WP/08/115

IMF Working Paper

Impact of Government Expenditure on Growth: The Case of Azerbaijan

Junko Koeda and Vitali Kramarenko

INTERNATIONAL MONETARY FUND

 WP/08/115

IMF Working Paper

Middle East and Central Asia Department

Impact of Government Expenditure on Growth: The Case of Azerbaijan

Prepared by Junko Koeda and Vitali Kramarenko[1]

Authorized for distribution by Marta Castello-Branco

May 2008

Abstract

This paper evaluates a fiscal scenario based on the assumption of a rapid scaling-up of expenditure to be followed by a rapid scaling-down in the context of Azerbaijan's current temporary oil production boom. To this end, it relies on a review of historical precedents and a neoclassical growth model. Based on both strands of analysis, the paper suggests that the evaluated fiscal scenario poses significant risks to growth sustainability.

JEL Classification Numbers: E22, O23, Q32, Q33

Keywords: Government expenditures, economic growth, public investment, oil revenues, Azerbaijan

Author's E-Mail Address: jkoeda@imf.org; vkramarenko@imf.org

[1] The authors are grateful to Lorenzo L. Pérez, Shuang Ding, Hajime Takizawa, Koba Gvenetadze, Valeria Fichera, Kenich Ueda, and Mayra Zermeño for their helpful comments and supports.

Contents Page

I. INTRODUCTION

Azerbaijan is experiencing a large but temporary oil production boom. Based on the current information on proven oil reserves, it is expected that oil production will increase from 0.3 million barrels per day in 2005 to about 1.1 million barrels per day by 2009 and will start to decline sharply thereafter. Despite its temporary nature, the oil boom presents a unique financial opportunity to lay the foundations for sustainable non-oil output growth and poverty reduction. Indeed, oil and gas wealth is estimated at about $200 billion in the 2004 constant U.S. dollar terms (20 times the level of 2004 GDP), assuming oil prices remain at about $60 per barrel in real terms in the long term.

During the initial stage of the oil boom (2005–07), the government of Azerbaijan opted for exceptionally large expenditure increases aimed at improving infrastructure and raising incomes. Total government expenditure increased by a cumulative 160 percent in nominal terms from 2005 to 2007 or from 41 percent of non-oil GDP[2] to 74 percent.[3] This scaling-up of expenditure raised the question whether the current level of expenditure is appropriate and whether it is sustainable from a long-term perspective. The theoretical literature, empirical studies, and country-specific analyses do not provide an easy answer to these questions.

This paper focuses on evaluating the impact of the most likely fiscal policy scenario on economic growth in Azerbaijan rather than defining optimal, sustainable fiscal rules suitable for the country. One of the policy options considered by the authorities is to continue to scale up expenditure, in particular in the area of infrastructure. Upon the completion of key infrastructure projects, the authorities intend to scale down capital expenditure significantly. Also, the authorities intend to raise the level of public sector wages and pensions and social benefits upfront with the understanding that the real growth of wages and transfers will also slow significantly over the medium term.

To evaluate the option of this additional scaling-up of expenditure, the paper relies on the analysis of historical precedents and a neoclassical growth model. The experiences of Nigeria and Saudi Arabia during the 1970s and 1980s provide valuable lessons on the potential risks associated with rapid expenditure increases and policies that can be used to mitigate these risks. These lessons are not specific to Azerbaijan as other new oil-producing countries are facing similar challenges. In addition, a neo-classical growth model customized to Azerbaijan-specific conditions is simulated to evaluate the impact of the scaled-up fiscal policy scenario on the growth of non-oil output and private consumption. In this model, fiscal policy decisions and the oil production profile are treated as exogenous. This approach stands in contrast with the previous research efforts on fiscal policy in oil-producing countries that focused on deriving optimal expenditure paths or assessing possible fiscal rules.

---

[2] Excluding oil and gas transportation.

[3] This paper is based on the information available as of September 2007.

The paper is organized as follows. Section II analyzes the experiences of Nigeria and Saudi Arabia in managing a significant increase and a subsequent decline in their oil revenues during the 1970s and 1980s with a view to determining potential medium- and long-term problems that Azerbaijan may face in a similar expenditure situation. Section III presents long-term simulations of a neoclassical growth model, which is calibrated using Azeri data. Section IV concludes by juxtaposing the findings of the analysis of historical precedents and the model simulation results.

The main finding of the paper is that the evaluated fiscal policy scenario carries significant medium- and long-term risks. The lessons from the historical experiences of managing large surges of oil revenue and expenditure, and the simulations of the Azerbaijan-specific model suggest that in Azerbaijan a significant growth deceleration could be unavoidable once oil production starts to decline in the middle of the 2010s.

II. Historical Lessons for New Oil-Producing Countries: The Cases of Nigeria and Saudi Arabia

This section describes the outcomes of the economic policies that were pursued by two oil-producing countries—Nigeria and Saudi Arabia—in response to large changes in their oil receipts during the 1970s and 1980s (Figure 1). These two countries were chosen because the size of their expenditure increases in the 1970s were commensurate with the orders of magnitude of the illustrative fiscal scenario for Azerbaijan that this paper evaluates. While initial economic conditions in Azerbaijan are very different from either Nigeria's or Saudi Arabia's conditions in the 1970s, the analysis of historical precedents is still useful because it shows potential risks associated with rapid expenditure increases and the role of institutions in managing oil booms.

