$

Let λ and $\hat{\lambda}$ denote the Lagrange multipliers associated with the budget constraints (A2) and (A3), respectively. The first order condition with respect to the insurance benefit b is expressed as

$$\pi\lambda = \hat{\lambda}. \tag{A4}$$

Because consumption and investment decisions are made contingent on survival or death, the first-order conditions for consumption and investments need to be derived for each case. The first-order condition for consumption of a survivor is

$$(1-\rho)\beta u'(c) = \lambda \tag{A5}$$

and for the newly born,

$$\rho\gamma u'(\hat{c}) = \hat{\lambda}. \tag{A6}$$

The condition for physical capital for survivors

$$(1-\rho)\beta w_1^{+} = \lambda \tag{A7}$$

and for the newly born,

$$\rho\gamma \hat{w}_1^{+} = \hat{\lambda}, \tag{A8}$$

where subscript 1 of the value function w denotes the partial derivative with respect to the first element (i.e., physical capital), superscript $+$ denotes the value function w with the values in the next period, and $\wedge$-bearing value function denotes the value function with variables $\hat{c}$, $\hat{k}$, and $\hat{h}$ for the newly born. Similarly, for human capital, the first-order conditions are

$$(1-\rho)\beta w_2^{+} = \lambda \tag{A9}$$

and

$$\rho\gamma \hat{w}_2^{+} = \hat{\lambda}, \tag{A10}$$

for survivors and the newly born, respectively.

The envelope condition with respect to current physical capital is

$$w_1 = \lambda(R_k + 1 - \delta_k) + \hat{\lambda}(R_k + 1 - \delta_k), \tag{A11}$$

and with respect to human capital,

$$w_2 = \lambda(R_h + 1 - \delta_w) + \hat{\lambda}(R_h + 1 - \delta_o). \tag{A12}$$

Using the first-order conditions for consumption (A5) and (A6), w_1 or w_2 can be substituted by u' and the *ex ante* Euler equation can be obtained as follows:

$$u'(c) = (R_k + 1 - \delta_k)\left((1-\rho)\beta u'(c^+) + \rho\gamma u'(\hat{c}^+)\right). \tag{A13}$$

When the altruism is perfect (i.e., $\gamma = \beta$), this Euler equation becomes equivalent to the one in the benchmark case and consumption growth rate is the same for both survivors and the newly born.

Because altruism is not perfect in this section and insufficient wealth is inherited over generations to smooth out consumption, survivors and the newly born consume different amount of goods even when those dynasties had the same consumption level in the previous period. To see this, note that the actuarially fair condition (9) should still hold, in addition to the first-order condition (A4). Then, the equilibrium premium is expressed as

$$\pi = \frac{\hat{\lambda}}{\lambda} = \frac{\rho}{1-\rho}. \tag{A14}$$

Based on this relationship, the envelope condition for physical capital (A11) can be rewritten as

$$w_1 = \lambda(1+\pi)(R_k + 1 - \delta_k), \tag{A15}$$

or

$$w_1 = \hat{\lambda}\left(\frac{1}{\pi} + 1\right)(R_k + 1 - \delta_k). \tag{A16}$$

Using the first order conditions (A5) and (A8), the *ex post* Euler equation for survivors and the newly born can be expressed as

$$\frac{u'(c^-)}{u'(c)} = \left(\frac{c}{c^-}\right)^\sigma = \beta(R_k + 1 - \delta_k), \tag{A17}$$

and for the newly born,

$$\frac{u'(c^-)}{u'(\hat{c})} = \left(\frac{\hat{c}}{c^-}\right)^\sigma = \gamma(R_k + 1 - \delta_k). \tag{A18}$$

These Euler equations are the proofs for the claims (i) and (ii) of Proposition 4. The consumption growth rate within a generation (A17) is the same as in the perfect altruism case (24). By comparing (A18) with (A17), it is easy to see that the consumption growth rate over generations is $\phi \equiv (\gamma/\beta)^{1/\sigma}$ times lower than the rate within a generation.

Now, I prove the claim (iii): The optimal human-to-physical-capital ratio is the same as in the benchmark case. Subtract both sides of the envelope condition for human capital (A12) from those for physical capital (A11) and obtain

$$0 = \lambda(R_k - R_h + \delta_w - \delta_k) + \hat{\lambda}(R_k - R_h + \delta_o - \delta_k), \tag{A19}$$

or equivalently,

$$R_h - R_k = \frac{\lambda}{\lambda + \hat{\lambda}}\delta_w + \frac{\hat{\lambda}}{\lambda + \hat{\lambda}}\delta_o - \delta_k. \tag{A20}$$

Substitute the equilibrium premium (A14) into (A20) and obtain,

$$\begin{aligned} R_h - R_k &= \frac{1}{1+\pi}\delta_w + \frac{1}{\frac{1}{\pi}+1}\delta_o - \delta_k \\ &= (1-\rho)\delta_w + \rho\delta_o - \delta_k. \end{aligned} \tag{A21}$$

In the general equilibrium, the return to human capital R_h is equal to the marginal product of human capital, $R_h(K,H) = (1-\alpha)A(H/K)^{\alpha}$, and the return to physical capital R_k is equal to the marginal product of physical capital, $R_k(K,H) = \alpha A(H/K)^{1-\alpha}$. Therefore, the condition for the optimal human-to-physical capital ratio in aggregate with imperfect altruism (A21) is the same as (18), the condition for the case with perfect altruism—recall that δ_h in (18) is defined as $(1-\rho)\delta_w + \rho\delta_o$. Hence, the aggregate human-to-physical-capital ratio is the same as in the perfect altruism case, $H/K = x$.