Nigeria and Saudi Arabia experienced large increases in oil revenues during the 1970s. In Saudi Arabia, oil exports increased from about $3 billion in 1970 to more than $100 billion in 1980. Nigeria experienced an equally impressive increase in oil export receipts from $0.6 billion in 1970 to $25 billion in 1978.

Saudi Arabia increased expenditure significantly during the 1970s, and achieved rapid improvements in infrastructure and exceptionally high non-oil GDP growth rates. Annual expenditure increases averaged about 90 percent during 1973–75, mainly to finance large infrastructure projects and increased welfare spending. In the late 1970s, expenditure continued to grow in double digits from the high base. In addition to the rapid expenditure increases, Saudi Arabia liberalized foreign trade and immigration rules to reduce capacity constraints. A large body of literature agrees that Saudi Arabia has successfully modernized its infrastructure, laying a sound foundation for the non-oil economy growth largely emanating from services and petrochemicals (e.g., Auty, 2001). Reflecting this remarkable transformation, Saudi Arabia's real non-oil GDP grew at a phenomenal pace of more than 40 percent a year on average during 1973–76 before stabilizing at 6–8 percent in the late 1970s.

Nigeria experienced much lower growth rates during the phase of large expenditure increases. It also increased expenditure by about 100 percent a year in 1974 and 1975 before entering a period of high expenditure volatility. Nigeria targeted sizable increases in both current (in particular wages) and capital expenditure. Poor management of the investment program and the ballooning wage bill in Nigeria intensified the negative impact of Dutch disease on agriculture and led to wasted resources, which is widely documented in the literature (e.g., Bevan et al, 1999). A relatively low Nigerian non-oil economy growth rate during the oil boom years—6.5 percent—compared with the size of expenditure increases, is an important piece of evidence on the low productivity of expenditure.

The decline in oil prices during the early 1980s necessitated large expenditure cuts, negatively affecting the non-oil growth performance of both countries (Figure 1). In response to declining oil prices, Saudi Arabia first started to accumulate large domestic debt, and then slashed expenditure. The generous welfare system developed during the oil boom years imposed rigid limits on the size of current expenditure cuts, and consequently capital expenditure was reduced by large amounts. The dramatic reduction in public investment and the crowding out of private sector activity because of the domestic debt build-up undermined Saudi Arabia's non-oil output growth in the 1980s, despite the fact that the country was able to develop world-class infrastructure prior to the decline in oil prices. In comparison with Saudi Arabia, Nigeria experienced an even deeper recession because its public investment had low rates of return and a significant increase in wages magnified the impact of Dutch disease.

Lessons from the Past for New Oil-Producing Countries

- While both Nigeria and Saudi Arabia experienced large initial expenditure increases in the mid-1970s, Saudi Arabia saw much higher real non-oil GDP growth than Nigeria. This is arguably because of more effective expenditure management, more liberal trade, and better access to low-wage foreign labor.

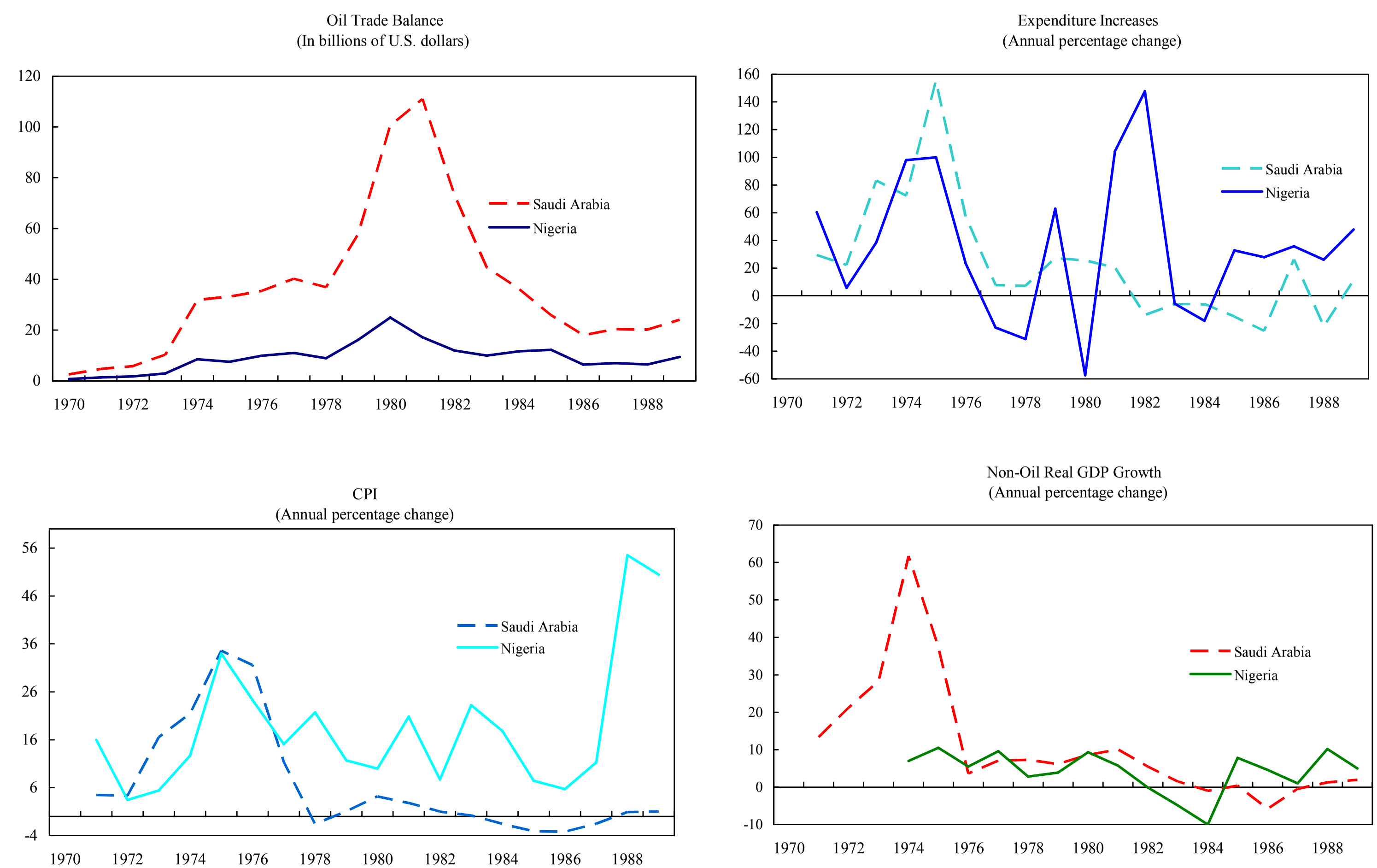
- Following the decline in oil revenues, even a country such as Saudi Arabia, that had successfully implemented its investment program, experienced a prolonged period of recession and stagnation resulting in expenditure cuts.

- Saudi Arabia's experience demonstrates that the generous welfare system may make it politically difficult to reduce current expenditure in response to falling revenue, contributing to the crowding-out of the private sector at a time when the resource constraint on the economy is tightening.

- In Saudi Arabia, government domestic borrowing aimed at containing expenditure cuts resulted in the crowding-out of private sector activity, further undermining economic growth.

- In the 1970s, Nigeria experienced a reallocation of resources from tradable sectors, including agriculture, to non-tradable activities, due to rapid wage growth in urban areas. At the end of the oil boom in the 1980s, Nigeria faced the difficult challenge of rebuilding its tradable sectors once oil revenue had started to decline.

Figure 1. Nigeria and Saudi Arabia : Selected Economic Indicators, 1970-89



Sources: IFS, WEO, and the Saudi Arabian Monetary Authority.

III. Model-Based Evaluation of Fiscal Policy Scenario

A. Literature Review

There are several neoclassical growth models allowing for the impact of government operations on resource allocation and growth. These models are based on various combinations of assumptions regarding the government, including the presence of lump sum or distortionary taxes, the inclusion of government purchases and transfers to households, the incorporation of public goods or public capital in the production function or the household utility function. Some models seek to derive optimal government expenditure or taxation paths (see literature reviews in Ljungqvist and Sargent, 2004; and Barro and Sala-i-Martin, 2004). Others aim to analyze different fiscal policy rules (e.g., Judd, 1985; Barro, 1989; and Baxter and King, 1993). Both types of models were applied to the analysis of public finances in the context of oil-producing countries (e.g., Engel and Valdés, 2000; and Takizawa, Gardner, and Ueda, 2004).

Drawing on the implications of theoretical models, empirical studies in the public finance literature focused on the impact of government consumption and public investment on growth. The majority of authors find strong negative correlations between the size of government consumption (i.e., expenses on goods and services, and wages) and economic growth for various groups of countries (Barro, 1991). However, there is significant controversy on the impact of public investment (i.e., capital expenditure) on growth. Some studies found that public investment had a similar or inferior impact on growth compared with private investment (e.g., Barro, 1991; Calderon, 2004; and Khan and Kumar, 1997). Others demonstrated that public capital may have a strong positive impact on growth if the growth equation is controlled for the effectiveness and type of public capital financing (e.g., Aschauer, 2000).

B. The Model

The main objective of the model presented in this paper is to evaluate the impact of government decisions on the timing and pace of spending out of oil revenues on private sector behavior, and ultimately on economic growth. This approach is different from the previous models developed for resource-rich countries in that it is mainly concerned with the policy evaluation of a specific fiscal scenario rather than with the derivation of optimal expenditure rules or an assessment of possible fiscal rules.

With this main objective in mind, a general equilibrium neoclassical growth model was specified to fit Azerbaijan-specific conditions. It has the following distinguishing features:[4] (i) all assumptions on the oil sector, including the production profile and prices, are given exogenously, and all oil revenues, net of production costs and profit payments to foreign oil companies, accrue to the public sector; (ii) the size and composition of expenditure are determined exogenously without conformity to any fiscal rules; (iii) public investment can

[4] Our model follows the spirit of Barro's (1990) model—and shares many features with the model presented in the IMF's 2006 Selected Issues Paper for the Russian Federation (IMF, 2006, Box A1).

increase the non-oil sector's total factor productivity (TFP), while current government consumption (i.e., current primary expenditure minus transfers) does not affect household utility or production functions; (iv) there are only lump sum taxes, and the Ricardian equivalence holds; (v) the public sector can incur external and domestic debt and hold external assets, but the private sector does not have access to international financial markets; and (vi) the utility-optimizing private non-oil sector reacts to the public sector's expenditure decisions and the evolution of the public sector's net wealth.