Now I consider the optimal human-to-physical capital ratio at the individual level. First, I would like to point out that the growth rate of physical capital of a survivor is different from that of a newly born, $k^+/k \neq \hat{k}^+/k$, in the case in which they had the same k. I prove this by contradiction. Suppose the physical capital growth rate is the same for both type of people. Then, human capital growth must be much less for a newly born, $h^+/h > \hat{h}^+/h$, because the consumption growth is less for a newly born as proven above. However, based on the equilibrium premium (A14), I obtain

$$w_1(k^+, h^+, K^+, H^+) = w_1(\hat{k}^+, \hat{h}^+, K^+, H^+). \tag{A22}$$

But this cannot be the case with $k^+ = \hat{k}^+$ and $h^+ > \hat{h}^+$, because w is strictly concave in the first two elements, $w_{12} < 0$, due to a characteristic of the production function.[20] This is a contradiction. Therefore, the growth rate of physical capital of a survivor must be different from that of a newly born. Similarly, if two w functions with different degrees of altruism γ are compared, it is easy to show that the physical capital growth rates vary across two types of people.

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[20]Note that w-function is the same for the both types and it is easy to show the monotonicity, $w_1 > 0$ and $w_2 > 0$, decreasing marginal returns, $w_{11} < 0$ and $w_{22} < 0$, and strict concavity, $w_{12} < 0$.

Second, consider the implication of the aggregate human-to-physical capital ratio.

$$x = \frac{H^+}{K^+} = \frac{(1-\rho)h^+ + \rho\hat{h}^+}{(1-\rho)k^+ + \rho\hat{k}^+}. \tag{A23}$$

Suppose the individual level optimal ratios are different and are denoted as follows: $h^+/k^+ = x_s$ and $\hat{h}^+/\hat{k}^+ = x_n$. Then, the aggregate condition (A23) can be expressed as

$$x((1-\rho)k^+ + \rho\hat{k}^+) = (1-\rho)x_s k^+ + \rho x_n \hat{k}^+. \tag{A24}$$

The equation (A23) implies that the degree of altruism γ does not affect the aggregate human-to-physical capital ratio. However, as I proved above, $\hat{k}^+ \neq k^+$ and $\hat{k}^+$ in equation (A24) varies with the altruism parameter γ that affect x_s and x_n. Since the equation (A24) must be satisfied for any degree of altruism γ, it must be the case that $x_s = x_n = x$. That is, the individually optimal human-to-physical capital ratio is equal to the aggregate one regardless of individual status, a newly born or a survivor.

Finally, properties (i)–(iii) in Proposition 2 imply (iv).

B. Proof of Proposition 3

Recall that $k^+ = gk$ for survivors and $\hat{k}^+ = \phi gk$ for the newly born (Proposition 2) and hence $K^+ = \tilde{g}K$, where $\tilde{g} = (1-\rho+\rho\phi)g$, for the aggregate capital. Assume $\tilde{V}(K)$ takes a form similar to $V(K)$, that is $\tilde{V}(K) = \tilde{\Psi}K^{1-\sigma}$. Then, the value function can be expressed as, based on (37),

$$\begin{aligned} \tilde{V}(K) &= \tilde{\beta}u(c) + \tilde{\beta}\tilde{\gamma}\tilde{V}(K^+), \\ &= \tilde{\beta}u(c) + \tilde{\beta}\tilde{\gamma}\tilde{g}^{1-\sigma}\tilde{V}(K). \end{aligned} \tag{A25}$$

Rearranging terms and using the savings rate s for survivors, the value function can be written as

$$\begin{aligned} \tilde{V}(K) &= \frac{1}{\frac{1}{\tilde{\beta}} - \tilde{\gamma}\tilde{g}^{1-\sigma}} u(c) \\ &= \frac{((1-s)Ax^{1-\alpha})^{1-\sigma}}{\left(\frac{1}{\tilde{\beta}} - \tilde{\gamma}\tilde{g}^{1-\sigma}\right)(1-\sigma)} K^{1-\sigma}. \end{aligned} \tag{A26}$$

This proves the guess on the form of $\tilde{V}(K)$ is correct, that is, $\tilde{V}(K) = \tilde{\Psi}K^{1-\sigma}$.

Moreover, I can pin down ζ and $\hat{\gamma}$. Consider the representative agent with $k = K$. Her value is $v(K, K) = \tilde{V}(K)$. Then by definition of ζ, $\zeta = \phi^{1-\sigma}$ and thus $\tilde{\gamma} = 1$.

Furthermore, substitute these values for ζ and $\tilde{\gamma}$ into (A26) and obtain

$$\begin{aligned}\tilde{V}(K) &= \tilde{\Psi}K^{1-\sigma} \\ &= \frac{\tilde{\beta}((1-s)Ax^{1-\alpha})^{1-\sigma}}{(1-\tilde{\beta}g^{1-\sigma})(1-\sigma)}K^{1-\sigma}.\end{aligned} \tag{A27}$$

Take

$$\kappa \equiv \tilde{\beta}\frac{1-\beta g^{1-\sigma}}{1-\tilde{\beta}\tilde{g}^{1-\sigma}}, \tag{A28}$$

then

$$\tilde{\Psi} = \kappa\Psi. \tag{A29}$$

Note that because the timing of the value function $\tilde{V}$ is just before deciding the insurance purchase, $\tilde{\beta}$ appears in front of the ratio of $(1-\beta g^{1-\sigma})$ to $(1-\tilde{\beta}\tilde{g}^{1-\sigma})$ in κ.