In the model, there are three types of agents: government, households, and firms.

Government

Government revenue, expenditure, and financing are exogenously given with the government budget constraint as follows:

(1) $$C_t^g + X_t^g - O_t^g - T_t = D_{t+1}^* - (1+i_t^*)D_t^*,$$

where C^g is government consumption, X^g is government investment, O^g is oil revenue, D^* is government net external debt, and i^* is the world interest rate. T is net transfers, which are defined as the lump-sum tax minus transfers to households plus domestic financing through the issuance of government bonds. Because of the existence of the lump-sum tax, the Ricardian equivalence holds.[5] The domestic real interest rate equals the marginal productivity of capital minus the depreciation rate by the firms' maximization.

Households

Households earn wages and rents on capital, allocate their income to consumption and investment, and also receive net transfers. The households maximize their utility subject to their budget constraint (3) and the transition equation for private capital[6] (4):

(2) $$\max_{C_t, K_{t+1}} \sum_{t=1}^{\infty} \beta^t u(C_t),$$

where $u(C) = \ln(C)$

(3) $$w_t L_t + i_t K_t = C_t + X_t + T_t,$$

(4) $$K_{t+1} = X_t + (1-\delta)K_t,$$

---

[5] In this model, raising the lump-sum tax or issuing government bonds has the same effect on the economy—thus the Ricardian equivalence holds.

[6] By the transition equation, private capital accumulation is equal to real private investment minus capital depreciation.

where C is private consumption, K is private capital, X is private investment, T is the net transfers, w is wage, i is the rental rate, and L is labor, β is time preference, and δ is the depreciation rate of private capital.

Firms

Hydrocarbon production is exogenously given, and firms produce only non-oil goods. They maximize their profits with the following non-oil production function:

(5) $f(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$, where $A_t = \left\{ z_t + \theta \frac{K_t^g}{L_t} \right\}^{1-\alpha}$

where A is total factor productivity (TFP), the growth rate of z is that of labor-augmented technological progress, K^g is public capital that follows the next transition equation:

(6) $K_{t+1}^g = X_t^g + (1-\delta^g) K_t^g$

θ is the coefficient that captures the impact of public capital on the TFP, and δ^g is the depreciation rate of public capital.

Because the model is very stylized, several caveats should be noted. First, it does not have nominal variables, such as prices or an exchange rate, and thus does not capture the real effect of these variables. Inflation and exchange rates are projected outside the model, and all nominal indicators are converted into real variables in local currency terms. Second, the model has only one non-oil sector and does not distinguish between tradable and non-tradable goods; thus it does not capture the potential impact of Dutch disease.

C. Simulations

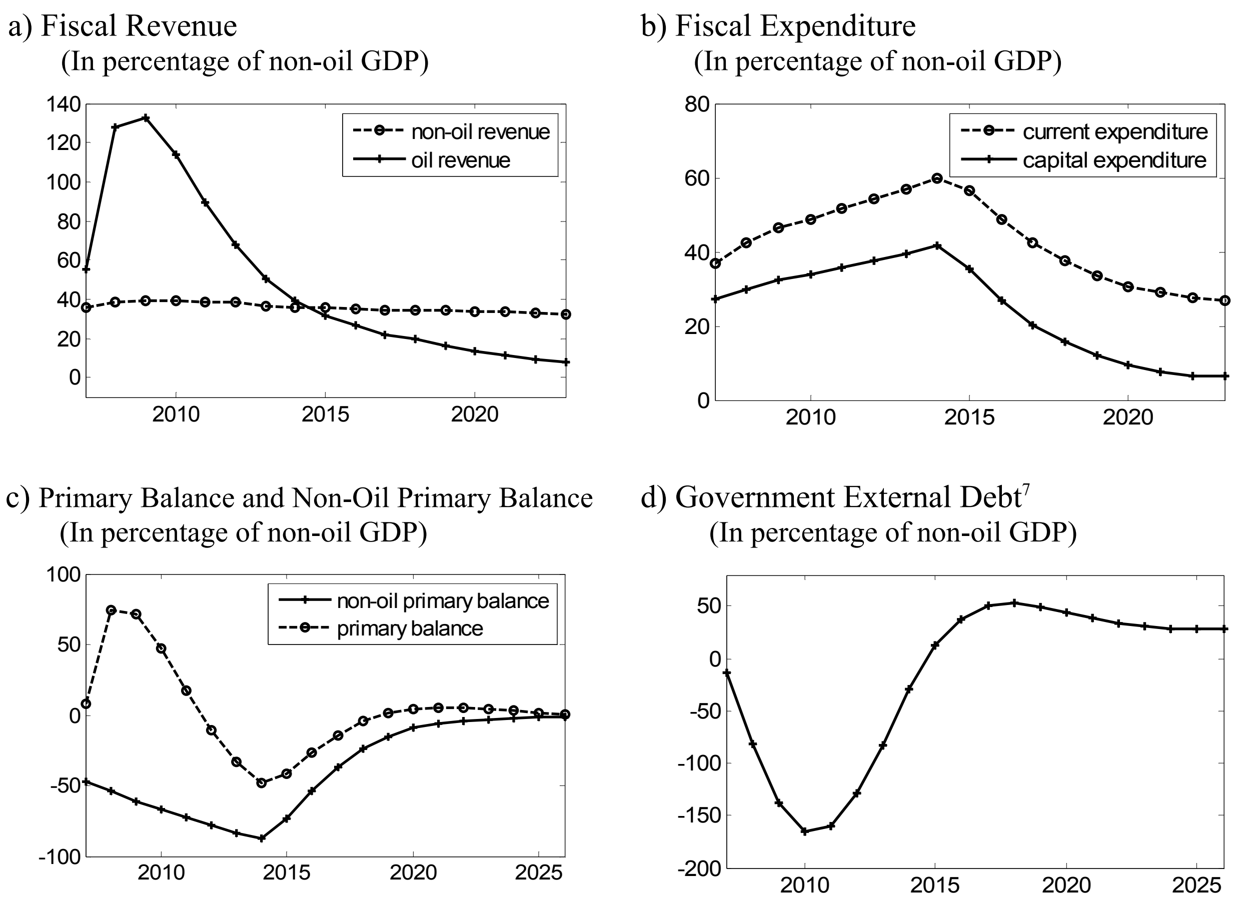
This sub-section describes (i) the baseline scenario which is used as an input to the paper's simulation exercise; (ii) calibrated parameters; and (iii) simulation results. The main objective of this simulation exercise is to evaluate whether the initial positive effects of the rapid increase in capital expenditure on growth, and current transfers on private consumption can be sustainable in the long term.

Baseline scenario: inputs to the simulation exercise
The baseline scenario is an extension of the IMF's medium-term projections (2007–12) used in the 2007 Article IV Staff Report for Azerbaijan (IMF, 2007). It assumes that the current policies of rapid increases in government consumption and capital expenditure will continue in the medium term.

- **The temporary nature of oil revenue** (Figure 2a). Under the January 2007 WEO oil price assumptions and the expected oil production profile, oil revenue is projected to rise rapidly through 2009 and to decrease continuously afterwards, as oil production declines. On current policies, non-oil revenue as a percentage of non-oil GDP is projected to remain relatively constant over time.

- **The continued fiscal expansion** (Figure 2b). Government expenditure is projected to increase by about 45 percent in 2007, and will increase by an additional 35 percent in 2008, 30 percent in 2009, and then by 25 percent annually during 2010–12. The composition of expenditure is relatively unchanged: current expenditure is about twice as large as capital expenditure.
- **High government transfers to households.** Generous transfers are available to households through pensions, social assistance, subsidies, and indirectly through energy subsidies.
- The State Oil Fund, which was set up to accumulate budget surpluses during the oil production boom years, will continue to be managed transparently, consistent with strict adherence to the Extractive Industry Transparency Initiative.

Figure 2. The Baseline Scenario.



Source: Authors' estimates.

[7] Nominal returns on foreign assets and liabilities vary in line with a given world real interest rate and the movement of the inflation and exchange rates. The level of world real interest rate is consistent with the model's steady state conditions.

The medium-term projections are extended to 2013–24.[8] For illustrative purposes, it is assumed that expenditure will continue to increase by 20 percent per year until the oil fund dries up in the mid-2010s. In the face of declining oil revenue and the depletion of the oil fund assets, a significant fiscal adjustment will be needed by the mid-2010s to avoid explosive debt dynamics (Figure 2b). During the fiscal adjustment period, for illustrative purposes, current expenditure is kept at the previous year's nominal level, and capital expenditure is reduced so as to keep primary fiscal balances consistent with the assumed public external debt ceiling of 40 percent of GDP.[9] Once the assumed external debt limit is reached, fiscal deficits are financed through domestic borrowing, which itself is limited to 10 percent of GDP.

Calibration

Table 1. Parameters and Initial Conditions (In annual frequency)

Parameter values			
Capital elasticity of output (α)	0.30	Depreciation rate of public capital (δ^g)	0.10
The intertemporal elasticity of substitution (σ)	0.45	Depreciation rate of private capital (δ)	0.10
Time preference (β)	0.98	Long-run rate of technological progress (g)	0.030
Coefficient of K_g for the augmented TFP (θ)	0.05	Population growth (n)	0.005
Initial conditions			
Private capital-non-oil GDP ratio	1.33		
Public capital-non-oil GDP ratio	0.30		

Source: Authors' estimates.

Table 1 lists calibrated parameter values and initial conditions for Azerbaijan. The value of the capital elasticity of the production function (0.25) is based on the existing cross-country studies;[10] this number is also in line with the labor share in non-oil GDP for Azerbaijan.[11] The initial levels of public and private capital are estimated using a perpetual inventory method with the public capital estimates based on the central government's capital expenditure in the non-oil sectors. Depreciation of capital is set at 0.1—a reasonable number in the real business cycle literature (e.g., Kydland and Prescott, 1982). The long-run population growth (0.005) is based on the historical average considering possible emigration. The long-term rate of labor augmented technological progress (0.03) is based on the sample average.[12] The time

[8] It is expected that oil production will end in 2024 in the absence of new discoveries.

[9] This debt threshold is in line with indicative sustainability thresholds suggested in IMF 2004a, 2004b, and 2006, where the debt sustainability threshold ranges between 30–60 percent in terms of the net present value of debt to GDP, depending on the strength of the policy framework.

[10] For example, see Gollin (2002).

[11] The labor share is estimated by the sum of employees in each non-oil sector (excluding social services) multiplied by the average wage in that sector, and then divided by non-oil GDP.

[12] Our estimate of the TFP (denoted as A) growth for 1995–2006 in Azerbaijan is about 0.04. This implies that the growth rate of labor-augmented technological progress (z) is about 0.03, given the calibrated parameter values for θ and α and the paths of K^g and L.

preference is set to 0.98, which implies a real interest rate of 2 percent at the steady state of the detrended model (Annex I).

Lastly, the value of the coefficient that captures the impact of public capital on the TFP (0.05) assumes a moderate impact of public investment on the TFP, given somewhat slow progress in improving expenditure management. However, because of the relatively short duration of the capital expenditure growth spurt, the model is more sensitive to the assumption on the pace of the subsequent capital expenditure decline rather than to the assumed measure of the public investment impact on the TFP. Indeed, even well-implemented infrastructure projects will have a limited impact on long-term growth if public investment falls short of depreciation and adequate maintenance is not undertaken following the completion of these projects.

Simulation results

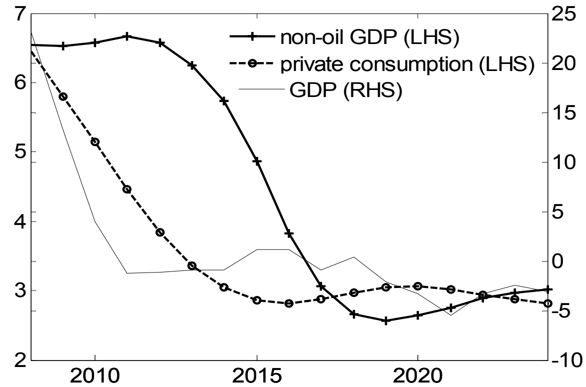
The main result of the model simulation is that once the fiscal adjustment starts in the mid-2010s, there is a risk of low non-oil GDP growth over a prolonged period (Figure 3). The relatively high non-oil growth projected for 2007–12 is driven by increased public sector investment and its attendant positive impact on the TFP, which also stimulates a gradual pick-up in private investment. Subsequently, there is a prolonged growth deceleration in the non-oil sector (to 2–3 percent). This deceleration mainly stems from a reduction in private investment induced by a rapid tightening of the resource constraint starting in the early 2010s, which is compounded by the negative effects of reduced government capital expenditure from the mid-2010s and crowding-out effects of the government's increased domestic borrowing.

By 2010–12 total real GDP starts to decline, as the buoyant real non-oil output growth rates simulated by the model are not sufficient to offset the projected decline in real oil GDP. Subsequently, the combination of the significant slowdown in the simulated non-oil GDP growth rate and the continued decline in real oil GDP is projected to result in a further significant decline in the level of total GDP. According to the simulations, by 2024, the level of real GDP will decline by about 20 percent from its peak in 2010.

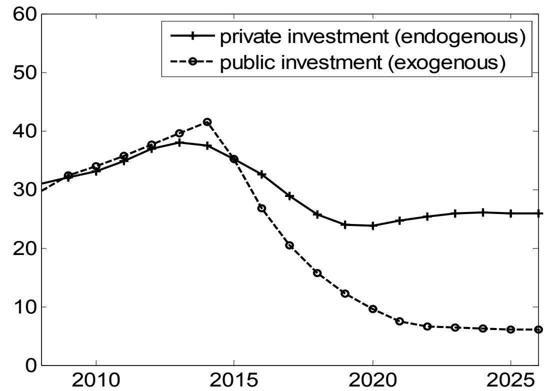
Real private consumption growth also decelerates significantly over the long term (Figure 3c). The initial brisk private consumption growth is mainly explained by a surge in transfers to households that can only smooth their consumption profile over time to the extent they can save by investing in physical capital in the presence of limited international capital mobility. By the mid-2010s, when the government enters the phase of the fiscal adjustment and cuts back transfers to household, real non-oil GDP growth also decelerates. Both factors contribute to a significant reduction in real private consumption growth.

Figure 3. Simulation Results

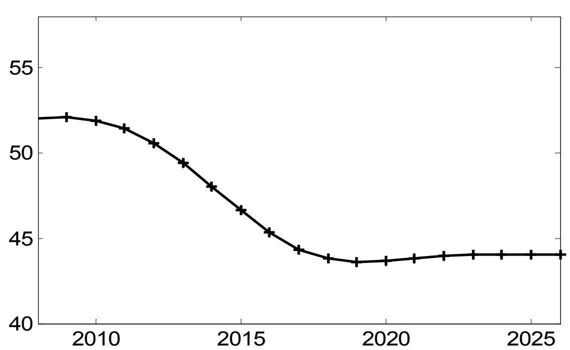
(a) Real GDP and Consumption Growth[13]
(Annual percentage growth rates)



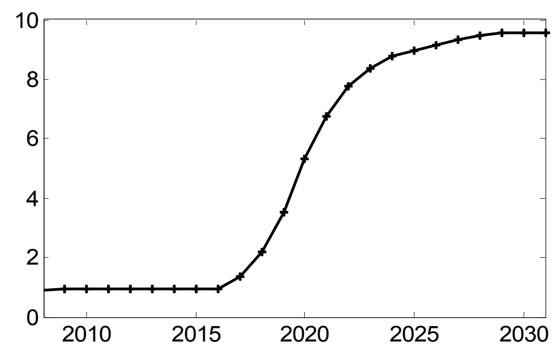
(b) Private Investment
(In percentage of non-oil GDP)



(c) Private consumption
(In percentage of non-oil GDP)



(d) Government Domestic Debt[14]
(In percentage of non-oil GDP)



Source: Authors' estimates.

IV. Conclusion

This paper evaluated the fiscal scenario based on the assumption of the rapid scaling-up of expenditure to be followed with its rapid scaling-down in the context of Azerbaijan's current temporary oil production boom. To this end, the paper reviewed the relevant experience of Nigeria and Saudi Arabia, and simulated the neo-classical growth model tailored to the Azeri conditions.

Both strands of analysis suggest that the evaluated fiscal scenario poses significant risks to growth sustainability. The historical experience indicates that the initial growth performance largely depends on the efficiency of scaled-up expenditure. It also sheds light on the risks

[13] Oil production is exogenous.

[14] Here, only domestic interest rates are endogenous.

associated with a sudden scaling-down of expenditure, including the political difficulties to undertake an orderly expenditure reduction strategy without undermining economic growth and the crowding-out effects of large government domestic borrowing. The results of the simulations of the Azerbaijan-specific model are similar to these lessons in many respects. Assuming moderate effectiveness of public capital expenditure, the initially robust non-oil growth performance is followed by a prolonged stagnation period. This stagnation is largely attributable to a significant tightening of the economy-wide resource constraint associated with the oil production decline, which is projected to coincide with cuts in public capital expenditure and crowding-out effects of domestic borrowing.

References

Aschauer, David A., 2000, "Public Capital and Economic Growth: Issues of Quantity, Finance, and Efficiency," *Economic Development and Cultural Change*, Vol. 48, No. 2, pp. 391-406.

Auty, R.M., 2001, *Resource Abundance and Economic Development* (Oxford: Oxford University Press).

Barro, Robert J., 1991, "Economic Growth in a Cross Section of Countries," *The Quarterly Journal of Economics*, Vol. 106, No. 2, pp. 407-443.

——, 1990, "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, Vol. 98, No. 5, pp.103-125.

——, 1989, "The Neoclassical Approach to Fiscal Policy," in R. J. Barro, ed., *Modern Business Cycle Theory,* pp. 178-235 (Cambridge, MA: Harvard University Press).

——; and Xavier Sala-i-Martin, 2004, *Economic Growth*, second edition (Cambridge, MA: MIT Press).

Baxter, Marianne and Robert G. King, 1993, "Fiscal Policy in General Equilibrium," *The American Economic Review*, Vol. 83, No. 3, pp. 315-334.

Bevan, David, Paul Collier, and Jan Willem Gunning, 1999, *Nigeria and Indonesia*, (New York: Oxford University Press).

Engel, Eduardo and Rodrigo O. Valdés, 2000, "Optimal Fiscal Strategy for Oil Exporting Countries," IMF Working Paper No. 00/118 (Washington: International Monetary Fund).

Gollin, Douglas, 2002, "Getting Income Shares Right," *Journal of Political Economy*, Vol. 110, No. 2, pp.458-474.

Judd, Kenneth L., 1985, "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model," *The Journal of Political Economy*, Vol. 93, No. 2, pp. 298–319.

Khan, Moshin S., and Manmohan S. Kumar, 1997, "Public and Private Investment and The Growth Process in Developing Countries," *Oxford Bulletin of Economics & Statistics*, Vol. 59, No. 1.

International Monetary Fund, 2007, Republic of Azerbaijan: The 2007 Article IV Consultation—Staff Report, Country Report No.07/191 (Washington: International Monetary Fund).

———————————, 2006, "Applying the Debt Sustainability Framework for Low-Income Countries Post Debt Relief," available at http://www.imf.org/external/np/pp/eng/2006/110606.pdf

———————————, 2006, "Managing Russia's Oil Wealth: An Assessment of Sustainable Expenditure Paths," in *Russian Federation: Selected Issues Papers*, IMF staff Country Report No. 06/430 (Washington: International Monetary Fund).

———————————, 2004a, "Debt Sustainability in Low-Income Countries—Proposal for an Operational Framework and Policy Implications," available at http://www.imf.org/external/np/pdr/sustain/2004/020304.htm

———————————, 2004b, "Debt Sustainability in Low-Income Countries—Further Considerations on an Operational Framework and Policy Implications," available at http://www.imf.org/external/np/pdr/sustain/2004/091004.htm

Kydland, Finn E. and Edward C. Presscott, 1982, "Time to Build and Aggregate Fluctuations," *Econometrica*, Vol. 50, No. 6, (Nov.), pp. 1345-1370.

Ljungqvist, Lars and Thomas J. Sargent, 2004, *Recursive Macroeconomic Theory*, 2nd Edition (Cambridge, MA: MIT Press).

Takizawa, Hajime, Edward H. Gardner and Kenichi Ueda, 2004, "Are Developing Countries Better Off Spending Their Oil Wealth Upfront?" IMF Working Paper, No. 04/141 (Washington: International Monetary Fund).

ANNEX I: Solving the Model

The model can be solved as follows: (i) rewrite it as the planner's problem; (ii) detrend it so that the system becomes stationary—the steady state of the detrended model is known as the balanced growth path; (iii) numerically solve the detrended model.[15]

The Planner's Problem:
The model's competitive equilibrium is efficient, and it can be rewritten as the following planner's problem, which is easier to solve. The planner maximizes the following lifetime utility function subject to (A-1) – (A-5):

$$\max_{C_t, K_{t+1}} \sum_{t=1}^{\infty} \beta^t u(C_t)$$

(A-1) $C_t + X_t = f(K_t, L_t) + H_t$,

where H is an exogenous variable given by

$H = (D_{t+1}^* - (1+i^*)D_t^*) - (C_t^g + X_t^g - O_t^g)$

(A-2) $f(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}$, where $A_t = \left\{ z_t + \theta \frac{K_t^g}{L_t} \right\}^{1-\alpha}$

where

(A-3) $K_{t+1} = X_t + (1-\delta)K_t$ (the transition equation for private capital),

(A-4) $K_{t+1}^g = X_t^g + (1-\delta^g)K_t^g$ (the transition equation for public capital).

(A-5) K_1^g, K_1, and the paths of X_t^g and H_t are given.

The Detrended Model:
The idea of detrending is to transform the model into a stationary system, that is, the one that does not have exogenous growth. Divide (A-1) by L_t so that other real variables are in terms of per capita (denoted in the lowercase). Assume that the growth rates of z_t and L_t are exogenously given at g and n respectively. Other real variables grow at $(1+g)$. The detrended model is given by (detrended variables are denoted with ^)

(A-6) $\max_{\hat{c}_t, \hat{k}_{t+1}} \sum_{t=1}^{\infty} \beta^t u(\hat{c}_t)$, where $u(.) = \log(.)$

(A-7) $\hat{c}_t + (1+g)(1+n)\hat{k}_{t+1} - (1-\delta)\hat{k}_t = \left(z_0 + \theta \hat{k}_t^g \right)^{1-\alpha} \hat{k}_t^\alpha + \hat{h}_t$,

[15] The sequential problem of maximizing the discounted utility in (A-6) subject to (A-7), (A-8), (A-9), and (A-10), is numerically solved over $\{\hat{c}_t, \hat{k}_{t+1}\}$ in a non-recursive way.

where $\hat{h}$ is an exogenous variable given by

$\hat{h} = ((1+g)(1+n)\hat{d}^*_{t+1} - (1+i^*)\hat{d}^*_t) - (\hat{c}^g_t + \hat{x}^g_t - \hat{o}^g_t)$

and the transition equation for public capital is given by

(A-8) $(1+g)(1+n)\hat{k}^g_{t+1} = \hat{x}^g_t + (1-\delta^g)\hat{k}^g_t$

(A-9) $\hat{k}^g_1$, $\hat{k}_1$, and the paths of $\hat{x}^g_t$ and $\hat{h}_t$ are given.

(A-10) The initial labor is normalized at 1.

The Lagrangian is given by:

$$L = \sum_{t=1}^{\infty} \beta^t u(\hat{c}_t) + \sum_{t=1}^{\infty} \beta^t \lambda_t \left[\left(z_0 + \theta \hat{k}^g_t \right)^{1-\alpha} \hat{k}^\alpha_t - \hat{c}_t - (1+g)(1+n)\hat{k}_{t+1} + (1-\delta)\hat{k}_t - \hat{h}_t \right],$$

where the first order conditions are:

$\hat{c}_t$: $u'(\hat{c}_t) = \lambda_t$

$\hat{k}_{t+1}$: $\lambda_t (1+g)(1+n) = \beta \lambda_{t+1} \left[\alpha \left(z_0 + \theta \hat{k}^g_t \right)^{1-\alpha} \hat{k}^{\alpha-1}_t + 1 - \delta \right]$

The Euler equation is given by combining the first order conditions:

(A-11) $u(\hat{c}_t)(1+g)(1+n) = \beta u(\hat{c}_{t+1}) \left[\alpha \left(z_0 + \theta \hat{k}^g_t \right)^{1-\alpha} \hat{k}^{\alpha-1}_t + 1 - \delta \right]$

The Balanced Growth Path:

The balanced growth path of the original model will correspond to a steady state of the detrended model. Along the balanced growth path, all detrended variables are constant. Such a steady state can be derived as follows: (i) normalize the TFP level at the steady state at a certain value; (ii) pin down the steady state level of private capital using the Euler equation; and (iii) pin down the steady state levels of consumption and output using the production function and resource constraint.

A Numerical Approach to Solving the Model:

In order to solve the model numerically, we assume that the country reaches the steady state of the detrended model in period T. We then solve for the optimal paths of capital and consumption for period $t = 1,\ldots,T$ that satisfy the Euler equation (A-11) and the resource constraint (A-7).